UT-BATTELLE

SAMMY Workshop Part 4.1g

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Part 4.1g. Angular Distributions

Updated July 2005

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Angular distributions formulae

- **The angular distribution for incident particle-pair α and exit particle pair α' is given by**

$$\frac{d\sigma_{\alpha\alpha'}}{d\Omega_{CM}} = \sum_L C_{L\alpha\alpha'}(E)\, P_L(\cos\beta)$$

where

- **β is the com angle**
- **the coefficient of the Legendre polynomial is a messy function –**

Formulae, continued

$$C_{L\alpha\alpha'}(E) = \frac{1}{4k_\alpha^2} \sum_{J_1} \sum_{J_2} \sum_{\substack{c_1 = \\ (\alpha\, l_1\, s_1\, J_1)}} \sum_{\substack{c_1' = \\ (\alpha'\, l_1'\, s_1'\, J_1)}} \sum_{\substack{c_2 = \\ (\alpha\, l_2\, s_2\, J_2)}}$$

$$\sum_{\substack{c_2' = \\ (\alpha'\, l_2'\, s_2'\, J_2)}} B_{\{l_1\, s_1\, l_1'\, s_1'\, J_1\}\{l_2\, s_2\, l_2'\, s_2'\, J_2\}\, L} \frac{1}{(2i+1)(2I+1)}$$

$$\times \quad \mathrm{Re}\left[(\delta_{c_1 c_1'} - U_{c_2 c_2'}) (\delta_{c_1 c_1'} - U^*_{c_2 c_2'}) \right]$$

for the non-Coulomb case, where

Formulae, cont.

$$B_{\{l_1 s_1 l_1' s_1' J_1\}\{l_2 s_2 l_2' s_2' J_2\} L} = A_{l_1 s_1 l_1' s_1'; J_1} \; A_{l_2 s_2 l_2' s_2'; J_2} \times D_{l_1 s_1 l_1' s_1' l_2 s_2 l_2' s_2'; L J_1 J_2}$$

with

$$A_{l_1 s_1 l_1' s_1'; J_1} = \sqrt{(2l_1+1)(2l_1'+1)} \times (2J_1+1)\,\Delta(l_1 J_1 s_1)\,\Delta(l_1' J_1 s_1')$$

Formulae, cont.

$$D_{l_1 s_1 l_1' s_1' l_2 s_2 l_2' s_2'; L J_1 J_2} = (2L+1)\ \Delta^2(J_1 J_2 L)\ \Delta^2(l_1 l_2 L)\ \Delta^2(l_1' l_2' L)$$

$$\times\ w(l_1 J_1 l_2 J_2, s_1 L)\ w(l_1' J_1 l_2' J_2, s_1' L)\ \delta_{s_1 s_2}\ \delta_{s_1' s_2'}\ (-1)^{s_1 - s_1'}$$

$$\times\ \frac{n!\ \ (-1)^n}{(n-1_1)!\ (n-l_2)!\ (n-L)!}\ \ \frac{n'!\ \ (-1)^{n'}}{(n'-l_1')!\ (n'-l_2')!\ (n'-L)!}$$

with $2n = l_1 + l_2 + L$

and $\Delta^2(abc) = \dfrac{(a+b-c)!\ \ (a+b+c)!\ \ (a+b+c)!}{(a+b+c+1)!}$

Formulae, cont.

$$w(l_1 J_1 l_2 J_2, s L) =$$

$$\sum_{k=k\min}^{k\max} \frac{(-1)^{k+l_1+J_1+l_2+J_2}\,(k+1)!}{\Big(k-(l_1+J_1+s)\Big)!\,\Big(k-(l_2+J_2+s)\Big)!}$$

$$\times \frac{1}{\Big(k-(l_1+l_2+L)\Big)!\,\Big(k-(J_1+J_2+L)\Big)!}$$

$$\times \frac{1}{(l_1+J_1+l_2+J_2-k)!\,(l_1+J_2+s+L-k)!}$$

$$\times \frac{1}{(l_2+J_1+s+L-k)!}$$

Formulae, cont.

with

$$kmin = \max\{ (l_1 + J_1 + s),\ (l_2 + J_2 + s),\ (l_1 + l_2 + L),\ (J_1 + J_2 + L) \}$$

and

$$kmax = \min\{ (l_1 + J_1 + l_2 + J_2),\ (l_1 + J_2 + s + L),\ (l_2 + J_1 + s + L) \}$$

Angular distributions

- **These equations work for elastic, inelastic, and other reaction angular distributions**
  - **Thresholds now included properly**
    - **for the next release of the code (had bug in earlier)**
- **Equations for Coulomb are also implemented**
- **Angular distribution data are sometimes important for determining *l*-values for an evaluation**