Biased Reconstruction for JPEG Decoding

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Abstract— **Assuming a Laplacian distribution, there exists a well known method for optimally biasing the reconstruction levels of the quantized AC DCT coefficients in the JPEG decoder. This, however, requires an estimate of the Laplacian distribution parameter. We derive a new, maximum likelihood estimate of the Laplacian parameter using only the quantized coefficients available at the decoder. We quantify the benefits of biased reconstruction through extensive simulations and demonstrate that such improvements are very close to the best possible resulting from centroid reconstruction.**

Keywords— **JPEG, JPEG decoding, JPEG reconstruction, JPEG quantization.**

I. Introduction

Despite its growing age, JPEG image compression [1] is still used in a wide variety of applications, including onboard digital camera storage and consumer digital imaging. Much recent work has focused on reducing artifacts in JPEG images via decoder modifications or postprocessing; some examples are reviewed in [2]. One decoder modification, noted previously in both [3] and [4], is to bias the reconstructed AC DCT coefficients since the standard, bin center reconstruction is suboptimal. Assuming a Laplacian distribution, the MSE optimal bias is well known [3]. First, however, the Laplacian parameter λ must be estimated. In this letter, we derive a new, maximum likelihood (ML) estimate for λ and perform experiments that demonstrate the benefits of biased reconstruction.

In the JPEG decoder, the reconstruction process assigns all coefficients in a given bin to the center value of that bin. For a given coefficient C_{ij}, the JPEG quantization-reconstruction process can be represented by the following equations:

$$n_{ij} = \operatorname{round}\left(\frac{C_{ij}}{Q_{ij}}\right), \tag{1a}$$

$$C^q_{ij} = n_{ij}Q_{ij}, \tag{1b}$$

where Q_{ij} indicates the quantization bin width for the given coefficient, n_{ij} indicates the bin index in which the coefficient falls, and C^q_{ij} represents the reconstructed coefficient. Considering the distribution of the AC ($i \neq 0$ or $j \neq 0$) coefficients, it is well known that bin center reconstruction is suboptimal (except for the zero bin). Referring to Fig. 1, any unquantized coefficient C_{ij} in the bin denoted by the shaded region ($n_{ij} = 1$) will be reconstructed to the bin center, Q_{ij}. The minimum MSE is achieved by reconstructing to the centroid of the distribution over the given bin, as indicated by the "×" symbol in Fig. 1.

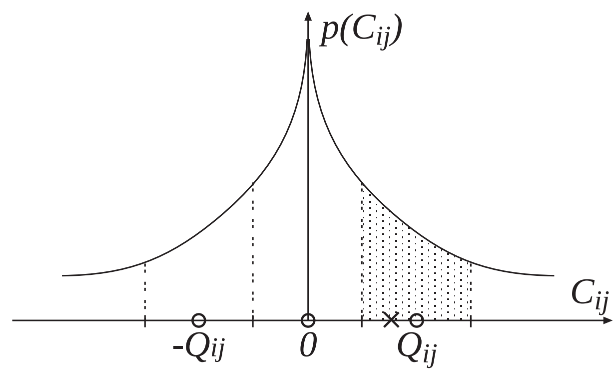

Fig. 1. Example AC coefficient distribution. Bin centers (i.e., standard reconstruction values $C^q_{ij} = n_{ij}Q_{ij}$) are indicated by the "∘" symbols. For $n_{ij} = 1$, the "×" symbol indicates the center of mass of the distribution over the shaded region.

II. Optimal Reconstruction

Although the results of previous studies [5]-[8] indicate that generalized Gaussians give the most accurate representations of the AC coefficient distributions, we employ the more commonly used Laplacian distribution. The Laplacian is more tractable both mathematically and computationally, and, as will be noted in Section IV, produces results very close to the best achievable.

The Laplacian distribution,

$$p(c) = \frac{\lambda}{2}\, e^{-\lambda|c|}, \tag{2}$$

is characterized by the single parameter λ. For a certain AC coefficient (i.e., fix i and j), we let $c = C_{ij}$ be in the n^{th} bin. Note that the n^{th} bin is given by the interval $I_n = \big[(n-1/2)Q, (n+1/2)Q\big]$, where Q indicates Q_{ij} for our fixed i and j. We seek $\hat{c} \in I_n$ to minimize the MSE, $\mathcal{E}\left\{|c-\hat{c}|^2\right\}$, for all $c \in I_n$. It is well known [3] that $\hat{c}$ is the just centroid of $p(c)$ over I_n and can be written

$$\hat{c} = nQ + b \tag{3}$$

where

$$b = -\operatorname{sgn}(n)\left[\frac{Q}{2}\left(\frac{1+e^{-\lambda Q}}{1-e^{-\lambda Q}}\right) - \frac{1}{\lambda}\right]. \tag{4}$$

Equation (3) states that $\hat{c}$ is just the bin center, nQ, plus the bias term b given from (4). The bias b depends only on

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the sign of n and therefore needs only be computed once for each of the 63 AC coefficients. The $-\mathrm{sgn}(n)$ term in (4) simply ensures that the bias is in the direction of the origin.

III. Laplacian Parameter Estimation

In [3], an estimate of λ is found by simply computing the variance, σ^2, of the dequantized coefficients reconstructed to bin center and then setting $\lambda = \sqrt{2}/\sigma$, which is a well known relation between the Laplacian parameter and the variance of the distribution. Here we derive a more rigorous, ML estimate for λ.

Assuming the Laplacian distribution, suppose we have a series of N observations of a given coefficient $c = C_{ij}$ (again i and j are fixed), prior to any quantization. Referring to these observations as c_k for $k = 1, \ldots, N$, it is easily shown that the ML estimate of λ is given by

$$\lambda_{ML} = \frac{N}{\sum_{k=1}^{N} |c_k|}, \tag{5}$$

where it is assumed that the summation in the denominator is not zero. On the decoder side, however, we do not have access to the original, unquantized coefficients. Therefore, we need to estimate λ from only the quantized coefficients. Referring to the quantized coefficients as c_k^q, we seek λ_{ML}^q. First, we note that quantization effectively transforms the continuous distribution of (2) into the discrete distribution given by

$$p^q(n) = \int_{(n-1/2)Q}^{(n+1/2)Q} \frac{\lambda}{2}\, e^{-\lambda|c|}\, dc, \tag{6}$$

where n indicates the bin index. Equation (6) leads to

$$p^q(n) = \begin{cases} \frac{1}{2}\, e^{-\lambda Q(|n|-1/2)}\, (1 - e^{-\lambda Q}), & \text{for } n \neq 0 \\ 1 - e^{-\frac{1}{2}\lambda Q}, & \text{for } n = 0. \end{cases} \tag{7}$$

To find λ_{ML}^q, we maximize (over λ) the log-likelihood function of $p^q(n)$ given by

$$L(\lambda; \{c_k^q\}_{k=1}^N) = \ln\Big[\prod_{k=1}^{N} p^q(n_k)\Big] = \sum_{k=1}^{N} \ln[p^q(n_k)] \tag{8}$$

where n_k indicates the bin index for the k^{th} observation. After some significant manipulation, omitted here for brevity, it can be shown that

$$\lambda_{ML}^q = -\frac{2}{Q}\ln(\gamma) \tag{9}$$

where

$$\gamma = \frac{-N_0 Q}{2NQ + 4S} + \frac{\sqrt{N_0^2 Q^2 - (2N_1 Q - 4S)(2NQ + 4S)}}{2NQ + 4S}, \tag{10}$$

and where N_0 is the number of observations that are zero, N_1 is the number of observations that are nonzero, N is the total number of observations ($N = N_0 + N_1$), and

$$S = \sum_{k=1}^{N} |c_k^q|. \tag{11}$$

If $S = 0$, which might occur at high compression ratios, then (9) is not valid. This, however, is not a problem since reconstruction to zero is optimal for the center bin.

As an example, we consider the well known "lena" image, encoded using the default JPEG quantization table (i.e., scale factor of 1.0). In (12)-(14) below, we show the bias magnitude tables computed using λ_{ML}, λ_{ML}^q, and $\lambda = \sqrt{2}/\sigma$ (as suggested in [3]), respectively.

0	0.21	0.40	1.83	5.49	14.71	21.56	27.23
0.43	0.62	1.28	3.19	7.47	24.58	26.48	24.55
1.48	1.29	2.21	5.90	15.08	24.56	31.34	25.30
2.45	3.50	5.62	9.53	21.39	40.19	37.20	28.57
5.12	7.00	14.40	24.10	30.72	51.72	49.08	36.28
8.91	14.48	24.49	29.07	37.83	49.64	54.29	43.92
22.05	29.54	36.67	41.17	49.23	58.36	57.97	48.57
33.75	43.82	45.43	46.96	53.97	48.02	49.62	47.68

(12)

0	0.21	0.39	1.79	5.18	13.14	19.64	25.43
0.43	0.61	1.25	3.03	6.92	22.17	24.42	23.18
1.45	1.25	2.11	5.40	13.41	21.90	29.98	24.12
2.37	3.34	5.30	8.59	19.27	42.39	35.19	29.89
5.03	6.67	13.37	22.12	28.35	53.39	50.39	37.39
8.66	13.76	22.93	30.89	39.39	50.89	55.39	44.89
23.39	30.89	37.89	42.39	50.39	59.39	58.89	49.39
34.89	44.89	46.39	47.89	54.89	48.89	50.39	48.39

(13)

0	0.17	0.29	1.35	4.25	13.03	21.40	28.37
0.32	0.46	0.92	2.34	6.22	24.08	27.11	25.89
1.07	0.92	1.55	4.56	13.56	23.79	33.42	26.93
1.86	2.72	4.57	8.13	20.82	42.39	39.12	29.89
4.62	6.46	14.14	24.34	31.62	53.39	50.39	37.39
9.15	15.12	25.58	30.89	39.39	50.89	55.39	44.89
23.39	30.89	37.89	42.39	50.39	59.39	58.89	49.39
34.89	44.89	46.39	47.89	54.89	48.89	50.39	48.39

(14)

IV. Experimental Results

Thirty-three monochrome images (five were 512×512 pixels, the rest were 768×512) were compressed using the default JPEG (luminance) quantization table, scaled by four different factors between 0.5 and 2.0. For scaling factors greater than 2.0, corresponding to increased compression ratios, we found that the benefits from biased reconstruction were outweighed by the large number of AC coefficients that were quantized to zero. The encoded images were decoded using the standard JPEG, bin center reconstruction as well as biased reconstruction, as given by (3).

TABLE I
AVERAGE PSNR IMPROVEMENTS, IN dB, OVER STANDARD JPEG FOR BIASED AND TRUE BIN CENTROID RECONSTRUCTIONS.

	Quantization Table Scaling			
Dequantization Type	0.5	0.75	1.0	2.0
Biased, λ_{ML}, Eq. (5)	0.30	0.27	0.25	0.20
Biased, λ^q_{ML}, Eq. (9)	0.35	0.32	0.30	0.24
Biased, λ from [3]	0.35	0.31	0.29	0.24
True Bin Centroid	0.42	0.38	0.36	0.30

For biased reconstruction, both λ_{ML} from (5) and λ^q_{ML} from (9) were used as estimates of the Laplacian parameters. Recall that λ_{ML} is computed from the DCT coefficients prior to quantization, which are only available at the encoder side, while λ^q_{ML} is computed from the quantized coefficients available at the decoder. The former method is not compatible with the JPEG standard, as it requires overhead information to convey the λ_{ML} values to the decoder. The bits resulting from this overhead have been ignored in our simulations, as we are using this case mainly as a comparison point. In our experiments, the coefficients of an entire image were used to compute the λ_{ML} and λ^q_{ML} values once. We also performed these experiments using the estimate of λ suggested by [3]. In the final test, a best case, albeit impractical, scenario was constructed. Prior to quantization on the encoder side, the true centroid of each bin (the average value of all the coefficients in that bin) for each coefficient was computed and stored. For decoding, each coefficient in a given bin was reconstructed to the true centroid for that bin. This method is impractical because of the large overhead (ignored in our simulations) and incompatibility with the JPEG standard. It does, however, provide us with the best possible improvement for the sake of comparison.

Quantitative results from these experiments are summarized in Table I. Let P_s represent the PSNR between the standard JPEG decoded (bin center reconstruction) image and the original, uncompressed image. Let P_m be the PSNR between the modified reconstruction (either biased or true bin centroid), image and the original, uncompressed image. We refer to PSNR improvement as the difference $P_m - P_s$. The quantities given in Table I are the average of this difference over the 33 test images for the indicated quantization table scaling.

As evident in Table I, biased reconstruction provides modest improvements in PSNR when compared to bin center reconstruction. There were no individual cases where biased reconstruction caused a relative loss in PSNR. It is well known, however, that such PSNR improvements do not necessarily imply subjective improvements in image quality. Careful analysis of our test images indicated that biased reconstruction produced little subjective improvement. In a few cases, some mild edge "ringing" artifacts were reduced. Generally, however, the differences between the standard JPEG decoded images and the biased reconstruction images were difficult to detect. Note also from Table I that using the Laplacian parameters estimated from the quantized coefficients (λ^q_{ML}) actually performs better than using the true ML parameters estimated from the unquantized coefficients (λ_{ML}). Table I also indicates that the method for estimating λ suggested in [3] performs just as well as our more rigorous ML approach and might therefore be more practical if computational complexity is a limiting factor. Finally, we note that the best case, true bin centroid reconstruction is not significantly better than any of the biased reconstructions. This validates the use of the Laplacian, as it performs almost as well as the best possible, and yet requires little computation when compared to the generalized Gaussian.

V. Conclusions

Assuming a Laplacian distribution for the unquantized, AC DCT coefficients, we derive the ML estimate of the Laplacian parameter using only the quantized coefficients available to the decoder. Experiments indicate that biased reconstruction with this estimate gives modest improvements in PSNR and that these improvements are close the the best possible, true bin centroid reconstruction. These experiments also show that a previously proposed, less rigorous estimate of the Laplacian parameter performs just as well as the ML estimate, and might therefore be the method of choice for minimal computation.

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