Physics of nonlinear ICRF-driven flows in 2D toroidal geometry *

J.R. Myra, D.A. D'Ippolito,

Lodestar Research Corp.,

E.F. Jaeger, L.A. Berry, D.B. Batchelor

ORNL

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Introduction

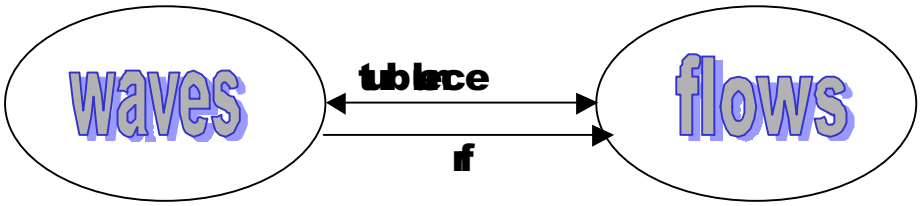
RF-driven sheared flows may be important

- control turbulence and transport barriers
- probe fundamental physics of nonlinear waves and flows

RF codes and experiments can help to understand turbulence & transport barrier formation

- rf driven flows are “open loop”, easier than “closed loop” turbulence problem
  - just understand how a given wave affects flows and macroscopic responses
  - separate off problem of how macroscopic changes affect instabilities, turbulence
    - flows modify the waves that create them – important but a separate issue

rf allows fundamental nonlinear physics in a controlled context



The challenge – ways in which the rf problem is difficult

- rf vs. turbulence problem:
  - turbulence driven flows are normally calculated for waves with $\omega << \Omega$; for ICRF must calculate nonlinear forces for waves with $\omega \sim \Omega_i$
- conventional "ponderomotive force" calculations are inadequate
  - the flow drive typically arises from dissipative terms
  - for inclusion into macroscopic evolution codes (e.g. transport codes) we need the drives for the moment equations on a fluid element (not a guiding center)
- recent 2D AORSA simulations of ICRF mode conversion scenarios under the rf SciDAC project [Jaeger et al., poster BP1.074] require a 2D calculation of the nonlinear forces that drive flows
- previous calculations are generalized here
  - eikonal result [Myra & D'Ippolito, PoP 7, 3600 (2000)] to multiple modes
  - bilinear form results [Jaeger, Berry & Batchelor, PoP 7, 3319 (2000)] to 2D covariant forms
- 2D AORSA calculations of power deposition and flow drive are computationally feasible but burdensome
  - run time typically dominated by post-processing
  - fast algorithms/formulations are desired

Theoretical calculations

Framework

- 2nd order in E, quasilinear calculation
- energy and momentum moments of Vlasov equations
- hot plasma, quasi-local theory
  - o $k_\perp \rho \sim 1$, gyrokinetic theory (nonlocal)
    - introduces bilinear operator $W(k, k')$
  - o 2D slab physics
    - direction along B is a symmetry direction
    - no toroidal corrections to dielectric response
  - o electromagnetic
  - o $\omega >> \omega_{drift}$
  - o retain first order in ρ/L where L = envelope of rf waves, or equilibrium scale length (local)
  - o weak absorption (damping rate $\gamma << \omega \sim \Omega$)
- energy moment recovers results of Smithe [1989] and Brambilla
  - o depends on gyrophase-averaged distribution $\langle f \rangle_\phi$
- momentum moment
  - o depends on gyrophase-dependent distribution

Gyrokinetic formalism

$$\frac{\partial f}{\partial t} + v_{\|}\nabla_{\|} f - \Omega \frac{\partial f}{\partial \phi} = -\nabla_v \cdot (\mathbf{a} f)$$

$$\mathbf{R} = \mathbf{r} + \frac{1}{\Omega}\mathbf{v} \times \mathbf{b}$$

$$\mathbf{a} = \frac{Ze}{m}\left[I\left(1 - \frac{\mathbf{k}\cdot\mathbf{v}}{\omega}\right) + \frac{\mathbf{kv}}{\omega}\right]\cdot \mathbf{E}_1 = \sum_k \mathbf{a}_k e^{i\mathbf{k}\cdot\mathbf{R} - i\delta_k}$$

$$\delta_k = \frac{1}{\Omega}\mathbf{k}\cdot\mathbf{v}\times\mathbf{b}$$

The X moment equation is given by

$$\int d^3v\, X f = \sum_k e^{i\mathbf{k}\cdot\mathbf{r}} \int d^3v\, X f_k e^{i\delta_k}$$

Energy moment: X = mv²/2

(recovers results of Smithe[PPCF 31, 1105 (1989)] and Brambilla)

The local power absorption is

$$\dot{w} = \frac{m}{4}\sum_{k,k'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} \int d^3v\, \mathbf{v}\cdot\mathbf{a}_k^* f_{k'} e^{i\delta_k} + cc$$

$$= \frac{1}{4}\sum_{k,k'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} \mathbf{E}_k^* \cdot W(k,k')\cdot \mathbf{E}_{k'} + cc$$

The tensor W is a symmetric bilinear operator that is related to the conductivity

$$W(k,k' \to k) = \sigma(k)$$

W is given explicitly in terms of orbit and $\int d^3v$ integrals

$$W = -\frac{2 n_0 Z^2 e^2}{\alpha^2 \Omega m}\int dv_{\parallel} f_{M\parallel} \int dv_{\perp} v_{\perp} f_{M\perp}\, L$$

$$L = \int_0^{2\pi} d\phi\, \mathbf{v}(\phi)\, e^{i\delta_k(\phi) - i\omega_{k'}\phi/\Omega} \int_{-\infty}^{\phi} d\phi'\, \mathbf{v}(\phi')\, e^{-i\delta_{k'}(\phi') + i\omega_{k'}\phi'/\Omega}$$

or, on carrying out the gyrophase integrals:

$$L = \sum_n \frac{2\pi\Omega e^{in(\theta'-\theta)}}{i(\omega - k_{\|}v_{\|} - n\Omega)} Q^*(k)Q(k')$$

$$Q(\mathbf{k}) = \hat{\mathbf{k}}_\perp v_\perp \frac{n}{a'} J_n(a) + \mathbf{b}\times\hat{\mathbf{k}}_\perp v_\perp i J_n'(a) + \mathbf{b} v_{\|} J_n(a)$$

It can be shown [L. Berry] from this form that the power absorption is positive definite.

Momentum moment: X = mv

The order $|E|^2$ terms arise from the Lorentz force

$$\mathbf{F}_L = Zen\mathbf{E} + \frac{1}{c}\mathbf{J}\times\mathbf{B}$$

or using Maxwell's equations

$$\mathbf{F}_L = \frac{1}{16\pi}\left[(\nabla\mathbf{E}^*)\cdot\mathbf{D} - \nabla\cdot(\mathbf{D}\mathbf{E}^*)\right] + cc$$

where $$\mathbf{D} = \frac{4\pi i}{\omega}\mathbf{J}$$

and from the nonlinear stress tensor

$$\Pi_{nl} = \frac{m}{4}\sum_{k''} e^{ik''r}\int d^3v(\mathbf{vv} - \langle\mathbf{vv}\rangle) f_{k''}^{(2)} e^{i\delta_{k''}} + cc$$

Note that gyrophase *dependent* parts of f are required, These are obtained by integrating **vv–<vv>** by parts.

The gyrophase average f gives rise to CGL type pressure tensor elements that we do not calculate – they are secular unless a sink for the rf heating is specified.

Suppressing the k, k′ sum and the phases, the total force can be expressed in the form

$$\mathbf{F} = \mathbf{F}_d - \nabla \cdot \Pi$$

The direct term contains the photon momentum absorption and a reactive term

$$\mathbf{F}_d = \frac{\mathbf{k}}{2\omega}\mathbf{E}^* \cdot W^H \cdot \mathbf{E} + \frac{i}{4\omega}\nabla(\mathbf{E}^*\mathbf{E}) : W^A$$

The remaining term combines parts of F_L and Π_{nl}

$$\Pi = \frac{\mu}{4\Omega}\left[\left(\mathbf{E}^*\mathbf{J} + \mathbf{J}\,\mathbf{E}^*\right)\times\mathbf{b} + \mathrm{tr} + \mathbf{b}\times\mathbf{J}\,\mathbf{E}^*\right] + \frac{1}{16\pi}\mathbf{D}_{\|}\mathbf{E}^* + cc$$

where μ = 1/4 (1) for $\perp,\perp$ ($\perp,\|$) components

$\mathbf{J}$ is a generalized plasma current

$$\mathbf{J}_k = \sum_{k'} W(k,k') \cdot \mathbf{E}_{k'}$$

Specialized limit: $\perp\perp$ dynamics

The $\perp$ force from $\perp$ gradients is

$$\mathbf{F} = \mathbf{F}_d - \nabla_\perp X_r + \mathbf{b}\times\nabla X_d$$

where

$$X_r = \frac{m}{8\Omega}\sum_{kk'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} \int d^3v\, e^{i\delta_k} f_{k'}\, \mathbf{b}\cdot\mathbf{v}\times\mathbf{a}_k^* + cc$$

is a measure of the parallel torques on the plasma, and

$$X_d = \frac{m}{8\Omega}\sum_{kk'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} \int d^3v\, e^{i\delta_k} f_{k'}\, \mathbf{v}_\perp \cdot \mathbf{a}_{k\perp}^* + cc = \frac{\dot{w}_\perp}{2\Omega}$$

is a measure of the perpendicular dissipation.

X_r and X_d are mostly from ions.

Specialized limit: electrostatic modes

$$\mathbf{E} = -i\mathbf{k}\Phi = -i\Phi(\mathbf{k}_\perp + k_\parallel \mathbf{b})$$

$$\mathbf{J} = -i\Phi(\alpha\mathbf{k}_\perp + \beta\mathbf{b}\times\mathbf{k} + \gamma k_\parallel \mathbf{b})$$

The total force is

$$\mathbf{F} = \mathbf{F}_d - \nabla_\perp X_r + \mathbf{b}\times\nabla X_d$$

$$+\mathbf{b}\nabla_\parallel\left(\frac{P_{R\parallel}}{\omega}\right) + \mathbf{b}\,\mathbf{b}\times\hat{\mathbf{k}}_\perp\cdot\nabla\left(\frac{k_\perp P_\parallel}{\Omega k_\parallel}\right) - \nabla_\parallel(B\mathbf{M}_\perp)$$

where

$$\mathbf{F}_d = \frac{\mathbf{k}}{\omega}P - \frac{1}{4\omega}(\alpha_i\nabla|\Phi|^2\ k_\perp^2 + \gamma_i\nabla|\Phi|^2\ k_\parallel^2)$$

$$X_r = -\frac{k_\perp^2\beta_r|\Phi|^2}{4\Omega}$$

$$X_d = \frac{k_\perp^2\alpha_r|\Phi|^2}{4\Omega} = \frac{P_\perp}{2\Omega}$$

and we define

$$P = \frac{1}{2}\mathrm{Re}\,\mathbf{J}\cdot\mathbf{E}^* = \frac{1}{2}|\Phi|^2\ (\alpha_r k_\perp^2 + \gamma_r k_\parallel^2)$$

$$P_R = -\frac{1}{2}\mathrm{Im}\,\mathbf{J}\cdot\mathbf{E}^* = -\frac{1}{2}|\Phi|^2\ (\alpha_i k_\perp^2 + \gamma_i k_\parallel^2)$$

$$B\mathbf{M}_\perp = \frac{|\Phi|^2}{2}k_\parallel\mathbf{k}_\perp\left(\frac{\beta_r}{\Omega} - \frac{\gamma_i}{\omega}\right) - \frac{|\Phi|^2}{2\Omega}k_\parallel\mathbf{b}\times\mathbf{k}_\perp(\alpha_r + \gamma_r)$$

What is required for driving flux-surface averaged flows?

let <> be a flux surface average

- toroidal rotation is driven by torque $\langle RF\zeta \rangle$
- poloidal rotation is driven by

$$\left\langle BF_{\|} \right\rangle - g\left\langle RF_{\zeta} \right\rangle$$

where

$$g(\psi) = RB_{\zeta} \Big/ \left\langle R^{2} \right\rangle_{x}$$

identities

$$\left\langle \nabla \cdot A \right\rangle = \frac{1}{\upsilon}\frac{\partial}{\partial\psi}\upsilon\left\langle RB_{\theta}A_{\psi} \right\rangle$$

$$\left\langle B\nabla_{\|}Q \right\rangle = \frac{1}{\upsilon}\int d\theta \int \frac{d\zeta}{2\pi}\frac{JB_{\zeta}}{R}\frac{\partial Q}{\partial\zeta} = 0$$

$\langle BF_{\|} \rangle$ vanishes when $F_{\|} = \nabla_{\|}$ (scalar)

$$\left\langle R\mathbf{e}_{\zeta} \cdot \nabla \cdot \Pi \right\rangle = \left\langle \nabla \cdot \Pi \cdot R\mathbf{e}_{\zeta} \right\rangle = \frac{1}{\upsilon}\frac{\partial}{\partial\psi}\upsilon\left\langle R^{2}B_{\theta}\Pi_{\psi\zeta} \right\rangle$$

$\langle RF\zeta \rangle$ vanishes when Π is a diagonal tensor

$$\langle \mathbf{B}\cdot\nabla\cdot\Pi\rangle = \langle \mathbf{B}\cdot\nabla\cdot Q\mathbf{bb}\rangle = -\langle Q\nabla_{\|}B\rangle \approx 0$$

$\langle BF_{\|}\rangle$ vanishes (to the order we care about) when Π is a CGL tensor

Therefore:

- diagonal Π tensors of the CGL form (in particular forces that are gradients) don't contribute to the forcing terms for flux-surface averaged flows
  - they group with the pressure and contribute to diamagnetic-like flows
- flows are driven by dissipative terms corresponding to
  - direct absorption of photon momentum $P\,\mathbf{k}/\omega$
  - dissipative stresses in Π_{xy} and to some extent in Π_{yz}

Forces driving flux-surface-averaged flows – electrostatic limit

$$\langle \mathbf{F}\rangle = \langle \mathbf{F}_d\rangle + \langle \mathbf{b}\times\nabla X_d\rangle + \left\langle \mathbf{b}\,\mathbf{b}\times\hat{\mathbf{k}}_{\perp}\cdot\nabla\left(\frac{k_{\perp}P_{\|}}{\Omega k_{\|}}\right)\right\rangle$$

1st term = photon momentum

2nd = Π_{xy}

3rd term = Π_{yz} is probably small

- Ω effectively restricts it to ions
- $P_{\|\ ion}$ is often small
- for poloidal flows, $k_{\perp}/k_{\|} >> 1$ is compensated by projection of F $\Rightarrow B_p/B$

Relation to the standard fluid ponderomotive force

Standard ponderomotive force in the fluid limit can be recovered from our general result upon adding the appropriate nonlinear CGL pressure tensor and ambipolar terms.

- keep only reactive terms
- in the fluid limit, W is independent of k
  - o $W(k, k') \to \sigma$ so that $W\mathbf{E} \to \mathbf{J} \to nm\mathbf{u}$

Our result above reduces to

$$\mathbf{F} = \mathbf{F}_d - \nabla_\perp\left(X_r + \frac{1}{2}nm\left\langle u_\perp^2\right\rangle\right) - \nabla_\parallel\left(\frac{inm\Omega}{4\omega}u_\parallel^*\mathbf{u}\times\mathbf{b} + cc\right)$$

Standard result is
[e.g. Lee and Parks, Phys. Fl. **26**, 724 (1983)]

$$\mathbf{F} = -n\nabla\psi + \mathbf{B}\times\nabla\times\mathbf{M} = -n\nabla\psi + B\nabla_\perp M_\parallel - B\nabla_\parallel\mathbf{M}_\perp$$

$$\psi = -\frac{1}{32\pi}\mathbf{E}\cdot\chi\cdot\mathbf{E}^* + cc$$

$$BM_\parallel = \frac{inm\Omega}{8\omega}\mathbf{b}\cdot\mathbf{u}\times\mathbf{u}^* + cc$$

The equivalence of these forms has been demonstrated in the straight B-field limit (after some algebra).

Computational issues and methods

[with acknowledgement to Ed D'Azevedo]

Issue

Consider the numerical computation of

$$\dot{w} = \frac{1}{4} \sum_{k,k'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} \mathbf{E}_k^* \cdot W(k,k') \cdot \mathbf{E}_{k'} + cc$$

- N Fourier modes for both k and k′ ⇒ N^2 computations per spatial point
- N spatial points ⇒ N^3 computations

Post-processing takes longer than solving for the wave fields (N×N matrix solve ⇒ N^2 ln N computations)

Observation

If W(k, k′) were separable: $N^3 \rightarrow N^2$

If W(k, k′) were expressible as a sum over p separable terms: $N^3 \rightarrow pN^2$ ⇒ computational savings for p < N

Possible method

The integrand of W(k, k′) is separable (before doing $v_\perp$ velocity integration)

One possible "summed separable" representation comes from doing velocity integral last numerically

- e.g. by Laguerre quadrature: can achieve 1% accuracy for k's of interest and p = 10

Other variations are also possible

- numerical for small $v_\perp$ & analytical asymptotics for large $v_\perp$ [L. Berry]
- singular value decomposition of numerical fit to inseparable parts of W(k, k′) [D'Azevedo]

Scenarios and the role of B_p

Global toroidal flows

- depend only on the input toroidal momentum ~ n P where n = toroidal mode number, P = absorbed power

Sheared poloidal flows

- most easily from power deposited to the bulk ions
  - o cyclotron absorption layer is narrow ⇒ strong shear

Strategies for ion power to bulk ions

- direct launch IBW – can be difficult to launch efficiently
- launch FW and mode convert
  - o want short wavelength MC wave to propagate into resonance
    - 1) heavy (μ/Z) minority ⇒ high field side resonance
      - standard MC IBW
    - 2) light (μ/Z) minority ⇒ low field side resonance
      - use B_p effects to get MC ICW to propagate towards resonance

RF SciDAC calculations show that 2) appears to work best.

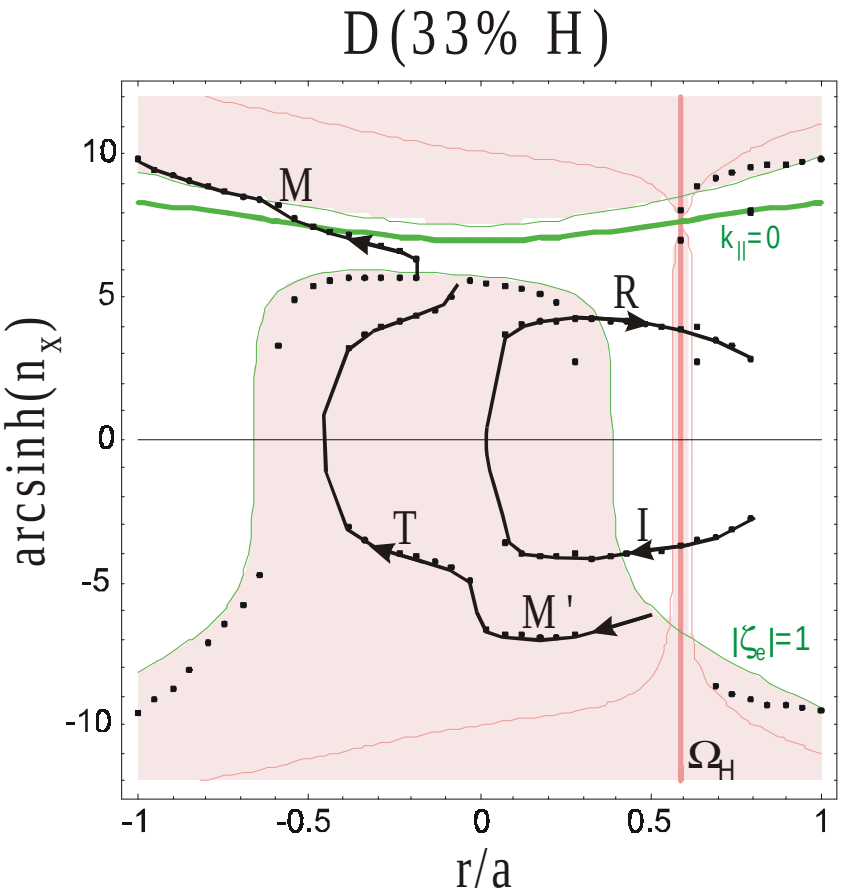
Effect of B_p on the mode-converted wave

(see posters BP1.074 Jaeger et al. and BP1.075 Berry et al. for AORSA-2D runs and more theory details)

1D hot plasma dispersion relation showing effect of poloidal magnetic field

- $k_{||} = B\zeta\ n/R + B_{px}\ k_x$ gives up/down asymmetry
  - o when $k_{||}$ is increased by $B_{px}\ k_x$ the MC wave is strongly affected

- mode converted wave:
  - o $B_{px}\, k_x < 0$: IBW propagates away from Ω_H
  - o $B_{px}\, k_x > 0$: ICW propagates towards Ω_H
- shown below $B_{px} < 0$. For $B_{px} > 0$ plot flips and arrows reverse



see also Perkins (NF, 1977); Ram & Bers (Phys. Fl., 1991)

This 2D B_p physics makes sheared flow drive by mode converted waves possible.

Summary and conclusions

Overview

ICRF waves are a promising means of controlling turbulence by flow drive.

RF also provides access to “open loop” experiments on the physics of flows and macroscopic plasma responses to waves.

Numerical tools for comparison with experiment are being developed under the RF SciDAC project.

Nonlinear theory of flow drive forces

A hot plasma, electromagnetic theory of nonlinear forces in 2D has been developed.

The forces can be expressed in terms of operator $W(k, k')$ which generalizes the conductivity tensor $\sigma(k)$.

Flux surface averaged flows are driven by photon momentum absorption and dissipative stresses.

Computation

The nonlocal nature of the force (and power deposition) operators leads to computationally expensive sums when implemented straightforwardly.

More computationally efficient algorithms have been explored and look encouraging.

Scenarios and tokamak physics

Minority mode conversion scenarios (as well as direct launch IBW) are suitable for driving sheared flows.

The poloidal magnetic field plays an essential role in allowing damping of the MC wave on ions, which is optimal for flow drive. (see posters BP1.074 & BP1.075)