Physics/Computational Issues for Transport Analysis of QPS devices

(DELTA5D vs. DKES)

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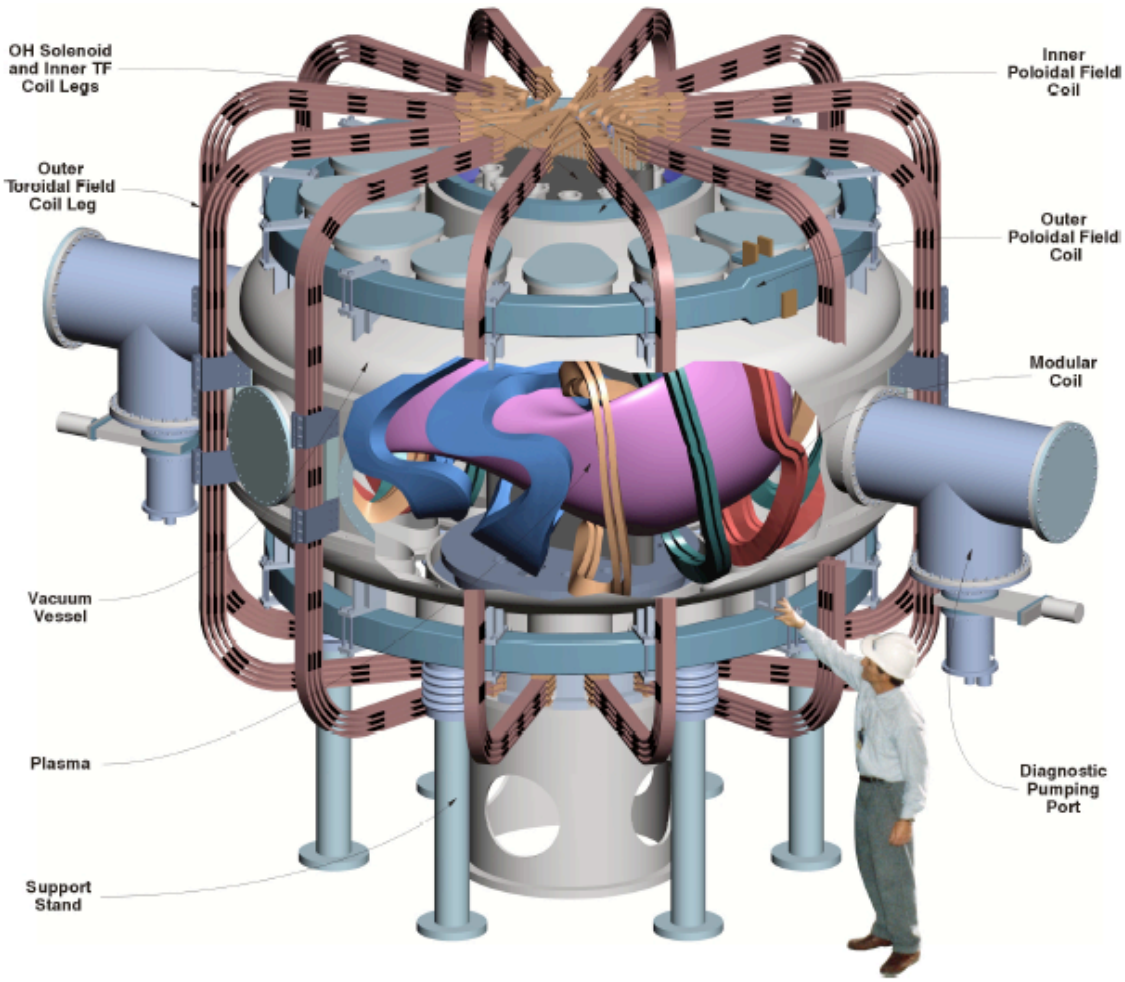
QPS is a very low aspect ratio (A = 2.7) Quasi-poloidal stellarator



- $\langle R_0 \rangle$ = 0.9 m, $\langle a \rangle$ = 0.33 m, $\langle B \rangle$ = 1 T ± 0.2T for 1sec, I_p □ 150 kA, P_{ECH} = 0.6-1.2 Mw, P_{ICH} = 1-3 Mw
- This experiment will test:
  - Equilibrium robustness
  - Neoclassical and anomalous transport
  - Stability limits up to $\langle \square \rangle$ = 2.5%
  - Bootstrap current effects
  - Reduced poloidal viscosity effects on shear flow transport reduction
  - Configurational flexibility

Our transport analysis has focused on optimization targets and comparisons between configurations

Optimization Strategy

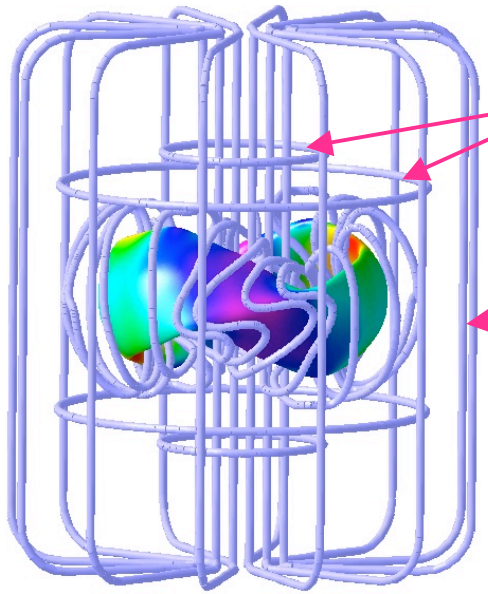
- QOS optimizations (1996 - 1999)
  - J^*, J, B_{min}, B_{max}
  - DKES
- QPS optimizations (2000 - present)
  - ϵ_{eff}
  - Poloidal symmetry
  - DKES

Evaluation Tools

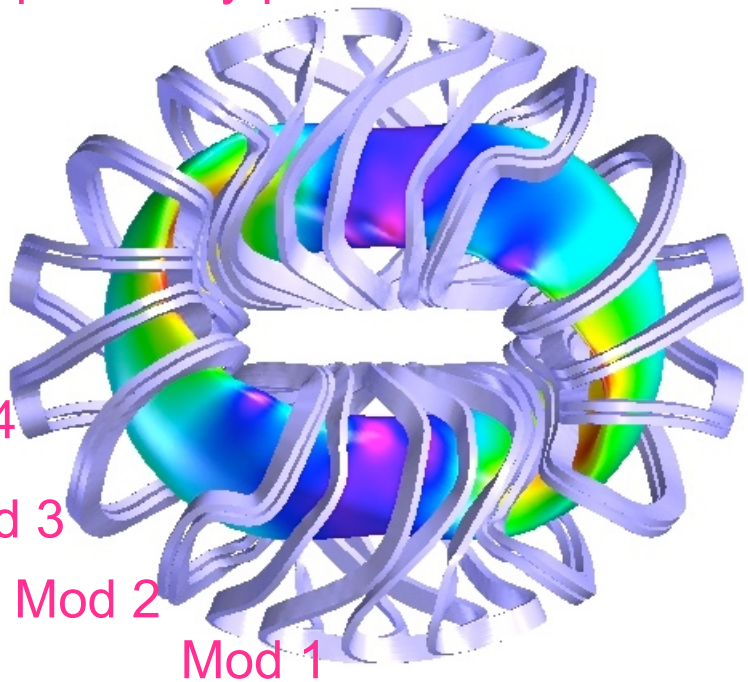
- NEO ϵ_{eff}, Poloidal symmetry
- DKES
  - Setup, mode selection
  - Parallel runs
  - Energy integration
- DELTA5D Monte Carlo
  - Global full-f model
  - ICRF heating
  - NBI heating efficiency
  - Alpha losses
  - Bootstrap current
- 1-1/2 D model

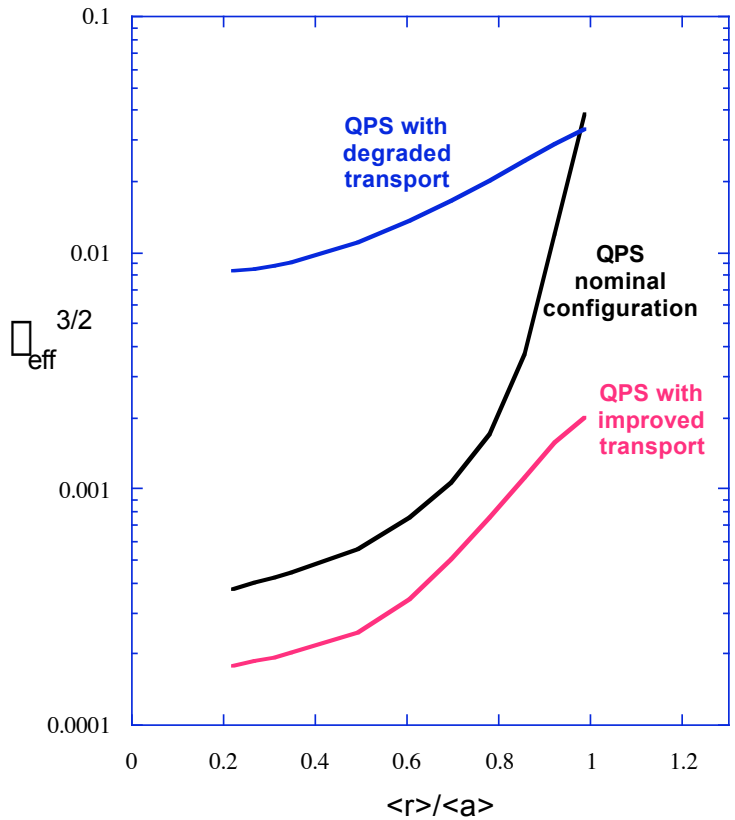
However, with the QPS configuration becoming fixed, we are shifting over to flexibility studies and transport predictions.

QPS Flexibility Studies



4 independently powered modular coils





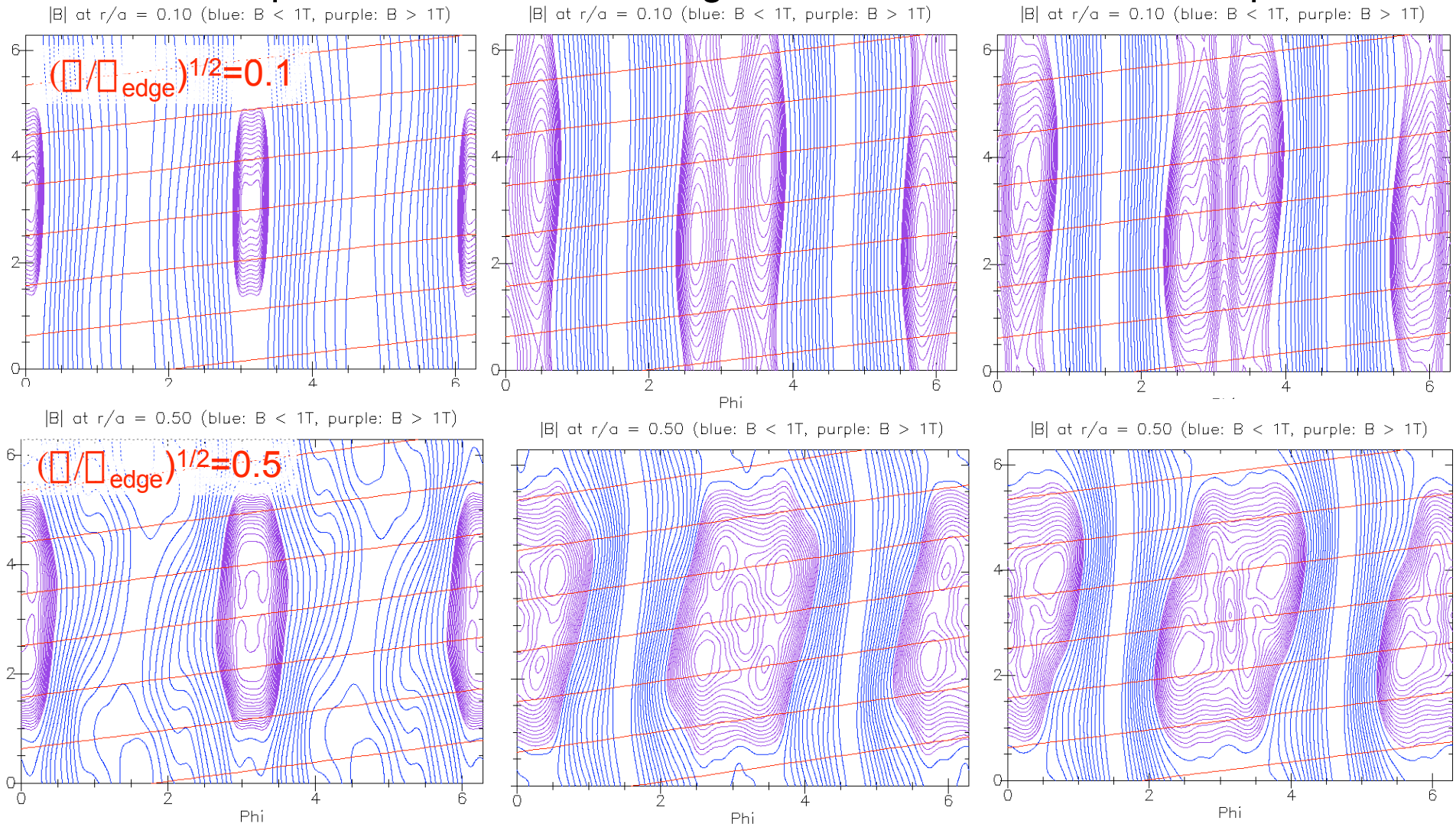
- Modular coil currents can vary ±25%
- Vertical field currents can vary ±100 kA
- Toroidal field currents can vary ±70 kA

QPS Flexibility Studies - effect on |B|

Degraded transport

Nominal configuration

Improved transport



The DKES (Drift Kinetic Equation Solver) provides the full neoclassical transport coefficient matrix (multi-helicity)

$$I_i = \begin{bmatrix} \vec{\square} \bullet \vec{\square}s \\ \frac{1}{T}\vec{Q} \bullet \vec{\square}s \\ n\langle(\vec{u} \square \vec{u}_S) \bullet \vec{B}\rangle \end{bmatrix} = \square \sum_{j=1}^{3} L_{ij} A_j \qquad A_j = \begin{bmatrix} \frac{n\square}{n} \square \frac{3}{2}\frac{T\square}{T} \square \frac{eE_r}{T} \\ \frac{T\square}{T} \\ \square\square\frac{e}{T}\square\frac{\langle \vec{E} \bullet \vec{B}\rangle}{\langle B^2\rangle} \end{bmatrix}$$

$$L_{ij} = n\frac{2}{\sqrt{\square}} \int_0^{\bullet} dK \sqrt{K}\, e^{\square K} g_i g_j D_{ij}$$

$$\text{where } g_1 = g_3 = 1,\ g_2 = K,\quad K = \left[\frac{v}{v_{th}}\right]^2$$

$$D_{11} = D_{12} = D_{21} = D_{22} = \square \frac{v_{th}}{2} \left[\frac{\square B v_{th}}{\square\ \square} \left[\frac{d\square}{dr}\right]^{\square 1}\right]^2 K\sqrt{K}\ \square_{11}$$

$$D_{31} = D_{32} = \square D_{13} = \square D_{23} = \square \frac{v_{th}}{2} \left[\frac{\square B v_{th}}{\square\ \square} \left[\frac{d\square}{dr}\right]^{\square 1}\right] K\ \square_{31}$$

$$D_{33} = \square \frac{v_{th}}{2} \sqrt{K}\ \square_{33}$$

$$\square_{ij} = \square_{ij} \left[\frac{\square}{v}, \frac{E_r}{v}\right]$$

- W. I. Van Rij, S. P. Hirshman, Phys. Fluids **B 1**, 563 (1989)
- Variational: provides upper and lower bounds on dS/dt
- Expands f in Fourier-Legendre series

(i.e., to carry out the above integrals, one will need to generate a 2-D matrix of □'s vs. these parameters for each flux surface)

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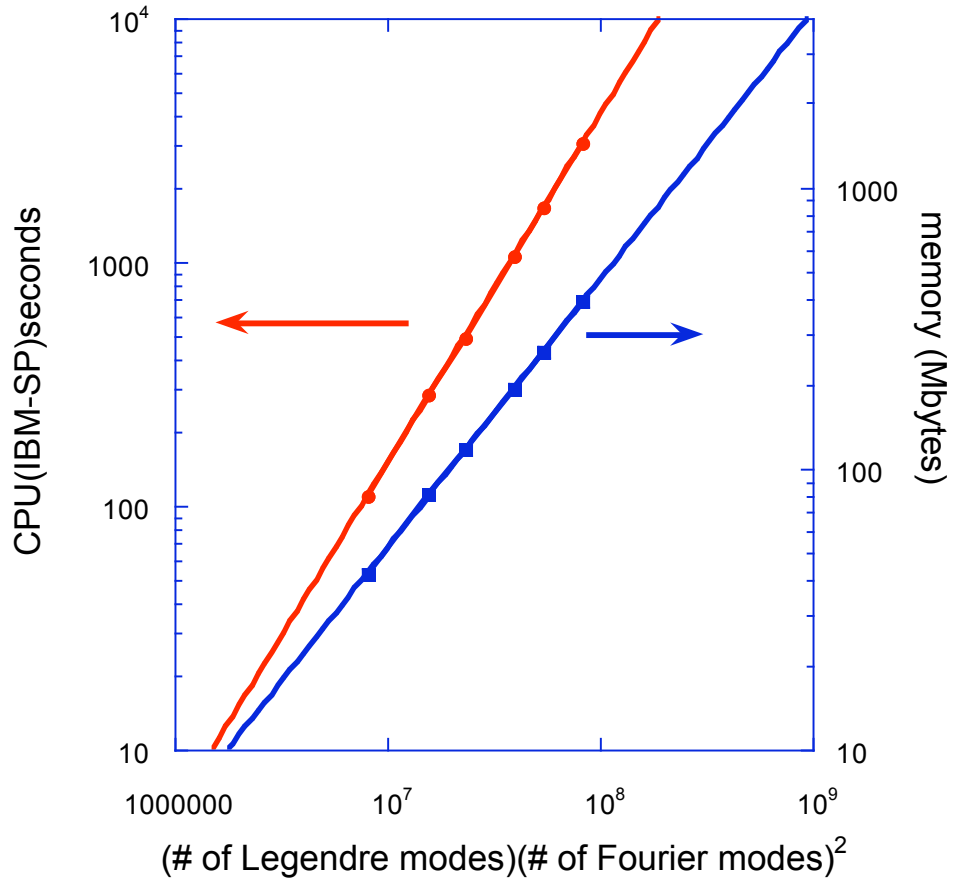
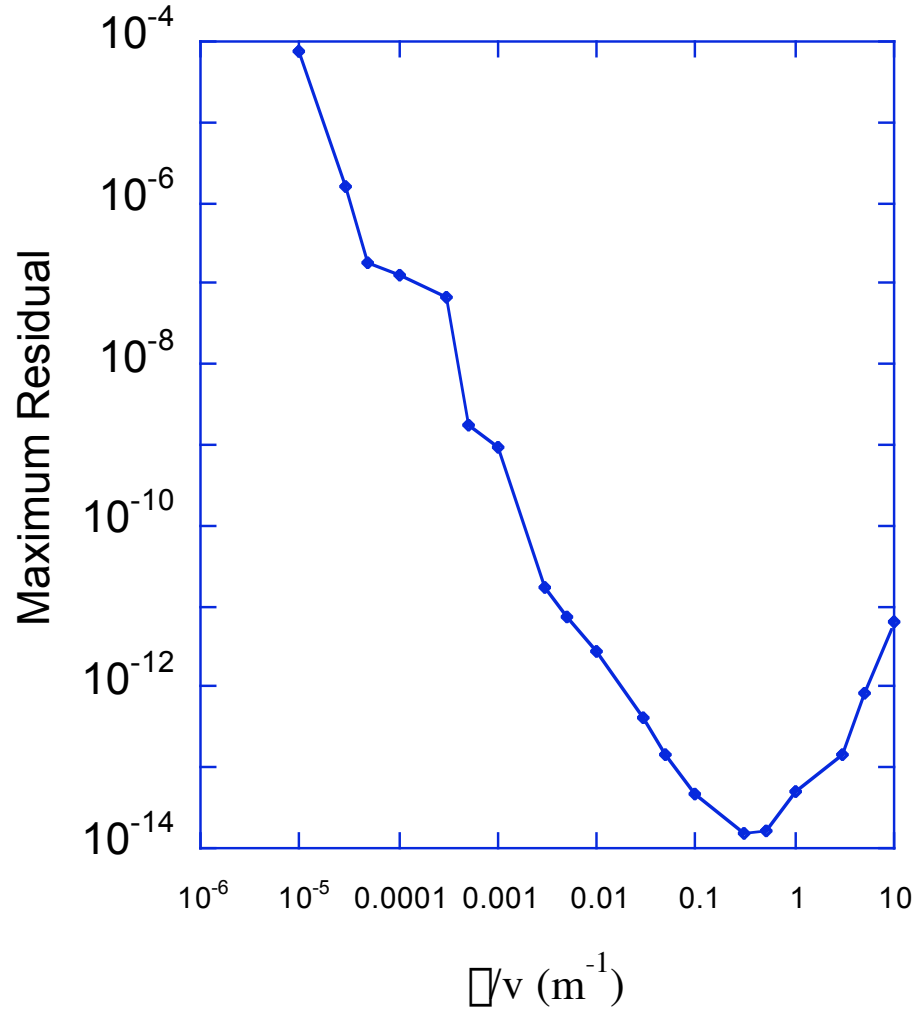
DKES perfomance, memory and residuals limitations



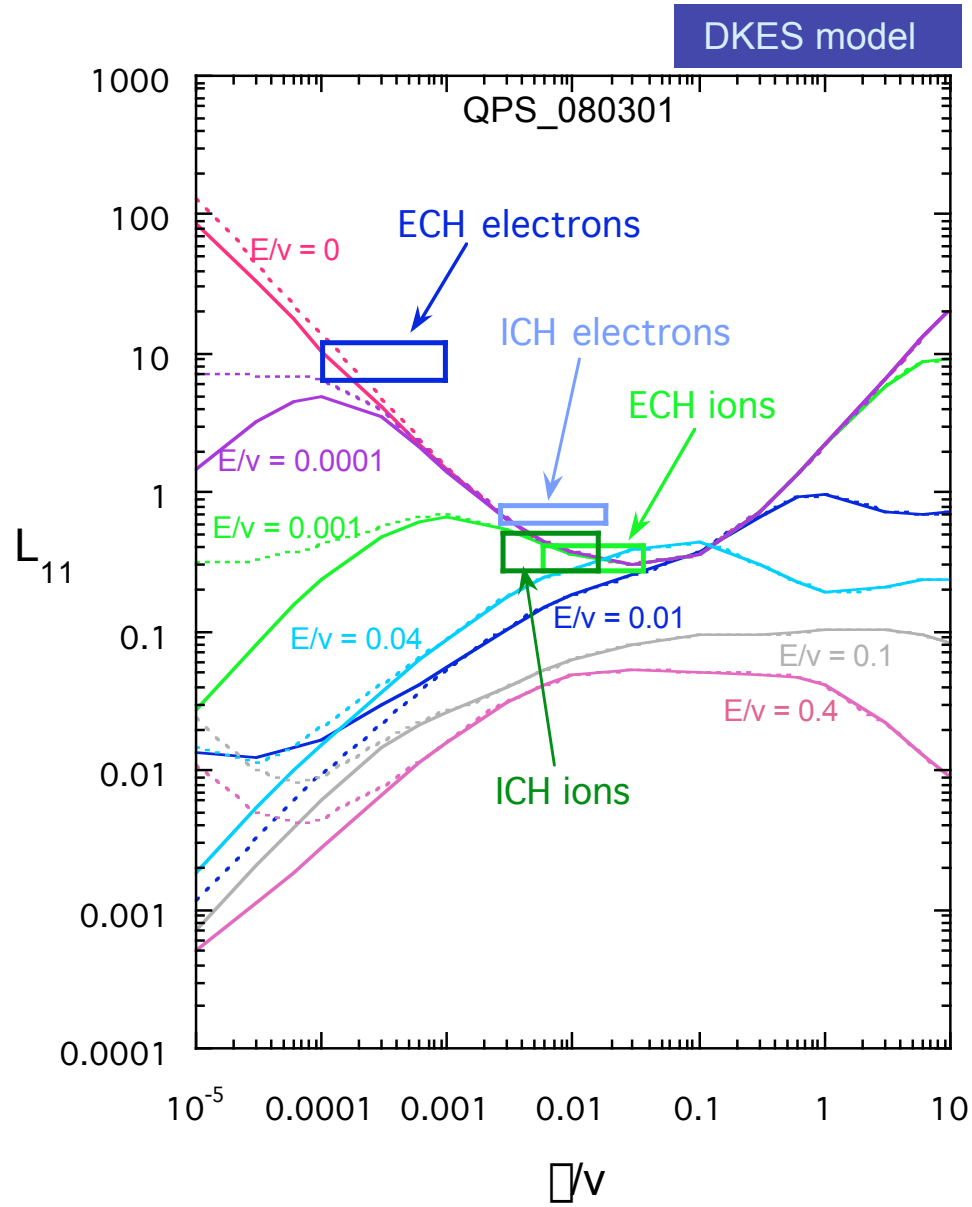


DKES transport dependencies show that both $L_{11} \mu \nu^{1/2}$ and $L_{11} \mu 1/\nu$ regimes are accessed.

These ranges are based on the maximum n, T in the profile, $e\Phi/kT(a) = 1$, and looking from thermal energy to 9 times thermal (as sampled by velocity integrals for ν)

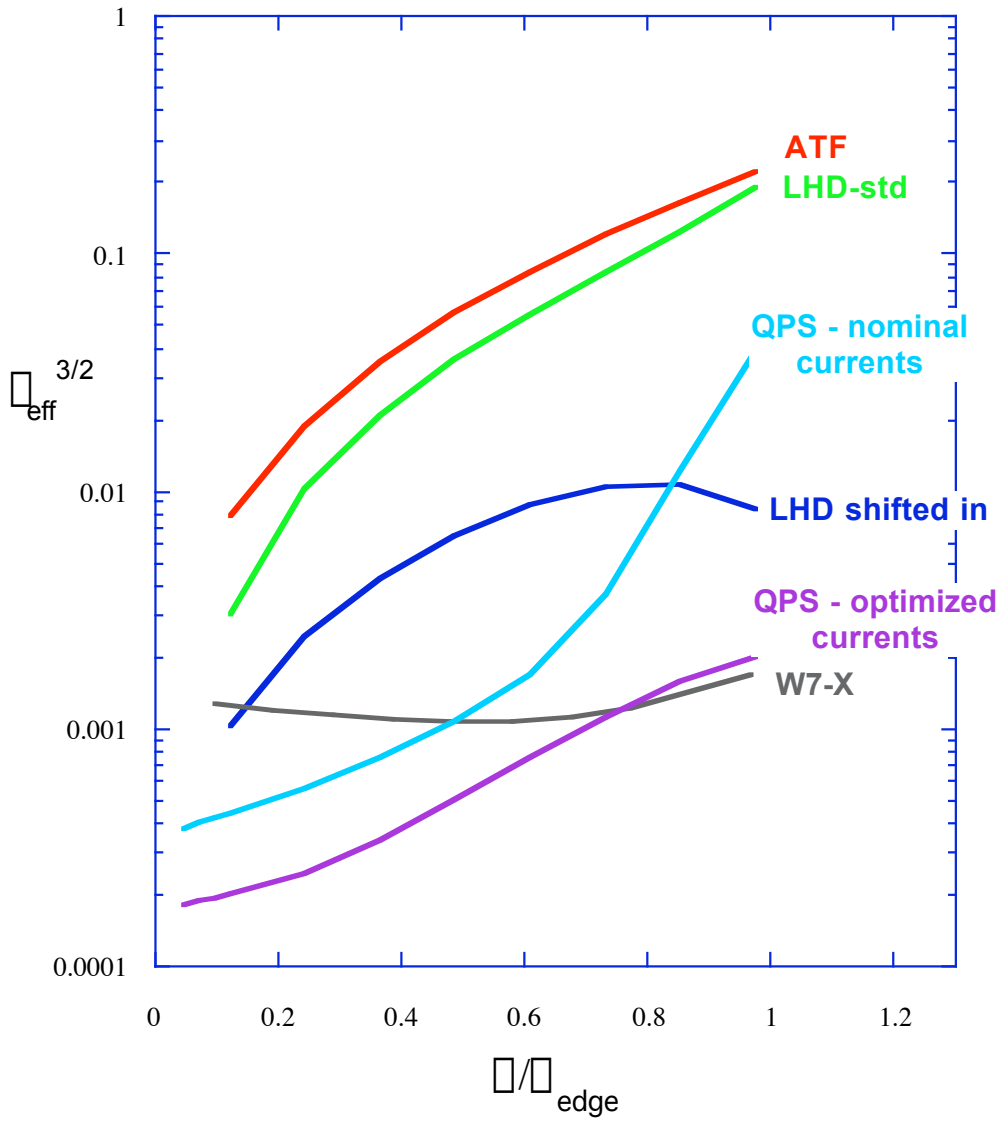
- **ECH electrons**:
  - $7\times10^{-5} < \nu/v < 10^{-3}$
  - $8\times10^{-5} < E/v < 2\times10^{-4}$
- **ECH ions**:
  - $6\times10^{-3} < \nu/v < 7\times10^{-2}$
  - $10^{-2} < E/v < 3\times10^{-2}$
- **ICH electrons:**
  - $3\times10^{-3} < \nu/v < 4\times10^{-2}$
  - $5\times10^{-5} < E/v < 2\times10^{-4}$
- **ICH ions:**
  - $2\times10^{-3} < \nu/v < 3\times10^{-2}$
  - $2\times10^{-3} < E/v < 6\times10^{-3}$

Note: these L11's are based on the gb4 device.



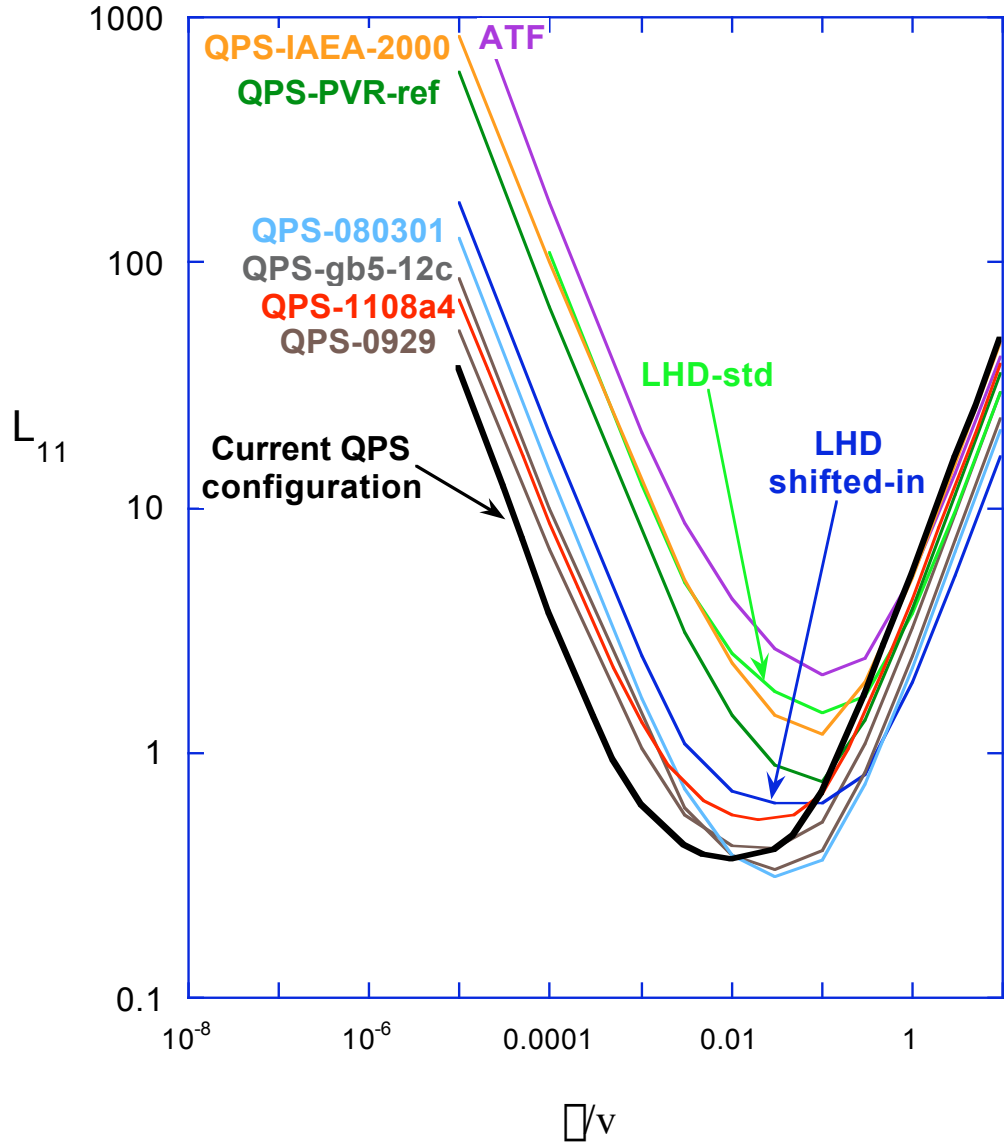
NEO code provides $\Gamma_{eff}^{3/2} \sim D^{1/\Gamma}, \Gamma^{1/\Gamma}$. Comparison of different configurations:

NEO Γ_{eff} code

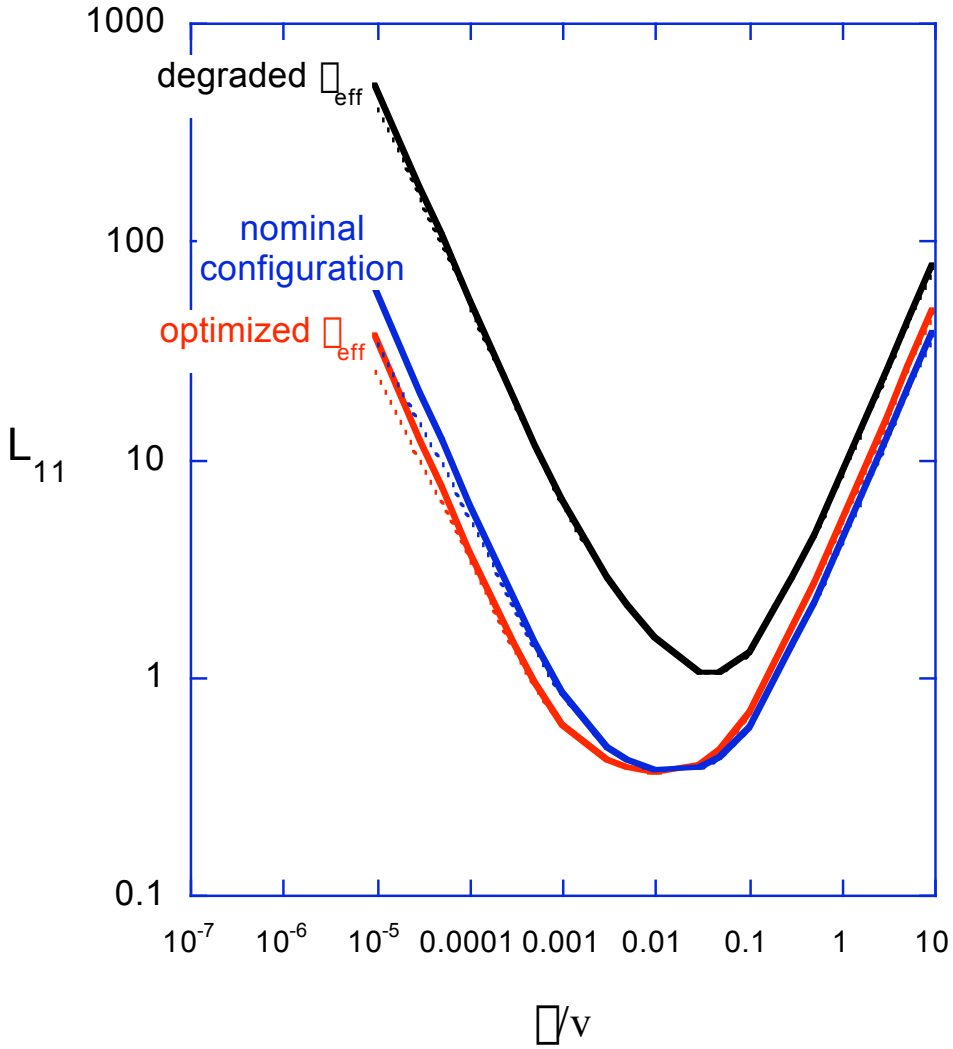


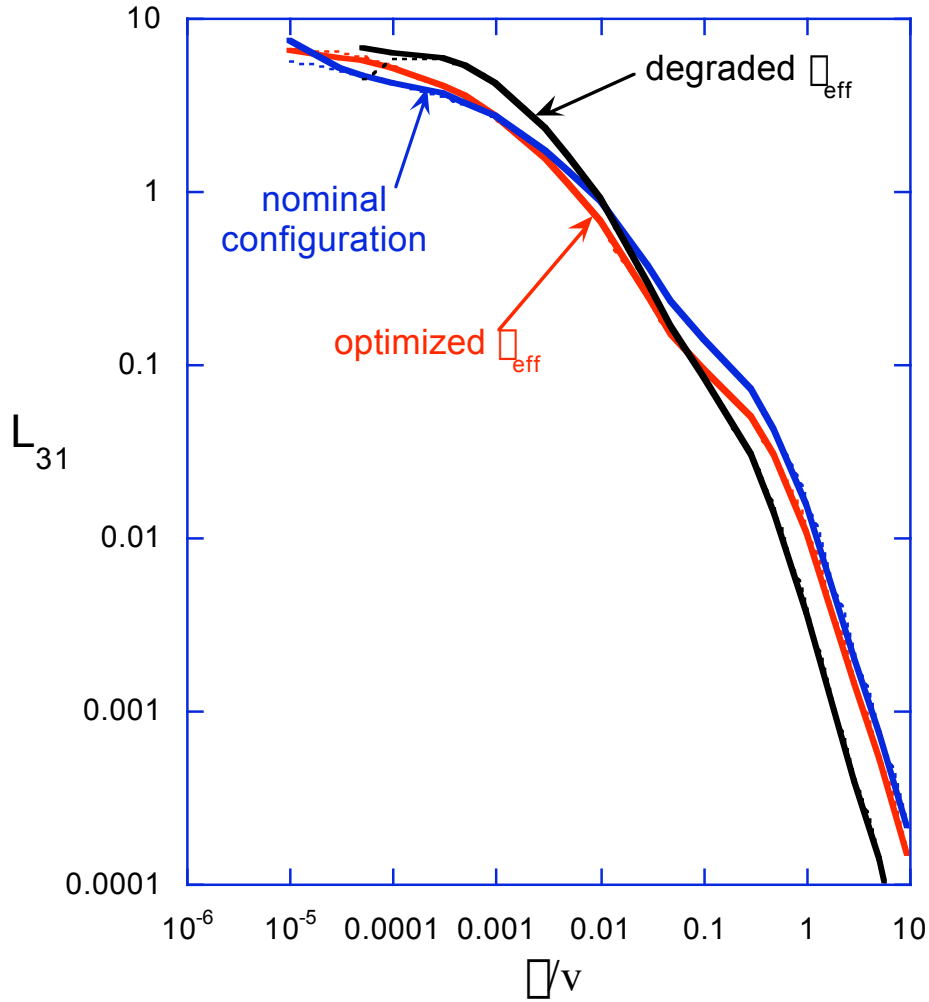
DKES L_{11} transport coefficient at $E_r/v = 0$ shows similar trends at low collisionality among QPS devices as NEO $\epsilon_{eff}^{3/2}$ coefficient

DKES model



DKES L_{11} coefficient demonstrates sensitivity of transport to coil current optimization

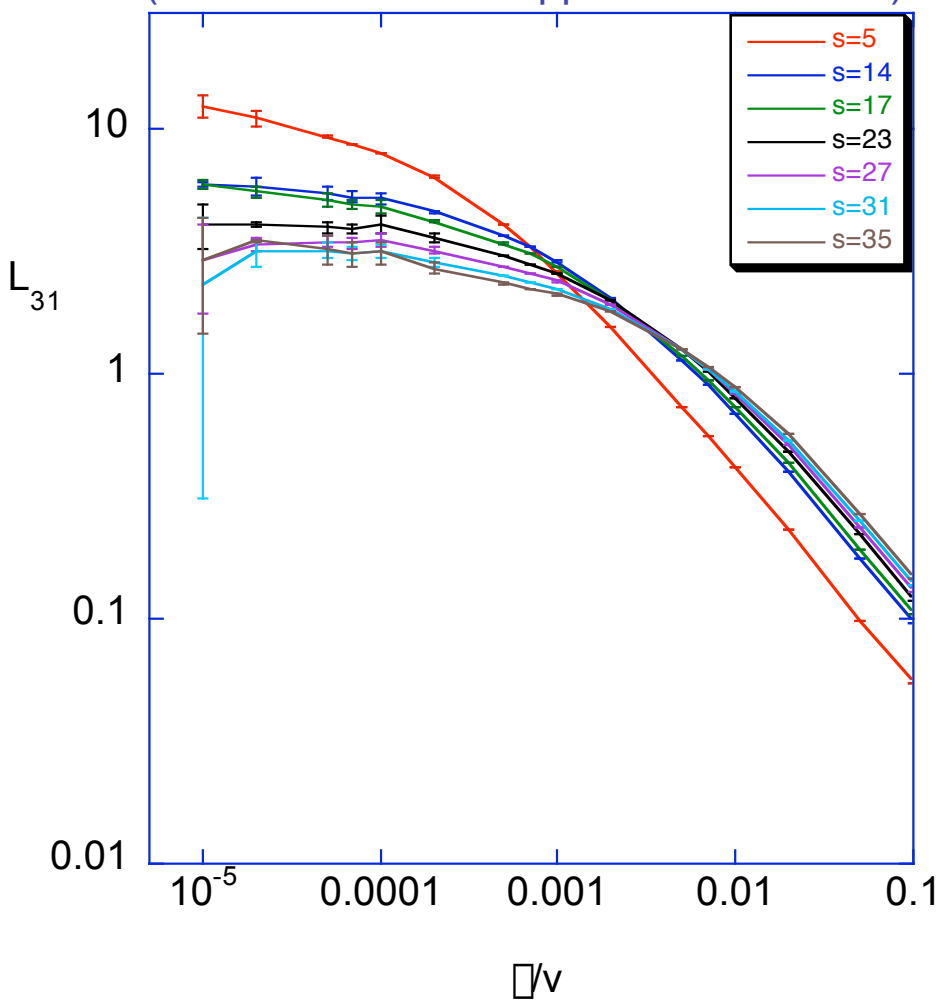




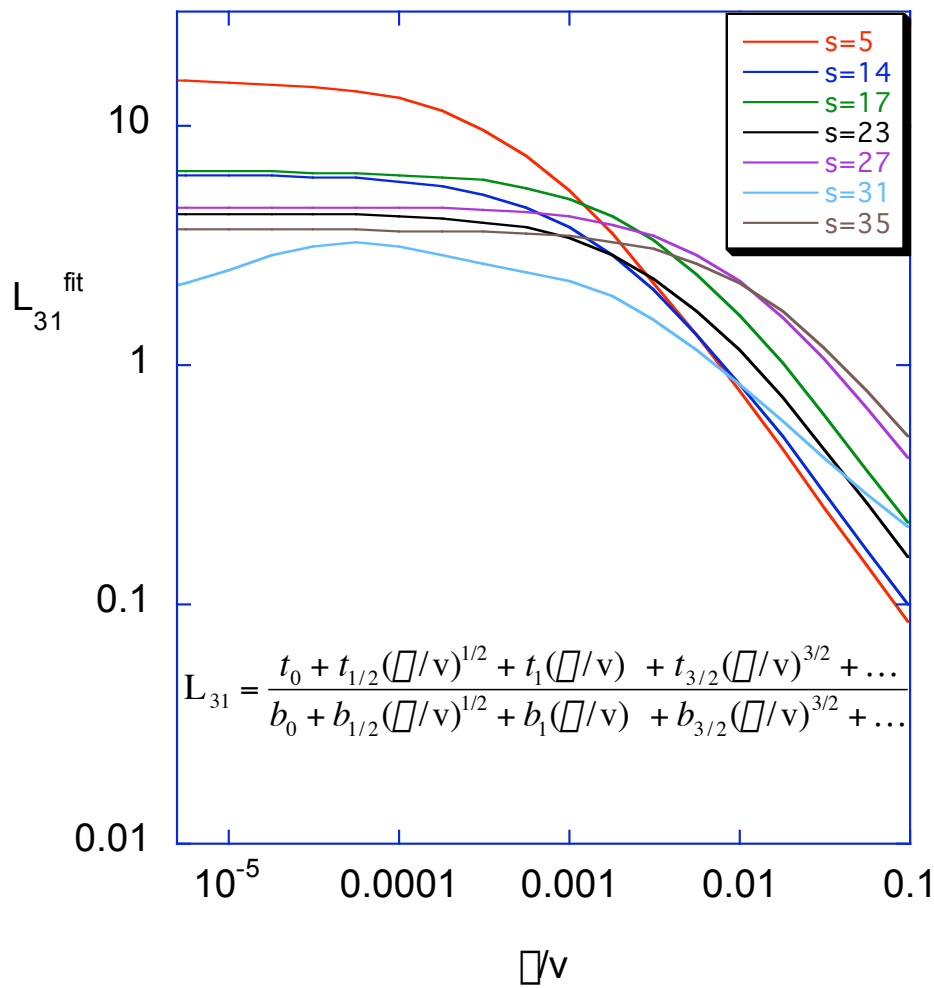
DKES L_{31} coefficient (bootstrap current for recent QPS device (411)

DKES results

(error bars indicate upper/lower bounds)



Fits using Padé approximant

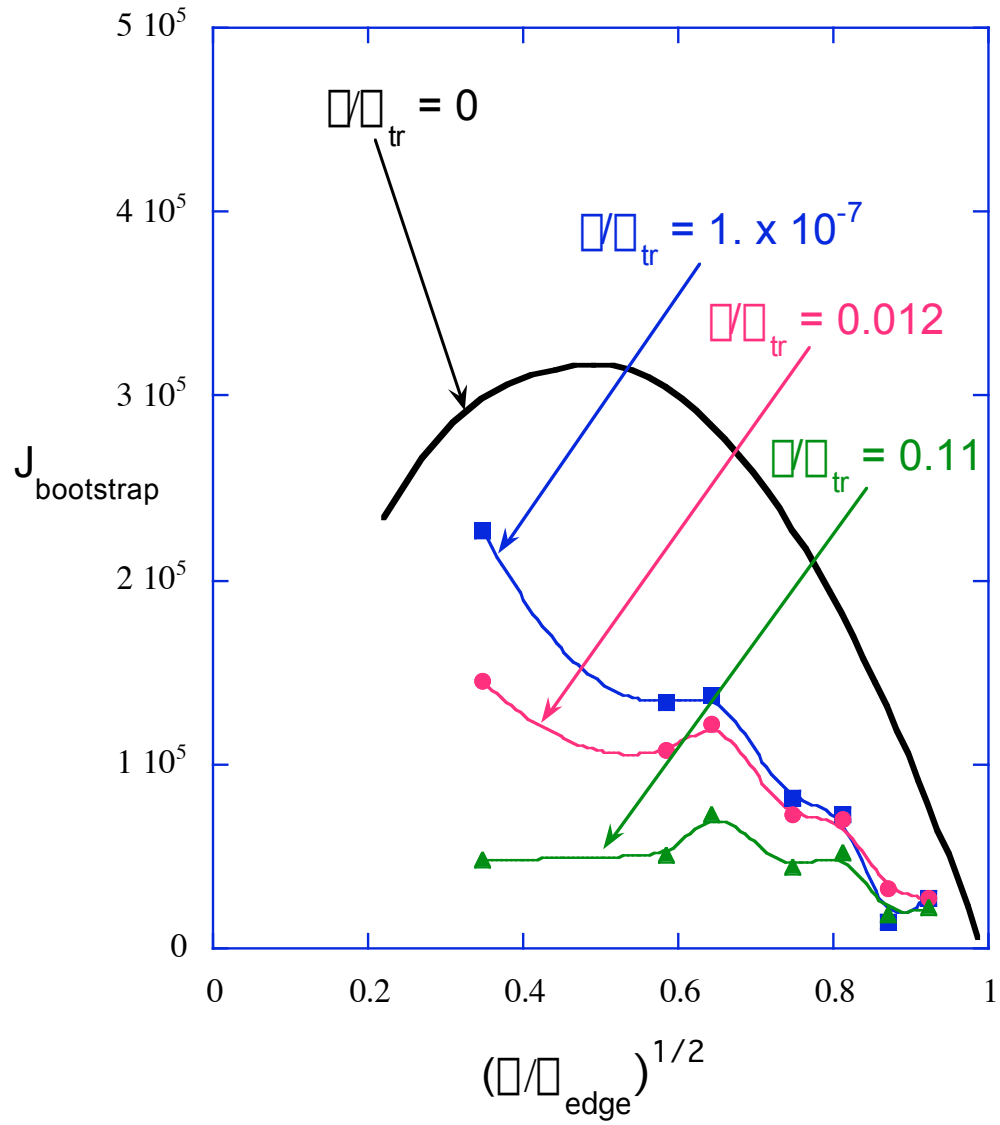


$$L_{31} = \frac{t_0 + t_{1/2}(\square/v)^{1/2} + t_1(\square/v) + t_{3/2}(\square/v)^{3/2} + \ldots}{b_0 + b_{1/2}(\square/v)^{1/2} + b_1(\square/v) + b_{3/2}(\square/v)^{3/2} + \ldots}$$

Comparison of DKES predicted bootstrap current profiles with collisionless limit

- Collisionless limit based on BOOTSJ code of L. Berry/J. Tolliver - includes damping to avoid singularities at rational surfaces
- Based on:

- K.C. Shaing, B.A. Carreras, N. Dominguez, V.E. Lynch, J.S. Tolliver, "Bootstrap current control in stellarators", Phys. Fluids B1, 1663 (1989)
- K.C. Shaing, E.C. Crume, Jr., J.S. Tolliver, S.P. Hirshman, W.I. van Rij. "Bootstrap current and parallel viscosity in the low collisionality regime in toroidal plasmas", Phys. Fluids B1, 148 (1989).
- K.C. Shaing, S.P. Hirshman, J.S. Tolliver "Parallel viscosity-driven neoclassical fluxes in the banana regime in nonsymmetrical Toroidal plasmas", Phys. Fluids 29, 2548 (1986).

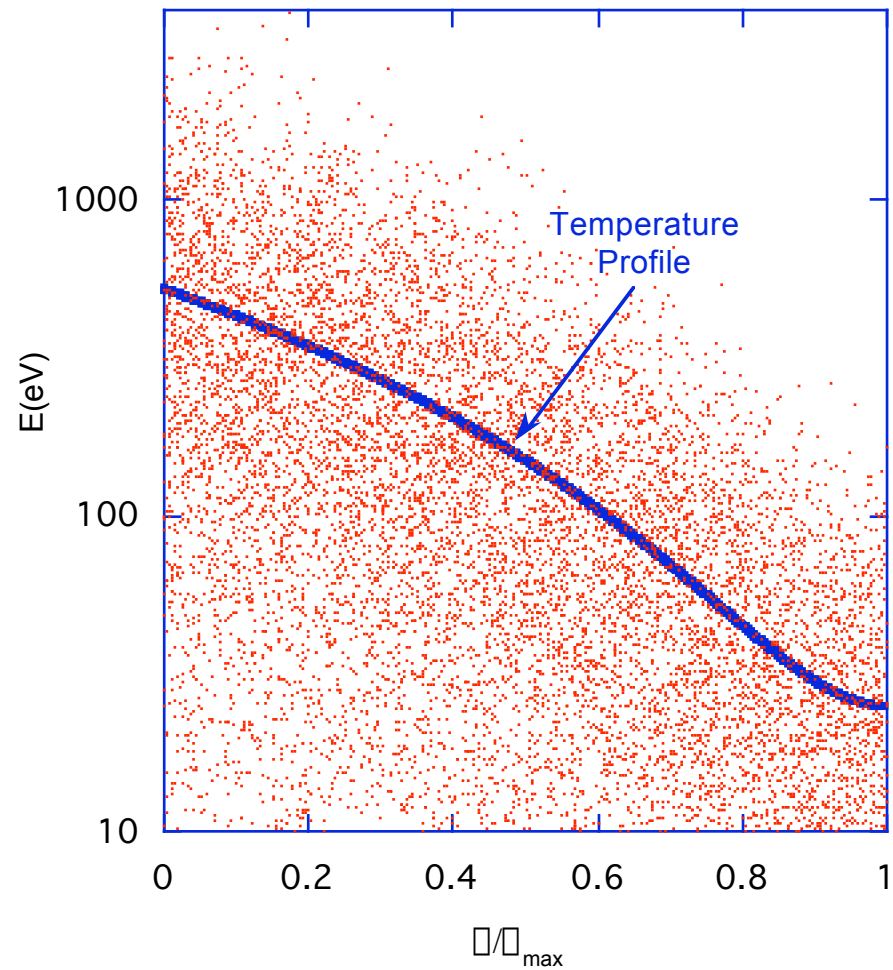


Monte Carlo procedure for estimating global energy lifetimes (DELTA5D code)

DELTA5D Monte Carlo

- start with particles distributed over cross section using PDF's consistent with assumed profiles and local Maxwellians
- follow ensemble in time, replacing particles (consistent with initial PDF's) as they leave outer surface
- Record energies of escaping particles - use to calculate ΔE
- Follow until approximate steady–state is achieved
- Vary potential (with fixed profile shape) to achieve global ambipolar balance of electron/ion particle loss rates

Typical initial Maxwellian particle loading for T = 500eV $(1 - \Phi/\Phi_{max})^2$



Power flows for global single species test particle simulations:

DELTA5D Monte Carlo

- To achieve steady state, in a reasonable simulation time, terms (1) and (2) need to be balanced:
- For $E_r = 0$
  - can include pitch angle and energy scattering if $Q_{ii} + Q_{ei}$ is weak during particle confinement time
  - or can include only pitch angle scattering
- For $E_r \neq 0$
  - rely on $Q_{ii} + Q_{ei}$ term (1) to redistribute energy loss/gain from term (2)
  - or can remove kinetic energy loss/gain (due to $e\Phi$) each time step

$$\frac{dW_{test,ion}}{dt} = \int \left(Q_{ii} + Q_{ei}\right) d^3v \qquad (+/-) \quad (1)$$

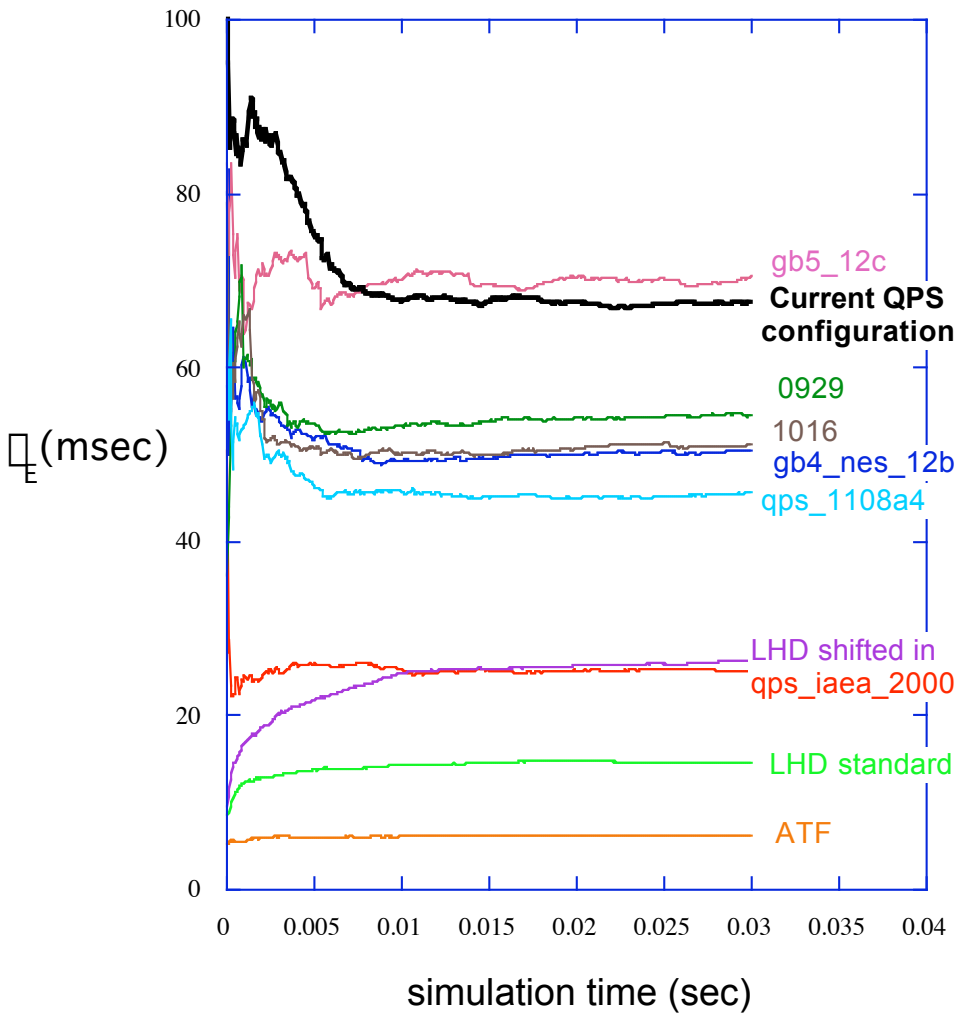
$$+ \int \vec{J} \bullet \vec{E}\, d^3v \qquad (+/-) \quad (2)$$

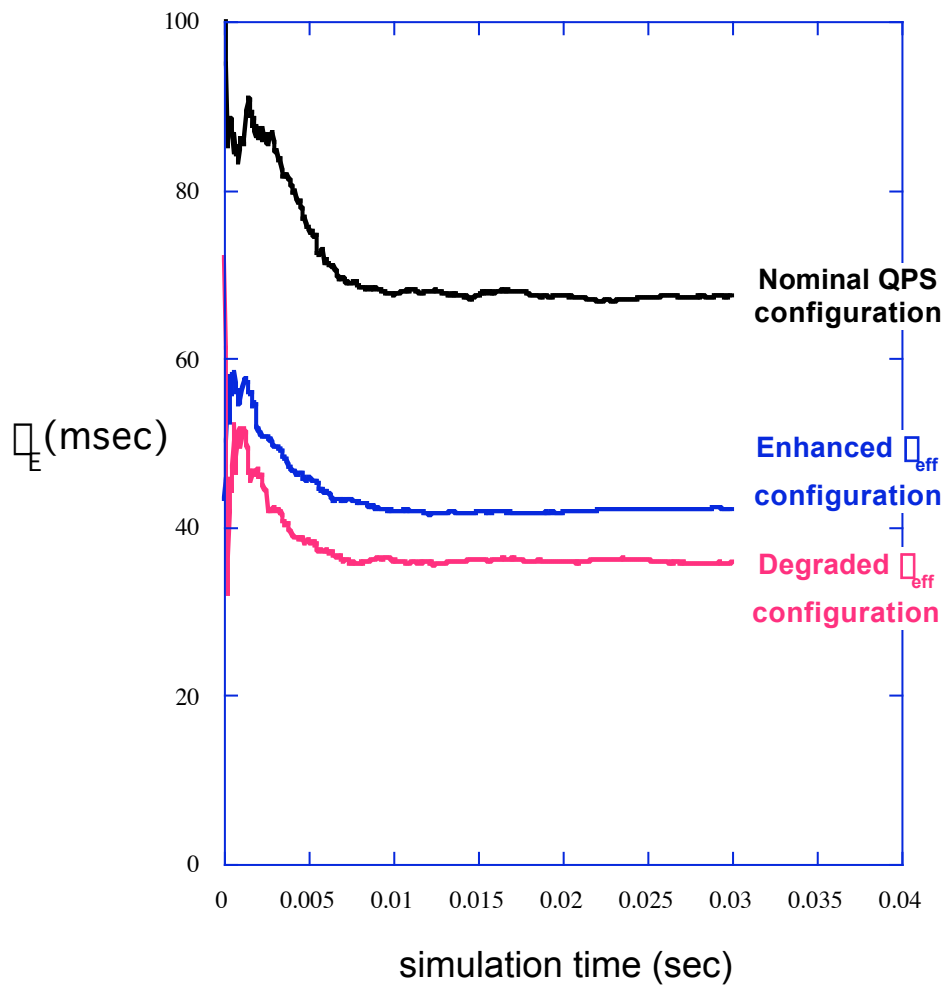
$$+ \int \frac{dW_{surface\_loss}}{dt} dS \qquad (-) \quad (3)$$

$$+ \int N_{replacement} \left[T(\Phi) + e\Phi\right] d^3v \qquad (+/-) \quad (4)$$

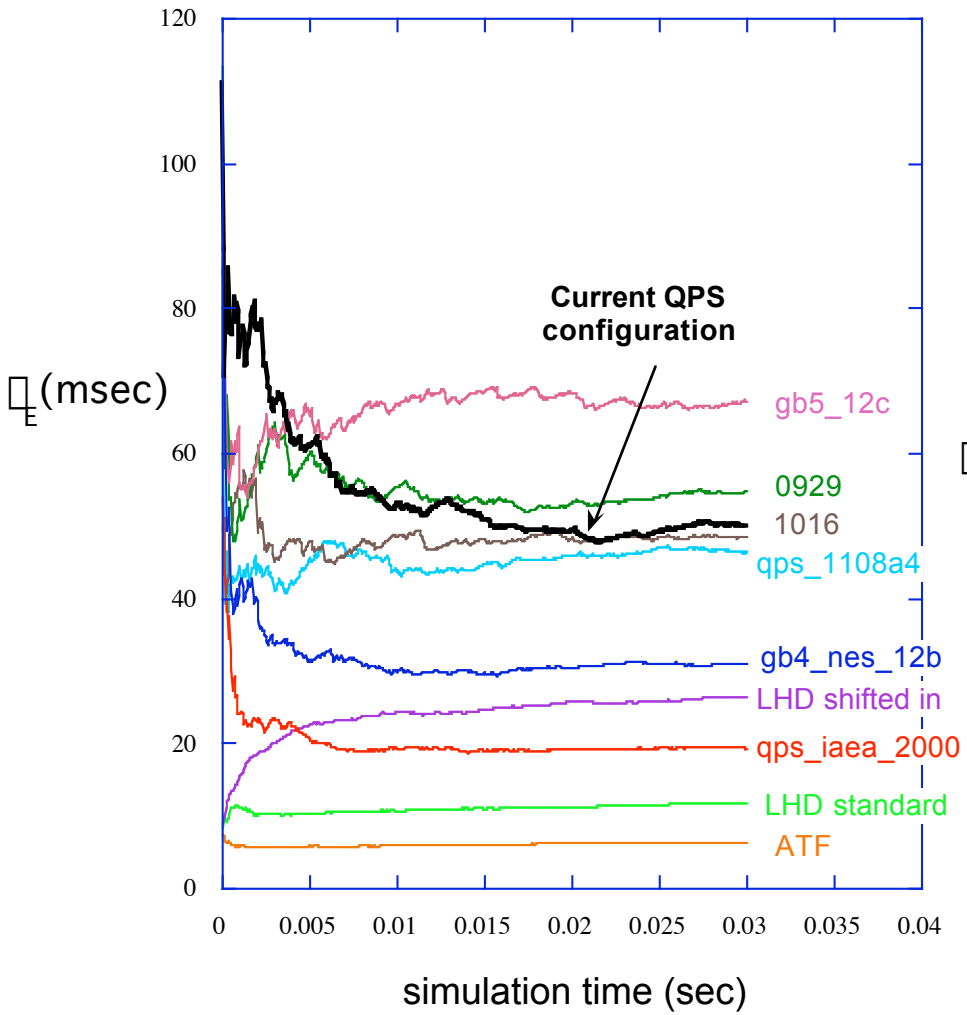
$$\frac{dW_{test,electron}}{dt} = \textit{Similar equations}$$

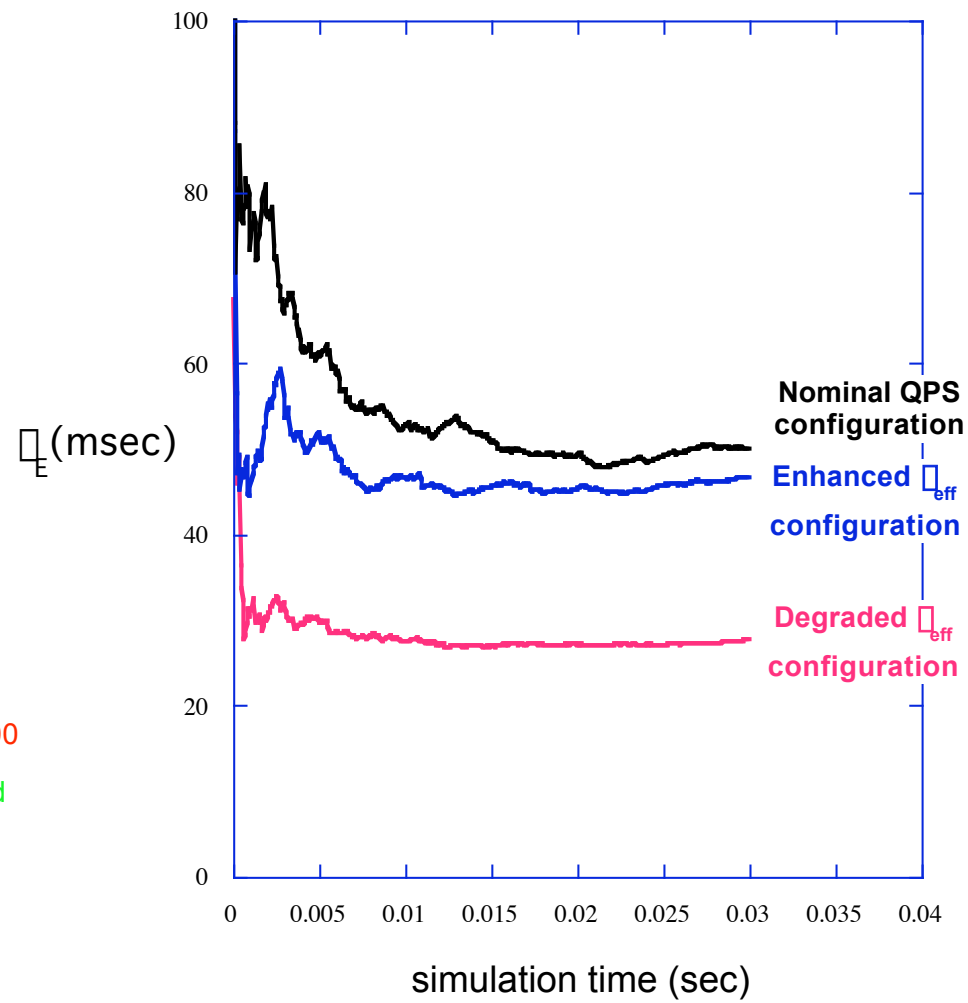
Monte Carlo studies of global ion confinement times for ICH parameters between devices (all at same R_0) and for different coil currents





Monte Carlo studies of global ion confinement times for ECH parameters between devices (all at same R_0) and in QPS for different coil currents

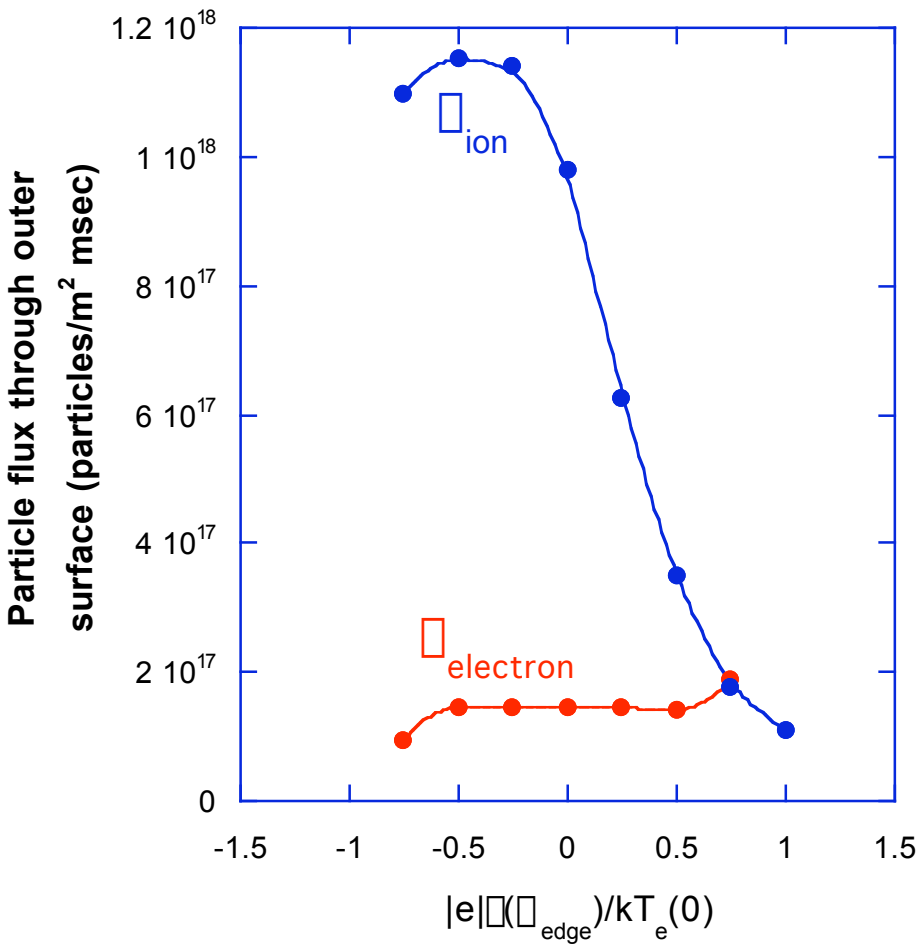




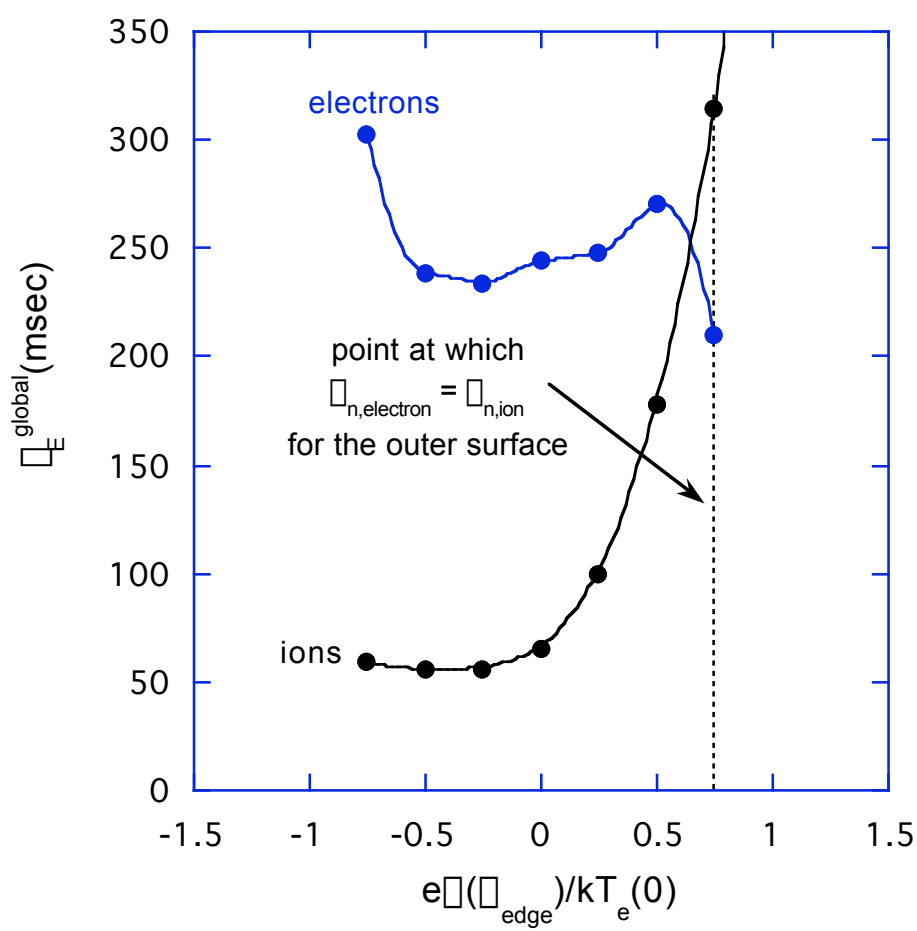
Monte Carlo lifetimes for ICH heated gb4 configuration

[n(0) = 8.3 x 10^{19} m^{-3}, $T_e(0)$ = 500eV, $T_i(0)$ = 500 eV, flat density profile, parabolic**2 temperature profile]

Global ambipolarity condition
[i.e., with □(r) profile fixed]



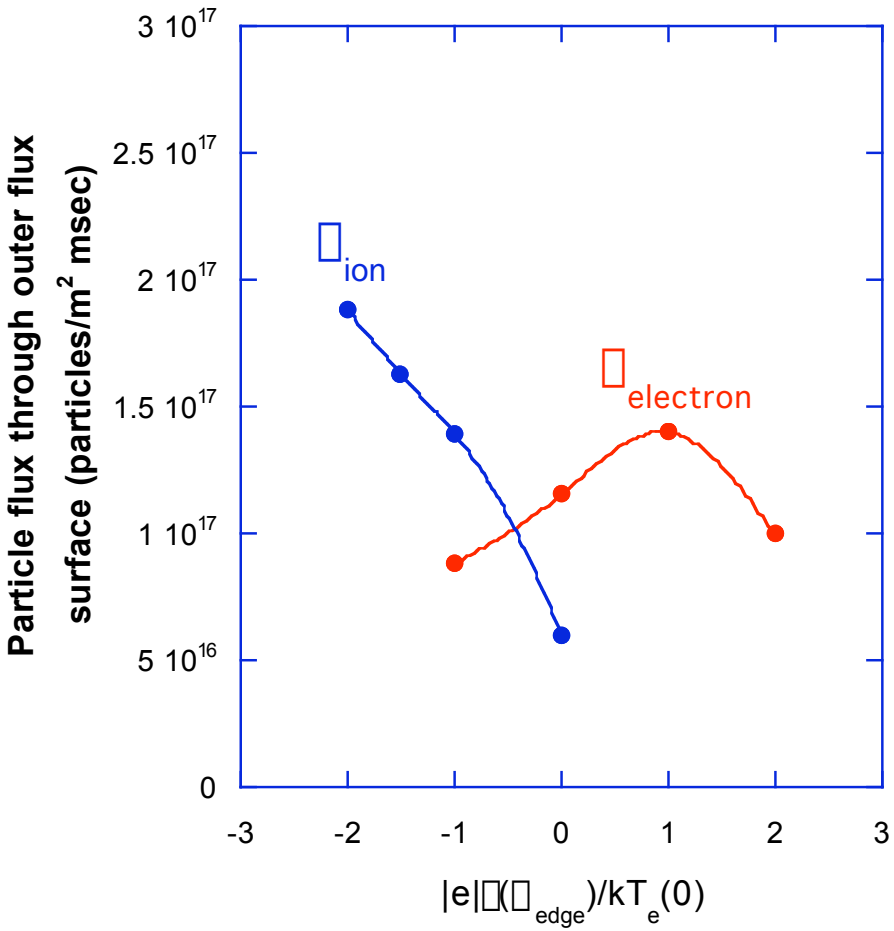
Global electron/ion energy lifetimes



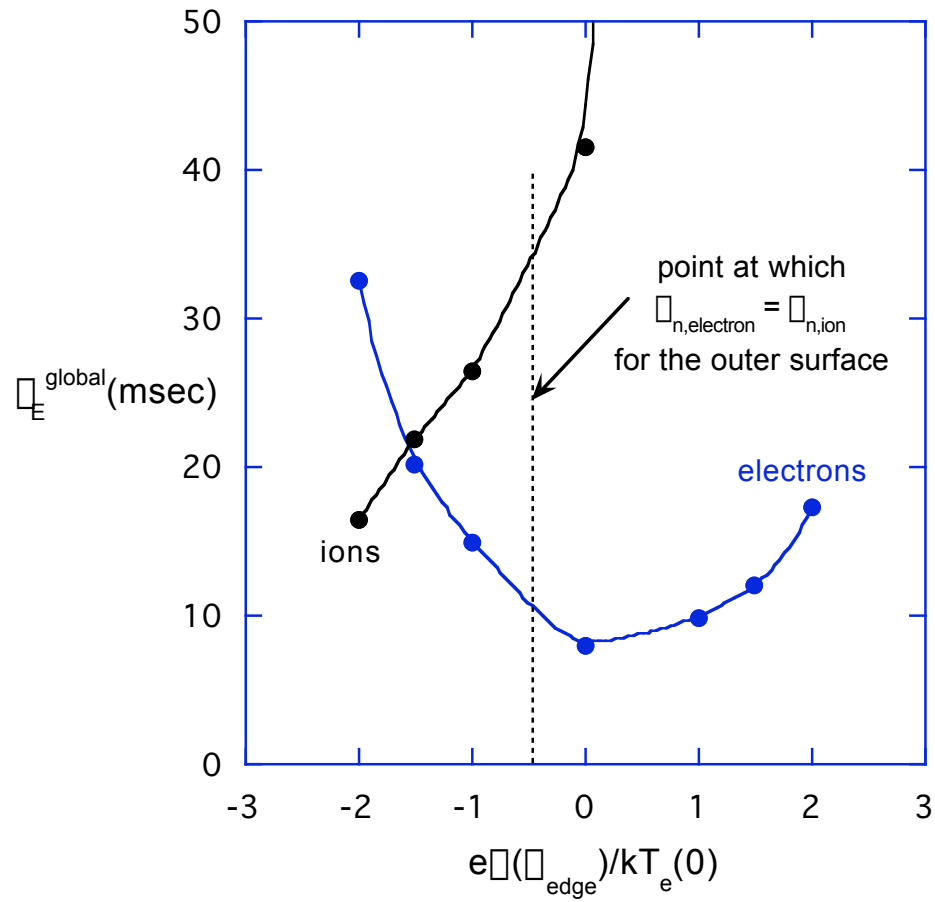
Monte Carlo lifetimes for ECH heated gb4 configuration

[n(0) = 1.8 x 10^{19} m^{-3}, $T_e(0)$ = 1400eV, $T_i(0)$ = 150 eV, flat density profile, parabolic**2 temperature profile]

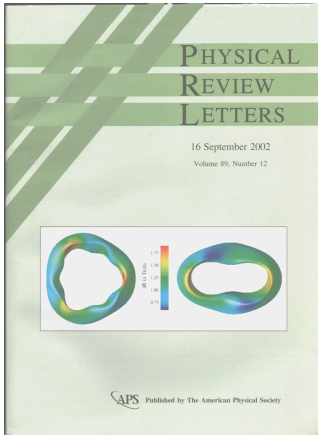
Global ambipolarity condition
[i.e., with □(r) profile fixed]



Global electron/ion energy lifetimes

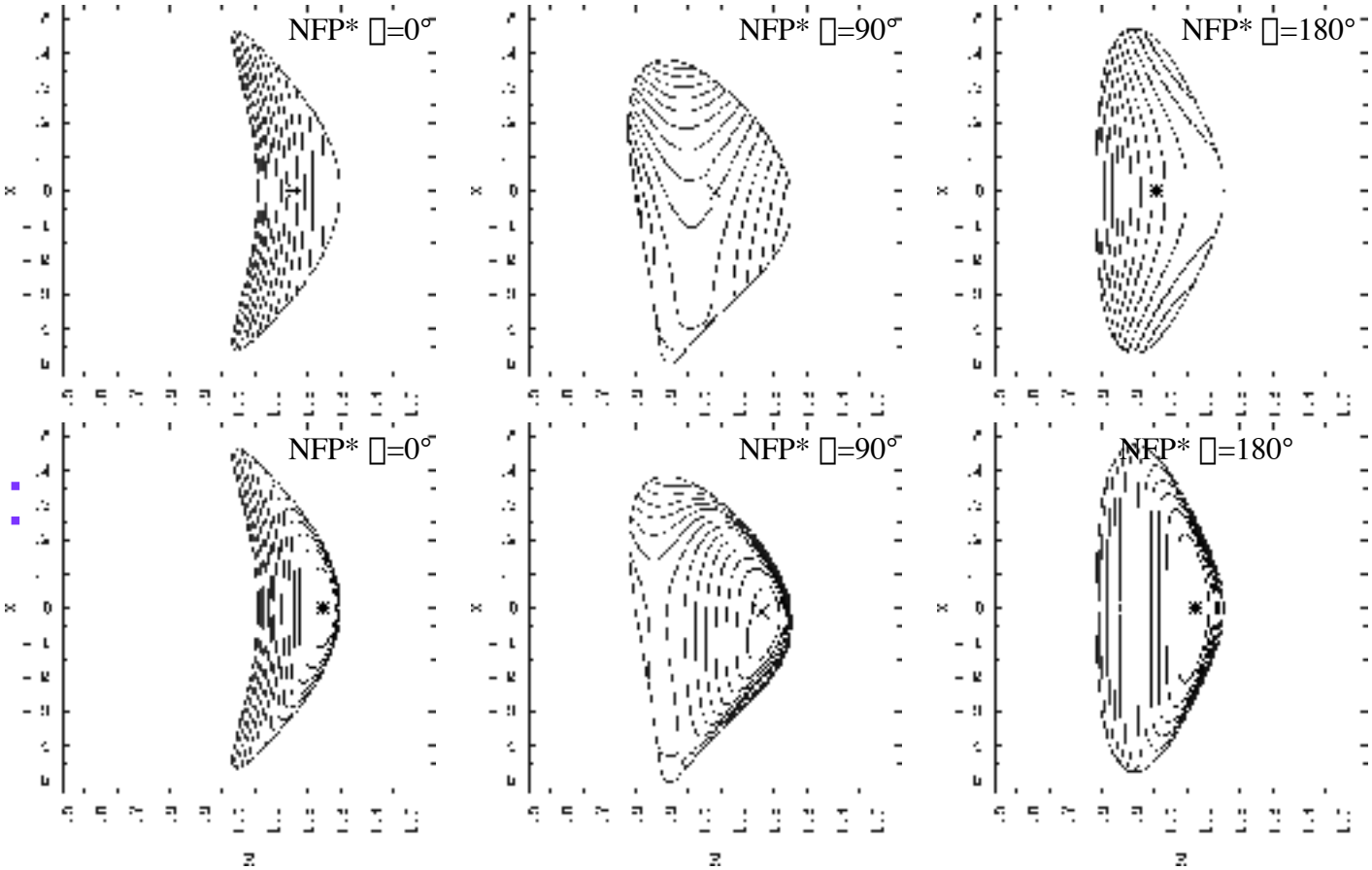


High □ configuration: |B| contours and flux surfaces become aligned



□=0%:

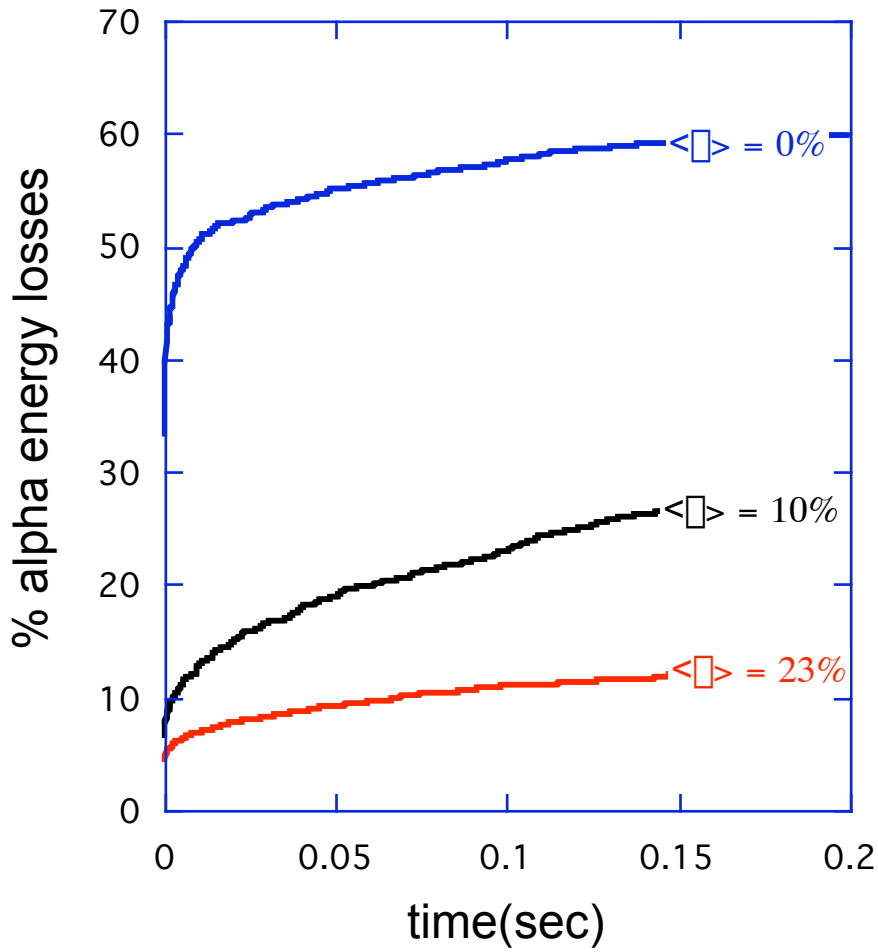
□=23%:

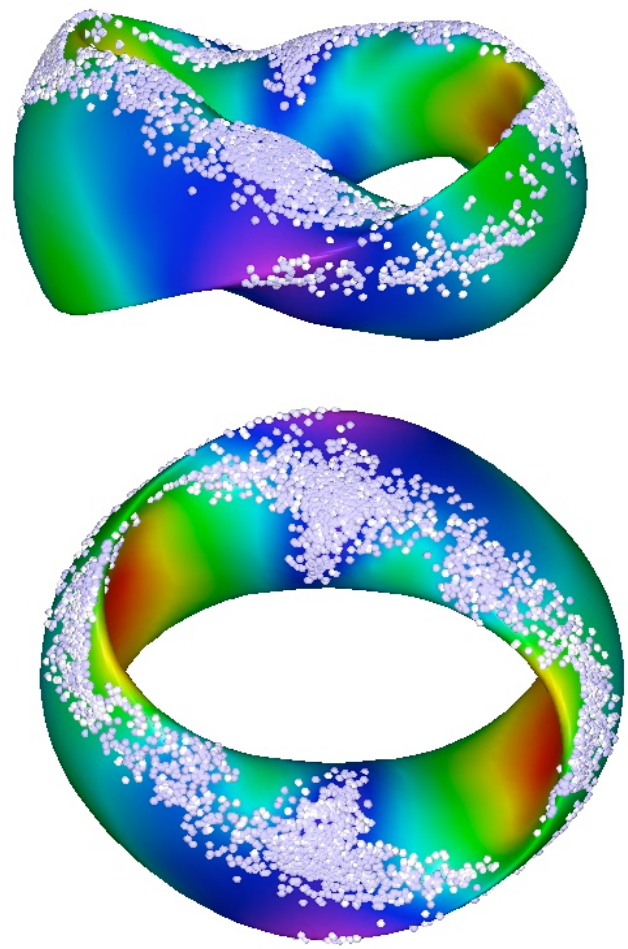


□-particle slowing-down simulations show these devices indicate very good confinement with increasing □.

Transport analysis

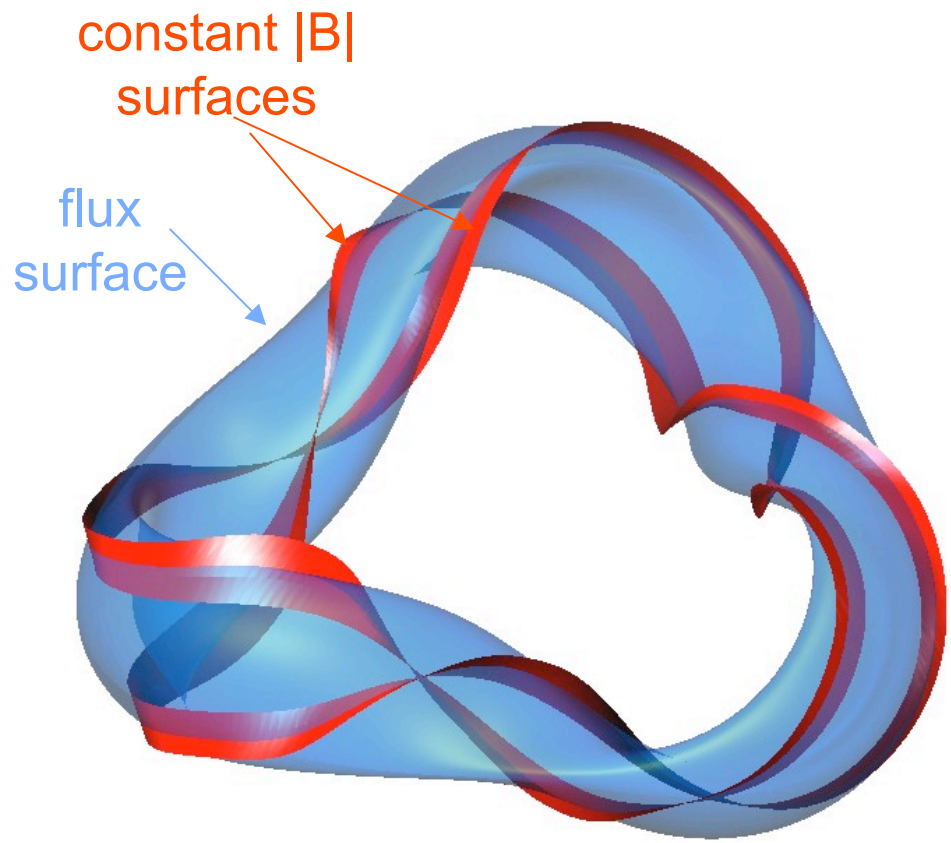
The configuration was scaled to <B> = 5T and R_0 = 10m for alpha confinement studies





ICRF heating in stellarators depends on wave propagation, geometry of the resonant regions and the orbit confinement of the resonant ions

- Confinement of ICRF heated ions is examined by following a collection of orbits
- Intersections of |B| contours with flux surfaces are determined for inner half of the plasma volume
- Ions are started out at $B = B_{res}$ with $v_{||0}/v = 0$ (equivalent to $\mu/\mu = B_{res}$)
- Ions leaving the outer surface are removed from the population



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Local Monte-Carlo equivalent quasilinear ICRF operator (developed by J. Carlsson)

$$E^{+} = E^{\square} + \square^{E} + \square\sqrt{\square^{EE}} \qquad \square^{+} = \square^{\square} + \square^{\square} + \square\sqrt{\square^{\square\square}}$$

$\square$ = a zero $\square$ mean, unit $\square$ variance random number (i.e., $\square^{\square} = 0$ and $\square^{\square} = 1$)

$$\square^{EE} = 2\, m^2 v_{\square}^2 \square v_0 \qquad \square^{\square\square} = 2\,\frac{\square k_{\parallel}}{\square\square}\,\square\,\frac{v_{\parallel}\square^2}{v^2\square}\,\frac{v_{\square}^3\square v_0}{v^2}$$

$$\square^{E} = 2\,\frac{\square}{\square}1\,\square\,\frac{k_{\parallel}v_{\parallel}\square}{\square\square}\,m v_{\square}\square v_0 \qquad \square^{\square} = \frac{\square}{\square}2\,\frac{\square}{\square}1\,\square\,\frac{k_{\parallel}v_{\parallel}\square}{\square\square}\,\square\,\frac{v_{\square}^2\square}{v^2\square}\,\frac{\square k_{\parallel}}{\square\square}\,\square\,\frac{v_{\parallel}\square}{v^2\square} + \frac{v_{\parallel}}{v^2}\,\frac{v_{\square}^2\square}{v^2\square}\,\frac{v_{\square}\square v_0}{v}$$

where

$$\square v_0 = \frac{1}{v_{\square}}\,\frac{\square eZ}{\square 2m}\left|E_{+}J_{n\square 1}(k_{\square}\square) + E_{\square}J_{n+1}(k_{\square}\square)\right|\frac{\square^2}{\square}\,\frac{2\square}{n|\dot{\square}|}$$

as $\dot{\square}\ \square\ 0$

$$\frac{2\square}{n|\dot{\square}|}\ \square\ 2\square^2\left|\frac{2}{n\ddot{\square}}\right|^{2/3}\ \square\ Ai^2\left[\frac{\square}{\square}\right]\square\,\frac{n^2\dot{\square}^2}{4}\left|\frac{2}{n\ddot{\square}}\right|^{4/3}\left[\frac{\square}{\square}\right]$$

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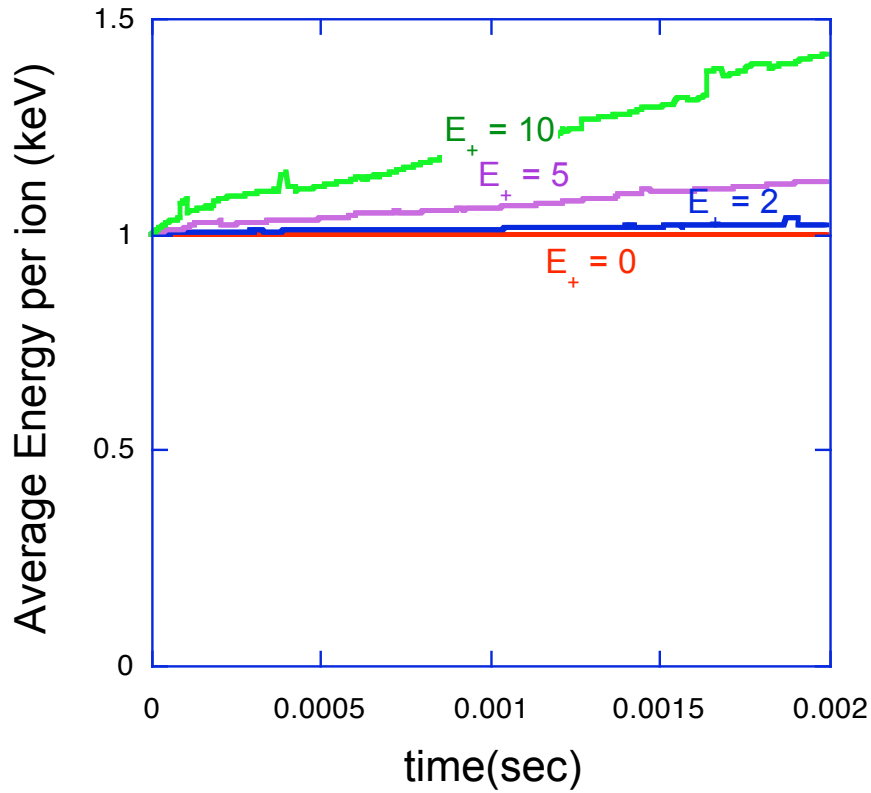
Monte Carlo simulation of ICRF heating in stellarators

- **Quasi-linear ICRF diffusion operator (follows particles as they are kicked up in energy)**
  - Intersections of |B| contours with flux surfaces determined
  - Ions are started out at $B = B_{res}$ with $v_{||0}/v = 0$ (equivalent to $\Omega/\omega = B_{res}$)
- Simple RF wave-field model:

  $k_{||} = n/2\pi R_0$, $n = 1$, $k_{\perp} = 0$

  $E_+ = E_+^0 \exp[-(r - 0.3)^2]$, $E_- = 0$
- Eventual goal is to incorporate particle simulation model with ORNL RF Full Wave codes



δf Bootstrap Current Calculation

[based on method of A. Boozer and M. Sasinowski, Phys. Plasmas **2** (1995) 610]

Substituting $f = f_M e^{\delta}$ into $\frac{df}{dt} = C(f)$ gives:

$$\frac{d\delta}{dt} - C_l(f) = -\frac{1}{f_M}\frac{\partial f_M}{\partial \delta}\frac{\partial \delta}{\partial t}$$

where C_l = linearized collision operator $= \frac{C(f)}{f}$

Now, define $\delta = -\frac{\delta}{\partial \ln f_M / \partial \delta}$ = particle deviation from initial δ surface

Expand f, assuming δ is small: $f = f_M \exp\left[-\frac{\partial \ln f_M}{\partial \delta}\delta\right] \approx f_M - \delta\frac{\partial f_M}{\partial \delta}$

[This allows one to take into explicitly account the cancellation of bootstrap current which occurs for f_M]

The plasma current is then given by:

$$j_{\parallel,mn} = q \int v_{\parallel} f \, e^{-2\pi i (n\theta - m\zeta)} d^3v \, d^3x$$

Which, using the above expansion for f, can be written as:

$$j_{\parallel,mn} = \frac{-q \int v_{\parallel} \frac{\sqrt{g}}{J} e^{-2\pi i (n\theta - m\zeta)} \frac{\partial f_M}{\partial \psi} 2\pi v^2 dv \, d\xi \, d^3x}{\int \frac{d^3x}{J}}$$

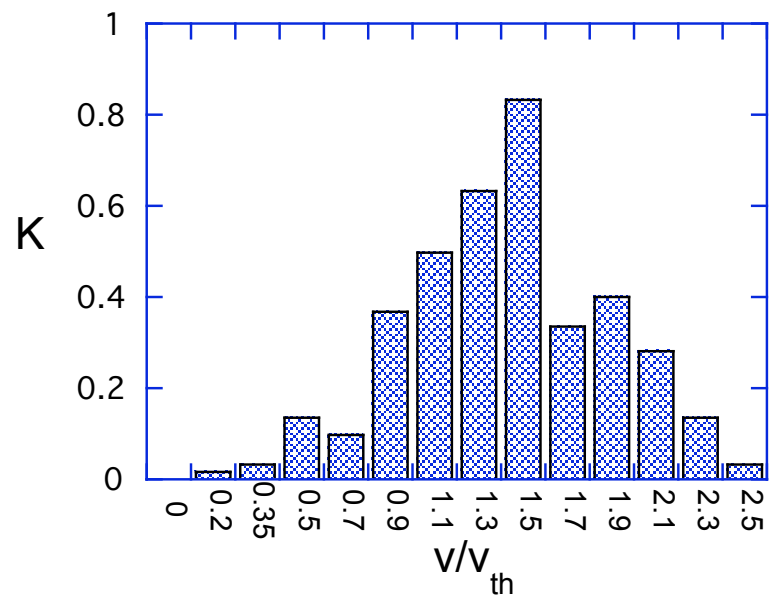
where $J = \mu_0 (G + \iota I) / B^2$

Specializing to the bootstrap current ($m = 0$, $n = 0$), the above integral can be reduced to a sum over particles, followed by an integration over velocity:

$$K(\psi, v, t) = \frac{\sum_{i=1}^{N} v_{\parallel,i} \frac{\psi_i}{J_i}}{2\sum_{i=1}^{N} \frac{1}{J_i}}$$

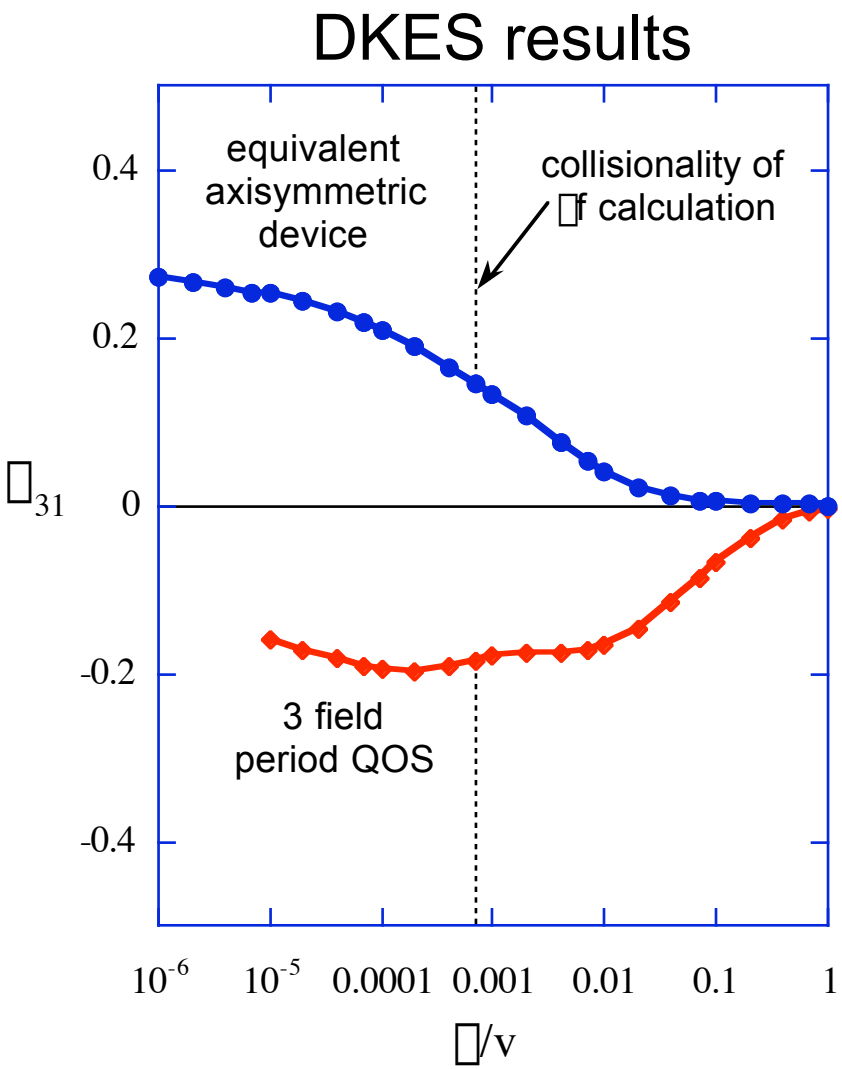
$K(\psi, v, t)$ is then sorted into velocity bins, all the processors communicate these results and a sum over processors is made. Finally, the bootstrap current is obtained by integrating over these bins:

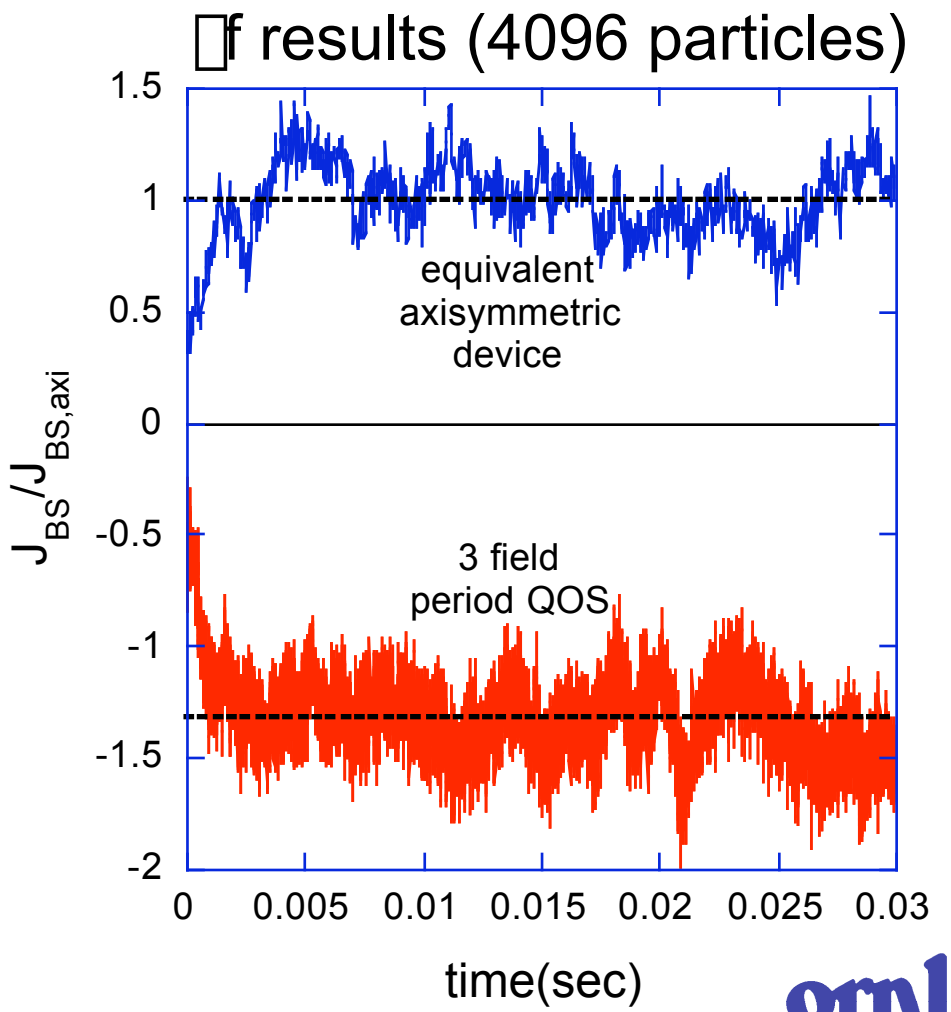
$$j_{bootstrap} = -4\pi\int v^2 dv K(\psi, v, t)\frac{\partial f_M}{\partial \psi}$$



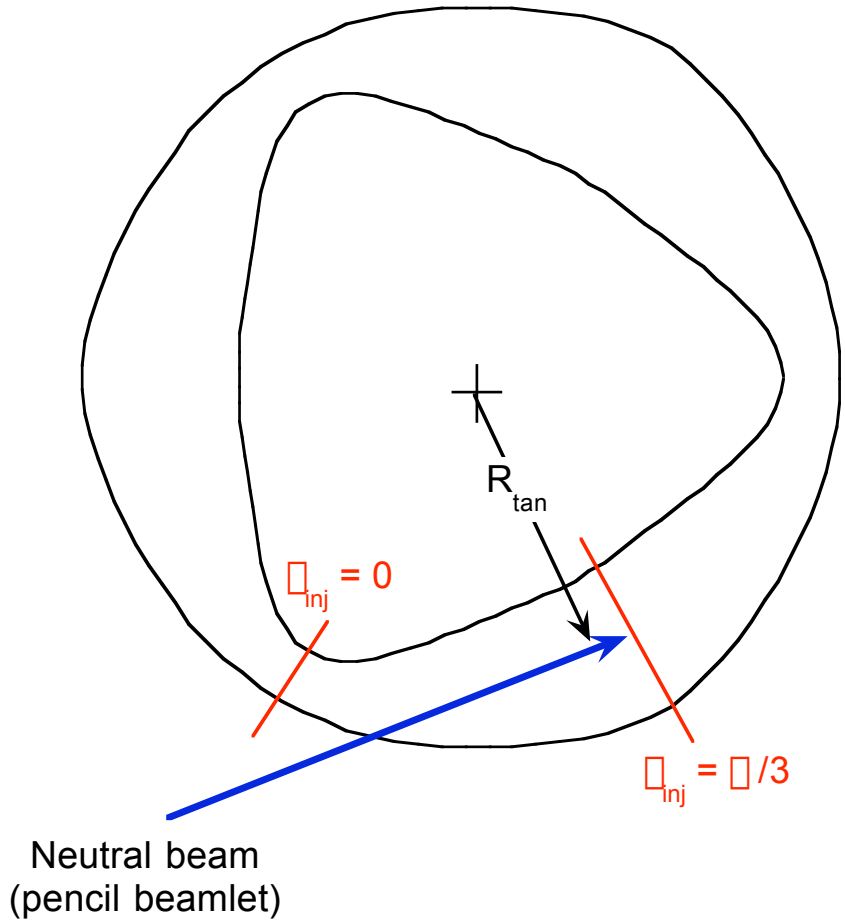
Bootstrap current levels for a QO device and its axisymmetric equivalent show agreement between DKES and the δf particle code:

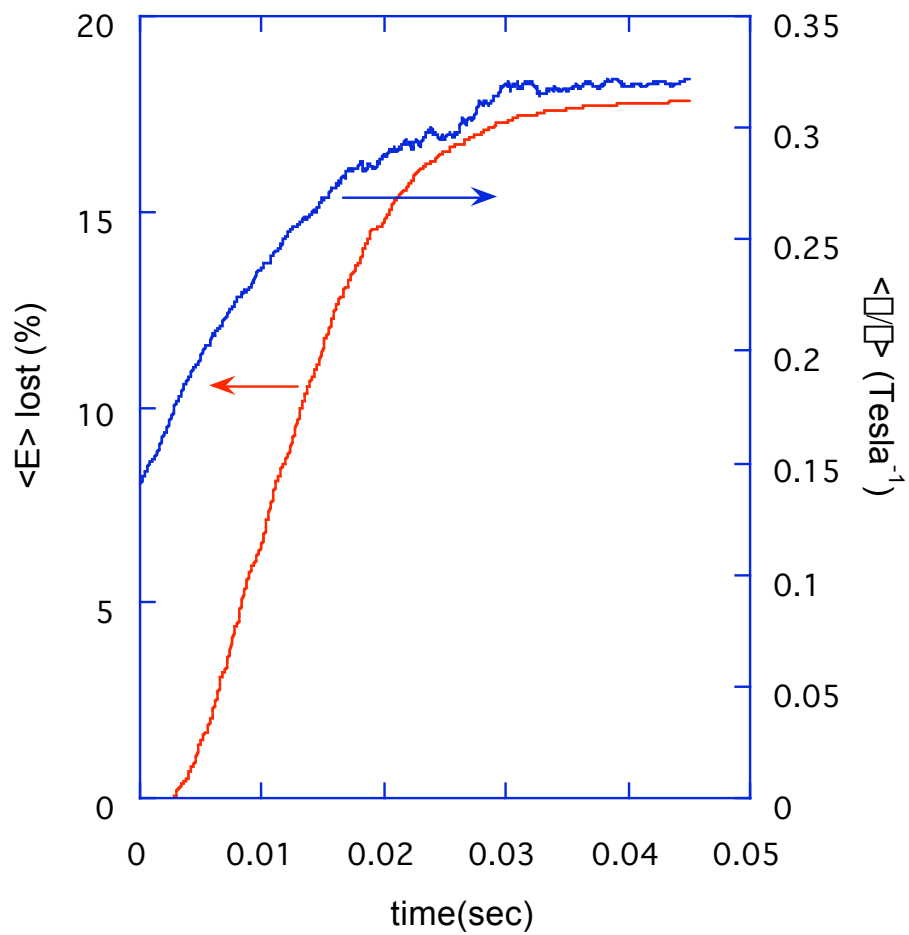
($J_{BS}^{non\text{-}axi}/J_{BS}^{axi}$ = -1.309 from DKES and = -1.32 from δf)



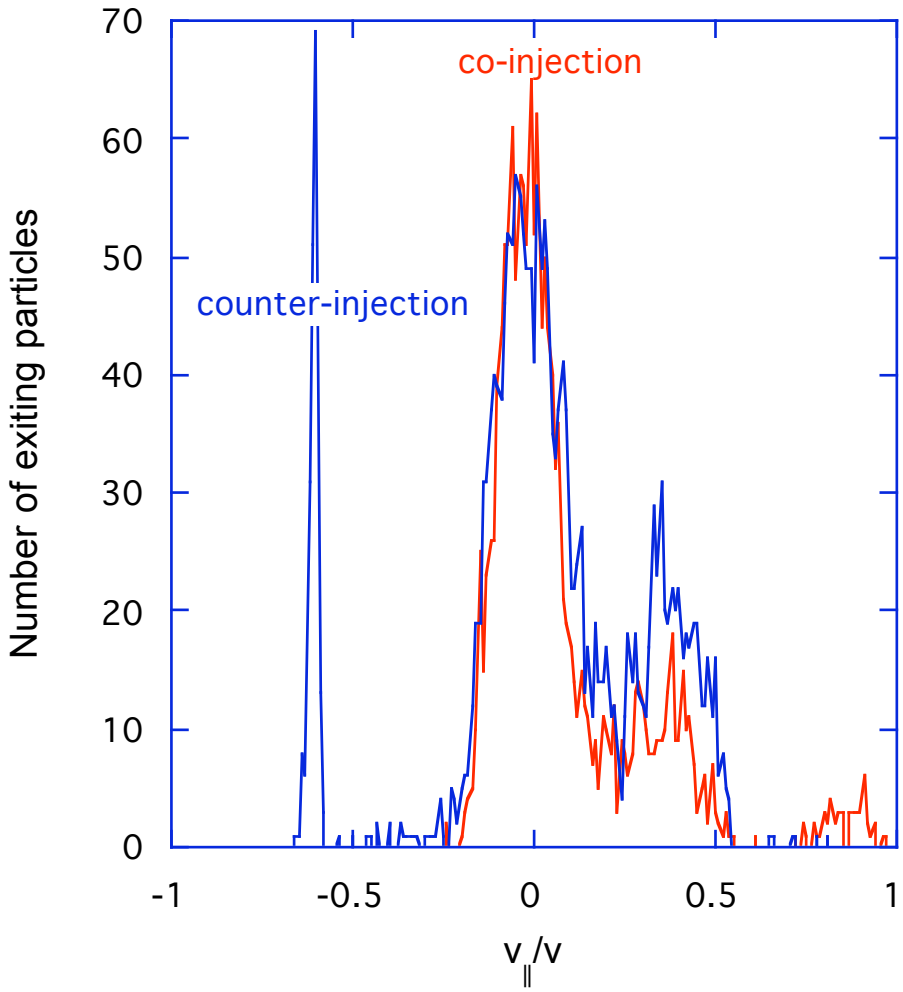


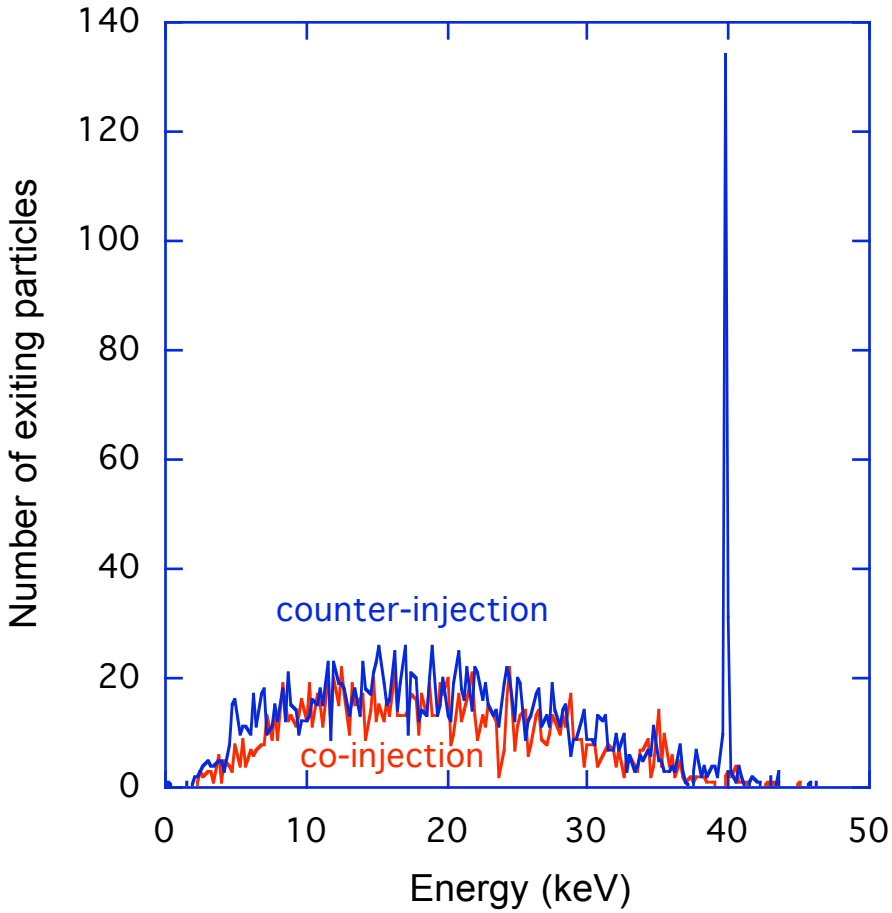
Neutral beam heating efficiency calculations: NCSX



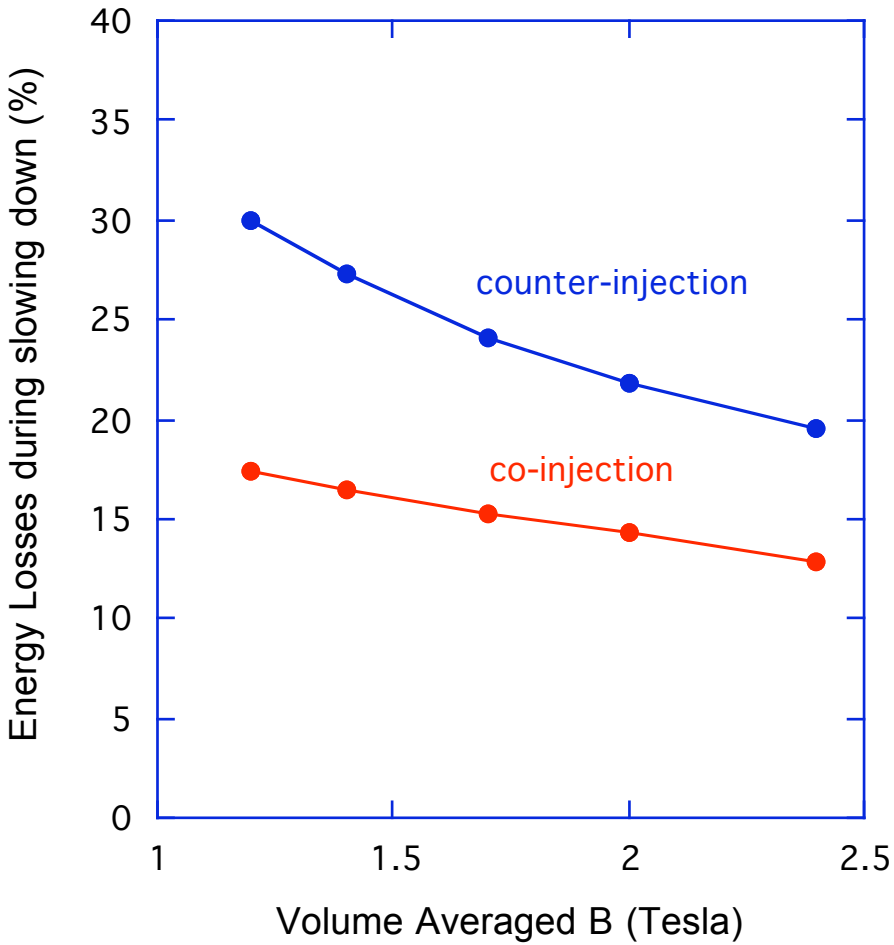


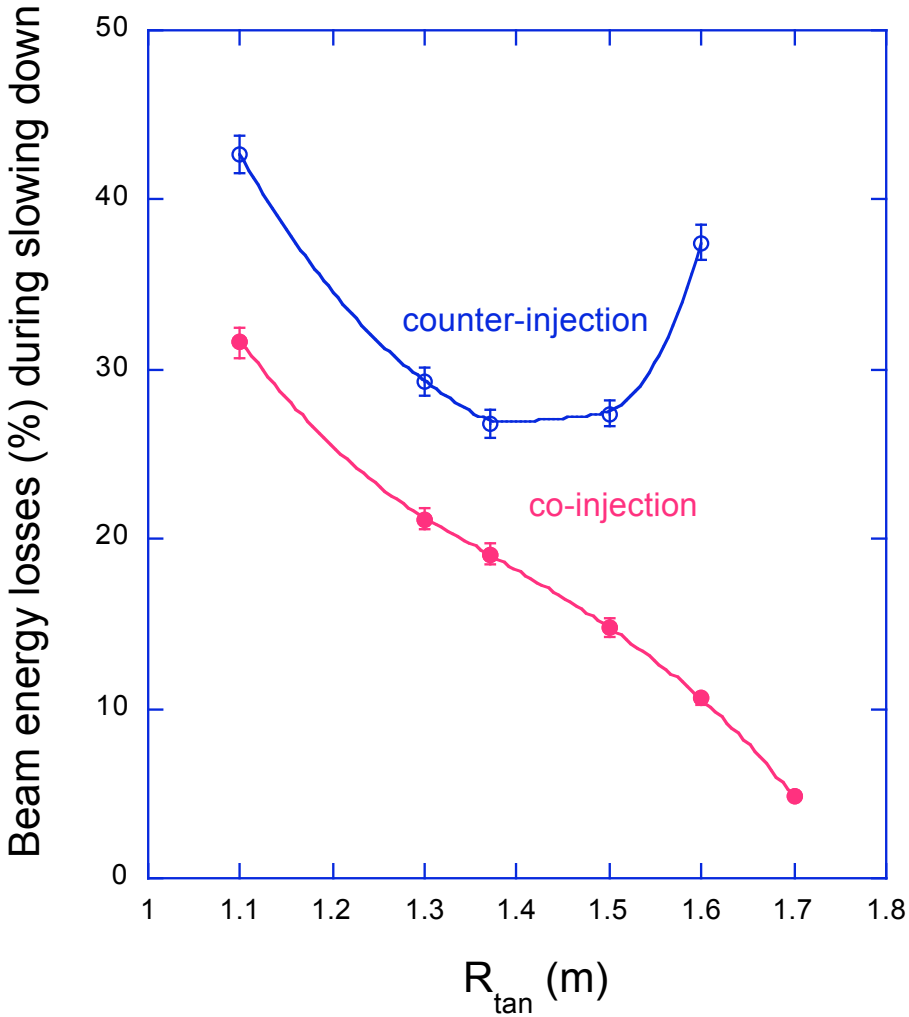
Neutral beam heating efficiency calculations: exit energies and pitch angles



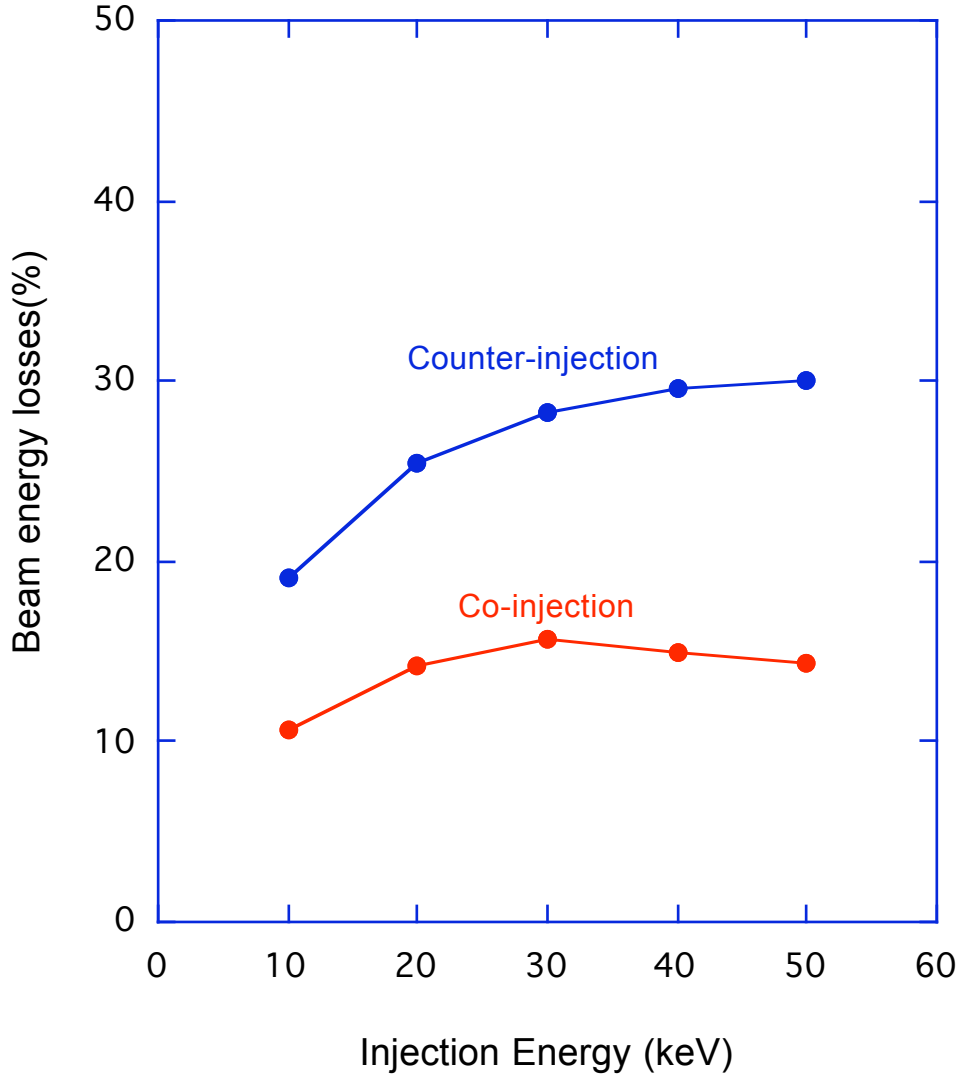


Neutral beam heating efficiency calculations: B and R_{tan} scaling

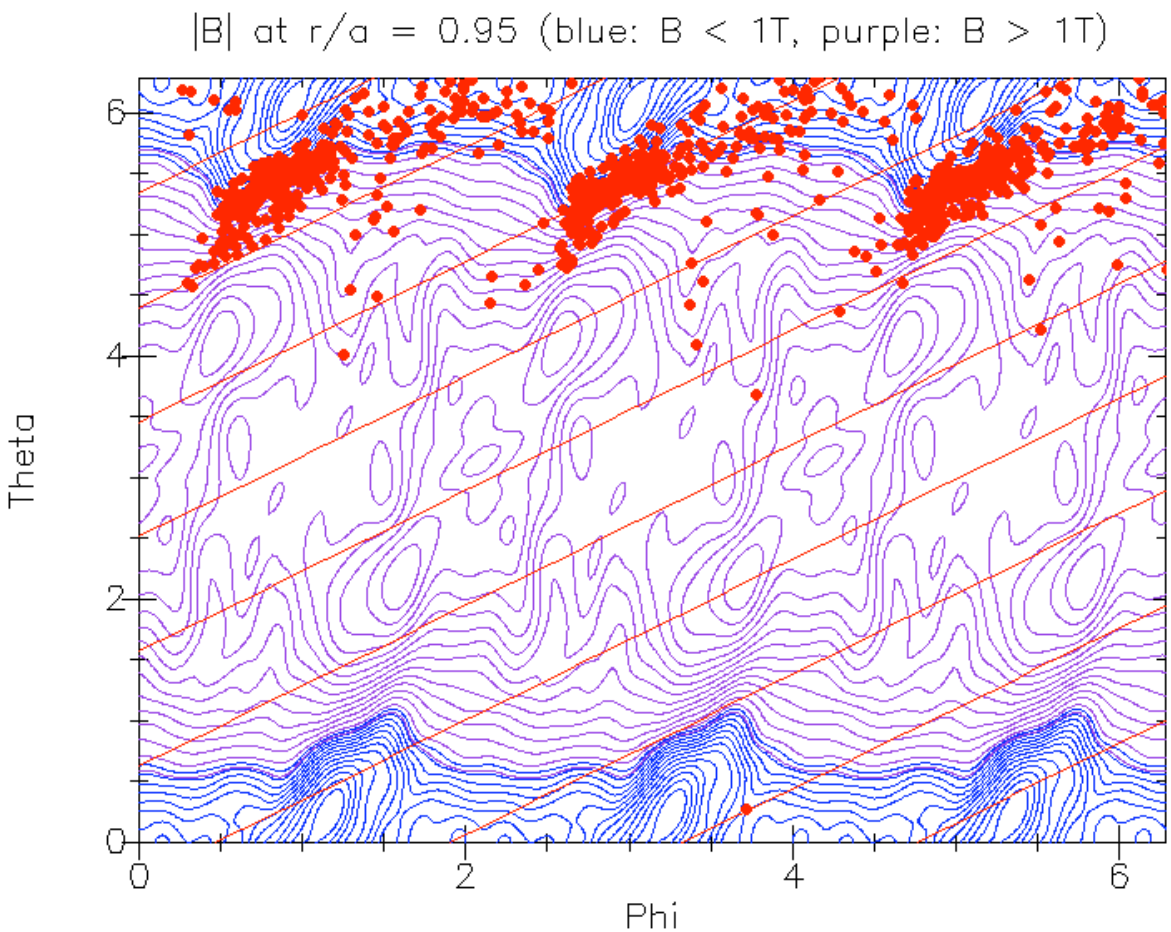


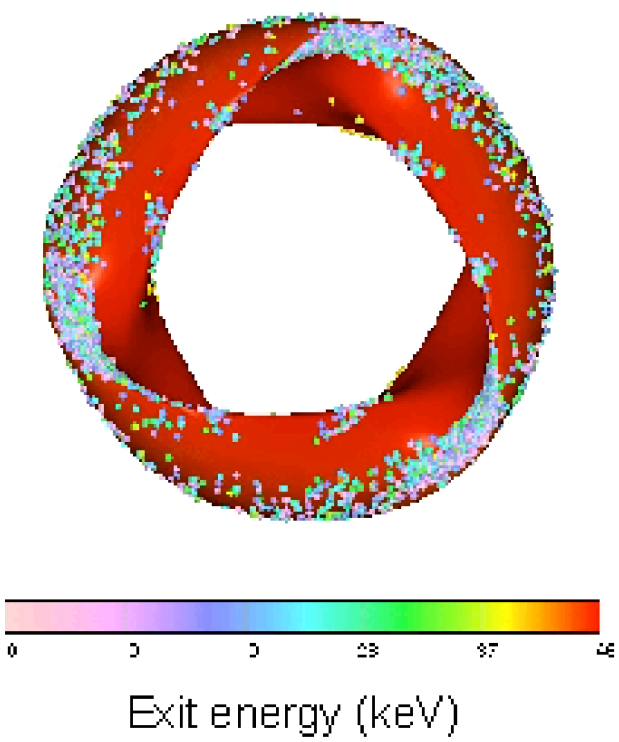


Neutral beam heating efficiency calculations: energy scaling

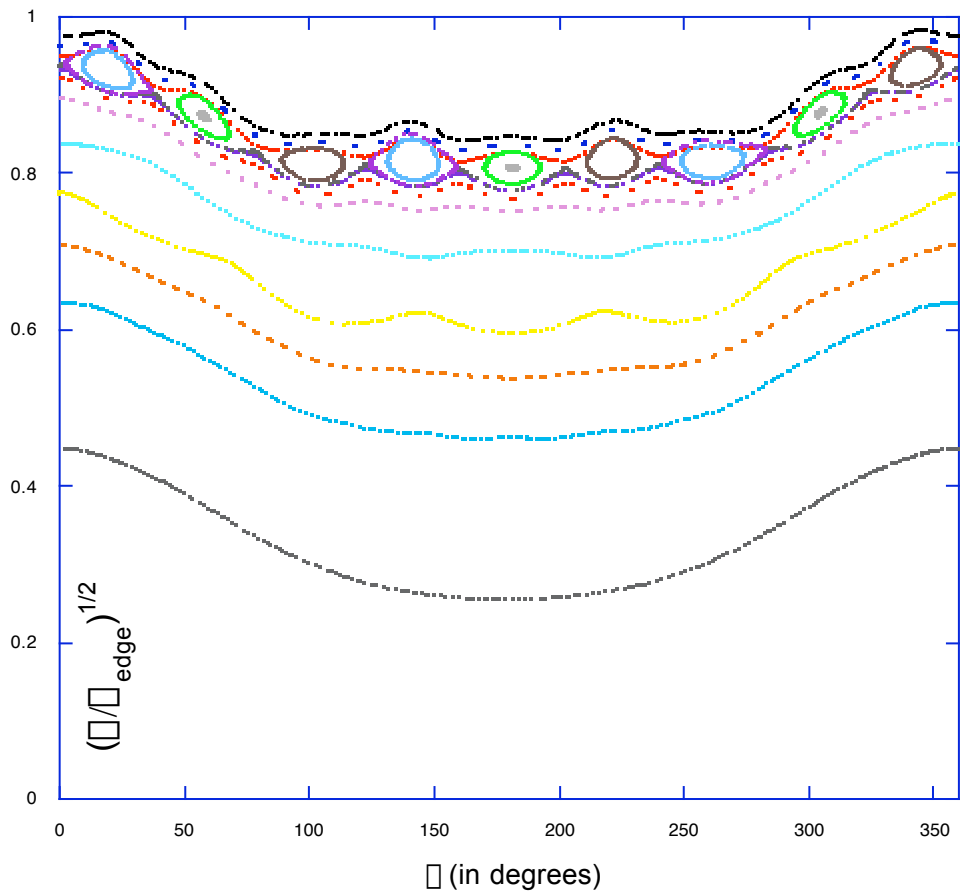


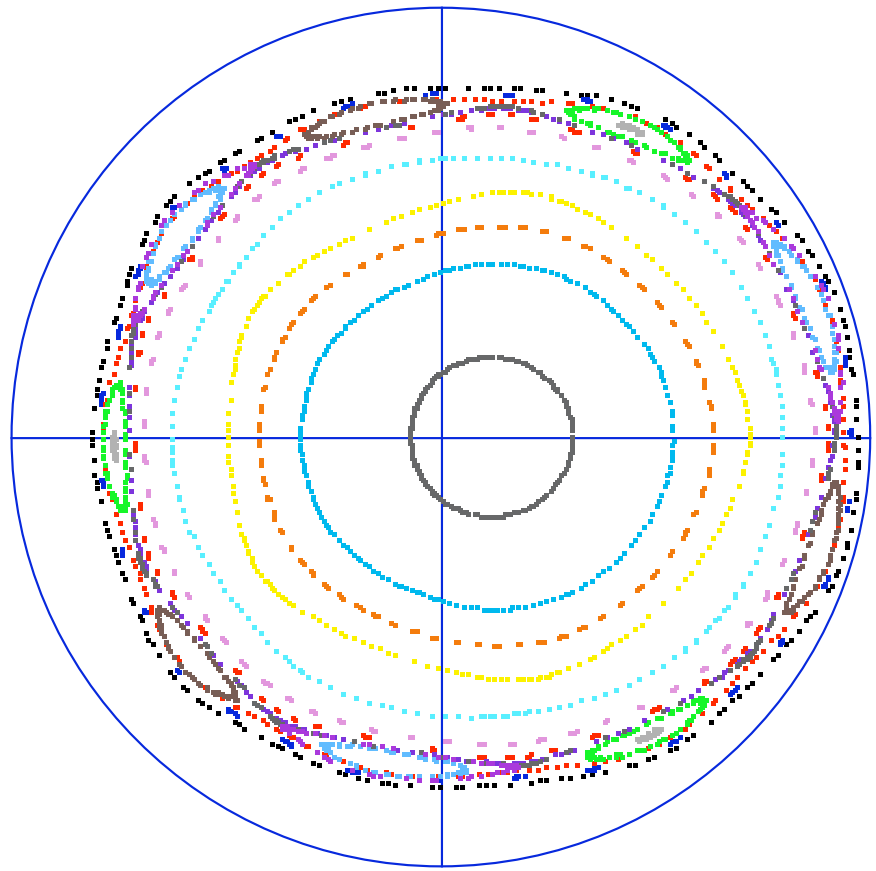
Neutral beam heating efficiency calculations: loss locations





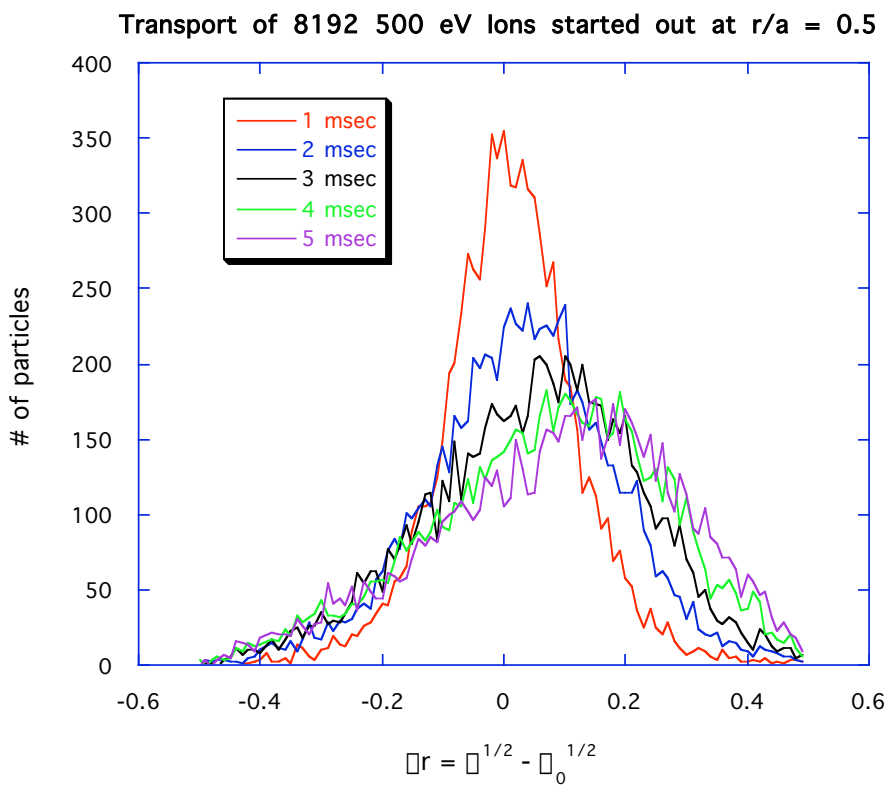
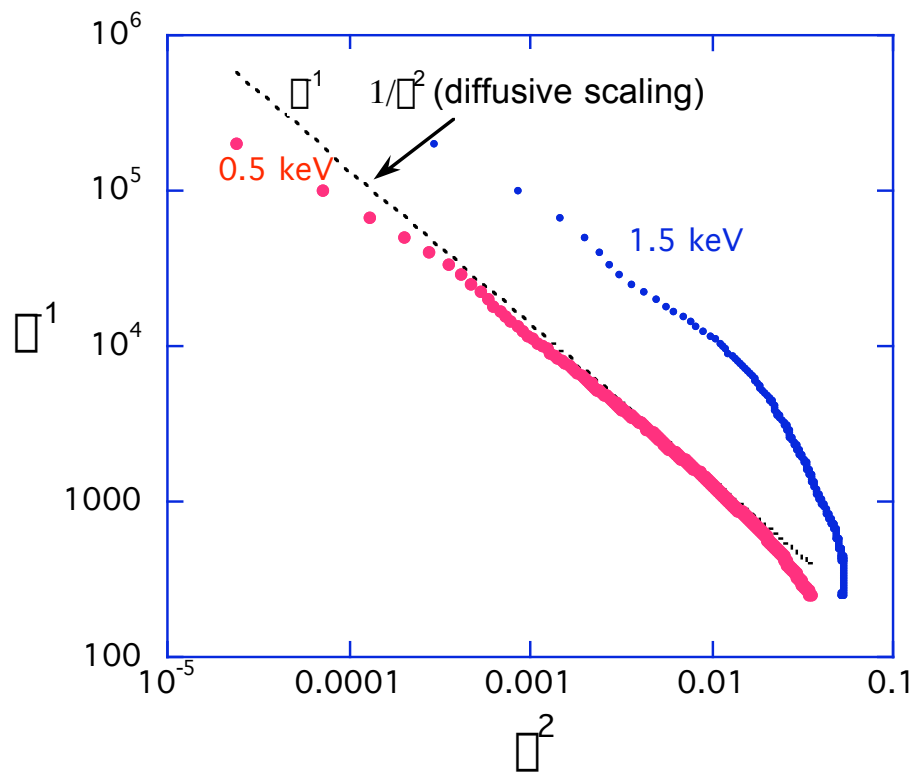
Drift island structures near edge of QA configuration for energetic (40 keV) passing ions





Conclusions and Future Directions

- QPS has attained low levels of neoclassical transport using a number of confinement based targets:
  - NEO ϵ_{eff}, QP-symmetry, DKES plateau regime coefficients
- Flexibility is attained using 7 independently variable coil currents
  - Factor of 40-50 variation in NEO ϵ_{eff}
  - Factor of ~10 variation in DKES L_{11} transport coefficient
  - Factor of ~2 variation in Monte Carlo global energy lifetime
- Future efforts will focus on
  - Benchmarking of different calculations for QPS configurations
  - Incorporate neoclassical viscous damping effects into DKES (H. Sugama, S. Nishimura, Phys. Plasmas Nov., 2002)
  - Develop non-diffusive models

New approaches to transport: non–diffusive scaling

- Verification of asymptotic diffusive behavior using tracer particles can be problematical in stellarators due to:
  - Finite system size
  - Convective losses
  - Characteristic step size for some particle classes not << system size
- Other approaches: PDF of times at which particles reach a fixed displacement, fractional derivative equations, finite size Lyapunov numbers