Alfven Continuum Studies of Existing and Planned Stellarators using the STELLGAP code

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Kinetic Theory Workshop 2002

Greifswald, Germany, Oct. 21 - 23

Introduction

- Motivations for study of stellarators Alfvén instabilities
  - Seen experimentally (W7-AS, CHS, LHD)
    - A. Weller, D. A. Spong, et al., Phys. Rev. Lett. **72**, 1220 (1994); K. Toi, et al., Nucl. Fusion **40**, 149 (2000); A. Weller, et al., Phys. of Plasmas **8** 931(2001)
  - Enhanced loss of fast ions
  - Diagnostic use (MHD spectroscopy)
  - Channeling of fast ion energy to thermal ions
- Low aspect ratio configurations provide a new environment for Alfvén studies
  - Stronger equilibrium mode couplings
  - Lower number of field periods lead to
    - More closely coupled toroidal modes (n_0, $\pm n_0 \pm N_{fp}$, etc.)
    - This leads to HAE (Helical Alfvén) couplings at lower frequencies

Introduction: Stellarator Alfvén Couplings

$$\text{Alfvén coupling condition:} \quad k_{\parallel,m,n} = -k_{\parallel,(m+\mu),(n+\nu N_{fp})}$$

$$\mu,\ \nu = \text{integers}$$

$$n - m\iota = -\left(n + \nu N_{fp} - m\iota - \mu\iota\right)$$

$$\iota = \frac{2n + \nu N_{fp}}{2m + \mu} \qquad \omega = \frac{v_A}{R}\,\frac{\mu n - \nu m N_{fp}}{2m + \mu}$$

- GAE (global Alfvén mode): $\nu = 0,\ \mu = 0$
- TAE (toroidal Alfvén mode): $\nu = 0,\ \mu = \pm 1$
- EAE (elliptical Alfvén mode): $\nu = 0,\ \mu = \pm 2$
- NAE (noncircular Alfvén mode): $\nu = 0,\ |\mu| > 2$
- MAE (mirror Alfvén mode): $\nu = 1,\ \mu = 0$
- HAE (helical Alfvén mode): $\nu = 1,\ \mu \neq 0$

Ideal MHD Shear Alfvén Equations

$$p_1 + \vec{\xi} \cdot \vec{\nabla} P + \gamma_s P \vec{\nabla} \cdot \vec{\xi} = 0$$

$$\rho \omega^2 \vec{\xi} = \vec{\nabla} p_1 + \vec{b} \times \left(\vec{\nabla} \times \vec{B} \right) + \vec{B} \times \left(\vec{\nabla} \times \vec{b} \right) \quad \boxed{+ \ \vec{\nabla} \cdot \overleftrightarrow{\mathbf{P}}_{hot}}$$

Fast particle drive

with $\vec{\xi}$ = *perturbed plasma displacement*

p_1 = *perturbed pressure*

$\vec{b} = \vec{\nabla} \times \left(\vec{\xi} \times \vec{B} \right)$ = *perturbed* $\vec{B}$ *field*

[e.g., see I. Bernstein, et al., Proc. Royal Soc. A244, 17(1958) with $\omega_0 = 0$]

For now we take $\gamma_s = 0$ and $\vec{\nabla} \cdot \vec{\xi} = 0$ (incompressible)

This leads to a set of 3 coupled equations. A singularity condition gives the Alfvén continuum

[A. Salat, J. A. Tataronis, Phys. Plasmas 8, 1200 (2001)]

$$L^{11}V = \frac{|B|^2}{G+\bar{\iota}I}\left(I\frac{\partial W_1}{\partial\Box} \Box G\frac{\partial W_1}{\partial\Box}\right) \Box L^{21}W_2 + \frac{\left(\vec{J}\cdot\vec{B}\right)}{G+\bar{\iota}I}\left(\bar{\iota}\frac{\partial W_2}{\partial\Box} + \frac{\partial W_2}{\partial\Box}\right)$$

$$\frac{\partial W_1}{\partial\Box} = \frac{1}{|B|^2}\left(L^{22}W_2 + L^{21}V\right) + \frac{\left(\vec{J}\cdot\vec{B}\right)}{G+\bar{\iota}I}\left(\bar{\iota}\frac{\partial V}{\partial\Box} + \frac{\partial V}{\partial\Box}\right)$$

$$\frac{\partial W_2}{\partial\Box} = \Box|B|^2 W_1 + \frac{1}{G+\bar{\iota}I}\left(G\frac{\partial V}{\partial\Box} \Box I\frac{\partial V}{\partial\Box}\right)$$

where $L^{11} = \Box_0\Box\Box^2\frac{g^{\Box\Box}}{|B|^2} + \vec{B}\bullet\Box\frac{g^{\Box\Box}}{|B|^2}\vec{B}\bullet\Box$

$$L^{21} = \Box_0\Box\Box^2\frac{\left(g^{\Box\Box}\Box\bar{\iota}g^{\Box\Box}\right)}{|B|^2} + \vec{B}\bullet\Box\frac{\left(g^{\Box\Box}\Box\bar{\iota}g^{\Box\Box}\right)}{|B|^2}\vec{B}\bullet\Box$$

$$L^{21} = \Box_0\Box\Box^2\frac{\left(g^{\Box\Box}\Box 2\bar{\iota}g^{\Box\Box} + \bar{\iota}^2 g^{\Box\Box}\right)}{|B|^2} + \vec{B}\bullet\Box\frac{\left(g^{\Box\Box}\Box 2\bar{\iota}g^{\Box\Box} + \bar{\iota}^2 g^{\Box\Box}\right)}{|B|^2}\vec{B}\bullet\Box$$

$V = E_1$

$W_1 = i\Box B_3$

$W_2 = E_2$

with $r^1 = \Box$

$r^2 = \Box\Box\bar{\iota}\Box$

$r^3 = I(\Box)\Box\Box G(\Box)\Box$

$\bar{\iota}$ = *rotational transform*

G = *poloidal current*

I = *toroidal current*

Singular for Alfvén continuum condition: $L^{11}V = 0$

Stellarator Alfvén Continuum Equation

The continuum equation in general geometry (low β) can be written:

$$\mu_0 \rho \omega^2 \frac{|\nabla\psi|^2}{B^2} E_\psi + \vec{B}\cdot\nabla\left[\frac{|\nabla\psi|^2}{B^2}(\vec{B}\cdot\nabla)E_\psi\right] = 0 \qquad (1)$$

This can be written in Boozer coordinates using the following:

$$\vec{B}\cdot\nabla = \frac{1}{\sqrt{g}}\left(\iota\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\zeta}\right) \quad \text{and} \quad \left|\nabla\psi\right|^2 = g^{\rho\rho}\left(\frac{d\psi}{d\rho}\right)^2$$

For devices with stellarator symmetry, the surface displacement can be expanded as follows:

$$E_\psi = \sum_{j=1}^{L} E_\psi^j e_j \quad \text{where} \quad e_j = \cos(n_j\zeta - m_j\theta)$$

Multiplying equation (1) by the Jacobian and integrating over a flux surface then leads to:

$$\mu_0 \omega^2 \rho \left\langle e_i \sqrt{g}\,\frac{g^{\rho\rho}}{B^2}\sum_{j=1}^{L} E_\psi^j e_j \right\rangle + \left\langle e_i\left(\iota\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\zeta}\right)\left[\frac{g^{\rho\rho}}{B^2\sqrt{g}}\left(\iota\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\zeta}\right)\sum_{j=1}^{L} E_\psi^j e_j\right]\right\rangle = 0$$

Stellarator Alfvén Continuum Equation (contd.)

Integrating by parts then leads to the following symmetric matrix eigenvalue problem:

$$\omega^2 \mathbf{Ax} = \mathbf{Bx}$$

$$where \quad \mathbf{A} = \left[a_{ij}\right] = \mu_0 \rho \left\langle e_i \sqrt{g}\, \frac{g^{\psi\psi}}{B^2} e_j \right\rangle$$

$$\mathbf{B} = \left[b_{ij}\right] = \left\langle \frac{g^{\psi\psi}}{B^2\sqrt{g}} \left(\iota \frac{\partial e_i}{\partial \theta} + \frac{\partial e_i}{\partial \zeta} \right) \left(\iota \frac{\partial e_j}{\partial \theta} + \frac{\partial e_j}{\partial \zeta} \right) \right\rangle$$

$$\mathbf{x} = \left[\xi_s^1 \;\; \xi_s^2 \cdots \xi_s^L\right]^T$$

STELLGAP code

The equilibrium coefficients in the continuum equation can be expanded in cos series:

$$\frac{g^{\psi\psi}\sqrt{g}}{B^2} = \sum_{k=1}^{K} E_k \cos(n_k\zeta - m_k\theta), \qquad \frac{g^{\psi\psi}}{B^2\sqrt{g}} = \sum_{k=1}^{K} F_k \cos(n_k\zeta - m_k\theta)$$

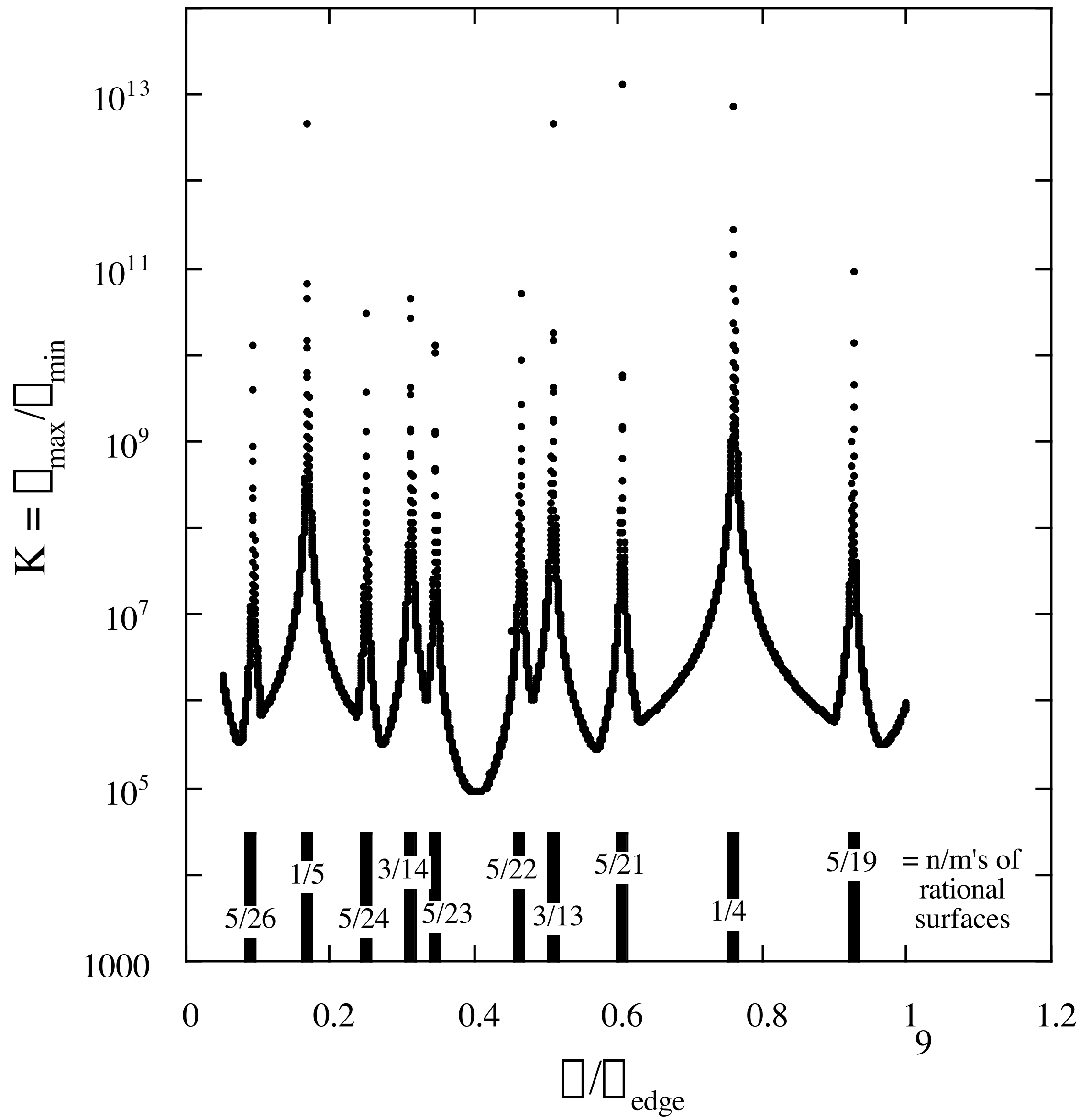
Using these expansions, the matrix elements then can be expressed in terms of the following convolution integrals:

$$I_{ccc}^{i,j,k} = \int_0^{2\pi} d\theta \int_0^{2\pi} d\zeta \cos\left(n_i\zeta - m_i\theta\right)\cos\left(n_j\zeta - m_j\theta\right)\cos\left(n_k\zeta - m_k\theta\right)$$

$$I_{ssc}^{i,j,k} = \int_0^{2\pi} d\theta \int_0^{2\pi} d\zeta \sin\left(n_i\zeta - m_i\theta\right)\sin\left(n_j\zeta - m_j\theta\right)\cos\left(n_k\zeta - m_k\theta\right)$$

Since our algorithm provides all eigenvalues of the system, the condition number is can be monitored.

- Peaks about rational surfaces because minimum eigenvalue goes to zero at $\iota = n/m$
- Since one is never precisely on a rational surface numerically, K remains finite
- Want to keep K < $10^{11} - 10^{12}$ for double precision calculations



Profiles, mode selection for Alfvén Continuum Plots

- **Profiles used:**
  - **For W7-AS discharges (40173, 42873, 43348), experimental profiles were used**
  - **For other devices: ion density $1 - \square^2$, $n_{ion}(0) = 1 \times 10^{20}\ m^{-3}$**
  - **ion density profile $(iota)^2$ aligns the gaps radially**
    - minimizes continuum damping
    - implies a hollow profile for stellarator iota profiles
- **Typical mode selections used:**

Equilibrium

$m = 0 - 19$

$n = -20N_{fp}$ to $20N_{fp}$

Eigenfunction

at least $n/\iota_{MAX} < m < n/\iota_{MIN}$

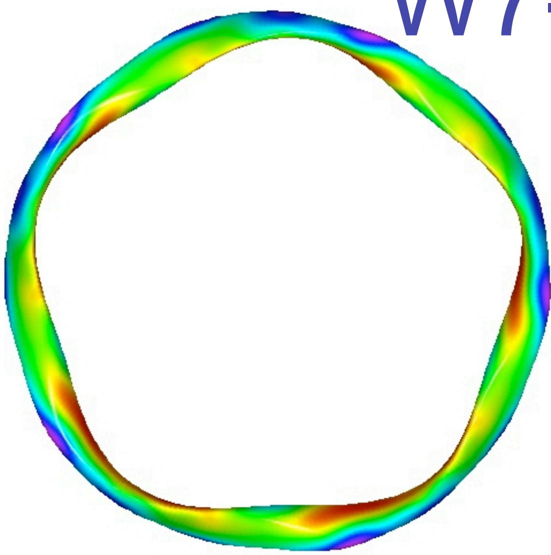
$n = \pm n_0, \pm n_0 \pm N_{fp}, \pm n_0 \pm 2N_{fp}, \pm n_0 \pm 3N_{fp}$ with n_0 = toroidal mode family

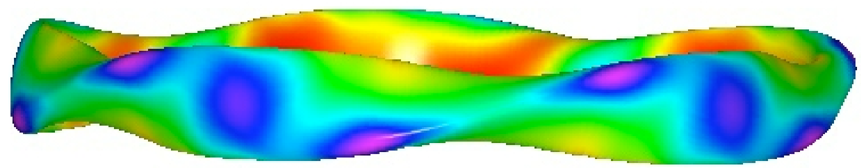
Examples:

W7-AS - $N_{fp} = 5$, $n_0 = 1$, $n = \ldots, -9, -6, -4, -1, 1, 4, 6, 9, \ldots$

QPS - $N_{fp} = 2$, $n_0 = 1$, $n = \ldots, -5, -3, -1, 1, 3, 5, \ldots$

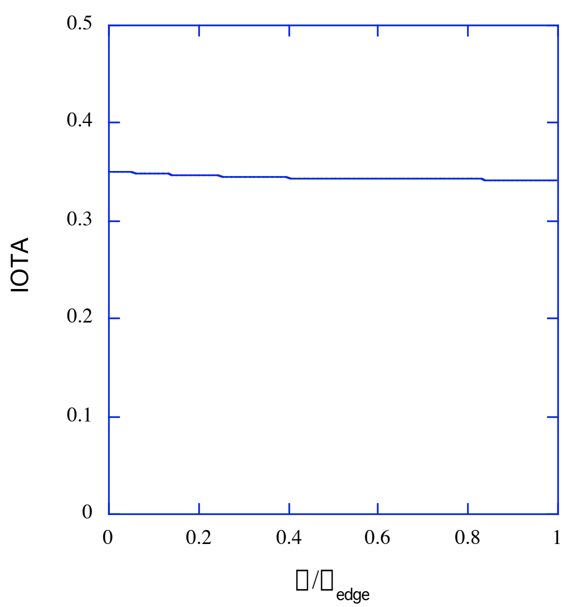
High aspect ratio drift-optimized W7-AS configuration



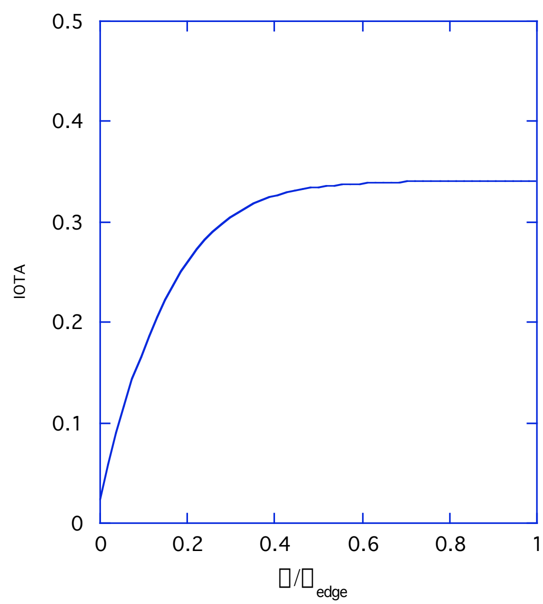


Iota profiles

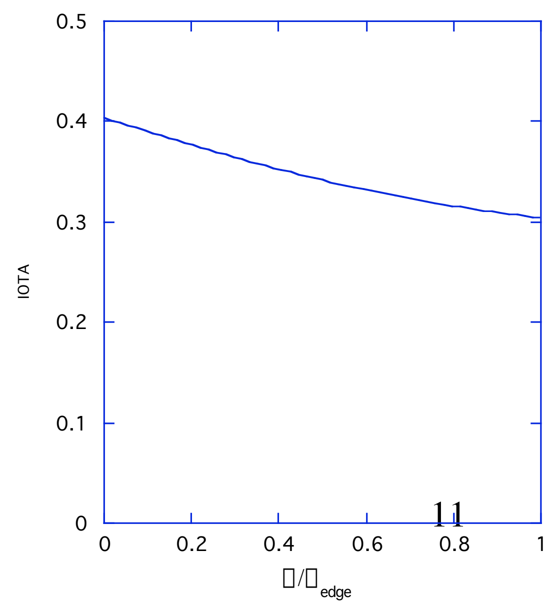
s40173



s42873



s43348

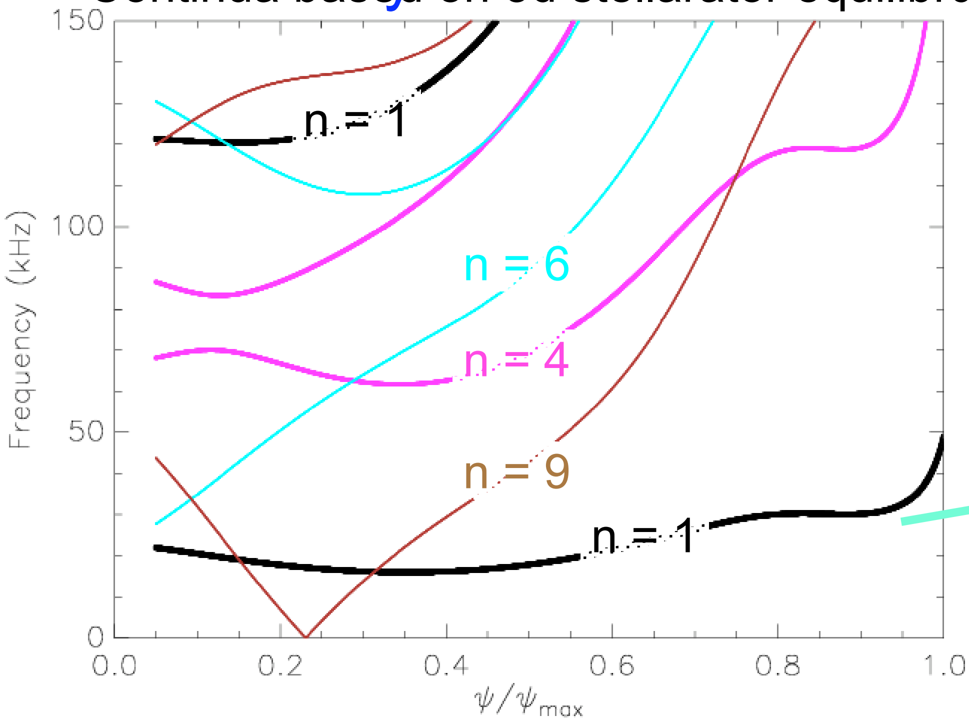


W7-AS: discharge 40173

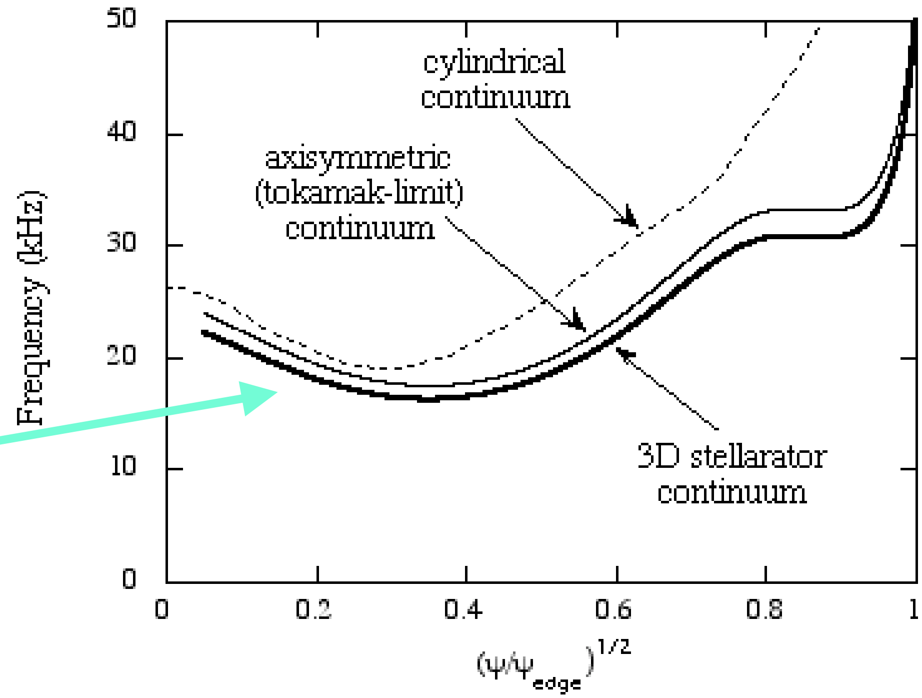
[Similar to cases analyzed in A. Weller, D. A. Spong, et al., Phys. Rev. Lett. **72** (1994) 1220]

- In experiment, fluctuations were observed at ~18 kHz
- GAE mode: below the lowest n = 1 continuum
- Such modes can be approximated by cylindrical or axisymmetric models

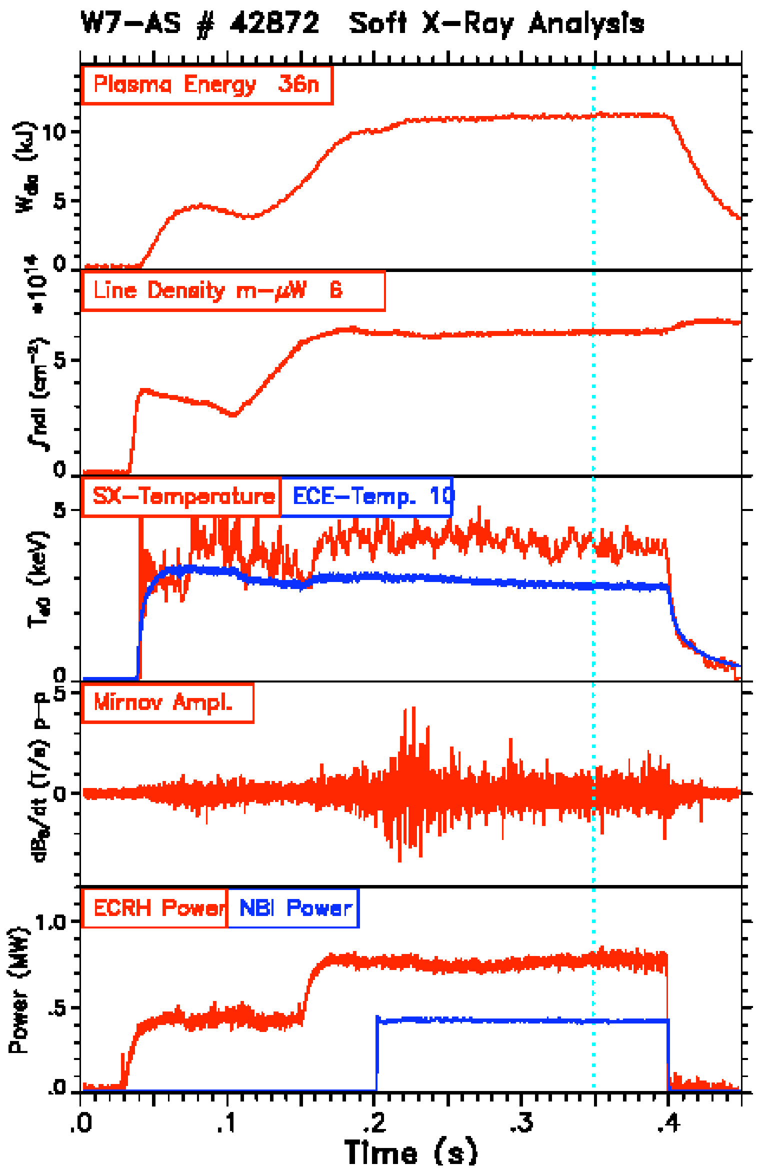
Continua based on 3d stellarator equilibruim



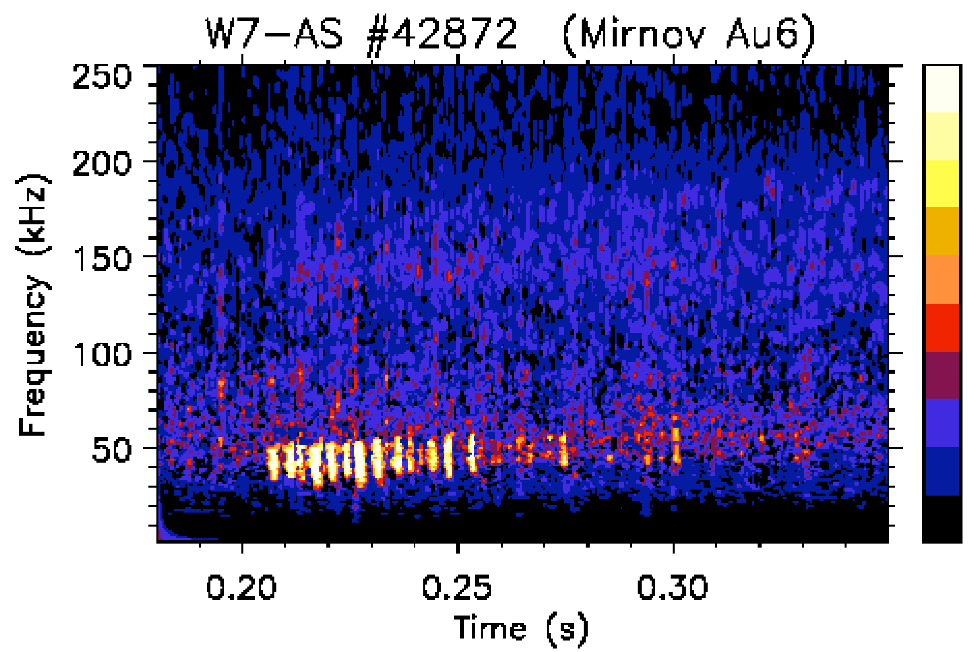
n = 1 GAE continua



W7-AS Experimental Results for #42872:



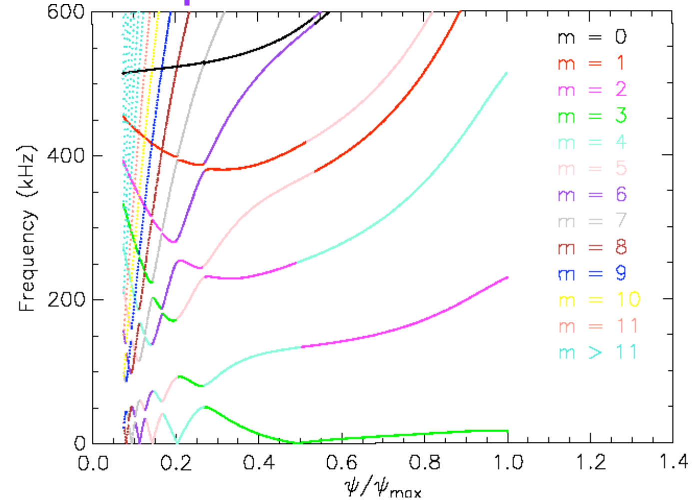
From A. Weller, et al.
Phys. of Plasmas **8** 931(2001)



W7-AS: discharge 42873

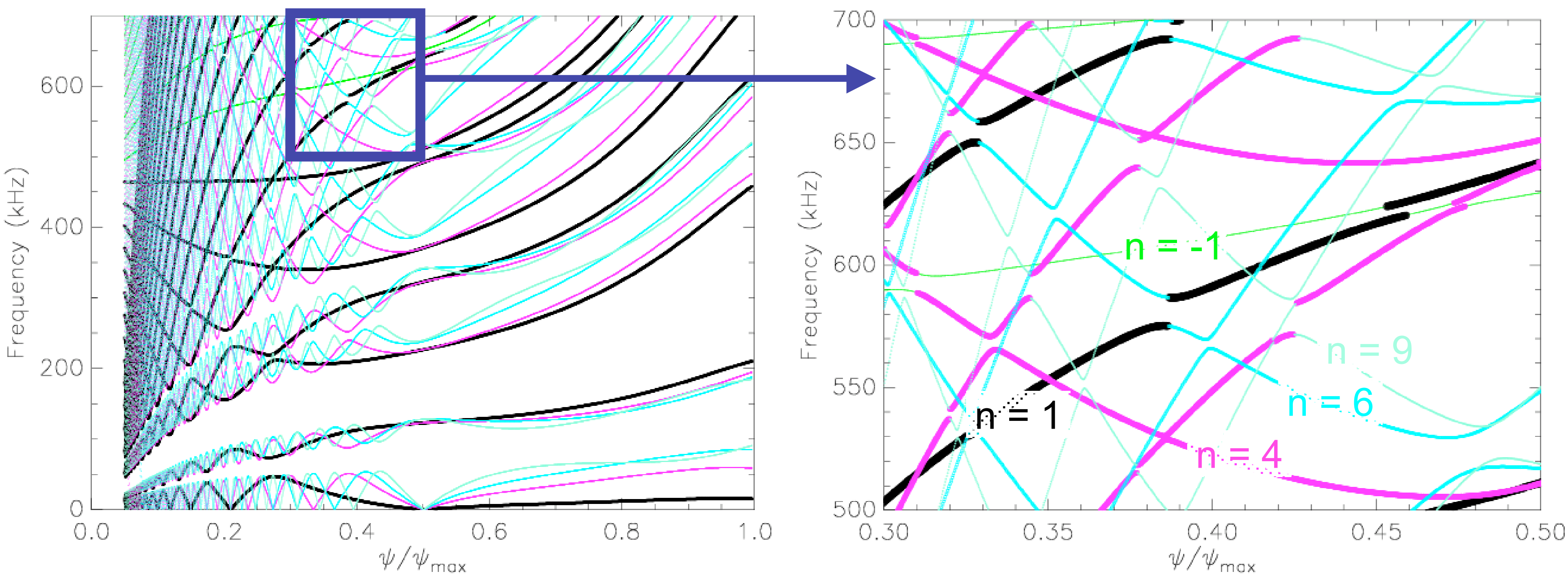
5 field periods R/<a> = 12
fluctuations observed at 30-50 kHz

Axisymmetric limit
dominant **poloidal** mode is color coded

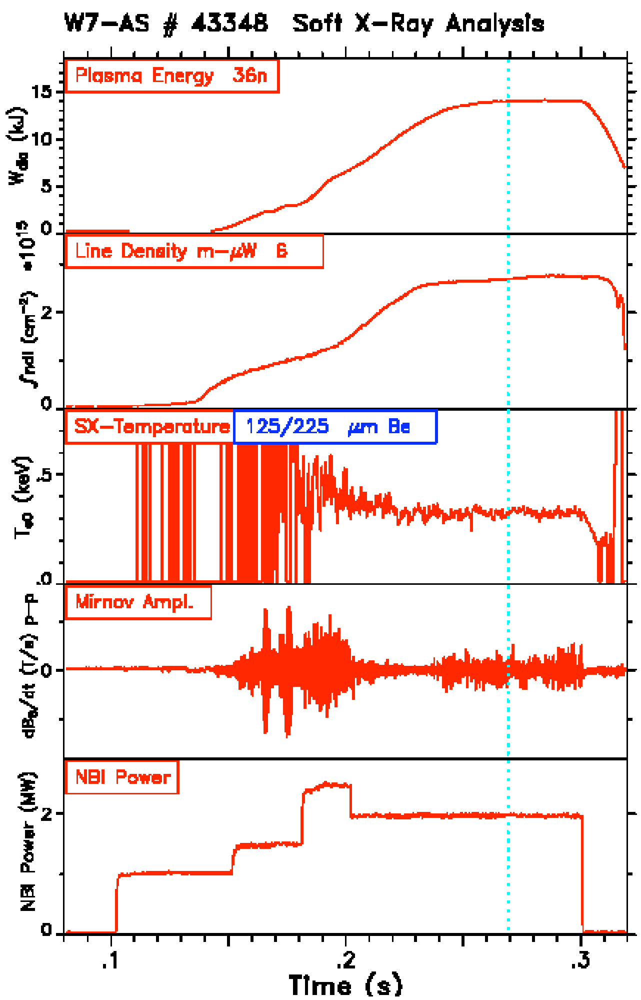


Continua with multiple toroidal modes

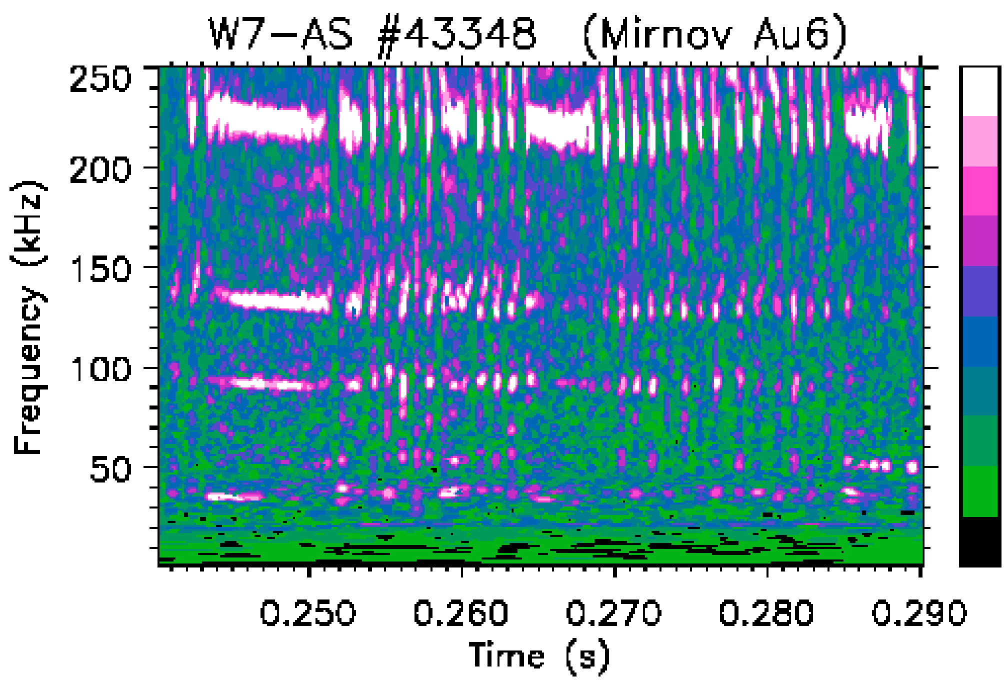
n = 1 mode family stellarator continua
dominant **toroidal** mode is color coded



W7-AS Experimental Results for #43348:



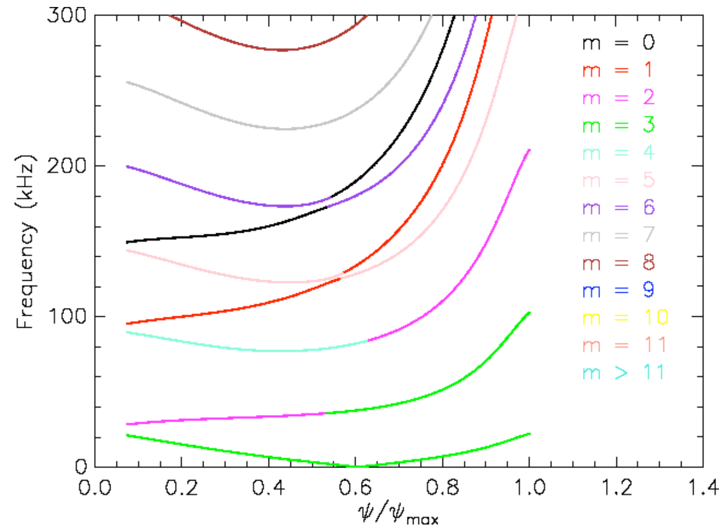
From A. Weller, et al.
Phys. of Plasmas **8** 931(2001)



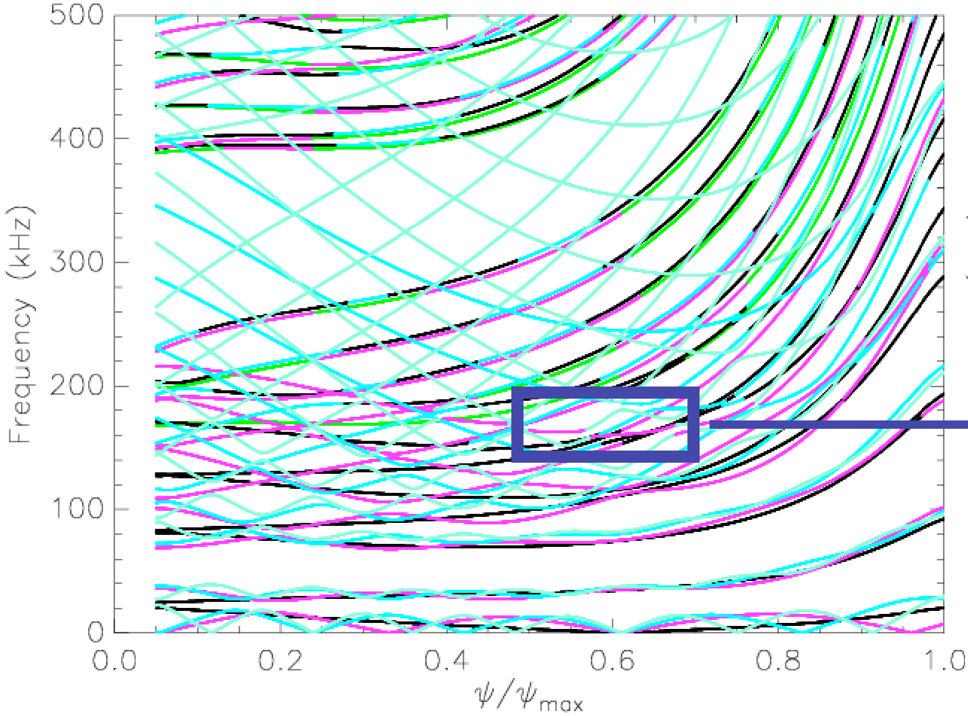
W7-AS: discharge 43348

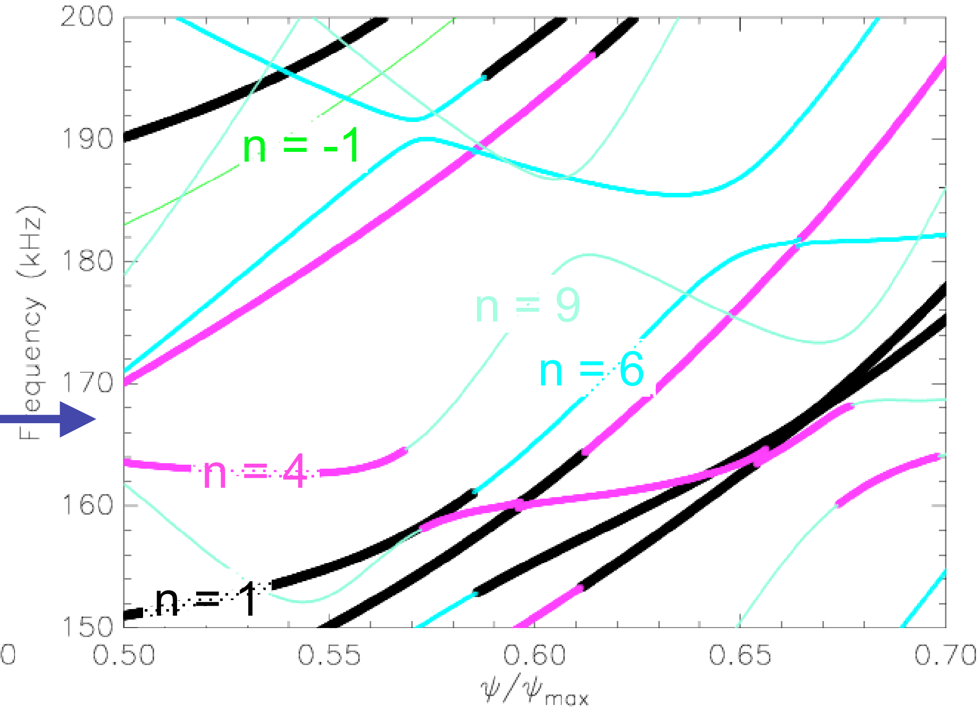
fluctuations observed at: 30-40, 50-60, 85-100, 125-150, 180-200, and 210-240 kHz

Axisymmetric limit
dominant **poloidal** mode is color coded

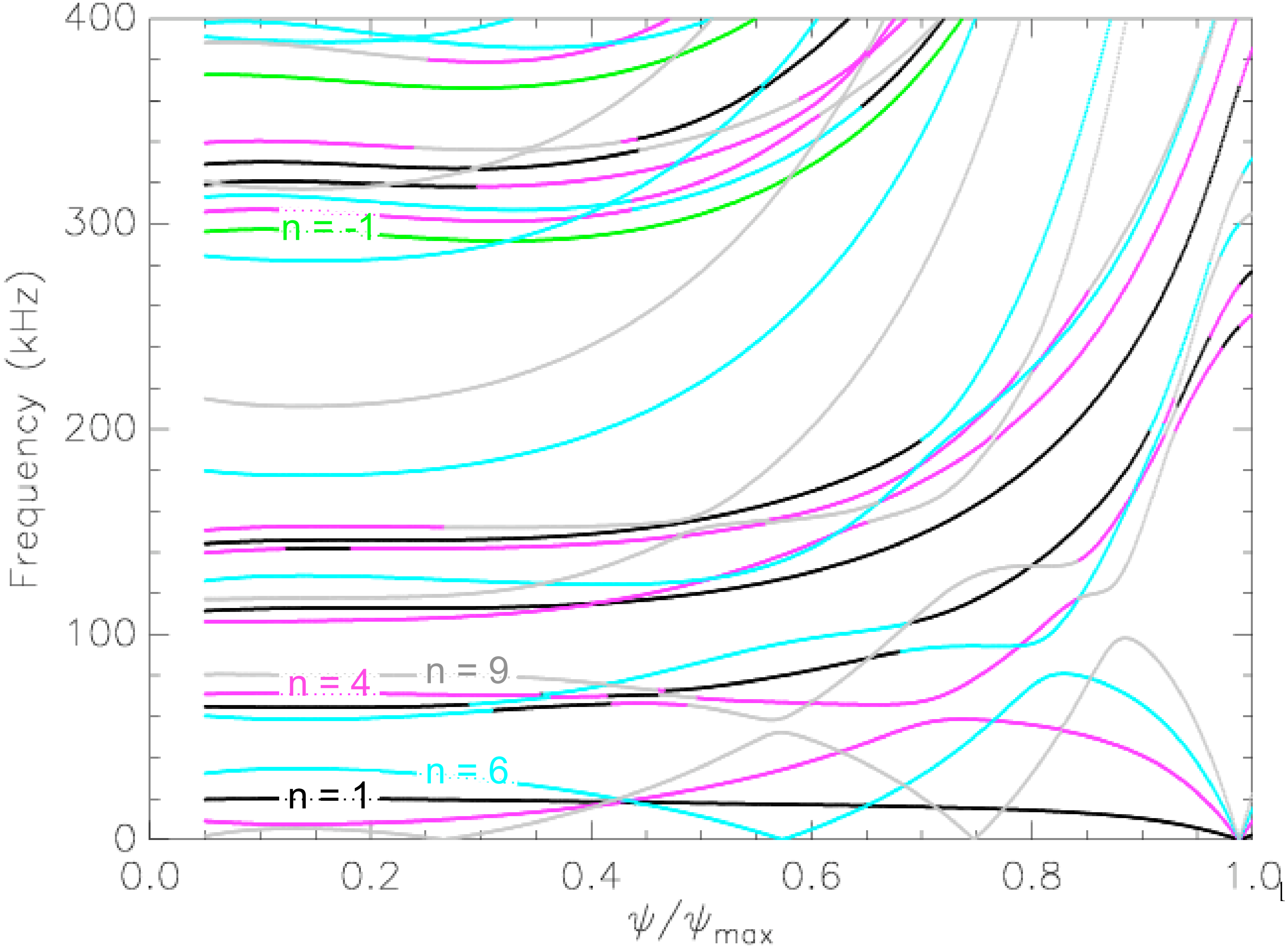


Continua with multiple toroidal modes
n = 1 mode family stellarator continua
dominant **toroidal** mode is color coded

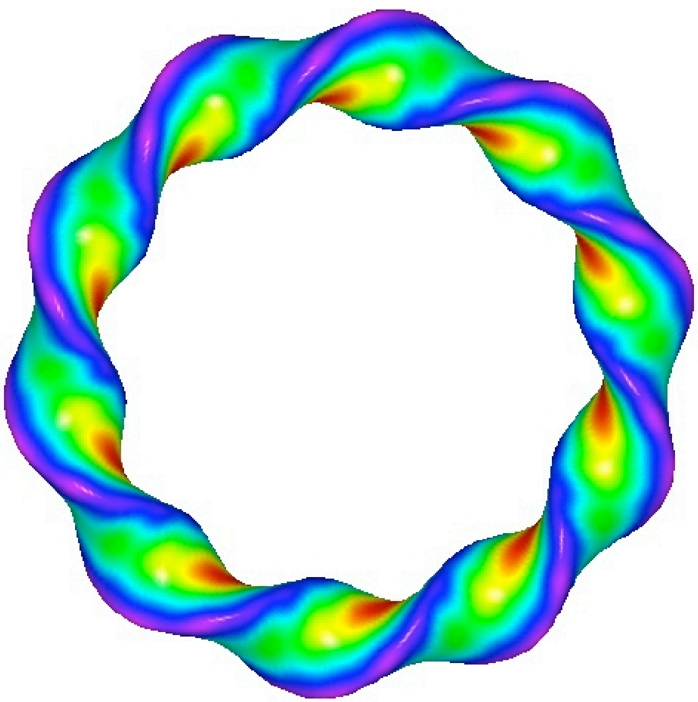


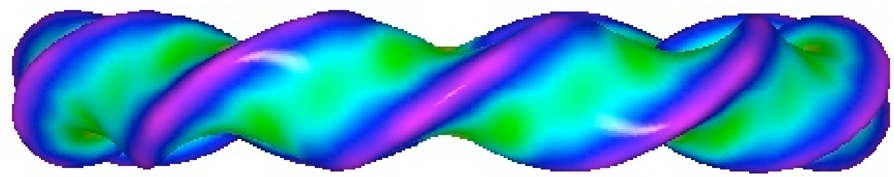


W7-X continuum structure

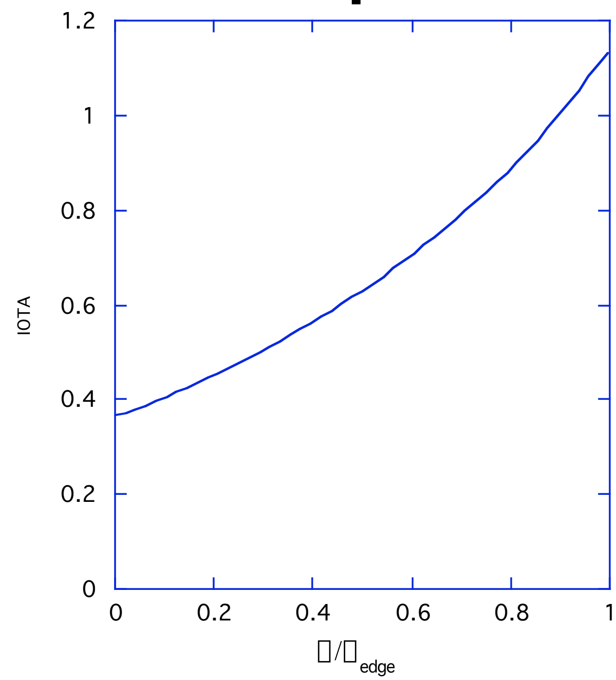


High aspect ratio torsatron
LHD configuration



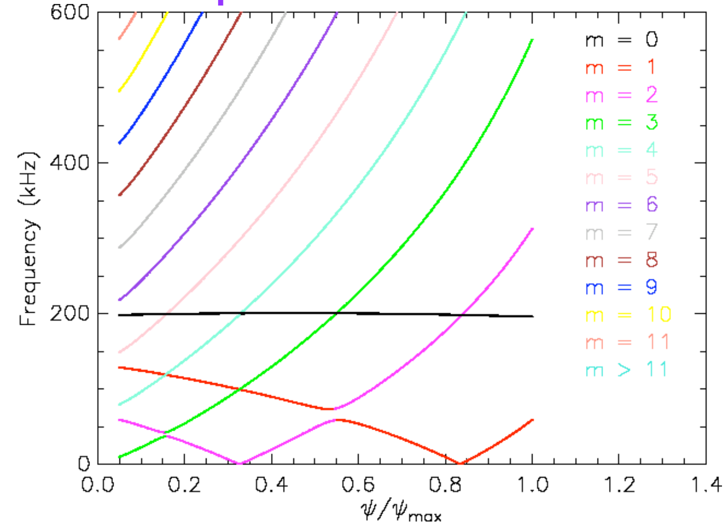


Iota profile



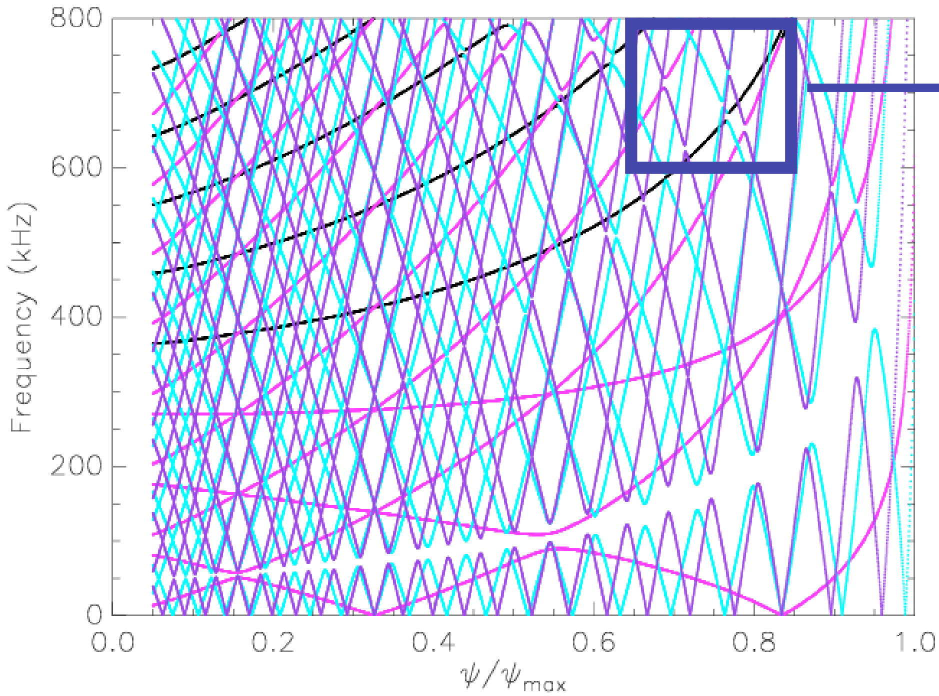
LHD
(10 field periods, R/<a> = 6, torsatron)

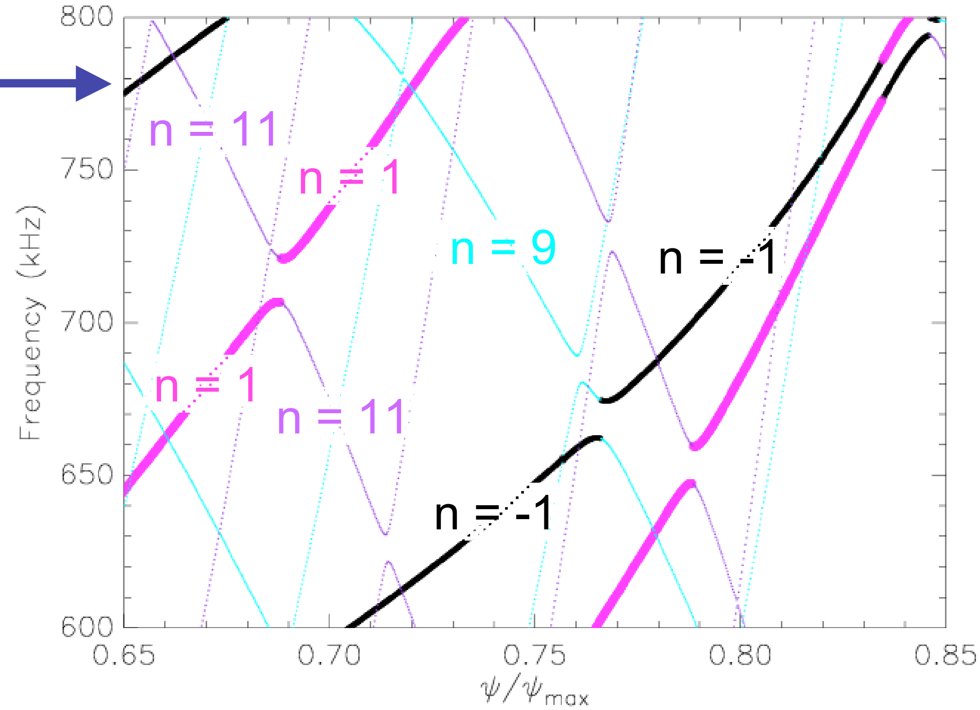
Axisymmetric limit
dominant **poloidal** mode is color coded



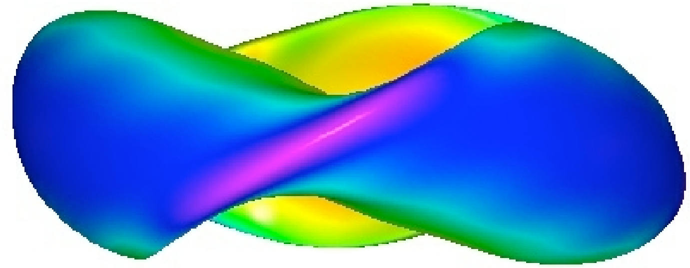
Continua with multiple toroidal modes

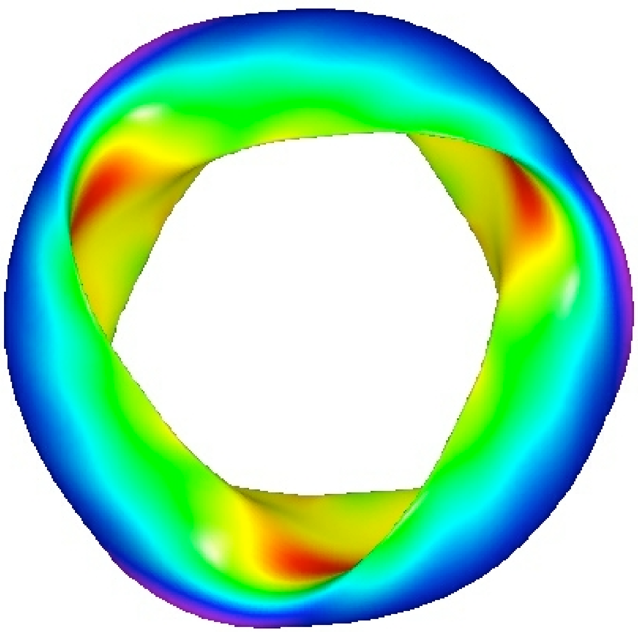
n = 1 mode family stellarator continua
<u>dominant **toroidal** mode is color coded</u>



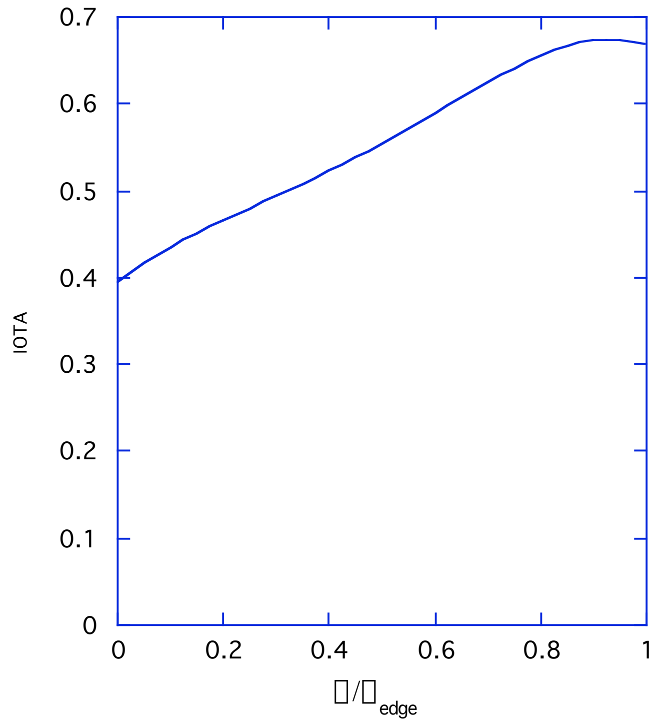


Low aspect ratio quasi-toroidal configuration LI383





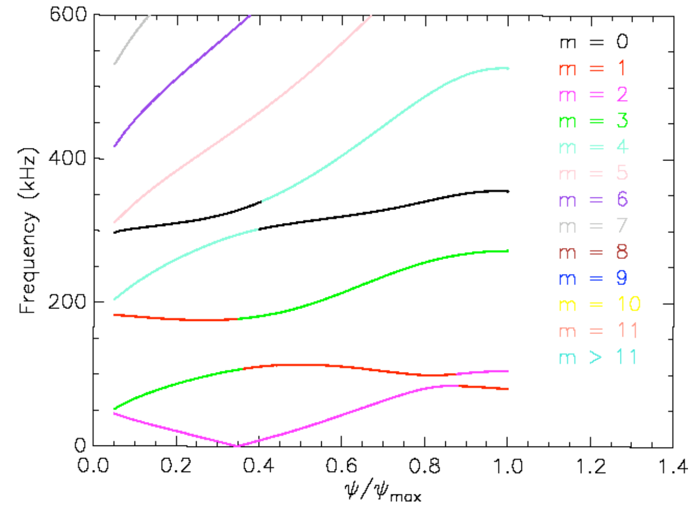
Iota profile



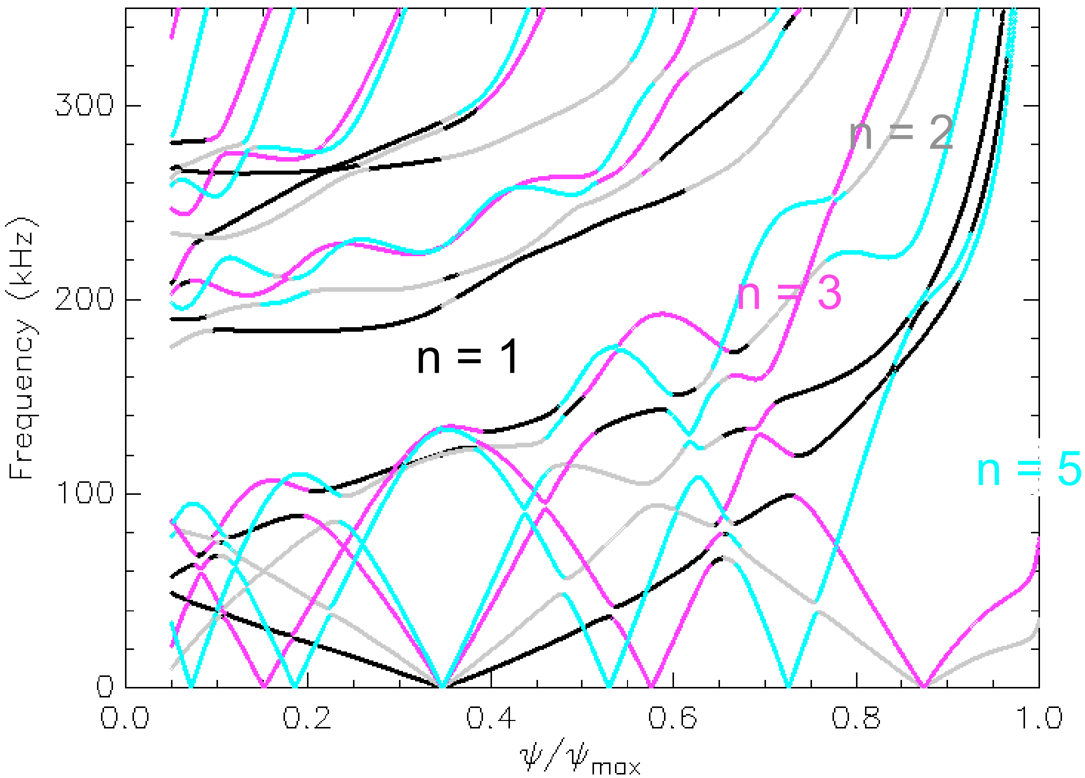
NCSX

3 field period, R/<a> = 4.4
quas-toroidal symmetry

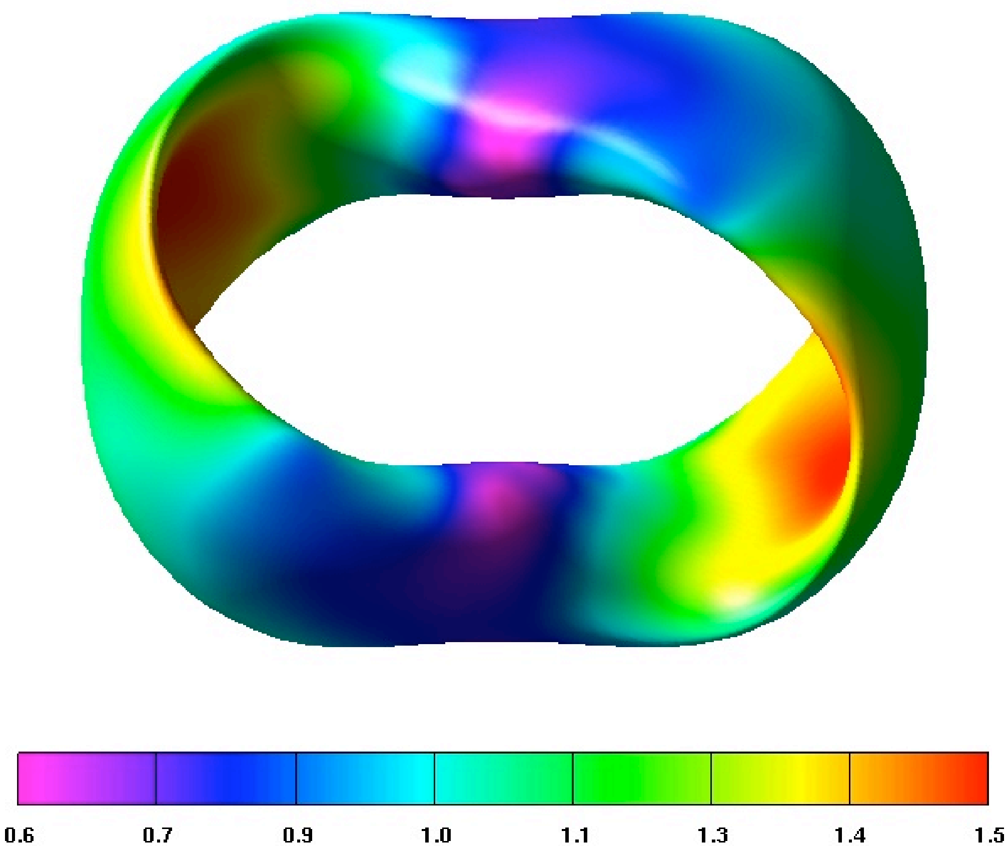
Axisymmetric limit
dominant **poloidal** mode is color coded

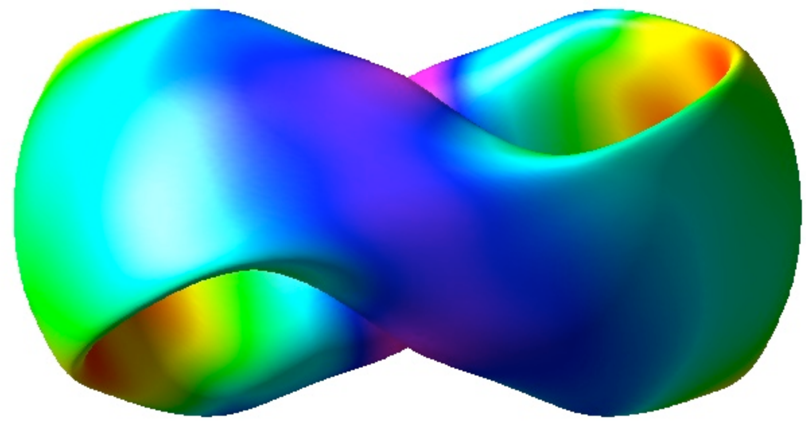


Continua with multiple toroidal modes
n = 1 mode family stellarator continua
dominant **toroidal** mode is color coded

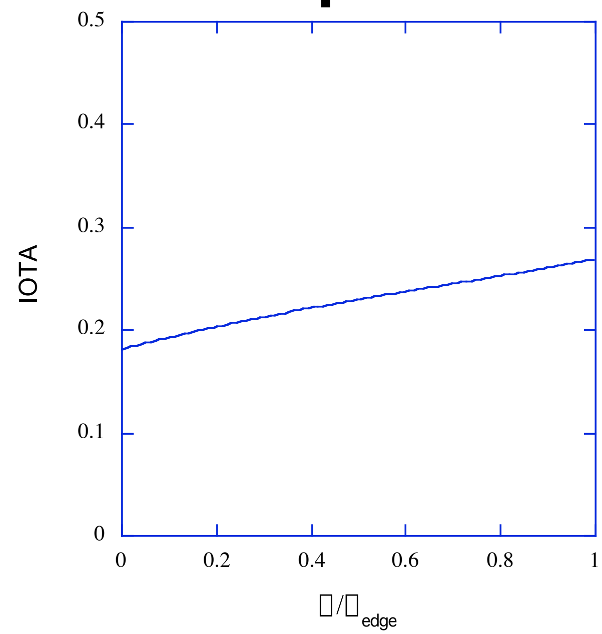


Low aspect ratio quasi-poloidal configuration QPS





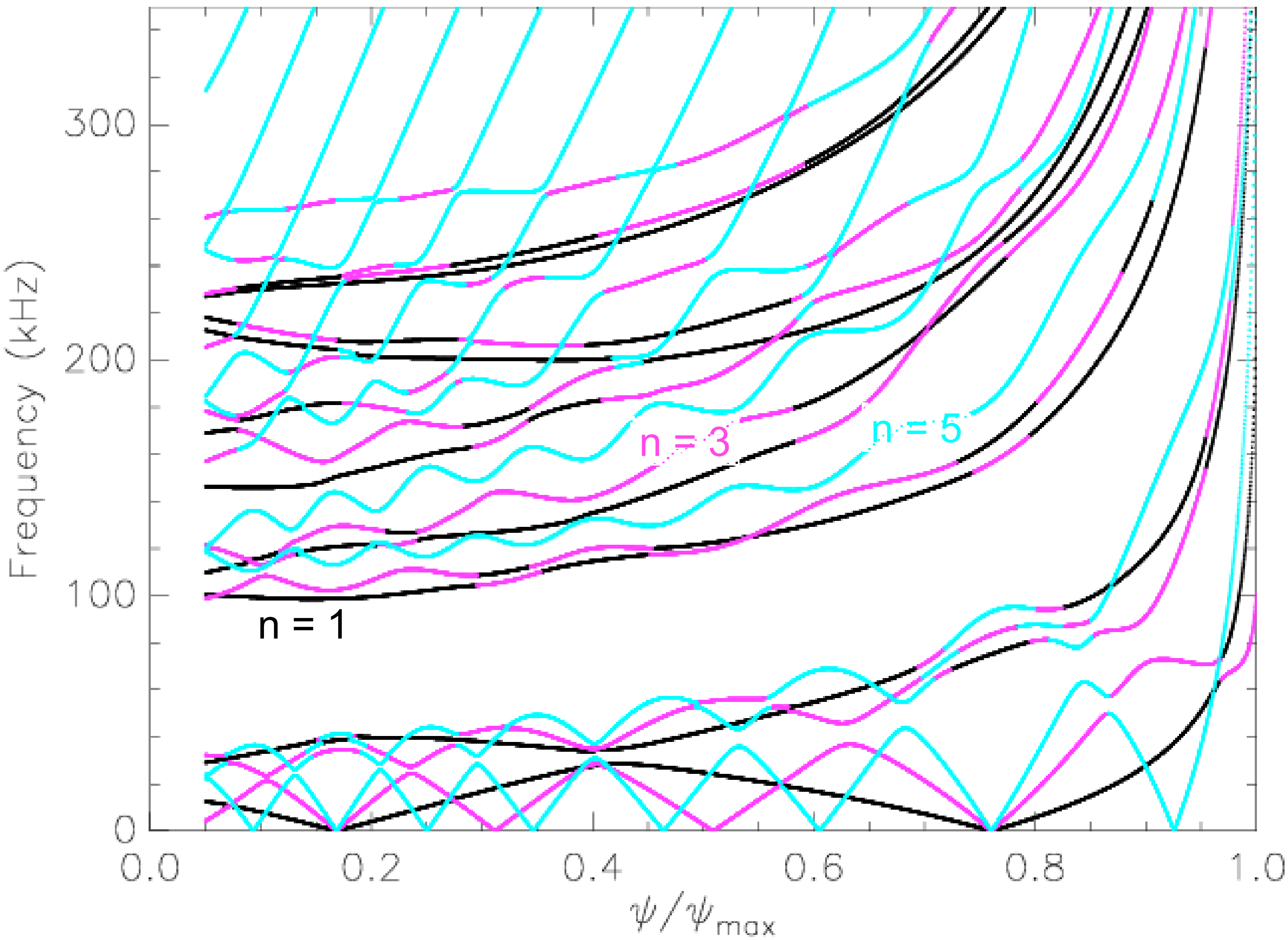
Iota profile



QPS (2 field periods, R/<a> = 2.7, quasi-poloidal symmetry)

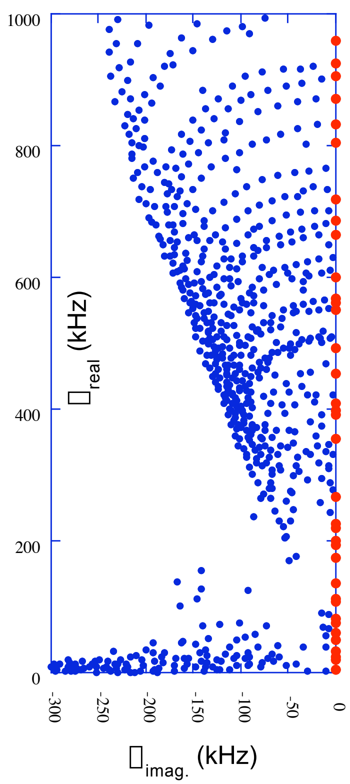
n = 1 mode family stellarator continua

dominant **toroidal** mode is color coded

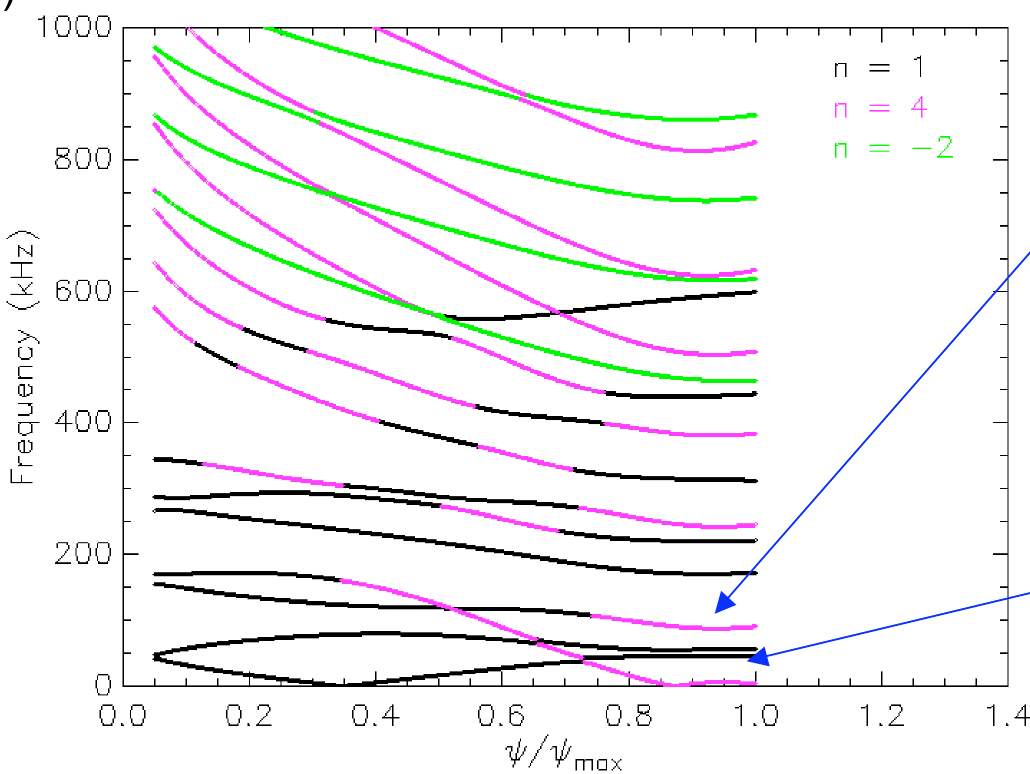


Recently the STELLGAP code has been upgraded to solve the 3 coupled equations for the stable Alfvén mode structure in compact 3D configurations

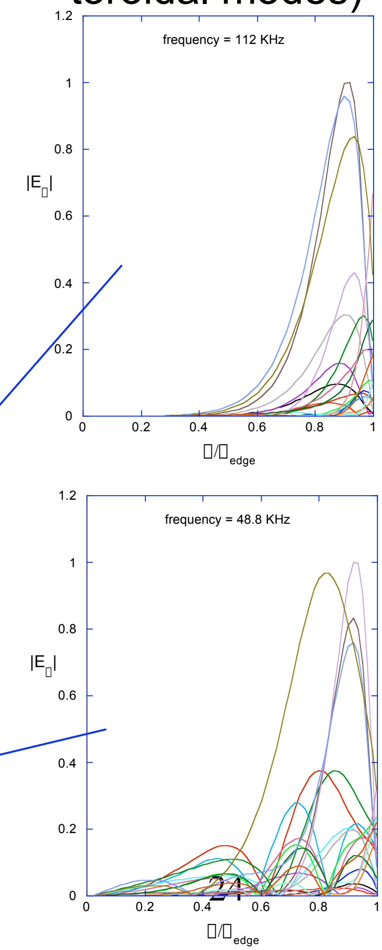
Discrete 3D Alfvén frequency spectrum (blue = damped red = neutrally stable)



Alfvén Continuum for QA-symmetric stellarator (color indicates dominant toroidal mode number)



AE radial mode Structure (multiple toroidal modes)



Conclusions

- The 3-dimensional structure of stellarator equilibria introduce new mode couplings in the Alfvén continuum spectrum:
  - Helical Alfvén mode (HAE) n, m with $n+\square N_{fp}$, $m + \square$
  - Mirror Alfvén mode (MAE) n, m with $n+\square N_{fp}$, m
- HAE modes are accessible at lower frequencies in compact systems (e.g.,QPS, NCSX)
  - Adjacent n's cross-link more readily (smaller N_{fp} - less separation)
  - Initial mode structure calculations show broad composite modes (wider coupling than simple TAE_{mn}, HAE_{mn}, MAE_{mn}, etc. categorization)

Conclusions

- W7-AS could access a wide range of Alfvén gap structures through iota-profile flexibility
  - Low shear (40173): GAE modes
    - Cylindrical, axisymmetric, stellarator models -> similar results
  - Moderate shear (43348): multiple modes
    - TAE, HAE/MAE also present in plasma interior and at medium frequency ranges
  - High shear (42873): single mode
    - Observed fluctuations likely TAE
    - HAE/MAE present, but only at higher frequencies
- 3-D effects only slightly change torsatron (LHD) low frequency continua away from that of the equivalent tokamak
  - HAE modes only occur near edge at higher frequency

Next Steps

- Develop methods for calculating linearized destabilization of these discrete modes by energetic particles.
  - compare mode structures and stability thresholds with stellarator experiments
  - Study proposed compact systems (QPS, NCSX)
  - apply to 3D effects on TAE’s in tokamaks (ripple, internal tearing and kink modes)
- MHD spectroscopy for iota/ion density profiles
  - NCSX, QPS applications
- Optimization of stellarators for AE mode minimization
- Develop nonlinear models