ON THE IDENTIFICATION OF SOC DYNAMICS IN THE SUN-EARTH SYSTEM

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Abstract. We use a self-organized criticality (SOC) model to show that 1) such a system can have different characteristic signatures depending on the level of external forcing while still having the same underlying dynamics and that 2) current time series of Sun–Earth processes are too short to compare with all dynamical regions of a SOC model. SOC is a concept that has been applied to various aspects of the Sun–Earth system, such as rearrangement of magnetic flux loops on the Sun, AE indices and substorm statistics. The basic tenet of SOC is that simple local interactions produce complex global signatures that are not simply predicted by the low level physics. Simple SOC models, such as the sandpile, are used to compare their signatures with those observed in a physical system.

1. Introduction

Self-organized criticality (SOC) *Bak et al.* [1987, 1988] has been suggested as a model for the dynamics of various aspects of the Sun–Earth system. The basic tenet of SOC is that simple local interactions produce complex global signatures that are not simply predicted by the low level physics. The Sun–Earth system and SOC models share similar dynamical and statistical signatures, such as power law scaling of event size distributions *Uritsky et al.* [2002] and of the power spectra of some characteristic time series, discussed below. Another shared signature is a Hurst exponent that indicates correlated dynamics over some time scales longer than an autocorrelation time *Price and Newman* [2001]. These very brief statements clearly do not do justice to all that is implied about the Sun-Earth system when viewed from the perspective of SOC. References *Uritsky and Pudovkin* [1998]; *Angelopoulos et al.* [1999]; *Chang* [1999]; *Klimas et al.* [2000]; *Lui et al.* [2000]; *Chapman and Watkins* [2001]; *Chang et al.* [2003] are some examples of more thorough treatments of the subject.

When deciding whether or not a system is SOC, a usual practice is to compare specific signatures of a defined SOC model and the physical system under study. Rather than saying: "If these signatures are similar, then the system must be SOC," the more cautious approach is to say: "If these signatures are similar, then this system is consistent with SOC dynamics." Fundamentally, it is the characteristics of the dynamics that are of interest. Here, we investigate two basic issues involved in whether or not even this second statement can be safely made. It is important to note

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that one cannot prove that a system is SOC unless it has been constructed as such. But it is the shared dynamics among systems that matters most, not the name.

Before presenting those two issues, we briefly mention the measures that we use in this study: the power spectrum and rescaled range (R/S) analysis. We will define and discuss them further below, but for now we simply say that they are two measures that quantify a time series in the frequency domain and time domain, respectively. Both measures can produce distinct straight line regions when plotted on doubly logarithmic axes, indicating power law scaling. Two important features of a SOC time series in this regard are that it 1) produces multiple power law regions and 2) the number and behavior of these power law regions are very different from any other known model, including random uniform or Gaussian noise, fractional Gaussian noise, fractional Brownian motion and random superposition of pulses. We elaborate on the significance of power laws below.

For our results, we first show that the spectral and R/S signatures of a SOC system can change drastically when the external forcing of the system changes. Because of this, there is no single reference signature that a system must match in order to be considered SOC. An application of this idea is in considering the fluctuations in the solar wind as the external driver of the magnetosphere, for which SOC has been suggested as a model. Since the solar wind can range from very strong and steady to practically nonexistent, the reaction to this changing forcing can easily produce different spectra over different time scales.

This result, changing spectrum with changing forcing, can be seen in two ways: by changing the strength of the external drive and by changing the level of correlations in the external drive. Strength of drive is the intuitive notion of how much of some quantity is being deposited in the system per unit time. In terms of the solar wind, for example, very weak drive was seen during the period 10-12 May 1999, when it almost disappeared *Le et al.* [2000b, a]. (In general, though, SOC is concerned with dynamics on much longer time scales.) The level of correlations deals with the issue of whether the external drive is completely random or not. For instance, the solar wind has been seen to be a correlated source so that the subsequent external driving of the magnetosphere is not completely random. We will only discuss strength of driving here. Refer to *Sánchez et al.* [2002] for studies of the effect of correlations in the drive in a SOC system.

Second, we show that the longest available time series of a space climate process is likely far too short to show all of the power law regions in the spectra and R/S measures associated with SOC. 'Too short' here implies a needed time scale on the order of a century. That is, many more generations of scientists will pass before long enough time series are acquired and this issue can be settled.

The rest of this paper is organized as follows. We first review power laws, the power spectrum and rescaled range analysis in Section 2 so that we can refer to them in Section 3, which is a brief overview of self-organized criticality, our model and some recent work in space physics from the perspective of SOC. We will present and discuss our results of the strength of forcing and the effects of the length of a time series in Sections 4 and 5. Conclusions are drawn in Section 6.

2. Power Laws, the Power Spectrum and R/S Analysis

A characteristic associated with SOC since its introduction have been power laws. A power law is any function of the form $y = cx^a$. On doubly logarithmic axes, this function appears as a straight line with slope a; a is also referred to as the scaling exponent. Power laws appear in many measures of many systems, such as probability distributions, power spectra and R/S analysis. We will only discuss the spectral and R/S analyses. For a thorough discussion of power laws in probability distributions refer to *Sornette* [2000] and references therein.

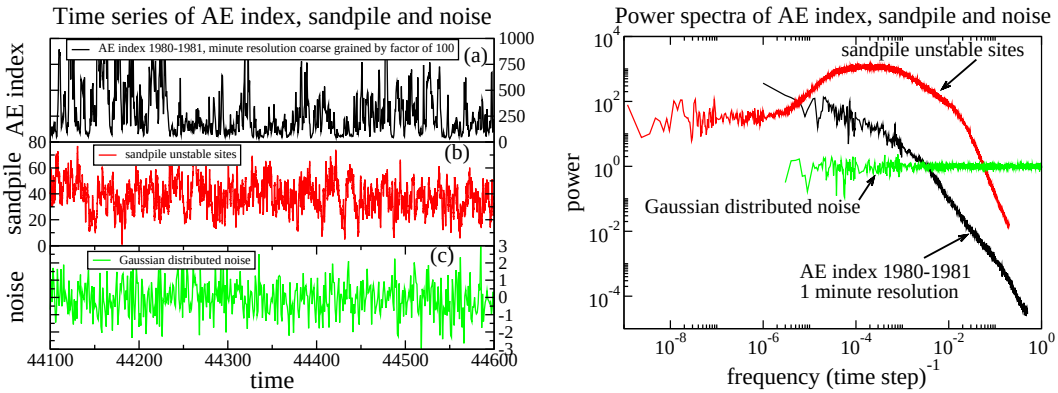

Figure 1. Time series and power spectra of AE index, sandpile and Gaussian noise.

When discussing power laws, a usual unstated assumption is that the power law is an appropriate fit of the data over a reasonably wide domain. 'Reasonably wide' is vague so this criteria must be separately established for each case. A common critical, yet reasonable, observation made in regard to these studies is that any function—power law, exponential, sine curve, etc.—can be fit very well to any data if one zooms in to a small enough scale on a plot. Moreover, when two neighboring power law regions are claimed, with different scaling exponents, the additional issue of whether or not there is a distinct breakpoint between the two regions is raised. Needless to say, these are critical issues and the scientist must be aware of them. We are, and elsewhere we have investigated them thoroughly for the data that we present here *Woodard et al.* [2004]. In general, we find that a decade (power of 10) is a reasonable minimum for establishing a power law as a good fit.

Having established that a power law is a good fit to a region and that the limits of the region are identified by breakpoints that separate neighboring regions that may or may not also be fit by a power law, the most important task is to identify the process or processes in the system that produce such a signature. This is where understanding of the particular measure is needed. In practice we find that using more than one measure is invaluable in attempts to understand such systems and signatures. This allows us to use multiple measures to distinguish and quantify separate regions. Hence we complement the power spectrum with the lesser known R/S analysis, with which we estimate the Hurst exponent H.

The power spectrum of a discrete time series $f(\tau)$ allows one to examine the data in frequency space. It is defined as $S(f) = |F(f)|^2$, where $F(f)$ is the Fourier transform of $f(\tau)$; power is plotted versus frequency. The spectrum of random noise (with Gaussian or uniform distribution) is flat ($\sim f^0$) so that the power at all frequencies is the same.

The task of any spectral analysis is to understand why a system has a spectrum that differs from the flat one of a completely random series. A simple example is the spectrum of a sine curve; it is the same flat spectrum as that of the random one except for a spike at the characteristic frequency, indicating where most of the power is concentrated. (Analytically, the spectrum is a delta function; we will deal mainly with finite discrete time series.) A spectrum that has more than one such peak indicates multiple periodic processes in the system.

Besides distinct peaks, another feature of some power spectra is a power law dependence on frequency, where the spectrum scales as $f^{-\beta}$ with $\beta \neq 0$; β can be positive or negative. The power law extends over a finite frequency band (for a finite time series) and there may be more than one such scaling region. Again, the task is to understand why this spectrum is different from that of a random one. Specifically, one must understand why each region has a particular value of β and why the breakpoints of the region occur where they do.

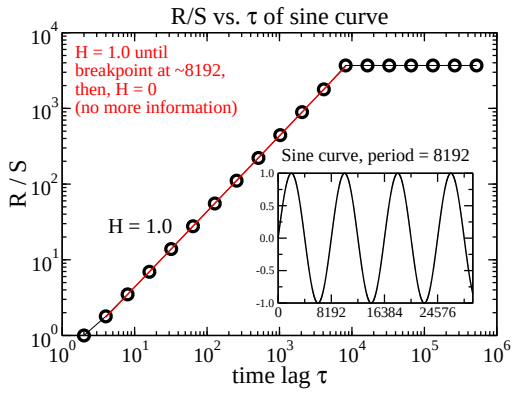

Figure 2. Time series and R/S analysis of sine curve.

Some spectral power laws are not due to dynamics in a system and can be analytically derived. We refer to this type of power law as one due to pulse shape, which is the highest frequency component of a series. The smoother a function is, the faster its spectrum falls off with increasing frequency. This is quantified by taking derivatives of the function. The spectrum falls off as f^{-2k} if the kth derivative becomes impulsive. For instance, a sawtooth wave (a triangular pulse) of width w has an impulsive second derivative so its spectrum scales as f^{-4} as $f \to \infty$. For frequencies below w^{-1}, the spectrum is flat. One can extend this and think of the sine wave as a function that does not have any impulsive derivatives; its spectrum, then, the delta function, falls off infinitely fast with increasing frequency *Bracewell* [2000]. Another analytically derived power law spectrum is that of a random superposition of square pulses, which falls off as f^{-2} for frequencies above the inverse of the widest pulse *Jensen et al.* [1989]; *Kertész and Kiss* [1990].

Part of the interest in physical systems that have power law scaling regions in their power spectra is because the time series themselves are far from the simple examples above. Most time series of 'real' physical processes, such as the AE index or fluctuations in the solar wind velocity, are not simple shapes or superpositions of simple shapes. They look noisy, almost random. Yet they still have spectra that are very different from that of a completely random process (Figure 1). Spikes in the spectra can usually be explained by known periodicities in the system (rotation of the earth, 11 year solar cycle, etc.). But the observed values of β and the locations of breakpoints between scaling regions, in most cases, are not well understood and are probably important indicators of the dynamics.

Qualitatively, a spectrum that falls off as $f^{-\beta}$ with $\beta > 0$, a negative slope on log-log axes, means that lower frequencies are most important in characterizing the time series. For values of $\beta < 0$, a positive slope, the higher frequencies dominate the signal. So signals with $\beta > 0$ are smoother than those with $\beta < 0$. Because of this, the smoother signals with positive β are said to be correlated and the rougher ones with negative β are anticorrelated. Here, correlated implies that a current trend in the data continues. The value of β, then, is often used as a measure of correlations in a time series.

A different and perhaps better measure of correlations in a time series is the Hurst exponent, $H \in [0, 1]$. A value of $H = 0.5$ implies a data set that is completely random, with no correlations. Series with $H > 0.5$ are correlated and those with $H < 0.5$ are anticorrelated. The closer H is to 0 the rougher and more anticorrelated is the signal; the closer to 0.5, the more uncorrelated; and the closer to 1 the smoother and more correlated. For instance, $H = 0.5$ for a time series of fair dice being rolled and $H = 1$ for a sine curve up to its period (Figure 2). *Price and Newman* [2001] has estimated the Hurst exponent of the AE index as $H \approx 0.7$, indicating that the AE process has long time correlations.

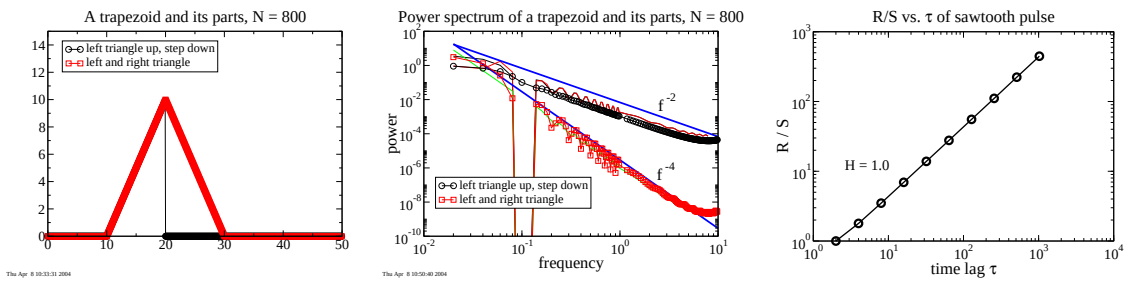

Figure 3. Time series, power spectrum and R/S analysisof a sawtooth pulse.

One technique of estimating H is through rescaled range (R/S) analysis *Hurst* [1951]; *Mandelbrot* [2002]; there are other methods *Bassingthwaighte and Raymond* [1994]; *Bassingthwaighte et al.* [1994]; *Bassingthwaighte and Raymond* [1995]. For a series of data ξ_t, the rescaled range is defined as:

$$R'(s) \equiv R(s)/S(s) \quad \text{(rescaled range)}$$
$$R(s) = \max_{1\le t\le s} X(t,s) - \min_{1\le t\le s} X(t,s), \quad \text{(range)}$$
$$X(t,s) = \sum_{u=1}^{t} (\xi_u - \langle\xi\rangle_s) \quad \text{(cumulative deviation)}$$
$$S(s) = \left[\frac{1}{s}\sum_{t=1}^{s}(\xi_t - \langle\xi\rangle_s)^2\right]^{1/2} \quad \text{(standard deviation)}$$
$$\langle\xi\rangle_s = \frac{1}{s}\sum_{i=1}^{s}\xi_i \quad \text{(mean)}$$

If the rescaled range of the time series scales as $R'(s) \sim s^H$, the slope of the plot of $R'(s)$ versus the time lag s on a doubly logarithmic plot is an estimate of the Hurst exponent, H.

H can be related to the more familiar measure, variance. Consider classical Brownian motion (a random walk), the increments of which are simply a Gaussian distributed random noise. For a large ensemble of random walks, the expectation value of the variance of the motion scales linearly with time, $\sigma^2 \sim t$. For the noise series, $H = 0.5$. In general, the variance of the motion is related to the Hurst exponent of the noise by $\sigma^2 \sim t^{2H}$.

Algorithms exist *Mandelbrot* [2002] that create synthetic time series with Gaussian distributions and a given value of $0 < H < 1$. Such a series is called fractional Gaussian noise (fGn) and when integrated produces fractional Brownian motion (fBm). For these series, H is analytically related to the slope of the power spectrum of the fGn via $\beta = 2H - 1$. For discrete fGn data, though, this relation does not hold so well at all values of H. A thorough discussion of H and β in the context of fGn is given in *Malamud and Turcotte* [1999].

It is very important to note that the relation $\beta = 2H - 1$ is not always true for physical data. This is equivalent to saying that fGn is not an appropriate model for the system under study. A basic example is the time series of a single sawtooth pulse mentioned above. It has a power spectrum that falls off as f^{-4} and a Hurst exponent, calculated via R/S analysis, of $H \approx 1$ (Figure 3). Here, $\beta \neq 2H - 1$; the spectrum and the R/S analysis describe two different aspects of the same series. $\beta = 4$ is a statement about the discontinuity of the first derivative of the sawtooth pulse and $H \approx 1$ is a statement of the strong correlation that a defined shape has with itself for time scales up to its width.

Self-organized criticality is a model where $\beta \neq 2H - 1$ over different time scale regions. But the reasons are not as simple as for that of a single pulse. Here, the dynamics of a SOC system has correlations, anticorrelations and lack of correlations on distinctly different time scales and on time scales larger than that of the largest fundamental single pulse. The changing levels of correlations appear in the power spectrum and R/S analysis of a SOC system as separate and distinct power law regions.

Looking at only the spectrum or the R/S alone does not adequately explain the regions of a SOC system. But together the measures give a clear picture of SOC dynamics. We emphasize this to encourage the use of multiple measures in any system; taken alone, the power spectrum does not always explain the dynamics of a system.

3. Self-Organized Criticality and the Sun-Earth System

The underlying idea behind self-organized criticality is that, in many complex systems, simple local interactions produce complex global signatures that are not easily predicted by the local low level physics. That is, the fundamental physics of a system is often understood but some of the observed signatures are not captured by models that are built on that physics. The problem is that, inevitably, all models must leave something out, must make some approximations. An example is the power spectrum of the AE index seen in Figure 1.

The main approximation that a SOC model makes is to reduce all of the local physics in a system to a simple physical rule: if the local gradient of some quantity between two nearest neighbors exceeds some critical gradient, then reduce the gradient by transporting some of the quantity from one neighbor to the other. A 'neighbor' is purposefully vague and can be different for each system. It represents the notion that gradients exist on macroscopic scales and the source of the gradients is not as important as the fact that they grow, shrink and interact. Crucial to the SOC dynamics are that the time scales of the driving and relaxing processes are very different: the gradients are reduced much faster than they are produced. Within these bounds, a plethora of models can be constructed but all adhere to this one rule; we describe our model, the sandpile, below. The interesting dynamics appears because the transport from one neighbor to the next may make the next local gradient exceed the critical gradient, causing a new transport event. In this way, disturbances can propagate throughout the model.

In the jargon of SOC, the disturbances are often referred to as avalanches. Avalanches can range in size from the smallest possible (one transport event) up to the size of the system. Over much of this range of scales, SOC avalanches are distributed as a power law, indicating no preferred spatial scale within that range. This is the criticality part of SOC. It refers to statistical mechanics, where disturbances in a material at a phase transition can propagate throughout the entire sample. In such a case, a control parameter—the temperature—must be tuned to reach this critical state. In contrast, a SOC system arrives at criticality with no external tuning of a parameter [†]. The system self-organizes to the critical and steady state.

Time series of the avalanches can be constructed and analyzed. Regions of the power spectra of these time series scale as power laws, indicating correlations and anticorrelations on different time scales. These temporal power laws together with the spatial power laws of the avalanche size distributions are important signatures of SOC systems that are similar to signatures of many observed physical systems. Because of these similarities and of the underlying physics, the Sun-Earth system has been studied as possibly SOC.

[†]This is a matter of discussion in the world of SOC. See, for example, *Jensen* [1998] and references therein for a discussion.

The model that we use is the one dimensional running sandpile, studied extensively in *Hwa and Kardar* [1992]; *Woodard et al.* [2004]. The sandpile consists of L cells labeled by an index $n \in [1, \mathrm{L}]$. Each cell stores an amount of sand h_n and the local gradient between two cells is defined as $Z_\mathrm{n} = h_\mathrm{n} - h_{\mathrm{n}+1}$. U_0 grains of sand are dropped randomly on every cell at each iteration with probability P_0. The external drive per cell is thus $S_0 = U_0 P_0$ grains per time step. SOC dynamics appears because of the existence of a critical slope Z_crit that, when locally overcome, triggers the removal of N_f grains of sand to the next cell in the downhill direction (increasing n). The sandpile is initialized with $h_\mathrm{n} = (Z_\mathrm{crit} - 1)(L - n)$ and run for T_max time steps. We study the sandpile in steady state so that transient time steps before T_T are ignored in the following analysis.[‡] We used $U_0 = 1$ (therefore $S_0 = P_0$), $Z_\mathrm{crit} = 8$ and $N_\mathrm{f} = 3$, the same parameters used in a study of confined plasmas *Newman et al.* [1996]. In this study, we use sandpile sizes of L = 200 and 1000 and driving rates of $P_0 L = 0.2$, 0.1 and 0.001. We study a more complete range of five orders of magnitude of driving rate and three orders of magnitude of system size in *Woodard et al.* [2004].

Effective driving rate is given by $P_0 L^2$ *Woodard et al.* [2004]. The idea of effective driving rate is that a fixed driving rate P_0, which is in units of grains per cell per time step, is effectively higher for a small sandpile than for a larger one. Since a larger sandpile has a greater capacity, P_0 will take longer to fill it up when compared with the same rate of sand falling onto a smaller system. In the past, $P_0 L$ has been used as the measure of driving rate because its units are in grains per time step for the entire system. But since the average avalanche size is larger in a larger sandpile, two sandpiles of different sizes but with the same $P_0 L$ can be in very different drive regimes because the quiet times will be shorter in the larger system. We find that a better measure, effective driving rate, should be used. Systems with different size and/or P_0 but with the same value of $P_0 L^2$ have power spectra that are related via a rescaling function. We elaborate on effective driving rate in *Woodard et al.* [2004].

The time series that we analyze is called the flips. Consider the total number of unstable sites (where $Z \geq Z_\mathrm{crit}$) at each time step in a sandpile model in steady-state. An unstable cell spills N_f grains of sand to the next cell; this action is a flip. The total flips at each time step can be thought of as the instantaneous (potential) energy dissipation in the system. The sandpile is driven by a random process but the number of flips fluctuates with time in a way that is not entirely random, as we show.

4. Effect of Strength of External Forcing and System Size

We now show that systems with very different driving rates can have very different spectral and R/S signatures. Figure 4 shows the power spectra from three different sandpile runs. Runs (a) and (b) differ in their driving rate $P_0 L$ and runs (b) and (c) differ in their driving rate and system size, L. Run (a) is the smallest system with the highest driving rate; run (b) is the same size but with a lower driving rate; run (c) has a lower driving rate and larger system size than (b). The effective driving rate $P_0 L^2$, then, decreases from run (a) through (c), with (c) being the lowest effective driving rate. The spectra can be seen to change systematically.

The spectra in Figure 4 show different power law scaling regions for all driving rates. Each spectrum actually has 4 or 5 separate regions, depending on effective driving rate *Woodard et al.* [2004]; we have sketched lines on each indicating only three of these regions. The highest frequency region with $\beta \approx 3.5$ indicates correlations at short time scales, where avalanche pulse shapes correlate with themselves. On longer time scales, the middle frequency region has a slope $0 < \beta \leq 1$ that changes with driving rate. These positive values of β indicate correlations among separate and overlapping avalanches. The time scales for this region are greater than the maximum duration of

[‡]The initialization saves computer time; the same results hold when the sandpile is started from any initial condition. The transient time T_T must be adjusted accordingly.

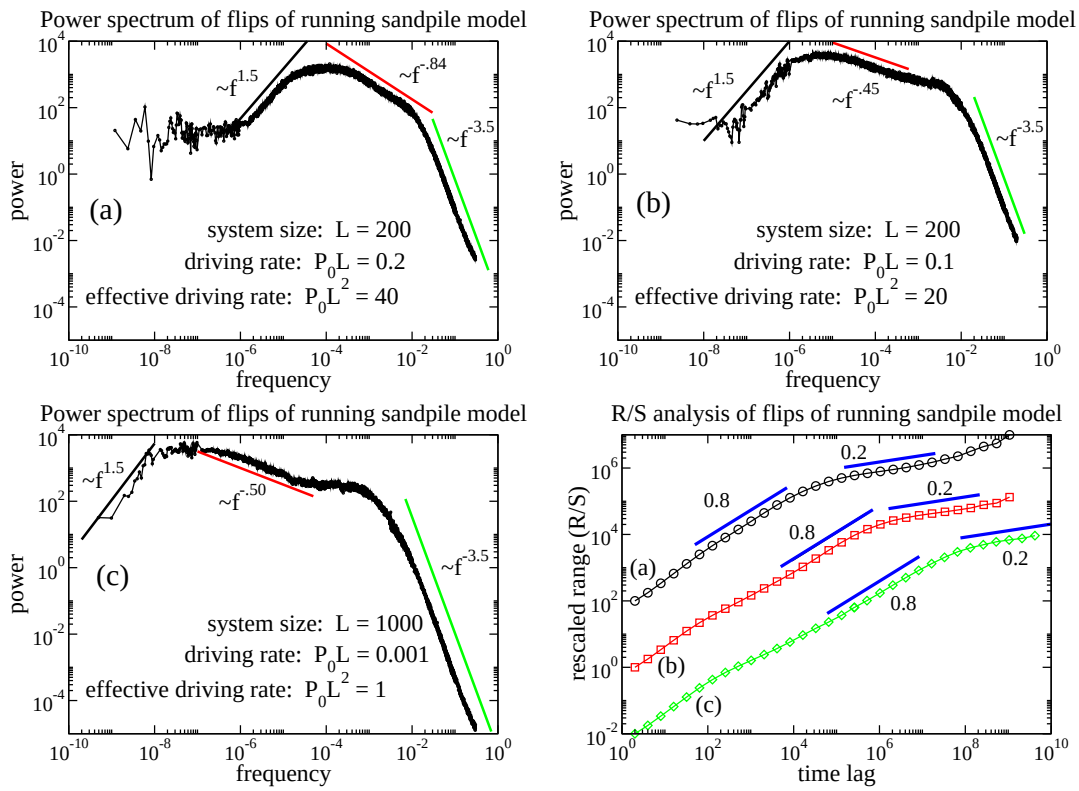

Figure 4. Power spectra and R/S analysis of flips of running sandpile model for three different values of effective drive, P_0L^2. Spectra and rescaled range have been shifted along the y-axes so that all plots are over the same range. Estimates of the Hurst exponent are shown for the straight line segments in the R/S plot. These are not fits of the data but lines of the indicated slope to guide the eye.

a single avalanche. The lowest frequency region with $\beta \approx -1.5$ indicates anticorrelations on the longest time scales. These are the time scales of global discharge events, where one such event is unlikely to be followed by another similar event.

The R/S analysis of sandpile flips also shows multiple regions. We have only indicated two in Figure 4. These correspond to the middle and long time scales of the power spectra. The estimated values of the Hurst exponent can be compared with the values of β in the spectra. For both cases, $\beta \neq 2H - 1$. Most importantly, the middle time scale region has a constant value of $H \approx 0.8$ while β changes with driving rate. This is because H measures correlations in the sandpile that are produced by the memory in the system due to the system rules. The rules do not change as driving rate changes, hence H stays relatively constant. Because of the changing values of β in this region, it is not clear what aspect of the correlations is measured by the power spectrum.

The goal of these results is to show that the SOC running sandpile model will produce very different power spectra when the level of external forcing changes. This is why comparing the values of the slope or the number of regions of the spectrum of a physical system like the Sun-Earth system with those of a single run of a SOC model can be misleading. If the values of the slope do not match or if the number of regions are different, this does not necessarily mean that the system is not SOC. Instead, it could be that the system is in a different drive regime than that of the model being used.

An important aspect of the running sandpile model is that over a very wide range of driving rate it is still in a state of SOC. That is, it still exhibits all of the qualitative signatures that collectively

are called SOC. These include power law distributions of event sizes, power law regions in the power spectra, correlated dynamics indicated by Hurst exponents $H \neq 0.5$ and regions and event sizes that scale with system size. More importantly, the physics in the models is identical *regardless of driving rate.* Sand is still added at a constant probability that triggers avalanches that may or may not spread throughout the entire system. The memory of the avalanches is retained in the local gradients of each cell and this is the source of long time correlations. The only difference is the rate at which the avalanches and the time scales over which the correlations occur.

This idea can be applied to data that is already in the literature. The first power spectrum of the AE index showing the broken power law behavior was seen in *Tsurutani et al.* [1990] for data from three separate periods, 1967-1970, 1971-1974 and 1978-1980. Similar spectra were shown subsequently by *Consolini et al.* [1996], *Uritsky and Pudovkin* [1998], *Price and Newman* [2001] and *N. W. Watkins* [2002]. The time periods studied in all of these works, as well as the slopes of the spectra in the two regions and the location of the breakpoints are shown in Table 1. That they vary over different periods is completely consistent with an SOC model that runs for a time at one driving rate and then for another time at a different driving rate. In general, the slopes found for shorter periods of observations differ the most from the average of all samples. In the language of the sandpile, there are two possible causes for this.

First, the driving rate can change over time so that the average input of the solar wind over the period 1 January through 19 February 1975 was very different from the period 1978-1979. These two periods were chosen as examples because they show the greatest difference in slopes for both spectral regions among the values presented. That one period is near the solar minimum and the other near the solar maximum should be noted and taken as a possible example of the level of external forcing changing the power spectrum, as in the running sandpile model.

A second and, in this case, more likely possibility that can account for changing spectra for the time periods shown is that over a long time scale, say the roughly 40 year period for which we have AE index data, the input to the system (the solar wind) is constant in the same way that the mean of a series of random numbers is constant. But when one looks at a small subset of the random series, the mean may be very different and there will certainly be fluctuations far from the overall mean. In other words, data is scattered within the errors bars of a sampling. All of the values of β in Table 1 are within 2σ of the mean.

Both of these possibilities are intuitively appealing because the Sun, the driver in the Sun-Earth system, is known to not have a constant output from year to year. But whether that change is due to a fundamental change in the drive regime of the Sun or to intrinsic fluctuations within a steady state is not clear.

As a quick test of this idea, we compared power spectra for three different three day periods, 10-12 May 1999, 26-28 January 1999 and 12-14 November 1999. The solar wind essentially vanished during the first period *Le et al.* [2000b, a], falling more than 98% for a period of approximately 30 hours. The other two periods were closer to the yearly mean. There is no significant difference in the spectra among the three cases. We attribute this to the very short observation time, as the spectra show much ringing at high frequencies, making determination of any sort of fit statistically irrelevant. We must wait for longer periods of no solar wind to pursue this idea further.

In addition to those of measured AE indices, the literature also holds power spectra of a continuum model that "provides a link between the sandpile model studies ... and a realistic plasma physical study of SOC dynamics in the plasma sheet *Klimas et al.* [2000]." That study refers to the Lu model of *Lu* [1995], which "can be viewed as an idealized one-dimensional resistive field reversal model in which the resistivity is generated self-consistently." This is a model where a scalar field is evolved in time while coupled to a variable diffusion coefficient and source term that are space

Table 1. Time period, resolution, slopes of first two spectral regions and breakpoint of AE index data found in *Tsurutani et al.* [1990], *Consolini et al.* [1996], *Uritsky and Pudovkin* [1998], *Price and Newman* [2001] and *N. W. Watkins* [2002]. Breakpoint for *Consolini et al.* [1996] taken between labeled second and third regions. Breakpoint for *Price and Newman* [2001] 1978-1979 estimated from plot at intersection of two power law fits. Slope for *N. W. Watkins* [2002] taken as best fit with a straight edge, slope estimated from axes.

Study	Period	Res.	β_A	β_B	Break (mHz)
Tsurutani et al. [1990]	1967–1970	5 min	2.42	1.02	0.059 (4.7 hr)
Tsurutani et al. [1990]	1971–1974	1 hr	2.2	0.98	0.050 (5.5 hr)
Uritsky and Pudovkin [1998]	1973–1974	1 hr	2.10	0.95	0.056 (5.0 hr)
Consolini et al. [1996]	1/1–19/2 1975	1 min	2.65	1.14	0.073 (3.8 hr)
N. W. Watkins [2002]	1978	5 min	2.1	1.1	0.056 (5.0 hr)
Price and Newman [2001]	1978–1979	1 min	1.85	0.82	0.033 (8.4 hr)
Tsurutani et al. [1990]	1978–1980	1 hr	2.2	1.00	0.056 (5.0 hr)
Price and Newman [2001]	March 1979	1 min	1.89	n/a	n/a
Mean $\pm\ \sigma$			2.4 ± 0.26	1.0 ± 0.10	

and time dependent. In terms of the sandpile model, the scalar field of the Lu model represents the height or gradient at each cell, the variable diffusion represents the avalanche rule and the source term represents the rain of sand. [§]

Power spectra of this model are shown in *Klimas et al.* [2000] for varying levels of the source term and the diffusion operator. As in the spectra of the running sandpile for varying driving rates, these spectra exhibit a wide range of behavior. Also, as in the sandpile model, the same physics from the same system produces these different spectra. This says that a system in a state of SOC can show very different signatures depending upon the drive regime in which it operates.

Concerning the anticorrelated region of the spectra, reference *Klimas et al.* [2000] states that the comparison of that model with the running sandpile appears to fail in this region because no system-wide discharges where all grid points are simultaneously unstable are seen in the continuous model. Instead, the largest events observed show wave-like behavior in the hydrodynamic regime. This behavior is, in fact, consistent with the sandpile model because of the following.

We have performed additional sandpile runs *Woodard et al.* [2004] that append the results of *Hwa and Kardar* [1992] and show that all sites are rarely, if ever, simultaneously unstable. System-wide discharges do not refer to a single time step where all cells are unstable. Instead, these large events occur over a short period of time. Recall that sand only exits the sandpile through the bottom cell, which has a small and fixed amount of sand, $N_{\rm f}$, that it can transport at a single time step. The nature of the sandpile is such that, in what is called the $N_{\rm f}/2$ limit, a cell in the middle of a spatially extended avalanche alternates between stable and unstable until the avalanche has either washed passed it or died. The drive regimes studied by *Hwa and Kardar* [1992] and *Woodard et al.* [2004] are well below the $N_{\rm f}/2$ limit. Furthermore, animated visualizations *Woodard* [2004] of the sandpile show that these system-wide discharge events are really a series of many avalanches in a short period of time and that they are very wave-like in nature. So then the observed behavior of the Lu model discussed in *Klimas et al.* [2000] is consistent with that of the running sandpile. This is another motivation for looking for similar SOC regions in physical data, as we next discuss.

[§] In the interest of cross-field communication, we mention that a very similar model to that of *Lu* [1995] has been applied to the study of the running sandpile model in the context of plasma transport in confined plasma devices *Garcia et al.* [2002]. Citations within that work refer to other studies of sandpile dynamics in confined plasmas.

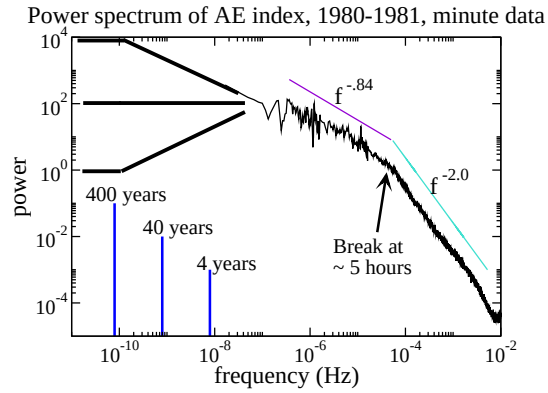

Figure 5. Power spectrum of AE index 1980-1981 and cartoon of time scales of additional spectrum.

5. Effect of Length of Time Series

The comparison of regions between the AE index and the sandpile model end at region B because of the limited time series for the AE index. The question arises: How long must the AE index time series be in order to see a new region at lower frequencies? This question assumes that the long-term process that drives the Sun-Earth system and that some properties of the Sun, magnetosphere and the space in between remain somewhat constant.

This question also assumes that the lower region seen in the spectrum of the AE index *will* end. Of course, the lifetime of a star ensures this on the longest time scales but this ending falls outside the bounds of the first assumption. We claim that this region will end for dynamical reasons. The positive slope $\beta > 0$ of a spectral region and Hurst exponent $H > 0.5$ imply long time correlations. But the correlations must end on some time scale because a system of finite size driven randomly will reflect the random drive at the longest time scales *Woodard et al.* [2004].

Given a steady state Sun-Earth system and a randomly driven yet deterministic universe, then the correlations must end at some time scale. Anticorrelations may arise in this system in the same way that they do in the running sandpile: all gradients in a finite system eventually grow close to critical and then trigger a system-wide discharge. This speaks to the finite capacity of the system. Perhaps an energy and mass balance calculation between the fluctuations of the input from the solar wind and output through the magnetotail, magnetopause and other parts of the system can give an estimate of a time scale on which the magnetosphere reaches capacity.

Figure 5 shows the same spectrum of the AE index as that in Figure 1 but now we have drawn lines showing possible behavior of the spectrum at longer time scales based on the behavior seen in sandpile spectra. We adamantly state here: the breakpoints and slopes of these lines are not predictions of breakpoints and slopes that would be seen given arbitrarily long AE time series. Instead, they are drawn to demonstrate how long the time series would have to be to see this behavior. At best, we now have roughly 40 years of AE index data. Since we look at scaling behavior on log-log axes, the full 40 years of data would only slightly extend the plot to the point labeled. An order of magnitude greater time series, 400 years, would extend the plot as shown. Observation of the spectra of sandpile models, AE index and other physical systems shows that nature does not cut her spectra off abruptly at low frequency. This is, of course, a rough and qualitative argument. But we do expect to see the spectrum of the AE index roll over to, possibly, an anticorrelated region with $\beta < 0$ and/or, definitely, a flat, uncorrelated region. Based on the slope of the best current spectra and longest available time series, we will not be around to see that new region.

6. Conclusions

We have shown that a rich diversity of spectral signatures can be produced by a self-organized critical system when the effective driving rate is changed. This has implications in the ongoing investigations of studying SOC as a model of various physical systems, including the Sun-Earth system. Our results show that failure of the power spectrum of a physical system to match that of the running sandpile model with a certain driving rate does not at all exclude SOC as an appropriate model for the system. A system can be in a wide range of drive regimes yet still be considered SOC even though the spectral signatures differ among the different regimes. This is a statement on the lack of tuning that is required for a SOC system to be critical; it is the self-organized portion of the name.

The system remains critical because the physics does not change when the driving rate changes. In the sandpile, changing the driving rate merely changes how fast or how slowly sand is added to the pile. The rules of the system have not changed. A memory of past events is stored in the local gradients of the system regardless of how fast or slowly sand is added. The analogy to a physical system is this: the underlying physics of the transport of a system, such as the Sun-Earth system, does not change when it is driven more strongly or weakly. Gradients still grow and shrink by the fundamental physical processes. But the rate at which this happens changes and this change is reflected in the different spectra of a process over different periods of time.

We also discussed that the current longest available time series of the AE index only shows two regions in the power spectrum. Inevitably, the region at lower frequency with $\beta \approx 1$ must either flatten or turn down to an anticorrelated region with $\beta < 0$ before flattening at the lowest frequencies. The flat f^0 spectrum is a signature of a fundamental random process that is driving the system. An anticorrelated region reflects a system of finite size and capacity that non-periodically relaxes in a series of long bursts over a short period of time.

The running sandpile model has a well-defined clock which makes it diffferent from other SOC models. More importantly, this feature, along with its dynamics that is similar to many physical processes, make it an appropriate model to use in the study of physical systems that are suspected to be SOC, such as that of the Sun and Earth.

Acknowledgments

R.W. thanks N. W. Watkins for noticing, H. J. Fletcher for comments and support and W. C. Ferenbaugh for assistance with unit conversion. We also thank the DOE for grants DE-FG03-99ER54551 and DE-FG03-00ER54599

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