All-Orders Spectral RF Calculations Using Local or "Windowed" Fourier Transforms

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Introduction

Full-wave spectral models have been developed for rf heating applications in one, two and three spatial dimensions (1-D, 2-D, and 3-D) [1]:

- These models are based on global Fourier expansions and are valid to all orders in the ratio of the ion Larmor radius to the perpendicular rf wavelength.
- However, because of the global nature of the Fourier basis set, the calculation of local power deposition ($\partial W/\partial t$) and poloidal force (flow drive) can take much longer than the wave solution itself

$$\frac{\partial W}{\partial t} = \frac{1}{2} Re \sum_{\vec{k}_R, \vec{k}_L} e^{i(\vec{k}_R - \vec{k}_L)\cdot \vec{r}} \sum_{l=-\infty}^{\infty} \vec{E}_L^{\,*} \cdot \widetilde{W}_l \cdot \vec{E}_R$$

- Sum over *k* requires 4 nested do loops

In 1-D, it has been shown that an eikonal-based formulation [2] can be used to calculate $\partial W/\partial t$ much more efficiently for certain cases with well-defined modes.

- However, the extension to 2-D is numerically difficult.

Here we investigate an alternate approach using local or "windowed" Fourier transforms

– The rf fields can be expanded in local sets of Fourier basis functions

Global expansion over interval: $0 \rightarrow x_{max}$ and $0 \rightarrow y_{max}$

$$E(x,y) = \sum_{n,\,m} E_{n,m} \; e^{i\,(k_n x_g + k_m y_g)}$$

Local expansion over interval: $X_1 \rightarrow X_2$ and $Y_1 \rightarrow Y_2$

$$E(x,y) = \sum_{\nu,\,\mu} E_{\nu,\mu} \; e^{i\,(k_\nu x_l + k_\mu y_l)}$$

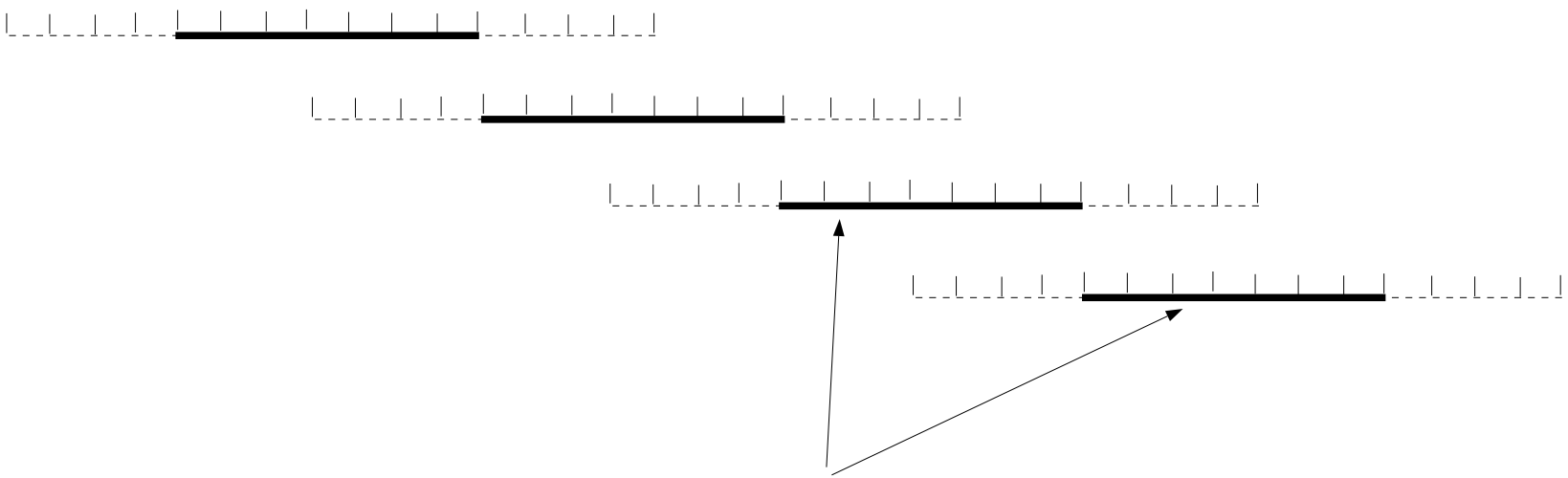
$E_{\nu,\mu}$ can be written in terms of $E_{n,m}$

$$E_{\nu,\mu} = \frac{-1}{(X_2 - X_1)(Y_2 - Y_1)} \sum_{n,m} E_{n,m} \frac{e^{i\,(k_n X_1 + k_m Y_1)}}{(k_m - k_\mu)(k_n - k_\nu)} \left[e^{i\,(k_n - k_\nu)(X_2 - X_1)} - 1 \right] \left[e^{i\,(k_m - k_\mu)(Y_2 - Y_1)} - 1 \right]$$

The local basis sets must be connected by a sufficient number of boundary conditions to ensure continuity

– This can be accomplished by combining a series of spatial intervals that overlap by several Larmor radii.

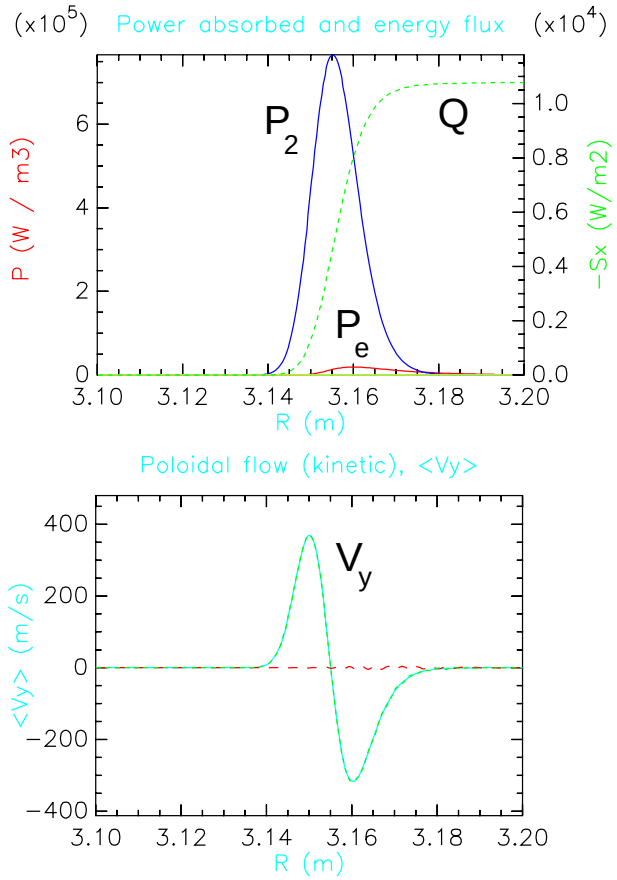
For 16 local modes with 4 overlapped, we have:



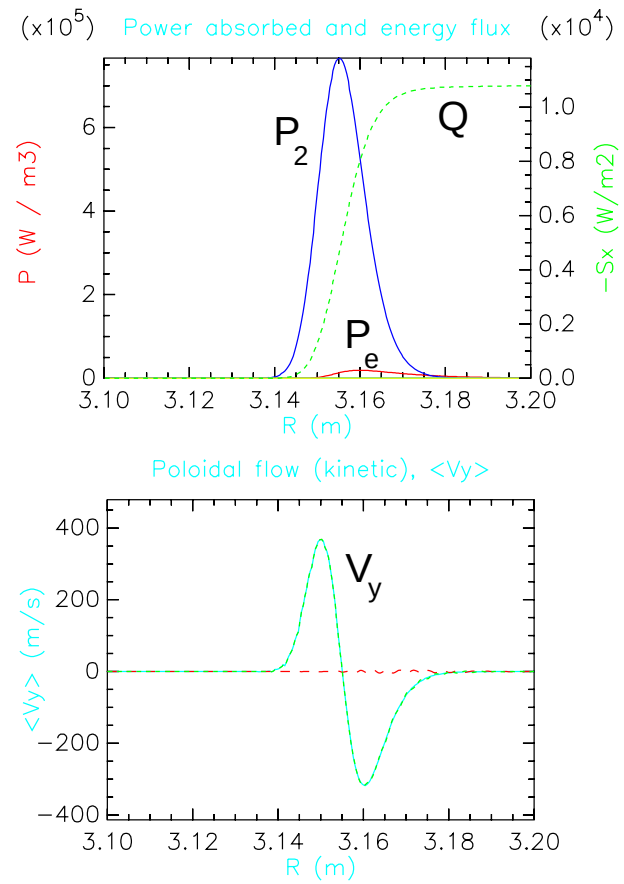
– Using only the interior (non-overlapping) regions, $\partial W/\partial t$ can be accurately calculated over the entire plasma volume.

Results show that with this approach, the time required to calculate $\partial W/\partial t$ is reduced by a factor of 200 in 1-D

Global (1024 global modes)
654.15 min total
632.28 min for wdot

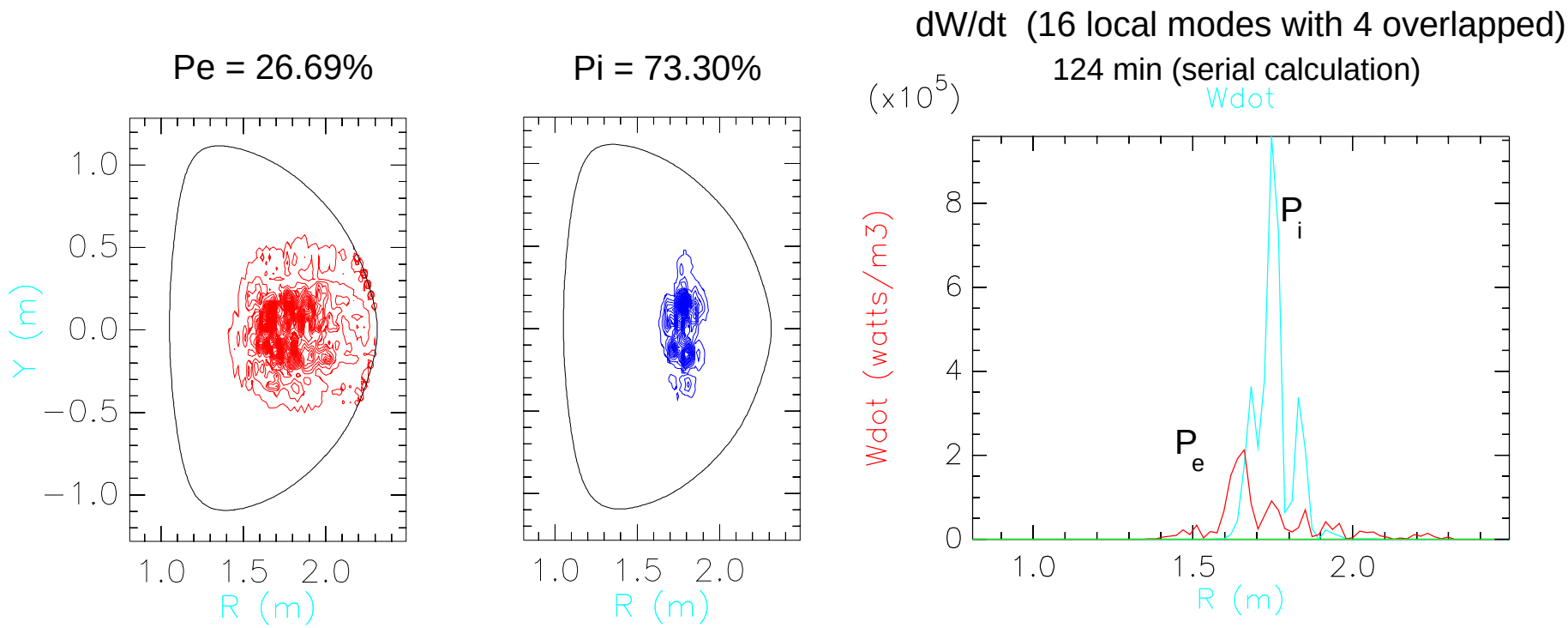


Local (64 local modes with 28 overlaped)
25.175 min total
3.31 min for wdot



The same method is directly applicable in 2-D with a speed-up of about 2000

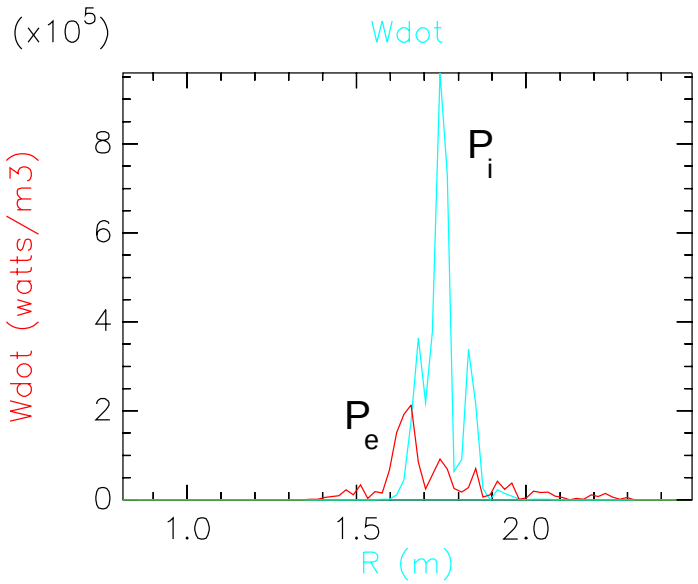
ASDEX: 2nd harmonic D using DIII-D eqdsk, shot #84293
and 80 x 80 modes in *x* and *y*



The ∂W/∂t calculation with windowed transforms can be easily parallelized

ASDEX: 2nd harmonic D with 80 x 80 modes in x and y
16 local modes with 4 overlapped are used for Wdot calculation

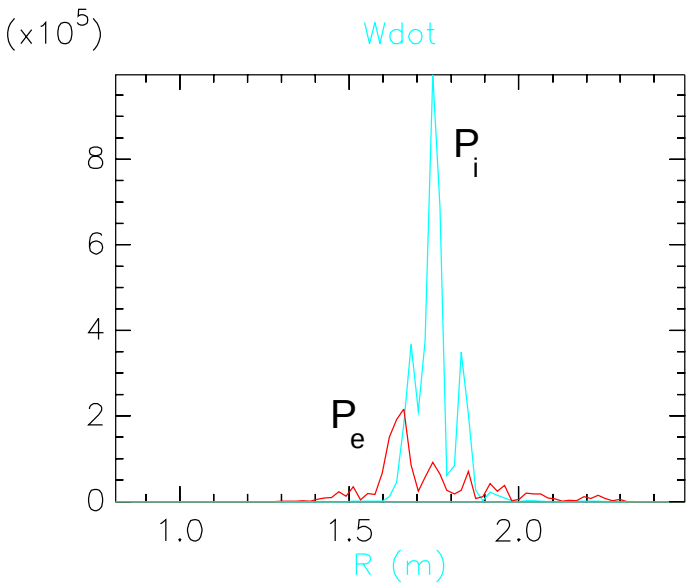
Serial (124 min)



Pe = 26.69%

Pi = 73.30%

Parallel (6.0 min)
(16 processors)



Pe = 26.31 %

Pi = 73.69 %

The poloidal force (flow drive) calculation can be generalized to 2-D

1-D result from *Phys. Fluids* 8, 3319 (2000), Eq. (20):

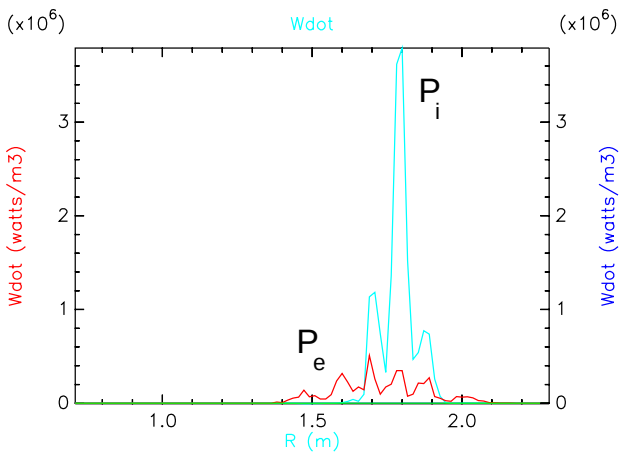
$$(F_{EM} - \nabla \cdot P_2)_y = \frac{\partial}{\partial x}\left[\frac{1}{2} Re \sum_{n,m} e^{i(\vec{k}_n - \vec{k}_m)\,x} \sum_{l=-\infty}^{\infty} \frac{l}{2\omega}\left(\vec{E}_m^{\,*} \cdot \widetilde{W}_l \cdot \vec{E}_n\right)\right]$$

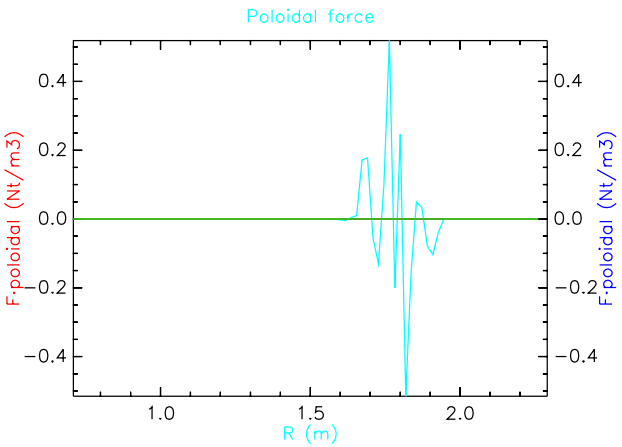
Generalize to 2-D: $\frac{\partial}{\partial x} \rightarrow \hat{\rho} \cdot \nabla = \hat{\rho}_x \frac{\partial}{\partial x} + \hat{\rho}_y \frac{\partial}{\partial y}$

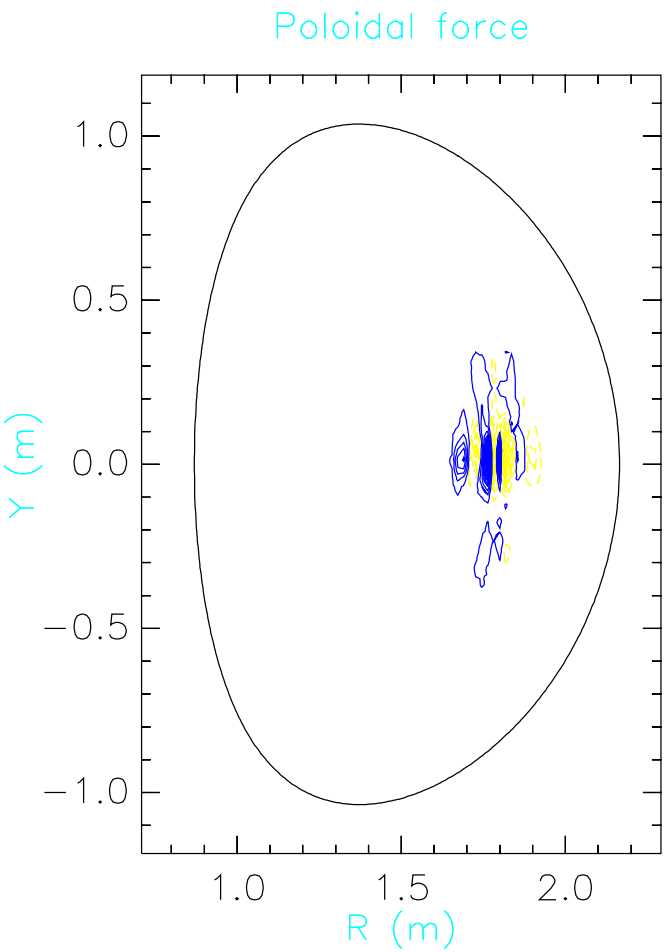
$$(F_{EM} - \nabla \cdot P_2)_\vartheta = \hat{\rho} \cdot \nabla \left[\underbrace{\frac{1}{2} Re \sum_{\vec{k}_R, \vec{k}_L} e^{i(\vec{k}_R - \vec{k}_L)\cdot \vec{r}} \sum_{l=-\infty}^{\infty} \frac{l}{2\omega}\left(\vec{E}_L^{\,*} \cdot \widetilde{W}_l \cdot \vec{E}_R\right)}_{\text{Similar to } \partial W/\partial t \text{ in 2-D:}}\right]$$

The new windowed $\partial W/\partial t$ calculation allows us to calculate the poloidal force (flow drive) in 2-D

ASDEX: 2nd harmonic D with 88 x 88 modes in x and y
16 local modes with 4 overlapped are used for $\partial W/\partial t$ and flow drive calculation







Future work

In addition, windowed Fourier transforms can be used as a basis set for the numerical wave field solution itself.

- The advantage is that these transforms yield banded rather than dense solution matrices
- In principle, they can be used to increase resolution for localized areas in space.

References:

[1] E. F. Jaeger, L. A. Berry, et. al., Phys. Plasmas **7**, 3319 (2000); Phys. Plasmas 8, 1573 (2001); Phys. Plasmas 9, 1873 (2002).

[2] J. R. Myra and D. A. D'Ippolito, Phys. Plasmas 7, 3600 (2000).