LINEAR HARMONIC ANALYSIS FOR STIRLING MACHINES AND SECOND LAW ANALYSIS OF FOUR IMPORTANT LOSSES*

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ABSTRACT

A simple thermodynamic theory for Stirling machine performance has been developed. By representing variables in terms of harmonic oscillations and representing the nonharmonic terms in the conservation equations with truncated Fourier series, the equations can be solved in a semi-closed form, leading to a better understanding of Stirling engine behavior. The theory further includes a Second Law analysis; therefore, the efficiency and power losses resulting from effects of adiabatic cylinders, transient heat transfer, pressure drop, and seal leakage can be allocated unambiguously, and the degree of loss coupling can be assessed.

NOMENCLATURE

A_s	heat-transfer surface area (m^2)
c_p	specific heat at constant pressure [J/(kg·K)]
c_v	specific heat at constant volume [J/(kg·K)]
h	cylinder-to-gas heat-transfer coefficient [W/(m^2·K)]
k_m	mass leakage coefficient [kg/(Pa·s)]
k_p	pressure drop coefficient [(Pa·s)/kg]
m	mass of gas (kg)
P	pressure (Pa)
$\dot{Q}_{in}$	total heat input (W)
$\dot{Q}_{out}$	total heat output (W)
R	gas constant [J/(kg·K)]
T	temperature (K)
$\lvert T \rvert$	amplitude of temperature fluctuation
T_f	temperature for enthalpy flux (K)
$\overline{T}_{in}$	average gas temperature at which heat is absorbed into the system (K)
$\overline{T}_{out}$	average gas temperature at which heat is rejected from the system (K)
t	time (s)
V	volume (m^3)
$\dot{W}$	PV work (W)
y_n	n=1,2,..., amplitudes of thermodynamic variable harmonic components

Superscripts

•	time rate of change
—	average over a cycle

Subscripts

c	compression space
d	dead space
e	expansion space
H	heater
K	cooler
w	working space or wall

Greek

$\Delta\dot{S}_{loss}$	entropy production by internal irreversibilities (W/K)
$\Delta\dot{S}_{mix}$	entropy production by mixing (W/K)
$\Delta\dot{S}_{ML}$	entropy production by mass leakage (W/K)
$\Delta\dot{S}_{FD}$	entropy production by pressure drop (W/K)
$\Delta\dot{S}_{Q}$	entropy production by heat transfer (W/K)
$\Delta\dot{S}_{Qw}$	entropy production by cylinder heat transfer (W/K)
$\Delta\dot{S}_{TQ}$	entropy production by transient heat transfer (W/K)

$\Delta\eta_{EHT}$	efficiency loss by external heat-transfer irreversibility (%)
$\Delta\eta_{loss}$	efficiency loss by internal irreversibilities (%)
η_{it}	indicated thermal efficiency (%)
η_{ca}	Carnot efficiency (%)
ω	frequency (rad/s)

INTRODUCTION

This paper presents a simple Stirling cycle analysis that allocates efficiency and power losses unambiguously to each of four loss mechanisms acting simultaneously. The engine thermodynamics are formulated by a control volume approach, and the resulting equations are solved by a linear harmonic analysis. The losses are allocated by a Second Law analysis. Although recent work has extended the analysis to include the dynamics of free-piston machines, this paper describes only the thermodynamic analysis. The engine used for the numerical example is essentially the Sunpower RE-1000 with the piston motions specified.

Four important loss mechanisms that significantly affect the engine performance are adiabatic cylinder effects, transient heat-transfer loss in semi-adiabatic cylinders, pressure drop across the heat exchangers, and gas leakage from the compression space. These four loss mechanisms are characterized by irreversible thermodynamic processes that occur when heat is transferred across a finite temperature difference, when gases at two different temperatures are mixed, or when there is a mass flow through a pressure difference. Each one contributes to a reduction of efficiency below the Carnot level.

Numerous cycle analyses with various complexities exist. Schmidt's (1) isothermal analysis with sinusoidal volume variations provides a closed-form solution and establishes a standard for Stirling engine theoretical studies. However, it is far from representing real engines because the gas inside the cylinders behaves more adiabatically than isothermally. Unfortunately, if any departures from the isothermal assumptions are made, then the equations describing the gas behavior become a nonlinear set of differential equations. Finkelstein (2) was the first to study the effects of adiabatic cylinders. Subsequently, Qvale (3), Rios (4), West (5), and Urieli and Berchowitz (6) all have contributed to the understanding of engine performance under various loss mechanisms. But surprisingly enough, none of these studies [except Ref. (5)] offers a nonnumerical solution.

In contrast, the linear harmonic analysis provides a semi-closed-form solution to the governing equations. It can greatly enhance theoretical understanding of Stirling engine because the cause and effect are related explicitly by nature of the closed-form solution. The linear harmonic analysis involves linearization of the waveforms and represents each term in the conservation equations, including the enthalpy flux discontinuity in the cylinder, by a truncated Fourier series.

In conjunction with the linear cycle analysis, the Second Law analysis provides a rational method for allocating overall efficiency losses to different loss mechanisms; it does this despite the fact that all the loss mechanisms are allowed to interact with each other and thereby combines some of the advantages of the second- and third-order methods. Through the use of Maxwell relations, the Second Law analysis can define precisely each irreversibility in terms of entropy production. Further, the Second Law analysis can relate entropy production to efficiency losses by linear harmonic analysis. Moreover, the Second Law analysis can show the degree of coupling when two or more mechanisms act at the same time.

FORMULATION AND LINEAR HARMONIC ANALYSIS

The engine under study is shown schematically in Fig. 1; it is a free-piston engine of the Beale type with the displacer sprung to ground. The buffer space contains a large volume of gas to maintain a nearly constant pressure. A dashpot-loading device is used to absorb power. This is similar to the RE-1000 engine tested by National Aeronautics and Space Administration-Lewis Research Center (NASA-LERC) (7). Some important operating conditions and basic dimensions are shown in Tables 1 and 2.

Table 1. RE-1000 nominal engine operating conditions

Working fluid	Helium
Frequency, Hz	30
Average pressure, MPa	7
Piston stroke, m	2.80×10^{-2}
Displacer stroke, m	2.80×10^{-2}
Power, W	1000
Displacer phase angle, deg	45
Heater temperature, K	900
Expansion cylinder wall temperature, K	900
Cooler temperature, K	300
Compression cylinder wall temperature, K	300

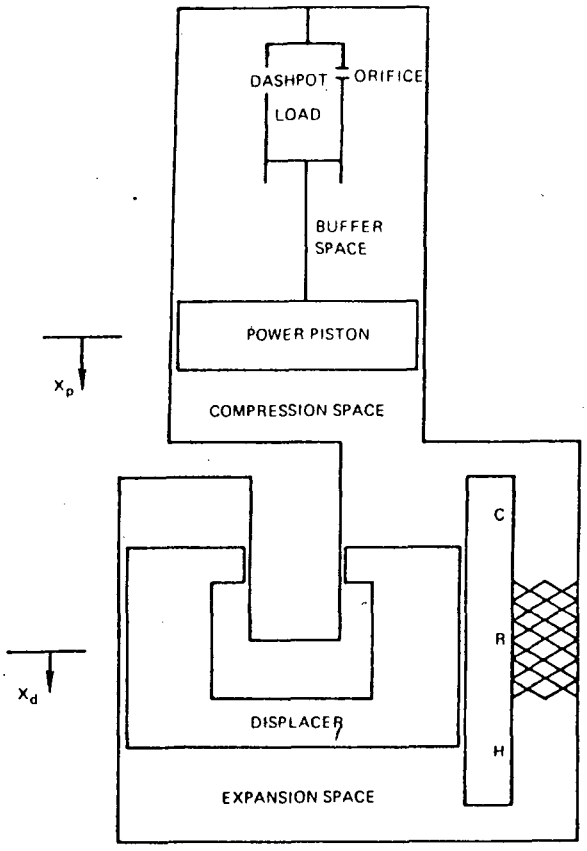

Fig. 1. Schematic of the RE-1000 free-piston Stirling engine.

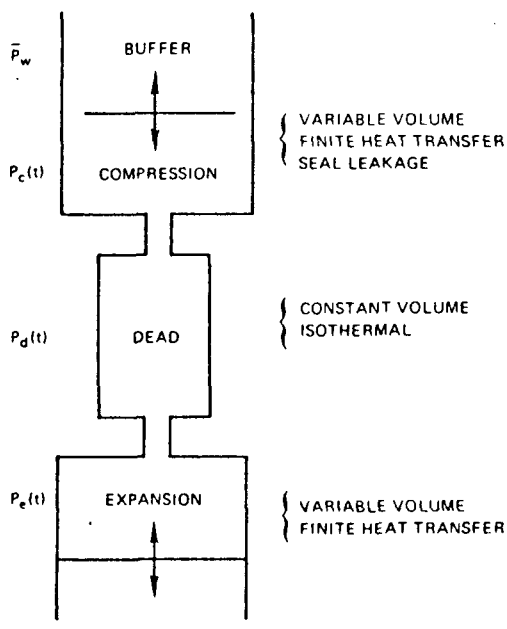

Fig. 2. Control volume representation of the RE-1000 engine.

Table 2. RE-1000 nominal engine dimensions and parameters

Maximum displacer stroke, m	4.04×10^{-2}
Maximum piston stroke, m	4.20×10^{-2}
Piston diameter, m	5.723×10^{-2}
Displacer diameter, m	5.723×10^{-2}
Displacer rod diameter, m	1.66×10^{-2}
Mean volume, m^3	
Expansion space	6.36×10^{-5}
Heater	3.96×10^{-5}
Regenerator	5.94×10^{-5}
Cooler	2.85×10^{-5}
Compression space	1.036×10^{-4}
Mean heat-transfer surface areas	
Expansion space, m^2	1.392×10^{-2}
Compression space, m^2	2.292×10^{-2}

For the analysis, the working space is divided into three control volumes depicted in Fig. 2: an expansion space, a compression space, and a dead volume. The volumes of the expansion and compression spaces are time varying and nonisothermal. On the other hand, the dead volume that consists of the heater, regenerator, and cooler is fixed and assumed to be isothermal. In addition, mass leakage is permitted between the compression and buffer spaces.

Five variables are chosen to define the thermodynamic states of the engine at any instant: the mass in each control volume and the instantaneous gas temperature in the cylinders. These five time-dependent unknowns require five governing equations that are provided by the conservation laws of mass and energy. The mass flow rate between one volume and another is assumed to be linearly proportional to the pressure difference between them. Thus, the mass conservation equations in the expansion, combined expansion and dead, and working spaces, respectively, are:

$$\frac{dm_e}{dt} = \frac{1}{k_{pe}} (P_d - P_e) , \tag{1}$$

$$\frac{dm_e}{dt} + \frac{dm_d}{dt} = \frac{1}{k_{pc}} (P_c - P_d) , \tag{2}$$

$$\frac{dm_e}{dt} + \frac{dm_c}{dt} + \frac{dm_d}{dt} = k_{mc} \cdot (\bar{P}_w - P_c) . \tag{3}$$

The first two equations represent pressure drop as a function of mass flow rate; the third represents mass leakage as a function of pressure difference.

By the first law of thermodynamics, the energy equations in the expansion and compression spaces are respectively:

$$c_p T_{fe} \frac{dm_e}{dt} + h_e \bar{A}_{se} (\bar{T}_{we} - T_e) = P_e \frac{dV_e}{dt} + c_v \frac{d(m_e T_e)}{dt} , \quad (4)$$

$$c_p T_{fc} \frac{dm_c}{dt} + h_c \bar{A}_{sc} (\bar{T}_{wc} - T_c) = P_c \frac{dV_c}{dt} + c_v \frac{d(m_c T_c)}{dt} , \quad (5)$$

where

$$T_{fe} = \begin{cases} T_H , & \text{if } \dot{m}_e > 0 \\ T_e , & \text{if } \dot{m}_e < 0 \end{cases} \quad (6)$$

and

$$T_{fc} = \begin{cases} T_K , & \text{if } \dot{m}_c > 0 \\ T_c , & \text{if } \dot{m}_c < 0 \end{cases} . \quad (7)$$

The first term in the left-hand side of Eqs. (4) and (5) represents the enthalpy flux discontinuity. When the gas flows out of the heat exchanger and enters the cylinder, it does so at the heat exchanger temperature (if the heater and cooler are perfect). When the gas flows out of the cylinder, it does so at the instantaneous gas temperature within the cylinder. It is the representation of this discontinuity by a truncated Fourier series that gives our approach the means to represent nonisothermal cylinders with semi-closed-form solutions. The second term in the left-hand side represents the effect of heat transfer between the cylinder wall and the gas inside the cylinder. In the analysis, the cylinder wall temperature and surface area are assumed constant. The first and second terms in the right-hand side are the PV work done by the gas and the rate of change of gas internal energy, respectively.

We seek only the steady-state solution of the governing equations, and the linear harmonic method is suitable for this. In such an analysis, it is assumed that the relevant variables can be adequately represented by a sum of sine and cosine terms; if the variables are indeed approximately sinusoidal, the accuracy of a linearized harmonic solution will be higher, the smaller the amplitudes of the variables are compared with their mean values. The methodology of the linear harmonic analysis has been explained elsewhere (8); therefore, no detail will be provided other than a summary. The linear harmonic analysis can be summarized by a nine-step procedure:

1. Divide the working space into control volumes.
2. Write the conservation equations (mass, momentum, and energy) for control volumes.
3. Assume that P, m, T, and V have small enough amplitudes relative to their mean values, and use a linear form of the ideal gas law to express P in terms of m, T, and V. Express V in terms of specified piston positions.
4. A complete transient solution is not pursued: the linear harmonic analysis method is restricted to steady-state solutions. Therefore, assume harmonic solutions for m and T in terms of undetermined coefficients (y_1, y_2, etc.), and substitute them into the governing equations.
5. Represent nonharmonic terms in the governing equations by truncated Fourier expansions.
6. The general form of each governing equation is now:
   Σ constant term + Σ sin (ωt) terms + Σ cos (ωt) terms = 0.
   A solution exists for all times only if:
   Σ constant term = 0
   Σ sin(ωt) terms = 0
   Σ cos(ωt) terms = 0
7. Most of the sin(ωt) and cos(ωt) terms are linear functions of the undetermined coefficients (y_1, etc.). Nonlinear combinations of y_1, y_2, etc., are treated in a quasi-linear manner. The resulting linear system of algebraic equations is solved using standard matrix theory to find the values of y_1, y_2, etc.
8. After y_1, y_2, etc. are obtained, the constant terms in the governing equations are solved to compute the values of other unknown parameters such as $\bar{T}_e$, $\bar{T}_c$, etc.
9. A few iterations may be needed because of coupling between $\bar{T}_e$, $\bar{T}_c$, etc., in (8) and the quasi-linear approximations in (7).

THE SECOND LAW ANALYSIS

As required by the steady-state solution under consideration, the entropy of the system should be conserved over a cycle. Thus,

$$\frac{\bar{\dot{Q}}_{in}}{\bar{T}_{in}} + \Delta\bar{\dot{S}}_{loss} + \frac{-\bar{\dot{Q}}_{out}}{\bar{T}_{out}} = 0 . \quad (8)$$

This is the general entropy equation where $\bar{T}_{in}$ and $\bar{T}_{out}$ are the average temperatures at which the heat enters and leaves the system. In addition, the cyclic entropy production ($\Delta\bar{\dot{S}}_{loss}$) refers to the sum of all internal losses due

to mixing, transient heat transfer, pressure drop, and mass leakage.

Efficiency losses

The indicated thermal efficiency (9) is defined as

$$\eta_{it} = 1 + \frac{-\bar{\dot{Q}}_{out}}{\bar{\dot{Q}}_{in}} . \qquad (9)$$

Substitute $\bar{\dot{Q}}_{out}$ from Eq. (8) into Eq. (9), then add and subtract a term involving the temperature ratio of heater and cooler, resulting in

$$\eta_{it} = \eta_{ca} - \Delta\eta_{EHT} - \Delta\eta_{loss} , \qquad (10)$$

where

$$\eta_{ca} = 1 - T_K/T_H , \qquad (11)$$

$$\Delta\eta_{EHT} = \bar{T}_{out}/\bar{T}_{in} - T_K/T_H , \qquad (12)$$

$$\Delta\eta_{loss} = \bar{T}_{out}\ \overline{\Delta\dot{S}}_{loss}/\bar{\dot{Q}}_{in} . \qquad (13)$$

Equation (10) states that the efficiency terms are additive and that the reference efficiency is the Carnot value. All the efficiency losses should be deducted from the Carnot value that is represented by the first term. The second term in Eq. (10) refers to the efficiency reduction due to external heat transfer. The external heat transfer is an irreversible thermodynamic process with four distinct components: heater, cooler, expansion- and compression-cylinder average heat transfer. The corresponding cyclic entropy productions for each component are derived from Maxwell relations (8) at a constant pressure and are listed in Table 3.

The last term in Eq. (10) represents the efficiency reduction caused by internal losses that are expressed in terms of entropy production and are also shown in Table 3. It is of great importance to note that $\bar{T}_{out}$ is the temperature to be used in the efficiency loss calculations.

Referring to Table 3, we see that adiabatic cylinders result in both external heat transfer and mixing. Heater (cooler) heat transfer takes place when gas flows out of the expansion (compression) space and enters the heater (cooler): in general, the gas will be at a different temperature from the heat exchanger that it is entering, leading to an irreversibility. Similarly, there is over a complete cycle an average heat transfer between the cylinder wall and the gas in the cylinders, which are not at the same temperature. In addition to these two external heat-transfer losses, the mixing of gas leaving the heater or cooler with gas already in the adjacent cylinder, at a different temperature, is characterized as an internal loss.

In addition to the average flow of heat between cylinder wall and gas, the time-dependent or transient heat transfer results in entropy increase because giving away heat at a high temperature and regaining it later in the cycle at a lower temperature is an irreversible thermodynamic process. The approximate formulas listed in Table 3 reveal that the cyclic entropy production due to transient heat transfer is proportional to the heat-transfer coefficient, the total heat-transfer surface area, and the square of the relative temperature amplitudes. The formula shows that no entropy is produced for either perfectly adiabatic or perfectly isothermal conditions. The physical reason is simple: in an adiabatic cylinder, there is a temperature difference between the gas and the wall, but (by definition) no heat is exchanged; in an isothermal cylinder heat is exchanged, but there is no temperature difference. There is, of course, a "worst case" between perfectly adiabatic and perfectly isothermal conditions.

Mass flowing across a pressure drop is an irreversible thermodynamic process. In the present study, pressure drop through the heat exchanger is represented by two throttling processes: one between the expansion and dead volume, the other between the dead and compression volumes. During a throttling process the enthalpy remains unchanged, and for an ideal gas the temperature remains unchanged too. Therefore, by use of the Maxwell relations, the resulting cyclic entropy production due to pressure drop is shown in Table 3.

Mass leakage between the compression space where pressure fluctuates and the buffer space where pressure stays constant is another irreversible thermodynamic process. Such a process is irreversible because mass flows out at a high pressure and returns at a low pressure. The cyclic entropy change associated with this process is no different in principle from that of pressure drop and is included in Table 3.

Power losses

Besides the efficiency loss, the concept of power loss is useful; in particular, the entropy change relates various losses to the energy balance. This can be calculated if Eq. (10) is multiplied by $\bar{\dot{Q}}_{in}$. Thus,

$$\bar{\dot{W}}_{out} = \bar{\dot{Q}}_{in}\ \eta_{ca} - \bar{\dot{Q}}_{in}\ \Delta\eta_{EHT} - \bar{T}_{out}\ \overline{\Delta\dot{S}}_{loss} . \qquad (14)$$

Table 3. Cyclic entropy production due to 11 basic irreversible thermodynamic processes

Irreversible process	Cyclic entropy production (W/K)
External heat transfer	
Heater heat transfer[a]	$\overline{\Delta\dot{S}_{QH}} = \frac{c_p}{2\pi}\int^{b} \frac{dm_e}{dt} \ln\left(\frac{T_e}{T_H}\right) d(\omega t)$
Cooler heat transfer[a]	$\overline{\Delta\dot{S}_{QK}} = \frac{c_p}{2\pi}\int^{b} \frac{dm_c}{dt} \ln\left(\frac{T_c}{T_K}\right) d(\omega t)$
Expansion-cylinder average heat transfer[a]	$\overline{\Delta\dot{S}_{Qwe}} \approx h_e\overline{A}_{se}\left(\frac{\overline{T}_{we}}{\overline{T}_e} - 1\right)$
Compression-cylinder average heat transfer[a]	$\overline{\Delta\dot{S}_{Qwc}} \approx h_c\overline{A}_{sc}\left(\frac{\overline{T}_{wc}}{\overline{T}_c} - 1\right)$
Internal loss	
Mixing in expansion cylinder[c]	$\overline{\Delta\dot{S}_{mixe}} = \frac{c_p}{2\pi}\int^{d} \frac{dm_e}{dt}\left[\ln\left(\frac{T_e}{T_H}\right) + \frac{T_H}{T_e} - 1\right] d(\omega t)$
Mixing in compression cylinder[c]	$\overline{\Delta\dot{S}_{mixc}} = \frac{c_p}{2\pi}\int^{d} \frac{dm_c}{dt}\left[\ln\left(\frac{T_c}{T_K}\right) + \frac{T_K}{T_c} - 1\right] d(\omega t)$
Transient heat transfer in expansion cylinder[a]	$\overline{\Delta\dot{S}_{TQe}} \approx \frac{h_e\overline{A_{se}}}{2}\left(\frac{\lvert T_e\rvert}{\overline{T}_e}\right)^2$
Transient heat transfer in compression cylinder[a]	$\overline{\Delta\dot{S}_{TQc}} \approx \frac{h_c\overline{A_{sc}}}{2}\left(\frac{\lvert T_c\rvert}{\overline{T}_c}\right)^2$
Pressure drop across expansion and dead spaces[e]	$\overline{\Delta\dot{S}_{PDe}} = \frac{R}{2\pi}\int_0^{2\pi} \frac{dm_e}{dt} \ln\left(\frac{P_d}{P_e}\right) d(\omega t)$
Pressure drop across compression and dead spaces[e]	$\overline{\Delta\dot{S}_{PDc}} = \frac{R}{2\pi}\int_0^{2\pi} \left(\frac{dm_e}{dt} + \frac{dm_d}{dt}\right) \ln\left(\frac{P_c}{P_d}\right) d(\omega t)$
Mass leakage across compression and buffer spaces[e]	$\overline{\Delta\dot{S}_{MLc}} = \frac{R}{2\pi}\int_0^{2\pi} \left(\frac{dm_e}{dt} + \frac{dm_d}{dt} + \frac{dm_c}{dt}\right) \ln\left(\frac{\overline{P}_w}{P_c}\right) d(\omega t)$

[a]Heat transfer across a finite temperature difference.

[b]Integration over the half cycle where the mass of gas leaves the cylinders and enters the adjacent heat exchangers.

[c]Mixing gases at two different temperatures.

[d]Integration over the half cycle where the mass of gas leaves the heat exchangers and enters the adjacent cylinders.

[e]Mass flowing over a pressure difference.

The first term on the right-hand side of Eq. (14) represents the maximum possible power output of a Carnot cycle that has a heat input $\bar{Q}_{in}$. This term $\bar{Q}_{in}\eta_{ca}$ is also the basic power output that can be defined as the reference value. However, $\bar{Q}_{in}$ strongly depends on the losses present and varies from case to case (see loss coupling in Tables 5 and 6 to be discussed later). The second term signifies the power loss due to the external heat-transfer irreversibility, and the last terms indicate power loss due to internal irreversibilities.

SPECIFIC RESULTS AND DISCUSSIONS

Combined Efficiency Effects of Adiabatic and Transient Heat Transfer Losses

The linear harmonic analysis supplemented with the Second Law analysis has been applied to the RE-1000 nominal engine to quantify the efficiency loss allocations. Figure 3 presents the effect on efficiency of the combined adiabatic and transient heat-transfer effects for a wide range of heat-transfer coefficients. In the figure, there are four curves. Curve 1, labeled THT, which is bell-shaped, represents the transient heat-transfer efficiency loss. There is no transient heat-transfer loss in either adiabatic (left side of plot) or isothermal (right side of plot) cylinders, but there is a maximum efficiency loss (worst case) in between.

Curve 2, marked EHT, represents the effects resulting from the external heat transfer for nonisothermal cylinders. This effect is most significant at the adiabatic limit and decreases gradually as heat transfer increases, because the heat transfer works to reduce the temperature amplitude within the cylinder.

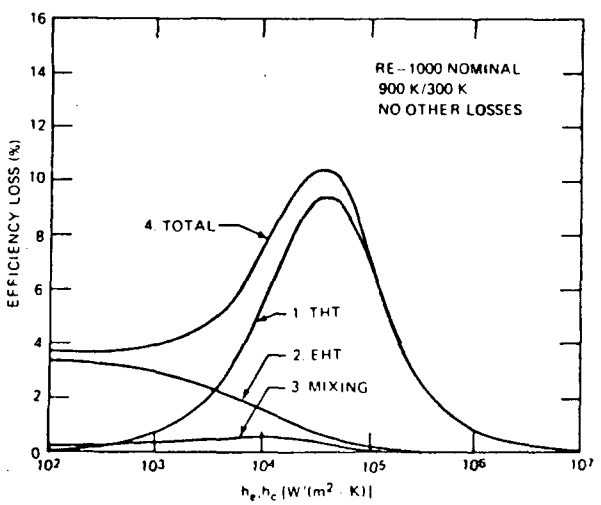

Fig. 3. Efficiency loss allocation versus cylinder heat transfer.

Curve 3 represents the effect of mixing. For this engine at least, mixing is less important than the external heat-transfer loss by almost an order of magnitude at the adiabatic limit. Note that mixing reaches a maximum away from the adiabatic limit; this can be attributed to the temperature overshoot in the expansion space and undershoot in the compression space.

Finally curve 4, labeled total, represents the sum of these individual loss mechanisms and shows the overall efficiency loss.

In summary, the following general conclusions concerning this particular engine size may be stated:

1. external heat-transfer irreversibilities predominate in the range of small heat-transfer coefficients,
2. transient heat transfer is by far the larger effect in the range of intermediate heat-transfer coefficients, and
3. temperature mixing losses remain relatively small in magnitude over the whole range of heat-transfer coefficients.

In addition, a specific conclusion is that in the RE-1000, which is believed to have a heat-transfer coefficient of about 1000 $W/(m^2 \cdot K)$ in both spaces at nominal operating conditions, the efficiency loss contributions resulting from external heat transfer, transient heat transfer, and mixing are in the ratio of approximately 10:2:1.

Coupling of Loss

The degree of coupling between various loss mechanisms is an important issue for numerical analysts seeking to make a choice between second- and third-order codes. The question can be addressed by the Second Law analysis. An extensive study considering a wide range of loss coefficients has been performed. The loss coefficients of 12 cases studied are listed in Table 4. In the table,

Table 4. Loss coefficients for coupling studies

Loss mechanism	Case	Coefficient
Cylinder heat transfer	None	0 $W/(m^2 \cdot K)$
	Medium	62,500 $W/(m^2 \cdot K)$
	High	3×10^8 $W/(m^2 \cdot K)$
Pressure drop	None	0 $(Pa \cdot s)/kg$
	High	6.7×10^6 $(Pa \cdot s)/kg$
Seal leakage	None	0 $kg/(Pa \cdot s)$
	High	1.0×10^{-8} $kg/(Pa \cdot s)$

the rate constants for both the pressure drop and seal leakage are chosen to exaggerate the coupling effects beyond those to be expected in a well designed and constructed engine. The highlights of the results are summarized in Tables 5 and 6.

The tables reveal some interesting loss interactions. First of all, pressure drop appears to have a significant effect on the other losses. Compare the cases that have seal leakage and no pressure drop with the cases that have both seal leakage and pressure drop. An increased pressure drop causes an increase in the seal leakage power and efficiency losses. These effects hold true for the adiabatic, semi-adiabatic, and isothermal cases. The physical explanation can be attributed to the fact that pressure drop causes the amplitude of the pressure wave in the compression space to increase. Therefore, a larger pressure difference exists across the piston seal and hence leads to a larger leakage loss. In addition, an increased pressure drop tends to increase the transient heat-transfer loss for those cylinders which are semi-adiabatic.

Second, seal leakage appears to have a reverse effect on pressure drop losses. Compare the cases that have pressure drop and no seal leakage with the cases that have both. An increased seal leakage causes a reduction in the pressure drop loss. However, it is difficult to infer a physical reasoning. In fact, the trends discussed so far may not hold true for all Stirling machines. Nevertheless, for the example configuration presented here, the results show rather convincingly that the losses do couple in a highly complex manner.

Table 5. Coupled efficiency loss of RE-1000 type engines

Cylinder heat transfer	Pressure drop	Seal leakage	Carnot efficiency (%)	Adiabatic		THT	Pressure drop	Seal leakage	Indicated thermal efficiency (%)
				EHT[a]	Mixing				
None (adiabatic)	Zero	Zero	66.67	−3.45	−0.25	0.0	−0.00	0.00	62.97
	Zero	High	66.67	−3.51	−0.21	0.0	0.00	−17.72	45.23
	High	Zero	66.67	−3.37	−0.29	0.0	−19.40	0.00	43.61
	High	High	66.67	−3.34	−0.23	0.0	−16.90	−26.18	20.02
Medium	Zero	Zero	66.67	−0.34	−0.18	−8.65	−0.00	0.00	57.50
	Zero	High	66.67	−0.32	−0.19	−8.94	−0.00	−14.01	43.21
	High	Zero	66.67	−0.32	−0.26	−13.12	−20.02	0.00	32.95
	High	High	66.67	−0.30	−0.26	−13.53	−18.17	−22.22	12.14
High (isothermal)	Zero	Zero	66.67	−0.00	−0.00	−0.00	−0.00	0.00	66.67
	Zero	High	66.67	−0.00	−0.00	−0.00	−0.00	−12.49	54.18
	High	Zero	66.67	−0.00	−0.00	−0.00	−22.50	0.00	44.17
	High	High	66.67	−0.00	−0.00	−0.00	−19.83	−20.40	26.44

[a]External heat-transfer irreversibility is made of four components: heater, cooler, expansion-cylinder, and compression-cylinder average heat transfer.

Table 6. Coupled power loss of RE-1000 type engines

Cylinder heat transfer	Pressure drop	Seal leakage	Reference[a] power (W)	Adiabatic		THT	Pressure drop	Seal leakage	Net power (W)
				EHT	Mixing				
None (adiabatic)	Zero	Zero	2636.9	−136.3	−9.9	0.0	−0.0	0.0	2490.7
	Zero	High	2363.7	−124.4	−7.4	0.0	0.0	−628.4	1603.5
	High	Zero	2137.9	−108.1	−9.3	0.0	−622.2	0.0	1398.3
	High	High	1867.6	−93.4	−6.5	0.0	−473.3	−733.5	560.9
Medium	Zero	Zero	2118.9	−10.9	−5.8	−275.0	−0.0	0.0	1827.2
	Zero	High	1904.2	−9.2	−5.3	−255.4	−0.0	−400.3	1234.0
	High	Zero	1622.9	−7.7	−6.3	−319.5	−487.3	0.0	802.1
	High	High	1419.0	−6.4	−5.6	−288.0	−386.7	−472.9	259.4
High (isothermal)	Zero	Zero	2027.0	−0.0	−0.0	−0.10	−0.0	0.0	2027.0
	Zero	High	1914.4	−0.0	−0.0	−0.10	−0.0	−358.7	1555.7
	High	Zero	1507.0	−0.0	−0.0	−0.1	−508.6	0.0	998.3
	High	High	1412.3	−0.0	−0.0	−0.10	−420.2	−432.2	559.8

[a]Reference power is defined as the heat input multiplied by the Carnot efficiency.

Some final comments about the Second Law analysis and loss allocation are in order. Efficiency losses are subtracted from the Carnot efficiency to arrive at an overall efficiency. The Carnot value depends on only the heater and cooler temperatures and represents the highest efficiency an engine can achieve. The 12 cases shown in Table 5 all have the same Carnot value. On the other hand, power losses are subtracted from a reference power output that is defined as the heat input to the engine multiplied by the Carnot efficiency. The reference powers shown in Table 6 vary from case to case because the engine heat inputs, which equal the expansion space pressure-volume integrals, are different. Thus, a comparison of power losses from case to case may be considered less meaningful than an efficiency loss comparison because the reference power outputs vary whereas the reference efficiencies remain the same.

CONCLUSION

A linear harmonic cyclic analysis, combined with a Second Law analysis of losses, provides a straightforward and physically meaningful way to calculate the efficiency and power losses due to various causes, even when several loss mechanisms may be acting simultaneously and interacting with each other. From the calculations on the particular engine that has the dimensions similar to the Sunpower RE-1000, the following observations are made:

1. Nonisothermal cylinders result in both mixing and external heat-transfer irreversibilities, but mixing loss is the smaller by an order of magnitude.
2. Transient heat-transfer loss is zero for both adiabatic and isothermal cylinders and reaches a maximum for intermediate cases.
3. The coupling between pressure drop and mass leakage losses can be significant.

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