EFFECTS OF PRESSURE-DROP CORRELATIONS ON STIRLING-ENGINE PREDICTED PERFORMANCE*

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ABSTRACT

Pressure-drop correlations for Stirling-engine applications were studied from three perspectives: (1) sensitivity studies of their effects on predicted engine performance, (2) presentation of an improved correlation for steady unidirectional flow through wire-screen matrices, and (3) a theoretical analysis of friction factors for oscillating laminar flow through circular tubes. The sensitivity studies of two reference engines show significant effects of regenerator pressure drop on engine performance. The improved friction factor correlation for steady unidirectional flow fits the data satisfactorily with an uncertainty of about ±10%. The derived friction factor for oscillating flow is shown to be characterized by two Reynolds numbers: steady and kinetic. When this friction factor is compared with the one for steady unidirectional flow, no significant difference is observed when the kinetic Reynolds number is small, but there is a considerable increase at high kinetic Reynolds numbers.

NOMENCLATURE

For woven-screen matrix

d	diameter of wire
f	friction factor, $(\Delta P/n)/(\rho U_{max}^2/2)$
ℓ	minimum screen opening, $s - d$
n	number of layers of screens in matrix
Re_ℓ	Reynolds number, $\rho U_{max} \ell/\mu$
s	distance between centerlines of wires in screens
U_f	velocity of fluid based on frontal area
U_{max}	maximum velocity of fluid based on cross-sectional area of minimum screen opening, $U_f s^2/\ell^2$
ΔP	total pressure drop across a packed, randomly stacked screen matrix
μ	viscosity of fluid
ρ	density of fluid

For pipe flow

B	nondimensional variable, $(1 - r^*)\sqrt{Re_\omega/2}$
f	oscillating Fanning friction factor, $4\sqrt{\overline{\tau_w^2}}/\rho\overline{U_a^2}$
f_s	steady Fanning friction factor, 16/Re for laminar flow
I_1, I_2	definite integrals, functions of Re_ω
k	ρk = magnitude of oscillating pressure gradient
r	radial position
r_o	radius of tube
r^*	nondimensional radial position, r/r_o
Re	steady Reynolds number, $2 r_o\sqrt{\overline{U_a^2}}/\nu$
Re_ω	kinetic Reynolds number, $\omega r_o^2/\nu$
t	time
U	fluid velocity
U_a	area-averaged fluid velocity
$\sqrt{\overline{U_a^2}}$	root-mean-squared fluid velocity, cyclic average of U_a^2
μ	viscosity of fluid
ν	kinematic viscosity of fluid
ρ	density of fluid
τ_w	wall shear stress, $-\mu(\partial U/\partial r)$ at $r = r_o$
$\sqrt{\overline{\tau_w^2}}$	root-mean-squared wall shear stress, cyclic average of τ_w^2
ω	frequency (rad/s)

INTRODUCTION

It is important to design the heat transfer components (heater, regenerator, and cooler) of a Stirling engine properly. Two major design goals are good heat transfer and low working-gas pressure drops across the components. These two objectives are somewhat contradictory because a component that is designed to have improved heat transfer will usually experience a higher pressure drop. The ability to estimate accurately heat transfer rates and pressure drops is a great aid in optimizing the design of the heat transfer components. This paper examines correlations for predicting pressure drops in Stirling engines. We intend to (1) present some results that quantify the effects of pressure drop on engine performance, (2) recommend an improved friction factor correlation for steady unidirectional flow through the

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woven-screen matrices in regenerators, and (3) derive a theoretical friction factor for oscillating flow, which is a complex and challenging phenomenon occurring in a real Stirling engine.

The effects of pressure drop on calculated performance were demonstrated by the Argonne National Laboratory (ANL) computer code (1) with respect to two well-documented kinematic engines, GPU-3 and 4L23 (2). Performance predictions are presented in the form of engine maps, which show the effects of pressure drop and engine speed on the predicted output power and efficiency of the engines.

The literature on the existing correlations for tubes, spheres, and woven-screen matrices was reviewed extensively. As a result, we found that there are abundant data pertinent to Stirling engine applications for steady unidirectional flow; but practically none exists for the oscillatory case. For steady unidirectional flow, we compared the published data on a unified nondimensional basis, and discrepancies were assessed wherever possible. Through this study, we were able to present an improved correlation for the wire-screen friction factor based on data published by Kays and London (3).

It became apparent that a fundamental analysis would be helpful as a first step in understanding the basics of oscillating flow and in proposing meaningful experimental work. We performed a simplified analysis of the laminar flow of an incompressible fluid in a circular pipe subject to a harmonic pressure gradient. The flow is fully characterized by two important parameters: steady and kinetic Reynolds numbers, provided that a root-mean-squared velocity is introduced to define the steady Reynolds number for an oscillating flow with a zero mean velocity. The ratio of friction factors between oscillating and unidirectional flow is presented as a function of kinetic Reynolds number, and some suggestions are included for applications to Stirling engines.

REFERENCE ENGINE PREDICTED PERFORMANCE

Sensitivity studies of the effects of internal working-fluid pressure drop on Stirling engine performance were made possible through the use of the ANL computer code (1), which uses a second-order adiabatic design method (4). ANL provided us with a tape copy of the Fortran code and input data files for two reference Stirling configurations, the GPU-3 and 4L23 engines. Keeping all parameters in the data files unchanged, computer predictions for the two reference engines indicated that (1) the pressure drop in the regenerator accounts for 70 to 90% of the total pressure drop through the heat transfer components, and (2) the pressure drop in the regenerator is nearly a linear function of velocity, as one would expect for laminar flow.

The effect of regenerator pressure drop on the performance of the two reference engines was analyzed from the viewpoint that the standard empirical friction factor correlations may not predict accurately the pressure drops that occur in actual regenerators. The regenerator friction factor correlations in the code were modified by introducing constant "multiplication factors," which were then varied to simulate the effect of using different correlations. The predicted performance of the GPU-3 and 4L23 engines is displayed in Figs. 1 and 2, respectively. In the performance maps, the solid curves are lines of constant fluid friction multiplication factor, and the broken curves are lines of constant frequency.

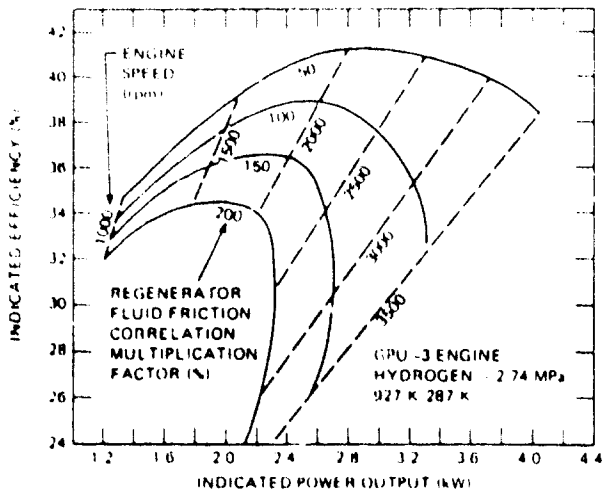

Fig. 1. Regenerator pressure drop has a significant effect on the performance of the GPU-3 engine.

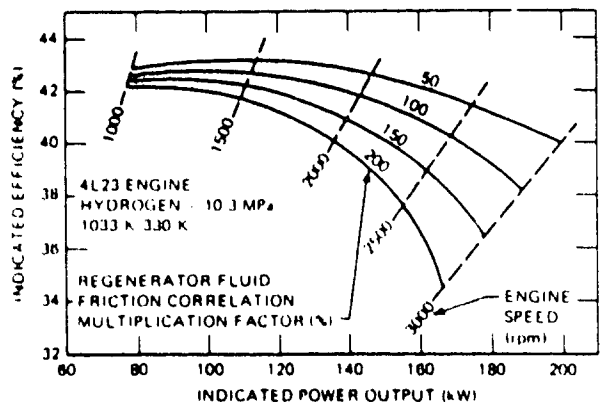

Fig. 2. Regenerator pressure drop has a significant effect on the performance of the 4L23 engine.

Regenerator pressure drop has a significant effect on both efficiency and power output, especially at high frequencies. For example, when the friction multiplier is increased from 50 to 200% (a factor of 4) in the GPU-3 engine operating at 3500 rpm, both efficiency and power output are reduced by about 50%. The trend in the 4L23 engine is the same, but the magnitude is somewhat less. This trend highlights the need for accurate pressure-drop correlations.

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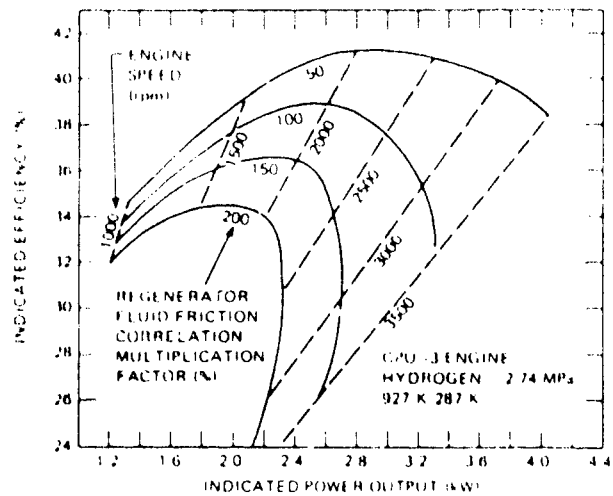

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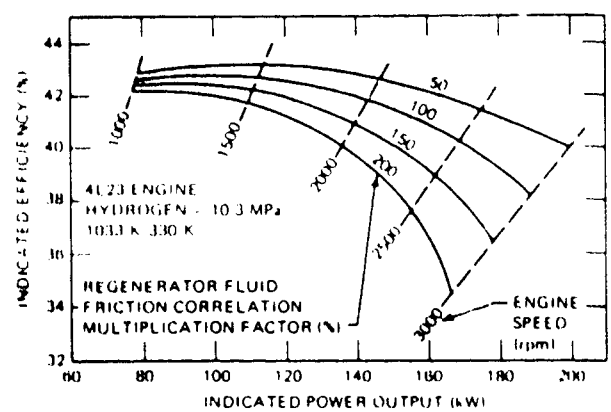

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NEW CORRELATION FOR STEADY UNIDIRECTIONAL FLOW

As stated earlier, the regenerator pressure drop in a typical Stirling engine predominates over the pressure drops in both the heater and cooler. Therefore, we focused on the review of friction factors associated with regenerators. Finegold and Sterrett prepared an excellent review of Stirling engine regenerator correlations, including a substantial bibliography (5). It is our intent to update this review by including the newly published screen matrix data by Miyabe et al. (6) and to present a better empirical correlation for the well-known Kays and London data.

Miyabe et al. (6) recently published new experimental pressure-drop data for steady unidirectional flow through generator screen matrices (6). The data covered screens from 10 to 300 mesh (openings per inch) over a wide range of Reynolds numbers. The nondimensional parameters used to present the data were defined in terms of the minimum screen opening and maximum fluid velocity. An empirical equation was presented:

$$f = \frac{33.6}{Re_\ell} + 0.337, \quad (4 < Re_\ell < 1000) .$$

When the empirical equation was compared with the data points, it was evident that the data scatter was very small at only about ±15%.

We reexamined the Kays and London screen matrix data for sizes from 5 to 60 mesh. The data were taken originally from Tong and London (7). In contrast to Miyabe et al., Kays and London defined the Reynolds number and friction factor in terms of a hydraulic diameter and an average fluid velocity. The Kays and London data, when plotted using their friction factor and Reynolds number definitions, displayed a large scatter.

For a meaningful comparison between the two sets of data, we converted the Kays and London data using the Miyabe nondimensional parameter definitions (Re_ℓ, f). The resulting plot of the friction factor vs Reynolds number displayed very little scatter. We derived a new empirical equation that fits the Kays and London data within about ±10%:

$$f = \frac{49.78}{Re_\ell^{0.945}} + 0.318, \quad (3 < Re_\ell < 2000) .$$

When this new empirical equation was compared with the Miyabe equation, we found that the predicted values resulting from the Kays and London data were about 50% higher when $Re_\ell = 10$, about 30% higher when $Re_\ell = 100$, and almost the same when $Re_\ell = 1000$.

In his design manual (8), Martini published independently an empirical equation that is based on the Kays and London data and fits the data no better than about ±40%. Interested readers should refer to the design manual for his derived equation and the details of the data fitting.

We applied our empirical equation to the sample case for pressure-drop calculation presented in the *Stirling Engine Newsletter* (9). In the example, an average pressure drop was computed from a half-cycle-averaged gas velocity. Based on the numbers provided, we calculated a pressure drop of 11.3 kPa. This is comparable with the pressure drops of 17.2 and 7.6 kPa computed from empirical equations of Martini and Miyabe, respectively.

The fact that the pressure drop computed from Martini's empirical equation tends to be higher than the one from Miyabe's correlation may be attributed to two potential sources of error. First, Martini's calculated pressure drop (17.2 kPa) differs from ours (11.3 kPa) because of a larger uncertainty of data fitting (±40% vs ±15%, respectively). Secondly, the difference between our figure and Miyabe's (7.6 kPa) is due to the differences between the experimental data of Kays and London and the experimental data of Miyabe (see Fig. 3).

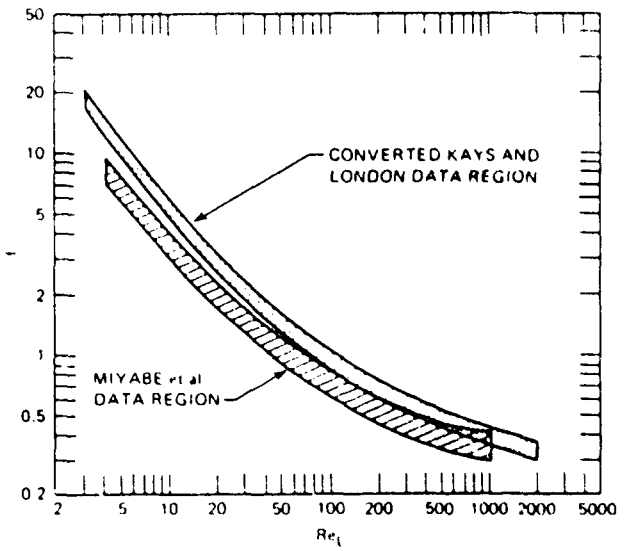

Fig. 3. Comparison of woven screen friction factor data between Kays and London (±10% uncertainty) and Miyabe et al. (±15% uncertainty) using Miyabe et al. nondimensional parameters.

THEORETICAL FRICTION FACTOR FOR OSCILLATING FLOW

Oscillating fluid flow, such as that generated by the combined movements of piston and displacer in a Stirling engine, is a very complicated problem. Therefore, a general treatment that accounts for the interactions of thermodynamics and dynamics is not our intent; rather, we will concentrate our efforts on the basic understandings of the oscillatory flow phenomena and its effect on the derivation of a friction factor from theoretical principles.

Fluid friction in oscillating flow has received much less attention than in steady unidirectional flow. There have been a number of theoretical studies on laminar oscillatory flow, including closed-form solutions (10–13) near flat plates and inside circular tubes. However, the

experimental work relevent to Stirling cycle machines is scarce. One of these measurements was published by Kim (14), who performed some pressure-drop and heat transfer measurements of an oscillating stream of air across a matrix of randomly packed spheres.

To gain a basic understanding of friction losses in oscillating flow, we derived the friction factor that is associated with an oscillating laminar flow of an incompressible fluid through a circular tube driven by a sinusoidal varying pressure gradient. This type of flow, based on a closed-form solution, is found to be fully characterized by two dimensionless parameters: the steady and kinetic Reynolds numbers. For a circular pipe, the steady Reynolds number is defined as

$$Re = \frac{2r_o\sqrt{\overline{U_a^2}}}{\nu} .$$

Because the oscillating flow has a zero mean velocity over one cycle, the root-mean-squared velocity is used as a representative value in the Reynolds number definition. Notice that the above definition will reduce to the more familiar form, $Re = 2r_oU_a/\nu$, if it is applied to unidirectional tube flow. The kinetic Reynolds number, defined as $Re_\omega = \omega r_o^2/\nu$, is a unique parameter that characterizes oscillating flow.

The exact solution for the velocity distribution involves Bessel functions with complex arguments. Numerical evaluation of the exact profiles would be very tedious; however, two limiting cases of very slow and rapid oscillations prove to be extremely simple (see Ref. 10):

$$U \simeq \frac{kr_o}{4\nu}\left[(1 - r^{*2})\cos(\omega t) + \frac{Re_\omega}{16}\right.$$

$$\left.\times (r^{*4} + 4r^{*2} - 5)\sin(\omega t)\right] , \quad (Re_\omega < 4) ,$$

and

$$U \simeq \frac{kr_o}{\nu Re_\omega}\left[\sin(\omega t) - \frac{e^{-B}}{\sqrt{r^*}}\sin(\omega t - B)\right] , \quad (Re_\omega > 4) ,$$

where

$$B = (1 - r^*)\sqrt{\frac{Re_\omega}{2}} .$$

For very small values of Re_ω, the velocity distribution is nearly parabolic, which is similar to steady state Hagen-Poiseuille flow. For very large values of Re_ω, the velocity is nearly constant across the core and decreases to zero across a relatively thin boundary layer close to the pipe wall.

The root-mean-squared wall shear stress and mean-squared velocity are used as representative values to define a Fanning friction factor for oscillating flow:

$$f = \frac{2\sqrt{\overline{\tau_w^2}}}{\rho\,\overline{U_a^2}} ,$$

where the subscript a refers to a cross-sectional area mean and the bar signifies a cyclic average. Note that the definition will simplify to the more familiar form $f = 2\tau_w/\rho U_a^2$ if it is applied to steady unidirectional flow. After considerable algebraic manipulation, the friction factor for oscillating flow can be derived from the two limiting solutions:

$$f = \frac{16}{Re}\left[\frac{1 + \left(\frac{3}{8}Re_\omega\right)^2}{1 + \left(\frac{1}{3}Re_\omega\right)^2}\right]^{1/2} , \quad (Re_\omega < 4) ,$$

and

$$f = \frac{2}{Re}\left[\frac{Re_\omega - \sqrt{\frac{Re_\omega}{2} + \frac{1}{4}}}{\left(\frac{1}{2} - I_1\right)^2 + I_2^2}\right]^{1/2} , \quad (Re_\omega > 4) ,$$

where I_1 and I_2 are definite integrals that are functions of Re_ω only:

$$I_1 = \int_0^1 \sqrt{r^*}\, e^{-B}\cos(B)\, dr^* ,$$

and

$$I_2 = -\int_0^1 \sqrt{r^*}\, e^{-B}\sin(B)\, dr^* .$$

I_1 and I_2 were evaluated easily by integrating numerically on a programmable hand calculator.

Notice that the friction factor for oscillating flow approaches the steady unidirectional value of

$$f_s = \frac{16}{Re} ,$$

when the kinetic Reynolds number approaches zero. A convenient comparison between unidirectional and oscillating flow can be made when the ratio of oscillating to steady friction factors is computed:

$$\frac{f}{f_s} = \left[\frac{1 + \left(\frac{3}{8}Re_\omega\right)^2}{1 + \left(\frac{1}{3}Re_\omega\right)^2}\right]^{1/2} , \quad (Re_\omega < 4) ,$$

and

$$\frac{f}{f_s} = \frac{1}{8}\left[\frac{Re_\omega - \sqrt{\frac{Re_\omega}{2} + \frac{1}{4}}}{\left(\frac{1}{2} - I_1\right)^2 + I_2^2}\right]^{1/2} , \quad (Re_\omega > 4) .$$

The friction factor ratio as a function of kinetic Reynolds number is plotted in Fig. 4. The ratio is a weak function of Re_ω for small values and a strong function of Re_ω for larger values. At low frequencies, the ratio is shown to remain constant and close to unity, while at higher frequencies, the ratio approaches a straight line with a slope of 1/2 as shown by the dotted line. At $Re_\omega = 4$, there is a discontinuity because the approximate expressions for the velocity distribution are discontinuous at that point.

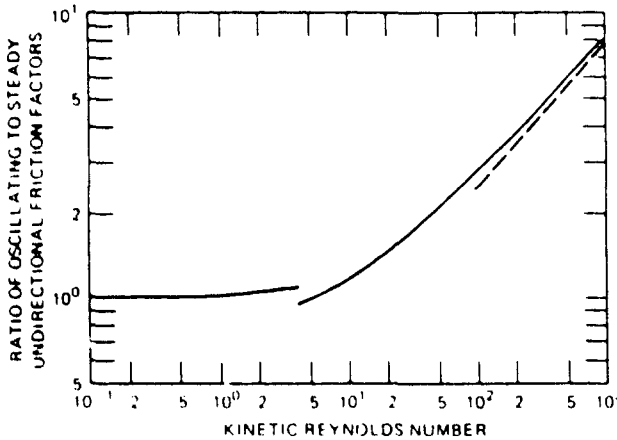

Fig. 4. Fluid friction comparison between oscillating and steady unidirectional flow.

For Stirling engine applications, the two reference engines mentioned earlier (Figs. 1 and 2) were used as typical examples. The estimated kinetic Reynolds numbers and the corresponding friction factor ratios obtained from Fig. 4 are listed in Table 1 for the heat transfer components of the reference engines. The values of Re_ω are based on a midrange frequency of 2000 rpm and a temperature-dependent viscosity (μ) for the hydrogen working fluid. Although the analytical solution for oscillating laminar flow in tubes may not apply directly to the wire screen matrices in the regenerators, the values of Re_ω were estimated from the hydraulic radii of the matrices to provide some approximate magnitudes. The kinetic Reynolds numbers of the heat transfer components span a wide range with a difference of more than 2 orders of magnitude. The regenerator Re_ω values are the smallest because of the radius-squared dependence.

As stated previously, the sensitivity studies of the two reference engines showed that the re-

Table 1. Estimated Re_ω and f/f_s for heat transfer components of reference engines operating at 2000 rpm

	Temperature (K)	r_o (m)	Re_ω	f/f_s
		GPU-3 (2.74 MPa)		
Heater tubes	880	1.51×10^{-3}	19	1.4
Regenerator	560	4.65×10^{-4}	0.04	1.0
Cooler tubes	340	5.4×10^{-4}	12	1.2
		4123 (10.3 MPa)		
Heater tubes	930	2.36×10^{-3}	160	3.5
Regenerator	600	7.5×10^{-4}	0.34	1.0
Cooler tubes	350	5.75×10^{-4}	49	2.0

generators account for most (70 to 90%) of the total pressure drop across the heat transfer components. Table 1 indicates that the friction factor ratio between oscillating and steady unidirectional flow for the regenerators is near unity. Thus, it appears that pressure drops in regenerators may be predicted adequately from the commonly used correlations that were developed for steady unidirectional flow. Table 1 also indicates that the friction factor ratio is above unity for all of the heater and cooler tubes. Consequently, the effects of oscillating flow may be important in these components. Fortunately, the accurate prediction of pressure drops in the heaters and coolers is less critical because they account for only a small portion of the total pressure drop.

CONCLUSION

Pressure drop has a significant effect on predicted engine performance, with the regenerator as the most dominant component. At a high frequency of 3500 rpm, quadrupling the friction factor in the regenerator reduces both the indicated efficiency and indicated power output by nearly 50% for the GPU-3 reference engine.

For steady unidirectional flow, an improved empirical equation was derived for the well-established Kays and London screen matrix data. This correlation appears to fit the data satisfactorily with an uncertainty of about ±10%.

For oscillating laminar flow, the flow is shown to be fully characterized by steady and kinetic Reynolds numbers. Moreover, the ratio of friction factor between oscillating and steady flow has been shown to vary with the kinetic Reynolds number only. No significant difference exists at slow oscillations, but there is significant enhancement at rapid oscillations.

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