# DYNAMIC ANALYSIS OF THE FLUIDYNE

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## ABSTRACT

The dynamic behavior of the liquid-piston Stirling engine is analyzed using the vector or phasor method of representing the motions of coupled systems. The result, for the first time, is a simple physical explanation of the feedback mechanism most frequently employed in these machines. In addition, the method leads to an easy derivation of certain results already known from experiment or from more complex analyses.

## NOMENCLATURE

| | |
|---|---|
| $A$ | ratio of tuning and displacer column areas |
| $A_d$ | area of displacer |
| $A_t$ | area of tuning column |
| $C_L$ | linear velocity-dependent force (load) on tuning column |
| $g$ | acceleration due to gravity |
| $H$ | equilibrium height of liquid surface above junction of tuning line and displacer |
| $h_c$ | displacement of cold column surface |
| $h_d$ | displacer movement ($h_e - h_c$) |
| $h_e$ | displacement of hot column surface |
| $h_t$ | displacement of tuning column surface |
| $\Delta L$ | horizontal length of displacer (difference between hot and cold column lengths) |
| $L_d$ | length of displacer column |
| $L_t$ | length of tuning column |

$P_g$ gas pressure

$P_o$ ambient pressure

$P_r$ pressure at junction of tuning line and displacer

$R_d$ viscous resistance coefficient in displacer tube

$R_t$ viscous resistance coefficient in tuning line

$\Delta T$ difference between expansion and compression space gas temperatures

$T_c$ compression space gas temperature

$T_e$ expansion space gas temperature

$V_m$ mean volume of gas

$\theta$ phase angle between displacer and tuning column movements

$\rho$ liquid density

$\omega$ operating frequency

$\omega_d$ natural frequency of lossless displacer oscillations

$\omega_t$ natural frequency of lossless tuning line oscillations

## INTRODUCTION

The dynamics of the liquid piston Stirling engine have been analyzed mathematically[1-3] and numerically.[4-6] The complexity of these excellent analyses and the lack of closed-form solutions inherent in computer models make it difficult to approach a clear physical understanding of the mechanisms at work in the operation of the simplest of all Fluidyne engines, the liquid feedback machine.[7] For the same reasons, there has been no equivalent for the Fluidyne of the methods available for approximate analyses of the expected behavior of conventional kinematic and free piston Stirling engines.[8-14]

One of the most successful methods of approximate dynamic analysis of the free piston engine has been the use of the vector or phasor method, pioneered for this purpose by Cooke-Yarborough, in which the amplitudes and phase angles of the various oscillating parts of the engine are represented as vectors on a two-dimensional plot; the length of the vector represents the amplitude of the oscillation, and the angle between the vectors representing different components of the movement corresponds to the phase angle between their oscillations, which are assumed to be sinusoidal. The method is entirely analogous to the standard method of describing oscillating electrical and mechanical circuits in forms of complex numbers, represented on an Argand diagram as two-dimensional vectors.

In this paper, the linearized equations of motion for the various components of a liquid feedback fluidyne are derived in a very simple way, and represented by their vector or phasor diagrams. The diagrams give a physical insight into the operation of the system and also, in this preliminary paper, some quantitative results for important parameters of the Fluidyne's operation are simply derived.

## EQUATIONS OF MOTION

Figure 1 shows the basic layout of the machine to be analyzed; it consists of liquid fitted U-tube displacer section, with a tuning or output column connected to it. The other end of the tuning column is open.

When the liquid columns are much larger than the amplitude of oscillation of the liquid surfaces, as is the case for all the engines described in the present literature, the change in the mass of liquid in each column as the surfaces oscillate is relatively small and may be ignored. In such circumstances, the equations of motion are easily written down.

We may take as a reference point for the lengths of the columns the junction of the displacer and tuning line, where the instantaneous pressure is called $P_r$ (see Fig. 1). The equations of motion generally take the following form: pressure difference x area - viscous force = average mass x acceleration. Expressed mathematically this becomes

Hot column

$$[P_g - P_r + \rho g(H - h_e)]A_d - R_d A_d H\dot{h}_e$$
$$= \rho A_d H\ddot{h}_e \ , \qquad (1)$$

Cold column

$$[P_g - P_r + \rho g(H - h_c)]A_d \qquad (2)$$
$$- R_d A_d (H + \Delta L)\dot{h}_c = \rho A_d (H + \Delta L)\ddot{h}_c \ ,$$

Tuning column

$$[P_o - P_r + \rho g(H - h_t)]\ A_t - R_t A_t L_t \dot{h}_t$$
$$= \rho A_t L_t \ddot{h}_t + C_L \dot{h}_t \ , \qquad (3)$$

$C_L$ represents a linear velocity dependent force (for example, from a dashpot or an electrical alternator with a resistive load) on the output column.

Subtracting Eq. (1) from Eq. (2) and dividing by $A_d$ yields

$$\rho g(h_e - h_c) + \rho H(\ddot{h}_e - \ddot{h}_c) - \rho \Delta L \ddot{h}_c$$
$$+ R_d H(\dot{h}_e - \dot{h}_c) - R_d \Delta L \dot{h}_c = 0 \ . \qquad (4)$$

## DISPLACER ACTION

The basic physics of the Stirling cycle are most simply analyzed by treating the displacer action and the total volume change separately.[10] This is easily done by defining a single variable to represent the displacer motion (the change in total volume of the working fluid is already represented by a single variable, being simply equal to $h_t$ multiplied by the cross-sectional area of the tuning column). As it turns out, this separation of the displacement and volume change components of the motion is also a convenient and simple approach to analyzing the dynamics of the liquid feedback fluidyne. The variable depicting the displacer action is $h_e - h_c$, the differential movement of the pistons in the hot and cold cylinders, which we shall call $h_d$. Similar variables were used by Stammers.[3] Note that $h_e$ and $h_c$ are both time dependent and even with the simplifying assumption that all the movements are sinusoidal, there will, in general, be a phase difference between them and $h_d$ will not be in phase with either $h_e$ or $h_c$.

Obviously, if there is a fixed amount of liquid in the system $h_e + h_c = -Ah_t$, where A is the ratio between the cross-sectional areas of the output tube and the displacer U-tube. A little further arithmetic shows that $h_c = -1/2(h_d + Ah_t)$. Substituting these relations into Eq. (4) and simplifying gives

$$\ddot{h}_d + (2g/L_d)h_d + (R_d/\rho)\dot{h}_d + A(\Delta L/L_d)\ddot{h}_t$$

$$+ A(R_d/\rho)(\Delta L/L_d)\dot{h}_t = 0 \quad . \qquad (5)$$

The first two terms represent the free oscillation of the liquid in the displacer U-tube at its natural frequency $\omega_d = \sqrt{2g/L_d}$ . The third term represents the flow losses associated with the displacement motion, and could easily be extended to include in addition the flow losses due to the displacement of gas through the heat exchangers and regenerator. The fourth term is the most interesting: it arises from the tuning column movement which is the source of power generation in the engine and has, as we shall see, a component in the right phase to overcome the displacer flow losses.

If the movements are reasonably sinusoidal, their phase relationships are conveniently represented on a phasor or vector diagram, a technique that is well established in electrical engineering and has also been applied to Stirling cycle analysis.[8,12-14] Figure 2 represents the displacer motion, as a solid line, and the tuning column motion, as a broken line. As in any Stirling engine, the displacer must move gas into the hot space before it is expanded, although of course with a sinusoidal motion of the pistons these phases of the cycle overlap somewhat. This phase relationship is represented in the diagram by the phase advance, $\theta$, of $h_t$ relative to $h_d$ (remember that a positive and increasing $h_t$ corresponds to lowering the liquid surface at the free end of the output tube, thereby decreasing the gas volume; in other words, $h_t$ and the gas volume are 180° out of phase, as shown by the dotted line in Fig. 2).

The velocity of a point moving sinusoidally leads its position by 90° (e.g., when the point is passing through its center position and its displacement is zero, its speed is maximum; conversely, when the point is at its extreme distance from the center position, its velocity is instantaneously zero). Similarly, the acceleration is 180° out of phase with the position. This is represented in Figs. 3(a) and 3(b), where the quantities pertaining to displacer action ($h_d$) are shown by solid lines, and the quantities relating to motion of the tuning column ($h_t$) are shown by broken lines. The relative lengths of the vectors representing position, velocity and acceleration are in the ratio $1:\omega:\omega^2$, corresponding to $\cos \omega t : d(\cos \omega t)/dt : d^2(\cos \omega t)/dt^2$.

With these relationships in mind, Eq. (5) can be represented on a single vector or phasor diagram (Fig. 4). The vectors represent the quantities appearing on the left-hand side of Eq. (5), and to satisfy the equation their resultant must be zero.

Here then is the physical basis for the success of the liquid feedback system. The $\ddot{h}_t$ term in Eq. (5), arising from forces exerted on the output column by the gas pressure inside the engine, clearly has a component that is opposite to, and can therefore compensate for, the flow losses arising from the velocity, $\dot{h}_d$, of the fluid taking part in the displacer action. There are also, as can be seen, vectors with a component able to overcome the smaller amount of dissipation in the displacer arising from the $\dot{h}_t$ term in Eq. (5).

## OPERATING FREQUENCY

The vector method is, as we have seen, a simple and useful way of visualizing the motions of the various liquid columns, and provides a satisfying explanation of the operation of the liquid feedback system. The method has, however, more power than we have used so far, and can provide quantitative predictions about the Fluidyne dynamics.

As a first example, covered here in outline only, it is easy to show that for most practical designs with a single-phase working fluid, the operating frequency will be within a few percent of the natural displacer frequency.

Referring to Fig. 4, we shall generally be designing the displacer so that the losses are small, and the $\dot{h}_t$ term in Eq. (5) will be negligible. The component of $(A\Delta L/L_d)\ddot{h}_t$ along the $h_d$ direction is $(A\Delta L/L_d)\ddot{h}_t \cos\theta$. For the vector components along the line of $h_d$ to equate to zero, and remembering that the amplitude of the acceleration vector is simply $\omega^2$ x the displacement, we must have

$$(A\Delta L/L_d)\omega^2 h_t \cos\theta + \omega^2 h_d - w_d^2 h_d = 0$$

$$\therefore \omega_d^2/\omega^2 = 1 + \frac{\Delta L}{L_d}\frac{Ah_t}{h_d}\cos\theta . \tag{6}$$

Figure 5 shows the relationship between $h_d$, which is by definition the vector difference between the hot and cold piston displacements, and $-Ah_t$, which is their vector sum. In practical machines, the phase angle between the hot and cold piston movements is in the range 90° to 150° for optimum output,[15] and the amplitudes of the two movements are made approximately equal — say, within ±20% of each other. Figure 5 illustrates the case where the movements are equal and with a 90° phase difference, and also the case where $h_c = 1.2\ h_e$ and the phase angle between them is 150°. By inspection of all such similar combinations, it may be seen that the maximum value of $(Ah_t/h_d)\cos\theta$ occurs when the hot and cold cylinders are moving with a 90° phase difference. In that case, for a 20% difference between $h_e$ and $h_c$, solution of the vector triangles shows that $(Ah_t/h_d)$ as $\theta = \pm 0.18$, according to whether $h_e$ or $h_c$ is the larger.

In a typical engine, the junction between displacer and timing line is not more than, say, 4/5 of the way from cold to the hot end of the U-tube - i.e., the cold leg is not more than four times longer than the hot. Therefore, the maximum value of $\Delta L/L_d$ is $(4/5 - 1/5) = 0.6$. Therefore, the extreme values of the operating frequency, as determined by Eq. (6), are given by

$$\omega_d^2/\omega^2 = 1 \pm 0.6 \times 0.18 = 1 \pm 0.11$$

$$\therefore \omega = \sqrt{1/(1 \pm 0.11)}\ \omega_o = (1.00 \pm 0.05)\omega_d .$$

Therefore, the operating frequency of a typical, practical engine will be close to the natural frequency of the displacer. This is borne out by experimental observation.[15]

## OUTPUT COLUMN MOTION

Next, let us consider the equation governing the motion of the tuning or output column, for which phasor analysis leads to a remarkably simple derivation of the minimum temperature difference that can give rise to self-sustaining oscillations. At the same time, the diagrams give a clear picture of the physics behind the well-known, but somewhat puzzling, result that a finite temperature difference is needed to initiate oscillations even in a completely lossless system.

Dividing Eq. (1) by $A_d$, Eq. (3) by $A_t$ and subtracting them eliminates the $P_r$ term from the equation of motion of the output column. Rewriting the results so that $h_e$ is expressed in terms of $h_d$ and $h_t$ yields

$$\begin{aligned}(P_g - P_o) &+ h_t \rho g(1 + A/2) + \dot{h}_t(C_L/A_t \\ &+ R_t L_t + AR_d\,H/2) \\ &+ \ddot{h}_t \rho(L_t + AH/2) - (h_d \rho g/2 + \dot{h}_d R_d H/2 \\ &+ \ddot{h}_d \rho H/2) = 0 \;. \qquad (7)\end{aligned}$$

Equation (7) looks complicated, but that is partly due to the large number of multiplying factors in the terms relating to $h_t$ and its derivatives; as we shall see, many of these terms are negligibly small in practical machines and the appearance, at least, of the equation can be greatly simplified.

To make use of the equation, we need an expression relating the gas pressure, $P_g$, to the position of the displacer and tuning column. Suppose that in the equilibrium position when all three liquid surfaces are at the same height, half of the gas volume is at temperature $T_e$ and half at $T_c$; then when the liquid surfaces are slightly displaced, and the gas pressure becomes $P_g$, the ideal gas laws require that

$$P_o \left( \frac{V_m/2}{T_e} + \frac{V_m/2}{T_c} \right) = P_g \left( \frac{V_m/2 + A_d h_e}{T_e} \right.$$

$$\left. + \frac{V_m/2 + A_d h_c}{T_c} \right) ,$$

therefore

$$(P_o - P_g) \ [(V_m/2 + A_d h_e)T_c$$
$$+ (V_m/2 + A_d h_c)T_e]$$
$$= P_o A_d (h_e T_c + h_c T_e) \ . \qquad (8)$$

For small movements of the liquid surface, we can neglect the volume change terms (such as $A_d$ $h_e$) compared with the mean volume $V_m$, in which case Eq. (8) simplifies to

$$(P_o - P_g) \ V_m \simeq 2P_o A_d \left( \frac{h_e T_c + h_c T_e}{T_e + T_c} \right) . \qquad (9)$$

And expressing this in terms of our preferred variables $h_t$ and $h_d$ leads to an impressively simple relationship between displacer action, volume change and gas pressure:

$$P_g - P_o \simeq \frac{P_o A_d}{V_m}\left(Ah_t + \frac{\Delta T}{T_e + T_c} h_d\right) . \qquad (10)$$

Substituting this into Eq. (7) yields

$$\begin{aligned} h_t\ &[P_o A_t/V_m + \rho g(1 + A/2)] + \ddot{h}_t \rho(L_t + AH/2) \\ &+ \dot{h}_t (C_L/A_t + R_t L_t + AR_d H/2) \qquad (11) \\ &+ h_d\left[\frac{P_o A_d \Delta T}{V_m(T_e + T_c)} - \frac{\rho g}{2}\right] - \dot{h}_d\ R_d H/2 \\ &- \ddot{h}_d \rho H/2 = 0 . \end{aligned}$$

The first two terms represent the free oscillation of the liquid in the output column at its natural frequency $\omega_t$

$$\omega_t = \sqrt{\frac{\dfrac{P_o A_t}{\rho V_m} + g(1 + A/2)}{L_t + AH/2}} . \qquad (12)$$

In practical machines, the compression of the gas usually gives rise to a much greater restoring force than gravity, and the total length of the tuning line is usually much greater than the length of the displacer uprights. In such cases the gravitational term and the term in H can be neglected in Eq. (12), so that

$$\omega_t \simeq \sqrt{\frac{P_o A_t}{\rho V_m L_t}} \; . \tag{13}$$

This is a familiar result.[15]

The directions of the vectors representing the various terms in Eq. (11) are shown in Fig. 6. Even ignoring losses (i.e., with all velocity dependent terms set to zero), only the $h_d$ and $\ddot{h}_d$ terms can provide a component of force lagging $h_t$ and so doing work on the tuning column to build up the energy, and hence the amplitude, of its oscillations. For this to happen, the diagram shows that the sum of the $h_d$ and $\ddot{h}_d$ terms must point in the $h_d$ direction, i.e., for the oscillation to build up we must have

$$\frac{P_o A_d \, \Delta T}{V_m (T_e + T_c)} - \frac{\rho g}{2} + \frac{\omega^2 \rho H}{2} > 0 \; ,$$

or

$$\frac{P_o A_d}{V_m} \frac{\Delta T}{T_e + T_c} > \frac{\rho g}{2} \left(1 - \frac{\omega^2 H}{g}\right) . \tag{14}$$

Remembering that $\omega \simeq \omega_d$, and that $\omega_d = \sqrt{2g/L_d}$ Eq. (14) can be rewritten as

$$\frac{\Delta T}{T_e + T_c} \gtrsim \frac{\rho g V_m}{2P_o A_d}\left(1 - \frac{2g}{L_d}\frac{H}{g}\right) ,$$

or

$$\frac{\Delta T}{T_e + T_c} \gtrsim \frac{\rho g V_m}{2P_o A_d}\frac{\Delta L}{L_d} \quad . \tag{15}$$

This is the same result as predicted by the more rigorous, and more complicated, analysis of Elrod[1] and Stammers.[3] Using Fig. 6, it is also possible to allow for the effects of viscous losses and a linearly velocity dependent load.

## CONCLUSION

The vector, or phasor, method of studying the linearized dynamics of Stirling machines, already used successfully for the design of free piston engines, can also be applied to the Fluidyne liquid piston Stirling engine. The result is an improved understanding of the physical principles of operation, a demonstration that the frequency of operation in practical engines is very nearly equal to the natural frequency of the displacer, and a very simple derivation of the minimum temperature difference needed for self-sustained oscillations.

Preliminary results indicate that the method can also deal with the effects of losses or loads, and can be used to give approximate predictions of the dynamic behavior of the active system.

## REFERENCES


1. H. G. Elrod, *The Fluidyne Heat Engine: How to Build One - How It Works*, ONR London Rept. R-14-74 (NTIS No. A D/A-006-367) (December 1974).
2. A. D. Geisow, *The Onset of Oscillations in a Lossless Fluidyne*, AERE-M2840 (October 1976).
3. C. W. Stammers, 'The Operation of the Fluidyne Heat Engine at Low Differential Temperatures' *Journal of Sound and Vibration*, 63(4) 507-516 (1979).
4. L. F. Goldberg, *A Computer Simulation and Experimental Development of Liquid Piston Stirling Cycle Engines*, MSc Thesis, University of the Witwatersrand, Johannesburg, South Africa, March 1979.
5. G. T. Reader et al., 'Modelling the Jet-Stream Fluidyne' Pap 819792, *Proc 16th IECEC*, Atlanta, Ga., August 1981.
6. T. M. Drzewiecki, *An Initial Model for the Finite Displacement Response Characteristics of a Fluidyne Pump*, HDL-TR-1868 (February 1979).

7. C. D. West, *Liquid Piston Stirling Engines,* Van Nostrand Reinhold, 1983.

8. E. H. Cooke-Yarborough, *The Thermomechanical Generator: A Proposal for a Heat-Powered, Non-Rotating Electrical Alternator,* AERE-M1881.

9. G. Walker, 'Elementary Design Guidelines for Stirling Engines' Pap. 799230, *Proc. 14th IECEC,* Boston, Mass., August 1979.

10. C. D. West, 'Theoretical Basis for the Beale Number' Pap. 819793, *Proc. 16th IECEC,* Atlanta, Ga., August 1981.

11. J. R. Senft, 'A Simple Derivation of the Generalized Beale Number' Pap. 829273, *Proc. 17th IECEC,* Los Angeles, Calif., 1982.

12. J. S. Rauch, 'Harmonic Analysis of Stirling Engine Thermodynamics' Pap. 809335, *Proc. 15th IECEC,* Seattle, Wash., August 1980.

13. G. Wood, 'Lecture Notes for Stirling Engine Workshop: Set 1,' Sunpower, Inc., October 1980.

14. H. Fokker and J. A. M. van Eekelen, 'The Description of the Stirling Cycle in a Vector Diagram' Pap. 789112, *Proc. 13th IECEC,* San Diego, Calif., August 1978.

15. C. D. West and R. B. Pandey, 'A Laboratory Prototype Fluidyne Water Pump,' Pap. 819787, *Proc. 16th IECEC*, Atlanta, August 1981.

16. C. D. West, *The Fluidyne Heat Engine*, AERE-R6775, May 1971.

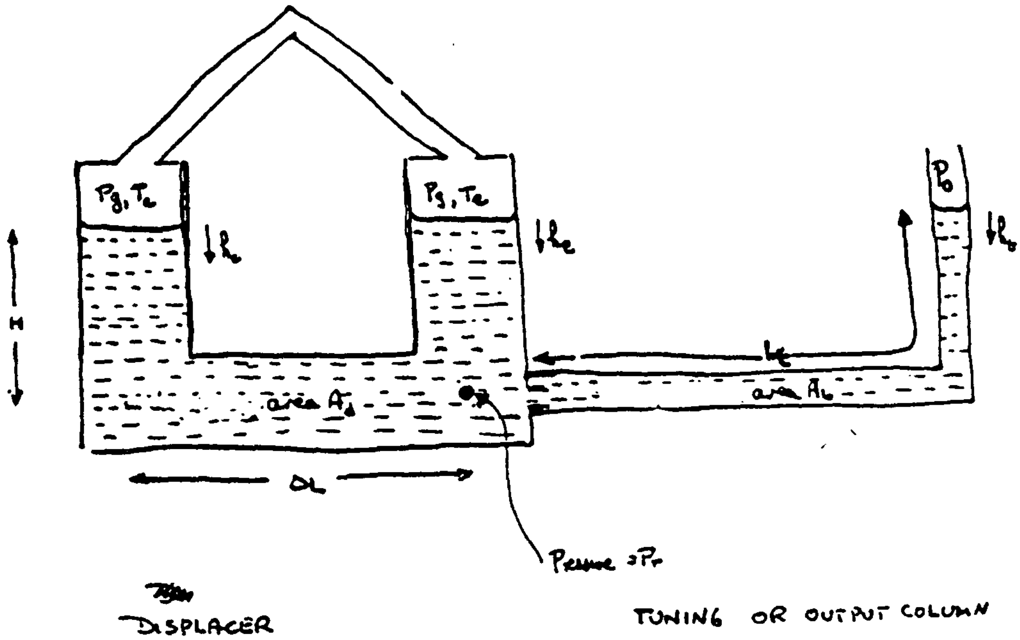


Figure 1 - Liquid feedback fluidyne and nomenclature used in the analysis.

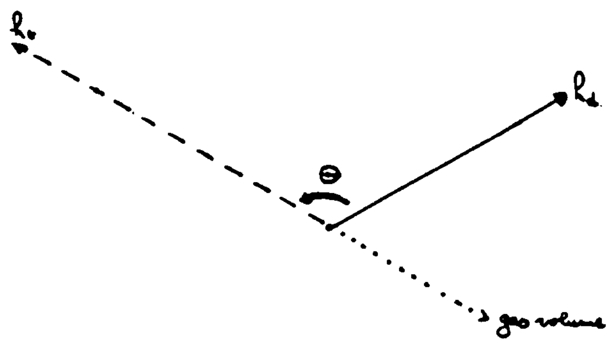


Figure 2 · Phase relationships between displacer action and tuning column movement, and between gas volume and tuning column position.

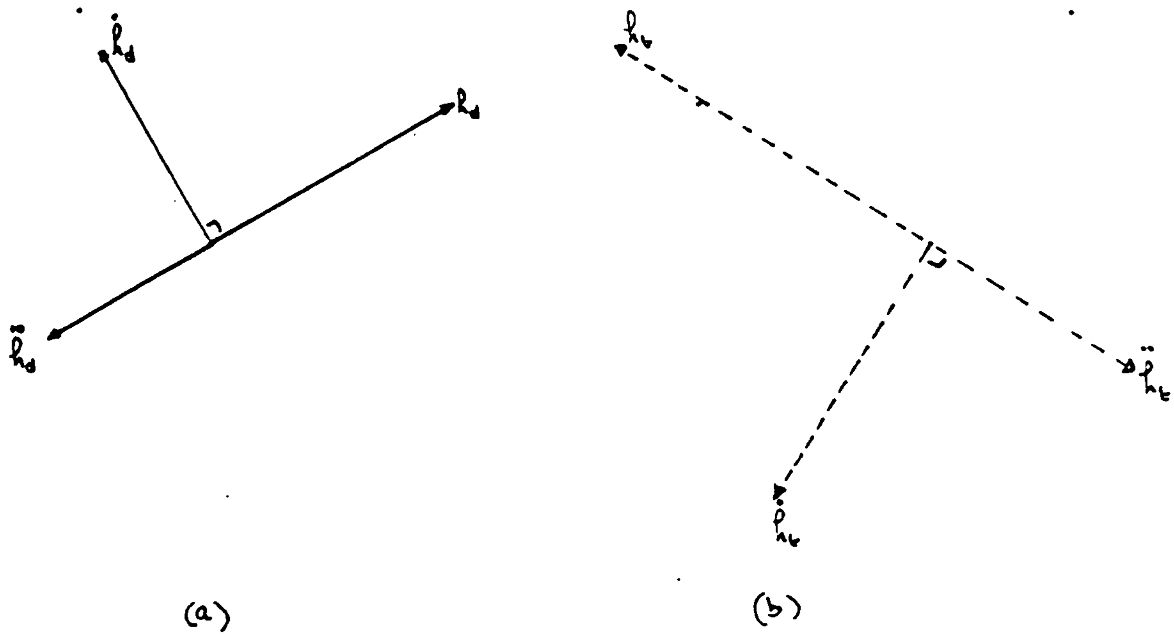

Figure 3 Position, velocity and acceleration vectors for the displacer, action and tuning columns.

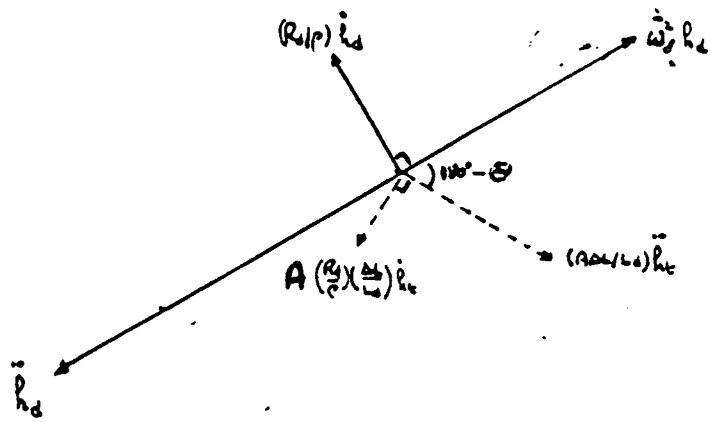


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Figure 4 Vectorial representation of the displacer equation of motion.

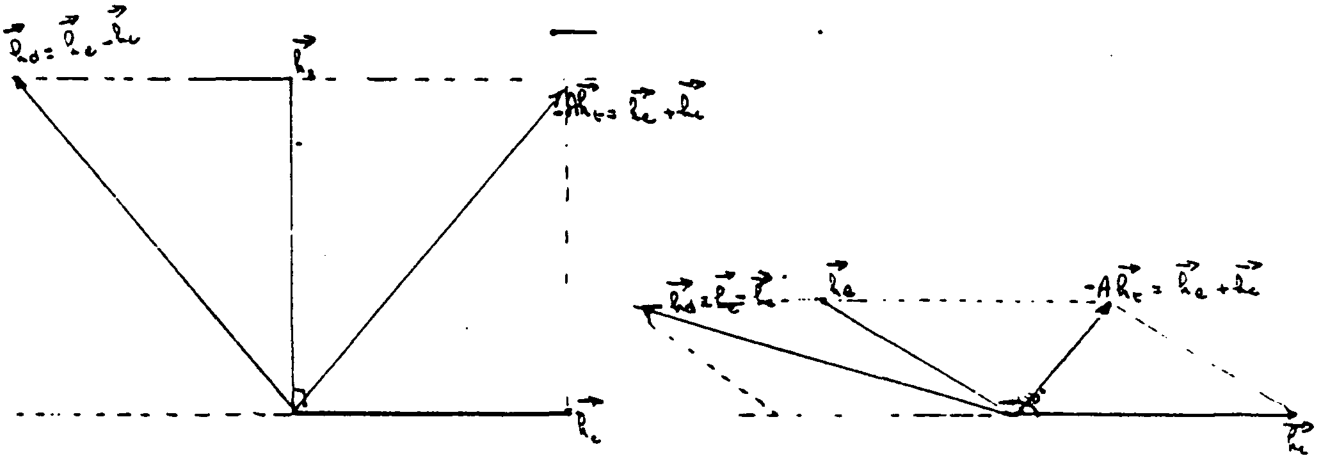

Figure 5 - Examples of the relationships between expansion and compression piston movements, and the timing line and displacer vectors

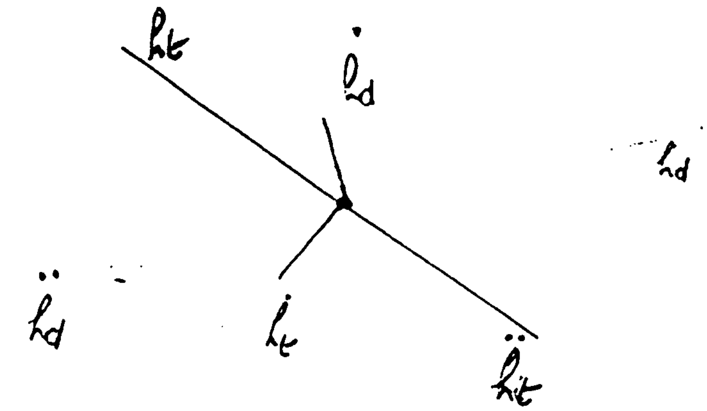


Figure 6.