A COMPARISON OF CYLINDER HEAT TRANSFER EXPRESSIONS BASED ON PREDICTION OF GAS SPRING HYSTERESIS LOSS

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ABSTRACT

Heat transfer-related gas spring hysteresis loss provides an objective criterion for comparing cylinder heat transfer expressions over a range of conditions. It is in itself of importance in Stirling engines and other devices. Hysteresis loss measurements were made on a helium gas spring at a volume ratio of 2.0 over a Peclet number range of 1.68 to 13,200. These measurements were compared with the losses predicted by a variety of cylinder heat transfer expressions.

NOMENCLATURE

A	Area
a	Constant in Nu formulae
A_o	Mean area
b	Constant in Nu formulae
c_v	Specific heat, constant volume
D	Cylinder diameter
D_h	Hydraulic diameter = $4V_o/A_o$
F	Breckenridge et al magnification factor
h	Convective heat transfer coefficient
i	Square root of -1
k	Thermal conductivity
K_1	Constant in time constant model
L_{nd}	Non-dimensional loss
L_o	Average length of gas space
L_s	Stroke length
m	Mass
Nu	Nusselt number
p	Pressure
p_a	Amplitude of pressure swing
Pe	Peclet Number
Pe_h	Peclet Number based on D_h
p_o	Mean cycle pressure
p_o'	Pressure at T_o and V_o
Pr	Prandtl number
q	Heat transfer rate
Re	Reynolds number
r_v	Volume ratio = V_{max}/V_{min}
r_v'	Modified $r_v = (V_{max}-V_o)/V_o$
T	Gas mixed mean temperature
t	Time
T_a	Amplitude of temperature swing
T_o	Wall temperature
V	Volume
V_o	Mean cycle volume
V_p	Mean piston speed
W_{adiab}	Work of adiabatic compression
y	Variable in Lee model = $(\pi Pe_h D_h/32L_s)^{1/2}$
z	Variable in Lee model = $(1+i)y$
α	Thermal diffusivity
α_o	Mean thermal diffusivity
γ	Specific heat ratio
τ	Thermal time constant
ω	Angular velocity

An asterisk superscript indicates a non-dimensional quantity.

INTRODUCTION

Heat transfer in the cylinders of reciprocating machines has been studied for more than sixty years. In this period a large number of experimental measurements of cylinder heat transfer have been made, and an even larger number of expressions have been proposed to explain and/or predict these measurements.

It is not surprising that no satisfactory expression has yet been found. Even without the added complications presented by combustion, the problem is intractable by any current analytical technique. The flow field is complex, three-dimensional, and unsteady. The convective heat transfer coefficient model may not be valid, since compression and expansion add and

remove energy in the boundary layer as well as the bulk gas. Numerical methods have not been successfully applied, since the nature of the flow requires very small grid spacing and time stepping for accurate representation.

It is difficult to discriminate among the expressions that exist by comparing plots of heat transfer vs. crank angle. As no satisfactory way of correlating such curves over a range of operating conditions has been found, they compare the performance of expressions at one condition only. In addition, it is not clear what guideline should be used to judge "goodness of fit" for these plots.

One objective criterion for comparing one expression with another is prediction of the cyclic work done on the gas due to heat transfer-related hysteresis in a gas spring or motored engine. This effect provides a measure of heat transfer integrated over time and area for the entire cycle. It is related to the indicated mean effective pressure for a fired engine. The hysteresis phenomenon is itself of importance in the design of gas springs, Stirling engines, and other reciprocating machines which have spaces operating at conditions where the loss is large.

This cyclic lost work provides a way of comparing expressions over a range of operating conditions. Faulkner and Smith [1,2] showed that for a given gas and geometry, proper non-dimensionalizing factors would correlate cyclic heat transfer loss over a wide range of speeds and mean pressures. Perhaps with further experimentation this correlation can be extended to cover a range of gases and geometries. Even in its present state, it allows comparison of cylinder heat transfer formulae over a wider range of conditions than do individual heat transfer vs. crank angle plots.

Accurate prediction of cyclic lost work does not, of course, guarantee that a heat transfer expression accurately predicts instantaneous heat transfer rates. Comparison of lost work plots does, however, provide a way of making a "first cut" among the expressions available.

This study compares the predictions of a number of expressions for heat transfer and cyclic loss with experimental measurements of heat transfer hysteresis loss.

OVERVIEW OF EXPRESSIONS

Because of the large amount of effort that has gone into finding a useful expression for cylinder heat transfer it is simplest to classify formulae according to the intended application of the work.

By far the largest body of research is that related to internal combustion engines. This research has been surveyed by Annand [3] and by Borman and Nishiwaki [4]. The expressions which will be examined here are those of Annand [op cit] and Woschni [5]. Both these expressions are the result of correlating experimental data with the form $Nu=aRe^b$. They are perhaps the most widely used expressions for engine heat transfer calculations.

Another area of research is that devoted to predicting the performance of gas springs. The expressions of Breckenridge et al [6] and of Otis [7] and Svoboda et al [8] will be examined. Breckenridge did not develop an explicit heat transfer expression, but analyzed one-dimensional transient heat flux in order to develop an expression for hysteresis loss. His result is widely quoted in the Stirling engine literature. Otis proposed modeling decay of temperature changes within a cylinder with a thermal time constant. Svoboda then developed a means for predicting this time constant based on a three-dimensional transient conduction model.

A third area of research is that intended for application to Stirling and other regenerative engines. The expressions of Lee [9] and of Cooke-Yarborough and Ryden [10] will be studied. These workers developed identical formulae for hysteresis loss using different approaches. Both studies were based on solution of a simplified form of the one dimensional energy equation.

DESCRIPTION OF EXPERIMENTS

There are two ways of measuring instantaneous heat transfer in the cylinder of a reciprocating machine. One is by the use of surface thermocouples, the other from cylinder pressure and volume measurements. The former method is the only useful one for a fired engine, since the gas properties and heat release rate during combustion are not precisely known. It has the disadvantage (for our purposes) of measuring only local heat fluxes which may not be representative of the heat transfer averaged over the entire cylinder and piston surface. Since this experiment was with a motored apparatus, the latter method was used.

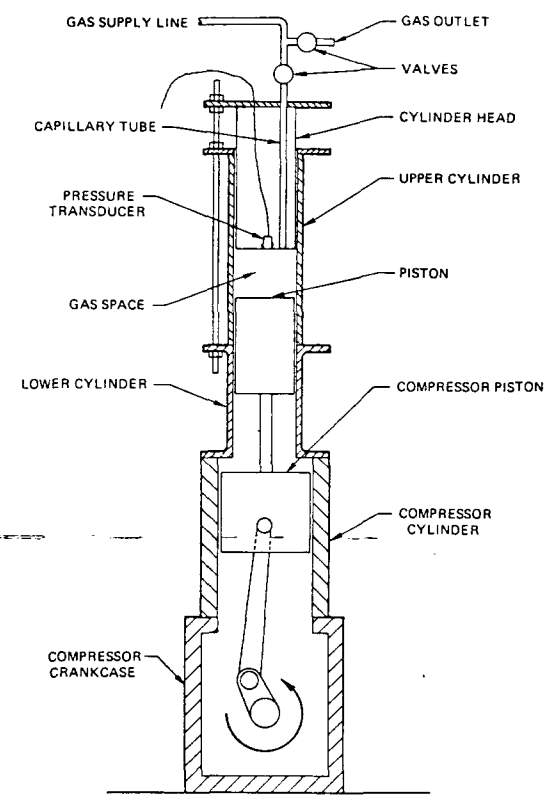

Figure 1. Equipment Schematic (from Ref [2]).

The apparatus used in these experiments was an improved version of that used by Faulkner and Smith. It is shown schematically in Figure 1. The apparatus was constructed by building a test section on top of an existing 7.62 cm (3.00 in.) stroke air compressor base, the cylinder of the compressor acting as crosshead. The test section consisted of a 5.08 cm (2.00 in.) piston and cylinder, with an adjustable flat head. The piston was sealed to the cylinder with a buna-n O-ring. The seal was located 9.5 cm (3.7 in.) from the piston face so that the frictional heating of the wall would effect results minimally. Piston and head surfaces were of brass, while the part of the cylinder wall

exposed to the gas was of micarta. The apparatus was belt-driven by a d.c. motor arranged to allow speed adjustment.

Pressure was measured with a wire strain gage pressure transducer mounted essentially flush with the cylinder head. Volume was calculated from the crank-connecting rod relationship and the crank angle signal provided by a 1200 count-per-turn optical encoder. Both these signals were recorded with a digital data acquisition system. At least 100 pressure-volume data points were collected per cycle. Ambient temperature was measured with a mercury-in-glass thermometer.

Measurements were recorded as soon after apparatus startup as both cycle-to-cycle speed and mean pressure variation settled to less than 1% of speed and mean pressure. The constant value of cyclic mean pressure indicated that the thermodynamic processes within the gas had reached cyclic equilibrium. The cylinder walls, however, had a much larger thermal time constant than the gas and were therefore still essentially uniformly at ambient temperature. (This was demonstrated by Faulkner in a similar apparatus equipped with wall thermocouples.) The number of cycles to reach steady state varied from approximately 5 to 50. In the cases where a large number of cycles were required the delay was a result of the time required for the motor to accelerate the apparatus, rather than a result of the time for the processes within the cylinder to reach cyclic equilibrium.

The cyclic lost work was calculated by trapezoidal rule integration of PdV around a cycle. The calculation was made for a single cycle rather than for data averaged over several cycles. Comparison of the results of the integration for adjacent cycles showed negligible cycle-to-cycle variation. At the time the experiment was designed it had been hoped that instantaneous space-averaged heat transfer rates could be calculated. The signal from the pressure transducer, however, had a noise level that would require a good deal of smoothing in the differentiation needed to calculate heat transfer. It was decided that heat transfer measurements should be postponed until pressure instrumentation could be improved. The noise did not effect the cyclic lost work calculation, in which it was effectively averaged out.

Experimental results were non-dimensionalized into non-dimensional loss and cyclic average Peclet number. Faulkner and Smith had shown that the results of experiments at different pressures and speeds could be correlated by plotting the loss divided by the measured work of compression against the cyclic average Reynolds number. In this study the loss has been non-dimensionalized against the work of adiabatic compression for the same mean pressure, the same mean volume, and the same volume ratio. This non-dimensionalizing factor, unlike that of Faulkner and Smith, can be calculated on the basis of independent variables alone and is thus more suitable for comparing experiment with analysis. A non-dimensionalizing factor of (mean pressure) x (mean volume) would correlate the data collected here equally well, but it was felt that the use of the work of adiabatic compression gave a better grasp of the magnitude of the effect. So we define

$$L_{nd} = \frac{p\,dV}{W_{adiab}} , \qquad (1)$$

where

$$W_{adiab} = \frac{1}{\gamma-1}\left(p_{max}V_{min}-p_{min}V_{max}\right)$$

$$= \frac{p_o V_o}{\gamma-1}\left(\frac{1+r_v}{2}\right)^{\gamma-1}\left(1-r_v^{1-\gamma}\right) . \qquad (2)$$

The cyclic average Peclet number has been chosen instead of the Reynolds number for the other non-dimensional parameter. In a sense, it represents the ratio of the rate of work input from the piston to heat transfer to the walls. This choice is supported by the fact that all expressions tested except those which assume Re dependence a priori predict loss as a function of Pe=RePr rather than Re. Since Prandtl number (Pr) is nearly the same for all gases, the point is perhaps moot. We define

$$Pe = \frac{V_p D}{\alpha_o} , \qquad (3)$$

where

V_p = mean piston speed,
D = cylinder diameter, and
α_o = gas cyclic mean thermal diffusivity.

All the experiments done in this study were at volume ratio 2.0 with helium as the working gas. Cyclic mean pressures ranged from 100 to 2500 kPa (15 to 360 psia) while speeds ranged from 0.038 to 15.8 Hz (2.3 to 950 rpm). The resulting Peclet numbers ranged from 1.68 to 13,200.

Potential sources of error have been examined. The most important was phase lag between the pressure and volume measurements. The lag of the pressure transducer was measured by comparing it to a much faster piezoelectric transducer. The lag of the volume instrumentation was measured with a strobotach. These lags, although small, were compensated for in the data processing. Another potential problem was transient thermal stress in the diaphragm of the pressure transducer. This was prevented by coating the transducer with a .02 cm (.01 in.) layer of silicone rubber. There was some deviation from a sinusoidal volume variation due to the crank-connecting rod geometry and due to motor loading at low speed and high pressure, but it was small. There was some variation in wall surface temperature of the micarta cylinder, but calculations based on the work of Pfriem [11] showed that the amplitude of the wall temperature variation was only about 5% of that of the bulk gas. Faulkner and Smith performed experiments on cylinders with walls made of copper, steel, and micarta and found that loss vs. Re plots were essentially identical.

EXPERIMENTAL RESULTS

The outcome of these experiments is shown in Figure 2. The plot shows low hysteresis loss at low Pe, where operation is nearly isothermal and temperature differences are small. Hysteresis loss is also low at high Pe, where operation is nearly adiabatic and heat fluxes are small. At intermediate Pe both temperature differences and heat fluxes are significant, resulting in much higher loss. The peak loss occurs at Pe approximately 70. At low Pe the loss varies roughly in proportion to Pe, while at high Pe it varies roughly in proportion to $Pe^{-1/2}$. The non-dimensional loss vs. Pe correlation is seen to break down in the region of maximum loss, where for the same Pe low-speed high-pressure points show greater loss than high-speed low-pressure points. These results agree well with those of Faulkner and Smith but show considerably less scatter.

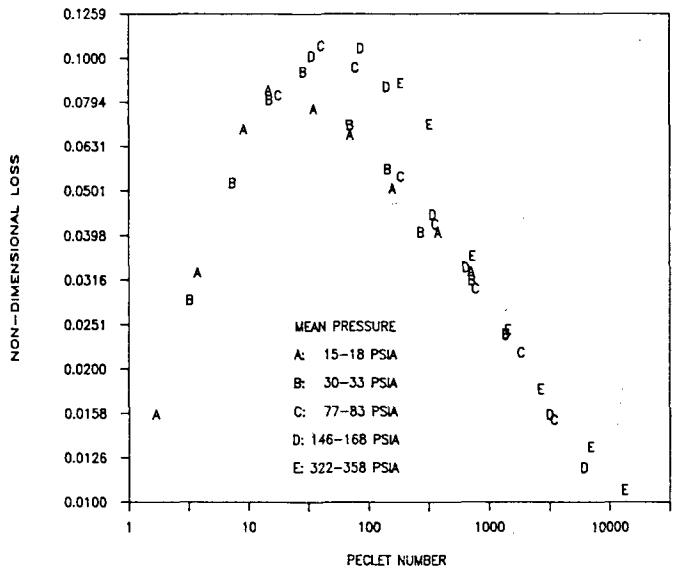

Figure 2. Experimental Results.

COMPARISON OF EXPRESSIONS WITH EXPERIMENTAL RESULTS

Expressions of Annand and of Woschni:

Annand proposed that cylinder convective heat transfer data could be correlated with expressions in the form $Nu=aRe^b$. He based Nusselt number on cylinder diameter and Reynolds number on mean piston speed and instantaneous volume-averaged gas properties. He found for two-stroke engines

$$Nu = .76\ Re^{.64\pm.10} \tag{4}$$

and for four-stroke engines

$$Nu = .26\ Re^{.75\pm.15}. \tag{5}$$

The +/- variations shown represent the spread of the data Annand analyzed. To fit the data, variations shown in the Re exponents require corresponding variations in the multipliers, but Annand did not explain those variations in detail.

Woschni modified Annand's expression by basing the Reynolds number on a mean gas velocity which was a different function of mean piston speed during compression, expansion, intake, and exhaust strokes. For this apparatus, where there is no combustion, intake, or exhaust, his expression becomes

$$Nu = .035\ Re^{.8} \tag{6}$$

where Re is based on a velocity 2.28 x mean piston speed. This is equivalent to

$$Nu = .068\ Re^{.8} \tag{7}$$

with Re based on mean piston speed.

No closed form solution for cyclic lost work appears to be possible using these expressions for heat transfer. Instead, the expressions were used in a numerical simulation of the experimental cycle from which cyclic lost work was predicted. This simulation took into consideration variations in exposed surface area and gas density over the cycle but neglected other property variations. A centered-difference time-stepping procedure was used so as to avoid numerically introduced phase shifts. The simulation is described more fully in the Appendix.

The results of these simulations are compared with the experimental data in Figure 3. None of the three correlations predicts the data well. However, the data at the high Peclet numbers for which correlations of this type are generally used is matched fairly well by

$$Nu = .26\ Re^{.6}. \tag{8}$$

The coefficients in this expression are in the range of those proposed by Annand. It is significant that relatively small changes in these coefficients result in major changes in the predicted non-dimensional loss vs. Pe plot.

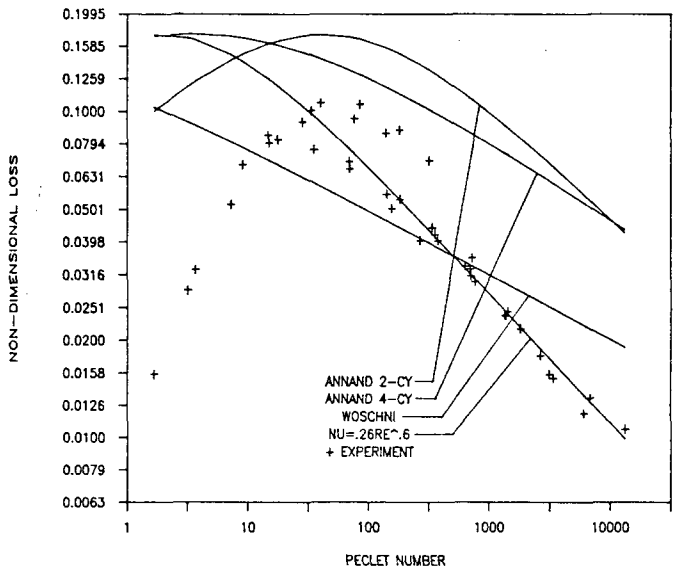

Figure 3. Annand and Woschni Expressions Compared with Experiment.

Expression of Breckenridge et al:

Breckenridge et al modeled the temperature field within a gas spring using exact solutions for one-dimensional solid conduction. The solution for a semi-infinite wall with sinusoidally varying surface temperature was superposed with a uniform sinusoidal temperature variation so that constant surface temperature resulted. The temperature variations in the solid conduction solution were used to predict volume variations in an ideal gas so that the integral of pdV could be calculated. An explicit heat transfer expression was not developed, but the cyclic lost work was predicted as

$$\frac{\text{Loss}}{\text{Cycle}} = F\ \frac{\pi}{\sqrt{2}}\ \frac{\gamma}{\gamma-1}\ A_o p_o \left(\frac{T_a}{T_o}\right)^2 \left(\frac{\alpha_o}{\omega}\right)^{1/2} \tag{9}$$

where F was a "magnification factor" meant to account for the effects of convection. F = 2.6 was recommended, but that value was based on experiments in an apparatus in which the piston rotated. In terms of dimensionless variables the expression becomes

$$L_{nd} = F \frac{\frac{r_v-1}{r_v+1} + \frac{L_s}{D}}{\left(\frac{Pe}{8\pi}\frac{L_s}{D}\right)^{1/2}} \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{T_a}{T_o}\right)^2 \frac{p_o V_o}{W_{adiab}}. \quad (10)$$

This is not truly predictive in that the temperature amplitude is one of the independent variables. In the original work, the loss was non-dimensionalized against the cyclic stored energy, which resulted in cancellation of the T_a term. In Figure 4, the expression has been compared with experimental results in two ways: once by using the measured temperature amplitude, and once using the temperature amplitude that would result from adiabatic compression:

$$\frac{T_a}{T_o}(\text{adiab}) = \frac{r_v^{\gamma-1}-1}{r_v^{\gamma-1}+1} . \quad (11)$$

Both these curves were plotted with F = 1. The curve based on the experimental temperature ratio fits the data well at high Pe but not at low Pe. As might be expected, the curve based on the adiabatic temperature ratio shows higher loss, approaching the other curve as Pe increases.

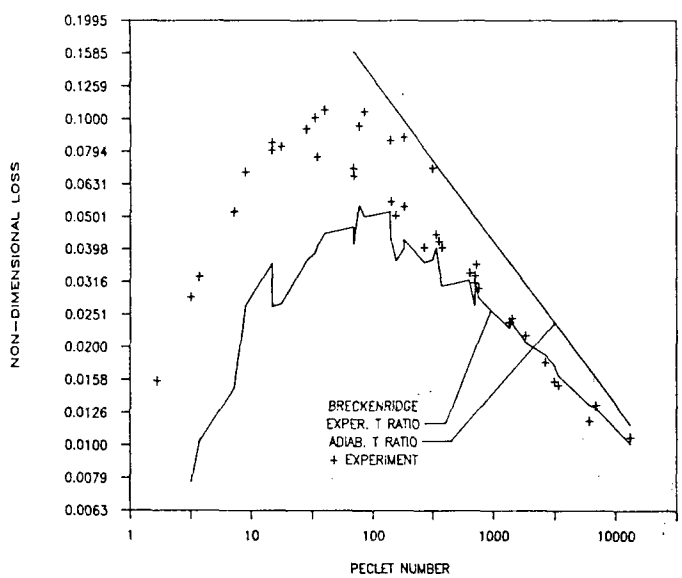

Figure 4. Breckenridge et al Expression Compared with Experiment.

Expression of Otis and of Svoboda et al:

Otis proposed that the heat transfer to the wall of a gas spring could be modeled by a thermal time constant so that the energy balance on the gas was

$$\frac{dT}{dt} = \frac{T_o-T}{\tau} - \frac{pdV}{mc_v dt} , \quad (12)$$

where

$$\tau = \begin{matrix}\text{thermal}\\ \text{time}\\ \text{constant}\end{matrix} = \frac{mc_v}{hA} . \quad (13)$$

The assumption that τ is constant throughout the cycle is equivalent to approximating the increase in h as the gas is compressed as inversely proportional to the decrease in wall area.

Svoboda et al examined the analytic solution for the temperature within a finite cylinder following a step change in wall temperature and proposed that the time constant might be approximated by

$$\tau = K_1 \frac{D^2}{8\alpha_o}\left(\frac{L_o/D}{1+2L_o/D}\right) , \quad (14)$$

where K_1 = constant to approximate analytic solution = 0.428.

This model was used to carry out a numerical cycle simulation similar to that for the Annand and Woschni models. It is described in the Appendix.

Svoboda had proposed that α_o be modified to account for natural convection in hydraulic accumulators, but that modification was not thought appropriate for this application.

Figure 5 compares experimental results with the predictions of the time-constant model, both with K_1=0.428 as proposed in the original work and with K_1=0.22. Neither curve fits the data over the whole range of Pe, but the K_1=0.22 curve fits well at low Pe. It was for low-speed phenomena that the time-constant model was originally developed.

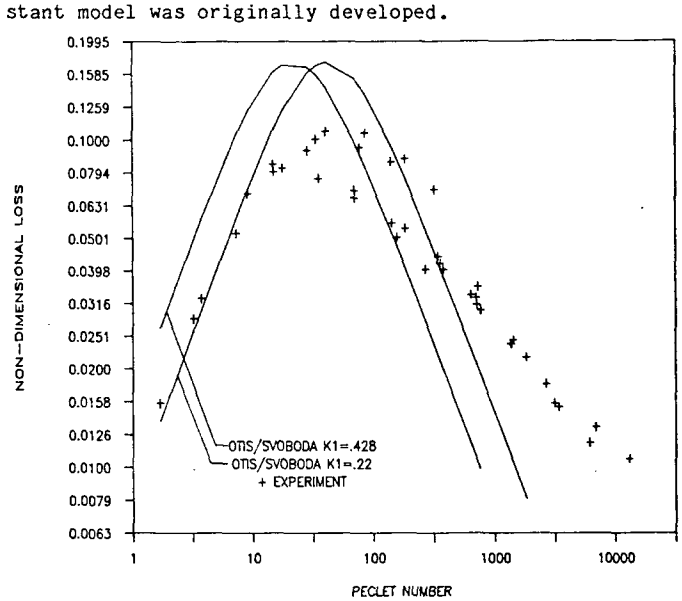

Figure 5. Otis and Svoboda et al Expression Compared with Experiment.

Expression of Lee and of Cooke-Yarborough and Ryden:

Both these studies were based on the exact conduction solution for a finite one-dimensional solid with a sinusoidal source-sink term. For a pressure variation

$$p = p_o + p_a e^{i\omega t} \quad (15)$$

the resulting wall heat transfer rate was

$$q(t) = \frac{-kAT_o}{(2\alpha_o/\omega)^{1/2}} \frac{p_a}{p_o} \frac{(\gamma-1)}{\gamma}(1+i)e^{i\omega t} \tanh z, \quad (16)$$

where

$$z = (1+i)D_h(\omega/32\alpha_o)^{1/2} = (1+i)(\pi Pe_h D_h/32L_s)^{1/2} . \quad (17)$$

Hydraulic diameter D_h is here defined as 4 x Volume / Wetted Area. Pe_h is Peclet number based on hydraulic diameter.

The resulting mixed mean temperature was

$$T = T_o + T_o \frac{p_a}{p_o} \frac{(\gamma-1)}{\gamma} \left(1 - \frac{\tanh z}{z} \right) e^{i\omega t}. \quad (18)$$

The heat transfer rate given by the conduction solution is a sinusoid symmetrical about zero. For a steady state, i.e. no net increase in gas temperature, a first law balance of work in and heat out appears to indicate no loss. In this model, however, the neglect of density variation causes the pressure variation to appear in the form of a heat source/heat sink term. The lost work appears in terms of heat being supplied to the system at a higher temperature than it is removed. Lee predicted loss by assuming that a gas would have the same mixed mean temperature as a solid. He calculated the integral of pdV for a cycle using the mixed mean temperature from Equation 18. Cooke-Yarborough and Ryden assumed the gas would have the same temperature profile as a solid, and calculated the integral of entropy generation over space and time. The result in both cases, expressed non-dimensionally, was

$$L_{nd} = \frac{p_o V_o}{W_{adiab}} \frac{\pi}{2} \left(\frac{p_a}{p_o} \right)^2 \frac{(\gamma-1)}{\gamma} \frac{1}{y} \left(\frac{\cosh y \sinh y - \sin y \cos y}{\cosh^2 y - \sin^2 y} \right). \quad (19)$$

where

$$y = D_h (\omega/32\alpha_o)^{1/2} = (\pi Pe_h D_h/32L_s)^{1/2}. \quad (20)$$

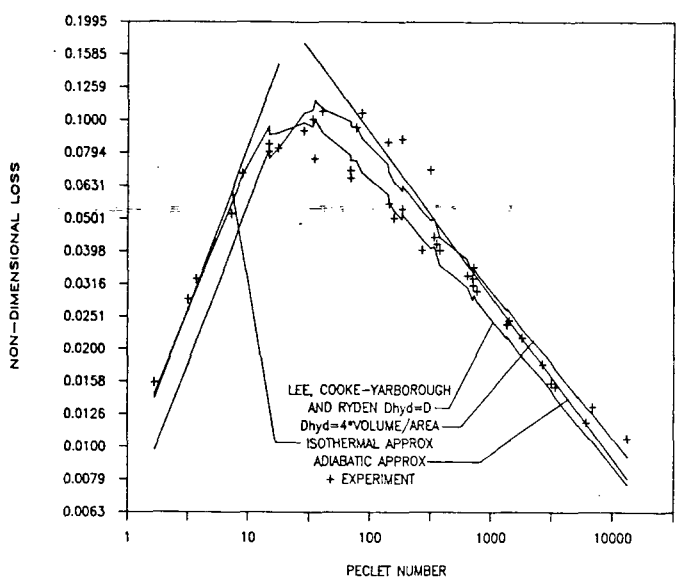

Figure 6. Lee and Cooke-Yarborough and Ryden Expression Compared with Experiment.

In Figure 6 this expression was plotted in two ways: once with hydraulic diameter based on cylinder wall area only (D_h=D) and once with it based on total area, including piston and head, at mid-stroke and volume at mid-stroke (D_h=0.82D for this geometry). While neither plot fits the experimental data exactly, the one in which D_h is based on wall area alone fits better, and in fact predicts experimental results fairly well except where the loss-vs-Pe correlation breaks down.

Lee suggested that a turbulent thermal diffusivity might be used to account for the effects of convection. The result of such a correction factor would be to shift the loss curves in the direction of higher Pe. No turbulent correction has been made here.

The pressure variation in the apparatus was not sinusoidal. The approximation

$$\frac{p_a}{p_o} = \frac{p_{max} - p_{min}}{p_{max} + p_{min}}. \quad (21)$$

was used to produce these curves, where p_{max} and p_{min} were experimentally measured pressures. Since p_a/p_o is one of the independent variables in the loss expression, it is not predictive for systems where only the volume variation is known.

Cooke-Yarborough and Ryden had approximated the group of hyperbolic functions above to form approximations for near-isothermal and near-adiabatic conditions. Near isothermal (low Pe)

$$L_{nd} = \frac{p_o V_o}{W_{adiab}} \left(\frac{p_a}{p_o} \right)^2 \frac{(\gamma-1)}{\gamma} \frac{\pi^2 Pe_h D_h}{48 L_s}, \quad (22)$$

while near adiabatic (high Pe)

$$L_{nd} = \frac{p_o V_o}{W_{adiab}} \left(\frac{p_a}{p_o} \right)^2 \frac{(\gamma-1)}{\gamma} \left(\frac{8 \pi L_s}{Pe_h D_h} \right)^{1/2}. \quad (23)$$

In an attempt to obtain a formula with volume ratio as independent variable, these expressions have been plotted with hydraulic diameter based on cylinder wall area only and with

$$\frac{p_a}{p_o} = \frac{r_v - 1}{r_v + 1} \quad (24)$$

for the near-isothermal, and

$$\frac{p_a}{p_o} = \frac{r_v^{\gamma} - 1}{r_v^{\gamma} + 1} \quad (25)$$

for the near-adiabatic.

These plots are shown in Figure 6. They approach the experimental results as Pe becomes very low and very high, and between them provide simple formulae for hysteresis loss over most of the Peclet number range.

SUMMARY AND CONCLUSIONS

The results of the comparisons between various expressions and experiment may be summarized as follows:

Expressions of the form $Nu=aRe^b$ appear to be capable of predicting heat transfer-related hysteresis loss for Pe>500. The values a=.26 and b=.6 appear to be appropriate. This Peclet number range encompasses the cylinder spaces of most reciprocating engines and compressors.

Expressions based on a thermal time constant model appear to be capable of predicting loss for

Pe<10. The success of such predictions depends on the proper selection of constant K_1: a value of 0.22 appears to be appropriate. This Peclet number range encompasses some low-speed gas springs.

Expressions based on one-dimensional transient conduction solutions appear to be suitable for predicting hysteresis loss over the full range of Peclet numbers, except in those regions where the L_{nd} vs. Pe correlation breaks down. The formula of Lee and of Cooke-Yarborough and Ryden is much superior to that of Breckenridge. A disadvantage of expressions of this type is that their proper application to prediction of instantaneous heat transfer is not clear. An advantage is that they provide closed-form expressions for loss.

The various results described above should be used with caution, as they are the result of experiments with a single gas in a single geometry. The success of the expression of Lee and of Cooke-Yarborough and Ryden suggests that data for a range of Peclet numbers, pressure ratios, specific heat ratios, and bore/stroke ratios might be correlated in the form

$$\frac{\text{Loss}}{p_o V_o (p_a/p_o)^{2(\gamma-1)/\gamma}} = f\,(Pe_h D_h/L_s). \qquad (26)$$

Future experiments must try this possibility.

Correct prediction of heat-transfer related hysteresis loss does not necessarily indicate correct prediction of instantaneous heat transfer throughout the cycle. Future experiments must compare heat transfer vs. crank angle plots in order to confirm accurate heat transfer prediction. Ideally, it will be possible to find a means of correlating these plots over a variety of conditions.

The one-dimensional transient conduction model must be further developed in order to provide useful heat transfer expressions. The expression for heat transfer given in Equation 16 is obviously incorrect; since it neglects the compressibility of the gas it shows zero time-average heat flow. A way must be found to incorporate compressibility effects into the heat transfer expression. Also, the model must be extended to include heat transfer driven by other energy sources in addition to compression.

As a final caution, it must be remembered that each of the expressions studied here neglects certain aspects of the problem in order to obtain a workable solution. The success of any one of them may very possibly represent, to some extent, a cancellation of errors. This cancellation may not occur when the expressions are applied to operating conditions significantly different from those of these experiments.

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APPENDIX: NUMERICAL CALCULATION OF HYSTERESIS LOSS

General Procedure:

In order to develop a general procedure for calculating cyclic loss from heat transfer correlations, the first law for the working gas was first written in nondimensional form. For an ideal gas

$$mc_v dT = q dt - p dV. \qquad (27)$$

Defining

$$T^* = (T-T_o)/T_o, \qquad V^* = V/V_o,$$
$$q^* = q/(p_o' V_o \omega), \qquad t^* = \omega t,$$
$$p^* = p/p_o' = (T^*+1)/V^* \qquad (28)$$

where V_o was volume at mid-stroke, T_o was wall temperature, and p_o' was pressure at V_o and T_o, this became

$$dT^* = (\gamma-1)q^* dt^* - (\gamma-1)(T^*+1)dV^*/V^*. \qquad (29)$$

With sinusoidal piston motion

$$V = V_o(1 + r_v' \sin(\omega t)) \quad \text{and} \quad V^* = 1 + r_v' \sin t^*, \qquad (30)$$

where

$$r_v' = (r_v-1)/(r_v+1).$$

Thus

$$\frac{dT^*}{dt^*} = (\gamma-1)q^* - \frac{(\gamma-1)(T^*+1)r_v'\cos t^*}{1+r_v'\sin t^*} . \quad (31)$$

An expression for q^* as a function of T^* and t^* was found from the heat transfer correlation being used. The first-law balance was then put in the form

$$dT^*/dt^* = F(t^*)T^* + G(t^*). \quad (32)$$

By expressing the derivative as a centered difference in time, the expression was put in discrete form:

$$T^*_{n+1} = \frac{G(t^*_n)+G(t^*_{n+1})+[F(t^*_n)+2/\Delta t^*]\,T^*_n}{2/\Delta t^* - F(t^*_{n+1})} . \quad (33)$$

Starting with uniform T^*, this time stepping was carried out until $T^*(0)$ and $T^*(2\pi)$ converged. The cyclic lost work could then be numerically calculated as

$$\text{Loss} = -\oint q\,dt = -p_o'V_o\oint q^*\,dt^* . \quad (34)$$

For the purpose of comparing the loss thus calculated with experimental results, it was assumed that $p_o=p_o'$, i.e. the cyclic mean pressure was approximately equal to the pressure of the gas at mean volume and wall temperature. This assumption is valid for low volume ratios.

An alternate means of numerically evaluating loss would be to apply an integrating factor to Equation 32. The resulting solution would contain integrals requiring numerical evaluation. Such a method might use less computer time than the one described here.

Evaluation of q^* for $Nu = aRe^b$ Models:

In these models Re was based on instantaneous rather than average gas properties. Neglecting viscosity variations:

$$Re_{inst} = Re_{avg}(V_o/V) = (Pe_{avg}/Pr)(V_o/V). \quad (35)$$

Thus $Nu = aRe^b$ was equivalent, using Pe_{avg}, to

$$q = -(aAk/D)(T-T_o)(Pe/Pr)^b(V_o/V)^b. \quad (36)$$

Using Equation 30 for V, non-dimensional variable definitions, and

$$A = (\pi D^2/2)[1 +(L_s/Dr_v')(1 + r_v'\sin t^*)], \quad (37)$$

the non-dimensional heat transfer was found to be

$$q^* = -\frac{4a}{\pi}\frac{\gamma}{\gamma-1}Pe^{b-1}Pr^{-b}r_v'T^*$$

$$[(1+r_v'\sin t^*)^{-b}+\frac{L_s}{Dr_v'}(1+r_v'\sin t^*)^{1-b}]. \quad (38)$$

Evaluation of q* for Time Constant Models:

By the definition of the time constant (Equation 13)

$$h = mc_v/\tau A, \quad (39)$$

thus

$$q = -hA(T-T_o) =- mc_v(T-T_o)/\tau . \quad (40)$$

Non-dimensionalizing the time constant as

$$\tau^* = \omega\tau = (\pi/16)K_1Pe/(r_v'+L_s/D), \quad (41)$$

and use of the ideal gas law, gave

$$q^* = \frac{-T^*}{(\gamma-1)\tau^*} . \quad (42)$$