Determining Effective Soil Formation Thermal Properties from Field Data Using a Parameter Estimation Technique

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ABSTRACT

A one-dimensional thermal model is derived to describe the temperature field around a vertical borehole heat exchanger (BHEx) for a geothermal heat pump. The inlet and outlet pipe flows are modeled as one, and an effective heat capacity is added to model the heat storage in the fluid and pipes. Parameter estimation techniques are then used to estimate various parameters associated with the model, including the thermal conductivity of the soil and of the grout that fills the borehole and surrounds the U-tube. The model is validated using test data from an experimental rig containing sand with known thermal conductivity. The estimates of the sand's thermal conductivity derived from the model are found to be in good agreement with independent measurements.

INTRODUCTION

Geothermal or ground-source heat pumps (GHPs) have been shown to be a very efficient method of providing heating and cooling for buildings. GHPs exchange heat (reject or extract) with the earth by way of a circulating fluid rather than circulating outdoor air as with an air-source heat pump. The entering water temperature to a GHP is generally cooler than outdoor air when space cooling is required and warmer than the outdoor air when space heating is required. Consequently, the temperature lift across a GHP is less than the lift across an air-source heat pump. This leads to greater efficiency, higher capacity at extreme outdoor air temperatures, and better indoor humidity control. These benefits are achieved, however, at the cost of installing a ground heat exchanger. In general, this cost is proportional to length, and for this reason there is an incentive to install the minimum possible length that meets design criteria.

The design of a ground heat exchanger is a complicated process, requiring at a minimum the operating characteristics of the heat pump, estimates of peak block and annual loads for the building, and some knowledge of the thermal properties of the soil. In the case of a vertical ground heat exchanger, these properties generally vary with depth; effective or average thermal properties over the length of the heat exchanger are usually sought. When the cost of doing so can be justified, these properties are measured experimentally at the site. In these experiments, a test well is drilled to a depth on the same order as the expected depth of the heat pump heat exchangers. A heat exchanger is inserted, and the borehole is grouted according to applicable state and local regulations. Water is heated and pumped through the heat exchanger, and the inlet and outlet water temperatures are measured as a function of time. A schematic of a typical experiment is presented in Figure 1. Data on inlet and outlet temperature, power input to the heater and pump, and water flow rate are collected at regular intervals—typically 10 to 15 minutes —for the duration of the experiment, which may be as long as 50 to 60 hours.

According to classical theory, at sufficiently large times the ground heat exchanger can be modeled as a line heat source in an infinite medium (Carslaw and Jaeger 1947; Ingersoll et al. 1954). Given the rate of heat input to the loop and the inlet and outlet temperatures as a function of time, the effective thermal conductivity of the soil formation can be determined. A problem with this method, however, is that it assumes the rate of heat input to the water loop to be constant. This is rarely the case, since in the field the heater is usually powered by a

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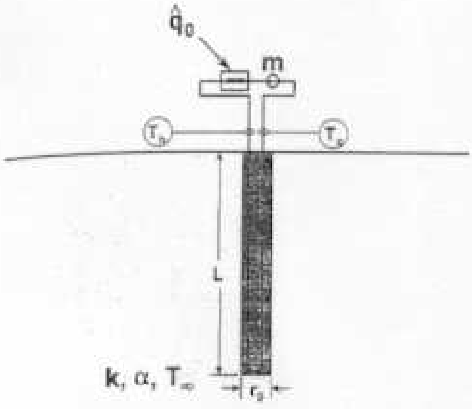

Figure 1 *Schematic of field test to measure effective soil thermal properties.*

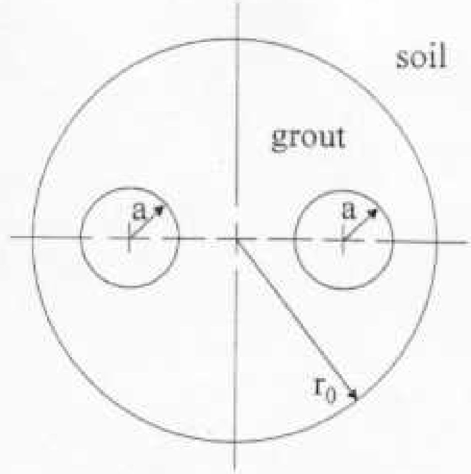

Figure 2 *Geometry of borehole heat exchanger.*

portable generator. Even where line electrical service is available, short-term sags and swells in voltage may cause variations in heat input to the water loop. The random variation in power input—as well as random errors in the temperature measurements—presumably cause variations in the value of thermal conductivity obtained from a particular experiment, but classical theory provides no information on the variance. Another area of uncertainty is when to begin measuring inlet and outlet temperatures and how long the data should be collected, i.e., the period during which the line source model is valid.

Ground heat exchangers have also been modeled as a cylindrical, constant heat source in an infinite medium (Ingersoll et al. 1954). Deerman and Kavanaugh (1991) extended this model to account for variable heat flux but in a manner not generally suitable for the analysis of short-term field data. Other authors (Eskilson 1987; Hellstrom 1991; Rottmayer et al. 1997; Austin 1998) have proposed more detailed two- and three-dimensional numerical models.

The objective of this paper is to show how parameter estimation techniques can be used to measure effective soil thermal properties based on field tests employing a vertical borehole heat exchanger and to develop confidence intervals for these measurements. A simple one-dimensional thermal model is derived to describe the temperature field around the borehole. The inlet and outlet pipe flows are modeled as one, and a thin film is added to account for the heat capacity of the pipes and the fluid. The estimation of the ground thermal conductivity and other parameters from fluid inlet and outlet temperature histories is investigated. It is shown that the ground conductivity can be relatively accurately estimated even though the conditions inside the borehole are uncertain.

DERIVATION OF MODEL

Figure 2 presents a cross section of a typical vertical borehole heat exchanger (BHEx), which is assumed to be filled with grout to a radius r_0. The BHEx contains a U-tube that consists of piping of radius a. Figure 3 is a section along the vertical axis of the borehole. It is assumed that a fluid with specific heat c_{pf} and flow rate $\dot{m}$ enters the heat exchanger at temperature T_h and exits the heat exchanger at a temperature T_c lower than the entering temperature. Heat is transferred from the inlet pipe to the grout at the rate of $q_h''(x,t)$ and from the outlet pipe at a rate of $q_c''(x,t)$. A steady-state heat balance for a control volume around the outside of the pipes gives

$$\dot{m}c_{pf}[T_h(0,t)-T_c(0,t)] = \int_{x=0}^{L} [q_h''(x,t)+q_c''(x,t)]2\pi a\,dx \quad (1)$$

Because Equation 1 does not account for the heat stored by the fluid and the pipes, it is accurate only for steady-state conditions; however, since the time constant for the fluid is assumed to be much shorter than that of the surrounding soil,

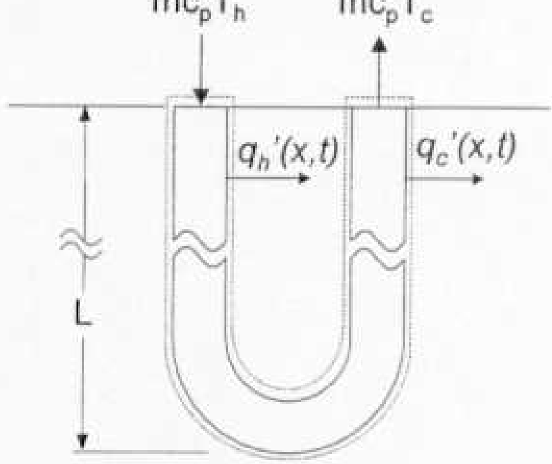

Figure 3 *Control volume around U-tube.*

modeling the fluid heat transfer as a steady-state process should not introduce large errors. As shown below, the heat stored by the fluid and piping will be included in another manner.

In the grout and the soil, heat transfer takes place through conduction. The energy transferred from the U-tube is absorbed by the grout and eventually by the surrounding soil. As the heating (or cooling) time increases, the thermally affected region becomes larger, that is, the thermal penetration increases with time. Moreover, as the distance from the borehole increases, the temperature distribution becomes one-dimensional in the r direction. This suggests that the temperature distribution in the soil at a large distance from the U-tube is similar to that which would be caused by a single pipe with some effective radius.

In Figure 4, the inlet and outlet pipes have been replaced by a single pipe of radius b. Note also that a thin film of thickness δ is included on the outer surface of the effective pipe. Since Equation 1 does not include the heat storage in the fluid and the pipes, this effect is introduced in the conduction equations though the use of the thin film.

The one-dimensional model for heat transfer in the soil is the transient heat conduction equation. One equation is required for the thermal resistive film, another for the grout, and another for the surrounding soil. The model for the soil is

$$\frac{k_s}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s}{\partial r}\right) = (\rho c_p)_s \frac{\partial T_s}{\partial t}, r_0 < r < \infty \tag{2}$$

where the s subscript denotes soil.

The grout is considered to extend from $b + \delta$ to r_0. The energy equation for the grout is similar to Equation 2 with s replaced with g for grout.

$$\frac{k_g}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_g}{\partial r}\right) = (\rho c_p)_g \frac{\partial T_g}{\partial t}, b+\delta < r < r_0 \tag{3}$$

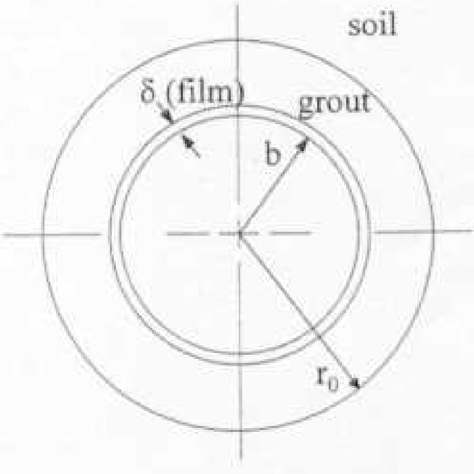

Figure 4 Geometry with U-tube replaced by a single pipe of effective radius b.

Finally, a similar equation is written for the film, which extends from b to $b + \delta$.

$$\frac{k_\phi}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_\phi}{\partial r}\right) = (\rho c_p)_\phi \frac{\partial T_\phi}{\partial t}, b < r < b+\delta \tag{4}$$

where the subscript ϕ denotes the thin film. In Equations 2, 3, and 4, the temperature is assumed to be a function of r, x, and t, but the heat conduction in the x direction is assumed to be negligible compared to that in the r direction. Hence, the net heat conduction term in the x direction, $k\partial^2T/\partial x^2$, is not included in these equations.

In general, the thermal conductivities and volumetric heat capacities in Equations 2 through 4 are also functions of depth. This is particularly true for the soil thermal conductivity, which may vary considerably across the various layers of soil and rock. The soil may also be anisotropic, i.e., its thermal conductivity may depend on the direction of heat flow. Heat transfer is also affected by the presence of moisture. Since inclusion of all of these effects would complicate the analysis considerably, it is assumed that heat transfer takes place in the soil and grout through conduction only and that the soil and the grout are both homogeneous and isotropic. The parameters k and ρc_p are then effective or average thermal properties for the given material over the length of the heat exchanger.

The boundary condition for Equation 3 at the effective pipe radius b is

$$-k_\phi \frac{\partial T_\phi(b,x,t)}{\partial r} = q''_h(x,t) + q''_c(x,t) \tag{5}$$

This applies at each x, but it is more convenient to average over x in Equation 5 and also in Equations 2, 3, and 4. Integrating Equation 5 from $x = 0$ to L gives

$$-k_\phi \frac{\partial \overline{T}_\phi(b,t)}{\partial r} = \frac{1}{L}\int_{x=0}^{L}(q''_h(x,t) + q''_c(x,t))dx \tag{6}$$

where

$$\overline{T}_\phi(b,t) = \frac{1}{L}\int_{x=0}^{L} T_\phi(b,x,t)dx \tag{7}$$

The differential equations for the conduction in the soil, grout, and film are now averaged over x from zero to L. Equations 2, 3, and 4 become

$$\frac{k_s}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{T}_s}{\partial r}\right) = (\rho c_p)_s \frac{\partial \overline{T}_s}{\partial t}, \quad r_0 < r < \infty, \tag{8}$$

$$\frac{k_g}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{T}_g}{\partial r}\right) = (\rho c_p)_g \frac{\partial \overline{T}_g}{\partial t}, \quad b+\delta < r < r_0, \tag{9}$$

$$\frac{k_\phi}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{T}_\phi}{\partial r}\right)=\left(\rho c_p\right)_\phi \frac{\partial \overline{T}_\phi}{\partial t}, \quad b<r<b+\delta \tag{10}$$

where the average temperatures are defined by

$$\overline{T}_s(r,t)=\frac{1}{L}\int_{x=0}^{L} T_s(r,x,t)dx, \tag{11}$$

$$\overline{T}_g(r,t)=\frac{1}{L}\int_{x=0}^{L} T_g(r,x,t)dx, \tag{12}$$

and the average film temperature is given by Equation 7.

The interface conditions are given by

$$k_\phi \frac{\partial \overline{T}_\phi(b+\delta,t)}{\partial r}=k_g\frac{\partial \overline{T}_g(b+\delta,t)}{\partial r}, \tag{13}$$

$$\overline{T}_\phi(b+\delta,t)=\overline{T}_g(b+\delta,t), \tag{14}$$

$$k_g\frac{\partial \overline{T}_g(r_0,t)}{\partial r}=k_s\frac{\partial \overline{T}_s(r_0,t)}{\partial r}, \tag{15}$$

$$\overline{T}_g(r_0,t)=\overline{T}_s(r_0,t). \tag{16}$$

The boundary condition at $x=\infty$ is

$$\overline{T}_s(\infty,t)=T_\infty, \tag{17}$$

and the initial conditions are

$$\overline{T}_s(r,0)=T_\infty, \overline{T}_\phi(r,0)=T_\infty, \overline{T}_g(r,t)=T_\infty . \tag{18}$$

Since the sum of the heat flow rates from the inlet and outlet pipes is equal to the rate of heat input to the system, the boundary condition given by Equation 6 can be written as

$$-k_\phi \frac{\partial \overline{T}_\phi(b,t)}{\partial r}=\frac{\hat{q}_0}{L} \tag{19}$$

where $\hat{q}_0$ is the field-measured heat input (including the net pumping power). Inlet and outlet temperatures are also measured in the field, and the condition given by Equation 19 could also be written as

$$-k_\phi \frac{\partial \overline{T}_\phi(b,t)}{\partial r}=\frac{\dot{m}c_p}{2\pi bL}\left[T_h(b,0)-T_c(b,0)\right]. \tag{20}$$

However, since the power measurement is generally more accurate than the measurements of temperature and flow rate, Equation 19 is used in this analysis. This presumes that heat losses between the pump/heater and the heat exchanger are negligible.

In summary, the direct problem is described by Equations 8, 9, and 10 with interface conditions (Equations 13 through 16), with condition at infinity (Equation 17), and with initial conditions (Equation 18). The boundary condition at $r=b$ is Equation 19.

PARAMETER ESTIMATION

Given the simplifications made in the analysis, the equations above are a complete mathematical statement of the direct problem. However, the problem cannot be solved using classical techniques because not all of the relevant parameters (such as the ground and film thermal conductivities) are known. Some extra information is available that allows these and other parameters to be estimated. The average fluid temperature defined by the average of the inlet and outlet temperatures is known:

$$\hat{T}_f=\left(T_h(t)+T_c(t)\right)/2 \tag{21}$$

(The "hat" symbol is used here and in Equation 19 to denote a measured quantity.) This average temperature may not be precisely equal to the integrated averages of the inlet and outlet fluid temperatures but is expected to be as accurate as the lumping of the two pipes into one. Furthermore, the inlet and outlet temperatures are the only ones that are feasible to measure in the field. This average temperature is assumed to be equal to the average of the surface film temperature given by Equation 7. Hence, two conditions are given at the film surface: heat flux and temperatures as a function of time. For direct problems, only one condition can be used, so there is excess information. This information can be used to estimate the value of one or more of the parameters. The calculated average surface temperature history given by Equation 7 is made to agree in a least squares sense with the measured average fluid temperature given by Equation 21 by minimizing a sum of squares function with respect to the parameters, for example, the ground thermal conductivity. The sum of squares function, S, is

$$S=\sum_{i=1}^{n}\left(\hat{T}_i-\overline{T}_{\phi,i}\right) \tag{22}$$

where i denotes a measurement time, $\hat{T}_i$ is the field-measured average fluid temperature, and $\overline{T}_{\phi,i}$ is the average temperature of the film at $r=b$, as predicted by the model.

Note that if the film thickness is taken to be zero, the film has no volumetric heat capacity and no resistance to heat transfer, and the grout has the same thermal properties as the soil, then the problem reduces to that of a cylindrical heat source in an infinite medium. For constant heat flux, this problem has an analytical solution involving a rather complicated integral of Bessel and other functions (Ingersoll et al. 1954). The method can be extended to the case of nonconstant heat flux using a convolution process, but this is difficult to implement numerically. For this reason, and so that the effect of the thin film and the grout can be included, the direct problem is solved numerically using a finite difference grid and a Crank-Nicolson integration scheme.

Altogether, the model presented above contains nine parameters: the thermal conductivities of the soil, grout, and thermal film; the volumetric heat capacities of the soil, grout, and thermal film; the thickness of the thermal film; the effective pipe radius; the far-field temperature. In general, it will not be possible to estimate all of these parameters with a single experiment, as some may be dependent on others. For example, since the thin film is used to account for the thermal capacitance of the fluid and pipes, one would expect the thickness δ to be related to the film's volumetric heat capacity.

The parameter estimation algorithm proceeds by first assuming trial values for the parameters in question. Given these values, the (measured) heat flux is used to drive the numerical model. The numerical model gives a predicted value of T_δ as a function of time for the duration of the experiment. The sum of squared errors between the predicted and measured temperatures is calculated, as in Equation 20. The Gauss method of minimization is then used to determine the parameter values that minimize the sum of the squared errors. A computer program is used to solve the problem for various thermal properties including the thermal conductivities of the soil and grout.

While the derivation is beyond the scope of this paper, use of the Gauss minimization technique enables calculation of approximate confidence regions for the parameters (Beck and Arnold 1976). The validity of these confidence intervals depends on a number of statistical assumptions that may or may not be satisfied in a given experiment. There are a number of other sources of error unaccounted for by these confidence intervals: the assumption that the soil properties are homogeneous, when in fact they vary with depth; the assumption that the far-field temperature T_∞ is accurately measured; and, of course, the assumption that a rather complicated three-dimensional heat transfer process can be represented by a one-dimensional model. For this reason, the confidence intervals derived from the data are expected to be somewhat smaller than the true confidence intervals; experience with other data indicates that the true confidence intervals on the parameter estimates may be as much as twice the value indicated. Nevertheless, these approximate confidence intervals are useful for qualitative assessment of the accuracy of property values and for comparing one experiment to another.

ANALYSIS OF TEST DATA

A major problem in validating a model such as the one presented here is that the true soil formation thermal properties at a given site are generally unknown. Where estimates of these properties are available, they are usually based on previous experiments and other simplified heat transfer models such as the line heat source. Recently at a university, however, a test rig was constructed that simulates the conditions experienced by a vertical heat exchanger. The rig consists of a box made of ¾ in. (1.90 cm) plywood with approximate dimensions of 4 ft × 4 ft × 48 ft (1.2 m × 1.2 m × 14.6 m). A U-tube heat exchanger consisting of nominal 1 in. (2.5 cm) diameter polyethylene piping is placed horizontally along the centerline of the box, parallel to the long axis. The U-tube is grouted to a diameter of 5 in. (12.7 cm) from the centerline, and the box is filled with a homogeneous material with known thermal conductivity. As in a field experiment, water is heated and pumped through the U-tube at a known flow rate, while the rate of heat input and the inlet and outlet water temperatures are measured at regular intervals. Although it is recognized that this rig does not exactly duplicate the heat transfer processes that take place in a vertical heat exchanger in a field installation, the device does provide an opportunity to build confidence in heat transfer models such as the one developed here because the material that represents the soil is homogeneous and its thermal conductivity can be measured independently.

A data set from a 63-hour experiment using the university test rig was obtained from Smith (1998). For the experiment, the rig was filled with wet sand and the U-tube was grouted along its entire length with a commercially available grout. The thermal conductivity of the sand was measured twice using a thermal probe, giving values of 1.40 and 1.45 Btu/h·ft·°F (2.42 and 2.51 W/m·K). At the beginning of the experiment, the sand was at a uniform temperature of 72.0°F (22.2°C). During the experiment, the average flow rate of water was approximately 4 gallons per minute (0.25 liters per second), and the average rate of heat input was 1238 Btu/h (363 W). Both of these values varied by only about ±0.5% of their average values over the 63-hour period. Water inlet and outlet temperature, flow rate, and power to the heater were measured at one-minute intervals.

To analyze the experimental data, some modifications were made to the model to account for the physical characteristics of the test rig. Since the model is radially symmetrical, the rig was modeled as a cylinder consisting of four regions with different materials. The U-tube was modeled as a pipe with an effective diameter of 0.75 in. (1.9 cm) (a value of 0.5 in. [1.3 cm] was also used; as discussed below, the effective pipe diameter appears to have only a small effect on the thermal property estimates). A thin film is assumed to exist at the surface of the pipe to simulate the heat capacity of the water and the pipes; the film is assumed to be 0.024 in. (0.61 mm) thick. The diameter of the grout is given as 5 in. (12.7 cm); thus, the material from the outer edge of the thin film to a radius of 2.5 in. (6.4 cm) is assumed to be grout. The bulk of the region is the sand—from the outer radius of the grout to an effective radius of 2.25 ft (0.686 m). Plywood with a thickness of 0.75 in. (1.9 cm) is at the outer radius. Summarizing in the radial direction, there are the film, grout, sand, and plywood. The finite difference model includes 72 nodes: 2 in the film, 6 in the grout, 60 in the sand, and 4 in the plywood. The internal time step was 15 seconds. In the analysis, the outer boundary of the plywood is assumed to be isothermal at the initial temperature of 72.0°F (22.2°C).

Experience with the model has shown that volumetric heat capacities of the materials are difficult to estimate using data from a typical field test. The error surface is relatively flat in the direction of volumetric heat capacity, resulting in large error bounds for the estimates. Fortunately, it has also been found that estimates of thermal conductivity are relatively insensitive to the value chosen for volumetric heat capacity. For this analysis, a volumetric heat capacity of 35 Btu/ft^3·°F (2300 KJ/m^3·K) was assumed for both the sand and the grout. For comparison, Kavanaugh and Rafferty (1997) report volumetric heat capacities of between 24 and 40 Btu/ft^3·°F (1600 and 2700 KJ/m^3·K) for 20% moist sand, and between 32 and 35 Btu/ft^3·°F (2100 and 2300 KJ/m^3·K) for 20% moist clay. (Bentonite grout is composed of a particular variety of moist clay.)

The thermal conductivity of the film was assumed to be a large value, so that all of the resistance to heat transfer is contained in the soil and the grout. Three parameters are estimated: the thermal conductivity of the soil, the thermal conductivity of the grout, and the volumetric heat capacity of the thin film.

Figure 5 plots the measured average water temperature and the model's predicted average temperature for the first thirty hours of the experiment, at the converged values of the parameters. There is excellent agreement between the two. In order to distinguish between model and data, the predicted water temperature is plotted only at intervals of one hour. Figure 6 presents the residuals, i.e., the difference between the measured and predicted average water temperature, as a function of time for the first thirty hours. The larger residuals at early times are likely due to the assumption of one-dimensional heat transfer. However, since the largest residual is only 0.2°F (0.1°C), the error is not large. Although the residuals are small overall, with an RMS value of 0.057°F (0.032°C), if the model were truly an accurate representation of the heat transfer process taking place in the test rig, the residuals would be random and uncorrelated, which they clearly are not. Their slow rise and fall over the 30 hours indicates that there may be some secondary effects that the model does not capture.

The estimate of thermal conductivity from the model is in excellent agreement with the measured values. After thirty hours, the converged value of thermal conductivity for the wet sand is 1.41 Btu/h·ft·°F (2.44 W/m·K), compared with the measured values of 1.40 and 1.45 Btu/h·ft·°F. The 95% confidence region for the thermal conductivity value is ±0.172 Btu/h·ft·°F (0.298 W/m·K), or about 12% of the estimated value.

The converged value for the thermal conductivity of the grout is 0.62 Btu/h·ft·°F (1.07 W/m·K) with a 95% confidence region of ±0.037 Btu/h·ft·°F (0.064 W/m·K), which is about 6% of the estimated value. The actual thermal conductivity of the grout was not available, but the estimated value is in the range of values reported by Kavanaugh and Rafferty (1997). In any case, it is presumed that the grout thermal conductivity estimate may account for other effects, such as the U-tube spacing and the contact resistance between the pipes and the grout.

The third parameter estimated was the volumetric heat capacity of the thin film. It was expected that this parameter would converge to a value close to the volumetric heat capacity of water, 62.3 Btu/ft^3·°F (4184 KJ/m^3·K). However, using 30 hours of data, the converged value was 116.5 Btu/ft^3·°F (7824 KJ/m^3·K). This indicates that the thin film may be accounting for other effects besides heat storage in the water and piping.

A second run was performed assuming an effective pipe diameter of 0.5 in. (1.3 cm). In this case, the converged value of the sand thermal conductivity was 1.42 ±0.133 Btu/h·ft·°F (2.46 ±0.230 W/m·K), which indicates that the effective pipe diameter has only a small effect on the thermal conductivity measurement. For comparison, Bose (1984) has recommended using an effective diameter of $\sqrt{2}$ times the diameter of the U-tube piping, which in this case would correspond to a radius of 0.707 in. (1.80 cm).

The thermal property estimates changed somewhat when data up to 60 hours were used. Assuming an effective pipe radius of 0.75 in. (1.9 cm), the RMS value of the residuals

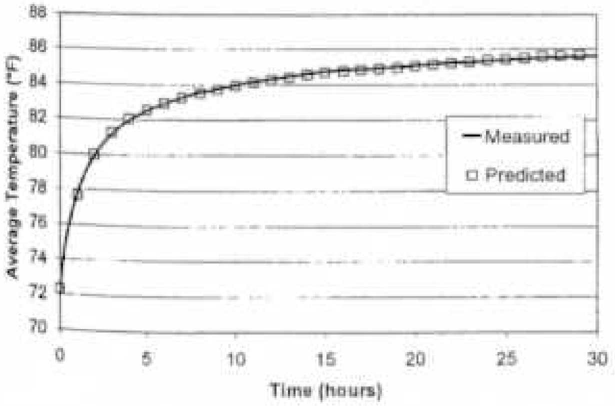

Figure 5 Measured and predicted average water temperatures for the experiment using a university test rig.

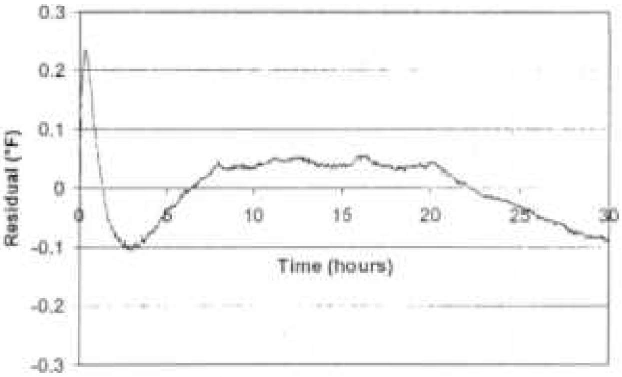

Figure 6 Residuals for the experiment on the university test rig.

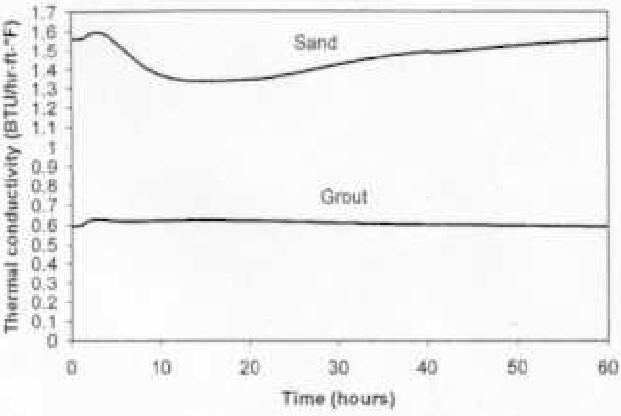

Figure 7 Sequential estimates of sand and grout thermal conductivity.

increased slightly to 0.064°F (0.036°C). The converged value for the thermal conductivity for the sand increased to 1.56 ±0.118 Btu/h·ft·°F (2.70 ± 0.204 W/m·K). This is higher than the value obtained after 30 hours, but the measured value of thermal conductivity still lies within the 95% confidence region. The converged value of the grout thermal conductivity was slightly lower in this case, 0.59 ±0.023 Btu/h·ft·°F (1.02 ± 0.040 W/m·K).

Sequential estimates of the sand and grout thermal conductivity are presented in Figure 7. What this shows, for each time, are the values of the thermal conductivities estimated using all data up to and including that time. If the model were an accurate representation of the heat transfer processes taking place in the test rig, both curves would be expected to flatten out at later times. In the case of the thermal conductivity of the grout, this does seem to be the case, but it appears that the estimate of the thermal conductivity of the sand is rising slightly even at 60 hours. This may be due to the assumptions made in the analysis, primarily the assumption that the rectangular test rig could be modeled as a cylinder. Errors associated with this assumption are expected to become more important at later times; however, this effect is caused by the finite dimensions of the test rig and would be of no concern in a field experiment.

The computer time required to converge to an estimate of the parameter values depends, of course, on the initial guesses for the parameter values. Nevertheless, even for guesses 50% larger than the true values, the model requires only about 60 seconds to converge on a 200 MHz Pentium II-based PC.

Obviously, some further work is required to validate the model and to gain experience in its use. Other parameters can be estimated as well, and it would be particularly useful to estimate the borehole resistance—i.e., the total resistance of heat transfer from the fluid to the borehole wall—a parameter that is required by many design algorithms. A series of field experiments is planned for the summer of 1998, in which the model's thermal conductivity estimates will be compared with estimates from other commonly used models and with the "true" or effective thermal conductivity of the soil, determined from data on active borefields at the sites.

CONCLUSIONS

A one-dimensional model has been developed to describe the thermal behavior of a borehole heat exchanger for a geothermal heat pump. The model lumps the inlet and outlet pipes into one pipe of an effective radius b and adds a film at the outer surface of the pipe to account for the heat transfer resistance of the surrounding grout and the convective heat transfer coefficient. The film also has an effective heat capacity to model the heat capacity of the fluid and different heat capacity of the grout compared to the ground. Parameter estimation techniques are used to derive values of soil and grout thermal conductivity from the model and experimental data. An important feature of this technique is that it provides approximate confidence intervals on the parameter estimates; these intervals can be used to assess the accuracy of experiments and the thermal conductivities derived from them. The technique was validated using data from an experiment at a university in which a U-tube heat exchanger was placed in a medium with independently measured thermal properties. After 30 hours, the model's predicted thermal conductivity is in excellent agreement with the measured value. After 60 hours the model predicts a slightly higher value of soil thermal conductivity (due perhaps to edge effects associated with the finite volume of the test rig), but the confidence interval still includes the measured value.

NOMENCLATURE

Abbreviations

BHEx = borehole heat exchanger
GHP = geothermal or ground-source heat pump

Variables and Constants

a = radius of U-tube pipes
b = radius of effective pipe
c_p = specific heat
k = thermal conductivity
L = borehole length
$\dot{m}$ = flow rate
q' = rate of heat flow per unit length
q'' = rate of heat flow per unit area
$\hat{q}_0$ = rate at which heat is added to the fluid
r = radial variable
r_0 = borehole radius
t = time
T = temperature
$\bar{T}$ = average temperature
$\hat{T}$ = measured temperature
x = axial variable

α = thermal diffusivity

δ = film thickness

ρ = density

ρc_p = volumetric heat capacity

Subscripts

c = cold or outlet side

h = hot or inlet side

i = time increment i

s = soil

g = grout

ϕ = thin film

∞ = infinity

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DISCUSSION

C. Yavuzturk: The authors present a very interesting and worthwhile study to predict thermal conductivity of ground formation using in situ collected temperature data as input into a parameter estimation model. I have benefited from the proposed concept and its findings in my own research activities. However, I have two questions to help further clarify some points made by the authors. (a) A confidence interval provided by the parameter estimation technique for the prediction of the thermal conductivity of ground formation is clearly very useful. The authors identify a number of other sources of errors that are not included in the confidence interval and state that the true confidence interval may be as much as twice the values indicated. Presumably, the confidence interval only accounts for purely random errors in the experimental data. Is this correct, or are the effects of any other uncertainties somehow included in confidence intervals? (b) The authors state that variations in the volumetric specific heat capacity of the ground formation have been found to be insensitive to the predictions of its thermal conductivity. Could the authors please provide the range for the specific volumetric heat capacity values of the ground formation where the thermal conductivity predictions are relatively insensitive? For example, how much change in the thermal conductivity prediction would be reasonable to expect when the volumetric specific heat is varied by about 10 BTU/ft3-F (670 kJ/m3-K)?

John A. Shonder: (a) Yes, the confidence interval accounts only for random errors in the experimental data. It is analogous to the confidence interval on slope that is obtained from the regression of linear data, and depends on the same statistical assumptions: that the experimental errors are normally distributed with a mean of zero, that these errors are uncorrelated with time, etc. We know however that in the case of time series data the errors usually are correlated with time. Field experiments to measure thermal conductivity are no exception, and a plot of residuals vs. time at the converged property values shows that there is correlation. Because this violates one of the statistical assumptions, we have to say that the confidence intervals are approximate.

Returning to the analogy of linear regression, it is always possible to plot a straight line through parabolic (or higher-order) data. Although a straight line is not the correct model for the data, the regression algorithm will blithely calculate a slope and a confidence interval for it. We recognize that something similar is going on when we use a simple one-dimensional model to estimate the thermal parameters, and this is another reason why we consider the confidence intervals to be approximate. The real heat transfer process is three dimensional, thermal conductivity varies with depth, etc., so a one-dimensional model simplifies the situation considerably. The search algorithm finds parameters that cause this model to follow the data, but there may be other models which, with different parameters, would follow the data more closely. Nevertheless we find that the one-dimensional approximation models the data quite well. (b) An example that covering about half that range should give an idea of the sensitivity. For one particular data set we analyzed, the converged values of thermal conductivity and volumetric heat capacity were 1.36 BTU/hr-ft-F (+/- 0.13) and 17.2 BTU/ft^3-F (+/- 9.0), respectively. Reducing the volumetric heat capacity by 10% to 15.5 BTU/ft^3-F caused the specific heat value to rise about 1.5% to 1.38 BTU/hr-ft-F. Raising the vol. heat capacity to 20.0 BTU/ft^3-F (about 16% higher than the converged value) resulted in a thermal conductivity of 1.33 BTU/hr-ft-F, about 2% lower than the converged value. So the percent change in thermal conductivity is around one-tenth the percent change in volumetric heat capacity. Note that even with a +/- 10% change in volumetric heat capacity, the thermal conductivity stays well within the confidence interval for the converged value of thermal conductivity.