Heat Pump Ground Coil Analysis With Thermal Interference*

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ABSTRACT

When two horizontal ground coils are buried in the same trench, one on top of the other, the total energy exchange between coils and ground increases drastically over that of a single coil for the same trench length. However, due to the thermal interference between the two coils, the overall performance of the two-coil trench is very difficult to determine. Traditionally this type of problem is handled by the line source mirror image approach. But this approach, due to the requirement of estimating the strength of the source, will result in extremely conservative ground coil design. This paper provides a detailed mathematical model to describe the operation of the two-coil system which can calculate more realistically the performance of the ground coils, which in turn could make the ground coil heat pump system more economically competitive with other heating and cooling systems. The effect of thermal interference is clearly shown in the calculated soil temperature profile. The effect of fluid inlet positions, from upper or lower coils, is also studied.

The comparison of measured and calculated daily energy absorbed from the ground indicated a maximum error of 27%, with the average error at less than 16%, with the calculated values on the conservative side.

This model can better calculate the performance of two coils in the same trench. It can be used for design purposes, or used to check such coil designs by other methods.

NOMENCLATURE

C_p	specific heat
DT	amplitude of annual surface temperature variation
h	convective heat transfer coefficient
K	thermal conductivity
r	radius
T	temperature
TA	annual average ground surface temperature
t	time
t_o	time of year since January 1, h
V	fluid velocity
x	distance along the ground coil
Z	distance below ground surface

Greek Symbols

ρ	density
θ	angular direction, defined in Fig. 1
ϕ	phase angle
α	thermal diffusivity, $K/\rho C_p$
ψ	angular direction, defined in Fig. 1

Subscripts

1	coil inside wall
2	coil outside wall
F	farfield
f	fluid
i	initial
l	lower
p	coil
s	soil
u	upper

INTRODUCTION

Heat pumps have long been recognized for their energy conservation potential. Low-grade heat from air, a convenient body of water, or the ground can be used as the heat source, or sink, for heat pump operation. Ground-coupled heat pumps can greatly improve the overall system's performance because of the stable year-round ground temperatures, the replacement of the outdoor fan with a small fluid-circulating pump, and the elimination of outdoor coil frost and defrost losses.

Much theoretical and experimental work in this field has been accomplished in the past years. However, comprehensive ground coil analyses are still very rare. The most popular current theoretical approach utilizes line source (or cylindrical source) theory [1]. The major drawbacks of this approach are (1) that one has to assume, or guess, the strength of the line source, which makes this approach more dependent on empirical data; (2) that the coil fluid-wall convective heat transfer resistance is generally

ignored even when the fluid flow is in the laminar region; (3) that the continuous coil fluid temperature change indicates that the strength of the line source will not be constant along the coil; and (4) that the effect of the seasonal temperature variation at depth is generally ignored so that the problem can be treated as radially symmetrical. Detailed modeling work of ground coil operation has been accomplished for vertical concentric tube type heat exchangers [2], and for a single horizontal coil [3], which have eliminated the drawbacks of line source application. Recently it was found that by burying multi-coils in the same trench, the trenching cost could be cut drastically [4], and thus make the ground coil heat pump systems more competitive. For analysis of multi-coils in a single trench, the effect of thermal interference could become very important. The thermal interference problem can be easily taken care of by line source theory. However, in order to describe the coil operation with thermal interference more realistically, it has to be modeled in detail mathematically.

In this paper, the operation of a two-coil trench ground-coupled heat pump system was analyzed for winter heating season application. A three-dimensional mathematical model based on energy conservation is formed. The model considers the fluid flow inside the coil, coil material and size, coil cyclic operation, and the effect of coil thermal interference.

A computer code, based on the model, has been completed. A simulation run of the University of Tennessee's field experimental data [5] for 28 days, by inputting the experimental ground coil inlet fluid temperature as a function of time, indicated excellent prediction of the test results of the energy exchange between the coil and ground. A parametric study of the effect of the fluid inlet position has also been performed.

MATHEMATICAL FORMULATION

The model is based on energy balance. Figure 1 shows the schematic of the coil arrangement. The following assumptions have been made to simplify the analysis: (1) the soil is homogeneous and the soil thermal properties are constant, (2) the fluid temperature and velocity are uniform at any coil cross section, (3) the temperature distribution in both coils is radially symmetrical, (4) the coils are buried deep enough so that a circular farfield boundary could be formed with the center located on lower coil and Z_2 as the radius, and (5) the axial heat transfer in soil and coil wall is neglected.

For winter operation, assumption (1) is close to the real ground conditions because the ground top layer is usually saturated with moisture. Assumption (2) is also true because of the large coil length-over-diameter ratio. The coil wall is usually very thin. With the well mixed fluid flow, assumption (3) would only cause a negligible error in coil wall temperature calculation. Since Z_1 is usually over 0.9 m, assumption (4) would be a good assumption for a single coil [6]. For a two-coil arrangement, there is not enough field experimental data to confirm the assumption. However, except for the part of the soil on top of the top coil, the distance from the top coil to farfield boundary is between Z_1 and Z_2. Since Z_2 is usually not less than 1.5 m, assumption (4) is probably true for most of the soil region around the two coils. Due to the small temperature differentials along the coil wall and soil in axial direction and low thermal conductivity values of the coil and soil, assumption (5) is acceptable.

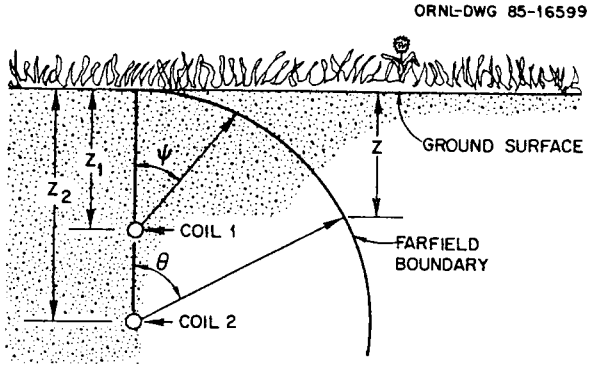

Fig. 1. Schematic of two-coil arrangement.

Upper coil: (a) Fluid:

$$-V\frac{\partial T_{fu}}{\partial x} + \frac{2K_p}{r_{1u}\rho_f C_{pf}}\frac{\partial T_{pu}}{\partial r}\Big|_{r_{1u}} = \frac{\partial T_{fu}}{\partial t} \tag{1}$$

(b) Coil wall:

$$\frac{\partial^2 T_{pu}}{\partial r^2} + \frac{1}{r}\frac{\partial T_{pu}}{\partial r} = \frac{1}{\alpha_p}\frac{\partial T_{pu}}{\partial t} \tag{2}$$

(c) Boundary conditions:

At $r = r_{1u}$:

$$h(T_{pu} - T_{fu}) = K_p \frac{\partial T_{pu}}{\partial r_u}\Big|_{r_{1u}} \tag{3}$$

At $r = r_{2u}$:

$$2\pi K_p \frac{\partial T_{pu}}{\partial r}\Big|_{r_{2u}} = K_s\int_0^{2\pi}\frac{\partial T_s(r,\psi)}{\partial r}\Big|_{r_{2u}} d\psi \tag{4}$$

$$T_s = T_{pu} \tag{5}$$

Equation (5) assumes that there is no contact resistance between the coil and the soil, which is true for grund coil winter operation where the voids between soil and coil will be filled by moisture.

Lower coil:

(d) Fluid:

$$V\frac{\partial T_{fl}}{\partial x} + \frac{2K_p}{r_{1l}\rho_f C_{pf}}\frac{\partial T_{pl}}{\partial r}\Big|_{r_{1l}} = \frac{\partial T_{fl}}{\partial t} \tag{6}$$

(e) Coil wall:

$$\frac{\partial^2 T_{pl}}{\partial r^2} + \frac{1}{r}\frac{\partial T_{pl}}{\partial r} = \frac{1}{\alpha_p}\frac{\partial T_{pl}}{\partial t} \tag{7}$$

(f) Soil:

$$\frac{\partial^2 T_s}{\partial r^2} + \frac{1}{r}\frac{\partial T_s}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T_s}{\partial \theta^2} = \frac{1}{\alpha_s}\frac{\partial T_s}{\partial t} \tag{8}$$

(g) Boundary conditions:

At $r = r_{1l}$:

$$h(T_{pl} - T_{fl}) = K_p \frac{\partial T_{pl}}{\partial r}\Big|_{r_{1l}} \tag{9}$$

At $r = r_{2l}$:

$$2\pi K_p \frac{\partial T_{pl}}{\partial r}\Big|_{r_{2l}} = K_s \int_0^{2\pi} \frac{\partial T_s(r,\theta)}{\partial r}\Big|_{r_{2l}} d\theta \tag{10}$$

$$T_s = T_{pl} \tag{11}$$

where Eq. (11) assumes that no contact resistance exists between coil and soil.

At $Z = Z_1$, $\theta = 0$:

$$T_s = T_{pu} \tag{12}$$

At $r = r_F$ (farfield):

$$T_{sF} = TA - DT \cdot \exp\left[-Z\sqrt{\frac{\pi}{8766\alpha_s}}\right] \cdot \cos\left[\frac{2\pi t_o}{8766} - Z\sqrt{\frac{\pi}{8766\alpha_s}} - \phi\right] \tag{13}$$

where Eq. (13) is the Kusuda and Achenbach correlation [7] for ground temperature calculation at different depths. If there is a known natural ground temperature profile below the surface to a depth of twice Z_2, Eq. (13) can be replaced by the known temperature profile.

(h) Initial conditions ($t = 0$):

$$T_{fu} = T_{fui}(x) \tag{14}$$

$$T_{pu} = T_{pui}(r,x)$$

$$T_{fl} = T_{fli}(x)$$

$$T_{pl} = T_{pli}(r,x)$$

$$T_s = T_{si}(r,x,\theta)$$

where T_{fui}, T_{pui}, T_{fli}, T_{pli}, and T_{si} are known functions of x, r and θ. For the coil to start the operation, these initial temperatures can be easily calculated by Eq. (13) or from the ground temperature profile if it is known.

(i) Fluid inlet conditions:

$$T_f(t,x=0) = T_{fo}(t) \tag{15}$$

where T_{fo} is a known function of time, t, which actually represents the heat pump model. For a given water-source heat pump, with known operating conditions and inlet fluid temperature, the exit fluid temperature (ground coil inlet fluid temperature) can be easily calculated from the heat pump published capacity data. While the line source theory needs to specify the strength of the source, this model only needs the ground coil inlet fluid temperature as the input. It will then calculate the amount of energy exchange between the coil and ground.

The model described so far is for the ground coil with circulating fluid. During the "off" cycle period, the fluid velocity is 0. Since the heat capacity of the fluid in the coil is very small, Eqs. (3) and (9) can be written into the form at $r = r_{1u}$:

$$T_{fu} = T_{pu} \tag{16}$$

and at $r = r_{1l}$:

$$T_{fl} = T_{pl} \tag{17}$$

Since the above equations are all of low order, they are relatively easy to solve numerically. A finite difference computer code based on the above model was written.

COMPUTER MODEL

A finite difference scheme was written to solve the mathematical model numerically. Figure 2 shows the construction of the mesh. In this case,

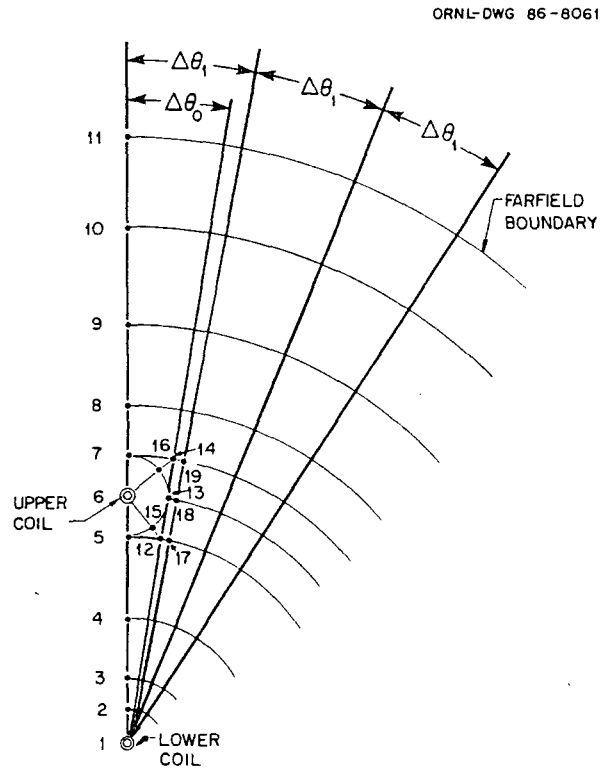

Fig. 2. Mesh scheme of two coil arrangement.

the radial interval between points 5 and 6 and between points 6 and 7 are selected to be equal. The angular interval $\Delta\theta_o$ is selected so that point 13 is very close to $\psi = \pi/2$ (see Fig. 1).

For the upper coil, the temperatures of fluid and coil wall are marched up to the coil outside edge. For the lower coil, the soil temperatures are also marched, except at point 6, which is equal to the upper coil outside edge temperature. Soil temperatures at points 15 and 16 can be marched because the temperatures at points 5, 6, 7, 12, 13, and 14 have already been calculated. Since points 5, 15, 13, 16, and 7 all have the same distance from the upper coil, the energy absorbed by the upper coil can be easily calculated. All the other calculations are fairly routine.

Two time steps are used. The first is quite small and is used for the fluid and coil wall, because the coil wall is very thin. The second time step, which is substantially larger, is used in the soil region.

MODEL VALIDATION DISCUSSION

The computer code was used to simulate the field test results provided by the University of Tennessee [5]. The hourly measurement of fluid and farfield ground temperatures (up to 3.05 m below surface) were provided. The hourly fraction of heat pump "on" time was determined by the fraction of heat pump power consumption over that at steady–state operation. Hourly energy absorbed from the ground, and coil fluid flow rate, were also provided. Figure 3 shows the coil layout. It can be seen that part of the layout is two-coil trench and the rest is a single coil trench. This ground coil setup made the computer simulation more difficult. The code was modified to include the simulation of the single coil section.

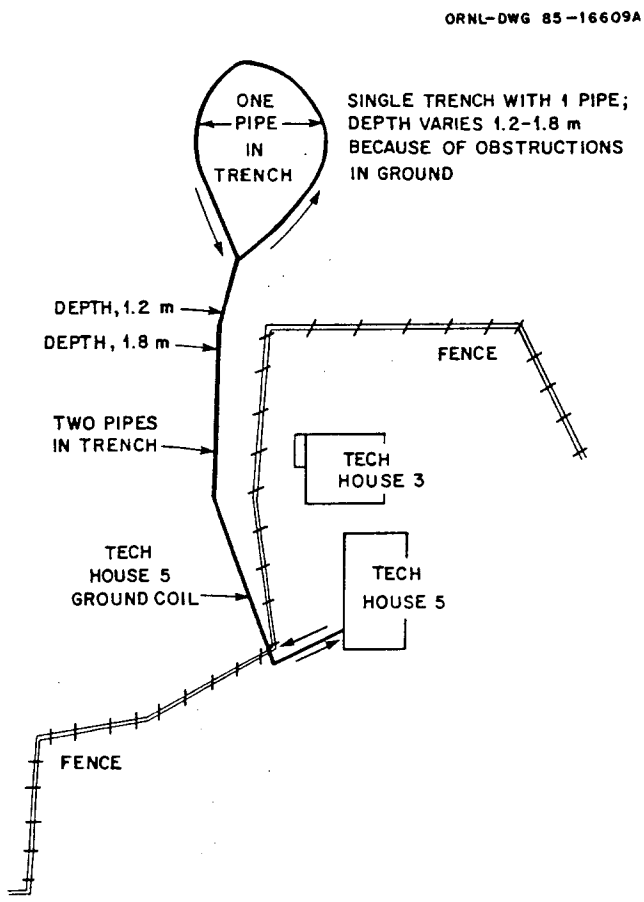

Fig. 3. Plan view of the University of Tennessee horizontal ground coil layout.

The following list provides the properties of the coil, soil, and fluid:

Coil length	116 m two coil trench and 116 m single coil trench
Coil burial depth	two coil section, 1.22 m top coil and 1.83 m lower coil; single coil section 1.52 m
Coil size	4.09 cm ID and 4.83 cm OD
Coil material	polybutylene
Coil thermal conductivity	0.216 W/(m·K)
Coil specific heat	2174 J/(kg·K)
Fluid	water
Thermal conductivity of fluid	0.594 W/(m·K)
Specific heat of fluid	4190 J/(kg·K)
Soil	saturated clay
Soil thermal conductivity	2.181 W/(m·K)
Soil thermal diffusivity	0.0035 m^2/h
Flow rate	1.522 m^3/h

The fluid flow entered the top ground coil and exited at the lower coil.

A total of 28 days were simulated. Figure 4 shows the comparison of calculated and measured daily energy absorbed from the ground. The calculated values are about 15% less than the measured ones. Figure 5 shows the comparison of calculated and measured fluid temperature at exit position. It can be seen that the match is good until the heat pump "off" period becomes long. Since the fluid temperatures were measured indoors, once the heat pump was "off," the fluid temperature started approaching

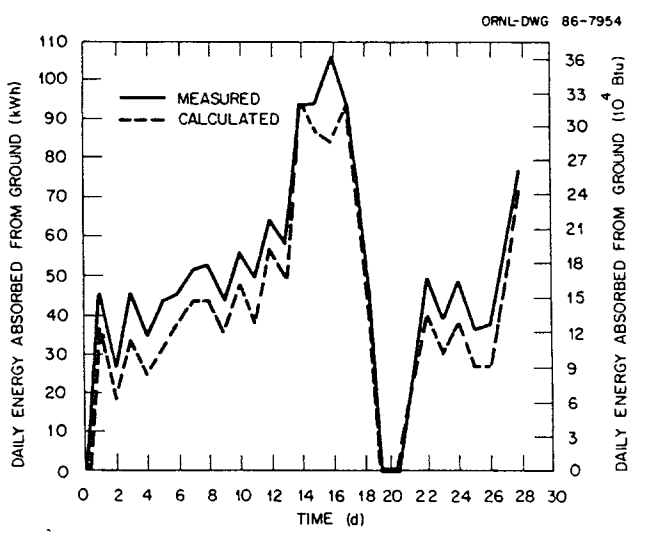

Fig. 4. Comparison of measured and calculated daily energy absorption from ground.

room temperature. However, the model still considers that the fluid temperature can only approach ground temperature, which is much lower than the room temperature. Figure 5 also indicates that calculated fluid temperatures are generally lower than the measured ones. One explanation is that the ground temperatures at different depths are actually higher than the measured farfield temperatures. Since part of the coil is buried alongside the house, as shown in Fig. 3, the effect of the house on the ground temperature should be considered. This also explains why the calculated

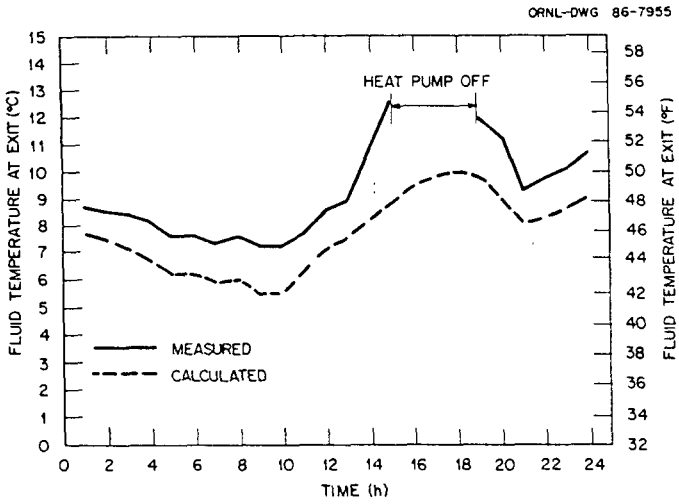

Fig. 5. Comparison of measured and calculated fluid temperature at coil exit.

daily energy absorption from the ground is conservative compared with the measured values.

Figure 6 shows the calculated soil temperature distribution after 28 days of coil operation in two coil section. It clearly indicates the thermal interference effect. The temperature profile cannot be validated due to the lack of field experimental data. However, it does indicate the ability of the computer code.

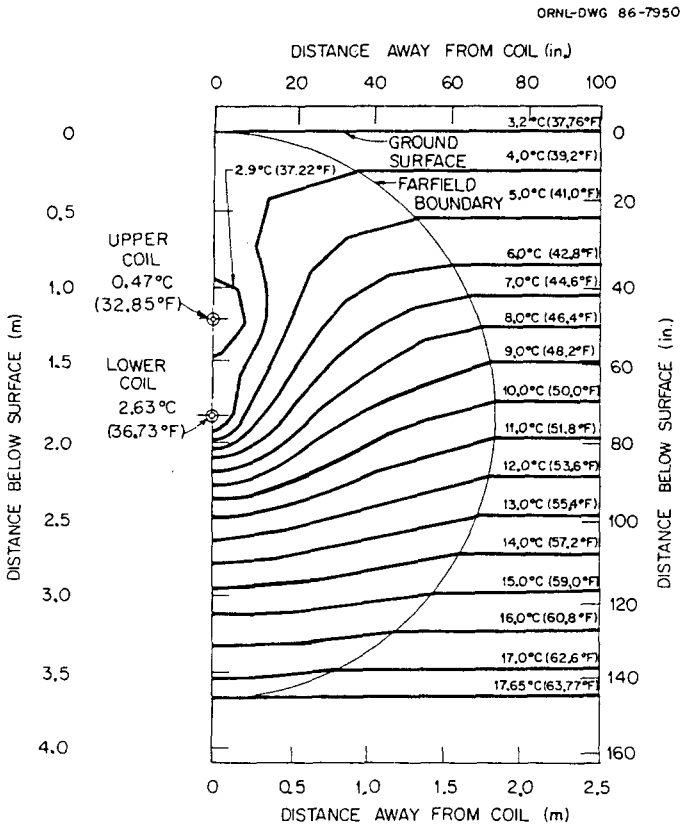

Fig. 6. Calculated ground temperature distribution after 28-day coil operation.

Figure 7 shows the cumulative energy absorbed from the ground by each coil section. Since the single-coil section has the same trench length as the two-coil section, it can be used to estimate the improvement of the coil performance due to an extra coil laid in the same trench by comparing the total energy exchange in the single-coil section to that of the two-coil section. Figure 7 indicates that the two-coil section, for 28-day coil operation, can absorb 60% more energy than the single-coil section. This figure is, however, expected to drop when the coil operation continues beyond 28 days, because the thermal interference effect becomes more and more serious. This result is consistent with the field experiment data reported by Hughes et al. [4].

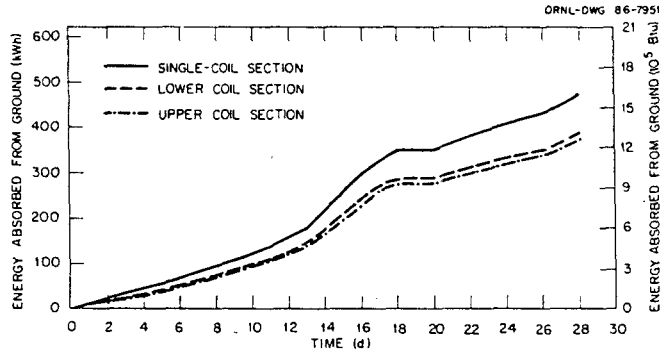

Fig. 7. Energy absorption from ground by each section of coils.

Figure 8 shows the effect of coil performance if the inlet fluid location is switched from the upper coil to the lower coil. The total energy absorbed was reduced by almost 25% compared with the simulation run, mainly due to ineffective use of the upper coil.

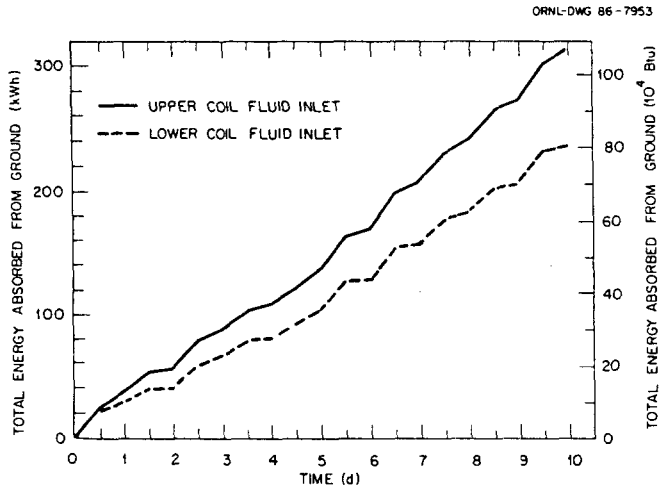

Fig. 8. Comparison of calculated total energy absorption from ground with different fluid inlet positions.

CONCLUSION

A set of partial differential equations describing the operation of a ground coil coupled heat pump system with thermal interference from the adjacent coil leg was solved numerically. This model differs from the traditional line source approach by calculating the energy exchange between the coil and the soil, while the line source approach requires knowledge of the strength of the source.

The model was used to simulate the University of Tennessee's ground coil field experimental results for 28 days with very good match in daily energy absorption from the ground. The average error over a 28-day period is less than 16% with the calculated values on the conservative side.

The two coil trench does increase the energy absorption over the single coil arrangement which occupies the same size of ground area. For better design, the fluid should enter the top coil. Otherwise, a decrease in coil performance would result. due to ineffective use of the upper coil.

Ground coil design has long been dominated by line source methods. They are simple to use, and can easily handle the thermal interference problem. However, the requirement to estimate the source strength makes this approach more or less dependent on past design experience. If no such local experience exists, the design of ground coil with line source theory tends to become extremely conservative. This model can more realistically simulate the ground coil operation with thermal interference effect and variable farfield boundary conditions. This model, although it consumes more computer time, can be used to check the ground coil design with other methods if it is not used for design purposes.

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