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A Theoretical and Experimental Analysis of Double-Spiral, Double-Wall Heat Exchangers

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ABSTRACT

A novel design for a double-spiral-coil counterflow heat exchanger, which satisfies the requirement for a double wall between the fluids for heat pump water heaters and some solar energy collecting systems, was built and tested. The temperature profiles for both fluids were found to be linear functions of coil length, even at flow rates as low as 1.5 gpm (0.095 l/s).

Based on the linear temperature profiles, a mathematical model was set up and an easy-to-use closed form solution was derived. The solution predicted the experimental fluid temperature profiles well, except at very low fluid flow rates where the calculated heat exchanger performance became conservative.

The performance of the conventional counterflow heat exchanger was calculated, based on the same amount of coil material and the same operating conditions, and compared with the novel design. It was found that the new design, for the tests performed so far, was at least 50% better in terms of heat exchanger effectiveness.

INTRODUCTION

For heat exchangers used in heat pump water heaters and some solar energy collecting systems, a double wall between the fluids is often required to prevent the contamination of potable water. Most heat exchangers for this kind of application are tube-in-tube type, with a third tube as a sheath around the inner tube, or with two tubes bonded tightly together. The former ones are more efficient, but the latter cost less.

This study discusses a novel design for double-wall heat exchangers. The concept of the design is to roll two coils in a double spiral fashion (Figure 1). This design will have more contact surface area than the traditional two-tube continuously bonded types with the same amount of coil material, yet the new design is still low in cost.

The theoretical analysis of the performance of this type of heat exchanger for water heater heat pump application is very simple, since the refrigerant-side temperature can be considered constant. However, for two noncondensing or nonevaporating fluids, the analysis becomes complicated because the fluids are cooled or heated on two sides of the coils with different temperatures. A mathematical model was derived for the performance of this heat exchanger design in counterflow operation, but the temperature variation along the coils had to be known before the model could be solved. The temperature profiles were determined experimentally by a prototype unit built for laboratory testing. The temperature profiles for both fluids were found to be linear functions of coil length even at very low flow rates. Based on the linear temperature profiles, a closed form solution was derived. The calculated results

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matched the experimental data very well. Furthermore, the analysis indicates that the new design outperforms the conventional simple counterflow heat exchanger by a wide margin, with the same amount of coil length and under the same operating conditions.

THEORETICAL ANALYSIS

Based on Figure 1, the following equations for coil length can be derived. The general equations are shown as follows for the counterflow arrangement:

$$R = \frac{\theta' t}{\pi} + r_o' \,, \tag{1}$$

$$L = \int_0^{\theta} R \, d\theta' = \frac{\theta^2 t}{2\pi} + r_o' \, \theta \,. \tag{2}$$

For coil 1, $r_o' = r_o + \frac{t}{2}$. For coil contact length 1, $r_o' = r_o$ (the contact starts at $\theta = 2\pi$). For coil contact length 2, $r_o' = r_o + t$. For coil 2, $r_o' = r_o + \frac{3t}{2}$. For coil 1:

$$L_1 = \frac{\theta^2 t}{2\pi} + (r_o + \frac{t}{2})\theta. \tag{3}$$

For coil 2:

$$L_2 = \frac{\theta^2 t}{2\pi} + (r_o + \frac{3t}{2})\theta. \tag{4}$$

The temperature gradient was found experimentally to be a linear function of coil length, so the temperature of fluid 1 can be expressed as follows:

$$T_1 = \frac{T_{1o} - T_{1i}}{L_{1T}} \left[\frac{t\theta^2}{2\pi} + (r_o + \frac{t}{2})\theta \right] + T_{1i} \,. \tag{5}$$

Similarly, the temperature of fluid 2 is:

$$T_2 = \frac{T_{2i} - T_{2o}}{L_{2T}} \left[\frac{t\theta^2}{2\pi} + (r_o + \frac{3}{2}t)\theta \right] + T_{2o} \,. \tag{6}$$

Total heat transfer for fluids 1 and 2 are as follows:

$$\begin{aligned} \dot{Q}_T &= \dot{m}_1 C_{p1}(T_{1o} - T_{1i}) \\ &= UW \int_0^{\theta} (\frac{\theta t}{\pi} + r_o + t)(T_2 - T_1)d\theta \\ &\quad + UW \int_{2\pi}^{\theta} (\frac{\theta t}{\pi} + r_o)(T_2 - T_1)d\theta \,. \end{aligned} \tag{7}$$

$$\dot{m}_2 C_{p2}(T_{2i} - T_{2o}) = \dot{m}_1 C_{p1}(T_{1o} - T_{1i}) \tag{8}$$

Equations 7 and 8 have only two unknowns, T_{2o} and T_{1o}. For simple counterflow heat exchangers, the second term at the right of Equation 7 does not exist.

By substituting Equations 5, 6 and 8 into Equation 7 and then rearranging the terms, we get:

$$
\begin{aligned}
&\left\{ \frac{m'_1 C_{p1}}{UW} + C_R(2r_o + t)\theta_o + C_R \frac{t\theta_o^2}{\pi} - C_R \frac{1}{L_{2T}} (2r_o + t)\left[\frac{t\theta_o^3}{6\pi} + (r_o + \frac{3}{2}t)\frac{\theta_o^2}{2} \right] \right. \\
&- C_R \frac{1}{L_{2T}} \left[\frac{t^2\theta_o^4}{4\pi^2} + \frac{2}{3\pi}(r_o t + \frac{3}{2}t^2)\theta_o^2 \right] + (2r_o + t)\frac{1}{L_{1T}} \left[\frac{t\theta_o^3}{6\pi} + (r_o + \frac{t}{2}) \frac{\theta_o^2}{2} \right] \\
&+ \frac{1}{L_{1T}} \left[\frac{t^2\theta_o^4}{4\pi^2} + (2r_o t + t^2) \frac{\theta_o^3}{3\pi} \right] - C_R 2\pi(r_o + t) + C_R \frac{r_o}{L_{2T}} 2\pi^2(r_o + \frac{13}{6}t) \\
&\left. + C_R \frac{2\pi^2}{L_{2T}} (\frac{4}{3} r_o t + 3t^2) - \frac{r_o}{L_{1T}} 2\pi^2(r_o + \frac{7}{6}t) - \frac{2\pi^2}{L_{1T}}(\frac{4}{3}r_o t + \frac{5}{3}t^2) \right\} T_{1o} \qquad (9) \\
&= \frac{m_1 C_{p1}}{UW} T_{1i} + \left\{ T_{2i} - (1 - C_R)T_{1i} \right](2r_o + t)\theta_o + \left[T_{2i} - (1 - C_R)T_{1i} \right] \frac{t\theta_o^2}{\pi} \\
&- C_R \frac{(2r_o + t)}{L_{2T}} \left[\frac{t\theta_o^3}{6\pi} + (r_o + \frac{3}{2}t) \frac{\theta_o^2}{2} \right] T_{1i} - C_R \frac{1}{L_{2T}} \left[\frac{t^2\theta_o^4}{4\pi^2} + \frac{2}{3\pi}(r_o t + \frac{3}{2}t^2)\theta_o^2 \right] T_{1i} \\
&+ (\frac{2r_o + t}{L_{1T}}) \left[\frac{t\theta_o^3}{6\pi} + (r_o + \frac{t}{2}) \frac{\theta_o^2}{2} \right] T_{1i} + \frac{1}{L_{1T}} \left[\frac{t^2\theta_o^4}{4\pi^2} + (2r_o t + t^2)\frac{\theta_o^2}{3\pi} \right] T_{1i} \\
&- \left[T_{2i} - (1 - C_R)T_{1i} \right] 2\pi(r_o + t) + \frac{C_R}{L_{2T}} r_o 2\pi^2(r_o + \frac{13}{6}t)T_{1i} \\
&+ \frac{C_R}{L_{2T}} 2\pi^2(\frac{4}{3}r_o t + 3t^2)T_{1i} - \frac{r_o}{L_{1T}} 2\pi^2(r_o + \frac{7}{6}t) T_{1i} - \frac{2\pi^2}{L_{1T}}(\frac{4}{3}r_o t + \frac{5}{3}t^2)T_{1i} .
\end{aligned}
$$

where $C_R = \dfrac{m'_1 C_{p1}}{m'_2 C_{p2}}$ and W is the width of the coil contact area. The calculation of the overall heat transfer coefficient, U, was based on the tube wall thickness and its thermal conductivity and the fluid Reynolds and Prandtl numbers (Kays 1966).

Equation 9 provides a closed form solution of T_{1o}. Once T_{1o} is found, T_{2o} can be easily calculated from Equation 8.

EXPERIMENTAL APPARATUS

The heat exchanger was built with 0.5 in (12.7 mm) copper tubing with a total of 8 3/4 windings for each coil ($\Theta_o = 17\frac{1}{2}\pi$). Hot and cold water were used as the fluids. Calibrated thermocouples were inserted into the coils for fluid temperature measurements. A data logger was used for data collection. The flow rates were measured by a scale and a stopwatch.

Figure 1 shows the schematic of the heat exchanger, and Figure 2 shows a photograph of the prototype unit.

MODEL VALIDATION

While the length of the coil is easy to calculate, the contact area of the two coils is very difficult to measure. Since the heat exchanger tested was a prototype, the two coils did not line up very well. It was estimated that "W" values varied from 0.2 to 0.3 in (5 to 7.6 mm).

In order to determine the average "W" value, it was first assumed that the mathematical model was correct. Then, with the measured fluid inlet and outlet temperatures and flow rate of one test, "W" was calculated from Equation 9 and found to be around 0.22 in (5.6 mm). This "W" value probably includes the contact resistance. If this "W" value was correct, the mathematical model should be able to predict the test results over a wide range of operating conditions. Failure to do so would indicate that the model was either wrong or inadequate.

Figures 3 through 7 show the excellent match between the calculated and tested results with W = 0.22 in (5.6 mm). The temperatures are linear functions of coil length for every test.

DISCUSSION AND CONCLUSION

While the calculated results match the test data very well, the deviation becomes more obvious when the flow rate is low, as shown in Figures 6 and 7. Since the model provides an exact solution and the temperature profiles are still linear as shown in the figures, the deviation is probably not caused by the model. It could be that the calculations of the Nusselt numbers for both coils were not accurate enough at low flow rates. With many turns of coil winding, the fluids at low flow rates could be more turbulent than the Reynolds numbers had indicated. Mori and Nakayama (1967, p. 37-59) and Shchukin (1969, p. 72-76) do provide ways to calculate Nu (Nusselt number) with the correction of coil curvature. However, Nu is then a function of coil length, which is not convenient to use. The way Nu is calculated here is roughly equal to the Nu calculated by cited references at about half coil length. However, Figures 6 and 7 still show good matches between experimental and theoretical results.

In order to compare the performance of this new design and conventional counterflow heat exchangers, the temperature profiles of the latter case were calculated (Jacob 1957, p. 208). Figures 3 through 8 indicate that the new design outperforms the simple counterflow heat exchanger for the same amount of coil material and same operating conditions by a wide margin. For the tests performed so far, the heat exchanger effectiveness of the new design is at least 50% higher than that of the conventional simple counterflow designs. Because the test unit is a prototype with very small coil contact width, the effectiveness is relatively low. Once the manufacturing procedures are set up, the effectiveness will be improved due to the increase of contact area between the coils.

At very low flow rates, less than 1.3 gpm (0.082 l/s) in this study, the fluid temperature profile becomes nonlinear along the coil length (Figure 8). This model for nonlinear temperature profiles cannot predict the heat exchanger performance accurately. However, the figure shows that even at this low flow rate, the double-spiral counterflow design is still much better than the conventional counterflow design.

Since this design is simple, efficient, easy to manufacture, and low in cost, the closed-form solution derived for this novel double-wall heat exchanger can be very useful in the nonmixing fluids heat transfer field.

NOMENCLATURE

C_{p1}	=	Specific heat of fluid 1
C_{p2}	=	Specific heat of fluid 2
C_R	=	Ratio of rate of heat capacity = $\dfrac{m_1 C_{p1}}{m_2 C_{p2}}$
L	=	Length of coil
L_1	=	Length of coil 1
L_{1T}	=	Total length of coil 1
L_{2T}	=	Total length of coil 2
m_1	=	Mass flow rate of fluid 1
m_2	=	Mass flow rate of fluid 2
Q_T	=	Total heat transfer rate
R	=	Coil bending radius

r	=	Initial coil bending radius
r_o^ρ	=	Coil bending radius (see description in Theoretical Analysis section)
t	=	Coil outer wall-to-wall thickness
T_1	=	Temperature of fluid 1
T_{1i}	=	Fluid 1 inlet temperature
T_{1o}	=	Fluid 1 outlet temperature
T_2	=	Temperature of fluid 2
T_{2i}	=	Fluid 2 inlet temperature
T_{2o}	=	Fluid 2 outlet temperature
U	=	"U" factor (overall heat transfer coefficent) between fluids
W	=	Width of coil contact area
θ	=	Radian
θ'	=	Dummy variable of radian
θ_o	=	Total radian of coil winding.

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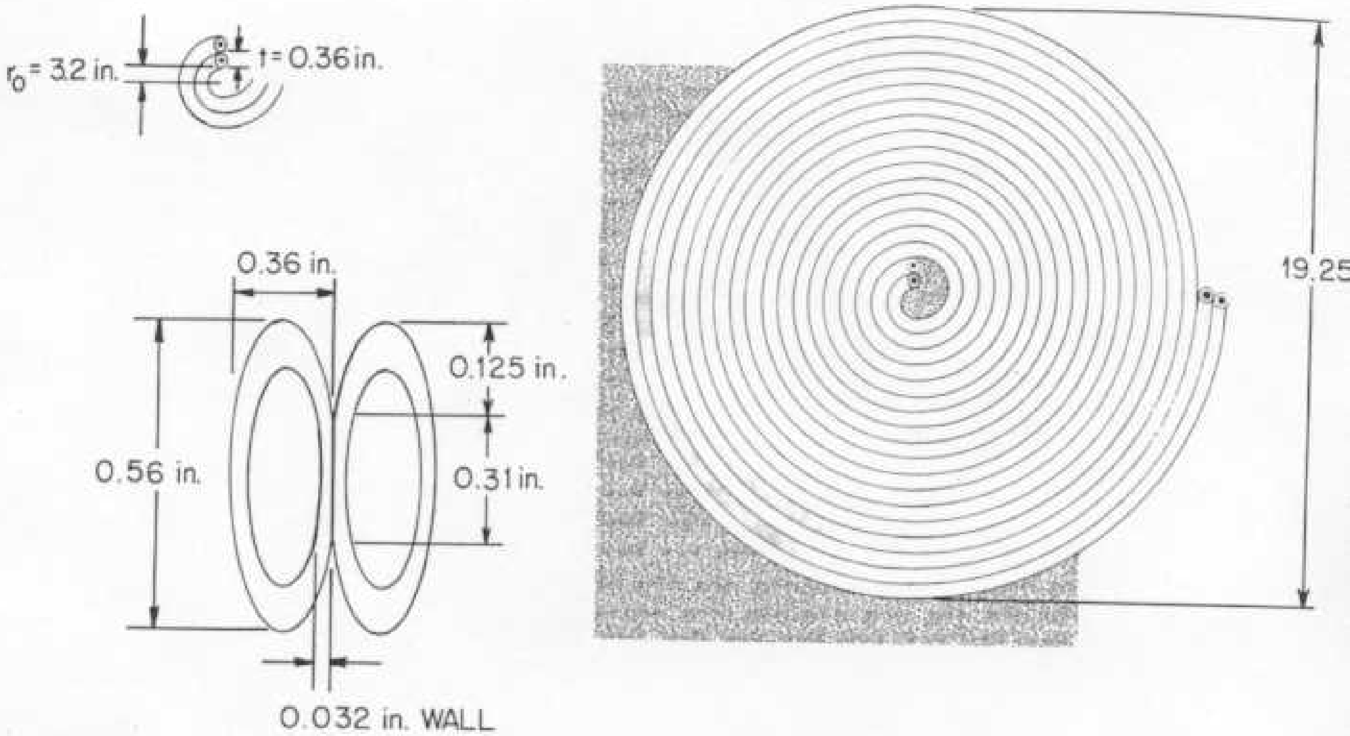

Figure 1. Schematic of double spiral coil heat exchanger

Figure 2. Picture of prototype test unit

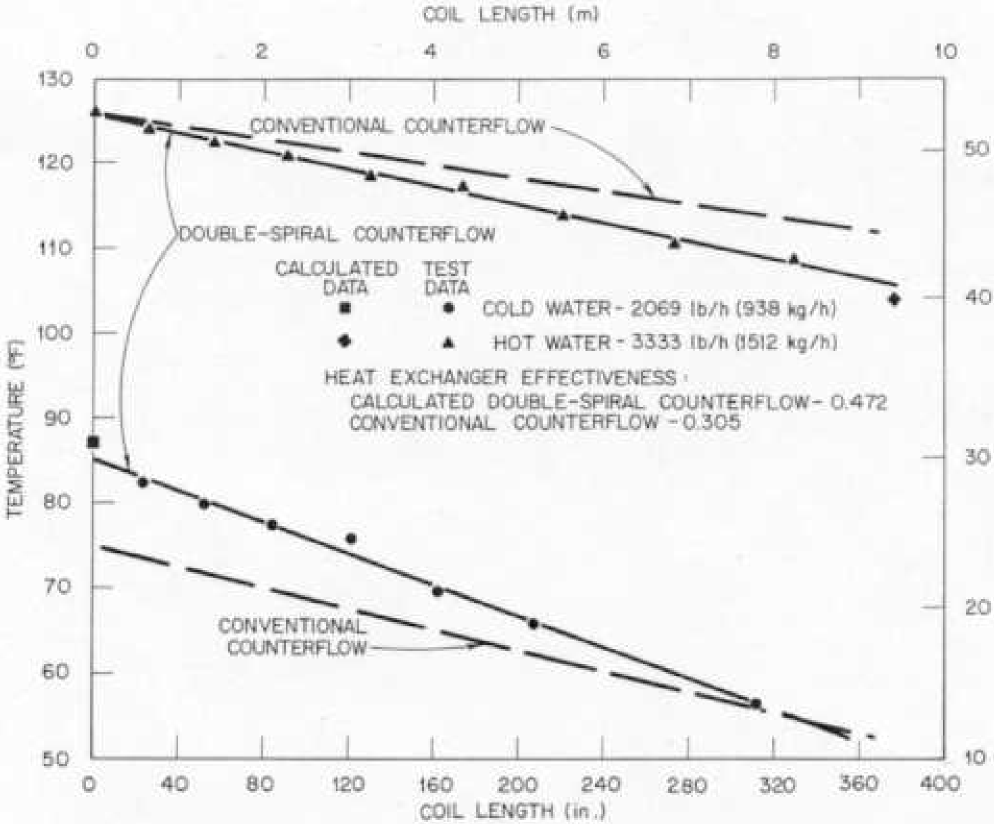

Figure 3. Comparison of experimental and theoretical results with conventional counterflow heat exchanger - hot and cold water flow rates at 3333 lb/h (1512 Kg/h) and 2069 lb/h (938 Kg/h)

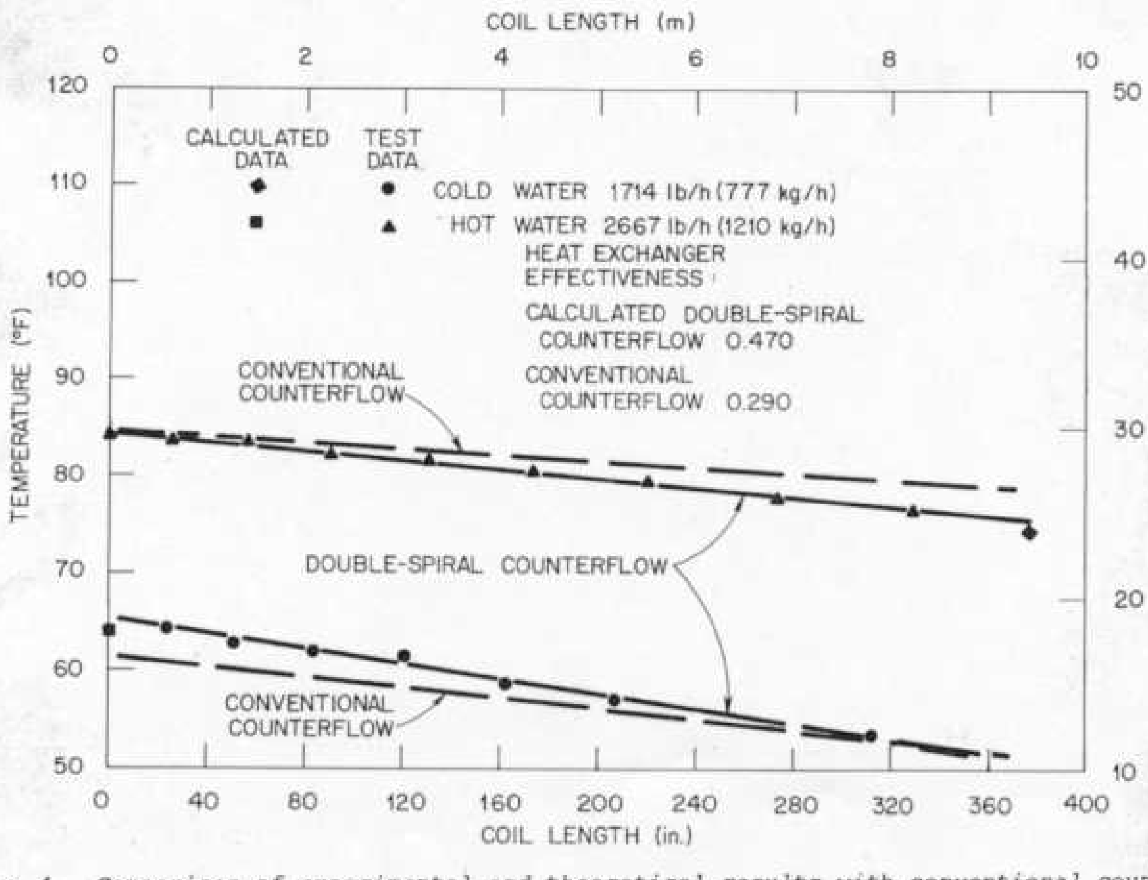

Figure 4. Comparison of experimental and theoretical results with conventional counterflow heat exchanger - hot and cold water flow rates at 2667 lb/h (1210 Kg/h) and 1714 lb/h (777 Kg/h)

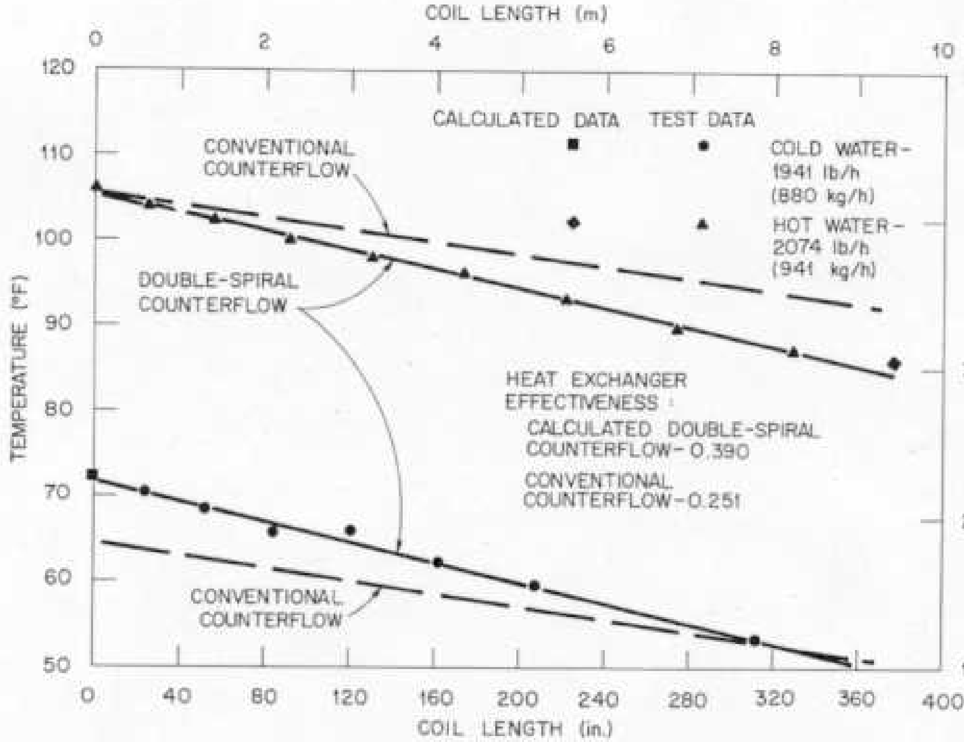

Figure 5. Comparison of experimental and theoretical results with conventional counterflow heat exchanger - hot and cold water flow rates at 2074 lb/h (941 Kg/h) and 1941 lb/h (880 Kg/h)

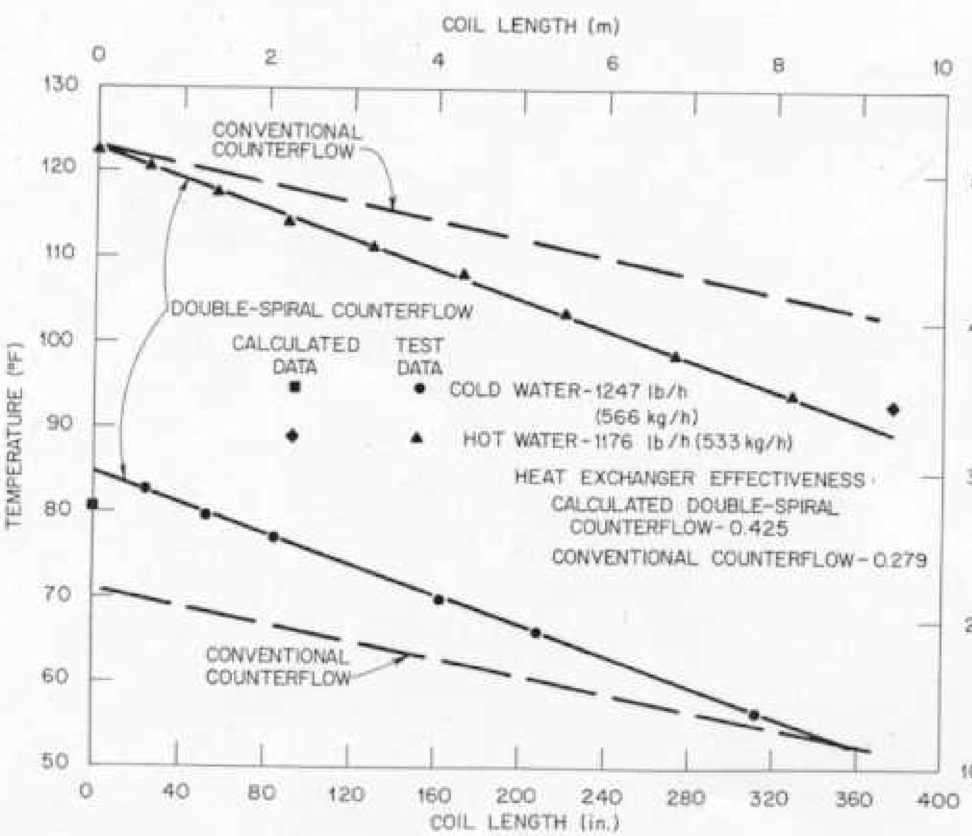

Figure 6. Comparison of experimental and theoretical results with conventional counterflow heat exchanger - hot and cold water flow rates at 1176 lb/h (533 Kg/h) and 1247 lb/h (566 Kg/h)

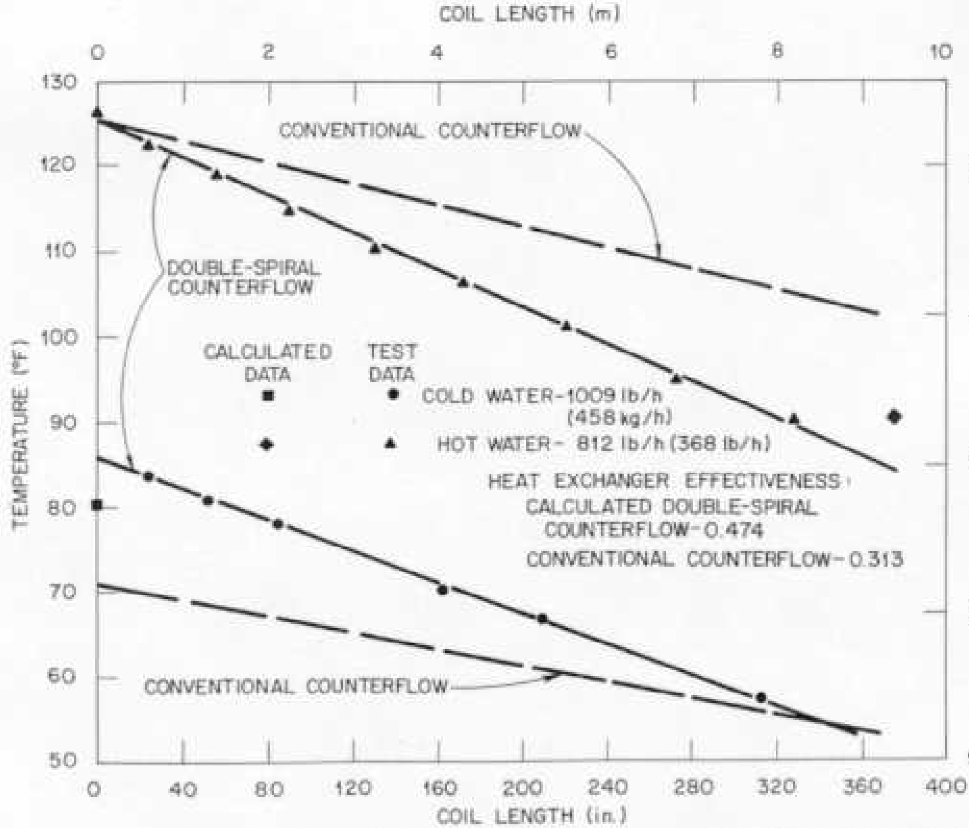

Figure 7. Comparison of experimental and theoretical results with conventional counterflow heat exchanger - hot and cold water flow rates at 812 lb/h (368 Kg/h) and 1007 lb/h (458 Kg/h)

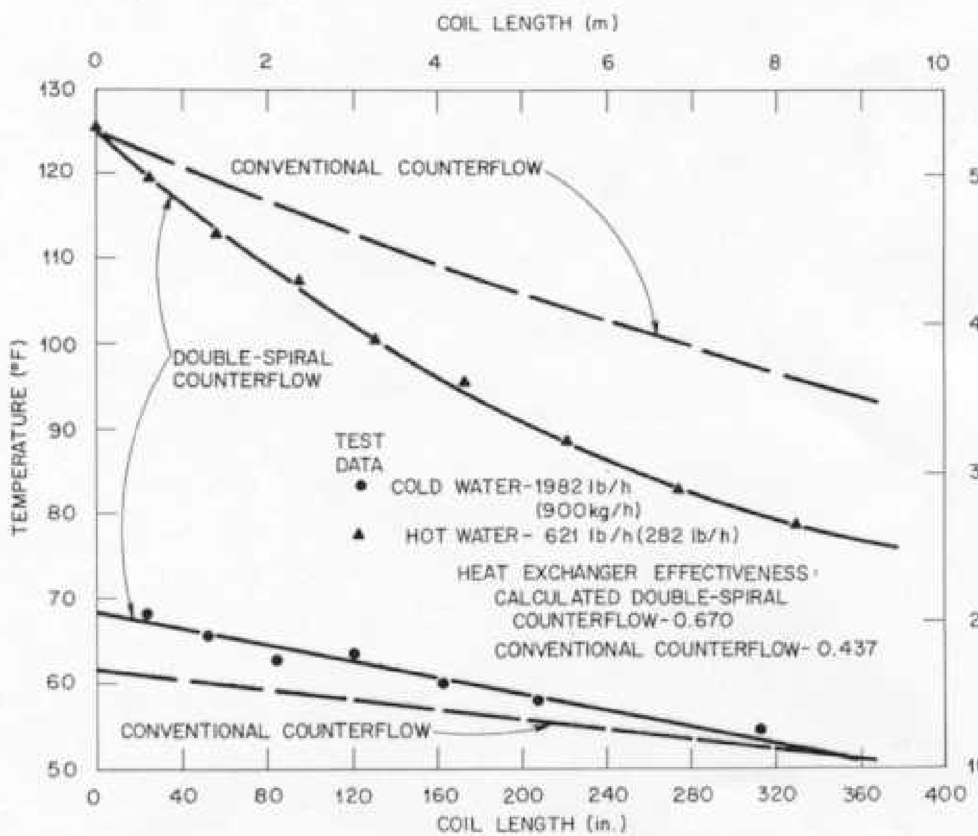

Figure 8. Comparison of double spiral coil heat exchanger experimental data with theoretical tesults of conventional counter flow heat exchanger - hot and cold water flow rates at 621 lb/h (282 Kg/h) and 1982 lb.h (kg/h)

Discussion

J.A. CLARK, JR., Patterson-Kelley, East Stroudsborg, PA: What is the method used to obtain a bond or contact between the two tube walls?

MEI: The most common method used to obtain a bond on contact between the two tube walls in this type of heat exchanger is to blow up the two tubes with high pressure air or liquid. Continuous soldering can be effective but will cost more.

CLARK: Was the experimental model similar to the "PARCA" heat exchanger now being marketed in the United States?

MEI: The experimental model was developed independently. Since the design is so simple, it is possible that some commercial products are being marketed already. However, the anlytical design model presented in the paper has never been shown before.