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Physical invariants of biosignatures

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Abstract

Biosignature is one of the most important evidences of life available to researchers. However, many complex physical and chemical phenomena can mimic prints of life so closely that special methods are required to make the distinction. In addition to that, life, in principle, can be composed of components which are fundamentally different from those known on Earth. That is why identification of biosignatures should be based upon some phenomenological invariants. Such invariants, within the framework of Newtonian formalism, are introduced and discussed. © 1999 Elsevier Science B.V. All rights reserved.

1. Introduction

It does not take much knowledge or experience to distinguish a living matter from inanimate in day-to-day situations. Paradoxically, there is no formal definition of life which would be free of exceptions and counter-examples [1]. There are at least two reasons for that. Firstly, many complex physical and chemical phenomena can mimic prints of life so closely that special methods are required to make the distinction. Secondly, extraterrestrial life, in principle, can be composed of components which are fundamentally different from those known on Earth.

One of the most important evidences of extraterrestrial life available to researchers is a biosignature, i.e., a print of a living system in environments or samples from planets. For the reasons mentioned above, identification of a biosignature requires development of special criteria which would allow one to disqualify any inanimate forgery, and at the same time to recognize life even if it is in a form fundamentally different from those on Earth. Since living systems belong to Newtonian macro-world, it is reasonable to start with Newtonian formalism which would capture some phenomenological invariants of living system based upon postulated definition of life. Such a definition has to reconcile experimental observations, Newtonian mechanics and thermodynamics.

Firstly, living systems have a sense of the direction of time [2], and therefore, their models must be irreversible. In addition to that, evolution of living systems is directed toward higher levels of complexity if complexity is associated with a number of different features. Combining Newtonian mechanics, thermodynamics and the phenomenon of instability, both of these properties can be implemented. However, they are necessary, but not sufficient for life: there are plenty of physical processes which possess

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the same characteristics: chaos, turbulence, convection, etc.

The third property of living systems is the capacity to stimulate their own replication; however, even that property is not sufficient since it cannot disqualify fire or other exponentially unstable physical processes.

The fourth property can be associated with a so called ''free will'', or in terms of mathematical formalism, with a probabilistic evolution. Again, there are plenty of physical phenomena (chaos, Langevin models) whose evolution can be described only probabilistically.

The fifth property can be stated as the ability to perform certain transitions or motions which are not directly controlled from outside. Such an autonomy must be supported by energy flux (with low entropy input and high entropy output), and as a ''side effect'', it can be accompanied by information processing. Indeed, autonomous systems can converge to a limit cycle (flutter), to chaotic attractor (Lorenz model) or to a static attractor (neural nets), and that can serve as a memory.

All the five properties listed above exhibit ''complexity without purpose'', and that is why they are necessary, but not sufficient for life identification. We will postulate now the last property: any living system has an objective of its activity. The global objective is always to survive, but local objectives can be different as long as they contribute to the global one. From the viewpoint of phenomenological formalism, a living system must possess its own model and interact with it to achieve the objective. For instance, it can run the model faster than real time thereby predicting its future state, compare this state with the objective and change the strategy if necessary. Actually, all the man-made controlled systems mimic this property of a living system (however, controlled systems usually do not possess all the previous five properties mentioned above).

The purpose of this paper is to develop a phenomenological model of living systems which would include all the properties listed above, and to establish physical invariants of the corresponding biosignatures. One of the challenges of the approach is to keep the model within the Newtonian formalism excluding any man-made devices (such as random number generators).

2. Background

The mathematical theory of active systems, both natural and artificial, has a relatively short history. The most general approach to it is presented in the monograph by Mikhailov [3]. On a time-scale of motions of simple individual elements, a one-component model is represented by a set of ODE:

$$\dot{a}_i = g_i(a_1, \ldots, a_n),\ i = 1, \ldots, n \tag{1}$$

where a_i are state variables, and g is a non-monotonous function with, at least, two extremal parts. An isotropic multicomponent active medium is described by a system of PDE:

$$\dot{a}_i = g_i(\{a_j\}) + \sum_j B_{ij}(\nabla a_j)^2 + \sum_{ij} D_{ij}\nabla^2 a_j \tag{2}$$

The main source of complexity of the models (1) and (2) is the multi-extremal configuration of the functions g_i which causes such phenomena as phase transition, trigger and spiral waves, travelling pulses, etc.

Eqs. (1) and (2) can be equipped by random forces (noise), and then they take form of the Langevin equations whose probability evolution is described by the associated Fokker–Planck equation.

Although solutions to Eqs. (1) and (2) can exhibit a variety of complex behaviors, this is still a ''complexity without a purpose'' and there are plenty of inanimate systems which are governed by the same equations (for instance, the whole class of reaction–diffusion processes).

The models (1) and (2) have other limitations, and one of them is continuous (non-punctuated) evolution. As many leading biologists suggest, the evolution of biological systems should include pauses for sensing the environment and choosing the direction of the next step; this paradigm controlled by a biological clock makes the evolution punctuated. From the mathematical viewpoint, it means that behaviors of living systems should include a random-walk component. Obviously such a component cannot be produced only by external noise since the solutions to the Fokker–Planck equations are continuous; in addition to that, external noise does not have enough power to drive a biological system.

In the next sections we will discuss a model in which all these limitations are removed.

3. Model of punctuated evolution

The model proposed in this section is based upon dynamical simulation of random walk introduced and discussed in [4] [5] [6].

Consider a rectilinear motion of a particle of unit mass described by the following differential equations:

$$\dot{v} = v v^{1/3} \sin \omega t + \varepsilon,\ v = \text{const.},$$
$$\omega = \text{const.}, |\varepsilon| \ll v|v^{1/3}| \tag{3}$$

$$\dot{x} = v \tag{4}$$

where x and v are position and velocity, respectively, and $\varepsilon(t)$ is vanishingly small noise.

As shown in [4–6] the particle performs a random walk with constant steps h and the transition periods τ:

$$h = 64(3\omega)^{-5/2} v^{3/2},\ \tau = \frac{2\pi}{\omega} \tag{5}$$

The probability density $f(x,t)$ is governed by the following difference equation:

$$f(x,t+\tau) = pf(x-h,t) + (1-p) f(x+h,t) \tag{6}$$

which represents a discrete version of the Fokker–Planck equation, while

$$\int_{-x}^{x} f(x,t)\,\mathrm{d}x = 1 \tag{7}$$

and

$$p = p_{\mathrm{r}}(\varepsilon > 0) \tag{8}$$

Several comments concerning a physical interpretation of the solution to Eq. (3) should be made.

Firstly, this solution has an infinite number of equilibrium points

$$\dot{v} = 0,\ x = \pm h \pm h \pm h \cdots \text{ etc.} \tag{9}$$

which are stable when $\sin \omega t < 0$, and unstable when $\sin \omega t > 0$.

Since

$$\left| \frac{\mathrm{d}\dot{v}}{\mathrm{d}v} \right| \to \infty \text{ at } v \to 0 \tag{10}$$

the Lipschitz conditions at the equilibrium points are violated, and that makes them terminal [4] attractors or repellers. As a result of that, the transition time τ is finite (see Eq. (5)).

Secondly, the noise ε is not driving the evolution: it only triggers the mechanism of instability which controls the energy supply via the harmonic oscillations $\sin \omega t$. As follows from Eq. (3), ε can be ignored when $\dot{v} = 0$ or when $v \neq 0$, but the equation is stable, i.e., at $t = \pi/\omega, \pi/3\omega \cdots$ etc. Indeed, at these instants, the solution to Eq. (3) has a choice to move left (if $sgn\,\varepsilon < 0$), or right (if $sgn\,\varepsilon > 0$). In other words, the sign of ε at the equilibrium points uniquely defines the evolution of the solution. But since ε is a random variable, the evolution becomes random too.

Thus, the solution to Eqs. (3) and (4) combines acting (the transition from one equilibrium point to another) and ''thinking'' (the decision making process based upon $sgn\,\varepsilon$). Both the energy reservoir for acting, i.e., the harmonic oscillations $\sin \omega t$, and the ''brain'', i.e., the noise ε can be simulated by different (autonomous) dynamical systems: the first system is supposed to converge to a periodic attractor, and the second one to a chaotic attractor.

4. The noise structure

In this section we will analyze possible structures of the noise ε which is responsible for triggering the transition from one equilibrium to another in the direction depending upon $sgn\,\varepsilon$.

Regardless of the type of the corresponding chaotic attractor, the noise can be derived from a sample of an underlying stationary stochastic process λ characterized by some probability density $\Phi(\lambda)$. Suppose that

$$\varepsilon = \lambda - \langle \lambda \rangle + \mu \tag{11}$$

where $\langle \lambda \rangle$ is the mean of λ

$$\langle \lambda \rangle = \int_{-\infty}^{\infty} \lambda \Phi(\lambda)\,\mathrm{d}\lambda \tag{12}$$

and μ is some deterministic variable.

Then

$$p_{\mathrm{r}}(\varepsilon > 0) = \int_{0}^{\infty} (\lambda - \langle \lambda \rangle + \mu)\,\mathrm{d}\lambda = \Theta(\mu) \tag{13}$$

while

$$\Theta(0) = \frac{1}{2}, \frac{d\Theta}{d\mu} \geq 0, 0 \leq \Theta \leq 1 \tag{14}$$

Obviously any stationary stochastic process uniquely defines the function $\Theta(\mu)$. This function, in many practical cases (for instance, when the stochastic process λ is generated by a logistic map [4]) can be approximated as:

$$\Theta = \begin{cases} 1 & \text{if } \mu > \lambda_{\max} - \langle \lambda \rangle \\ \frac{1}{2}\left(1 + \frac{\mu}{\lambda_{\max} - \langle \lambda \rangle}\right) & \text{if } \mu \leq \lambda_{\max} = \lambda \end{cases} \tag{15}$$

where $\lambda_{\{\text{func } \{\max\}\}}$ is the largest term in the time series $\lambda(t)$.

In general, μ can depend upon the particle coordinate x, its probability distribution $f(x)$, and the functionals of $f(x)$ such as the mean

$$m = \langle x \rangle = \Sigma x f(x) \tag{16}$$

the variance

$$\sigma = \Sigma(x - m)^2 f(x) \tag{17}$$

or the Shannon uncertainty (entropy)

$$H = -\Sigma f(x) \ln f(x) \tag{18}$$

i.e.,

$$\mu = \mu\{x, f(x), m[f(x)], \sigma[f(x)], H[f(x)]\} \tag{19}$$

Actually, all the variety and complexity of the particle behavior described by Eqs. (3) and (4) is defined by the structure of the function (19).

In the next sections, a variety of structures of Eq. (19) as well as specific characteristics of the corresponding behaviors will be analyzed.

5. Complexity without objective

We will start with the simplest case of Eq. (19) when μ depends only upon the state variable x

$$\mu = \mu(x) \tag{20}$$

Then Eqs. (3), (4) and (6) reduce to

$$\dot{v} = v v^{1/3} \sin \omega t + \lambda - \langle \lambda \rangle - \mu(x), \dot{x} = v \tag{21}$$

$$f(x, t + \tau) = \Theta[\mu(x)] f(x - h, t) + \{1 - \Theta[\mu(x)]\} f(x + h, t) \tag{22}$$

Since a closed form solution of Eq. (21) is not available, we will confine ourselves with a qualitative analysis.

Suppose first that

$$\mu = 0 \tag{23}$$

Then, as follows from Eq. (14),

$$\Theta = \frac{1}{2} \tag{24}$$

and the solution to Eq. (22) subject to the initial conditions

$$f(0,0) = 1, f(x,0) = 0 \text{ if } x \neq 0 \tag{25}$$

describes a symmetric unrestricted random walk:

$$f(x,t) = C_n^k 2^{-n}, k = \frac{1}{2}(n+1); n = \text{integer}\left(\frac{2\omega t}{\pi}\right) \tag{26}$$

Here the binomial coefficient C_n^k should be interpreted as zero whenever k is not an integer in the interval $[0, n]$, and n is the total number of steps.

At this point we have to clarify the relationships between Eq. (21) and Eq. (22) which are the following: if one fixes the initial conditions as

$$x = 0, v = 0 \tag{27}$$

and run Eq. (21) many times, he will get different chaotic-like time series as solutions; but if he performs a statistical analysis of these solutions and find the evolution of the probability density, this evolution will coincide with the solution to Eq. (21). In other words, the probabilities described by Eq. (22) are simulated by the dynamical system (21).

Let us assume now that, instead of Eq. (23),

$$\mu = -ax, a = \text{const.} > 0 \tag{28}$$

Then the number of negative signs in the string of numbers (11) will prevail if $x > 0$ since the effective zero-crossing line moves down away from the middle. Similarly, the number of positive signs in

(11) will prevail if $x < 0$. Thus, when $x = 0$ at $t = 0$, the system starts with an unrestricted random walk (26), and $|x|$ grows. However, this growth as a feedback changes signs in Eq. (11) such that

$$\dot{x} < 0 \text{ if } x > 0, \text{and } \dot{x} > 0 \text{ if } x < 0 \tag{29}$$

Moreover, as follows from Eq. (15), with probability one the system will escape the domain where

$$\mu > \lambda_{\max} - \langle \lambda \rangle \tag{30}$$

As a result of that,

$$|x| \leq \frac{1}{a}\left(\lambda_{\max} - \langle \lambda \rangle\right) \tag{31}$$

Hence, the dynamical system (21), (28) subject to the initial condition (25) simulates a symmetric random walk restricted by the boundaries (31). The probability evolution for this stochastic process is described by the solution to Eq. (22).

It has to be emphasized that although the noise ε is vanishingly small (see Eq. (3)), the boundaries (31) can be sufficiently large since a^2 must have the same order as ε. This should not be surprising because, as mentioned earlier, the noise only triggers the mechanism of instability while the energy for the motion is supplied by the harmonic oscillations $\sin \omega t$.

As a next step toward a higher complexity, replace Eq. (28) by the following:

$$\mu = -a \sin \gamma x, \; a = \text{const.} > 0 \tag{32}$$

For

$$|x| < \frac{\pi}{2\gamma}, \text{i.e.,} \; \frac{1}{a}\left(\lambda_{\max} - \langle \lambda \rangle\right) < \frac{\pi}{2\gamma} \tag{33}$$

the behavior of the solution to Eq. (21) qualitatively remains the same, i.e., it is described by a symmetric random walk restricted by the boundaries (31).

However, if

$$|x| > \frac{\pi}{2\gamma}, \text{i.e.,} \; \frac{1}{a}\left(\lambda_{\max} - \langle \lambda \rangle\right) > \frac{\pi}{2\gamma} \tag{34}$$

the motion becomes unstable since

$$sgn\, x = sgn\, \dot{x} \tag{35}$$

and x will grow until it approaches the basin of the next attractor:

$$\frac{3\pi}{2\gamma} < |x| < \frac{5\pi}{2\gamma} \tag{36}$$

The probability of this transition is:

$$p_{\mathrm{T}} = \int_{\pi a/2\gamma}^{\infty} \Phi(\lambda - \langle \lambda \rangle)\, \mathrm{d}\lambda \tag{37}$$

Hence, with the probability p_{T} the boundaries of the random walk are shifted from

$$|x| < \frac{\pi}{2\gamma} \text{ to } \frac{3\pi}{2\gamma} < |x| < \frac{5\pi}{2\gamma},$$

$$\text{and then to } \frac{7\pi}{2\gamma} < |x| < \frac{g\pi}{2\gamma}, \text{etc.} \tag{38}$$

Thus, the noise structure (32) leads to much more complex behavior of the solutions to Eq. (21), and this complexity is associated with the alternation of the effects of stability $sgn\, x = -sgn\, \dot{x}$ and instability $sgn\, x = sgn\, \dot{x}$. Actually similar phenomenon (but on a lower level of complexity) follows from the multi-extremal function (1) discussed in [3].

Further increase of complexity can be associated with introducing memory by replacing Eq. (4) with the following:

$$\dot{x} = v(t) + \alpha_l v(t - \tau_0) + \alpha_2 v(t - 2\tau_0) + \cdots \text{ etc.} \tag{39}$$

providing non-Markovian correlations between present and past.

However, there is a more fundamental way to enhance the dynamical complexity. Indeed, consider the following noise structure:

$$\mu = a\langle x \rangle, \; a = \text{const.} > 0 \tag{40}$$

Now Eq. (21) and Eq. (22) are coupled (see Eq. (16)). Moreover, Eq. (22) becomes nonlinear.

Subject to the initial conditions

$$f(0,0) = 1, \; f(x,0) = 0 \text{ if } x \neq 0, \; x = 0 \text{ at } t = 0 \tag{41}$$

the solution to Eqs. (21) and (22) describes a symmetric unrestricted random walk (26) since for this process

$$\langle x \rangle \equiv 0 \tag{42}$$

However, for different initial conditions:

$$f(1,0) = 1,\ f(x,0) = 0 \text{ if } x \neq 1,\ x = 1 \text{ at } t = 0 \tag{43}$$

the solution to the same system is fully deterministic: it is described by a travelling wave of the delta-function:

$$f = \delta\left(x - 1 + \frac{h}{\tau}t\right) \text{ at } t \to \infty \tag{44}$$

Thus, one can observe a fundamental non-linear effect: the dependence of the attractor upon the initial conditions. In general, Eqs. (21) and (22) with the noise structure depending upon the probability f (as in Eq. (40), or Eq. (19) may have different attractors and repellers, i.e., different asymptotic solutions for different initial conditions. It should be recalled that in all the previous cases when Eq. (22) was linear, the solution had only one type of asymptotic behavior regardless of initial conditions.

6. Systems with prescribed objectives.

As has been demonstrated in the previous section, relatively simple structures of noise (11) (see Eqs. (23), (28), (32) and (40)) lead to a high level of behavioral complexity which, in principle, can match the complexity of living systems. However, all these systems do not exploit their complexity for the purpose of survival, or to any other purpose, and that disqualifies them as models of living systems.

In this section we will introduce systems with objectives starting with the simplest case when the objective is prescribed. As a rule, such systems must include some additional parameters which represent the ''master'' enforcing the objective.

Let us turn to Eq. (28) and assume that the coefficient a depends upon an external parameter φ

$$a = a(\varphi), \text{ while } a_{\max} = a(\varphi_0), \text{ and } \varphi = \varphi(x,t) \tag{45}$$

where φ can represent the ambient temperature or concentration of some chemicals which are important for the system survival.

Now the objective can be formulated as follows: find such a small region of x where φ is sufficiently close to its optimal value of φ_0.

For the sake of mathematical simplicity, we will assume that $a(\varphi)$ and $\varphi(x,t)$ are slow changing functions, i.e.,

$$\left|\frac{\mathrm{d}a}{\mathrm{d}t}\right| \ll \frac{h}{\tau} \tag{46}$$

Then eventually the solution to Eqs. (3), (4), (28) and (45) will be trapped in the region following from Eq. (31):

$$|x| \leq \frac{1}{a(\varphi_0)}(\lambda_{\max} - \langle\lambda\rangle) \tag{47}$$

As follows from Eq. (45)

$$|x|_{\varphi=\varphi_0} < |x|_{\varphi\neq\varphi_0} \tag{48}$$

i.e., the stochastic attractor (47) has the least uncertainty among all the possible stochastic processes (31). In other words, the process of approaching the objective is characterized by decrease of the entropy:

$$\frac{\mathrm{d}H}{\mathrm{d}t} < 0 \tag{49}$$

It should be noticed that such an ''inverse'' evolution of the entropy is due to the fact that the system (3), (4) is not isolated as a result of enforcement of the objective via the external parameter φ. Nevertheless, the condition (49) is necessary, but not sufficient for a living system. Indeed, there is plenty of physical (non-isolated) systems with self-organization properties (i.e., with different type of attractors) for which the inequality (49) holds. However, one can argue that such physical systems do not ''benefit'' from the property (49), and therefore, they do not have an objective. Unfortunately, the detection of the attractor which represents an objective for the system cannot be made based only upon biosignatures: some additional information will be required.

7. Systems with emerging objectives

The main difficulties in detection of life start with the fact that there is no definition of life. In this section we will try to find such a level of complexity at which we can draw a sharp boundary between living and inanimate systems in terms of phe-

nomenological invariants. For that purpose we will turn to the concept of reflection introduced in psychology [7]. Reflection is traditionally understood as the human ability to take the position of an observer in relation to one's own thoughts. In other words, the reflection is a self-awareness via the interaction with the ''image of the self''. In terms of the phenomenological formalism proposed above, Eq. (6) represent the probabilistic ''image'' of the dynamical system (3), (4). If this system ''possesses'' its own image, then it can predict, for instance, future expected values of its parameters, and, by interacting with the image, change the expectations if they are not consistent with the objective. In this context, Eqs. (3) and (4) simulate ''acting'', and Eq. (6) simulates ''thinking''. Their interaction can be implemented by incorporating probabilities, its functions and functionals into the noise structure (see Eq. (19)).

Now we are approaching the central point of our discussion: the feedback (19) makes the probability evolution (6) (with reference to Eqs. (8) and (13)) nonlinear. It should be recalled that any stochastic process without memory (i.e., Markov process) can be described by linear Fokker–Planck equation [8], or its discrete version, the Chapman–Kolmogorov equations. Only that type of processes have been observed in physical (i.e., inanimate) world. But coupling between the dynamical equations and their probabilistic ''image'' does not contradict any law of physics: for instance, the Langevin equations can, in principle, interact with the corresponding Fokker–Planck equation if the external noise depends upon the probability distribution of the state variable. Strictly speaking, such processes are Markovian since the future still depends only upon the present, but not the past. However, now present includes not only values of the state variable, but also its probability distribution, and that leads to nonlinear evolution of random walk.

For the proof-of-concept, suppose that the noise structure (19) is presented in the following form:

$$\mu = \begin{cases} f(\lambda_{\max} - \langle \lambda \rangle) & \text{if } f \le 1 \\ (\lambda_{\max} - \langle \lambda \rangle) & \text{if } f > 1 \end{cases} \tag{50}$$

In addition to that, suppose that in Eq. (5)

$$h \to 0, \tau \to 0, \frac{h}{\tau} = 1 \tag{51}$$

Then one arrives at the continuous form of Eq. (6):

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{2} f^2\right) = 0 \tag{52}$$

which simulates a traffic flow.

The solution to this equation is well-known: starting with a flat distribution, it forms shock waves [9].

Hence, if

$$f = \begin{cases} \dfrac{1}{2\angle} & \text{at} |x| \le \angle \\ 0 & \text{otherwise} \end{cases} \tag{53}$$

and therefore, the entropy

$$H = H_{\max} \text{ at } t = 0 \tag{54}$$

then, eventually

$$\frac{\partial H}{\partial t} < 0 \text{ and } H \to 0 \text{ at } t \to \infty \tag{55}$$

(It can be verified [10] that the normalization condition for $f(x,t)$ is preserved.)

Thus, as a result of interaction with its own ''image'', and without any external enforcement, the system (3), (4) decreases its entropy in the course of nonlinear evolution of probability, and, according to our definition, that is the privilege of living systems. It should be emphasized that the system (3), (4) is not isolated: it consumes energy via the harmonic oscillations $\sin \omega t$, and therefore, the condition (55) does not violate the second law of thermodynamics.

Eqs. (3), (4), (50) and (52) illuminate another remarkable property of living systems: their ability to predict future. Indeed, with the noise structure (50), Eq. (6) (as well as its continuous version, Eq. (52)) does not depend upon Eqs. (3) and (4), and therefore, it can be run faster than real time. As a result of that, future probability distributions as well as its invariants (expectation, variance, etc.) can be predicted and compared with the objective. Based upon that comparison, the noise structure (50) can be changed if needed.

Thus, in general, living systems are better equipped for dealing with future uncertainties. In other words, their present motion is ''correlated with future'' in terms of the probability invariants. Such a remarkable property which increases survivability could be acquired accidentally and then be strengthened in the process of natural selection.

It should be emphasized that the ability to predict results from a special type of nonlinear probability evolution generated by the noise structure which does not depend explicitly upon the state variable x.

In general when Eq. (50) is replaced by Eq. (19), and the shift operator E is replaced by the differential operator D as

$$E_h = e^{hD}, E_\tau = e^{\tau D} \tag{56}$$

Eq. (52) will be replaced by a nonlinear parabolo-hyperbolic equation which can describe shock waves, the Burger's waves, solitons and even chaos, in the probability space. However, for the ability to predict, Eq. (19) should not depend explicitly upon x. Nevertheless, dependence of μ upon both x and f does not disqualify the system from being ''alive'': it only means that the system interacts not only with its own image, but also with an outside observer.

So far we avoided formulations of objectives for systems with reflections, and we did it deliberately. Indeed, based upon the signature in the Newtonian world, one can detect only apparent objective which, in the case of Eq. (52), is to concentrate the motion within certain domains of the state variable. Although such information is not sufficient for finding the real objective, it is still sufficient for detecting life.

8. Discussion and conclusion

There has been demonstrated that a nonlinearity of the evolution in the probabilistic space represents a physical invariant of the living systems which distinguishes them from inanimate ones. This property results from the assumption that a living system, by definition, possesses an ''image of the self'' and interacts with it in order to approach the objective. One should notice that the image has to represent some sort of abstraction of the original system which preserves only the most fundamental and predictable properties such as expectation, variance, etc.; these properties become available after ''projecting'' the system from physical into probability space.

One of the most remarkable consequences of nonlinear evolution of probability is the ability of systems with reflection to predict future, and that makes them more adaptable to uncertainties. A phenomenological characteristic of such a property can be captured in terms of correlation between present and future, which eventually results in a spontaneous ability to concentrate the motion within certain domains of the state variable according to the objective.

The results discussed in this paper can be generalized in two different ways.

Firstly, the same invariant stands if Eq. (4) is replaced by Eq. (38), i.e., if the system (3), (39) has memory.

Secondly, instead of the single variable x, one can introduce a set $\{x\} = x_l, \cdots, x_n$. Then each variable x_i will be governed by Eqs. (3), (4) and (6) interacting with the other variables via the structured noise (compare with Eq. (19)):

$$\mu_i = \mu_i(\{x\},\{f\} \cdots \text{ etc.}) \tag{57}$$

if one assumes that any single-variable system possesses not only an image of the self, but images of other systems as well.

It should be recalled that all the models discussed in this paper do not include a capacity of self-replication, since they describe the behaviors on the life-time scale while modelling self-replication requires the time-scale of many generations. That is why models of self-replications based upon the logistic equation and its modifications are not coupled with the behavioral models, and therefore, they can be considered separately.

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