ELSEVIER

26 January 1998

PHYSICS LETTERS A

Physics Letters A 238 (1998) 1–7

Incompatible stochastic processes and complex probabilities

Michail Zak

Center for Space Microelectronics Technology, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA

Received 14 July 1997; revised manuscript received 12 September 1997; accepted for publication 16 September 1997
Communicated by C.R. Doering

Abstract

The definition of conditional probabilities is based upon the existence of a joint probability. However, a reconstruction of the joint probability from given conditional probabilities imposes certain constraints upon the latter, so that if several conditional probabilities are chosen arbitrarily, the corresponding joint probability may not exist. Such an incompleteness in conditional probabilities can be eliminated by introducing complex probabilities. The physical meaning of the new mathematical formalism, as well as its relation to quantum probabilities, is discussed. © 1998 Elsevier Science B.V.

One of the oldest and still unsolved problems in the field of multivariate stochastic processes is to reconstruct a joint probability from several correlated conditional probabilities. This problem has been discussed in Refs. [1–3]. Its origin lies in the fact that classical probability theory defines conditional probabilities based upon the existence of a joint probability. At the same time, one can observe correlated stochastic processes which are represented by conditional probabilities. And then the inverse problem of reconstructing the underlying joint probability arises.

As an illustration to this point, consider two coupled diffusion equations,

$$\frac{\partial \rho_1(x_1,x_2)}{\partial t} = D_1(x_2)\frac{\partial^2 \rho_1(x_1,x_2)}{\partial x_1^2}, \qquad \frac{\partial \rho_2(x_1,x_2)}{\partial t} = D_2(x_1)\frac{\partial^2 \rho_2(x_1,x_2)}{\partial x_2^2}, \tag{1}$$

with the initial conditions

$$\rho_i(x_i,t'|x_i',t') = \delta(x_i - x_i'), \qquad i = 1,2. \tag{2}$$

The solution for $t > t'$ reads

$$\rho_1(x_1,x_2) = \frac{1}{\sqrt{4\pi D_1(x_2)(t-t')}} \exp\left(-\frac{(x_1-x_1')^2}{4D_1(x_2)(t-t')}\right), \tag{3}$$

$$\rho_2(x_1,x_2) = \frac{1}{\sqrt{4\pi D_2(x_1)(t-t')}} \exp\left(-\frac{(x_2-x_2')^2}{4D_2(x_1)(t-t')}\right). \tag{4}$$

Obviously, ρ_1 and ρ_2 are interpreted as the density of x_1 given x_2 and the density of x_2 given x_1, respectively, i.e. they represent conditional probabilities,

0375-9601/98/$19.00 © 1998 Elsevier Science B.V. All rights reserved.
PII S0375-9601(97)00763-9