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Versatile signal generation in chaotic optical communication devices modeled by delay-differential equations

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1 Introduction

Chaotic devices have many potential applications in communications systems as generators of signals with complex waveforms. Random or pseudo-random signals are widely used in communication from the coding of data and waveforms for security, error-correcting and avoiding interference, to applications in communication protocols where random generation of addresses and timing are necessary. Since chaotic nonlinear devices have the ability to generate a variety of complicated waveforms, chaotic devices may offer effective alternatives to conventional random and pseudo random signal generation modules, particularly as the demand increases for signal generators which are more compact and which can operate at higher speeds.

This paper is concerned with generation of laser light signals with complex waveforms for applications in communication systems. There are two approaches to generating laser light signals with complex waveforms. One is to generate a modulation pattern electronically and use it to modulate a laser source. The other is to use dynamics within a laser light source or circuit to generate the waveform directly. In this paper, we deal with the latter approach. We present some recent results concerning generation and control of signals in a particular class of laser systems with delayed feedback. These systems have in common that they can be modeled by delay-differential equations. We propose that laser device and systems that can be modeled by delay-differential equations are promising as signal generators in the sense of being able to gen-

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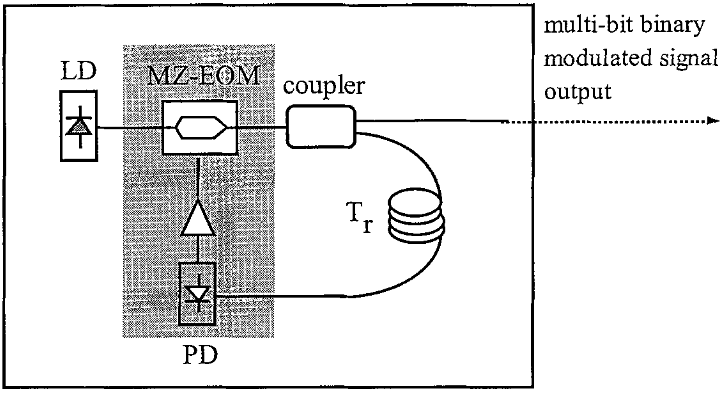

Fig. 1. Laser with nonlinear intensity modulation driven by delayed-feedback. LD: Laser Diode, EOM: electro-optic intensity modulator. PD: photodiode, T_r: delay time in fiber delay line. The shaded part could in principle be replaced by an all-optical nonlinear intensity modulator.

erate a variety of robust signals and also being amenable to systematic design and control.

2 Laser intensity-modulated by delayed feedback

The first system we consider is the system shown schematically in Fig. 1. This system has been extensively studied in numerical and physical experiments by the authors from the point of view of application as a signal generator [1–11]. In this system, signals are generated by intensity modulation of light from a laser source using an electro-optic modulation device. The modulation device is driven by delayed feedback from the output of the modulator. Fig. 2 shows an example from one of the many different classes of signals that can be generated in this way. The signals in this class have a binary modulation of peak height, corresponding to arbitrary n-bit sequences.

The system can be modeled with a delay-differential equation as follows,

$$\tau dX(t)/dt = -X(t) + Y(t), \tag{1}$$

$$Y(t) = F(X(t - T_r)). \tag{2}$$

The variable X represents the intensity of the light output. It tends to relax with time τ toward a level Y which is a nonlinear function of X at a time T_r earlier. For an electro-optic intensity modulator (EOM), this nonlinear feedback function takes the following form

$$F(X) = a\cos(X + b) + c \tag{3}$$

where a,b and c are constants or control parameters.

It was known from the work of Ikeda and co-workers ([12] and references therein) that in systems of the type described by Eqns (1-3), when the response time τ is short compared to the delay time T_r, a huge variety of signals can be generated just by changing initial conditions. The longer the delay, the greater the variety. Many different types of attractors had been found and charted through numerical experiments. Moreover, a systematic hierarchical structure to the waveforms and their bifurcations had been elucidated. References to more recent work on properties of delay-differential equations and corresponding difference equations can be found in [13].

In a focus on applications for optical signal generators in an optical communications context, we were looking for a reasonable combination of properties of variety, robustness, and controllability [1,2]. We focused on the particular class of oscillations of the type shown in Fig. 2 and proposed methods for stably and controllably generating signals corresponding to arbitrary binary sequences. These oscillations have a binary modulation of peak height, corresponding to arbitrary n-bit sequences. The oscillations form a complete set which can all be stable at the same parameter values. Pairs of peaks and valleys separated in time by T_r are roughly related by the corresponding difference equation

$$X(t) = F(X(t - T_r)) \tag{4}$$

with the levels corresponding to the period-4 solutions of this difference equation. The complete set is obtained by the 2^n different ways to select the levels of the n successive peaks and valleys in a delay interval T_r. The bit number n can be systematically increased by selecting higher harmonics of the fundamental oscillation at $4T_r$. The frequency of the highest harmonic is constrained by the response frequency $1/\tau$, and the highest harmonic number is constrained by the ratio T_r/τ.

In 1988, Aida designed and built a system of this type that generated signals at bit rates of up to 20MHz. This was used for various subsequent switching and control experiments [3–5,8,10,11], and is still functioning today with the original laser, modulator and fiber components. Later, a faster version was built to generate optical control sequences in an all-optical gate-control experiment which operated at up to 2GHz [7]. If we replace the nonlinear intensity modu-

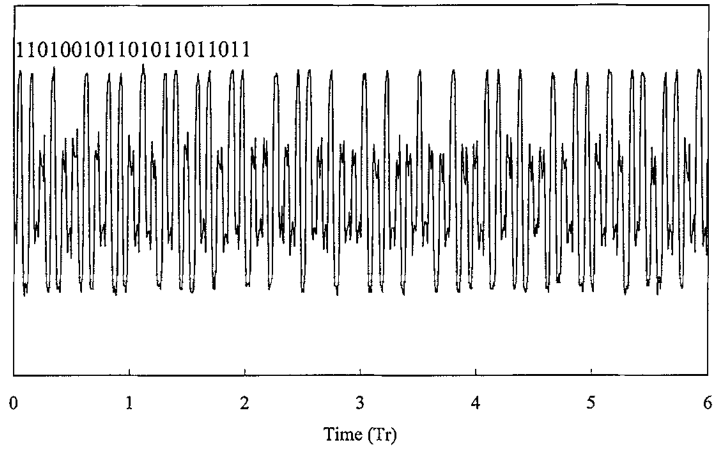

Fig. 2. Self-oscillating laser signal with n-bit binary peak modulation. The intensity of the laser signal output from the EOM in Fig. 1 is self-modulated to repeatedly generate a series of peak levels which correspond to an n-bit sequence. The n-bit pattern is inverted in every $2T_r$ interval. Signals corresponding to any arbitrary n-bit sequence can be generated by this system. These are one of many types of attracting solutions of Eqs. (1-3).

lation with an optical mechanism [1,25], then all-optical sequence generation would be possible, at even higher speeds.

Experiments first confirmed the effectiveness of two proposed practical ways to switch. The first way is by input of a specific control signal, such as a binary or quaternary sequence over one or two round-trip times, which selects a specific targeted pattern [3,4]. The second way to switch is by a longer pulse, of the order of multiple round trip times, which results in a random choice of a new pattern [4,5,8,10,11].

Let us describe this second method in more detail. Let us start with the parameter set at the value P^- for stable modes. Let P^+ be the parameter value for chaotic mode hopping. Set P^+ for time T^+, then set P^- for time T^-. In the sense of the difference equation, Eq. (4), this corresponds to a temporary switch between period-4 regime and the period-2 chaos regime. During the pulse of length T^+, the oscillations go chaotic, resulting in mode hopping, or chaotic itinerancy. Now, in order to get n new random bits, we need to take T^+ large enough for sufficient randomization of the hop position, typically up to $10T_r$. In practice, we also need to take T^- large enough for re-stabilization of the waveform, typically a few T_r. These times together with the value of n determine the maximum rate of truly random bit generation. In the first implemented version, the rate was 2 Mbps [4]. In the second implemented version, the rate was 200 Mbps [7]. Another key point to note is that regardless

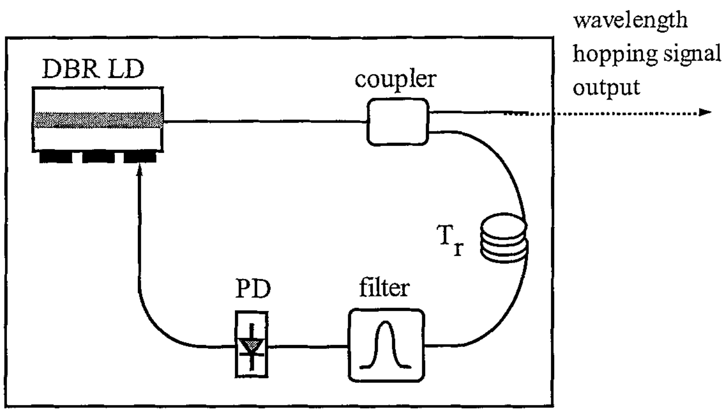

Fig. 3. Delayed feedback via optical filter to tunable laser. DBR LD: Laser Diode with Distributed Bragg Reflector for selective single mode operation. Injection of tuning current into DBR allows tuning of wavelength.

of whether the modulation pattern is stable or hopping chaotically, the carrier oscillation can be phase-locked to an external clock by injecting a small periodic signal. This means that the clock reference can be maintained during the generation of random bit patterns [3–5,10,11].

3 Wavelength-tunable laser modulated by delayed feedback

Next we describe a signal generator which obeys a similar dynamical model, but which generates not only intensity-hopping signals like the previous system, but also wavelength-hopping signals. An example is shown in Fig. 3. The key new components here are a wavelength-tunable laser, and an optical filter with a pass-band extending over the tuning range of the laser. The intensity of the laser light detected after it has passed through the filter is used to drive the wavelength-tuning of the laser.

Systems of this type have been studied by Liu and co-workers [14–16], and Goedgebuer and co-workers [17,18]. Depending on the laser and the tuning method, the tuning may affect power as well as wavelength, but so long as we can assume the laser state follows rapidly a single, more slowly varying tuning variable X, such as the tuning current in the DBR section of the laser, we can include these effects into a single feedback function and write a dynamical equation similar to the previous case, Eqns. (1-2). In simplest form, the nonlinear feedback function now looks like

$$F(X) \propto T(\lambda(X)) \tag{5}$$

where $\lambda(X)$ is the laser wavelength tuning characteristic, and $T(\lambda)$ is the transmission characteristic of the optical filter, which for an interferometric optical filter is typically of the form

$$T(\lambda) \propto (1 + a\sin^2(\lambda - \lambda^c)/\Delta)^{-1} \tag{6}$$

where λ^c is the center wavelength and Δ is the band-width of the pass-band.

The DBR laser used in our recent experiments [15,16] lased in single mode, with mode and wavelength determined by the current injected in to tune the refractive index in the DBR and only small power dependence on mode. In the lasing state, the wavelength is fixed so there is a one-to-one and typically monotonic relationship $\lambda(X)$ between tuning current and wavelength. The non-monotonic feedback necessary for oscillation and chaos comes from the convex filter function $T(\lambda)$.

In this system, solutions of the delay-differential equation of the type shown in Fig. 4, can correspond to hopping among discrete levels of intensity or discrete levels of wavelength, depending on where and how the signal is detected. The signal output of the optical filter shows variations in power and wavelength. The signal output from the laser shows variations in wavelength but not in power. This new feature of wavelength hopping could be very useful for spread-spectrum and multiplexing in optical networks.

If the laser stays in a single mode over the tuning range, the function $\lambda(X)$ is a continuous monotonic function and the nonlinear feedback function is a continuous convex function, so we expect dynamics similar to the previous system with feedback to an external intensity modulator. Specifically, we should be able to generate various different sequences of hopping among 4 wavelengths, and randomly select a sequence by switching for a time into the chaotic mode hop regime.

In this context, we can propose another simple way to generate a randomly selected wavelength, as follows. Turn on feedback for chaotic oscillation for a time, say T^+ , and then turn off the feedback and hold the last value of tuning current. If T^+ is long enough, successive choices of wavelength will be random. The probability of generating different wavelengths is directly observable as the long-time optical spectrum (as seen with a standard optical spectrum analyzer) during the chaotic oscillation. In this regard it would be an interesting challenge for nonlinear analysts to see how the probability of

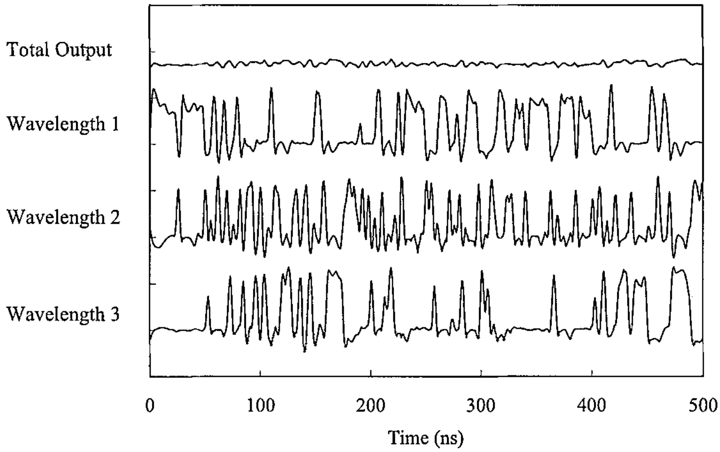

Fig. 4. Chaotic wavelength hopping. The output of the laser in Fig. 3 as detected by multiple detectors responding to different wavelength bands.

the different wavelengths can be estimated from the invariant density of the corresponding difference map.

In recent experiments, we used a laser that was tunable over multiple longitudinal modes, and a broad band optical filter, with band-pass range extending over a large part of the multi-mode tuning range of the laser [15,16]. In this case, the feedback function F is a piecewise-continuous function, and we obtained chaotic hopping among discrete wavelength bands corresponding to the different modes of the laser (Fig. 5). The use of optical filters with variable transmission characteristics offers many new possibilities for design and control of the dynamics in this system.

4 Synchronization of chaos

Proposed uses of chaotic signals in communications as carriers or modulations for hiding and tagging transmissions typically require transmitter and receiver to be able to generate the same waveform, and hence require a means to synchronize them [19–21]. Even though they have a variety of dynamics, delayed-feedback systems of the type described in this paper are easy to synchronize. It is sufficient to replace some or all of the feedback Y_R in the receiver with a signal Y_T sent from a corresponding part of the circuit of the transmitter [15–18]. Then, the dynamics of the signal V_R in the receiver system become as,

$$\tau dV_R/dt = -V_R + \varepsilon Y_T + (1-\varepsilon)Y_R \qquad (0 \le \varepsilon \le 1) \tag{7}$$

Typically there is a threshold value of coupling parameter ε above V_S which converges to the transmitter signal V_T (allowing for time lag due to propagation from transmitter to receiver). Numerical studies have shown that this scheme is quite robust with respect to noise and small mismatches in parameters of the two systems [16].

In the case of the wavelength hopping system, we emphasize that this means that the chaotic on-off dynamics of each pair of wavelengths in master and slave will be synchronized [15,16] . In the context of applications for transmitting coded data, this type of synchronized system allows multiple sets of data to be coded separately in the multiple wavelengths [16] .

Recently, we found that delay-differential systems allow a peculiar type of dual synchronization [22] that is possible when the signals from two independent oscillators are combined and sent to a receiver system containing a corresponding pair of oscillators. By appropriate injection into the receiver oscillators, we can get them to synchronize to their respective masters, even though they are receiving only one common signal.

The theoretical model for the two oscillators on the receiver side is as follows,

$$\tau dX_{Ri}/dt = -X_{Ri} + Y_{Ri} + S_T - S_R \tag{8}$$

where $i = 1, 2$ is the oscillator label. Here S_T is the superposed transmission signal received on the receiver side,

$$S_T = \varepsilon_1 Y_{T1} + \varepsilon_2 Y_{T2} \tag{9}$$

and S_R is a similar superposition of the signals from the slave oscillators

$$S_R = \varepsilon_1 Y_{R1} + \varepsilon_2 Y_{R2} \tag{10}$$

with summation constant coefficients ε_1 and ε_2. The trick is to make a reference signal from the outputs of the receiver oscillators combined in the same way as the sum signal from the transmitters, subtract this from the transmitted signal, and inject the difference into the receiver oscillators. It is obvious that a dual synchronization state exists for pairs with identical dynamics, since if they start in a dual synchronized state, then the injected signals will be zero, and they will continue to follow the same evolution. However, it is not obvious that this is an attracting state i.e. that it will converge to the dual synchronization state from arbitrary initial conditions. In numerical simulations of delay-differential systems using various nonlinear feedback functions, we found robust regimes where chaotic dynamics can indeed be dual synchronized [22].

In the case of corresponding difference equations defined on discrete time (such as Eqn. (4)) it is possible to prove explicitly the existence of stable dual synchronization states for particular pairs of chaotic feedback functions [22,23]. In the delay-differential systems, under the condition of $\varepsilon_1 = \varepsilon_2 = 1/2$, we can show small deviations from the dual synchronized state, $x = X_{T1} - X_{R1}$, $y = X_{T2} - X_{R2}$, converge quickly to the line $x = -y$, where they obey

$$\tau dx(t)/dt = -x(t) + 0.5(D_1(t - T_r) + D_2(t - T_r))x(t - T_r) \tag{11}$$

Here D_1 and D_2 are the derivatives of the nonlinear feedback functions for the two transmitter oscillators. The deviations will converge to zero if there is a statistical balance of the positive and negative fluctuations of D_1 and D_2, under similar conditions to the difference equations. This is another situation where properties of difference equations can be used in the design and control of laser systems obeying delay-differential equations.

5 Adaptive signal generation using chaos

Finally, we mention an adaptive control technique which promises to be useful for selective generation of signals in a signal generator modeled by a delay-differential system [2,6,9]. This technique has been demonstrated in experiments on adaptive generation of multi-bit binary sequences [4,10,11]. It has also been tested in numerical simulations of adaptive wavelength selection in a multi-mode laser with optical feedback [24].

The key point feature of this technique is that no extra logic or switch control is needed other than feedback of the scalar response to a single control parameter. This is possible due to the particular nature of the multi-stability and chaotic mode hopping that occurs in these delay-differential systems. The technique assumes that we can switch between a multistable regime and a chaos mode hopping regime, as we described in Section 2. It also assumes that the signal generator can receive a scalar external response signal, which tells it whether its current output signal is satisfactory or not. The technique, described simply, is: If the output is not satisfactory, the control parameter is set to the value for chaotic mode transitions. If it is satisfactory, the parameter is set to a value for which oscillations are stable. The result is a self-consistent situation, where the system will continue to hop around until it finds a mode for which the response is good.

6 Conclusion

We have described how signal generator modules consisting of laser devices with delayed feedback, modeled by delay-differential equations can be used to generate a variety of complex signals in simple and tunable configurations. We described methods for generating both intensity modulated and wavelength modulated laser signals. We pointed out that these systems can have a variety of robust, multi-stable attractors corresponding to different signals or signal patterns. We emphasized that not only do these systems generate a variety of robust signals, but this generation is also amenable to systematic design and control. Finally, we mentioned novel techniques for synchronization and adaptive control of signal generation, which make use of particular dynamical properties found in systems modeled by delay-differential equations.

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