Effects of viscous flow on residual stresses in film/substrate systems

Chun-Hway Hsueh[a)]
Metals and Ceramics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

Sanboh Lee
Department of Materials Science and Engineering, National Tsing Hua University, Hsinchu, Taiwan

(Received 22 October 2001; accepted for publication 21 November 2001)

An analytical model is developed to analyze the effects of viscous flow on residual stresses in film/substrate systems. This is achieved by utilizing the analogy between the governing field equation of elasticity and the Laplace transform with respect to time of the viscoelastic field equation. While viscous flow can occur in either the film or the substrate, analyses of how viscous flow in the substrate relaxes residual stresses in the film have not been performed. Also, the film thickness is often ignored in analyzing stress relaxation due to viscous flow in the film. The above two issues are studied in the present analysis. For the case of viscous flow in the film, the stress relaxation rate decreases with increasing film thickness. Conversely, for the case of viscous flow in the substrate, the stress relaxation rate increases with increasing film thickness. Compared to viscous flow in the film, viscous flow in the substrate results in slower stress relaxation in the system. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1445282]

I. INTRODUCTION

Film/substrate systems have extensive applications as microelectronic, optical, and structural components.[1–3] However, these systems are generally subjected to residual stresses, which can result from (1) mismatch in the coefficients of thermal expansion between the film and the substrate,[1–5] (2) lattice mismatch at coherent interfaces for single crystal materials,[1,6–8] and (3) phase transformation,[9,10] oxidation,[11] or densification[12–14] of the constituents. Because the configuration of the system is asymmetric, the residual stresses in the system are also asymmetric which, in turn, results in bending of the system. Considerable effort has been devoted to analyzing residual stresses and bending in film/substrate systems;[4,5,15–18] however, the analyses are generally based on linear elasticity. It should be noted that viscous flow in materials can occur at high temperatures. In silicon semiconductor devices, high temperature treatments are indispensable in fabrication processes. Therefore, understanding of the deformation characteristics of film/substrate systems is essential. Viscous flow in film/substrate systems has been documented. Observations of viscous flow in films include that in SiO_2/Si,[19,20] Pd–Si/Si,[21] Mo–Si/Si,[22] amorphous Si/crystalline Si,[9,10] and polyimide/Ge;[23] observations of viscous flow in substrates include that in Si/Ge[24,25] and Si_3N_4/Si.[26,27] In the presence of viscous flow, stress relaxation occurs and both the residual stresses and the curvature in the system are modified.[9,10,19–27] While rigorous analytical solutions have been obtained for elastic systems,[4,5,16,17] analyses for viscous flow systems are few in number, elementary, and often limited to the case of viscous flow in a thin film constrained by a rigid substrate.[21,22,24,27] In this case, the film thickness is ignored in the analyses and an analytical model describing how viscous flow in substrates relaxes residual stresses in films has not yet been developed.

The purpose of the present study is to develop an analytical model to analyze the effects of viscous flow on residual stresses in film/substrate systems. This is achieved by circumventing the need to solve complex partial differential equations for the viscoelastic problem and by utilizing the analogy between the governing field equation of elasticity and the Laplace transform with respect to time of the viscoelastic field equation.[28–30] First, the elastic solutions for the residual stresses and bending in the system are derived. Then, the elastic solutions are converted into viscoelastic solutions via the Laplace (and the inverse Laplace) transform. Finally, the present solutions are compared to existing solutions and the difference between viscous flow in the film and in the substrate is discussed.

II. ELASTIC SOLUTIONS

A cross section of a film/substrate system is schematically shown in Fig. 1(a), where a film with thickness, h_f, is bonded to a substrate with thickness, h_s. The system has planar geometry, and the coordinate system is defined such that the plane of the film/substrate system is normal to the z axis and the interface between the film and the substrate is located at $z=0$. In this case, the two free surfaces of the system are located at $z=h_f$ and $z=-h_s$. The film is subjected to mismatch strain, $\Delta\varepsilon$, relative to the substrate, which can result from thermal mismatch, lattice mismatch, phase transformation, or other mechanisms. To determine the stress field induced by this mismatch strain in the system, the following logic is invoked. First, without being bonded at the interface, the film is subjected to a unconstrained mismatch strain, $\Delta\varepsilon$, while the strain in the substrate remains zero [Fig. 1(b)]. Then, because of bonding at the interface, displacement compatibility is required. To fulfill this, uniform tensile

[a)]Electronic mail: hsuehc@ornl.gov

 © 2002 American Institute of Physics

Downloaded 24 Apr 2002 to 128.219.47.162. Redistribution subject to AIP license or copyright, see http://ojps.aip.org/japo/japcr.jsp

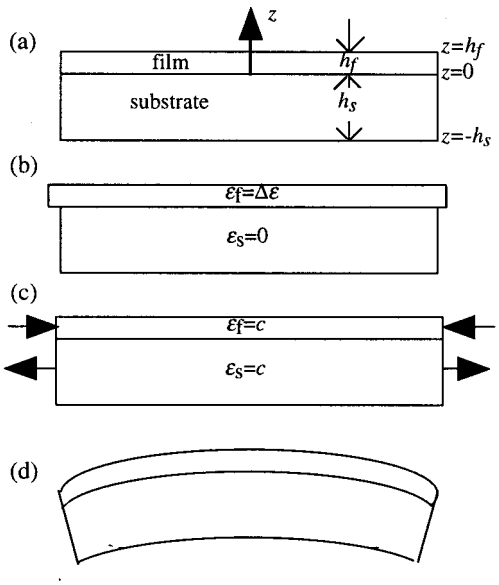

FIG. 1. Schematics showing cross sections of a film/substrate system subjected to mismatch strain: (a) stress-free condition, (b) unconstrained condition for the film subjected to mismatch strain, $\Delta\varepsilon$, (c) constrained strain, $\varepsilon_f=\varepsilon_s=c$, to maintain displacement compatibility, and (d) bending induced by asymmetric stresses.

and compressive stresses are imposed on the individual layers, such that the strain in the system is a constant, c, and the total force on the system remains zero [Fig. 1(c)]. Finally, to balance the bending moment induced by the asymmetric stresses in the system, bending of the system occurs [Fig. 1(d)].

Based on the logic described in Fig. 1, the strain in the system, ε, can be decomposed into a uniform component and a bending component. While the uniform component, c, is described in Fig. 1(c), the bending component is described in Fig. 1(d). The bending component can be formulated based on classical beam bending theory.[31] The neutral axis was defined in classical beam bending theory such that it is the line in the cross section of the beam where the normal stress is zero.[31] To avoid confusion with the neutral axis, a new term—the bending axis—is defined as the line in the cross section where the bending strain component is zero. Then, based on classical beam bending theory, the bending strain component is proportional to the distance from the bending axis and inversely proportional to the radius of curvature.[31] Hence, the total strain in the system can be formulated as

$$\varepsilon=c+\frac{z-b}{r} \quad (\text{for } -h_s\leqslant z\leqslant h_f), \tag{1}$$

where $z=b$ indicates the location of the bending axis, and r is the radius of curvature of the system. When b is negative, the bending axis is located in the substrate.

Since the mismatch strain, $\Delta\varepsilon$, is a stress-free strain [i.e., stresses in Fig. 1(b) are zero], the normal stresses in the film and the substrate, σ_f and σ_s, are related to strains by

$$\sigma_f=B_f(\varepsilon-\Delta\varepsilon) \quad (\text{for } 0\leqslant z\leqslant h_f), \tag{2a}$$

$$\sigma_s=B_s\varepsilon \quad (\text{for } -h_s\leqslant z\leqslant 0), \tag{2b}$$

where ε is given by Eq. (1), B is the biaxial modulus, and the subscripts, f and s, denote the film and the substrate, respectively. The biaxial modulus is related to the Young's modulus, E, and Poisson ratio, ν, by

$$B=\frac{E}{1-\nu}. \tag{3}$$

If the system is a strip instead of planar geometry, the biaxial modulus in Eqs. (2a) and (2b) should be replaced by the Young's modulus.

The strain/stress distributions in the system [i.e., Eqs. (1) and (2)] are contingent upon solutions of the three parameters, c, b, and r, which can be determined from the following three boundary conditions. First, the resultant force due to the uniform strain component [i.e., the total force in Fig. 1(c)] is zero:

$$B_f(c-\Delta\varepsilon)h_f+B_sch_s=0. \tag{4a}$$

Second, the resultant force due to the bending strain component is zero:

$$\int_0^{h_f}\frac{B_f(z-b)}{r}dz+\int_{-h_s}^{0}\frac{B_s(z-b)}{r}dz=0. \tag{4b}$$

Third, the sum of bending moments is zero, such that

$$\int_0^{h_f}\sigma_f z dz+\int_{-h_s}^{0}\sigma_s z d=0. \tag{4c}$$

Solutions of Eqs. (1), (2), and (4) yield

$$c=\frac{B_fh_f\Delta\varepsilon}{B_fh_f+B_sh_s}, \tag{5a}$$

$$b=\frac{B_fh_f^2-B_sh_s^2}{2(B_fh_f+B_sh_s)}, \tag{5b}$$

$$\frac{1}{r}=\frac{6B_fB_sh_fh_s(h_f+h_s)\Delta\varepsilon}{B_f^2h_f^4+B_s^2h_s^4+2B_fB_sh_fh_s(2h_f^2+2h_s^2+3h_fh_s)}. \tag{5c}$$

It should be noted that while both the uniform strain component, c, and the curvature, $1/r$, are linearly proportional to the mismatch strain, $\Delta\varepsilon$, the position of the bending axis is independent of $\Delta\varepsilon$.

The stress distribution in the system is nonuniform because of bending, and conversion of elastic solutions to viscoelastic solutions via the Laplace transform is very complex. However, this can be simplified if the average stress in the film is considered. This consideration is appropriate because the film is relatively thin and knowledge of the detailed stress distribution through the thickness is not essential. The average film stress is defined by

$$\bar{\sigma}_f=\frac{1}{h_f}\int_0^{h_f}\sigma_f dz. \tag{6}$$

By combining Eqs. (1), (2a), (5), and (6), the average film stress is

Downloaded 24 Apr 2002 to 128.219.47.162. Redistribution subject to AIP license or copyright, see http://ojps.aip.org/japo/japcr.jsp

$$\bar{\sigma}_f=\frac{-B_fB_sh_s(B_fh_f^3+B_sh_s^3)\Delta\varepsilon}{B_f^2h_f^4+B_s^2h_s^4+2B_fB_sh_fh_s(2h_f^2+2h_s^2+3h_fh_s)}. \tag{7a}$$

Combined with Eq. (5c), Eq. (7a) can also be expressed as

$$\bar{\sigma}_f=\frac{-(B_fh_f^3+B_sh_s^3)}{6h_f(h_f+h_s)r}. \tag{7b}$$

While $\bar{\sigma}_f$ is expressed as a function of the mismatch strain, $\Delta\varepsilon$, in Eq. (7a), it is expressed as a function of the curvature, $1/r$, in Eq. (7b). Compared to Eq. (7a), Eq. (7b) is sometimes more desirable because the film stress can be obtained from the curvature measured without knowing the mismatch strain. However, to obtain viscoelastic solutions from elastic solutions, Eq. (7a) should be used because the curvature in Eq. (7b) is not a constant during viscous flow.

It should also be noted that the average film stress described by Eq. (7a) is still complex for taking the inverse Laplace transform to derive its corresponding viscoelastic stress. Further simplification can be made because the film is generally much thinner than the substrate. When $h_f \ll h_s$, Eqs. (7a) and (7b) can be simplified and their first-order approximations are, respectively,

$$\bar{\sigma}_f=\frac{-B_fB_sh_s\Delta\varepsilon}{4B_fh_f+B_sh_s} \quad (\text{for } h_f\ll h_s), \tag{8a}$$

$$\bar{\sigma}_f=\frac{-B_sh_s^2}{6h_f(1+h_f/h_s)r} \quad (\text{for } h_f\ll h_s). \tag{8b}$$

When a zero-order approximation is taken, Eqs. (7a) and (7b) become

$$\bar{\sigma}_f=-B_f\Delta\varepsilon \quad (\text{for } h_f\ll h_s), \tag{9a}$$

$$\bar{\sigma}_f=\frac{-B_sh_s^2}{6h_fr} \quad (\text{for } h_f\ll h_s). \tag{9b}$$

Equation (9b) is identical to the well-known Stoney's equation[15] but opposite in sign, which results from the different conventions for defining curvature. The positive curvature is defined for a concave surface by Stoney's equation but is defined for a convex surface in the present study (e.g., $\Delta\varepsilon$ and $1/r$ shown in Fig. 1 are positive). Note that the zero-order approximation, Eq. (9a), contains no functional dependence on the film thickness. In order to examine how the film thickness influences stress relaxation due to viscous flow in the system, the first-order approximation, Eq. (8a), is adopted in Sec. III to derive viscoelastic solutions.

III. VISCOELASTIC SOLUTIONS

While the stress in an elastic material is related directly to the strain, the stress in a viscous material depends on the strain rate. Hence, the constitutive equation describing the stress–strain relation for a viscoelastic material is a time-dependent differential equation. When the stress in a viscoelastic material has spatial variation, the solution for the position-dependent and time-dependent partial differential equation becomes formidable. Fortunately, the Laplace transform, which is an integral transform, has been developed to help in solving differential equations.[28–30]

The Laplace transform converts a function of t, $F(t)$, to a function of s and is defined by

$$L[F(t)]=\int_0^\infty \exp(-st)F(t)\mathrm{d}t, \tag{10}$$

where $L[F(t)]$ denotes the Laplace transform of $F(t)$, and s is a transform parameter. It has been shown that the governing field equation of elasticity and the Laplace transform with respect to time, t, of the viscoelastic field equation are analogous in form.[28–30] Hence, knowing the stresses in an elastic system [e.g., Eq. (8a)], the Laplace transform of the stresses in a corresponding viscoelastic system can be formulated. Then, by taking the inverse Laplace transform, the viscoelastic stresses can be obtained.

Adopting the usual assumptions that the viscoelastic behavior can be described by a Maxwell model, it was derived that the elastic constants in the elastic system have the following transforms in formulating the Laplace transform of the stresses in a corresponding viscoelastic system.[13,28]

$$E\rightarrow\frac{9KG\eta s}{3K(G+\eta s)+G\eta s}, \tag{11a}$$

$$\nu\rightarrow\frac{3K(G+\eta s)-2G\eta s}{6K(G+\eta s)+2G\eta s}, \tag{11b}$$

where K and G are the bulk modulus and shear modulus, respectively, and η is the Newtonian shear viscosity. Substituting Eqs. (11a) and (11b) into Eq. (3), the corresponding transform of the biaxial modulus is

$$B\rightarrow\frac{18KG\eta s}{3K(G+\eta s)+4G\eta s}. \tag{11c}$$

Using the above transforms, the solutions for viscoelastic film/elastic substrate and elastic film/viscoelastic substrate, respectively, are derived as follows.

A. Viscoelastic film/elastic substrate

The biaxial modulus of the film, B_f, in Eq. (8a) is replaced by Eq. (11c) and the Laplace transform of the viscoelastic stress in the film, $L[\bar{\sigma}_f(t)]$, becomes

$$L[\bar{\sigma}_f(t)]=\frac{\left(\dfrac{-18K_fG_f\eta_fB_sh_ss}{3K_f(G_f+\eta_fs)+4G_f\eta_fs}\right)L[\Delta\varepsilon(t)]}{\left(\dfrac{72K_fG_f\eta_fh_fs}{3K_f(G_f+\eta_fs)+4G_f\eta_fs}\right)+B_sh_s}, \tag{12}$$

where $L[\Delta\varepsilon(t)]$ is the Laplace transform of $\Delta\varepsilon(t)$. The inverse Laplace transform of Eq. (12) gives the viscoelastic stress in the film, $\bar{\sigma}_f(t)$, such that

$$\bar{\sigma}_f(t)=\frac{-B_fB_sh_s}{4B_fh_f+B_sh_s}\left[\Delta\varepsilon_0\exp\left(\frac{-B_fB_sh_st}{6(4B_fh_f+B_sh_s)\eta_f}\right)+\int_0^t\frac{\mathrm{d}\Delta\varepsilon}{\mathrm{d}u}\exp\left(\frac{-B_fB_sh_s(t-u)}{6(4B_fh_f+B_sh_s)\eta_f}\right)\mathrm{d}u\right], \tag{13}$$

Downloaded 24 Apr 2002 to 128.219.47.162. Redistribution subject to AIP license or copyright, see http://ojps.aip.org/japo/japcr.jsp

where $\Delta\varepsilon_0$ is $\Delta\varepsilon(t)$ at $t=0$. Equation (13) is a general equation describing the time-dependent viscoelastic stress in the film. The limitation in applying this equation is that both the biaxial modulus and the shear viscosity are constant. For nonconstant viscoelastic properties, complications arise in applying the inverse Laplace transform to Eq. (12).

Depending upon the experimental conditions, Eq. (13) can be simplified accordingly and two examples are given as follows. First, the system subjected to initial mismatch strain is kept at a constant temperature to allow viscous flow in the film. In this case, the mismatch strain is a constant (i.e., $d\Delta\varepsilon/dt=0$), and Eq. (13) becomes

$$\bar{\sigma}_f(t)=\frac{-B_fB_sh_s\Delta\varepsilon_0}{4B_fh_f+B_sh_s}\exp\left(\frac{-B_fB_sh_st}{6(4B_fh_f+B_sh_s)\eta_f}\right). \quad (14)$$

Furthermore, if the zero-order approximation is taken, Eq. (14) can be simplified to

$$\bar{\sigma}_f(t)=-B_f\Delta\varepsilon_0\exp\left(\frac{-B_ft}{6\eta_f}\right). \quad (15)$$

Second, the system is initially stress free (i.e., $\Delta\varepsilon_0=0$) and oxidation of the film occurs at high temperatures. In this case, the oxidation strain develops as a function of time while the stress is relaxed simultaneously due to viscous flow at high temperatures and Eq. (13) becomes

$$\bar{\sigma}_f(t)=\frac{-B_fB_sh_s}{4B_fh_f+B_sh_s}\int_0^t\frac{d\Delta\varepsilon}{du}\exp\left(\frac{-B_fB_sh_s(t-u)}{6(4B_fh_f+B_sh_s)\eta_f}\right)du. \quad (16)$$

An example of analyzing oxidation-induced stresses has been shown previously.[11]

We note that the stress relaxation in the film is accompanied by stress relaxation in the substrate. Because there is no force applied to the system, the average stresses in the film and the substrate, $\bar{\sigma}_f$ and $\bar{\sigma}_s$, satisfy the following equilibrium condition:

$$\bar{\sigma}_fh_f+\bar{\sigma}_sh_s=0. \quad (17)$$

Hence, $\bar{\sigma}_s$ becomes relaxed as $\bar{\sigma}_f$ relaxes.

B. Elastic film/viscoelastic substrate

The analysis is similar to that in Sec. III B. In this case, the biaxial modulus of the substrate, B_s, in Eq. (8a) is replaced by Eq. (11c) and the Laplace transform of the viscoelastic stress in the film, $L[\bar{\sigma}_f(t)]$, becomes

$$L[\bar{\sigma}_f(t)]=\frac{\left(\frac{-18K_sG_s\eta_sB_fh_ss}{3K_s(G_s+\eta_ss)+4G_s\eta_ss}\right)L[\Delta\varepsilon(t)]}{4B_fh_f+\left(\frac{18K_sG_s\eta_sh_ss}{3K_s(G_s+\eta_ss)+4G_s\eta_ss}\right)}. \quad (18)$$

The inverse Laplace transform of Eq. (18) gives the viscoelastic stress in the film, $\bar{\sigma}_f(t)$, such that

$$\bar{\sigma}_f(t)=\frac{-B_fB_sh_s}{4B_fh_f+B_sh_s}\left[\Delta\varepsilon_0\exp\left(\frac{-2B_fB_sh_ft}{3(4B_fh_f+B_sh_s)\eta_s}\right)+\int_0^t\frac{d\Delta\varepsilon}{du}\exp\left(\frac{-2B_fB_sh_f(t-u)}{3(4B_fh_f+B_sh_s)\eta_s}\right)du\right]. \quad (19)$$

If the mismatch strain is a constant, Eq. (19) becomes

$$\bar{\sigma}_f(t)=\frac{-B_fB_sh_s\Delta\varepsilon_0}{4B_fh_f+B_sh_s}\exp\left(\frac{-2B_fB_sh_ft}{3(4B_fh_f+B_sh_s)\eta_s}\right). \quad (20)$$

Furthermore, if h_f is ignored, Eq. (20) can be simplified to

$$\bar{\sigma}_f(t)=-B_f\Delta\varepsilon_0\exp\left(\frac{-2B_fh_ft}{3h_s\eta_s}\right). \quad (21)$$

Because of the factor h_f/h_s in the exponential term in Eq. (21), stress relaxation is very slow when h_f is much smaller than h_s.

IV. EXISTING SOLUTIONS

When viscous flow occurs in the film, the resulting differential equation for stress relaxation in the film constrained by a rigid substrate has been shown, such that[21,22,24,27]

$$\frac{1}{B}\frac{d\sigma}{dt}+\frac{\sigma}{6\eta}=0, \quad (22)$$

where σ is the stress in the film. When B and η are constant, the solution of Eq. (22) is

$$\sigma(t)=\sigma_0\exp\left(-\frac{Bt}{6\eta}\right), \quad (23)$$

where σ_0 is $\sigma(t)$ at $t=0$. Equations (15) and (23) are identical when $-B_f\Delta\varepsilon_0$ in Eq. (15) is considered as initial stress, σ_0, in Eq. (23). To the best of the authors' knowledge, the film thickness is always considered to be infinitesimal and has not been included in existing analyses. Also, an analytical equation describing how the viscous flow of the substrate relaxes stresses in the film has not been derived before. However, a comprehensive analysis considering temperature changes, volume changes, viscous flow, and anelastic relaxation (e.g., annihilation of free volume) during annealing of a viscoelastic thin film constrained by a rigid substrate was performed by Loopstra *et al.*[22]

V. RESULTS

The effects of viscous flow in the film and the substrate on the relaxation of film stresses are studied, respectively, using Eqs. (14) and (20). The film stress, $\bar{\sigma}_f$, is normalized by its initial stress, $\bar{\sigma}_f(t=0)$, and plotted as a function of normalized time, B_ft/η, to elucidate the essential trends. The material properties of viscoelastic amorphous Si films on elastic crystalline Si substrates and elastic Si films on viscoelastic Ge substrates are used as examples. For amorphous Si/crystalline Si systems, the biaxial modulus ratio of amorphous Si to crystalline Si is 0.7.[9] Using $B_f/B_s=0.7$ and Eq. (14), the stress relaxation is shown in Fig. 2 for different relative film thicknesses, h_f/h_s. When $h_f/h_s=0$, the solu-

Downloaded 24 Apr 2002 to 128.219.47.162. Redistribution subject to AIP license or copyright, see http://ojps.aip.org/japo/japcr.jsp

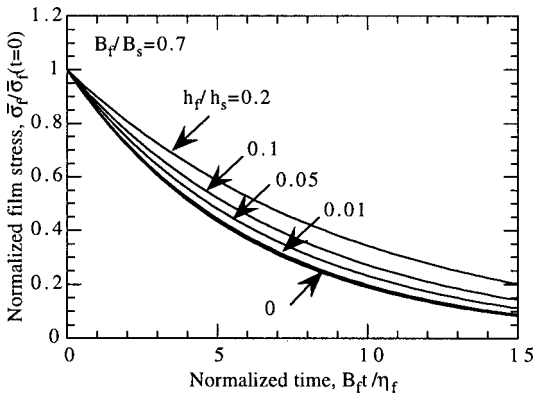

FIG. 2. Normalized film stress, $\bar{\sigma}_f/\bar{\sigma}_f(t=0)$, as a function of the normalized time, $B_f t/\eta_f$, at different film thicknesses, h_f/h_s, for viscoelastic amorphous Si/elastic crystalline Si systems.

tion is the same as Eq. (23). It is shown in Fig. 2 that the rate of stress relaxation decreases with an increase of h_f/h_s.

For Si/Ge systems, the biaxial moduli are $B_f=107$ GPa and $B_s=180$ GPa (i.e., $B_f/B_s=0.6$).[24] Using $B_f/B_s=0.6$ and Eq. (20), the stress relaxation is shown in Fig. 3 for different relative film thicknesses, h_f/h_s. When $h_f/h_s=0$, the stress in the film cannot be relaxed; i.e., $\bar{\sigma}_f/\bar{\sigma}_f(t=0)=1$ in Fig. 3. This result can be visualized by examining Eq. (17). In the limiting case of $h_f/h_s=0$, $\bar{\sigma}_s=0$ and hence viscous flow in the substrate cannot relax stresses in the system at all. It is shown in Fig. 3 that the rate of stress relaxation increases with an increase of h_f/h_s. However, compared to Fig. 2, Fig. 3 reveals slower stress relaxation. This can also be visualized by comparing the exponential terms in Eqs. (14) and (20). If we ignore the difference between the film viscosity and the substrate viscosity, Eqs. (14) and (20) have the same stress relaxation rate when $h_f/h_s=0.25$. For $h_f/h_s<0.25$, viscous flow in the substrate gives slower stress relaxation than viscous flow in the film.

VI. CONCLUSIONS

Film/substrate systems are generally subjected to residual stresses that can be relaxed by viscous flow in the film or the substrate. In existing analyses, the stress relaxation is generally obtained by solving a differential equation, which describes the stress–strain relation during viscoelastic deformation [e.g., Eq. (22)]. To have a solvable differential equation, many factors are ignored. To the best of the authors' knowledge, the film thickness has not been included in the analysis. Also, an analytical equation describing how viscous flow in the substrate relaxes stresses in the film has not been derived. The above problems are resolved in the present study by utilizing the analogy between the governing field equation of elasticity and the Laplace transform with respect to time of the viscoelastic field equation. The viscoelastic solution is obtained by (1) deriving the elastic solution, (2) formulating the corresponding Laplace transform of the viscoelastic solution, and (3) inverting the Laplace transform. However, the analyses are complex and the following simplifications are adopted: (1) a first-order approximation is taken for average film stress, (2) the viscoelastic behavior can be described by a Maxwell model, and (3) the viscoelastic properties are assumed to be constant.

For the case of viscous flow in the film, the solution based on the zero-order approximation [Eq. (15)] is identical to the existing solution [Eq. (23)] and the solution based on first-order approximation [Eq. (14)] shows the effects of the film thickness. In this case, the stress relaxation rate in the film decreases with an increase in film thickness. Conversely, in the case of viscous flow in the substrate, the stress relaxation rate in the film increases with an increase in film thickness [Eq. (20)]. Compared to viscous flow in the film, viscous flow in the substrate results in slower stress relaxation in the system.

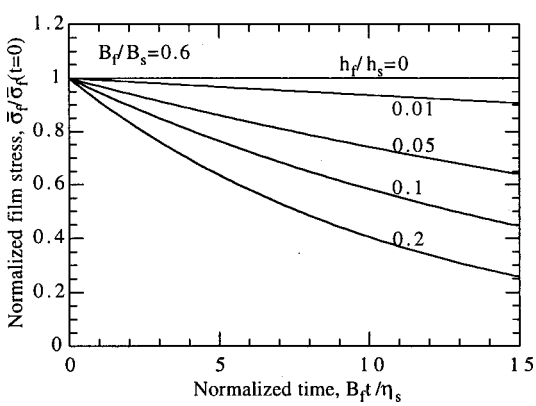

FIG. 3. Normalized film stress, $\bar{\sigma}_f/\bar{\sigma}_f(t=0)$, as a function of the normalized time, $B_f t/\eta_s$, at different film thicknesses, h_f/h_s, for elastic Si/viscoelastic Ge systems.

ACKNOWLEDGMENTS

The authors thank Dr. P. F. Becher, Dr. R. D. Carneim, and Dr. M. J. Lance for reviewing this article. This research was sponsored by the Division of Materials Sciences and Engineering, U.S. Department of Energy, under Contract No. DE-AC05-00OR22725 with UT–Battelle.

[1] W. D. Nix, Metall. Trans. A **20A**, 2217 (1989).
[2] S. M. Hu, J. Appl. Phys. **70**, R53 (1991).
[3] A. G. Evans and J. W. Hutchinson, Acta Metall. Mater. **43**, 2507 (1995).
[4] S. Timoshenko, J. Opt. Soc. Am. **11**, 233 (1925).
[5] C. H. Hsueh and A. G. Evans, J. Am. Ceram. Soc. **68**, 241 (1985).
[6] J. W. Mattews and A. E. Blakeslee, J. Cryst. Growth **27**, 118 (1974).
[7] L. B. Freund and W. D. Nix, Appl. Phys. Lett. **69**, 173 (1996).
[8] T. Y. Zhang, S. Lee, L. J. Guido, and C. H. Hsueh, J. Appl. Phys. **85**, 7579 (1999).
[9] C. A. Volkert, J. Appl. Phys. **70**, 3521 (1991).
[10] C. A. Volkert, J. Appl. Phys. **74**, 7107 (1993).
[11] C. H. Hsueh and A. G. Evans, J. Appl. Phys. **54**, 6672 (1983).
[12] G. W. Scherer and T. Garino, J. Am. Ceram. Soc. **68**, 216 (1985).
[13] C. H. Hsueh, Scr. Metall. **19**, 1213 (1985).
[14] P. Z. Cai, D. J. Green, and G. L. Messing, J. Am. Ceram. Soc. **80**, 1940 (1997).
[15] G. G. Stoney, Proc. R. Soc. London **82**, 172 (1909).
[16] P. H. Townsend, D. M. Barnett, and T. A. Brunner, J. Appl. Phys. **62**, 4438 (1987).
[17] C. H. Hsueh, J. Am. Ceram. Soc. **74**, 1646 (1991).
[18] A. Atkinson, Br. Ceram. Proc. **54**, 1 (1995).
[19] E. P. EerNisse, Appl. Phys. Lett. **30**, 290 (1977).
[20] H. C. Liu and S. P. Muraka, J. Appl. Phys. **72**, 3458 (1992).

Downloaded 24 Apr 2002 to 128.219.47.162. Redistribution subject to AIP license or copyright, see http://ojps.aip.org/japo/japcr.jsp

[21] A. Witvrouw, C. A. Volkert, and F. Spaepen, Mater. Sci. Eng., A **134**, 1274 (1991).
[22] O. B. Loopstra, E. R. van Snek, T. H. de Keijser, and E. J. Mittemeijer, Phys. Rev. B **44**, 13 519 1991).
[23] S. D. Bluestein, E. K. Chan, I. N. Miaoulis, and P. Y. Wong, IEEE Trans. Compon. Packag. Technol., **22**, 421 (1999).
[24] S. E. Rosenberg, C. G. Madras, P. Y. Wong, and I. N. Miaoulis, J. Mater. Res. **12**, 1706 (1997).
[25] C. G. Madras, P. Y. Wong, I. N. Miaoulis, L. Goldman, and R. Korenstein, Thin Solid Films **320**, 260 (1998).
[26] S. Isomae, M. Nanba, Y. Tamaki, and M. Maki, Appl. Phys. Lett. **30**, 564 (1977).
[27] S. E. Rosenberg, P. Y. Wong, and I. N. Miaoulis, Thin Solid Films **269**, 64 (1995).
[28] G. E. Mase, *Schaum's Outline Series: Theory and Problems of Continuum Mechanics* (McGraw–Hill, New York 1970), p. 196.
[29] G. B. Arfken, *Mathematical Methods for Physicists* (Academic, New York, 1970), p. 688.
[30] W. Nowacki, *Thermoelasticity*, 2nd ed. (Pergamon, Oxford, 1986), p. 476.
[31] J. M. Gere and S. P. Timoshenko, *Mechanics of Materials*, 3rd ed. (Chapman and Hall, London, 1991), p. 295.

Downloaded 24 Apr 2002 to 128.219.47.162. Redistribution subject to AIP license or copyright, see http://ojps.aip.org/japo/japcr.jsp