# Analyses of edge effects on residual stresses in film strip/substrate systems

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The residual stress distribution in a thin-film strip overlaid on a substrate is influenced by the edges of the strip. An analytical model is developed to derive a closed-form solution for the stress distribution along the film width. Because the film is much thinner than the substrate, the stress variation through the film thickness is ignored; however, the stress variation through the substrate thickness is considered in the analysis. Compared to the existing analytical models, the present model is more rigorous and the analytical results agree better with both finite element results and experimental measurements. © *2000 American Institute of Physics.* [S0021-8979(00)02218-0]

## I. INTRODUCTION

Thin-film strips on a substrate have extensive applications in microelectronic devices.[1–5] The film/substrate system is subjected to residual stresses, which result from the film deposition process, the lattice mismatch, and the thermal expansion mismatch. These stresses provide a driving force to form defects which, in turn, degrade the device performance. If the film has an infinite lateral dimension, the stress in the film would be uniform when the substrate is much thicker than the film and bending of the system is ignored. However, a finite film strip has free surfaces at the edges which, in turn, modify the stress distribution in the film/substrate system. There have been many studies to analyze the film/substrate stresses arising from the edges of an otherwise uniformly stressed film.[6,7] Both analytical modeling[6,8–12] and the finite element analysis (FEA)[7,13–15] have been performed to predict the film-edge effects. However, by comparing the analytical solutions with finite elements results and experimental measurements, it is concluded that predictions for the film-edge effects from the existing analytical models are not satisfactory.[7,15] Also, numerical procedures are required in some of the existing analytical models in order to solve some complex equations.

The purpose of this study is to develop a better analytical model with a closed-form analytical solution for the film-edge effects. Unlike the FEA which requires a finite element package and extensive numerical computations, the closed-form analytical solution is ready to be used to examine the roles of the essential material parameters in the system. First, the existing analyses are reviewed. Then, an analytical model is developed to derive a closed-form solution to predict the stress distribution in the film strip/substrate system. Finally, comparison is made between the present analytical results, existing analytical/numerical results,[7–11,15] and experimental measurements.[16–19]

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## II. SUMMARY OF EXISTING ANALYSES

Both analytical modeling[6,8–12] and the FEA[7,13–15] have been performed to analyze the film-edge effects, and they are summarized as follows.

### A. Analytical modeling

Analytical solutions for the stress distribution in a film strip similar to that shown in Fig. 1 have been previously derived. A film strip with a width $2l$ and a thickness $t$ is overlaid on a substrate with a thickness $s$. The film is subjected to a mismatch strain, $\Delta\varepsilon$. The Cartesian coordinates, $x$, $y$, and $z$, are shown in Fig. 1, such that the film is bonded to the substrate at $x=0$, and the edges of the film are located at $y=\pm l$. The dimension of the film/substrate system along the $z$ axis is assumed to be infinitely long. Among the existing analytical models, the most cited ones are developed by Hu[8,9] and Suhir.[10,11] However, in all analytical models, the variation of the stress through the film thickness is neglected because the film is much thinner than the substrate. Other approximations are also used to simplify the problem in order to obtain analytical solutions.

In Hu's analysis,[8,9] an infinitely thick substrate (i.e., $s\rightarrow\infty$) is considered and the solution is derived for a single-edged film having a semi-infinite width. A concentrated force model was derived first, such that a concentrated body force (per unit length in the $z$ direction), $F$, is induced at the film edge and acts tangentially on the substrate.[8] In this case, the stress distribution in the substrate can be obtained readily from Timoshenko and Goodier's solutions,[20] in which the stress distribution in a half space subjected to a concentrated force tangential to the surface is analyzed. The solution obtained from the concentrated force model lacks self-consistency because the film stress becomes distributed as it relaxes with the substrate deformation. Consequently, the concentrated force model[8] was modified to a distributed force model,[9] in which the change in film stress is balanced by a distributed tangential force on the substrate. However, for the distributed force model, the formulation for the stress distribution in the system is an integral equation and a numerical method is required to obtain the solutions.[9]

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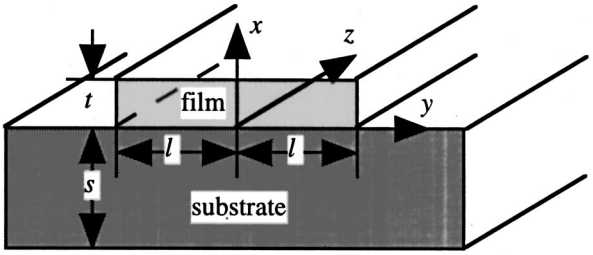


FIG. 1. Schematic showing a thin-film strip deposited on a substrate.

When the film becomes a strip, Hu's solutions for a single-edged film and the principle of superposition are used to obtain the solutions for a two-edged film.[2,12] It has been noted that the stress distribution in the film obtained by Hu for a single-edged film satisfies the free surface condition at the edge. However, the stress distribution in the film obtained by using Hu's solutions and the principle of superposition for a two-edged film does not satisfy the free surface condition at the edges.[7] A simple analytical solution for the stress distribution in the substrate can be obtained when Timoshenko and Goodier's solutions (i.e., the concentrated force model)[20] and the principle of superposition are used, such that the stress component in the $y$ direction in the substrate, $\sigma_y^s$, for the geometry described by Fig. 1 is given by[2,7]

$$\sigma_y^s = \frac{-2F}{\pi}\left\{\frac{(y+l)^3}{[(y+l)^2+x^2]^2} - \frac{(y-l)^3}{[(y-l)^2+x^2]^2}\right\}. \tag{1}$$

Suhir's analysis is a modification of Timoshenko's model[21] on analyzing the residual stresses in a bi-metal thermostat, in which the width is infinite (i.e., $l \to \infty$). By considering a finite film width, $2l$, Timoshenko's model is modified by Suhir. Because of the mismatch, $\Delta\varepsilon$, between the film, the substrate, and the edge effect, a nonuniform force along the $y$ axis (per unit length in the $z$ direction) $P(y)$ is induced in the film and a force of the same magnitude but of the opposite direction is induced in the substrate. These forces can be regarded as concentrated forces at the half thicknesses of the film (i.e., at $x=t/2$) and the substrate (i.e., at $x=-s/2$), respectively. They can also be treated as forces distributed uniformly through the thicknesses of the film and the substrate. A shear force $q(y)$ is induced at the interface such that $q(y)$ and $P(y)$ are related by an equilibrium equation. Bending of the system is considered. Then, the displacements of the film and the substrate at the interface are formulated, respectively, based on the mismatch strain, the two force distributions, and the curvature of the system. It is noted that a Fourier series is adopted in formulating the displacement due to the interfacial shear force, and $t/l$ is assumed to be small (e.g., $t/l<0.2$) in order to simplify the equations.[10] By satisfying the displacement compatibility condition at the interface and the boundary conditions that $P(l)=0$ and $q(0)=0$, the solution for $P(y)$ [and hence $q(y)$] can be obtained, such that[11]

$$P(y) = \frac{-tE_f\Delta\varepsilon}{1-\nu_f}\left[1-\frac{\cosh(\kappa y)}{\cosh(\kappa l)}\right]. \tag{2}$$

The parameter, $\kappa$, is defined by

$$\kappa = \left[\frac{\dfrac{1-\nu_f}{tE_f}+\dfrac{1-\nu_s}{sE_s}+\dfrac{(t+s)^2}{\dfrac{t^3E_f}{1-\nu_f}+\dfrac{s^3E_s}{1-\nu_s}}}{\dfrac{2t(1+\nu_f)}{3E_f}+\dfrac{2s(1+\nu_s)}{3E_s}}\right]^{1/2}, \tag{3}$$

where $E$ and $\nu$ are Young's modulus and Poisson's ratio, and the subscripts, $f$ and $s$, denote the film and the substrate, respectively. Because the film is relatively thin, the bending moment in the film (proportional to $t^3$) is small, and the stress in the film, $\sigma(y)$, is due to the force $P(y)$ only, such that

$$\sigma(y) = \frac{-E_f\Delta\varepsilon}{1-\nu_f}\left[1-\frac{\cosh(\kappa y)}{\cosh(\kappa l)}\right]. \tag{4}$$

Unlike the stress in the film, the stress in the substrate is due to both the force and the bending moment. While the stress component due to the force, $P$, is uniform through the substrate thickness, the stress component due to the bending moment varies through the substrate thickness. However, when the thickness of the substrate is infinite, the bending moment becomes zero, and the stress in the substrate becomes zero although the force is nonzero.

### B. Finite element analyses

The stress distribution in the geometry depicted by Fig. 1 has been analyzed using the FEA.[7,14,15] The tractions on all free surfaces are zero except at the bottom of the substrate (i.e., at $x=-s$) which is assumed to be rigid (i.e., zero displacement). An initially uniform mismatch strain, $\Delta\varepsilon$, is assigned to the film. Then, this mismatch strain is allowed to relax, and a numerical code is used to find the minimum energy configuration for the whole system. It is noted that singularities exist at the points where the film edges meet the substrate, and a refined finite element mesh is used in the region close to the edges.

## III. THE PRESENT ANALYTICAL MODEL

The present analytical model preserves the essence of Suhir's analysis but adopts different analytical procedures to derive the stress distribution in the film/substrate system. Also, since the substrate is much thicker than the film, the variation of the stress through the thickness of the substrate is included in the analysis; however, bending of the system is ignored. Incidentally, the present model can also be regarded as a shear lag model in a planar geometry. The shear lag model was first developed by Cox to analyze the stress transfer problem for a cylindrical geometry (i.e., a fiber embedded in a coaxial cylindrical matrix).[22] It has then been modified and improved by many others.[23–27] The analytical procedures in deriving the present model are as follows.

### A. Condition for a bonded interface

The film and the substrate are bonded at the interface, and the film is subjected to a mismatch strain, $\Delta\varepsilon$. Without the constraint in the $x$ direction (see Fig. 1), the stress normal to the interface is zero, and the displacement compatibility condition at the interface requires

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$$\frac{1}{E_f}(\sigma_y^f-\nu_f\sigma_z^f)+\Delta\varepsilon=\frac{1}{E_s}(\sigma_y^s-\nu_s\sigma_z^s)\quad(\text{at } x=0), \tag{5a}$$

$$\frac{1}{E_f}(\sigma_z^f-\nu_f\sigma_y^f)+\Delta\varepsilon=\frac{1}{E_s}(\sigma_z^s-\nu_s\sigma_y^s)\quad(\text{at } x=0), \tag{5b}$$

where $\sigma_y$ and $\sigma_z$ are the stresses in the $y$ and the $z$ directions, and the superscripts, $f$ and $s$, denote the film and the substrate, respectively. When the substrate is much thicker than the film, bending of the system can be ignored. Furthermore, when the system has an infinite length in the $z$ direction, $\sigma_z^f$ is uniform in the film and $\sigma_z^s$ is zero in the substrate. With $\sigma_z^s=0$, the combination of Eqs. (5a) and (5b) yields

$$\sigma_y^s=\frac{(1-\nu_f{}^2)E_s}{(1-\nu_f\nu_s)E_f}\sigma_y^f+\frac{(1+\nu_f)E_s}{1-\nu_f\nu_s}\Delta\varepsilon\quad(\text{at } x=0). \tag{6}$$

If the system is also infinitely long in the $y$ direction, $\sigma_y^f$ is uniform in the film, $\sigma_y^s$ is zero in the substrate, and the solution of $\sigma_y^f$ from Eq. (6) is

$$\sigma_y^f=\sigma_{\text{fb}}=\frac{-E_f}{1-\nu_f}\Delta\varepsilon\quad(\text{for } 0\leqslant x\leqslant t). \tag{7}$$

Hence, for the special case that the substrate is much thicker than the film and the system is infinitely long in both the $y$ and the $z$ directions, the film is subjected to a biaxial stress ($\sigma_y^f=\sigma_z^f$) described by Eq. (7) and the substrate is stress-free. For convenience, this biaxial stress in the film is redefined as $\sigma_{\text{fb}}$. However, when the film has a finite width in the $y$ direction (e.g., a strip), the displacement compatibility condition at the interface is described by Eq. (6).

### B. Stress gradient in the film

When the system has an infinite length in the $z$ direction, there is no stress variation in the $z$ direction. The equilibrium equation between the normal stress, $\sigma_y$, and the shear stress, $\tau_{xy}(=\tau)$, is

$$\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau}{\partial x}=0. \tag{8}$$

Integration of Eq. (8) over the thickness of the film and then division by its thickness yields

$$\frac{\partial}{\partial y}\left(\frac{1}{t}\int_0^t\sigma_y dx\right)+\frac{1}{t}\int_0^t\frac{\partial\tau}{\partial x}dx=0. \tag{9}$$

Because the film is relatively thin, the variation of $\sigma_y$ through the thickness can be ignored. Letting $\sigma_f$ represent the average value of $\sigma_y$ over the thickness of the film and using the free surface condition that $\tau=0$ at $x=t$, Eq. (9) becomes

$$\frac{d\sigma_f}{dy}=\frac{\tau_0}{t}, \tag{10}$$

where $\tau_0$ is the shear stress at the interface (i.e., $\tau=\tau_0$ at $x=0$). The stress gradient in the film is described by Eq. (10).

### C. Stress distribution in the substrate

To strictly satisfy the equilibrium equation, the stress–displacement relation, and the essential boundary conditions, solutions for the stress distribution require extensive numerical analysis. However, simple analytical solutions are attainable when some of the above equations are approximately satisfied and the rest of the equations are exactly satisfied. Because the main interest in the system is the variation of $\sigma_y$ in the $y$ direction, the variation of $\sigma_y$ in the $x$ direction is ignored in the equilibrium equation such that the gradient of $\sigma_y$ in the $y$ direction in the substrate can be approximated by a function of $y$; i.e.,

$$\frac{\partial\sigma_y}{\partial y}=f(y)\quad(\text{for } -s\leqslant x\leqslant 0). \tag{11}$$

The physical meaning of this approximation [i.e., Eq. (11)] in the equilibrium equation [Eq. (8)] is that the equilibrium equation is satisfied in an average sense (with respect to the $x$ direction). A similar approach has been adopted in the shear lag model in analyzing the stress transfer problem in a cylindrical geometry. The approximate $x$ dependence of $\sigma_y$ can be obtained by solving the approximate equilibrium equation and is shown as follows.

Continuity of the shear stress at the interface (i.e., $\tau=\tau_0$ at $x=0$) is required, and solution of Eqs. (8) and (11) subjected to the continuity condition gives

$$\tau=\left(1+\frac{x}{s}\right)\tau_0-\frac{x}{s}\tau_s\quad(\text{for } -s\leqslant x\leqslant 0), \tag{12}$$

where $\tau_s$ is the shear stress at $x=-s$.

With $\sigma_x=0$ and the displacement in the $x$ direction being negligible, $\tau$ can be related to the displacement in the $y$ direction, $w$, by

$$\tau=\frac{E_s}{2(1+\nu_s)}\frac{dw}{dx}\quad(\text{for } -s\leqslant x\leqslant 0). \tag{13}$$

Combination of Eqs. (12) and (13) and integration yields

$$\tau_0=\frac{E_s(w_0-w_s)}{s(1+\nu_s)}-\tau_s, \tag{14a}$$

$$\tau=\left(1+\frac{x}{s}\right)\frac{E_s(w_0-w_s)}{s(1+\nu_s)}-\left(1+\frac{2x}{s}\right)\tau_s\quad(\text{for } -s\leqslant x\leqslant 0), \tag{14b}$$

where $w_0$ and $w_s$ are the displacements in the $y$ direction at $x=0$ and $-s$, respectively. When the substrate is fixed at $x=-s$, $w_s=0$ and the displacement, $w$, can be derived from Eqs. (13) and (14b) such that

$$w=\left(1+\frac{x}{s}\right)^2 w_0-\frac{2(1+\nu_s)}{E_s}\left(x+\frac{x^2}{s}\right)\tau_s\quad(\text{for } -s\leqslant x\leqslant 0). \tag{15}$$

Using the conditions that $\sigma_x=\sigma_z=0$ in the substrate, differentiation of Eq. (15) with respect to $y$ and then multiplication by $E_s$ gives

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$$\sigma_y=\left(1+\frac{x}{s}\right)^2\sigma_0-2(1+\nu_s)\left(x+\frac{x^2}{s}\right)\frac{d\tau_s}{dy}$$
$$(\text{for } -s\leqslant x\leqslant 0), \tag{16}$$

where $\sigma_0$ is the substrate stress in the $y$ direction at $x=0$. The approximate $x$ dependence of $\sigma_y$ in the substrate is described by Eq. (16).

The mechanical equilibrium condition for the stress distribution in the system requires that

$$t\sigma_f+\int_{-s}^{0}\sigma_y dx=0. \tag{17}$$

Solution of Eqs. (16) and (17) gives

$$\frac{d\tau_s}{dy}=\frac{-3t}{s^2(1+\nu_s)}\left(\sigma_f+\frac{s}{3t}\sigma_0\right). \tag{18}$$

The aforementioned equation is required in deriving the governing differential equation for $\sigma_f$.

### D. Stress distribution in the film

Combination of Eqs. (10), (14a), and (18) and differentiation with respect to $y$ yields

$$\frac{d^2\sigma_f}{dy^2}=\frac{2}{st(1+\nu_s)}\left(\frac{3t}{2s}\sigma_f+\sigma_0\right). \tag{19}$$

The solution of the stress distribution $\sigma_f$ is contingent upon the determination of $\sigma_0$. For a bonded interface, $\sigma_0$ is related to $\sigma_f$ by Eq. (6), where $\sigma_y^s$ at $x=0$ becomes $\sigma_0$ and $\sigma_y^f$ at $x=0$ becomes $\sigma_f$ when the variation of $\sigma_y^f$ through the film thickness is ignored. Hence, the differential equation governing the stress distribution in the film can be obtained by substituting Eq. (6) into Eq. (19), such that

$$\frac{d^2\sigma_f}{dy^2}=\frac{2}{st(1+\nu_s)}\left[\left(\frac{3t}{2s}+\frac{(1-\nu_f^2)E_s}{(1-\nu_f\nu_s)E_f}\right)\sigma_f-\frac{(1-\nu_f^2)E_s}{(1-\nu_f\nu_s)E_f}\sigma_{\text{fb}}\right], \tag{20}$$

where $\sigma_{\text{fb}}$ is defined by Eq. (7).

The solution of $\sigma_f$ from Eq. (20) is subject to the boundary conditions that the film is stress-free at the edge and the system is symmetric with respect to $y=0$; i.e.,

$$\sigma_f=0 \quad (\text{at } y=l), \tag{21a}$$

$$\tau_0=0 \quad (\text{at } y=0). \tag{21b}$$

With the above two boundary conditions, the solution of $\sigma_f$ is

$$\sigma_f(y)=\frac{\sigma_{\text{fb}}}{1+\dfrac{3t(1-\nu_f\nu_s)E_f}{2s(1-\nu_f^2)E_s}}\left[1-\frac{\cosh(\alpha y)}{\cosh(\alpha l)}\right], \tag{22}$$

where

$$\alpha=\left[\frac{2}{st(1+\nu_s)}\left(\frac{3t}{2s}+\frac{(1-\nu_f^2)E_s}{(1-\nu_f\nu_s)E_f}\right)\right]^{1/2}. \tag{23}$$

It is noted that the present solution for the film stress distribution [Eq. (22)] has a similar formulation as that in Suhir's model [Eq. (4)].

The maximum stress within the film occurs at the center of the film (i.e., at $y=0$), such that

$$\sigma_f(0)=\frac{\sigma_{\text{fb}}}{1+\dfrac{3t}{2s}\dfrac{(1-\nu_f\nu_s)E_f}{(1-\nu_f^2)E_s}}\left[1-\frac{1}{\cosh(\alpha l)}\right]. \tag{24}$$

When the film strip is sufficiently wide such that $\cosh(\alpha l)\rightarrow\infty$, this maximum stress has a constant value: i.e.,

$$\sigma_f(0)=\frac{\sigma_{\text{fb}}}{1+\dfrac{3t}{2s}\dfrac{(1-\nu_f\nu_s)E_f}{(1-\nu_f^2)E_s}} \quad (\text{for } l\gg 0). \tag{25}$$

Furthermore, when the substrate is much thicker than the film, this constant maximum stress becomes

$$\sigma_f(0)=\sigma_{\text{fb}} \quad (\text{for } l\gg 0 \text{ and } s\gg t). \tag{26}$$

## IV. RESULTS

The FEA has been performed to predict the stress distribution in GeSi film strips on silicon substrates.[7,14,15] The material properties of this system are: $E_f=147.2$ GPa, $E_s=163.1$ GPa. $v_f=0.2765$, and $v_s=0.28$. The thickness and the width of the substrate are 50 and 100 units respectively, and different thicknesses and widths are used for the film in the FEA. The width of the substrate is not involved in all the analytical solutions. Unless noted otherwise, the above material properties and a thickness of 50 units for the substrate are used to calculate the results from the analytical models. First, comparison of the stress distribution along the film width is made between the present analytical results, Suhir's analytical results, and finite element results. Second, the stress at the center of the film (i.e., at $y=0$) is calculated and compared with the results obtained from other models and experimental measurements. Third, comparison of the stress distribution in the substrate is made between the results obtained from the present analytical model, Hu's model, and the FEA. Finally, the effects of the Young's modulus ratio of substrate to film and the aspect ratio of the film on the stress distribution in the film are shown. Because the geometry of the system is symmetric with respect to $y=0$, only the solutions for $y\geqslant 0$ are shown in the following.

### A. Stress distribution along the film width

Based on the finite element results, the stress is not uniform throughout the thickness of the film. The film stress is largest at the interface and decreases with the increasing distance from the substrate. However, the stress at the half thickness of the film (i.e., at $x=t/2$) is a good approximation of the average film stress.[28] Since the analytical solutions give only the average stress throughout the film thickness, the stresses at $x=t/2$ from the finite element results are chosen to compare with the analytical results. Using the film thickness $t=1$ unit and the half width $l=20$ units, the present analytical results, Suhir's analytical results, and the finite elements results are shown in Fig. 2. Compared to

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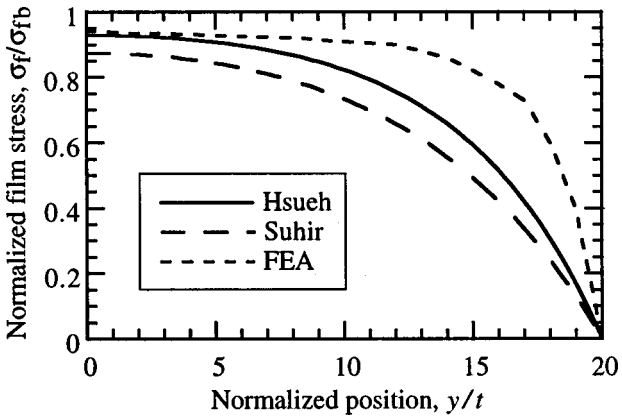


FIG. 2. Comparison of variations of normalized film stress, $\sigma_f/\sigma_{fb}$, along the film width between the results obtained from the present model, Suhir's model, and the FEA for $s=50$ units, $t=1$ units, and $l=20$ units.

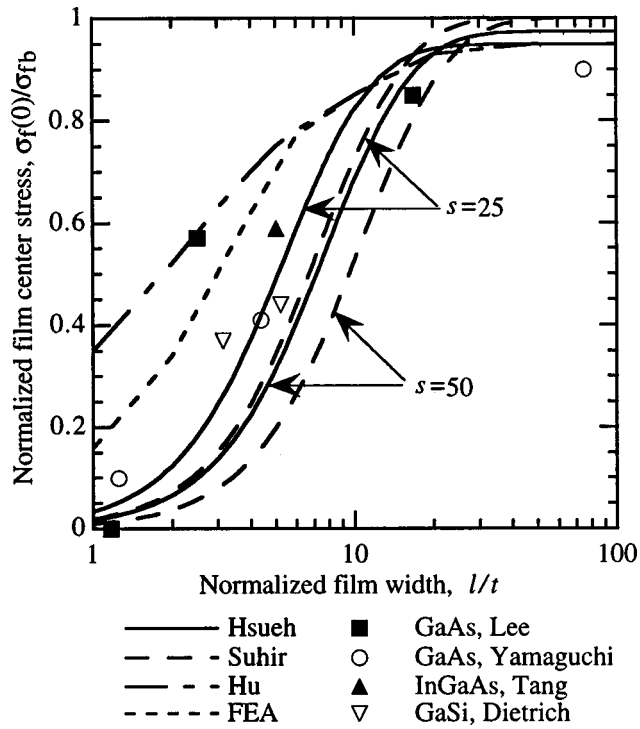


FIG. 3. Comparison of the normalized stress at the center of the film, $\sigma_f(0)/\sigma_{fb}$, as a function of $l/t$ between the results obtained from the present model, Suhir's model, Hu's model, the FEA, and experimental measurements.

Suhir's results, the present results are closer to the finite elements results. Suhir's model predicts a lower stress in the film. This discrepancy could result from the different boundary conditions adopted in the modeling. While the substrate is fixed at $x=-s$ in the present analytical model and the finite element model, the system is free to bend in Suhir's model which, in turn, results in stress relaxation. However, when the substrate is much thicker than the film (e.g., when $s/t=50$ in Fig. 2), the difference in the predicted results caused by the different boundary condition assigned at $x=-s$ is negligible. Hence, the lower stresses obtained in Suhir's model are believed to be due to the assumption of the force $P(y)$ being uniformly distributed through the substrate thickness.

### B. Stress at the center of the film

Stress measurements have been made on several semiconductor films deposited on Si substrates. Using cathodoluminescence, the stresses in GaAs films were measured by Lee (with $t=3$ $\mu$m, $2l=7$, 15, and 100 $\mu$m)[16] and Yamaguchi (with $t=4$ $\mu$m, $2l=10$, 35, and 600 $\mu$m),[17] respectively. Using micro-Raman, the stress in a $In_{0.25}Ga_{0.75}As$ film (with $t=0.1$ $\mu$m, $2l=1$ $\mu$m) was measured by Tang,[18] and the stresses in $Ge_{0.14}Si_{0.86}$ films (with $t=96$ nm, $2l=0.6$ $\mu$m and 1 $\mu$m) were measured by Dietrich.[19] It is noted that a sharp film edge is difficult to obtain in film processing due to the formation of ''bird's beak,''[2,7] which distorts the film edge. This complicates the stress measurement at the film edges; nevertheless, the stresses have been measured at the film center (i.e., at $y=0$). However, because of the finite beam spot diameter (e.g., 1 $\mu$m for cathodoluminescence and 0.7 $\mu$m for micro-Raman), the measured stress is not just at the center but some average around the center.

The FEA[7] has been performed to predict the stress at the center of the film as a function of $l/t$ to compare with the measurements. Not all the films involved in the stress measurements are GeSi. However, the elastic constants are neither sensitive to small changes in Ge concentration for GeSi films nor are they very different from GaAS and InGaAs films,[7] and the material properties of GeSi film are used to calculate the results from modeling. In Hu's model, the substrate thickness is assumed to be infinite, and the ratio, $l/t$, is sufficient to determine the solutions. However, the parameters, $s$, $t$, and $l$, are involved in both the present model and Suhir's model. Using $s=50$ units, $t=1$ unit, and different units for $l$, the results from the present model and Suhir's model are calculated for comparison. The normalized stress at the center of the film, $\sigma_f(0)/\sigma_{fb}$, as a function of $l/t$ is shown in Fig. 3. Higher stresses are predicted based on Hu's model. Compared to Suhir's model, the present model predicts results closer to the finite element results. While the stresses predicted from the present model are lower than those predicted from FEA, the present predicted results qualitatively agree with the measurements.

It has been argued that for the geometry described by Fig. 1, the thin film influences the stress field in the substrate only within a certain boundary zone in the neighborhood of the film. Outside the boundary zone, the stress field in the substrate is not perturbed by the presence of the film.[29] Hence, an effective thickness should be defined, such that this effective thickness instead of the actual substrate thickness should be used in analytical modeling. However, a criterion for defining the effective substrate thickness has not been established. Based on the finite element results for the film thickness being one unit, the perturbation of the stress field in the substrate is negligible when the position in the substrate is $>25$ units away from the interface.[7] Using 25 units instead of 50 units as the substrate thickness, the results from the present model and Suhir's model are also shown in Fig. 3 and they agree better with both the finite element results and the measurements.

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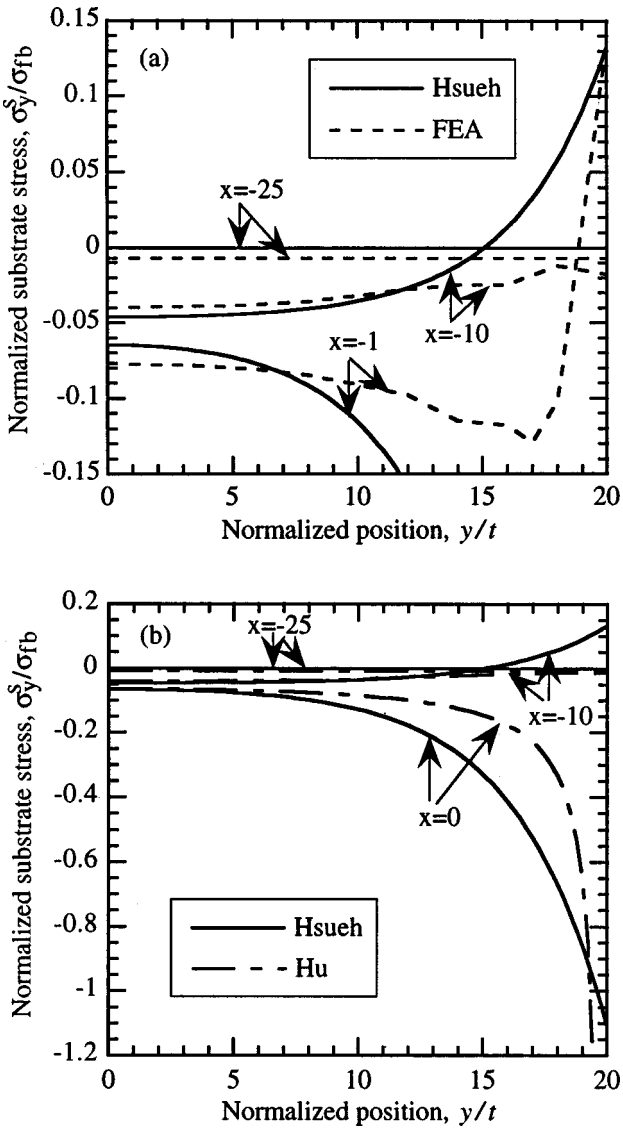


FIG. 4. Comparison of the variations of normalized substrate stress, $\sigma_y^s/\sigma_{fb}$, along the film width between the results obtained from (a) the present model and the FEA, and (b) the present model and Hu's model.

### C. Stress distribution in the substrate

The stress distribution in the substrate has been analyzed using the FEA. Using one unit for the film thickness and 20 units for the half film width $l$, the stresses in the substrate have been calculated at positions 1, 10, and 25 units away from the interface, (i.e., at $x=-1$, $-10$, and $-25$) respectively, and the results are shown in Fig. 4(a). Choosing 25 units as the effective substrate thickness, the results predicted from the present model are also shown in Fig. 4(a). The stress distribution in the substrate can also be obtained from Hu's model [Eq. (1)], in which the concentrated edge force acting on the substrate, $F$, becomes $t\sigma_{fb}$. The comparison between the present results and Hu's results is shown in Fig. 4(b). For $y$ positions away from the edge position (e.g., $y/t<10$ for $l/t=20$), the stresses in the substrate predicted from the present model, the FEA, and Hu's model are similar. However, very different stresses are predicted from the three models for $y$ positions close to the edge, and these differences are discussed as follows. First, without the edge effect, mechanical equilibrium of the system requires that the stresses in the substrate and the film have the opposite sign, such that the normalized substrate stress, $\sigma_y^s/\sigma_{fb}$, is negative. The condition that the film has a free surface at the edge (i.e., $\sigma_f=0$ at $y=l$) is satisfied in both the present model and the finite element model. Hence, the integral of the substrate stresses at $y=l$ over the substrate thickness is zero which, in turn, results in both positive and negative signs for $\sigma_y^s/\sigma_{fb}$ at $y=l$ depending on the $x$ position [Fig. 4(a)]. However, a concentrated force, $F$, at the edge position acting on the surface of the substrate [i.e., at $(x,y)=(0,l)$] is considered in Hu's model, the integral of the substrate stresses at $y=l$ over the substrate thickness must balance $F$, and $\sigma_y^s/\sigma_{fb}$ at $y=l$ is always negative through the substrate thickness [see Eq. (1) and Fig. 4(b)]. Second, Hu's results show that the normalized substrate stress has a negative singularity at $(x,y)=(0,l)$. In the present analytical model, the stress singularity cannot be handled exactly; however, the magnitude of the negative $\sigma_y^s/\sigma_{fb}$ reaches a maximum at $(x,y)=(0,l)$. In the finite element results, the stress singularity is reflected by an abrupt change of the sign for $\sigma_y^s/\sigma_{fb}$ at positions close to the interface [see $x=-1$ in Fig. 4(a)] and near the edge. Considering $\sigma_y^s/\sigma_{fb}$ at $y=l$, while the finite element results show positive values at locations close to the interface and negative values at locations away from the interface, the present results show the opposite [Fig. 4(a)].

### D. Effects of $E_s/E_f$ and $l/t$ on the stress distribution in the film

To examine the effects of Young's modulus ratio on the stress distribution in the film, the normalized stress in the film, $\sigma_f/\sigma_{fb}$, as a function of the normalized position, $y/t$, are shown in Fig. 5(a) for $s/t=25$, and $l/t=20$ at different values of $E_s/E_f$. The stress in the film, $\sigma_f$, is zero at the free edge and increases with the distance from the edge; however, the rate of increase of $\sigma_f$ with the distance from the edge increases with increasing $E_s/E_f$.

To examine the effects of the film aspect ratio on the stress distribution in the film, the normalized stress in the film, $\sigma_f/\sigma_{fb}$, as a function of the normalized position, $y/t$, is shown in Fig. 5(b) for $s/t=25$ at different values of $l/t$. The stress at the center of the film reaches an equilibrium value defined by Eq. (25) if the film is sufficiently wide [e.g., $l/t \geqslant 30$ in Fig. 5(b)].

## V. CONCLUSIONS

A closed-form analytical solution is derived in the present study to analyze the edge effects on the residual stress distribution in a thin-film strip on a substrate. In the existing analytical models, the stress variation through the film thickness is always ignored, and there are two major approaches; i.e., Hu's model and Suhir's model. The first approach starts with a single-edged uniformly stressed film on an infinitely thick substrate and considers a concentrated force exerted by the film edge on the substrate.[8] Because of the deformation of the substrate under the force, the stress in the film is modified. In this case, the stress distribution in the

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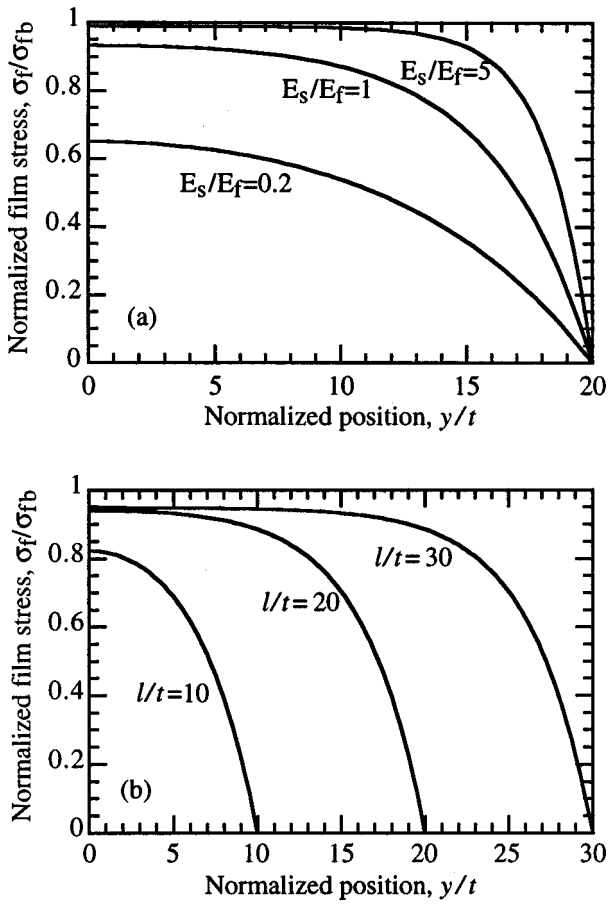


FIG. 5. Effects of (a) $E_s/E_f$, and (b) $l/t$ on the stress distribution in the film.

substrate has been formulated by Timoshenko and Goodier.[20] The concentrated force model[8] lacks self-consistency due to the film stress relaxation and is, consequently, improved to a distributed force model.[9] In this case, the stress distribution in the system is an integral equation and a numerical method is required to obtain the solution. When the film is a strip with two edges, the principle of superposition is used to obtain the solutions from the solutions for the single-edged film. It has been noted that the solution obtained from the principle of superposition does not satisfy the free surface condition at the film edges. The second approach considers a distributed force $P(y)$ within the film and a distributed force of the opposite sign within the substrate. A distributed shear force $q(y)$ is introduced at the interface between the film and the substrate to equilibrate the nonuniform force distribution $P(y)$. However, the force $P(y)$ is assumed to be uniformly distributed through the thickness of the substrate.

The present analytical model keeps the essence of the second approach but adds the stress variation through the substrate thickness in the analysis. Compared to the existing analytical models, the present model is more rigorous and the analytical results agree better with the finite element results. Also, a qualitative agreement for the stress at the center of the film is obtained between the present analytical results and the experimental measurements. However, two issues remain unsolved. First, the stress singularity, which occurs at the points where the film edges meet the substrate, cannot be exactly handled in analytical models. Second, the thin film influences the stress field in the substrate only within a certain boundary zone in the neighborhood of the film. Hence, when the substrate is much thicker than the film, an effective substrate thickness instead of the actual substrate thickness should be used in analytical modeling. This effective substrate thickness can be extracted from the finite element results. Using the effective substrate thickness, the analytical results agree better with both the finite element results and the experimental measurements. However, an analytical model capable of predicting the effective substrate thickness has not been established.

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