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Implementation of Multivariable Control Techniques with Application to Experimental Breeder Reactor II

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IMPLEMENTATION OF MULTIVARIABLE CONTROL TECHNIQUES
WITH APPLICATION TO EXPERIMENTAL
BREEDER REACTOR II

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ABBREVIATIONS

ANL	Argonne National Laboratory
CAD	Computer-aided design
EBR-II	Experimental Breeder Reactor-II
PWR	Pressurized water reactor
PI	Proportional-integral
LQR	Linear quadratic regulator
IHX	Intermediate heat exchanger
MIMO	Multiple-input, multiple-output

ABSTRACT

After several successful applications to aerospace industry, the modern control theory methods have recently attracted many control engineers from other engineering disciplines. For advanced nuclear reactors, the modern control theory may provide major advantages in safety, availability, and economic aspects. This report is intended to illustrate the feasibility of applying the linear quadratic Gaussian (LQG) compensator in nuclear reactor applications. The LQG design is compared with the existing classical control schemes. Both approaches are tested using the Experimental Breeder Reactor II (EBR-II) as the system. The experiments are performed using a mathematical model of the EBR-II plant. Despite the fact that the controller and plant models do not include all known physical constraints, the results are encouraging. This preliminary study provides an informative, introductory picture for future considerations of using modern control theory methods in nuclear industry.

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1. INTRODUCTION

1.1 PURPOSE OF RESEARCH

The development of advanced liquid metal reactor technology involves improving control strategies for several purposes. Recovery from unanticipated transients requires safe operational maneuvers. Besides handling the consequences of nonoptimal control actions, advanced strategies may minimize control efforts and may improve system performance in daily operation. Using new techniques, the control may be less dependent on the availability of sensory information compared to existing classical control systems. The modern controllers may prove to be more robust than classical schemes, leading to reliable plant operations.

The possibility of improving system performance using the modern control theory design methods has been a subject of extensive investigation for the last three decades. Applications in the aerospace and chemical industries have proved the high-performance characteristics of modern controllers. The question of feasibility of using modern controllers in advanced reactors is yet unanswered. One of the reasons is that conventional controllers are efficient and reliable in controlling nuclear systems and there is no current need for the new concepts. Another reason is related to the limitations of computational tools. Nuclear reactors are large-scale systems, and the representation of their dynamic behavior requires modeling with a large number of state variables. Control design studies of large-scale systems depend strictly on the software development. The computer-aided design (CAD) package MATRIX_x* is a good example of the latest development in this area. The objective of this study includes investigating the modern control theory algorithms of MATRIX_x, applying standard design methods using the EBR-II model as a test bed, and evaluating the controller performance. The objective is not to propose a new control strategy for the EBR-II, but instead to study the capabilities of modern controllers and their advantages and disadvantages with a limited comparison to existing schemes. This approach may find further applications in the control strategy design of advanced modular reactors.

In modern control theory, it is possible to design a compensator for any process by placing the closed-loop poles in appropriate locations, provided that the process is observable and controllable. An appropriate choice of the closed-loop poles yields satisfactory performance in general. The question of seeking an optimal solution to the design problem may seem to be a redundant effort; however, optimization is necessary for the following reasons. In a multiple-input multiple-output (MIMO) system, the pole placement method may not completely specify the controller parameters. Fortunately, attaining

*MATRIX_xTM, Version 5.0, ©1985 by Integrated Systems, Inc., Palo Alto, California.

the desired closed-loop pole locations can be done in many different ways for MIMO systems. An n -th order system with m inputs has $n \times m$ parameters to be designed for a nondynamic compensator in a full-state feedback arrangement. This means finding the same n desired closed-loop poles in m different ways. The abundance of adjustable parameters can be a great benefit in terms of satisfying several design requirements if an optimization technique is used. When an unfamiliar process is considered, an arbitrary choice of having the closed-loop pole locations far from the origin of the S -plane may require control inputs too big to be generated by the process actuators. Thus, minimization of control input can be achieved by optimizing performance. Another reason for the use of the optimal control theory arises when uncontrollable systems are concerned. It is possible to design a control system using optimal control theory so that the overall system behavior will be acceptable, provided the uncontrollable part of the system is stable.¹

1.2 BACKGROUND OF MODERN CONTROL THEORY

The development of the modern control theory began in the late 1950s and has been expanding since then as new discoveries are made by control and system engineers. The discovery of the state-space approach and theories like Liapunov's stability theory constituted the basis of the modern control theory. Pontryagin's contribution to the optimization of system performance using the variational calculus technique was a remarkable achievement in the short history of the modern control theory. Recent developments² provide an initial value solution overcoming the classical dilemma of the two-point boundary problem of the Pontryagin's theorem. Application of this algorithm to simple nuclear reactor models showed the validity of the approach within the limits of real-time implementation. For linear systems, the development of the LQG method using a state estimation technique such as the Kalman filter design has provided the most widely used tool in control system design during the last two decades. Recent progress in the theory has enabled set-point tracking applications using the LQG method.¹

One of the latest efforts in advanced control strategy development has been the focus on global control concepts of large-scale systems³ in the form of decentralized and hierarchical control structures. Using methods such as the interaction prediction method, a more global task can be achieved in an optimal fashion. The nuclear reactor control problem is a potentially open field for such applications.

1.3 SCOPE OF RESEARCH AND DEVELOPMENT STUDY

The study reported here focuses on the implementational aspects of modern multivariable control techniques using the MATRIX_x design package. This application highlights the steam generator drum level control for the EBR-II. A comparison of the classical three-element controller and the linear quadratic Gaussian (LQG) regulator design is presented, as is the extension of the controller design to degraded

conditions and set-point tracking. The robustness of the LQG design to parameter uncertainties is also discussed. Note that the EBR-II plant is highly stable at steady state normal operations, and it does not exhibit complex dynamic behavior from the control point of view. Thus, the comparisons made using EBR-II models may not be true for systems with more complicated dynamic behavior.

The optimal control theory solution minimizes the error between the actual state and the target value, but does not ensure zero error. Modification to the controller design by augmenting an integrator dynamics is sometimes necessary to assure that the steady state error goes to zero. This modification is needed because the dynamic systems under consideration are not always of type-1 or higher.

The various capabilities of the computer-aided control design system, MATRIX_x, have been explored and applied to the regulation and set-point control of the EBR-II steam generator drum level.

2. MODERN CONTROL FOR LINEAR SYSTEMS

2.1 LINEAR QUADRATIC GAUSSIAN COMPENSATOR

The LQG compensator design problem includes designing an optimal regulator in conjunction with an optimal observer design. The regulator problem is concerned with the design of a control policy $U(t)$ to take a system from a nonzero state to zero state in which the zero state is the steady state of the perturbed linear system. Consider the linear system described by a set of matrix differential equations.

$$\dot{\underline{x}}(t) = A \underline{x}(t) + B \underline{u} \quad ; \quad (2.1)$$

$$\underline{y}(t) = C \underline{x}(t) \quad . \quad (2.2)$$

Define the regulator performance index,

$$J[\underline{x}(t), \underline{u}, t] = \int_{t_0}^T [\underline{u}^T R \underline{u} + \underline{x}^T Q \underline{x}] dt \quad , \quad (2.3)$$

where

R = input weighting matrix,

Q = state weighting matrix.

The problem is to find an optimal control $u^*(t)$, $t_0 < t < T$, such that J is minimized. The solution to Eq. (2.3) with a linear control law is given by

$$\underline{u}^*(t) = -K_R \underline{x}(t) = -R^{-1} B^T P \underline{x}(t) \quad , \quad (2.4)$$

where K_R is the regulator gain vector which includes the solution P to the matrix Riccati equation:⁴

$$-\dot{P}(t) = P(t) A + A^T P(t) - P(t) B R^{-1} B^T P(t) + Q \quad . \quad (2.5)$$

Equation (2.3) represents a quadratic criterion with matrices Q and R weighting appropriate state and input variables. The diagonal elements of matrix Q reflect the relative importance of the state variable whose steady state error is to be minimized. It is important to note that the minimization of J does not necessarily guarantee that the error will be exactly zero, but it is always possible to attain a desired error.

The Riccati [Eq. (2.5)] is a condition obtained as a result of the minimization of J and the representation of the optimal input as a linear function of the state variables (or their estimates). Very often the steady state solution of $P(t)$ is obtained from

$$P A + A^T P - P B R^{-1} B^T P + Q = 0 \quad . \quad (2.6)$$

The closed-loop system is given by

$$\dot{\underline{X}} = (A - B K_R) \underline{X} \quad , \quad (2.7)$$

which is asymptotically stable. The minimum cost function is given by

$$J^*(\underline{X}(t_0), t_0) = \underline{X}^T(t_0) P \underline{X}(t_0) \quad . \quad (2.8)$$

2.2 STATE ESTIMATION USING THE KALMAN FILTER

The compound structure of an output feedback control system can be formed by including the state estimation features in the regulator problem. The control law can be written as

$$\underline{u} = -K_R \hat{\underline{X}}(t) \quad , \quad (2.9)$$

where $\hat{\underline{X}}(t)$ is the estimate of $\underline{X}(t)$. The solution to $\hat{\underline{X}}(t)$ is obtained by solving the equation,

$$\dot{\hat{\underline{X}}}(t) = A \hat{\underline{X}}(t) + B \underline{u}(t) + K_E [Y(t) - C \hat{\underline{X}}(t)] \quad (2.10)$$

$$= A \hat{\underline{X}}(t) - B K_R \hat{\underline{X}}(t) + K_E [C \underline{X}(t) - C \hat{\underline{X}}(t)] \quad ,$$

$$\dot{\hat{\underline{X}}}(t) = [A - B K_R - K_E C] \hat{\underline{X}}(t) + K_E \underline{Y} \quad (2.11)$$

$$\underline{u}^*(t) = -K_R \hat{\underline{X}}(t) \quad , \quad (2.12)$$

where K_E is the estimator gain matrix. The design of K_E includes an optimization with respect to process noise and observation noise, and it is known as the Kalman filter design. The resulting $2n \times 2n$ system is formed in the following manner.

$$\begin{bmatrix} \dot{\underline{X}} \\ \dot{\hat{\underline{X}}} \end{bmatrix} = \left[\begin{array}{c|c} A & -B K_R \\ \hline K_E C & A - K_E C - B K_R \end{array} \right] \begin{bmatrix} \underline{X} \\ \hat{\underline{X}} \end{bmatrix} \quad . \quad (2.13)$$

System Eq. (2.13) represents the plant and appended output feedback compensator as shown in Fig. 2.1.

2.3 SET-POINT TRACKING LQG DESIGN

One of the main objectives of the control design studies is to design an optimal controller for which the closed-loop behavior can be directed from one set point to another. Such a controller can be used to change

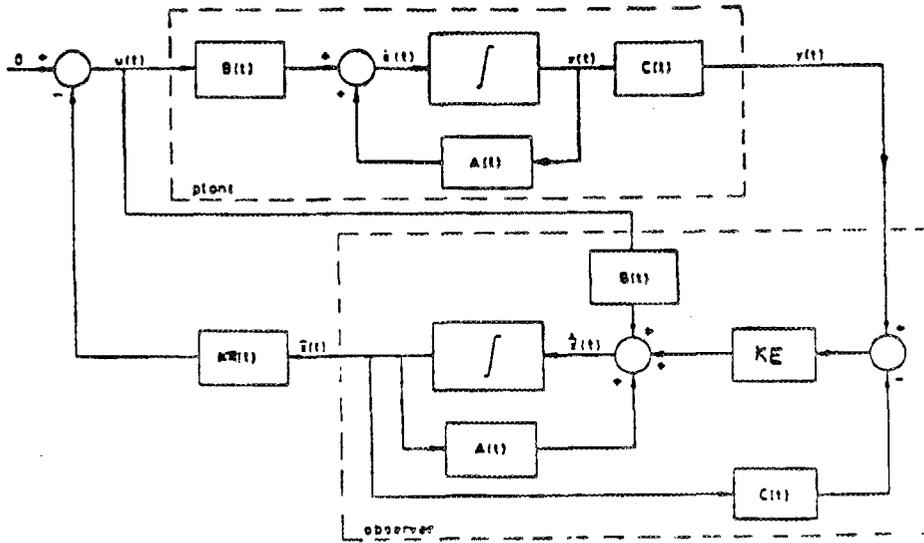


Fig. 2.1. Output feedback compensator.

the steady state value of the reactor power. The controller's task is to adjust the system behavior in an optimized manner so that a chosen output reaches the desired steady state with a negligible error.

A tracking optimal controller can be designed using the following formulation, which is based on the metastate representation of the system.¹ Consider a dynamic system,

$$\dot{\underline{X}} = A \underline{X} + B \underline{u} \quad . \quad (2.14)$$

Assume that the reference state is known:

$$\dot{\underline{X}}_r = A_r \underline{X}_r \quad . \quad (2.15)$$

Define an error vector,

$$\underline{e} = \underline{X} - \underline{X}_r \quad , \quad (2.16)$$

$$\dot{\underline{e}} = A \underline{e} + E \underline{X}_r + B \underline{u} \quad , \quad (2.17)$$

where

$$E = [A \quad -A_r] \quad .$$

The metastate representation is given by

$$\dot{\underline{X}} = A \underline{X} + B \underline{u} \quad , \quad (2.18)$$

$$y = C \underline{X} \quad , \quad (2.19)$$

where

$$\underline{X} = \begin{bmatrix} e \\ X_r \end{bmatrix}, \quad A = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad C = [C \ C_r] .$$

The formulation of the linear, quadratic optimum control problem using the metastate representation can be carried out as follows. An appropriate performance integral for the problem is

$$V = \int_t^T (e^T Q e + u^T R u) \, d\tau . \quad (2.20)$$

For the metastate \underline{X} , the state weighting matrix is

$$Q = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix} . \quad (2.21)$$

The following equation can be derived for zero steady state error:

$$B u^* + E X_r = 0 . \quad (2.22)$$

The total control action can be written as a combination of two components, namely, the steady state control and the corrective control:

$$U = u^* + v . \quad (2.23)$$

The performance integral for the corrective control can be written as

$$\bar{v} = \int_t^\infty (e^T Q e + v^T R v) \, d\tau . \quad (2.24)$$

Let M be the equivalent Ricatti matrix for the metastate system:

$$M = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} . \quad (2.25)$$

The gain matrix G for this system is given by

$$G = R^{-1} [B^T \ 0] \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = [R^{-1} B^T M_1 \mid R^{-1} B^T M_2] . \quad (2.26)$$

The corresponding equations for M_1 and M_2 are obtained from the following general equation:

$$-\dot{M} = M A + A^T M - M B R^{-1} B^T M + Q . \quad (2.27)$$

They are:

$$-\dot{M}_1 = M_1 A + A^T M_1 - M_1 B R^{-1} B^T M_1 + Q ; \quad (2.28)$$

$$-\dot{M}_2 = M_1 E + [A^T - M_1 B R^{-1} B^T] M_2 . \quad (2.29)$$

3. PRELIMINARY PERFORMANCE EVALUATION OF MULTIVARIATE CONTROL METHODS

The following aspects of the multivariate control are studied using the LQG design: (1) the regulation capability for low-level perturbations, (2) robustness to modeling errors, (3) state estimation in case of degradation, and (4) set-point tracking capability. Section 3 describes design considerations for using the tools of MATRIX_x and for implementing performance tests in the form of computer simulations using mathematical models and discusses results obtained by classical and modern methods. The conclusions of the study are based on preliminary studies using physical models. Actual system-related information will be added in the future.

3.1 LINEAR EBR-II MODEL AS A TEST BED

Experimental Breeder Reactor-II is a 62.5-MWth liquid metal fast breeder reactor located at Idaho Falls, Idaho. Initially constructed as a prototype central station power plant with a breeding ratio greater than unity, its purpose was shifted to provide irradiation services for the development of fuels and structural materials. As an engineering test facility, EBR-II recently played a major role in the demonstration of inherently safe characteristics of LMRs.

EBR-II is an unmoderated, heterogeneous, sodium cooled, pool-type reactor with an intermediate sodium coolant loop and a steam plant that produces 20 MW of electrical power. Control in the plant is provided mainly by operator action in the console room, which is supported by the data acquisition system and a few automatic control schemes such as the computerized power regulator and four-element and single-element feedwater controllers. Briefly, other control mechanisms include the feedwater temperature control, steam pressure control, level controls in feedwater heaters, pump speed control, and turbine generator control. Existing controllers of the EBR-II are classical designs used in almost every commercial nuclear power plant.

A previously developed linear EBR-II model⁵ includes primary-side and steam generator subsystems. This model uses 57 state variables and includes a PI controller to stabilize the drum level. The PI controller in the model uses three signals instead of four, as in the actual plant controller. The excluded signal, which is the blowdown flow, has a negligible effect on steady state dynamics. Since the linear models are valid for low-level perturbations around the steady state, the PI controller gains are deliberately chosen big so that it performs better than the actual controller; therefore, comparisons using new techniques are made against a PI controller that performs at least better than the actual plant. Several low-level perturbations are successfully represented by linearization about steady state dynamics. This model is used for LQG applications, as presented in the following sections. The new control philosophy is applied to reactor power regulation and drum

level stabilization tasks. Note that this application is valid only for small variations (20% maximum) around the steady state.

3.2 GENERAL GUIDELINE IN DESIGNING LQG

The LQG design includes determining the appropriate weights for the states and for the control input in Eq. (2.3). The relationship between the Q and R matrices and the dynamic behavior of the closed-loop system dependent on the matrices A , B , and C are quite complicated. "It is impractical to predict the effect on closed-loop behavior of a given pair of weighting matrices."¹ The approach used in this study is to solve for the gain matrices that result from a range of weighting matrices and simulate the corresponding closed-loop responses. The gain matrix K that produces the response closest to design objectives is considered the best choice. Despite the fact that the design procedure is done in a trial-and-error fashion, the following background knowledge has been used as a guide.¹

The weighting matrix Q specifies the importance of the various components of the state vector relative to each other. If a state variable changes unreasonably then that state has to be suppressed by increasing the corresponding weighting. However, the choice of the input weighting matrix R can not be established easily. The main idea is the following. With a desire to avoid saturation and its consequences, the control signal weighting matrix is selected large enough to avoid saturation of the control signal under normal operational conditions. As the control weights tend to infinity, the gains are such that the closed-loop poles tend to open-loop poles when the latter are stable, or to their "mirror images" when they are unstable. In a single-input system the zeros of the system are defined by $\underline{c}^T(sI-A)^{-1}\underline{b}=0$. If there are r such zeros, then as the weights go to infinity r of the closed-loop poles approach these zeros and the remaining $k-r$ closed-loop poles radiate outward to infinity in a configuration of a Butterworth polynomial of order $k-r$.

3.3 LQR DESIGN FOR EBR-II REACTIVITY CONTROL

The first modern controller design to be considered is for the EBR-II primary system model. The design objective is to regulate the system behavior in an allowable range, that is, not to exceed the safety limit of 2°F/min in reactor ΔT .⁶ The design uses a single control rod as the actuator. To avoid saturation of the actuator, the control input is designed not to exceed 0.01 \$/s reactivity limit (for a single-speed gear train). It is assumed that all measurements are available without any noise. The design of an optimal output feedback compensator reduces to an optimal regulator design since there is no need for an observer under the perfect measurement assumption. The state weights, a , in the quadratic cost function are chosen to be equal to one for the first state (reactor power), and the remaining states are excluded (weighted zero). The input weighting, R , is scalar and equal to 0.0007. The

compensator design is performed using the design routines in MATRIX_x. Closed-loop system responses to a -5-cent reactivity insertion show a drastic improvement over open-loop system behavior. The fractional power response to a -5-cent reactivity step perturbation is shown in Fig. 3.1, which also includes the open-loop step response to the same input. The temperature step response of the fuel material to the same input is shown in Fig. 3.2. Figure 3.3 shows the reactor inlet and outlet temperature responses of the open-loop primary system model. A similar comparison for the closed-loop system model is shown in Fig. 3.4. The temperature difference between reactor outlet and inlet is changing faster than 2°F/min for the open-loop system shown in Fig. 3.3. The closed-loop system responses shown in Fig. 3.4 indicate that the variation in reactor ΔT does not exceed the allowable range, which is in complete agreement with the EBR-II design specification. It can also be seen from these figures that closed-loop responses are quite fast and steady state errors are very small compared to the open-loop case. Figure 3.5 shows the control input generated by the optimal control strategy. The safety limitation of 0.01 \$/s is not violated, and the control performance is achieved using approximately 50% of the maximum rod speed. The nonzero steady state errors in the responses are caused by the type-0 structure of the closed-loop system.⁷ One approach to drive the error to zero is to convert the type-0 system to a type-1 system by the addition of an extra state variable that represents the integration of the error corresponding to the variable of interest, in this case, integration of reactor power variation.

3.4 LQR DESIGN FOR EBR-II DRUM LEVEL CONTROL

The optimal regulator design for the EBR-II steam generator subsystem is performed with a philosophy similar to that for the primary system. The design objective is to maintain the drum level deviation in an allowable range, that is, 4 in. above or below the steady state level.⁶ The closed-loop model includes an additional state variable (20th state variable) in the form of an integrator. Since the optimal control theory does not guarantee zero steady state errors, the addition of an external integrator is one of the possible solutions to the nonzero steady state error problem. The 18th and the 20th states of the model are weighted to be equal to 10, while the rest of the states are excluded (weights are zero). The input weighting is scalar and equal to 10^{-5} , which is chosen after several trials to achieve a performance level close to the three-element controller performance. For this specific application, it is assumed that all measurements are available without any noise. The detailed structure of the MATRIX_x programming is listed in Appendix A. The closed-loop responses of the steam generator model with a three-element controller and optimal regulator but without any control action are plotted in the same figures. The drum level response to a +10% steam valve opening step perturbation is shown in Fig. 3.6. As can be seen, the level is controlled efficiently by both control strategies. Figure 3.7 compares the control signals (feedwater flow) generated by the three-element controller and optimal regulator. Figure 3.8 shows the drum pressure response to the same perturbation.

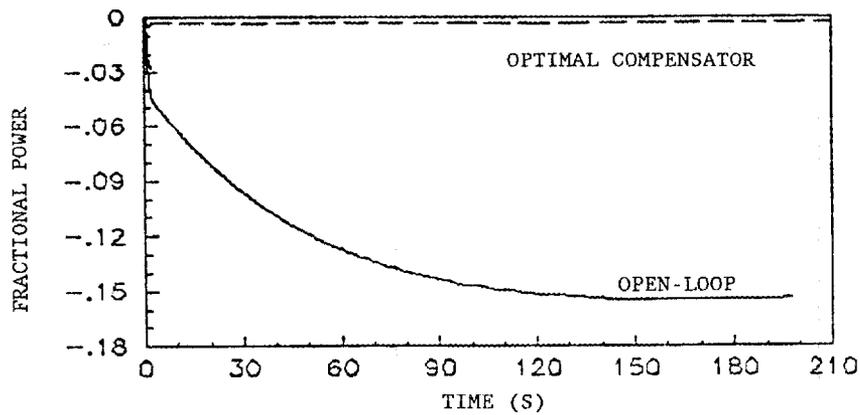


Fig. 3.1. Step response of fractional reactor power to a -5-cent reactivity perturbation (optimal compensator and open-loop models).

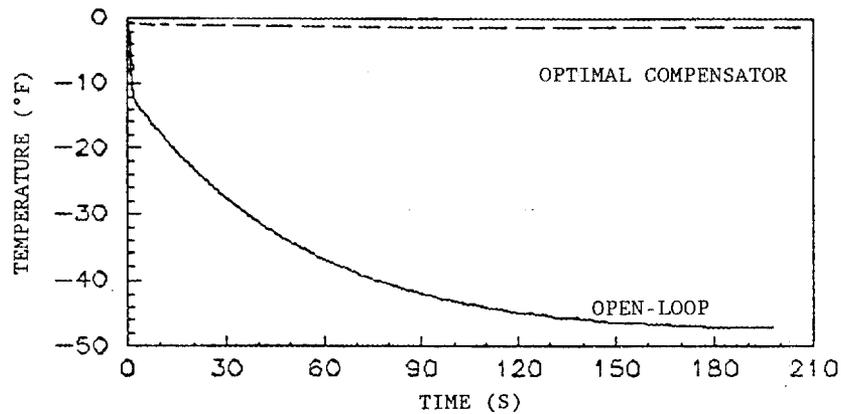


Fig. 3.2. Step response of fuel temperature to a -5-cent reactivity perturbation (optimal compensator and open-loop models).

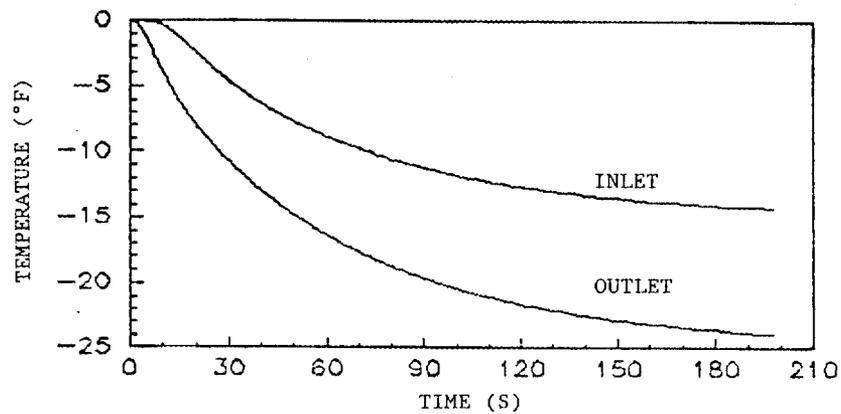


Fig. 3.3. Step response of reactor inlet and outlet temperatures to a -5-cent reactivity insertion (open-loop model).

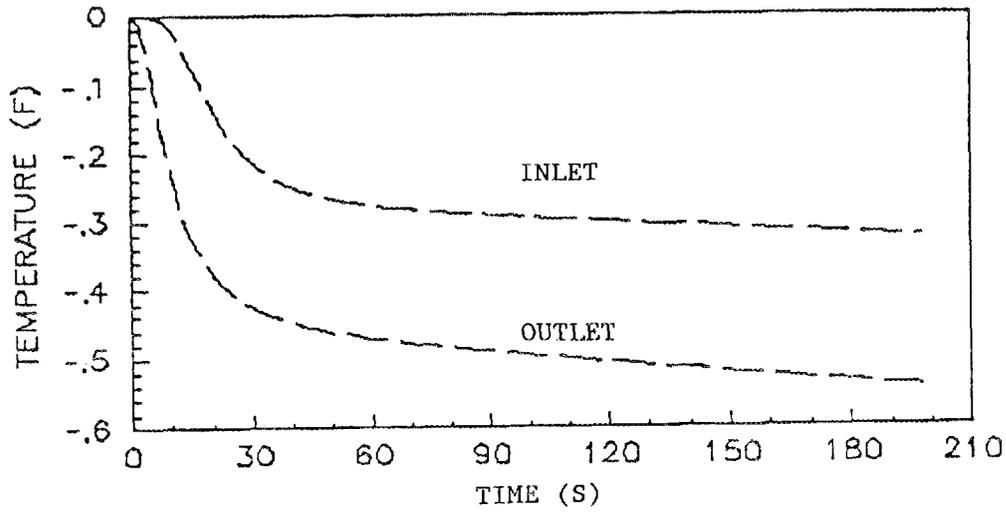


Fig. 3.4. Step response of reactor inlet and outlet temperatures to a -5-cent reactivity insertion (closed-loop model, optimal compensator).

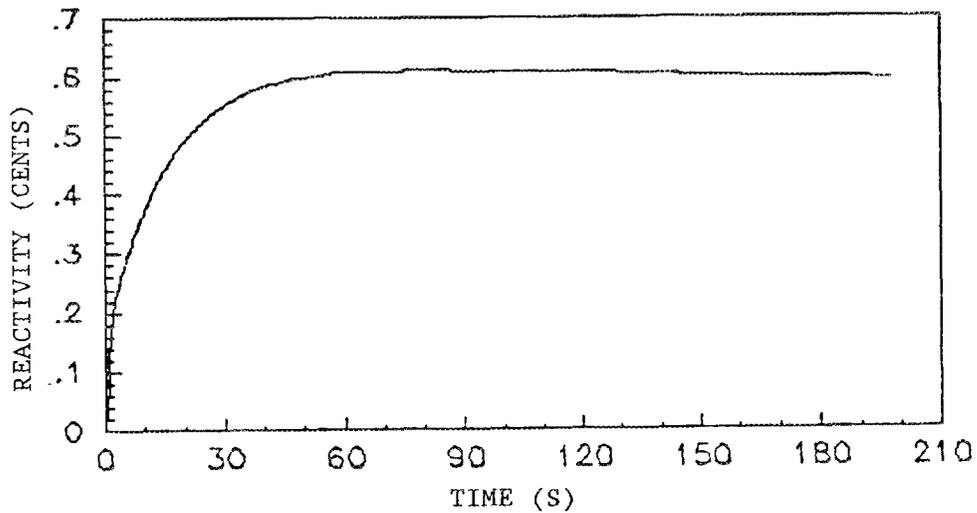


Fig. 3.5. Control input (cents) generated by the optimal compensator for a -5-cent reactivity perturbation.

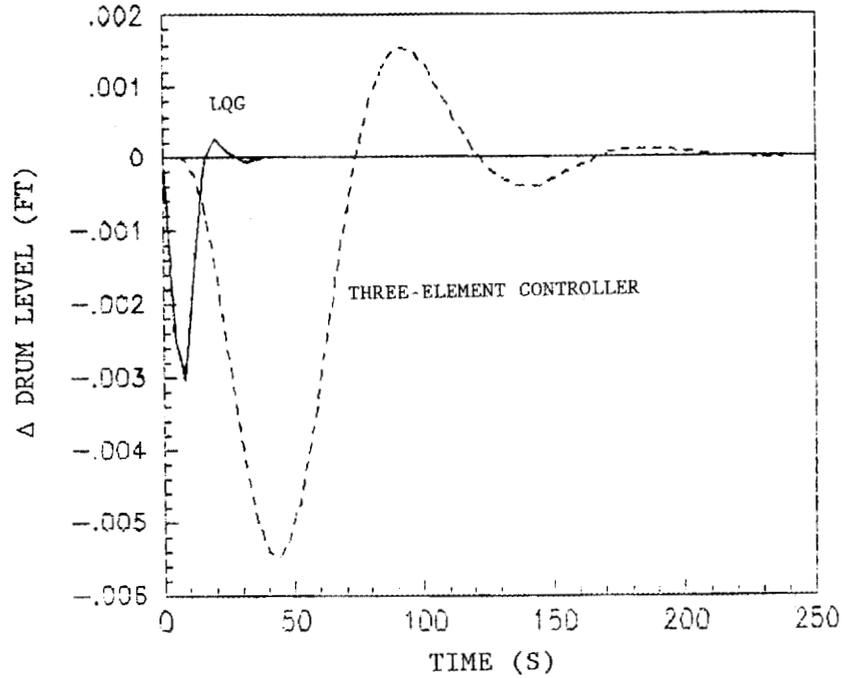


Fig. 3.6. Step response of drum level to a +10% steam valve opening perturbation (optimal compensator and three-element controller).

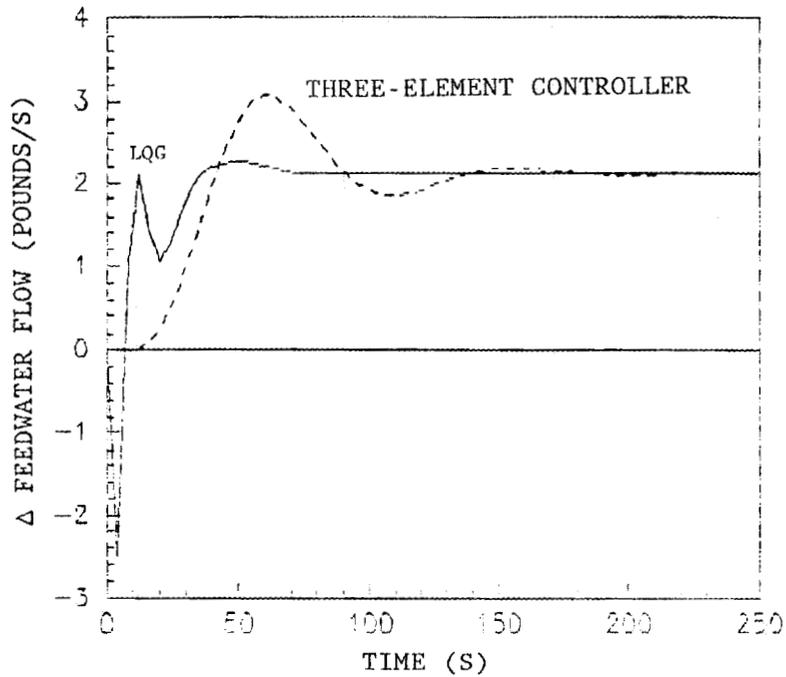


Fig. 3.7. Control signals generated by three-element controller and optimal compensator for a +10% change in steam valve position.

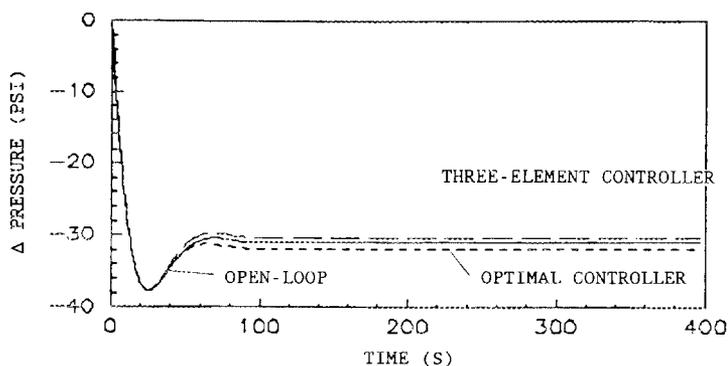


Fig. 3.8. Step response of the drum pressure to a +10% steam valve opening perturbation (optimal compensator, three-element controller, and open-loop models).

The drum level step response to a +10°F inlet sodium temperature perturbation is shown in Fig. 3.9. The control signals are compared in Fig. 3.10. Figures 3.11 and 3.12 show the drum level step response and the control signals for a +10°F feedwater temperature perturbation. The results show that the controller performances using classical and modern control design methods differ considerably. Compared to the PI controller that uses 3 plant signals, the LQR uses 18 measurements and exhibits better performance. The improvement is caused strictly by the fast response characteristic of the LQR, which is due to some of the 18 signals used (obviously ones that the 3-element controller does not use). See Sect. 3.7 for further discussions. A set of closed-loop responses of the EBR-II subsystem models is shown in Appendix B.

3.5 IMPROVED WEIGHTS FOR BETTER PERFORMANCE

As previously stated, an optimal regulator design includes assessing an appropriate combination of weightings in the corresponding quadratic cost function. A simple approach to the problem of finding the best combination requires some knowledge about open-loop system dynamics. The problem becomes complicated for partially uncontrollable or unobservable systems. A basic practical method of choosing the state weightings is to examine the open-loop system behavior and determine the "penalties" for those states in which deviations from steady state exceed some predefined limits. A criterion can be defined using the standard performance features such as overshoot, steady state error, bandwidth, or gain/phase margins. Once the starting combination is predicted, then a finer tuning can be accomplished through several closed-loop simulations in trial-and-error fashion. If the open-loop system has unstable poles, then the unstable states (states causing or contributing to overall instability) have greater importance compared to others. In such cases, a change in the weights of the stable states may not affect the performance significantly because the selected weights of the unstable states are necessarily much higher than those of the stable states. Improving the optimal regulator design also includes selecting

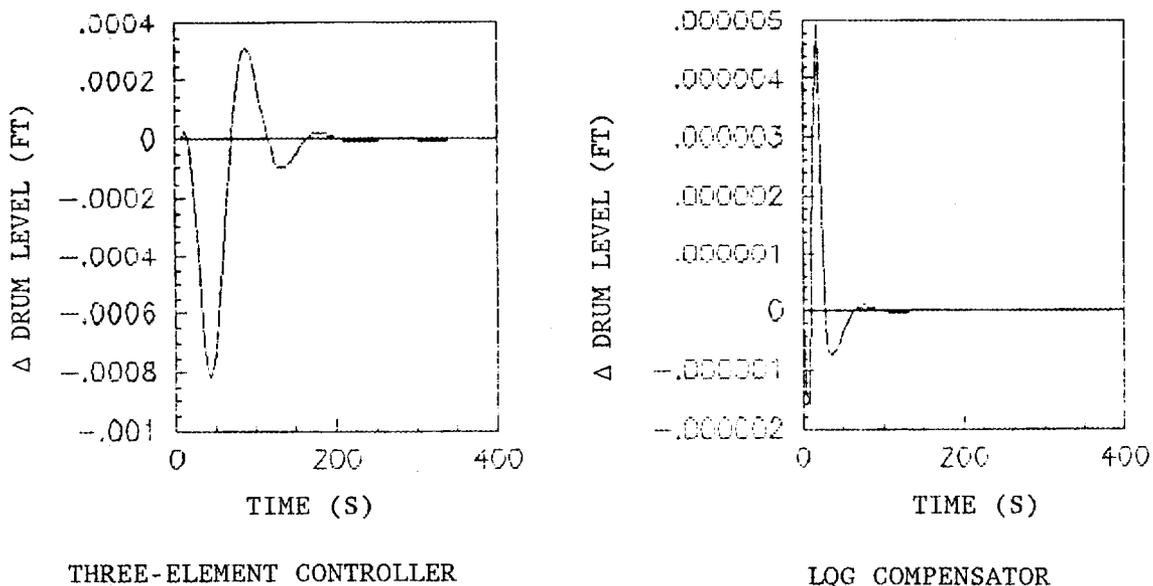


Fig. 3.9. Step response of drum level to a $+10^{\circ}\text{F}$ inlet sodium temperature perturbation (optimal compensator and three-element controller).

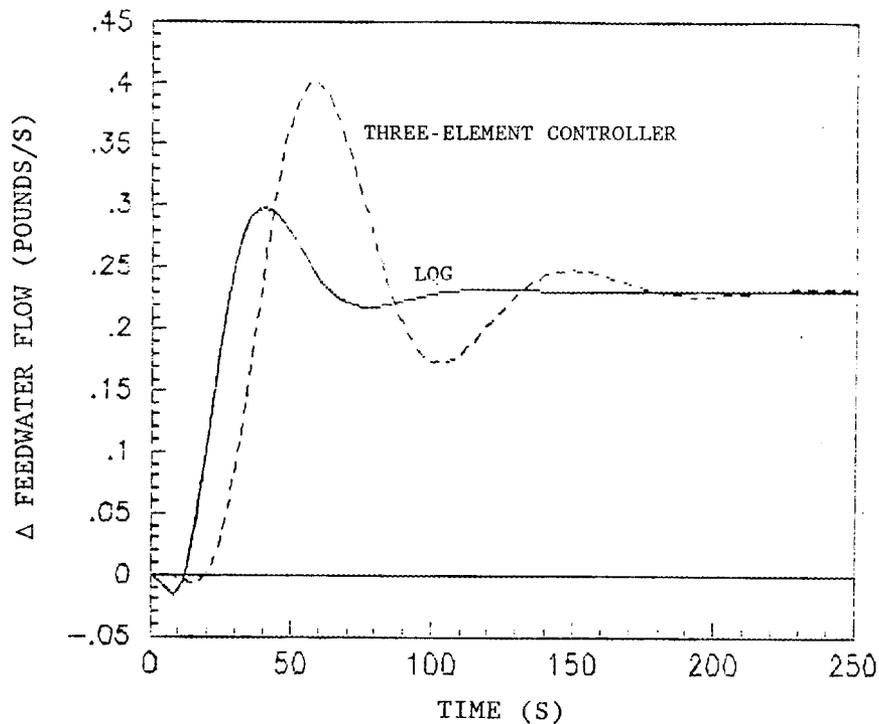


Fig. 3.10. Control signals generated by three-element controller and optimal compensator for a $+10^{\circ}\text{F}$ inlet sodium temperature perturbation.

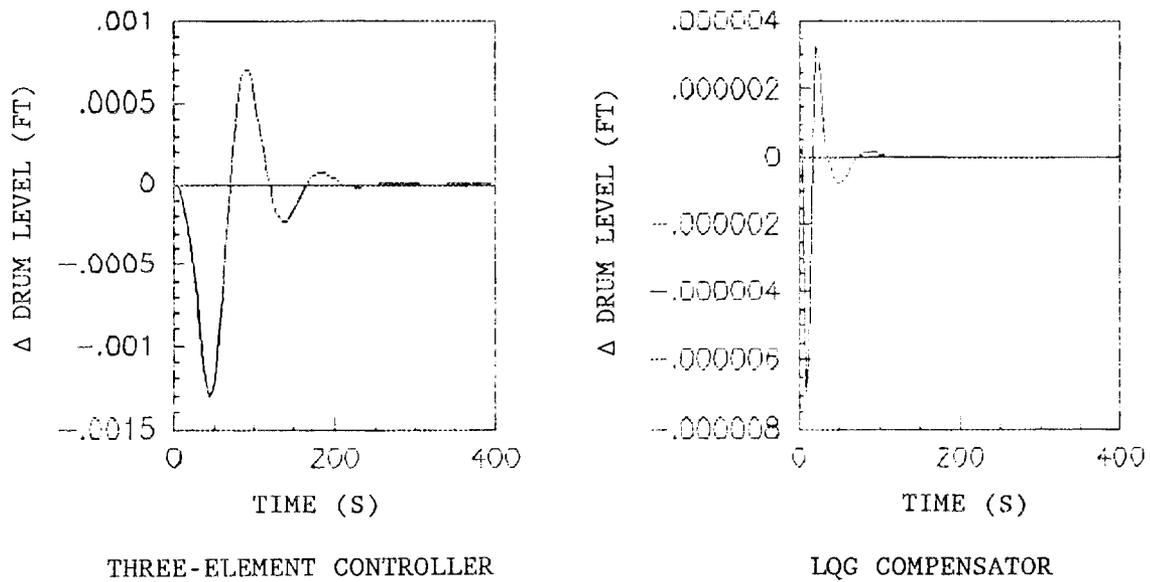


Fig. 3.11. Step response of drum level to a $+10^{\circ}\text{F}$ feedwater temperature perturbation (optimal compensator and three-element controller).

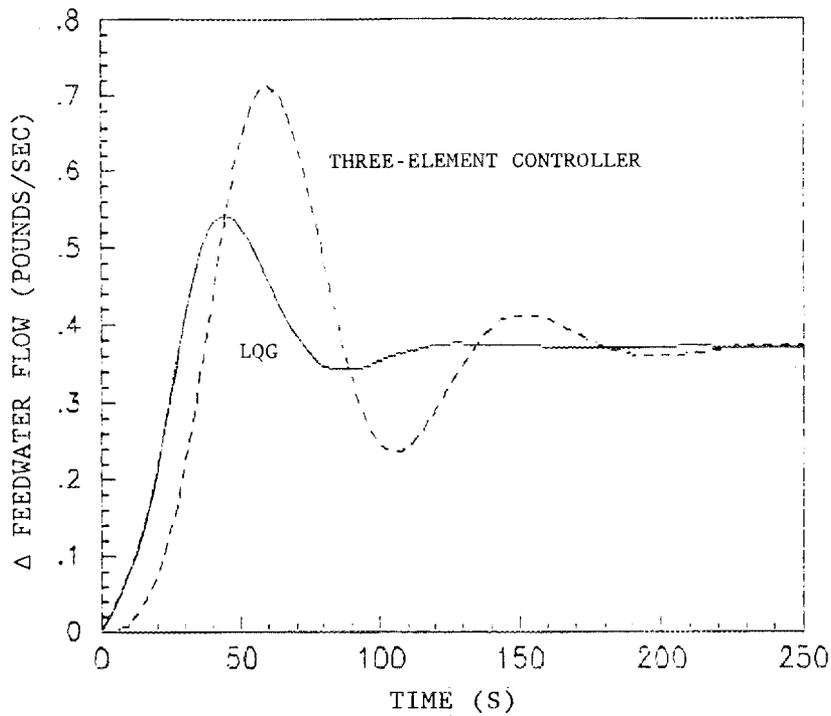


Fig. 3.12. Control signals generated by three-element controller and optimal compensator for a $+10^{\circ}\text{F}$ feedwater temperature perturbation.

the control input weights. Since we are interested in only single-input systems, the control input weighting selection becomes less complicated. However, an appropriate choice cannot be made easily unless the actuator dynamics are completely known. It is very important to design a control input such that the process actuators are not saturated.

In light of the above discussion, the optimal controllers previously developed for the EBR-II subsystem models are reconsidered. The primary system model has eigenvalues widely spaced (stiff system). The reactor power changes very rapidly for reactivity perturbations compared with the changes in the other state variables. This change implies that the state weight for the reactor power must be dominant. However, several optimal regulator design applications indicate that different combinations of state weights do not improve the system behavior as much as the simple design (using single weight for the reactor power). An improvement in the optimal regulator design without violating the safety limitations is made by adjusting the control-input weight. Figure 3.13 shows the step responses of fractional reactor power to a -5 cent reactivity insertion using three different optimal regulators. The designs differ only in the input weights (0.007, 0.002, 0.0007). The single state weight (reactor power) is equal to one. As shown in Fig. 3.13, the reactor power is regulated most powerfully when the input weight is 0.0007, which generates the fastest control input among the others. Figure 3.14 shows the control inputs corresponding to the three cases of input weights. Note that when the reactor power is controlled using the fastest control input, the 1-cent/s safety limitation is not violated and the rod speed is around 50% of its maximum value.

The EBR-II open-loop steam generator model includes the level dynamics. The previously developed optimal regulator model for controlling the EBR-II steam generator uses a weight only on the 18th state, which is the drum level. The design uses feedwater valve position as the control input with a scalar weight of 10^{-5} . A set of different combinations of state weights is tested to improve the optimal regulator performance. The primary task of stabilizing the drum level requires a relatively large weight on the drum level when all the state weights are used. The list of state weights given in Table 3.1 is found to be the best combination after several trials. The overshoot ratios of each state are used for the distribution of weights. Despite the fact that 17 of the state weights are close to each other and exhibit a big contrast ($\times 10^6$) compared to the level weighting, the controller performance is affected by a small change in one of the weights of the 17 states. The step responses of the drum level to a +10% steam bypass valve opening perturbation using two different optimal controllers, (1) single weight on drum level and (2) the combination of state weights of Table 3.1, are shown in Fig. 3.15. The improvement in the drum level response can be seen in Fig. 3.15. A similar comparison made for the drum pressure response to the same perturbation is shown in Fig. 3.16.

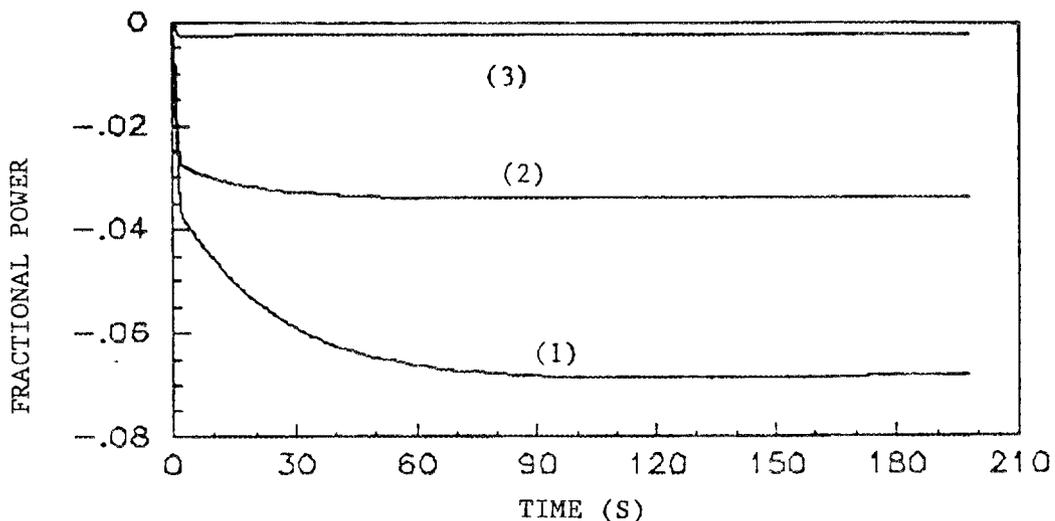


Fig. 3.13. Step response of fractional reactor power to a -5-cent reactivity perturbation [closed-loop primary system with an optimal compensator using (1) 5%, (2) 10%, and (3) 50% of the maximum control rod speed].

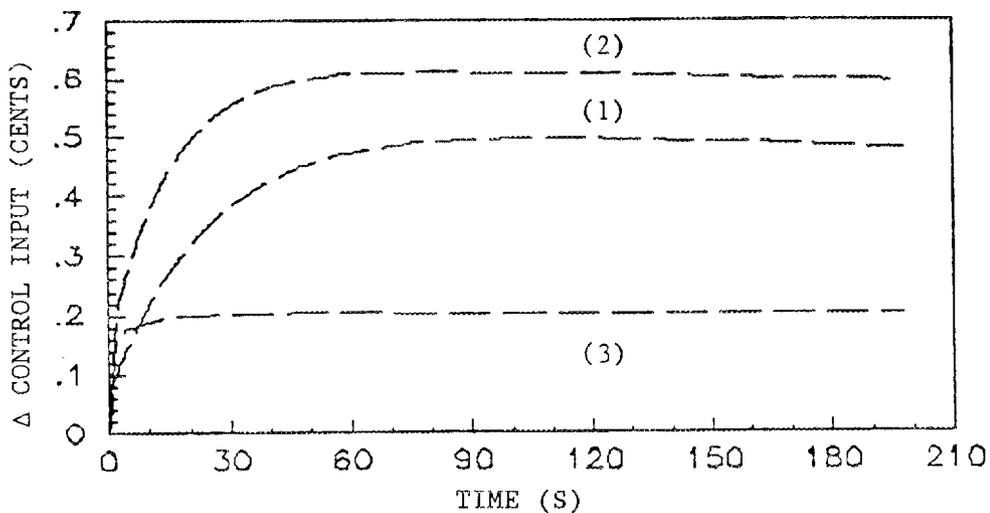


Fig. 3.14. Control signals generated by the optimal compensator appended to the primary model. Step perturbation is a -5-cent reactivity insertion. (1) 5%, (2) 10%, and (3) 50% of the maximum rod speed.

Table 3.1. State weights of the quadratic cost function; LQG design for the EBR-II drum level control

Q(I,J)	Weight	Q(I,J)	Weight
1 1	1.3	10 10	3.0
2 2	2.0	11 11	1.0
3 3	1.0	12 12	1.0
4 4	1.4	13 13	1.0
5 5	1.3	14 14	1.0
6 6	1.4	15 15	1.0
7 7	1.3	16 16	1.0
8 8	1.0	17 17	1.0
9 9	1.0	18 18	1.0D + 06

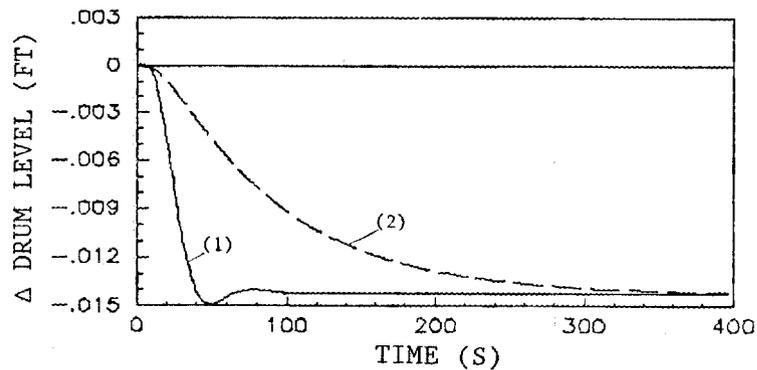


Fig. 3.15. Step response of drum level to a +10% steam valve opening perturbation [optimal compensator: (1) simple design and (2) improved design].

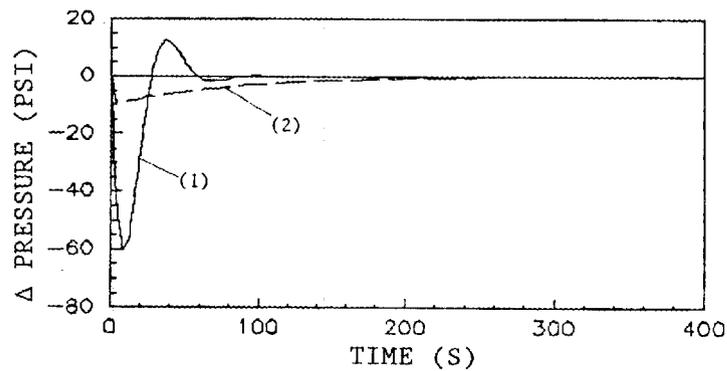


Fig. 3.16. Step response of drum pressure to a +10% steam valve opening perturbation [optimal compensator: (1) simple design and (2) improved design].

3.6 ROBUSTNESS TO MODELING ERRORS

One of the most important aspects of a feedback control system is its ability to maintain stability in the presence of any plant uncertainty.⁸ "This uncertainty may arise from variations in the actual plant itself due to changes in operating conditions, or may arise from the differences between the actual plant and a control design model used to represent it at any given operating point." Mathematical methods of assessing stability robustness include matrix norms, measures, and transformation techniques using only the design model.

A simple robustness test is considered for the three-element controller and optimal regulator previously designed for the EBR-II steam generator subsystem. The test includes simulation of the closed-loop system for a predefined plant uncertainty. Initially the three-element controller and the optimal regulator designs are performed using the existing EBR-II steam generator model. The controllers are then appended to a new open-loop model that includes a 5% deviation in the drum geometry calculations. We assume that the latter represents the actual plant. The step responses of both closed-loop system models to a +10% steam valve opening perturbation are shown in Figs. 3.17 and 3.18. Figure 3.17 shows the drum level step responses of the three-element controller model appended (1) to the original open-loop model and (2) to the model with uncertainty. Figure 3.18 shows the drum level step responses of the optimal regulator appended as (1) and (2) above. The closed-loop results indicate that the three-element controller is more sensitive to a plant uncertainty than the optimal regulator. Note that an increase in overshoot is encountered in the three-element controller case when an uncertainty is present. The drum level behavior is less affected in the optimal regulator case. The three-element controller is a local controller, and its performance is affected more by a local parameter uncertainty than the full-state feedback optimal regulator which has a global-type control behavior. This is one example of robustness testing. Further study is necessary to establish a complete robustness analysis including all important uncertainties.

3.7 LQG COMPENSATOR DESIGN FOR DEGRADED CONDITIONS

In a real process, all measurements will not be available, and available measurements may contain noise. Under these conditions an optimal output feedback controller design requires the estimation of those states that are missing in the measurement. Constant, optimal state-estimator gain matrices can be calculated using the Kalman filter design routines in MATRIX_x (see Appendix A). A Kalman filter design optimizes the estimator performance with respect to measurement noise and input disturbance. An LQG compensator design can be completed by designing an optimal regulator and appending it to the optimal estimator, arranged as shown in Fig. 2.1.

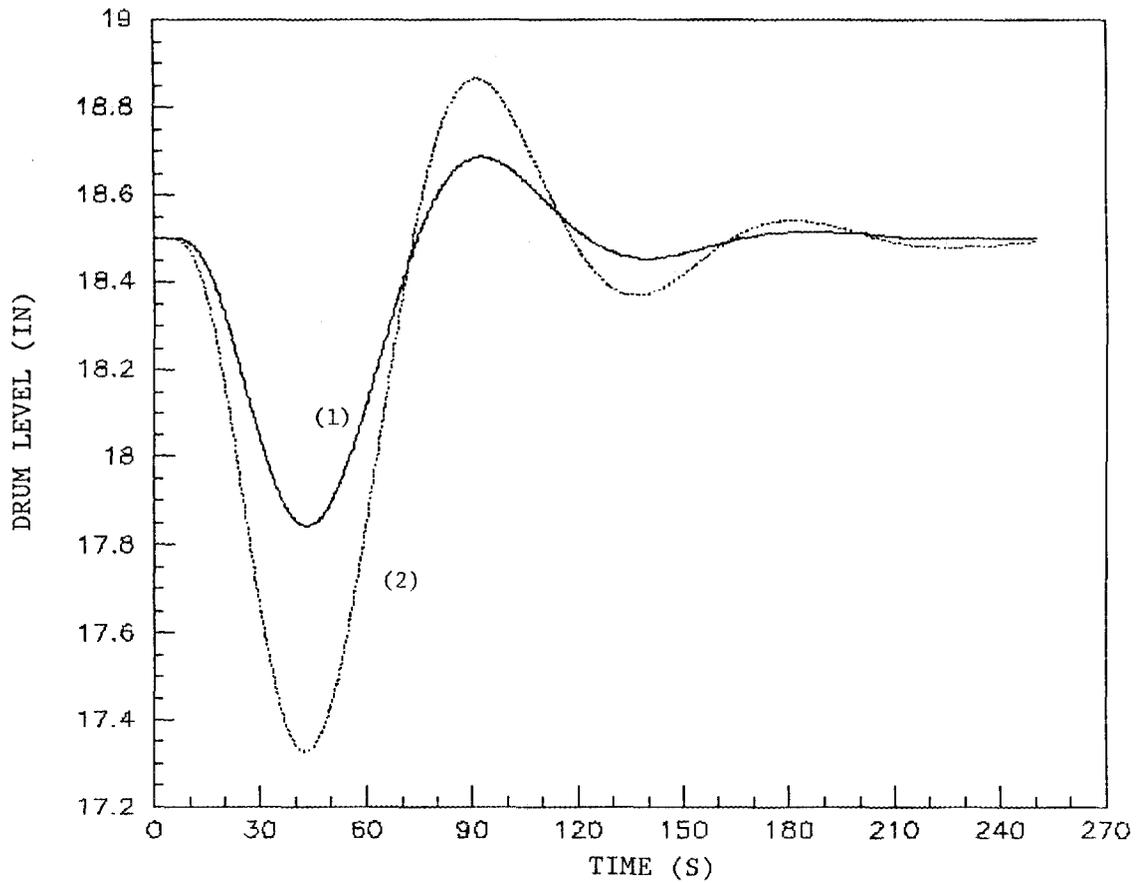


Fig. 3.17. Step response of drum level to a +10% steam valve opening perturbation [three-element controller appended to (1) original model and (2) model with uncertainty].

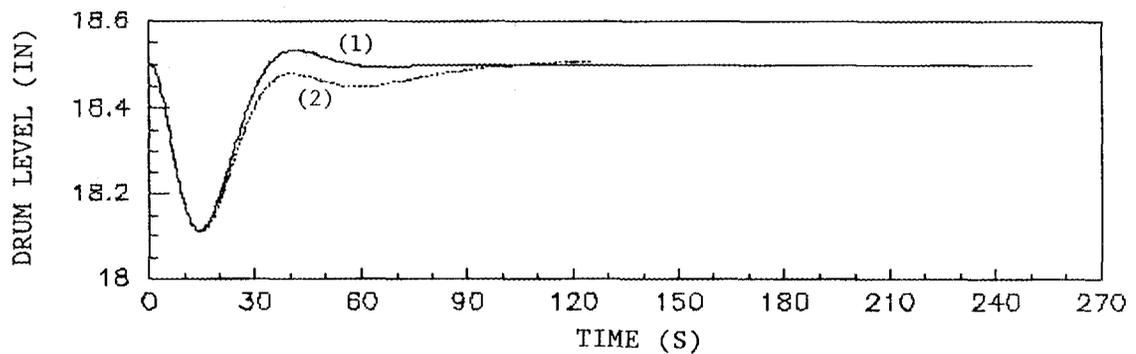


Fig. 3.18. Step response of drum level to a +10% steam valve opening perturbation [optimal compensator appended to (1) original model and (2) model with uncertainty].

The EBR-II steam generator model is used to study the LQG compensator design under degraded conditions. It is assumed that some measurements are not available: the subcooled height, steam quality, superheater and evaporator tube wall temperatures and the evaporator primary sodium temperatures (states 2, 3, 4, 5, 6, 7, 15). It is also assumed that noise intensities on the available measurements are about 10% of the steady state values. A small input disturbance of 1% is assumed to exist for each state. An optimal estimator for the steam generator model is designed with the specifications stated above. The optimal regulator, designed as described in Sect. 3.4, is used to complete the LQG compensator design. The step responses of the closed-loop model to a 10% steam bypass valve opening perturbation show that the controller performance is satisfactory. The drum level step responses of the two closed-loop system models (one designed for degraded condition and the other with perfect measurement assumption) are shown in Fig. 3.19. The drum steam pressure step responses of the two same closed-loop system models are shown in Fig. 3.20.

As Fig. 3.19 indicates, the LQG performance departs from the LQR performance when the plant measurements are not available and the state estimation is required. Note that the response time gets larger using the Kalman filter; however, the LQG still performs better than the PI design in terms of overshoot and response time. The difference in performances between the PI and LQG designs is due to the number of state variables utilized in implementing the control law. In this application, the LQG uses temperature and pressure signals in addition to the three signals that the PI design uses (feedwater flow, steam flow, and drum level). The LQG design also includes estimations of several state variables such as the evaporator outlet steam quality and subcooled height. These additional state variables provide information about the fast components of overall steam generator dynamics. Thus, a three-element controller that does not utilize such information exhibits a phase-lag with respect to the fast components of overall dynamics and results in slower compensation.

The LQG design with different combinations of observation matrix provides very useful knowledge about the importance of plant signals. After several trials, it is found that the steam-drum pressure signal plays a dramatic role in improving the controller performance. Figure 3.21 shows the drum-level response to 10% steam valve opening perturbation using LQR, LQG and PI designs. In this application, the LQG estimates the drum pressure rather than using a direct measurement. The time behavior of the drum level is slower in this case, compared to the LQG design in Fig. 3.19. The feedwater flow (control input) is shown in Fig. 3.22.

3.8 SET-POINT TRACKING LQG APPLICATIONS

The isolated EBR-II steam generator is considered for the reference input tracking control problem. The design procedure using the MATRIX_x routines can be summarized in five steps.

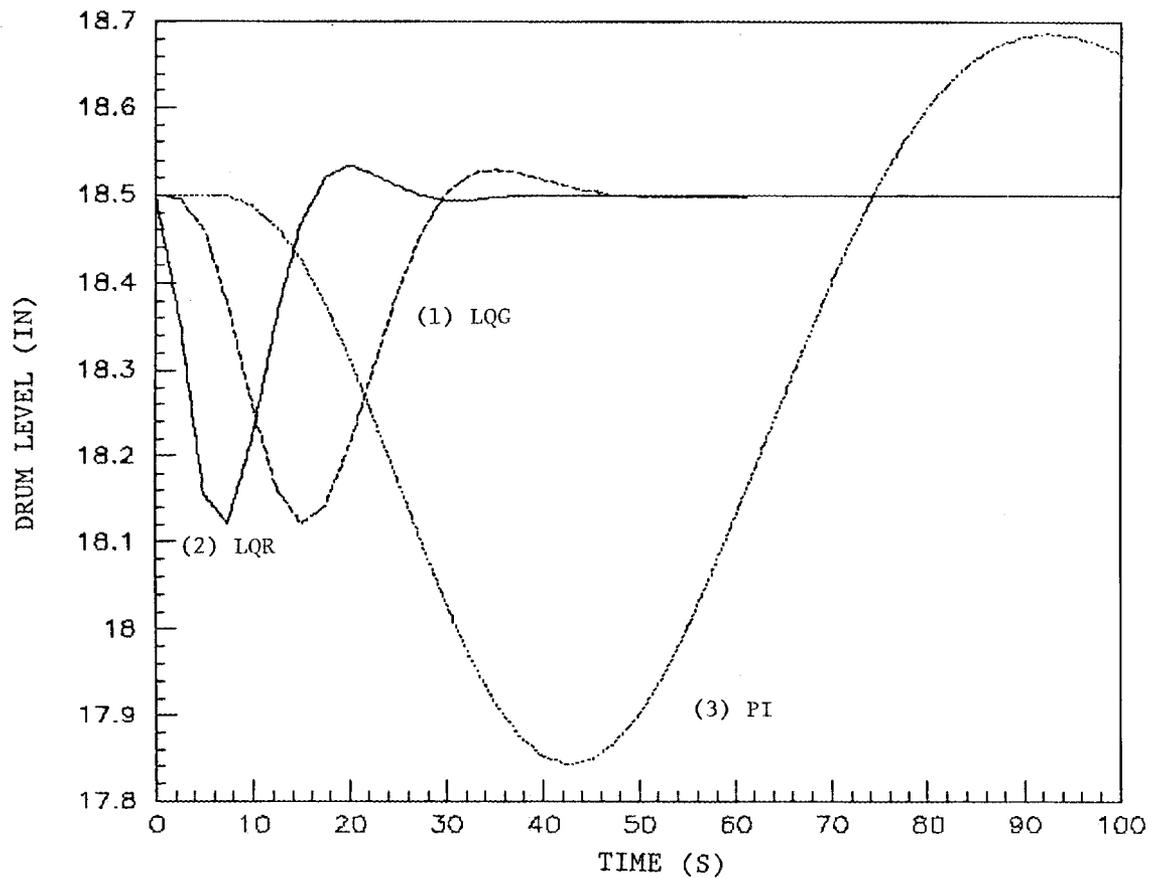


Fig. 3.19. Step response of drum level to a +10% steam valve opening perturbation [(1) LQG, (2) full-state feedback (LQR), and (3) PI controllers].

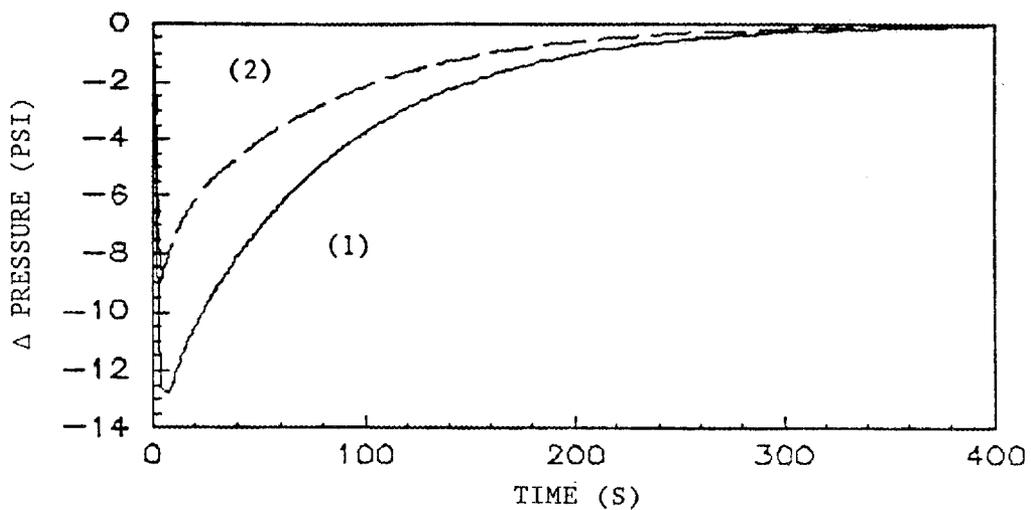


Fig. 3.20. Step response of drum pressure to a +10% steam valve opening perturbation [LQG compensator: (1) degraded conditions and (2) full-state feedback, ideal case].

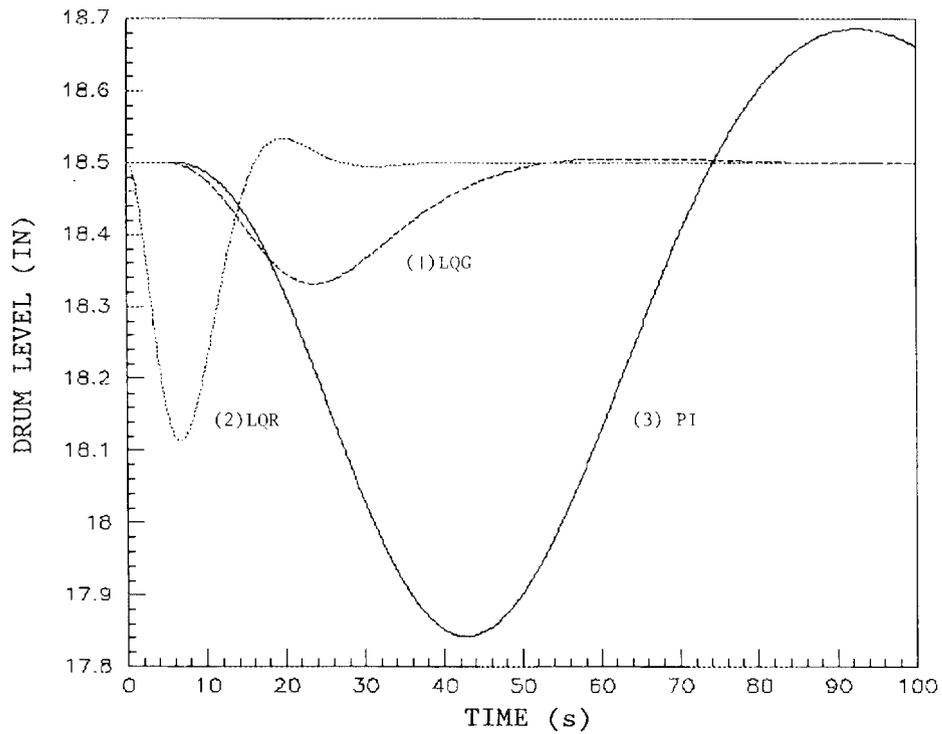


Fig. 3.21. Drum level response to a 10% steam valve opening perturbation [(1) LQG, (2) LQR, and (3) PI controllers].

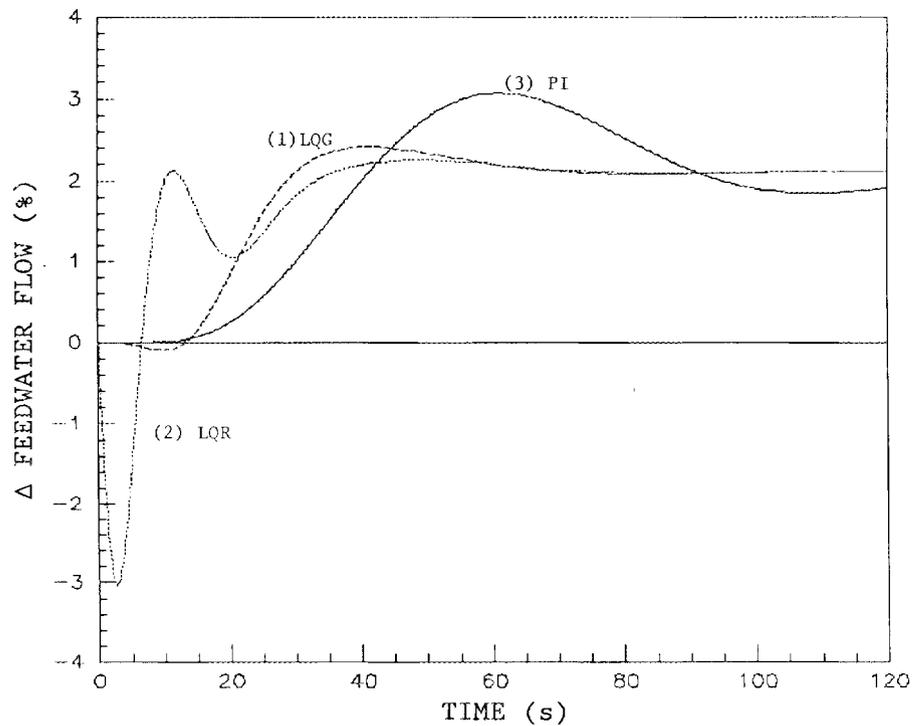


Fig. 3.22. Feedwater flow (control input) to a 10% steam valve opening perturbation [(1) LQG, (2) LQR, and (3) PI controllers].

1. The first term in Eq. (2.26) represents the regulator gains obtained using the standard regulator design method. Using the original model, these steady state control gains can be calculated with the MATRIX_x commands.
2. The closed-loop system matrix is then calculated as follows:

$$A_c = A - B K_r \quad , \quad (3.1)$$

where

K_r = Regulator gain matrix calculated by MATRIX_x.

3. To calculate the gain matrix corresponding to the "corrective control":

$$\begin{aligned} \dot{M}_2 &= 0 \quad ; \\ M_2 &= -(A_c^T)^{-1} M_1 E \quad . \end{aligned} \quad (3.2)$$

M_1 is obtained from the MATRIX_x command REGULATOR. The corrective gain matrix is given by

$$K_c = R^{-1} B^T M_2 \quad . \quad (3.3)$$

4. The total gain matrix is constructed:

$$K = [K_r \quad K_c] \quad . \quad (3.4)$$

5. The metastate representation of the system is made by calculating E matrix for the desired reference state. The observation matrix has diagonal elements equal to one that enable us to observe error outputs. It also includes an additional entry for the reference state, which is the steady state value of the corresponding output. This addition is done only to observe the error in the actual output. The final stage is to form the closed-loop system using K and the estimator gain matrix K_E in the LQG compensator design steps. Note that all the outputs are measured without any disturbance or noise; therefore, there is no state estimation in the practical implementation.

An application of the reference input tracking control problem is considered using the EBR-II steam generator system as an example. The first reference input is a step change in the drum pressure (the derivation stated above is valid for constant reference inputs). It is possible to increase the power generation in nuclear reactors by extracting more steam from the steam generator (load-following property), which results in a steam pressure drop in the drum. It is assumed that a 100-psi steam pressure drop is the reference input to be tracked. The closed-loop response of the steam pressure in the drum to the reference input is shown in Fig. 3.23. The remaining error outputs are found to be close to zero. It can be seen from this figure that the reference tracking optimum control is accomplished with a small steady

state error. A second application to the EBR-II steam generator model is performed using a reference input of a 4-in. level increase to the reference input. The steady state error is practically zero, as can be seen from the Fig. 3.24. Other error outputs are also close to zero. Note that the optimal regulator design for the steady state portion of the total control action uses a weight (equal to one) only on the drum level variable, which is presumably the reason for a relatively smaller steady state error compared to the pressure tracking controller case.

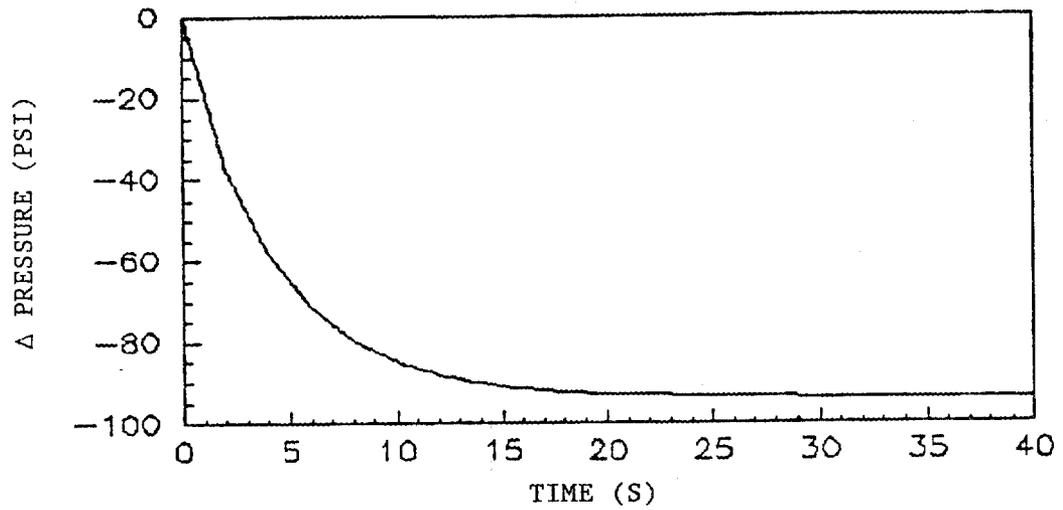


Fig. 3.23. Drum pressure response to a -100-psi step reference input (optimal tracking controller).

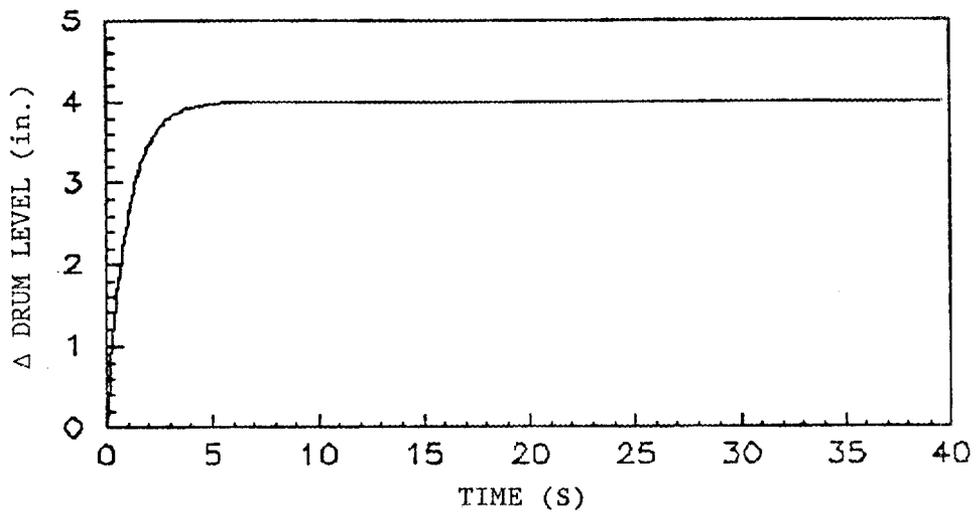


Fig. 3.24. Drum level response to a +4-in. step reference input (optimal tracking controller).

4. USE OF COMPUTER-AIDED DESIGN SOFTWARE MATRIX_x

Engineering applications of modern control strategies involve on-line computer usage, as in the dynamic process control and extensive off-line usage for design purposes. Improving the capabilities of modern controllers is, therefore, dependent on software development and construction of fast computers with large memories. One of the latest CAD software developments, known as MATRIX_x, provides a few modern control design routines as well as several traditional classical design tools.⁹ The capabilities of MATRIX_x were previously studied by Jamshidi and others,¹⁰ and several properties were compared with other commercially available menu-driven CAD packages.

The LQG designs in this study were accomplished using the built-in routines of MATRIX_x. Since this software also includes simulation routines, the complete model-buildings using compensators on the feedback loop and simulations were performed within the software environment and without any external programming efforts. (See Appendix A for programming in Matrix_x). The following describes the control design and simulation steps.

4.1 CONTROL DESIGN SCHEMES

Among the several control design routines, we concentrated our attention on commands that are used for LQG design.

The REGULATOR command computes optimal constant gain, state-feedback matrices for continuous-time system under the assumption of full-state feedback. Examples of the REGULATOR command are

$$[EVAL, KR] = \text{REGULATOR}(A, B, R_{xx}, R_{uu}, R_{xu}) \quad ;$$

$$[EVAL, KR, P] = \text{REGULATOR}(A, B, R_{xx}, R_{uu}, R_{xu}) \quad .$$

Inputs to the REGULATOR command include the A and B (plant and input) components of the continuous-time system matrix,

$$S = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad ,$$

and the design weighting matrices R_{xx} , R_{uu} , and R_{xu} , where R_{xu} is optional. The design weighting matrices provide weights on the states (x) and controls (u) as defined by the quadratic cost function. R_{uu} must be positive definite, and R_{xx} must be positive semidefinite.

Outputs from the REGULATOR command are the closed-loop eigenvalues ($EVAL$), the optimal regulator state-feedback gain matrix (KR), and optionally, the Riccati solution matrix (P). The closed-loop eigenvalues are obtained from the closed-loop plant ($A-B*KR$). The

matrix dimensions of the gain matrix, KR , are equal to the number of system inputs by the number of system states.

The ESTIMATOR command calculates constant, optimal state-estimator gain matrices for continuous-time systems. Examples of the ESTIMATOR command are:

$$[EVAL,KE] = ESTIMATOR(A,C,QXX,QYY,QXY) \quad ;$$

$$[EVAL,KE,P] = ESTIMATOR(A,C,QXX,QYY,QXY) \quad .$$

Inputs to the ESTIMATOR command include the A and C (plant and observation) components of the continuous-time system matrix,

$$S = \begin{bmatrix} A & B \\ C & D \end{bmatrix} ,$$

and the noise covariance matrices Q_{xx} , Q_{yy} , and Q_{xy} are optional. Q_{yy} has matrix dimensions equal to the number of plant outputs and must be positive definite, while Q_{xx} has matrix dimensions equal to the number of plant states and must be positive semidefinite. In many cases the input disturbances and output noises are uncorrelated so that $Q_{xy} = 0$.

Outputs from the ESTIMATOR command are the estimator eigenvalues ($EVAL$), the optimal estimator gain matrix (KE), and optionally, the state estimation error covariance matrix (P). The estimator eigenvalues are obtained from $(A-KE*C)$. The dimensions of the gain matrix KE are equal to the number of system outputs.

The LQGCOMP command computes the system matrix for the optimal output feedback compensator. The optimal compensator is a combination of the optimal state estimator, which reconstructs the system states, and the optimal regulator, which provides a linear state-feedback control law. The form of the LQGCOMP command is:

$$[SC,NSC] = LQGCOMP(S,NS,KR,KE) \quad .$$

Inputs to the LQGCOMP command are the optimal regulator and estimator gain matrices (KR and KE) and the continuous-time design system (S) with number of states (NS). The LQGCOMP assumes that both KR and KE were designed from system matrix S .

Output from the LQGCOMP includes the continuous-time compensator system matrix (SC) and the corresponding number of states (NSC):

$$SC = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} ,$$

$$A_2 = A_1 - K_E \cdot C_1 - B_1 \cdot K_R \quad ,$$

$$B_2 = K_E \quad ,$$

$$C_2 = K_R \quad ,$$

$$D_2 = 0 \quad ,$$

where A_1 , B_1 , and C_1 are the open-loop system matrices.

The closed-loop dynamic system can be constructed with the FEEDBACK, APPEND, and CONNECT commands or with the SYSTEM-BUILD graphical modeling and simulation tool. Using the FEEDBACK command, the components of the closed-loop system matrix (SCL) are as follows:

$$SCL = \left[\begin{array}{cc|c} A_1 & -B_1 C_2 & B_1 \\ B_2 C_1 & A_2 & 0 \\ \hline C_1 & 0 & 0 \end{array} \right] .$$

4.2 SIMULATION CAPABILITIES

Consider the following linear system,

$$\dot{\underline{X}} = \underline{A}\underline{X} + \underline{B}u \quad ,$$

$$y = \underline{C}\underline{X} + \underline{D}u \quad ,$$

where \underline{X} is a vector of state variables, y is a vector of measurements, and u is a forcing function. A , B , C , and D are the constant system matrices. The above state-space representation of a linear dynamic system can be loaded to the computer using the following MATRIX_x syntax,

$$S = [A,B;C,D] \quad ,$$

where S is a matrix representing the overall system. The number of states should be specified by an arbitrary variable.

The representation of systems in the transfer function domain is also possible. Consider the following transfer function:

$$G(s) = \frac{a_1 s^2 + a_2 s + a_3}{b_1 s^3 + b_2 s^2 + b_3 s + b_4} \quad ,$$

where

$a_i, i = 1, \dots, 3$ = numerator coefficients,

$b_i, i = 1, \dots, 4$ = denominator coefficients,

$G(s)$ = transfer function in Laplace domain.

The $MATRIX_x$ syntax for the representation of this transfer function can be made as follows:

$$NUM = [0 \ a_1 \ a_2 \ a_3] \ ,$$
$$DEN = [b_1 \ b_2 \ b_3 \ b_4] \ ,$$

where NUM and DEN are arbitrary variable names. The order of the transfer function should be specified by an arbitrary variable.

A transformation from one type of representation to another is also possible with $MATRIX_x$.

Once the linear system representation in $MATRIX_x$ is completed, the transient simulations can be performed using the following commands:

$$[T, Y] = STEP(S, NS, TMAX, NPTS) \ ,$$

or

$$[T, Y] = STEP(NUM, DEN, TMAX, NPTS) \ .$$

The outputs of this command are the time vector T and the Y matrix which are the step responses of the linear system. Y matrix has the number of columns equal to the number of states and the number of rows equal to the number of time points.

The inputs of the simulation commands are the overall system representation, either in state-space form or in transfer function form. $TMAX$ specifies the length of time and $NPTS$ specifies the number of points to be calculated. The default integration algorithm is the "variable Kutta-Merson,"⁹ an explicit, fourth-order, one-step method. The integration step is optimized to provide the largest step while remaining within the local error tolerances. The maximum step size is equal to the time increment. The choice of $NPTS$ determines the time increment, and this integration technique retains the stability of the solution regardless of any arbitrary choice of $NPTS$.

5. SUMMARY AND RECOMMENDATIONS

5.1 SUMMARY OF WORK

In this study the modern control theory design algorithms are examined. Design effort is concentrated mainly on the linear, quadratic Gaussian compensator design, which includes an optimal regulator design and an optimal estimator (Kalman filter) design.⁹ A full-state feedback optimal regulator is designed for the EBR-II primary system model for which measurement noise and input disturbance are assumed to be zero. The fractional reactor power is weighted in the corresponding quadratic cost function. The design uses reactivity as the control input with a small scalar weight. Closed-loop step responses to several perturbations show improvement in system behavior. The safety limitations of 2°F/min reactor ΔT change and 1 cent/s control rod speed are not violated.

A similar design is devised to control the drum level in the EBR-II steam generator model. The control input for this case is the feedwater flow. The states corresponding to the drum level and the dummy variable are weighted in the quadratic cost function. The design also uses all the measurements without noise or input disturbance. The closed-loop step responses to several perturbations show consistently high performance. Comparison with the three-element controller results indicates that both strategies are favorable. The drum level design requirements of 3 in. above and 4 in. below the center line drum level variation limit are not exceeded.

When control input and system disturbance units are the same, the optimal regulator controller exhibits a global control behavior. This is the case when the main steam valve is used as an actuator to control the system for a steam bypass valve-opening perturbation (or when control rods are used to control the primary system for small rod-drop perturbations). Note that the closed-loop system model construction in the MATRIX_x algorithms is made in a pole-placement fashion using the optimal gains when the closed-loop system with the three element-controller includes the actuator dynamics as an additional state. The nonzero steady state errors encountered in modern control design applications are related to the type-0 structure of the closed-loop systems. That observation is explicitly detailed in this study. Thus it cannot be assumed that the LQG compensator will result in a zero steady state error. However, an error less than a specified value can be achieved by the LQG design. Theoretically the LQG design does not guarantee a zero error for a step perturbation, if the corresponding transfer function is of type-0. The problem can be solved, however, using a dummy state variable in the form of an integrator. The implementation of the LQG design for the EBR-II drum level controller includes this modification.

A simple robustness test is applied to the controllers of the EBR-II steam generator model. Instead of assessing robustness stability by analytical methods, a practical method is considered in determining the robustness levels of each controller. The test method includes controller models that are developed using the open-loop model and a "plant" model that includes several uncertainties in the system parameters. It is assumed that the drum geometry parameters of the open-loop model are 5% different from actual dimensions. The optimal regulator and the three-element controller are appended to the "plant" model (one at a time). The closed-loop step responses to a +10% steam bypass valve-opening perturbation show a large degradation in the drum level using the three-element controller. The closed-loop model with an optimal regulator responds to the same input in a more favorable manner. The zero steady state error in the drum level is practically unchanged; consequently, the optimal regulator is more robust to plant uncertainties.

The closed-loop system performance using an optimal regulator design is related to the state and control input weights in the corresponding quadratic cost function. The best combination of the state weights is selected for the EBR-II steam generator controller in a trial-and-error fashion. A starting combination is predicted using the knowledge of the open-loop system dynamics. The drum level is suppressed by a relatively large weight when the other states are weighted based on the overshoot ratios (open-loop case vs simple optimal regulator design case). The closed-loop step responses to a +10% steam bypass valve-opening perturbation show that a drastic improvement can be achieved. However, the controller performance evaluation does not include the limitations on the control signal. The actuator constraints bring certain limitations to the control input weight as well as to the state weights.

A complete LQG compensator design that includes an optimal estimator (Kalman filter) design is considered for the EBR-II steam generator system. The measurement set is assumed to be incomplete, and available measurements are assumed to contain noise. A small input disturbance on the states is also considered. An optimal estimator is designed and appended to a previously designed optimal regulator. The LQG compensator synthesis is the final form of the modern controller. The closed-loop step responses to a +10% steam bypass valve-opening perturbation show that the linear observer design yields high performance in terms of estimating the states and regulating the closed-loop behavior in an optimal manner. Study also showed that the drum steam-pressure is an important state variable and its measurement or its estimation must be used in order to improve the controller performance.

The last task presented in this study is an optimal reference-tracking controller design which has a major importance in plant operation. The analytical methods¹ are studied, and an optimal tracking controller is designed using the metastate representation of the EBR-II steam generator model. It is assumed that the measurements are complete and perfect. The closed-loop responses to the given step reference inputs such as drum level increase and drum pressure drop indicate that the

optimal tracking controller is efficient in set-point tracking. Steady state errors are negligible and response times are short.

MATRIX_x is one of the most advanced members of the MATLAB CAD package family. All of the simulations in this work are performed using the special features of MATRIX_x. The modern control theory design algorithms, such as the optimal regulator design (or the LQG design) routines, are very powerful design tools. The classical methods such as root-locus or Bode plots are also available in MATRIX_x. Using the menu-driven System-Build option, a number of modules can be coupled regardless of the complexity of input-output relationships.

5.2 RECOMMENDATIONS

It is the authors' belief that each of the control design studies presented in this work must be investigated in more detail. The selection of the best combination of weights in an optimal regulator design is one of the issues to be investigated. Despite the fact that finding the best weights can be done intuitively, provided the designer is familiar with the open-loop dynamics, an analytical method based on system matrices A and B would be very useful. LQG compensator design applications to the EBR-II model shows that such controllers are quite efficient. A comparison between the full-state feedback compensator results and the estimator results assures that the practical implementation of the LQG compensators can handle the control problem, even under degraded conditions. The applications of the optimal estimator design should be extended to more general cases of measurement degradation. The optimal tracking design problem can be incorporated with the estimator design problem. An optimal tracking controller design study under degraded conditions would be a topic for future study. When the closed-loop system structure has a type-0 form, the steady state errors can be driven to zero by appending an additional integrator to the system.

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APPENDIX A

MATRIX_x MACROS FOR CONTROL DESIGN

APPENDIX A

MATRIX_x MACROS FOR CONTROL DESIGN

LQG Design

```
BI=[BFPD;DD],...
[EVAL,KR]=REGULATOR(A20,B20,RXX,RUU),...
[EVAL,KE]=ESTIMATOR(A20,C20,QXX,QYY),...
[SC,NSC]=LQGCOMP(S20,20,KR,KE),...
[SCL,NSCL]=FEEDBACK(S20,20,SC,NSC),...
[AA,BB,CC,DD]=SPLIT(SCL,NSCL),...
PLANT=[AA,BI;CC,DD],...
[T,Y]=STEP(PLANT,NSCL,400),...
I=18;]MP1[,...
I=19;]MP3[,...
BI=[],...
```

A20= System matrix listed in Table 4.5 with one modification,...
The last row elements are equal to zero, except the 18th column,
which is equal to 1. (dummy state),...

B20= 20x1 vector all zeros except the 19th row equal to 1,...

BFPD=Forcing vectors of Table 5.2 extended to row number 20,...

C20 =20x20 unity matrix,...

D20 =B20*0,...

S20 =System matrix=[A20,B20;C20,D20],...

MP1 =Graphic macro,...

MP2 =Graphic macro,...

Tuning Three-Element Controller Gains

```
> M(20,18)=-K1,...
M(20,10)=-K2*0.05371,...
M(20,19)=K2,...
N(20,18)=KI*K1,...
N(20,10)=K2*KI*0.05371,...
N(20,19)=-K2*KI,...
AFIN=INV(M)*N,...
EIG(AFIN),...
K1K2KI=[K1,K2,KI],...
(if the eigenvalues have negative real parts then:),...
SFIN=[AFIN,BF;CF,DF],...
(testing the controller performance),...
[T,YFIN]=STEP(SFIN,20,TMAX),...
Y=YFIN,...
]MPL[,...
```

Optimal Tracking Design

```

◇ MA=[A,E;Z1],...
  [EVAL,KR,P]=REGULATOR(A,B,RXX,RUU),...
  AC=A-B*KR,...
  AC=AC',...
  M2=-INV(AC)*P*E,...
  KR2=INV(RUU1)*B'*M2,...
  MKR=[KR,KR2],...
  MS=[MA,MB;MC,MD],...
  [EVAL,KE]=ESTIMATOR(MA,MC,MQXX,MQYY),...
  [SC,NSC]=LQGCOMP(MS,NMS,MKR,KE),...
  [SCL,NSCL]=FEEDBACK(MS,NMS,SC,NSC),...
  [T,MY]=STEP(SCL,NSCL,TMAX),...
  Y=MY,...
  ]MPL[,...

```

Graphics

```

◇ ]MP1[,I=I+1;]MP2[,I=I+1;]MP3[,I=I+1;]MP4[,...
  (macro MP),...
  PLOT(T,Y(:,I),'UPPER LEFT/NOGRID'),...
  (macro MP2),...
  PLOT(T,Y(:,I),'LOWER LEFT/NOGRID'),...
  (macro MP3),...
  PLOT(T,Y(:,I),'UPPER RIGHT/NOGRID'),...
  (macro MP4),...
  PLOT(T,Y(:,I),'LOWER RIGHT/NOGRID'),...

```

APPENDIX B

EBR-II PRIMARY SYSTEM CLOSED-LOOP RESPONSES USING LQG

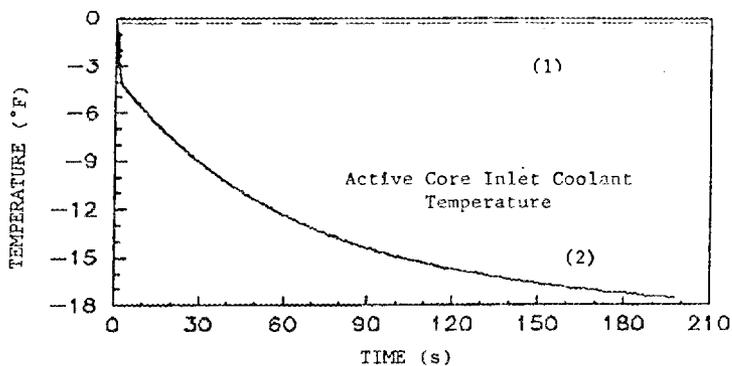
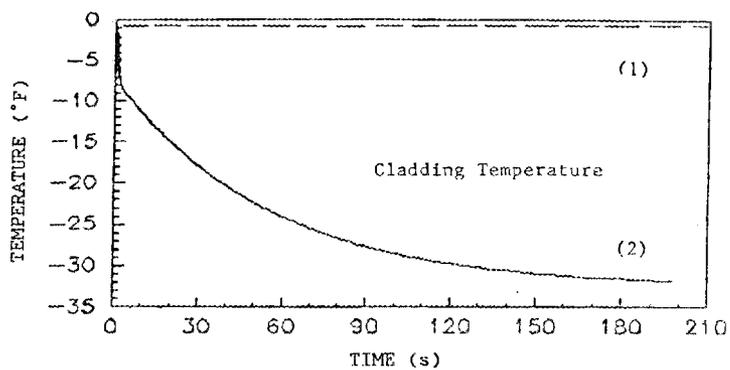
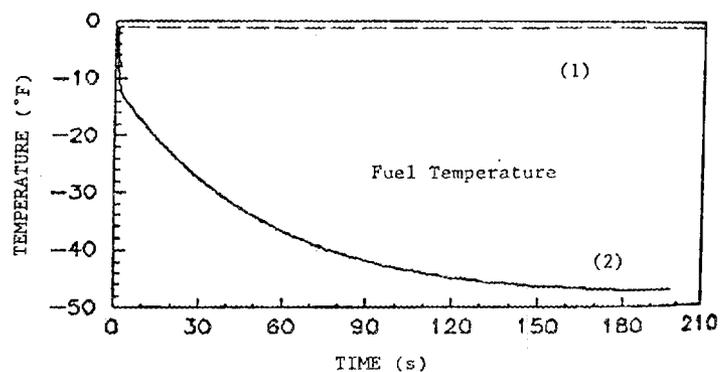
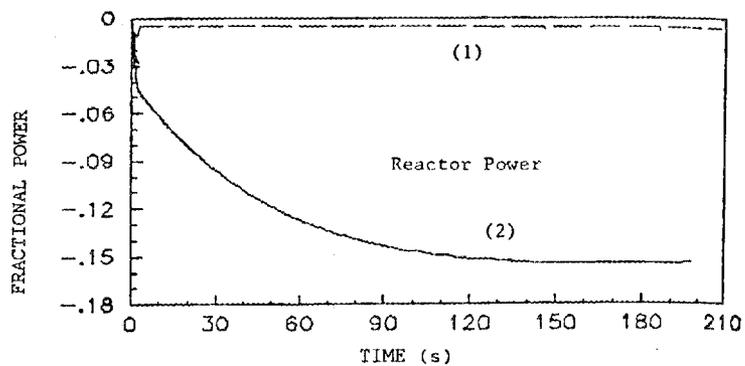


Fig. B.1. Step responses of closed-loop primary model to a -5-cent reactivity perturbation [(1) optimal compensator and (2) open-loop response]. (Deviations from steady-state.)

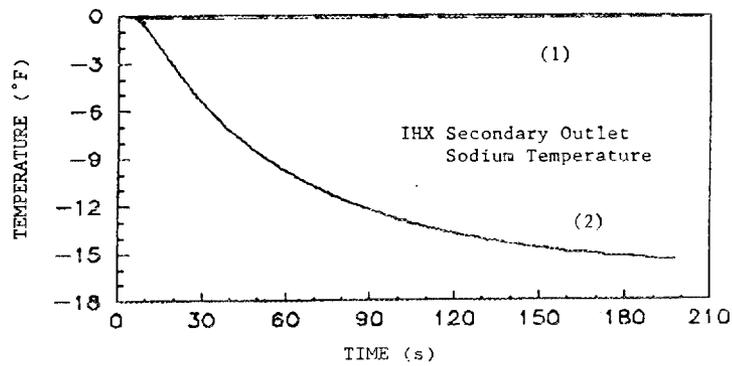
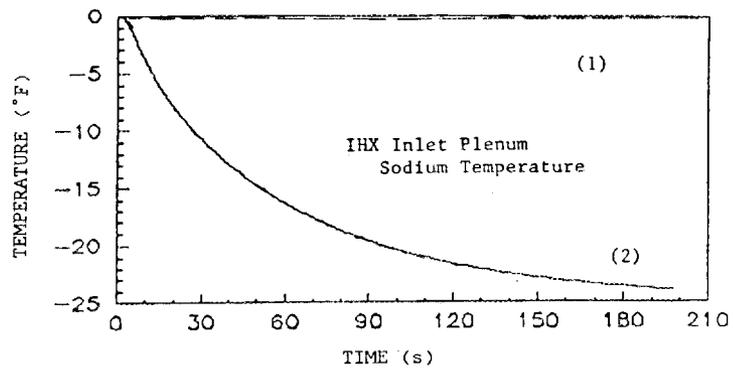
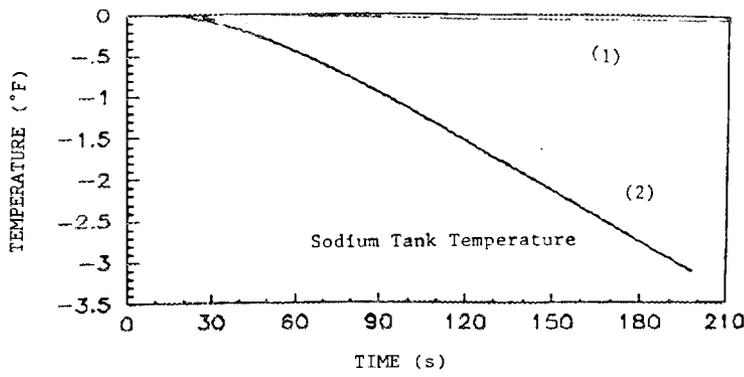
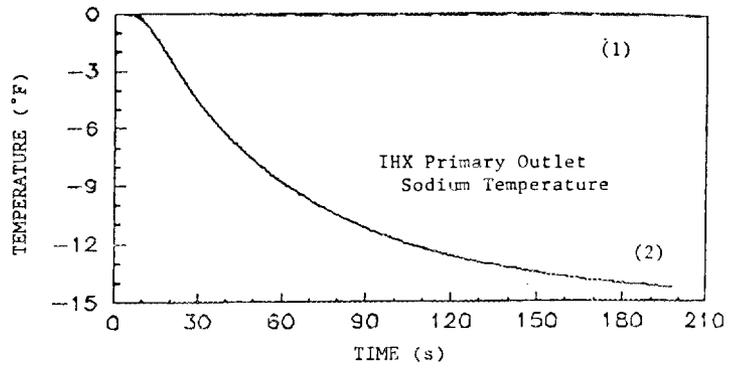


Fig. B.1. (continued).

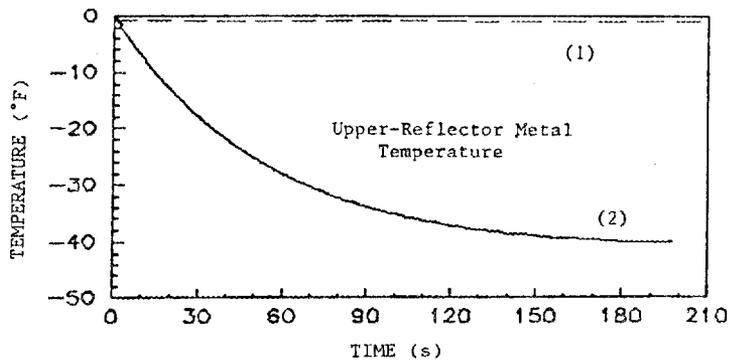
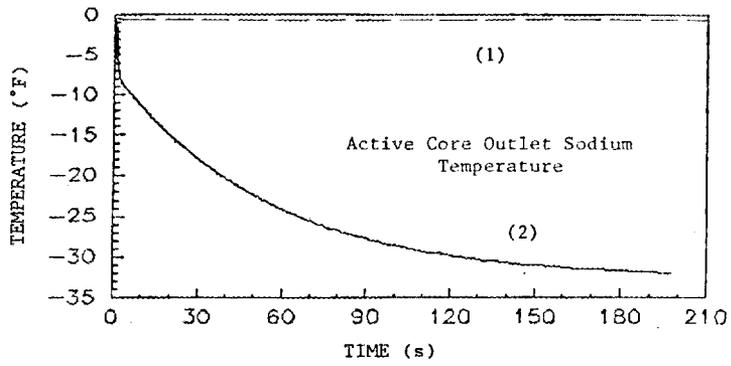
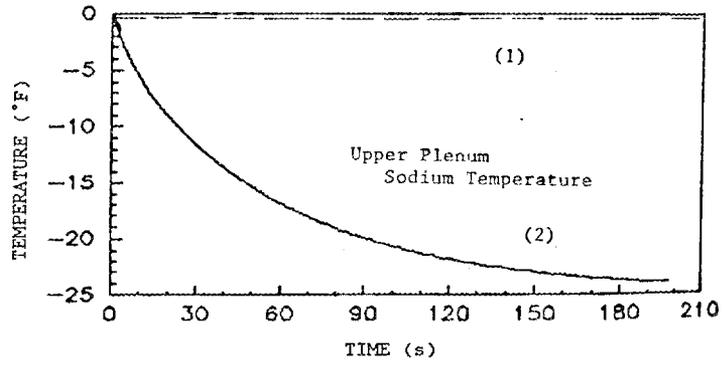
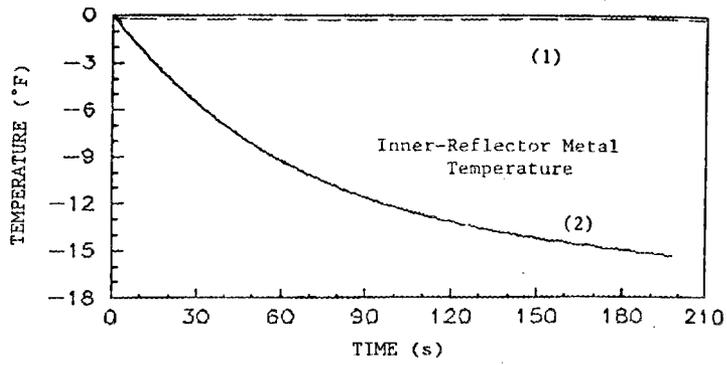


Fig. B.1. (continued).

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