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Maxwellian Cascade Model

R. L. Macklin

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MAXWELLIAN CASCADE MODEL

R. L. Macklin

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ABSTRACT

A model for gamma-ray cascade de-excitation of a nucleus derived from the Maxwellian energy distribution function but imposing energy conservation was investigated. Energy distributions and multiplicities and their averages were found over a range of nuclear temperatures and excitation energies appropriate to neutron capture. The model was compared to existing measurements for tantalum, a case where the level density was high and thus a good approximation to the model.

1. INTRODUCTION

Nuclei excited to energy levels from a few keV to several MeV above their ground states emit prompt gamma rays. At higher excitation energies neutrons and charged particles may also be emitted, depending on their binding energies. The neutron binding energy is usually the lowest, thus neutron capture leads to an excited nucleus which then may de-excite by emitting one or more gamma rays. Hauser-Feshbach reaction codes[1] have incorporated a gamma-ray transmission coefficient to parametrize cascade probabilities and competition with particle emission at higher energies. Theoretical estimates of gamma-ray transition probabilities are based on extrapolation of $E1$ giant dipole resonances[2,3] parameters.

Measurements of neutron capture gamma rays have been most successful where few nuclear levels are involved, i.e. for high-energy gamma rays to either the ground state or to one of the low-lying states near it. For cases where many levels are involved, measurements[4] are more difficult but indicate the overall distribution of typical cascade gamma-ray energies resembles the Maxwellian energy distribution.

$$p(x)dx = (2/\sqrt{(\pi)}) * \sqrt{(x)} * \exp(-x)dx \tag{1}$$

$$x = E/kT \tag{2}$$

where E is the gamma-ray energy and kT the nuclear temperature. A plot of the distribution is shown in Fig. 1. The integral of $P(x)$ from a variable lower limit X to infinity is shown in Fig. 2.

The Maxwellian energy distribution extends to infinity and thus cannot directly represent a neutron capture gamma-ray distribution. The present investigation concerns a more restrictive model in which the finite excitation energy is conserved. A constant nuclear temperature with many virtual gamma-ray photons in equilibrium is assumed. A series of typically 200,000 gamma-ray cascades is constructed to approximate the model distribution. For each cascade, an energy budget is kept, starting with the full excitation energy. Successive energies are chosen from the thermal distribution. If the chosen energy does not exceed the current energy budget, it is accepted as a member of the cascade and the budget decreased by that amount of energy. Otherwise, the remaining budget energy is accepted as the last member of the cascade and the next cascade selection is initiated.

Superimposed on this sort of distribution may be other distinct gamma rays. High energy transitions to a low-lying state and their successors have been mentioned above. There may also be a prominent gamma ray from a low-lying level fed by many cascades, in cases where decay to the ground state is hindered. Thus, the model may be used in several ways to supplement detailed experimental information from energy level evaluations to give a comprehensive picture of prompt gamma-ray cascades. The original motivation for developing the model was for extrapolating large liquid scintillator pulse amplitude spectra to energies below 1 MeV where high background rates obscured neutron capture spectra.[5]

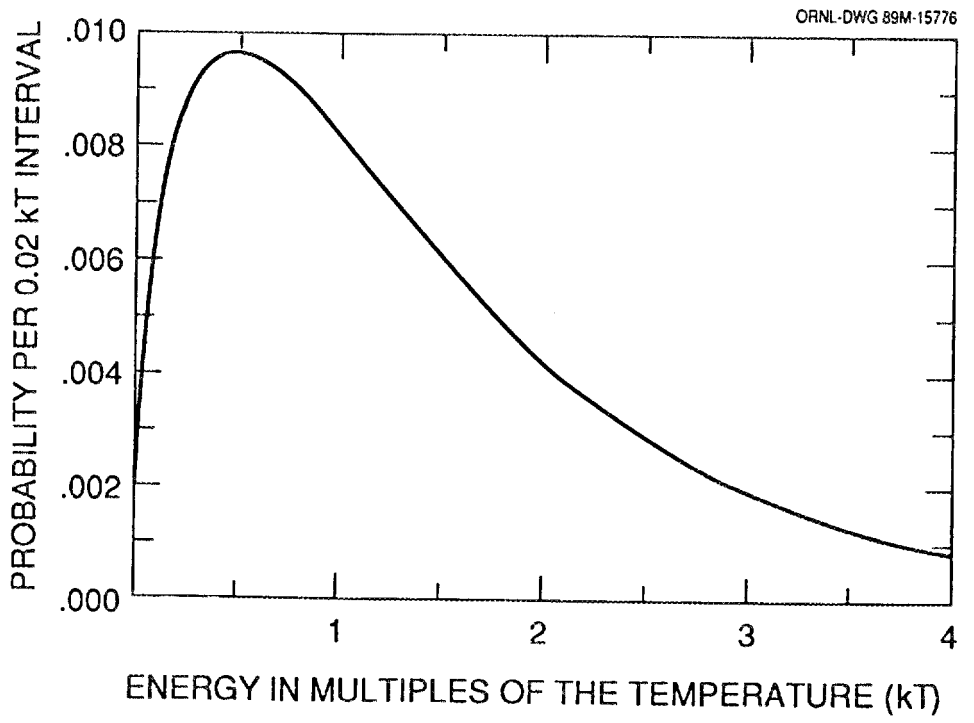

Fig. 1. The Maxwellian energy distribution.

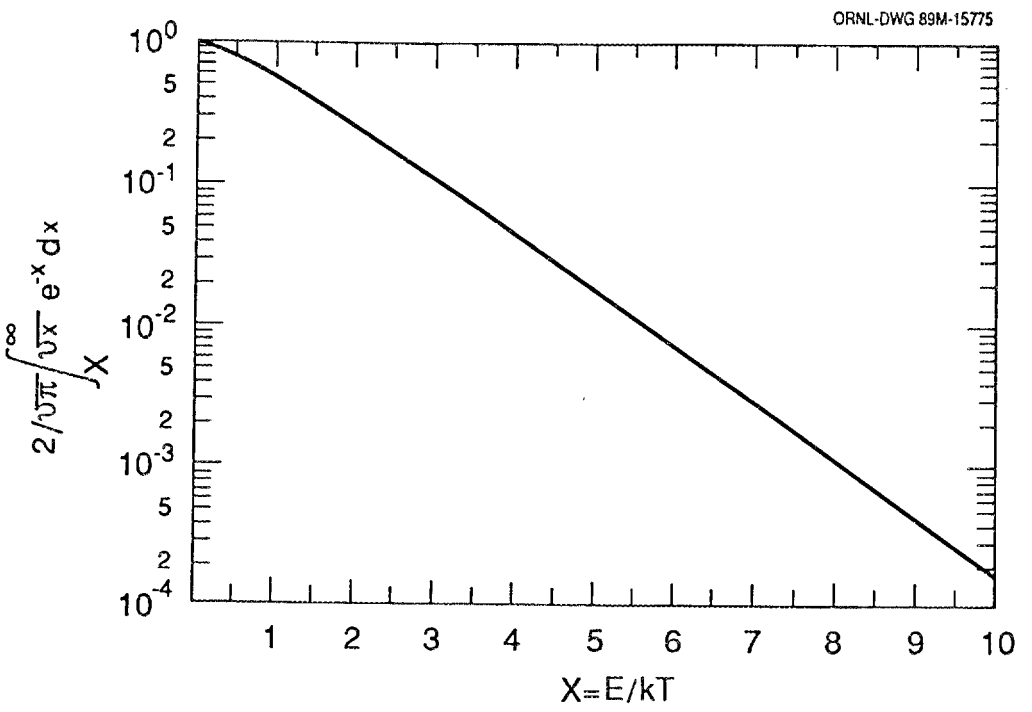

Fig. 2. The integral of the Maxwellian distribution from a variable lower limit to infinity.

2. MAXWELLIAN INTEGRAL

A FORTRAN code MAX has been written to sample the Maxwellian distribution and construct random cascades, imposing a cutoff on the last energy chosen to match each cascade to the available nuclear excitation energy. The code was constructed with an upper limit of 10 kT in steps of 0.02 kT. $P(x)$ was integrated from zero to the top of each step, X_i, using the formula

$$Erf(\sqrt{(X_i)}) - (2/\sqrt{(\pi)}) * \sqrt{(X_i)} * \exp(-X_i) \tag{3}$$

where Erf is the error function, for which computer algorithms have been developed. The Hastings algorithm for Erf is in a form which allowed direct incorporation of Eq. (3) as may be seen in the subroutine eMaxint (see Appendix A). The stored values of the integral (3) represent the probability up to each step and were searched to find the step matching successive random numbers uniformly distributed from zero to one.

3. RANDOM SAMPLING CODE

It is possible that analytic methods exist for calculating the probability distribution for the model, as the distribution up to each successive cutoff is drawn from the same Maxwellian. More transparent to me is a numerical approximation using the small finite energy bins and many sample cascades. Pseudo-random numbers, uniformly distributed from zero to one were used to select gamma-ray energy bins from the Maxwellian distribution by checking each against the cumulative stepwise integration [Eq. (3)]. Cascades were constructed by adding the randomly selected energies together until the sum exceeded an assigned cutoff value X_c. The energy of the last gamma-ray selected (dubbed the lopped gamma ray) was reduced to bring the total for each cascade down to X_c. If the first gamma-ray energy selected in a cascade exceeded X_c, it was counted as a single, or ground state gamma-ray transition, rather than a lopped one. The random number selection function was the most time-consuming process in the code. To economize on it the random numbers were scaled to the cumulative Maxwellian integral at the cutoff X_c and the singles probability corresponding to higher energies was calculated separately. Typical code execution is 3 minutes of central processing unit (CPU) time on a VAX 11/785 computer.

4. MODEL ENERGY DISTRIBUTIONS

The energy distribution of the lopped gamma rays found by random sampling is illustrated in Figs. 3–6 for several values of X_c. Those distributions have more low-energy gamma rays than the Maxwellian and decrease markedly as X_c is approached. For $X_c = 9.98$ so few samples are selected near X_c that the decrease is masked by the statistics and the distribution shape is seen to closely follow the integral of the Maxwellian from X to infinity shown in Fig. 2.

The average energy of the lopped gamma rays was calculated from the energy distributions. It is shown as a function of X_c, the excitation energy available to each cascade in Fig. 7. As X_c is increased, the average appears to approach 5/4 kT, distinctly below the 3/2 kT average energy of the Maxwellian distribution. For $X_c = 9.98$ the average was 1.245 kT on the longest test run, only 0.2% below the average energy up to 9.98 kT for the distribution shown in Fig. 2.

The full energy distributions of the gamma rays in the model cascades are formed by adding the randomly sampled lopped-gamma distribution, the cut-off Maxwellian and the single gamma rays to the ground state, in the proportions determined by the random selection process. Examples are shown in Fig. 8 of the distributions below the excitation energy. The proportion of single gamma rays to the ground state, while fixed in the model, can readily be modified to match a case of interest. Figure 9 shows the ratio between the excitation energy and the energy at which the probability has fallen to half its peak value as a function of the model parameter X_c.

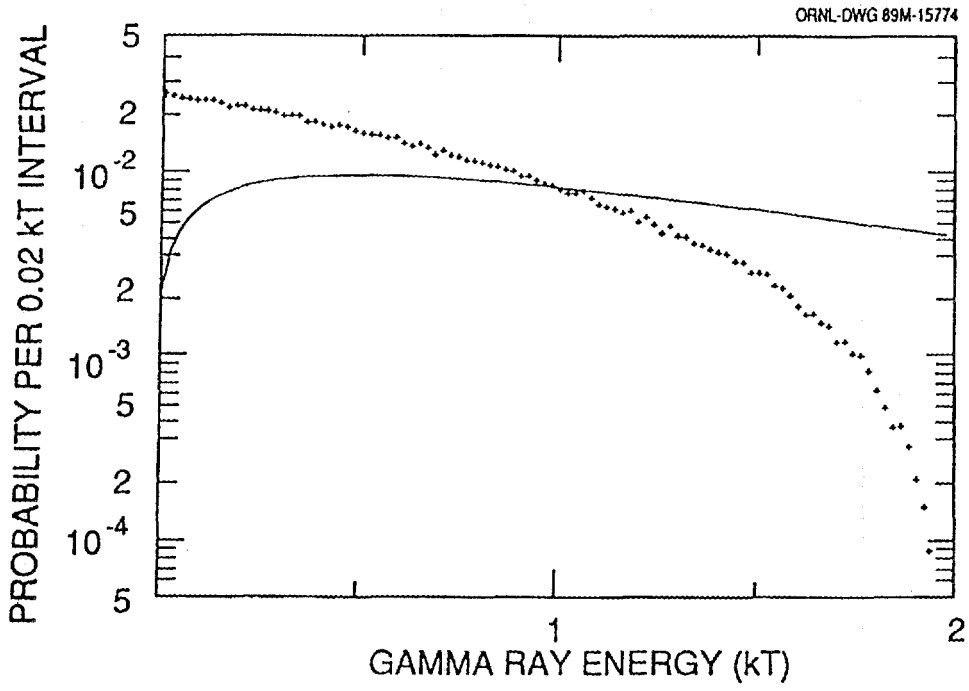

Fig. 3. The distribution of lopped gamma-ray energies for a cascade energy of 2 kT. The crosses represent 200,000 randomly sampled cascades. The Maxwellian distribution sampled for the other gamma rays in a cascade is shown as a solid line.

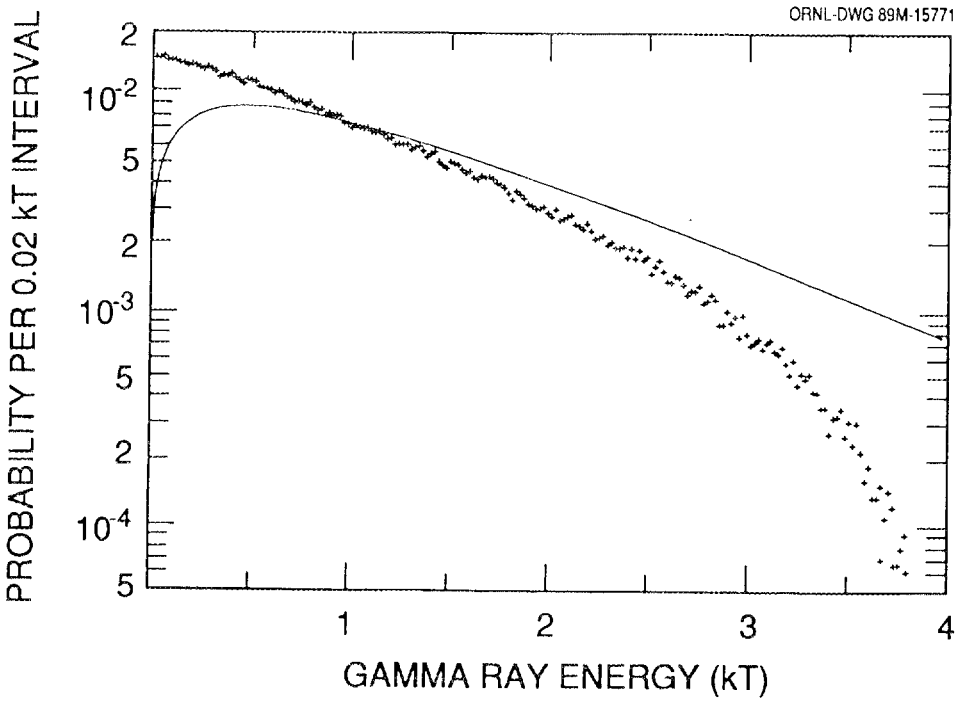

Fig. 4. The distribution of lopped gamma-ray energies for a cascade energy of 4 kT. The crosses represent 200,000 randomly sampled cascades. The Maxwellian distribution sampled for the other gamma rays in a cascade is shown as a solid line.

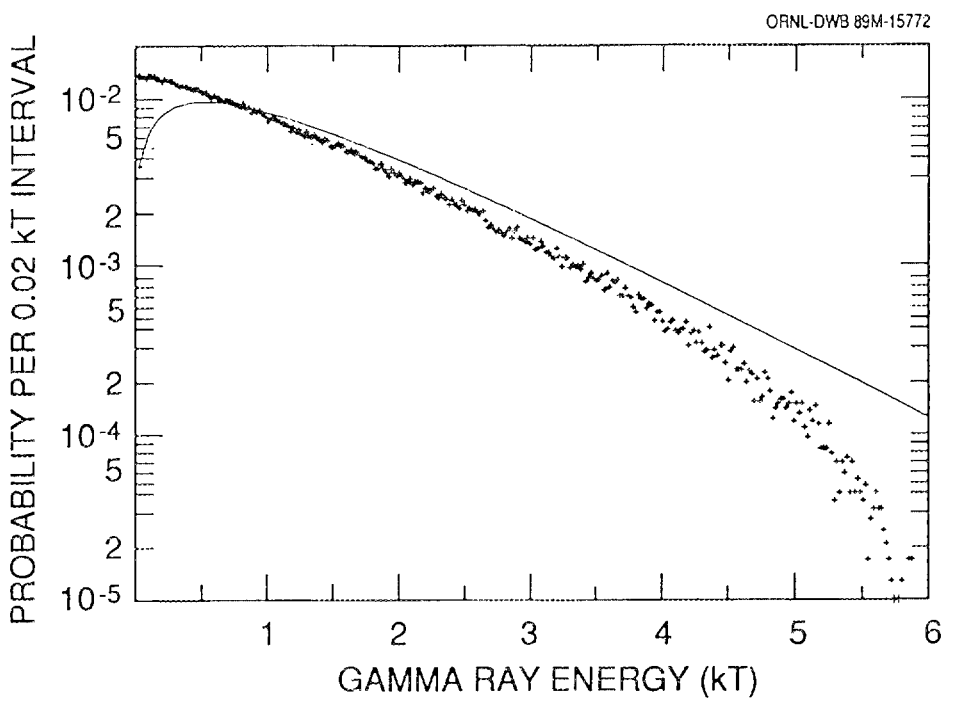

Fig. 5. The distribution of lopped gamma-ray energies for a cascade energy of 6 kT. The crosses represent 200,000 randomly sampled cascades. The Maxwellian distribution sampled for the other gamma rays in a cascade is shown as a solid line.

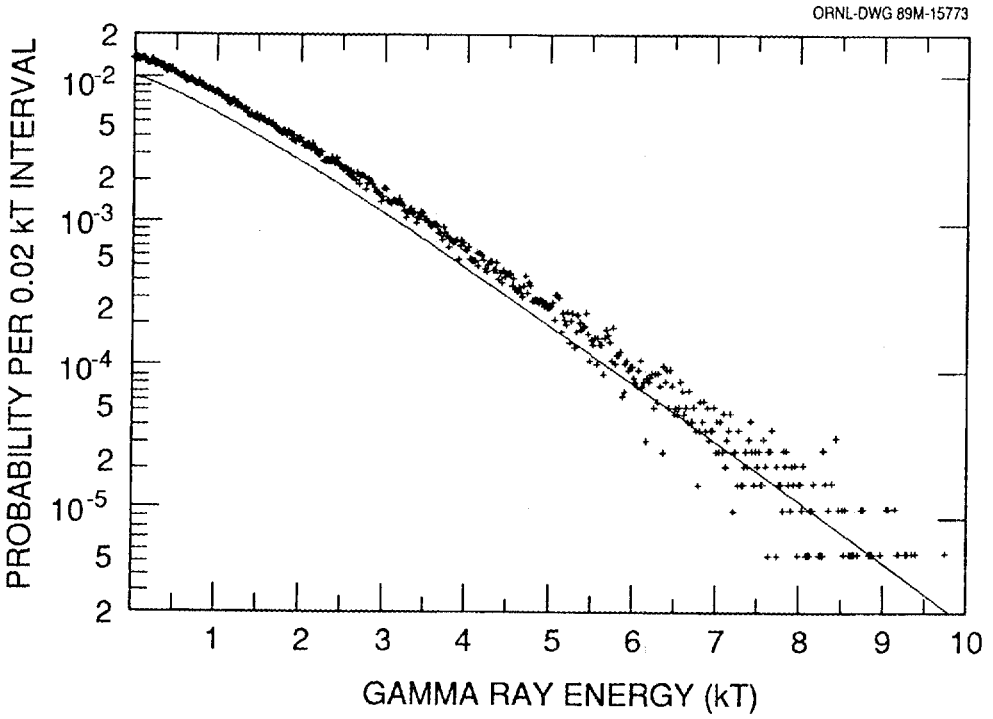

Fig. 6. The distribution of lopped gama-ray energies for a cascade energy of 9.98 kT. The crosses represent 200,000 randomly sampled cascades. The integral of the Maxwellian distribution from X to infinity and scaled down by a factor of 100 is shown as a solid line to indicate the similarity in shape.

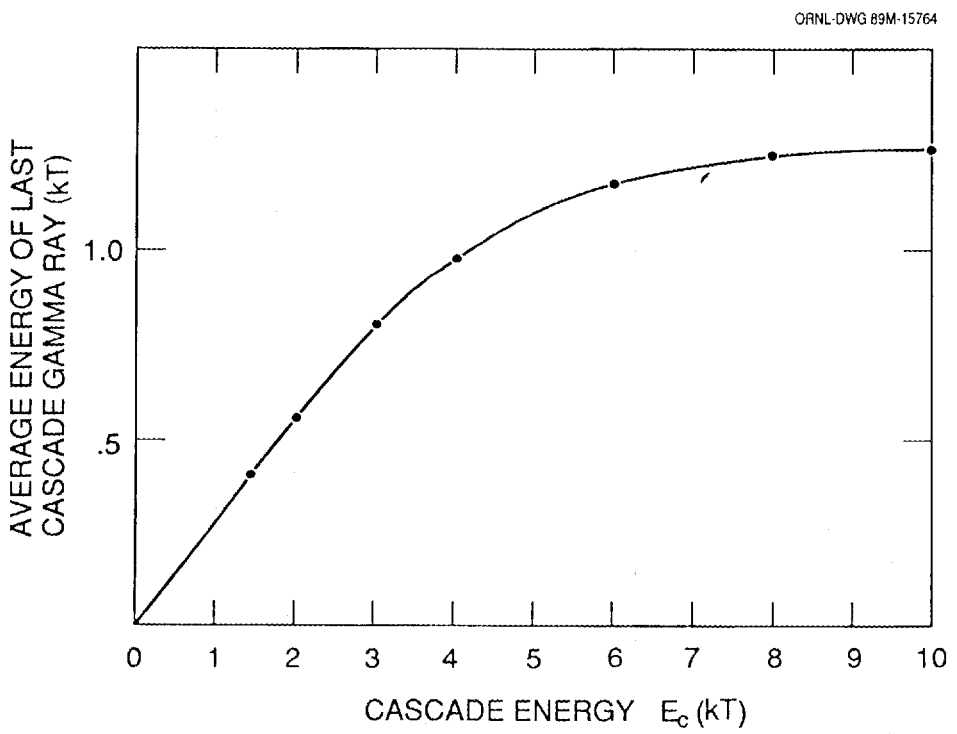

Fig. 7. The average energy of the lopped gamma-ray energy distributions as a function of the excitation energy available to each cascade.. The average appears to approach an asymptote distinctly below the 3/2 kT of the Maxwellian distribution.

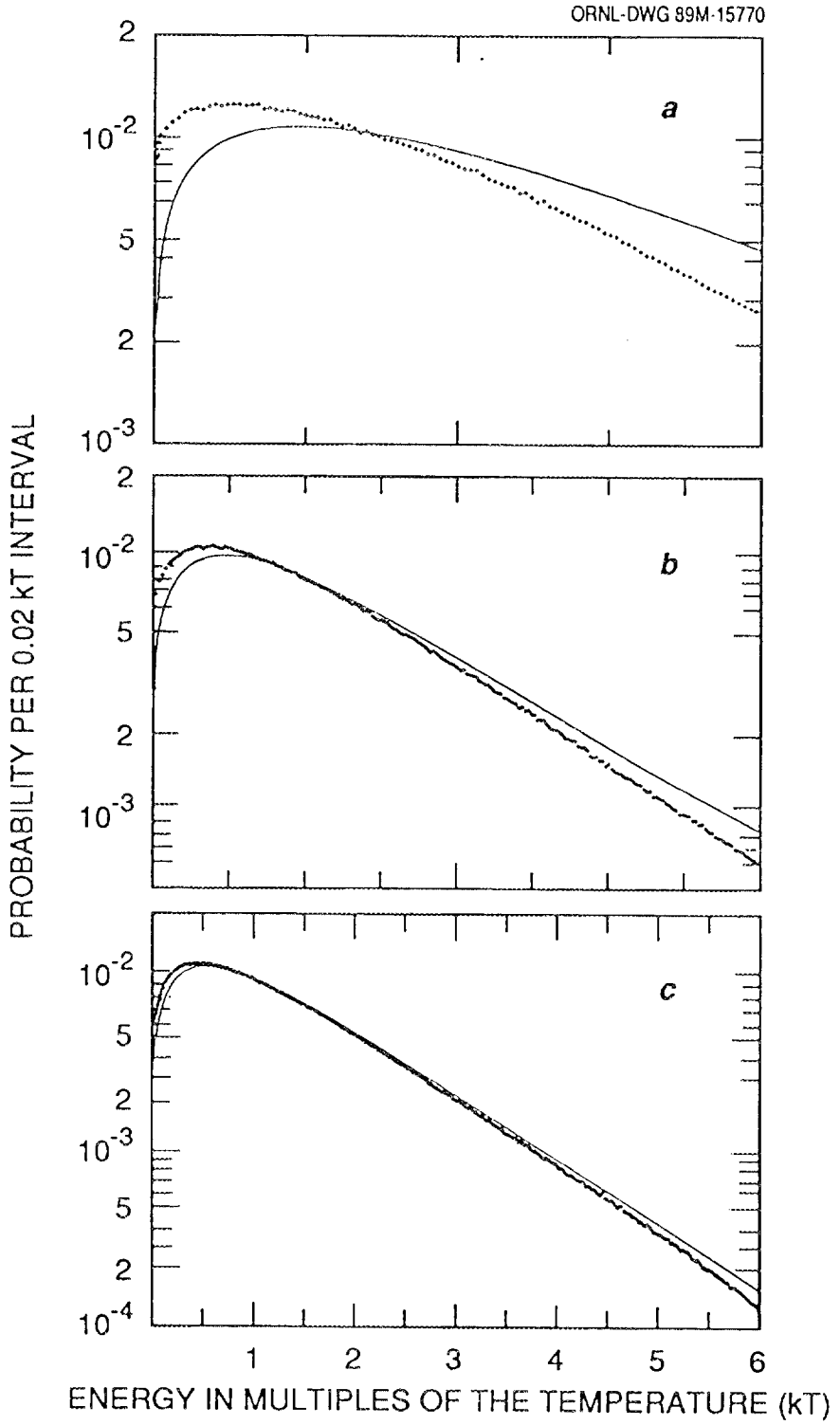

Fig. 8. Gamma-ray energy distributions for the model as a function of the excitation energy X_c available to each cascade. (a) 2 kT, (b) 4 kT, and (c) 6 kT. The Maxwellian distribution, shown as a solid line, contributes an increasing proportion of the total as X_c and the average multiplicity increase.

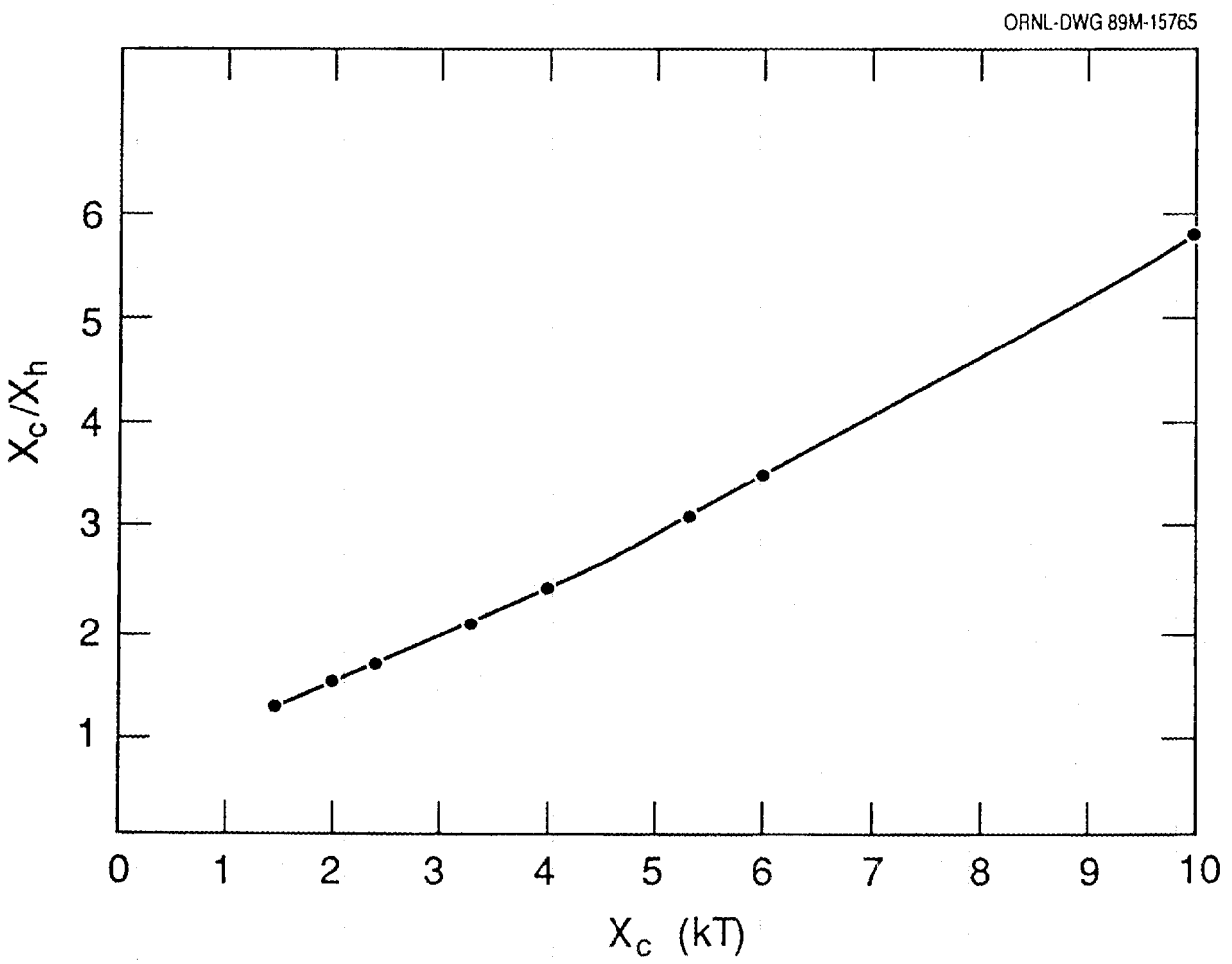

Fig. 9. The ratio of excitation energy to half-peak-height point on the model energy distribution as a function of X_c.

5. MODEL MULTIPLICITIES

Samples of the multiplicity distributions for a range of values of X_c are shown in Fig. 10. Experimental multiplicities are expected to be smaller as the lowest gamma-ray energies are not detected. Multiplicities for a fixed gamma-ray energy bias can be readily calculated in the code.

As the nuclear temperature is the free parameter of the model but the average gamma-ray multiplicity may be the parameter of interest, a number of cases were run to determine their relation. It can be approximated as

$$M = 1 + f * X_c \ . \tag{4}$$

As X_c approaches zero the probability of selecting a gamma ray exceeding X_c approaches unity as seen in Fig. 2; thus, the first term in Eq. (4) is 1. The function f is an empirical coefficient. If X_c is allowed to go to extremely high values, the selected gamma rays are drawn from a nearly complete Maxwellian distribution for which the average energy is 3/2 kT. Already at $X_c = 9.98$, the average Maxwellian energy is 1.498 kT. Thus we might expect the factor f to approach 2/3 as X_c goes to infinity. However, the probability of selecting an energy below the average is over 61%, so deviations from a simple linear relation at low values of X_c are expected.

While mathematically the multiplicity approaches one as the excitation energy goes to zero, the useful range of the model is where it preserves a semblance of the Maxwellian distribution with a well defined peak. The range in X_c is thus about 1.4 to 10, with the larger values more appropriate to fission gamma rays for example.

The random selection code MAX was run for 100000 cascades or more for integral values of X_c from 1 to 9 and for one million cascades at 9.98. Figure 11 shows the average multiplicities found. That for $X_c = 3.000$ was (2.996 ± 0.004) where the uncertainty is based on the square root of the number of cascade gamma rays selected. A constant value of $f = 0.6436$ in the formula above misses the multiplicities for X_c from 2 to 5 by as much as 3%. A modified linear relation

$$M = 1.070 + 0.6380 * X_c \tag{5}$$

fits the useful range $X_c = 1.4$ to 10 to within 1%.

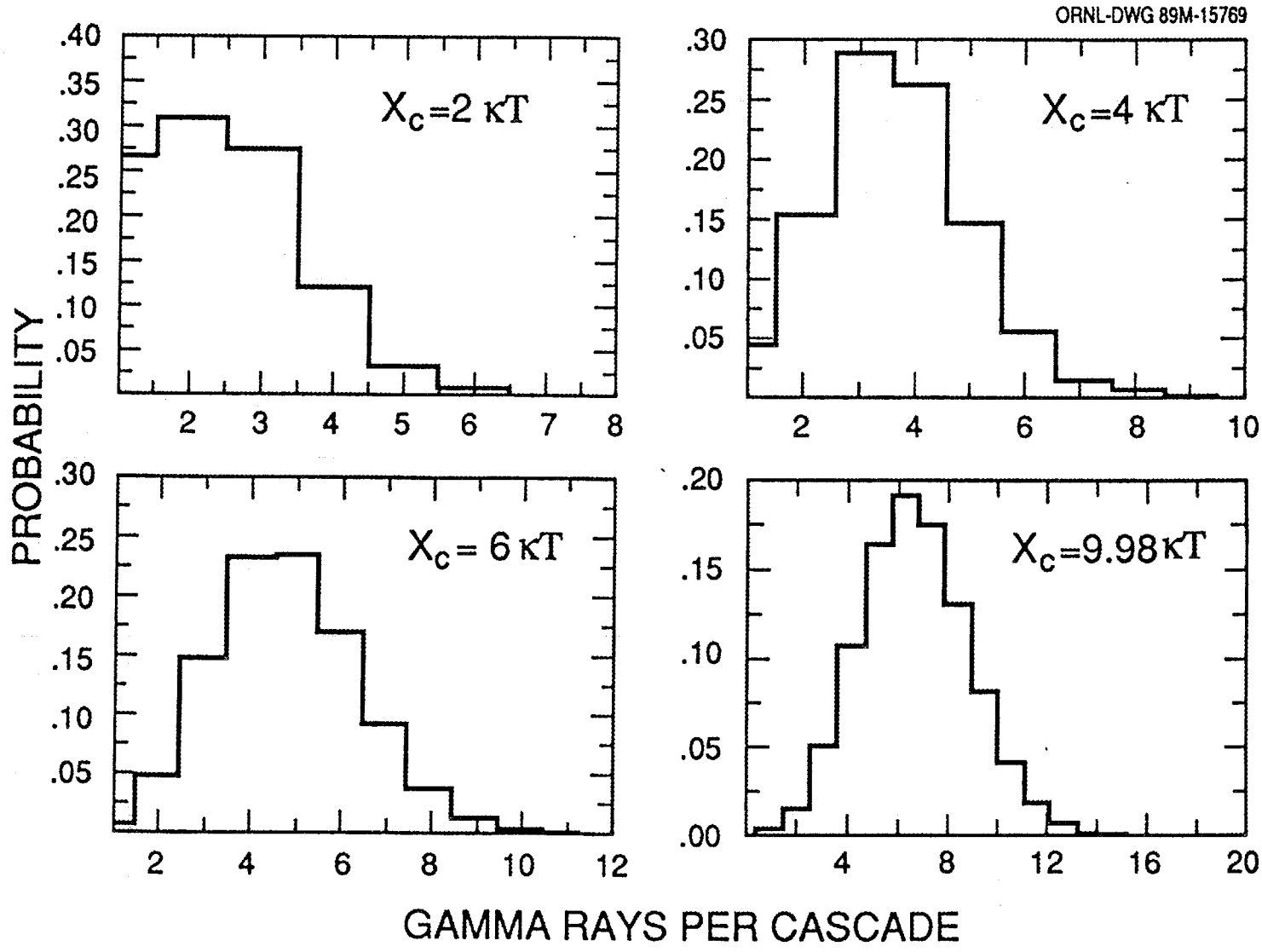

Fig. 10. The Multiplicity distributions for various maximum cascade energies X_c. The proportion of single gamma rays directly to the ground state can be readily modified without recomputing the model.

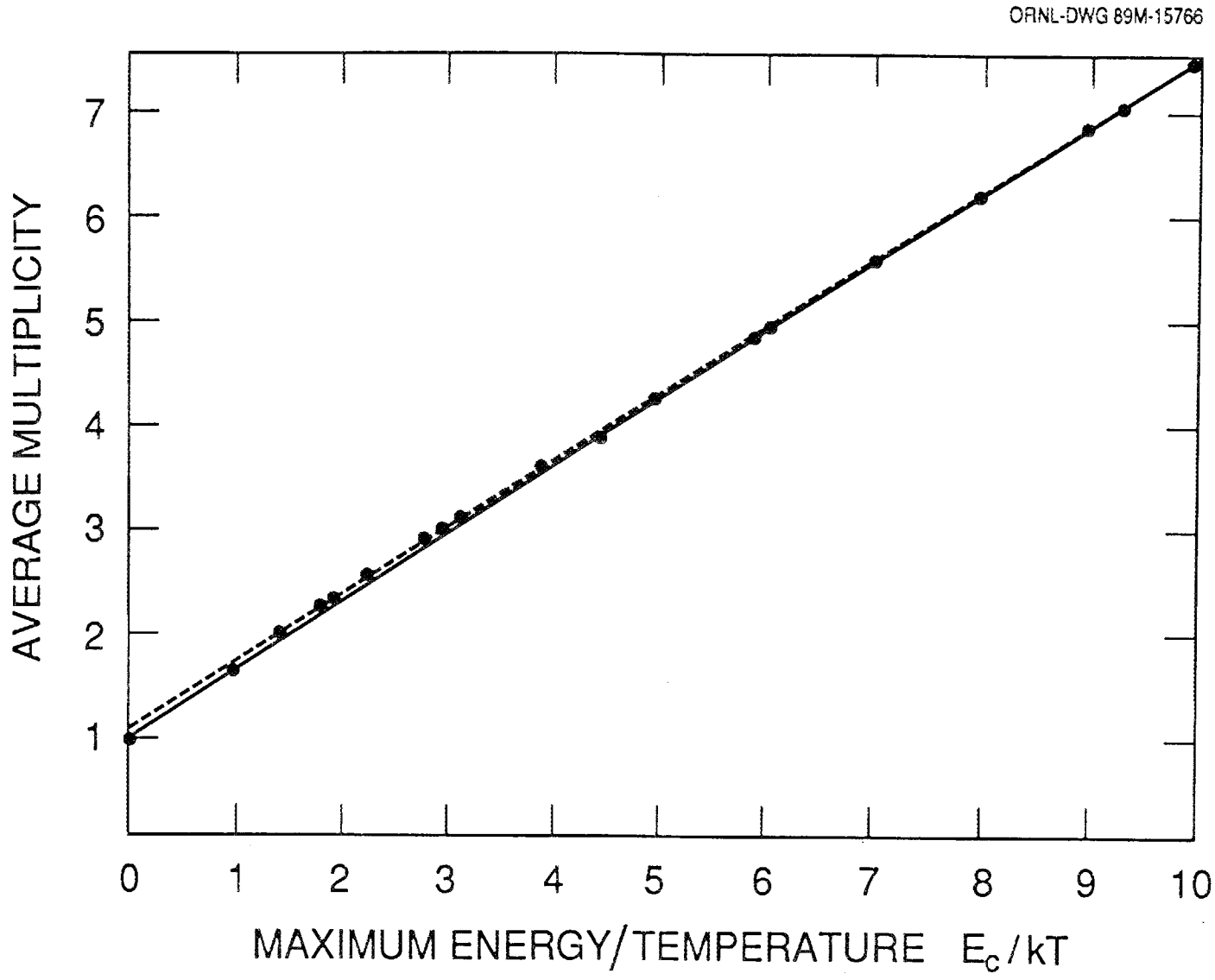

Fig. 11. Average multiplicities found for various maximum cascade energies. The solid line fits fairly well. In the range $X_c = 2$ to $X_c = 3$, it lies as much as 3% below the points. The dashed line covers the range $X_c = 1.4 - 10$ within 1%, allowing a closer estimation of the model temperature required to fit a desired multiplicity at a given excitation energy.

6. COMPARISON WITH EXPERIMENT

A number of isotopes above atomic mass 100 show thermal neutron capture gamma ray spectra with substantial unresolved components similar to the model spectra investigated here. In particular, spectra of rhodium, silver, samarium, europium, gadolinium, erbium, hafnium, tantalum, rhenium, iridium and thorium[4] show such a shape. Where a single isotope predominates, the available excitation energy is essentially equal to its neutron binding energy, and thus one feature of the model is automatically satisfied.

Thermal neutron capture on tantalum offers a case with high-level density where relevant data exist.[4] The binding energy for the principal isotope ^{181}Ta is 6.0629 MeV, and compound nuclear spins 3 and 4 can be formed. Prompt gamma rays lead to the 115 day $t_{1/2}$ ground state and a 280-ms isomer at 16.5 keV. The early work with magnetic analysis of electrons from a thin converter led quite directly to average gamma-ray spectra. Note that the corrected spectra of Groshev et al.[4] include the radius of curvature and magnetic field strength of his apparatus to facilitate line strength comparison. That feature has been factored out in the comparison to be shown. Most of the more recent work has used germanium crystal gamma spectrometers and their peak response to measure relative strengths of individual gamma rays. For tantalum a great many capture gamma rays below 1 MeV and above 3.88 MeV are now known.[6] From 3.88 MeV to the binding energy at 6.063 MeV 70 gamma rays are listed as primary. Since the probability of primary transitions per neutron captured must sum to unity, a normalization factor of 0.072 was given[6] corresponding to the intensities of these gamma rays (and no others). The normalized intensities per neutron capture have been grouped in approximately half-MeV intervals and shown in Fig. 12 as a histogram.

The model parameters were chosen to come close to the data in the range above 1 MeV, and include 11.5% of the gamma-rays at the 6.06 MeV cutoff energy, corresponding to the two highest energy primary transitions.[6] The model shows more gamma-rays below 1 MeV than the data, from which, however, the strong peak at 0.511 MeV has been deleted as if it were entirely due to positron annihilation gamma rays. The model multiplicity distribution is shown in Fig. 13 and its average is 3.56. The average multiplicity for the model with gamma-rays below 0.75 MeV not counted is 2.54, corresponding more nearly to a coincidence experiment with biased gamma-ray detectors.

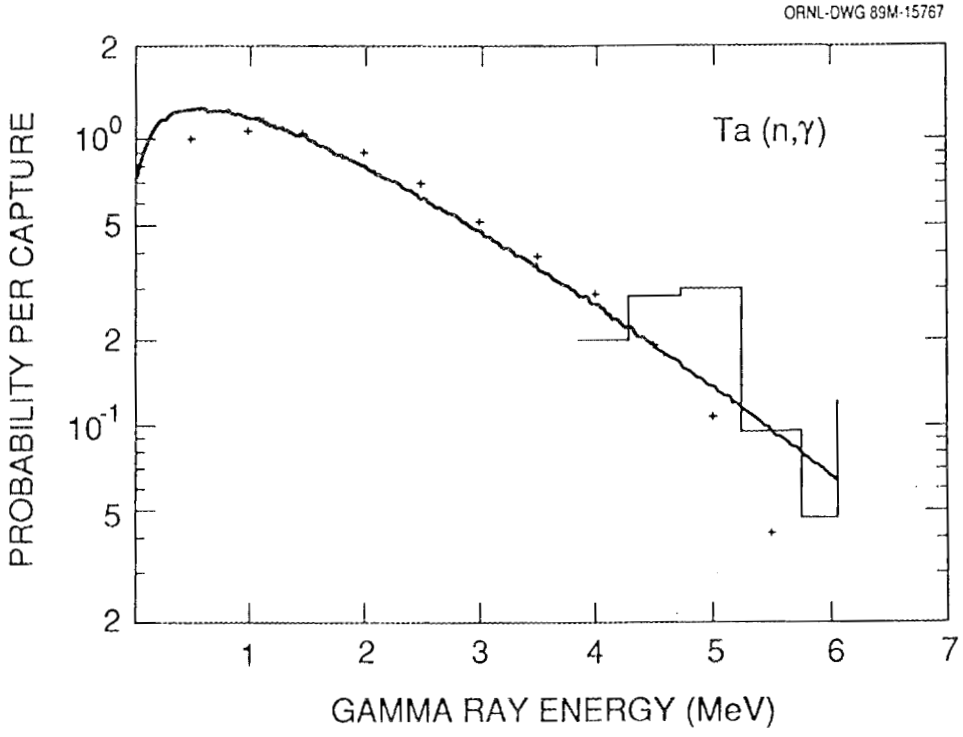

Fig. 12. Gamma-ray energy distribution per thermal neutron capture by tantalum. The crosses approximate the measurements of Groshev et al.[4] The histogram represents primary gamma-ray transition probabilities as evaluated by Firestone.[6] The probabilities for 6.063 MeV and 5.965 MeV gamma rays were summed to give a 0.115 "singles" probability used with the Maxwellian model shown as a solid line.

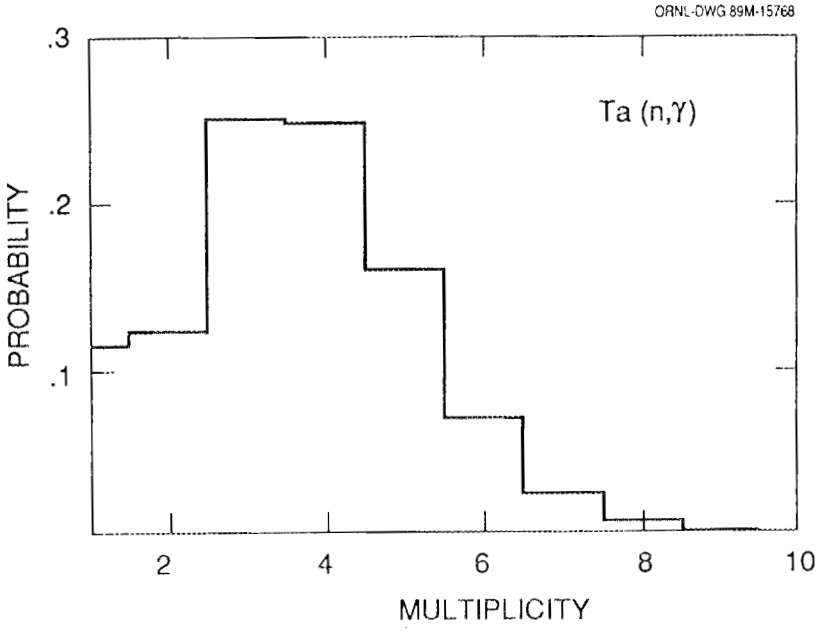

Fig. 13. The gamma-ray multiciplicity distribution for the model parameters of Fig. 12. Single gamma rays at the 6.06-MeV cutoff were set at 11.5% of the total. The average multiplicity for all gamma rays in the model was 3.56.

7. CONCLUSIONS

The excitation energy limited Maxwellian cascade model investigated provides a plausible description of nuclear gamma-ray cascades in the limit of high level densities. As such it can be used to supplement detailed but partial level-to-level transition data where a more complete description of neutron capture gamma-ray cascades is needed, or where an average prediction is sufficient. Complete energy and multiplicity distributions for the model were investigated for a range of values of the free parameter, the nuclear temperature. A satisfactory comparison with existing measurements for thermal neutron capture by tantalum was found. Further work suggested involves incorporating the pulse-height response matrix of a detector system and adjusting the model parameter by least squares or other criteria to measured spectra.

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APPENDIX A

The FORTRAN coding was developed in several stages. The model behavior was studied in terms of the dimensionless parameter $x = E/kT$ in 2% steps with arrays of 500, even for the multiplicity. The ASCII output in a MAX.LOG file was read into an ORELA Data File (ODF) with a simple command file MAXREAD.CMD where plots could be studied and a few simple algebraic transformations could be done with the FORODF program.[7] The integration over steps was originally done with Gaussian quadratures but replaced by the more analytic form (3) as suggested by Peter Fu. When the model behavior study was completed, the code was adapted to accept and generate output in energy units. The step size was set at 20 keV and the maximum excitation energy was set at 15 MeV, thus increasing the output arrays to 750. This led to a concern with the multiplicity array as it was found that 25 was a practical upper limit, but the ODF format required equal length for all arrays in one file. Rather than form a separate file for the multiplicity, the energy range was cut back to 14.5 MeV and the last 26 channels used to store the multiplicity. MAXR.ODF and MAXRREAD.CMD accommodate the longer files and leave two sections for experimental data and/or uncertainties. Vestiges of the earlier use of the program probably remain in the FORTRAN listing which follows.

```
c     set up a Maxwellian array for use in checking the gamma ray
c     cascade model where energies are picked from it at random until
c     their sum exceeds an excitation energy.  rlm  6/2/89
c     Files max.for, max.bat, max.odf, maxread.cmd, max.rpt etc.
c
      Dimension ehi(750),eav(750),a5(500),sa5(500)
      Dimension test(500),dist(25),spect(500),rspec(750)
c
C     set channel width .02 (*kT); to be scaled later to MeV.
      E=0.02
      E2=0.5*E
C     Maxwellian infinite area normalization constant *2/√(π)
      anorm=1.12837917
c
c
c
      x=-e2
      do 1 i=1,500
      x=x+e
      ehi(i)=x+e2
      eav(i)=x
      test(i)=0.0
      if (i.le.25) dist(i)=0.0
      spect(i)=0.0
      sa5(i)=eMaxint(ehi(i))
      if (i.eq.1)a5(i)=sa5(i)
      if (i.gt.1)a5(i)=sa5(i)-sa5(i-1)
    1 continue
c
      do 2 i=501,725
      eav(i)=eav(i-1)+e
    2 ehi(i)=ehi(i-1)+e
      do 3 i=726,750
    3 eav(i)=i-725
```

```
c
      ncasc=200000
      rn=1.0/float(ncasc)
      rnsum=0.0
c     initialize ran function before use
      ivar=6553657
      rx=ran(ivar)
c     generate cascades with maximum excitation energy  MeV
      accept 110,ExMeV
  110 format (f10.0)
      limtag=0
c     set a reasonable range of excitation energies
      if (ExMeV.lt.4.5) limtag=1
      if (ExMeV.lt.4.5) ExMeV=4.5
      if (ExMeV.gt.14.5) limtag=2
      if (ExMeV.gt.14.5) ExMeV=14.5
c     reset ExMeV to top of nearest channel.
      mmax=nint(ExMeV/e)
      ExMeV=ehi(mmax)
      mmaxm1=mmax-1
c     generate cascades with cutoff so many times kT
      accept 110,EkT
c     set a range of cutoffs 1.38 to 9.98
      if (EkT.lt.1.38) limtag=1
      if (EkT.lt.1.38) EkT=1.38
      if (EkT.gt.9.98) limtag=2
      if (EkT.gt.9.98) EkT=9.98
c     reset EkT to top of nearest channel.
      nmax=nint(EkT/e)
      EkT=ehi(nmax)
c     set bias at 0.75 MeV to get biased average multiplicity
      bx=0.75*EkT/ExMeV
      nbx=ifix(0.5+bx/e)
c
c
c     accept 110,avm
c     set a reasonable range of model+singles multiplicity
c     if (avm.lt.1.05) limtag=3
c     if (avm.lt.1.05) avm=1.05
c     if (avm.gt.7.457) limtag=4
c     if (avm.gt.7.457) avm=7.457
c
      accept 110,sngl
c     set a reasonable range of desired singles fraction
      modtag=1
      if (sngl.gt.0.00) go to 9
      modtag=-1
c     negative sngl to be replaced with model sngl
c
c     assuming avm is the desired model multiplicity
c     xcest=(avm-1.07)/0.638
c     nmax=ifix(0.5+(xcest/e))
c     go to 33
c
    9 if (sngl.gt.0.95) limtag=5
      if (sngl.gt.0.95) sngl=0.95
c
c     calculate the desired multiplicity of multiple-step cascades
c     avmm=(avm-sngl)/(1.0-sngl)
c     check for avmm in reasonable model range X_c = 1.4 - 9.98
c     if (avmm.lt.2.690) limtag=6
```

```
c     if (avmm.lt.2.690) avmm=2.690
c     if (avmm.gt.7.457) limtag=7
c     if (avmm.gt.7.457) avmm=7.457
c     estimate the model parameter to give it.
c     do 30 i=499,58,-1
c     nmax=i
c     linear formula for multiplicity vs Xc good to 1%
c     but inversion gives a minimum avmmc=2.674815 at i=58  6/29/89
c     avmmc=(1.070+(0.6380*ehi(i+1))+sa5(i)-1.0)/sa5(i)
c     if (avmmc.le.avmm) go to 32
c  30 continue
c     type 117,nmax,avmmc
c 117 format (' Finder loop exceeded, nmax= ',i4,' avmmc= ',f8.5)
c     stop
c  32 continue
   33 continue
      if (nmax.lt.3) limtag=9
      if (limtag.eq.9) type 119,limtag
      if (nmax.lt.3) stop
      nmaxm1=nmax-1
      if (modtag.eq.-1) sngl=1.0-sa5(nmaxm1)
      umsngl=1.0-sngl
      cutoff=ehi(nmax)
      if (cutoff.gt.10.0) limtag=8
      if (limtag.eq.8) type 119,limtag
      if (cutoff.gt.10.0) stop
c     reserve last channel for singles = full energy bin
c
c     find probability up to nmax and set ran scale to match
      scale=sa5(nmaxm1)
      rns=rn*umsngl
      rnssum=0.0
c
      ksum=0
      kbsum=0
c     ksum to total the number of gamma rays summed over all cascades
c     kbsum for those above a bias (0.75 Mev set above)
      do 10 i=1,ncasc
      nsum=0
c     nsum to total the energies for each cascade in terms of channels
      do 13 k=1,500
      ik=k
      rx=ran(ivar)*scale
      do 11 j=1,500
      ii=j
      if (rx.le.sa5(j)) go to 12
   11 continue
c
   12 continue
      nsum=nsum+ii
      if (nsum.ge.nmax) go to 14
      if (ii.ge.nbx) kbsum=kbsum+1
   13 continue
   14 continue
c     if the last gamma energy overflowed, lop it and store lopped
c     gamma distribution in test
      ilast=nmax-nsum+ii
      if (ilast.gt.0) test(ilast)=test(ilast)+rn
      if (ilast.gt.0.and.ilast.gt.nbx) kbsum=kbsum+1
      if (ik.gt.25) stop
      dist(ik)=dist(ik)+rns
```

```
      ksum=ksum+ik
   10 continue
c     put in fraction of singles called for
      dist(1)=sngl
      do 22 i=1,nmaxm1
      rnsum=rnsum+test(i)
      if (i.le.25) rnssum=rnssum+dist(i)
   22 continue
c
c     calculate the average multiplicity for more than one gamma-ray
c     in the cascade
      avmult=float(ksum)/float(ncasc)
      Bmult=float(kbsum)/float(ncasc)
      avmm=avmult
      fcmaxwl=avmult-1.0
      cmodmult=(avmult*scale)+1.0-scale
c     calculate the factors for the full energy distribution
      flop=umsngl/avmult
      fmaxwl=fcmaxwl*umsngl/(avmult*scale)
      spect(nmax)=sngl
c
      rnck=1.0/rnsum
      rnsck=umsngl/(rnssum-sngl)
      avxlop=0.0
c
      do 20 i=1,500
c     renormalize for repeated roundoff errors on rn and rns
      if (test(i).ge.rn) test(i)=test(i)*rnck
      avxlop=avxlop+test(i)*eav(i)
      if (i.gt.25) go to 21
      if (dist(i).ge.rns.and.i.gt.1) dist(i)=dist(i)*rnsck
   21 continue
c
      if (i.lt.nmax) spect(i)=(a5(i)*fmaxwl)+(test(i)*flop)
   20 continue
      rspec(726)=sngl
      do 24 i=727,750
   24 rspec(i)=dist(i-725)
      avmult=(avmult*umsngl)+sngl
c     reset biassed multiplicity to include singles fraction
      Bmult=(Bmult*umsngl)+sngl
      avx=(1.5*fmaxwl+avxlop*flop)*umsngl+cutoff*sngl
c
c     section to rebin the calculated spectrum into MeV units.
c     remember channel nmax contains the singles probability.
      rspec(mmax)=spect(nmax)
c     calculate the rebinned channel width in kT units
      r=e*ehi(nmaxm1)/ehi(mmaxm1)
      i=0
      reb=0.0
      esum=0.0
      erest=0.0
      rrest=0.0
      do 40 j=1,mmaxm1
      esum=erest
      reb=rrest
c     check whether the last overage produces another also
      if (esum.ge.r) go to 44
   42 continue
      i=i+1
      esum=esum+e
```

```
      reb=reb+spect(i)
      if (esum.lt.r)go to 42
   44 continue
c     calculate overage in energy and channel probability content
      erest=esum-r
      rrest=erest*spect(i)/e
c     store in rebinned channel j and go on to j+1
c     renormalize to per MeV units; one gamma per capture
      rspec(j)=(reb-rrest)/e
   40 continue
      do 46 i=1,750
      type 100,eav(i),rspec(i)
  100 format (2e16.8)
   46 continue
      if (modtag.lt.0) type 118
  118 format (' Singles fraction reset to model estimate.')
      if (limtag.gt.0) type 119,limtag
  119 format (' A parameter has  reached a programmed limit.',i2)
      type 120,ExMeV,cutoff,avmult,avx,dist(1)
  120 format(' ExMeV',2f6.2,'*kT avmult',f8.5,' avx',f8.5,' sngls',f8.5)
      type 130,mmax,avmm,Bmult
  130 format (' mmax',i4,' multiples alone',f8.5,' bias .75 MeV',f8.5)
      stop
      end
      Function eMaxint (z)
c     calculate the integral of the Maxwellian energy distribution
c     from 0 to x, using the parametrized error function erf
C     Maxwellian infinite area normalization constant *2/√(π)
      anorm=1.12837917
      x=ABS(z)
      y=sqrt(x)
c     emaxint=erf(y)-anorm*y*exp(-x)
c     incorporate algorithm for erf(y)
C     APPROXIMATE ERROR INTEGRAL ERF(y) GOOD TO 1.5 E-07 ABS.
C     FROM HASTINGS VIA ABRAMOWITZ AND STEGUN  7.1.26  PG 299     RLM
      P=0.3275911
      A1=.254829592
      A2=-.284496736
      A3=1.421413741
      A4=-1.453152027
      A5=1.061405429
      T=1./(1.+P*y)
      POL=A1*T+A2*T*T+A3*T*T*T+A4*T*T*T*T+A5*T*T*T*T*T
c     Erf=1.0-POL*EXP(-y*y)
      emaxint=1.0-(POL+anorm*y)*exp(-x)
      RETURN
      END
```

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