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Optimal Control Theory of Load-Following and Parameter-Tracking of Nonlinear Systems: An Application of Pontryagin Maximum Principle to Reactor Dynamics

Carlos March-Leuba
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OPTIMAL CONTROL THEORY OF LOAD-FOLLOWING AND PARAMETER-TRACKING OF NONLINEAR SYSTEMS: AN APPLICATION OF PONTRYAGIN MAXIMUM PRINCIPLE TO REACTOR DYNAMICS

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OPTIMAL CONTROL THEORY OF LOAD-FOLLOWING AND PARAMETER-TRACKING OF NONLINEAR SYSTEMS: AN APPLICATION OF PONTRYAGIN MAXIMUM PRINCIPLE TO REACTOR DYNAMICS

ABSTRACT

A demand-following and parameter tracking algorithm has been developed which utilizes the Pontryagin Maximum Principle (PMP). Starting from a variational principle, we have derived the methodology for the construction of a Hamiltonian function, with the analytical properties required by the application of the Pontryagin Maximum Principle method to Free Terminal Time optimization problems. A crucial result has been the conversion of the two point boundary value problem, typical of the Pontryagin Maximum Principle method for nonlinear systems, into a noniterative initial value problem. The introduction of sensor signals information, as a set of differential equations complementing the model's equation, allows the reformulation of parameter tracking algorithms as control optimization problems, where the demands are the plant signals, and the controls the time-varying plant parameters. The present algorithm allows correction for the time delays which affect the information flowing from the plant. The control algorithm presented in this paper has been validated against a nonlinear model of a nuclear power plant with time-varying parameters.

I. INTRODUCTION

Important new issues emerge and will continue to arise in the future as new reactor concepts are being investigated, and as the problems related to reactor safety, control, and operation become more sharply focused. The "unforgiving" nature of nuclear technology, vis-a-vis public safety and acceptance, demands the use of reliable surveillance and diagnostic methodologies.

Historically, the control philosophies followed in the nuclear industry have been based on Proportional-Integral-Differential (PID) methodologies or on the application of Pontryagin's Maximum Principle (PMP)[1,2] to linearized plant models. Although these technologies were very successful, as shown by the excellent safety record enjoyed by the Nuclear Industry, the control philosophies derived from them are suboptimal, lack flexibility, and do not have plant parameter tracking capabilities.

Any advanced adaptive control algorithm should:

1. Provide an optimal control philosophy.

2. Provide control flexibility, in the sense that, if one or more controls fail, the system can be kept in the homeostatic region (robustness against control degradation).

3. Provide parameter tracking so that the plant model reproduces the signals coming from the actual plant sensors (robustness against plant condition changes).

4. Treat and compensate for sensor degradation and time delays in the plant signals (robustness against sensor failure).

The PMP methodology coupled with nonlinear dynamical plant models provides a convenient technique to construct control algorithms satisfying the above requirements.

There is a large body of literature in the field of optimal control for nuclear reactors (for the interested reader we recommend the excellent text book by Mohler and Shen[3]). The application of PMP methods to nuclear reactor technology can be divided into two broad categories, the first dealing with applications to critical mass calculations and fuel management problems,[4-7] and the second dealing with reactor control and load-following algorithms. To our knowledge, the first application of the PMP methodology to the latter category, has been the work of Rosztocy *et al.*[8] Since that early work, a growing literature has appeared in this particular field, which has been competently summarized by Lin and Grossman.[9]

A survey of the literature on PMP methods revealed that a major problem arises in connection with the numerical implementation of this methodology for nonlinear systems, having to do with the two-point boundary value problem which arises from the simultaneous solution of two sets of state and adjoint equations, the first set going forward in time the second backwards. The iterative techniques for the solution of this two-point boundary value problem may become too time consuming for on-line computer calculations. Our survey also revealed the lack of PMP applications to dynamical parameter tracking, a feature of control algorithms which is of substantial interest to the operation of load-following, signal-driven dynamical systems.

The purpose of this work is to develop a PMP-based control algorithm, capable of performing simultaneously load-following and parameter-tracking tasks, while avoiding the iterative solution of a two-point boundary value problem. In the present study, the plant and controls are assumed to be in

equilibrium at the beginning ($t=T_0$) and at the end ($t=T_F$) of a transient (load-following period). Hence, the state of the dynamical system is known at the final time, T_F, however, the value of this quantity is not available. We are then dealing with a Free Terminal Time (FTT) problem[2] where the final (terminal) time is not specified.

Section II of this paper deals with the FTT problem using a variational principle to redefine the PMP methodology and to establish the proper setting for its application to the problem at hand. In Section III we develop a formalism which avoids the two-point boundary value problem. The general theory in Section III, is applied in Section IV to the development of the load-following and parameter-tracking algorithm. This algorithm is implemented in Section V, using as an illustration a nonlinear model of a nuclear power plant. Section VI contains a summary of the present work.

II. THE FREE TERMINAL TIME CONTROL PROBLEM

In the Free Terminal Time Control Problem (FTT), the state vector describing the dynamical system is known at the beginning and end of the transient, but the duration of the transient is unknown, which leaves the final time, T_F, as an unspecified quantity. The FTT problem is stated as follows: given a mathematical model of the dynamical system

$$\frac{dX}{dt} = F(X,U) \tag{II.1}$$

with the initial and final conditions

$$X(t=T_0) = X_0 \tag{II.2}$$

$$X(t=T_F) = X_F \tag{II.3}$$

where

$X(t)$ ≡ I_s-dimensional state vector
$U(t)$ ≡ I_c-dimensional control vector
F ≡ I_s-dimensional vector-valued function

and a cost function

$$J = \int_{T_0}^{T_F} dt\ V(X,U) \tag{II.4}$$

where

T_0 = initial time of the transient
T_F = final time of the transient
$V(X,U)$ = a positive definite scalar function

find the necessary condition for a minimum of the J-functional with the constraint that the describing Eq. (II.1) be satisfied at all times.

Following a well known prescription we write the extended cost function

$$J' = \int_{T_0}^{T_F} dt \left\{ V(X,U) - W^T \left(\frac{dX}{dt} - F(X,U) \right) \right\} \tag{II.5}$$

where we introduced the vector, $W(t)$, of Lagrange multipliers, also known in the literature as the adjoint state vector. It is now convenient to rewrite Eq. (II.5) in the form

$$J' = \int_{T_0}^{T_F} dt \left\{ H(X,U,W) - W^T \frac{dX}{dt} \right\} \tag{II.6}$$

where following Pontryagin's work, we introduced the Hamiltonian function

$$H(X,U,W) = V(X,U) + W^T F(X,U) \tag{II.7}$$

Since the duration of the transient has not been specified, the final time, T_F, is a free parameter and its variation has to be accounted for in the calculation of the variation, $\delta J'$, of the cost function.

With this proviso, we take variations in Eq. (II.6) to obtain:

$$\delta J' = \int_{T_0}^{T_F} dt \left\{ \delta H(X,U,W) - \delta W^T \frac{dX}{dt} - W^T \frac{d}{dt} \delta X \right\} \tag{II.8}$$

$$+ \left\{ H(T_F) - W(T_F) \left[\frac{dX}{dt}\right]_{T_F} \right\} \delta T_F$$

where we have

$$\delta H(X,U,W) = \frac{\partial H}{\partial X} \delta X^T + \frac{\partial H}{\partial U} \delta U^T + \frac{\partial H}{\partial W} \delta W^T \tag{II.9}$$

Next we perform the integration by parts below:

$$\int_{T_0}^{T_F} dt\, W^T \frac{d}{dt} \delta X = W^T(T_F)\, \delta X(T_F) - W^T\, (T_0) \delta X(T_0)$$

$$- \int_{T_0}^{T_F} dt\, \frac{dW^T}{dt} \delta X \tag{II.10}$$

Insertion of Eqs. (II.9) and (II.10) into Eq. (II.8), yields:

$$\delta J' = \int_{T_0}^{T_F} dt \left\{ \delta X^T \left[\frac{dW}{dt} + \frac{\partial H}{\partial X}\right] + \delta W^T \left[- \frac{dX}{dt} + \frac{\partial H}{\partial W}\right] + \delta U^T \frac{\partial H}{\partial U} \right\}$$

$$+ H[X(T_F),\ W(T_F),\ U(T_F)]\ \delta T_F + W^T(T_0) \delta X(T_0)$$

$$- W^T(T_F) \left[\frac{dX}{dt} \delta T_F + \delta X(T_F)\right] \tag{II.11}$$

We now pause to investigate the last term in Eq. (II.11), which depends on the free terminal time T_F, and ask ourselves about the relationship between the end-points of state vector trajectories upon a small change δT_F, in the final time. We have (by expanding in Taylor series):

$$X(T_F + \delta T_F) = X(T_F) + \left(\frac{dX}{dt}\right)_{T_F} \delta T_F \tag{II.12}$$

which after taking variations becomes

$$\delta X(T_F + \delta T_F) = \frac{\partial}{\partial T_F} [X(T_F + \delta T_F)]\delta T_F = \left[\frac{\partial X(T_F)}{\partial T_F} + \frac{d}{dt}\frac{\partial X}{\partial T_F}\delta T_F + \left(\frac{dX}{dt}\right)_{T_F}\right]\delta T_F$$

or neglecting second order terms

$$\delta X(T_F + \delta T_F) = \delta X(T_F) + \left(\frac{dX}{dt}\right)_{T_F} \delta T_F \tag{II.13}$$

Insertion of the result in Eq. (II.13) into Eq. (II.11) yields:

$$\begin{aligned} \delta J' = \int_{T_0}^{T_F} dt \left\{ \delta X^T \left[\frac{dW}{dt} + \frac{\partial H}{\partial X}\right] + \delta W^T \left[-\frac{dX}{dt} + \frac{\partial H}{\partial W}\right] + \delta U^T \frac{\partial H}{\partial U} \right\} \\ + H[X(T_F), U(T_F), W(T_F)]\delta T_F + W^T(T_0)\ \delta X(T_0) \\ - W(T_F)\ \delta X(T_F + \delta T_F) \end{aligned} \tag{II.14}$$

All the variations in Eq. (II.14) are arbitrary variations, except for $\delta X(T_0)$ and $\delta X(T_F + \delta T_F)$, which from Eq. (II.3) are fixed (i.e., set equal to zero). Hence, with $\delta X(T_0) = \delta X(T_F + \delta T_F) = 0$, the stationary condition $\delta J' = 0$, is obtained by the vanishing of the coefficients of the arbitrary variations, δX, δW, δU, and δT_F.

We obtain in this way the following set of relations:

$$\frac{dX}{dt} = \frac{\partial H}{\partial W} \tag{II.15}$$

$$\frac{dW}{dt} = -\frac{\partial H}{\partial X} \tag{II.16}$$

$$\frac{\partial H}{\partial U} = 0 \tag{II.17}$$

$$H[X(T_F),U(T_F),W(T_F)] = 0 \tag{II.18}$$

The relations in Eqs. (II.15) and (II.16) are similar in form to the Hamilton equations of Analytical Mechanics, with the adjoint vector, W, playing the role of the conjugate momenta. The result Eq. (II.17) reproduces Pontryagin's Principle for the determination of the optimal controls.

The condition (II.18) for the Hamiltonian evaluated at the final time, T_F, is characteristic of free terminal time problems. Care has to be exercised in satisfying this condition while constructing the cost function.

Since the time variable does not enter explicitly in the Hamiltonian, it is easily shown on account of Eqs. (II.15), (II.16), and (II.17) that the Hamiltonian function is a conserved quantity and equal to zero, because of Eq. (II.18).

The classical solution of the present control problem proceeds as follows: (a) use Pontryagin's Principle [i.e., Eq. (II.17)] to obtain the optimal control vector, U(t) in terms of the adjoint state vector W(t); (b) insert the result into the Hamilton equations (II.15) and (II.16); (c) solve the resulting equation for the state vector, X, forward in time, and the equation for the adjoint state vector backward in time, using the initial and final conditions for the state vector [Eqs. (II.2) and (II.3)]; and (d) go back to step (a) and insert the now known forward and adjoint state vectors into Eq. (II.17) to calculate the optimal control vector.

The classical procedure presented above involves the simultaneous solution of two sets of coupled nonlinear equations, one of the sets going forward in time, the other backwards. Although there are several numerical methods to solve this problem ,their iterative nature makes them slow and difficult to converge to the desired solution.

III. THE OPTIMAL CONTROL ALGORITHM ACTOR

The implementation of the optimal control algorithm discussed in Section II is hindered by two difficulties: (a) the numerical solution of two coupled sets of nonlinear equations, as a boundary value problem, and (b) the construction of a Hamiltonian which must vanish at the end of the transient, when the values of the control vector may not be a known quantity. We develop in this section a modified (FTT) algorithm, named ACTOR, which avoids the above problems. The idea behind ACTOR is to solve the

control problem by starting from the final state of the system, thus following backwards in time the sequence of events in the (FTT) algorithm. The basis of the method is the following time reversal formulation of the PMP theorem: "For system Hamiltonians which are invariant against time-reversal the control problem is solved by the set of equations:"

$$\frac{dX}{dt} = -\frac{\partial H}{\partial W} \tag{III.1}$$

$$\frac{dW}{dt} = \frac{\partial H}{\partial X} \tag{III.2}$$

$$\frac{\partial H}{\partial U} = 0 \tag{III.3}$$

$$H[X(T_0),U(T_0),W(T_0)] = 0 \tag{III.4}$$

The proof of this formulation is easily obtained by first applying the time-reversal operation on the cost function, J', [Eq. (II.6)], which results in

$$J' = \int_{T_0}^{T_F} dt\left\{H(X,U,W) + W^T \frac{dX}{dt}\right\} \tag{III.5}$$

then taking variations on the variables X, U, W, and T_0, which is now the free initial time of the transient and proceeding as in Section II. The time-reversal theorem, in view of Eq. (III.4), avoids the construction of the Hamiltonian at $t=T_F$, satisfying the condition (II.18). However, as expected, the theorem now implies forward time-integration of the adjoint Eqs. (III.2) and backward time-integration of the state vector Eq. (III.1). Notice though, that from the definition (II.7), of the Hamiltonian and from Pontryagin's Principle, [Eq. (III.3)], the optimal control vector, U, does not have any functional dependence on the time-derivative of the state vector. Needed is just the correct value for the vector function, X(t), which can be arrived at by either backward time-integration from the final condition, X_F, or by forward time-integration from the initial condition X_0. By choosing the latter time direction for the integration of the state vector equations, the ACTOR algorithm simply reverses the sign of the derivative, dX/dt in Eq. (III.1). It remains now to find the initial condition for the adjoint vector, W. To this end we use the following initial value theorem:

"If $\partial V/\partial U$ is a function of **U** and the vector-valued function **F(U,X)** is an explicit function of U, and if the system and controls are at equilibrium, it follows that the vector **W**, is also at equilibrium; i.e., $dW/dt=0$." To prove this theorem we take the time derivative of Pointryagin's Principle, Eq. (III.3), to obtain, on account of Eq. (II.7) for the Hamiltonian, the following expression:

$$\sum_{i,j,k} \left[\left(\frac{\partial^2 V}{\partial U_i \partial X_j} \delta jk + \frac{\partial^2 F_i}{\partial U_j \partial X_k} \right) \frac{dX_k}{dt} + \left[\frac{\partial^2 V}{\partial U_i \partial U_j} \delta_{jk} + \frac{\partial^2 F_i}{\partial U_j \partial U_k} \right] \frac{dU_k}{dt} + \frac{dW_i}{dt} \left(\frac{\partial F_i}{\partial U_j} \delta_{jk} \right) \right] = 0 \qquad \text{(III.6)}$$

The conditions set forth in the theorem for the quantities, V, and F_i, prevent the vanishing of the bracketed quantities in (III.6), hence in equilibrium conditions (i.e.: $dX/dt = dU_k/dt=0$), the conclusion of the theorem follows.

In summary the ACTOR algorithm is based on the following scheme

$$\frac{d\mathbf{X}}{dt} = \frac{\partial H}{\partial \mathbf{W}}; \ \frac{d\mathbf{W}}{dt} = \frac{\partial H}{\partial \mathbf{X}}; \ \frac{\partial H}{\partial \mathbf{U}} = 0; \ H[X(T_0),U(T_0),W(T_0)] = 0 \qquad \text{(III.7)}$$

with initial conditions, $\mathbf{X}_0$ and $\mathbf{W}_0$, where the latter is obtained from the initial value theorem, i.e., as the solution of

$$\left(\frac{\partial H}{\partial X}\right)_{T_0} = 0 \qquad \text{(III.8)}$$

The validity of the ACTOR algorithm demands the fulfillment of the following conditions: (a) the Hamiltonian must be invariant against time-reversal; (b) the initial value of the Hamiltonian must be zero; (c) the scalar V(X,U) must be at least quadratic in U; and (d) the vector-valued function, F, must satisfy the condition $\partial F/\partial U \neq 0$.

It is interesting to note that in the ACTOR algorithm the Hamiltonian is not a conserved quantity. Indeed from the relation

$$\frac{dH}{dt} = \left(\frac{\partial H}{\partial \mathbf{X}}\right)^T \frac{d\mathbf{X}}{dt} + \left(\frac{\partial H}{\partial \mathbf{U}}\right)^T \frac{d\mathbf{U}}{dt} + \left(\frac{\partial H}{\partial \mathbf{W}}\right)^T \frac{d\mathbf{W}}{dt} \qquad \text{(III.9)}$$

and on account of Eqs. (III.7) and (II.7) one obtains

$$\frac{dH}{dt} = \frac{dW^T}{dt}\left[\frac{dX}{dt} + F(X,U)\right] \tag{III.10}$$

which shows that the time derivative of the Hamiltonian is zero only at the beginning and end of the transient where the system is at equilibrium.

IV. OPTIMAL CONTROL AND PARAMETER TRACKING

The purpose of this section is to apply the general theory of Section III to develop the ACTOR algorithm for the simultaneous tasks of optimal control and parameter tracking.

We formulate the following problem: "Given a demand, D(t), and a dynamical system with time-varying parameters: (a) determine the optimal controls to match the demand in the least square sense, and (b) perform parameter tracking by matching a set of sensor signals, Y_p, also in the least square sense."

The basis of our approach to the solution of this problem is to couple two (FTT) algorithms, the first FTT optimizes the controls to follow the demand, D(T), keeping all plant parameters fixed, whereas the second FTT optimizes the time-varying parameters to follow the sensor signals, Y(t), keeping the controls fixed. We have the following information: the plant model is a set of coupled nonlinear ordinary differential equations

$$\frac{dX}{dt} = F_1(X,U,a,b) \tag{IV.1}$$

with initial conditions

$$X(t=0) = X_0 \tag{IV.2}$$

where

X	≡	I_s-dimensional state vector (dimensionless)
F_1	≡	I_s-dimensional real vector-valued nonlinear operator
U	≡	I_c-dimensional control vector ($i=1...I_i$) (dimensionless)
a	≡	I_f-dimensional vector of fixed plant parameters (s^{-1})
b	≡	I_v-dimensional vector of time-varying plant parameters (s^{-1})

The vector, Y, is given by the relation

$$Y(t) = \int_0^t dt_1 \, \underline{h}(t-t_1)\left\{\underline{A}_1\underline{P}_m X + Y_0\right\} \tag{IV.3}$$

with

$$Y(t=0) = Y_0 \tag{IV.4}$$

where

Y	$\equiv$	I_m-dimensional vector of sensor signals (dimensionless)
$\underline{h}(t)$	$\equiv$	Response function matrix associated with the measuring system $(I_m X I_m)$ (dimensionless)
$\underline{P}_m$	$\equiv$	projection operator matrix which picks out the subset of measured state variables $(I_m X I_m)$ (dimensionless)
$\underline{A}_1$	$\equiv$	matrix whose elements are rate constants $(I_m X I_m)$ (s^{-1})

For computational purposes it is more convenient to use the differential form equivalent to Eq. (IV.3) below:

$$\frac{dY}{dt} = \underline{A}Y(t) + \underline{A}_1\underline{P}_m X \tag{IV.5}$$

where

$\underline{A} \equiv$ matrix whose elements are rate constants (s^{-1})

with

$$\underline{h}(t) = \frac{1}{2\pi i}\int ds(s\underline{I}-\underline{A})^{-1}e^{st} \tag{IV.6}$$

Based on the previous information, we construct the Hamiltonians, H_1, for the demand following step, and H_2 for the parameter tracking step:

$$H_1 = \frac{1}{2}(D - \underline{P}_d X)^T \, \underline{Q} \, (D - \underline{P}_d X)$$

$$+ \frac{1}{2}(U-U_0)^T \, \underline{R} \, (U-U_0) + W^T F[X,U,b] \tag{IV.7}$$

$$H_2 = \frac{1}{2}\,(Y_p - Y)^T\,\underline{M}\,(Y_p - Y) + \frac{1}{2}\,(b - b_0)^T\,\underline{L}\,(b - b_0)$$

$$+ E^T\,F[X,U,b] + Z^T\,G(Y,X,b) \qquad \text{(IV.8)}$$

with

$$G[Y,X,b] = \underline{A}Y + \underline{A}_1\underline{P}_m X \qquad \text{(IV.9)}$$

and where

Y_p	=	I_m-dimensional vector of plant signals
$\underline{P}_d$	=	projection operator matrix which picks out the subset of state variables are marked to follow a specified demand
$\underline{Q},\underline{R},\underline{M}$	=	weighting matrices whose matrix elements are real, positive and symmetric quantities (s^{-1})
$\underline{L}$	=	as above (seconds)
D	=	I_d-dimensional vector of demands (dimensionless)
U_0	=	control vector at equilibrium (dimensionless)
b_0	=	time varying parameter vector at equilibrium (s^{-1})
W,E,Z	=	adjoint vectors (dimensionless)

Note that the Hamiltonians Eqs. (IV.7) and (IV.8) fulfill the requirements established in Section III for the validity of the ACTOR algorithm. Indeed, since the Hamiltonians do not depend explicitly on the time variable or on the time derivatives of any of the state and control variables, the time-reversal invariance is satisfied. Also, by construction, the Hamiltonians vanish at $t=T_0$, and are quadratic in U.

The next step is to use the Hamiltonians, H_1 and H_2, and the ACTOR algorithm, as shown in Eq. (III.7) to derive the following set of equations:

$$\frac{dX}{dt} = \frac{\partial H_1}{\partial W} = \frac{\partial H_2}{\partial V} = F_1[X,U,b] \qquad \text{(IV.10)}$$

$$\frac{dY}{dt} = \frac{\partial H_2}{\partial Z} = G[Y,X,b] \qquad \text{(IV.11)}$$

$$\frac{dW}{dt} = \frac{\partial H_1}{\partial X} = \left[\frac{\partial F}{\partial X}\right]^T W - P_{\underline{d}}\,\underline{Q}\,(D-X) \qquad \text{(IV.12)}$$

$$\frac{dE}{dt} = \frac{\partial H_2}{\partial X} = \left[\frac{\partial F}{\partial X}\right]^T E + \left[\frac{\partial G}{\partial X}\right]^T Z \tag{IV.13}$$

$$\frac{dZ}{dt} = \frac{\partial H_2}{\partial Y} = \left[\frac{\partial G}{\partial Y}\right]^T Z - \underline{M}\ (Y_p - Y) \tag{IV.14}$$

$$\frac{\partial H_1}{\partial U} = \underline{R}(U-U_0) + \left[\frac{\partial F_1}{\partial U}\right]^T W = 0 \tag{IV.15}$$

$$\frac{\partial H_2}{\partial b} = \left[\frac{\partial F}{\partial b}\right]^T E + \left[\frac{\partial G}{\partial b}\right]^T Z + \underline{L}\ (b-b_0) \tag{IV.16}$$

with the initial conditions

$$X_i(T_0) = 1 \quad (i=1\ldots I_s) \tag{IV.17}$$

$$Y_i(T_0) = 1 \quad (i=1\ldots I_m) \tag{IV.18}$$

$$\left(\frac{dW_i}{dt}\right)_{T_0} = 0 \quad (i=1\ldots I_s) \tag{IV.19}$$

$$\left(\frac{dE_i}{dt}\right)_{T_0} = 0 \quad (i=1\ldots I_s) \tag{IV.20}$$

$$\left(\frac{dZ_i}{dt}\right)_{T_0} = 0 \quad (i=1\ldots I_m) \tag{IV.21}$$

The results of Eqs. (IV.10) and (IV.11) recover the model Eq. (IV.1) and the sensor signal vector, **Y**, as calculated by the model. Equation (IV.12), defines the adjoint vector, **W**. The source term on the rhs of Eq. (IV.12) is proportional to the deviation from the prescribed demand. The vector, **Z**, adjoint to the signal vector, **Y**, is defined by Eq. (IV.14),

with a source proportional to the model's departure from the sensor signals flowing from the real plant, $\mathbf{Y}_p$. The adjoint vector, **E**, defined by Eq. (IV.13), plays the role of coupling the demand following and signal matching tasks of the algorithm. Equation (IV.15) is Pointryagin's Principle for the determination of the optimal controls, whereas the result (IV.16) provides an expression for the calculation of the time-varying parameters. Note that this result shows that the parameter vector, **b**, is determined as the set of controls, to be manipulated so that the plant model actually matches the plant sensors signals.

V. IMPLEMENTATION OF THE ACTOR ALGORITHM

In this section we illustrate the use of the ACTOR algorithm for the control and parameter tracking of a nonlinear dynamical system. For this illustration we use a one-point reactor kinetics model with a single group of delayed neutron precursors, and temperature feedback. The thermohydraulic loop is described by heat balances in the fuel and coolant lumps. The system is controlled by adjustment of core reactivity and the value of the inlet core temperature. The system equations are:

$$\frac{dX_1}{dt} = \frac{1}{\Lambda}\,(\rho_0 + U_1 - \alpha_F X_3 - \alpha_c X_4 - \beta)X_1 + \lambda_p X_2 \tag{V.1}$$

$$\frac{dX_2}{dt} = \frac{1}{\Lambda}\,\beta X_1 - \lambda_p X_2 \tag{V.2}$$

$$\frac{dX_3}{dt} = \lambda_q\, X_1 - \lambda_F X_3 + \lambda_F X_4 \tag{V.3}$$

$$\frac{dX_4}{dt} = \lambda_c X_3 - (\lambda_c + \lambda_R)X_4 + \lambda_R(X_{CL} + U_2) \tag{V.4}$$

where all state variables are dimensionless i.e.:

P_0 ≡ Steady state reactor power
T_{0c} ≡ Steady state coolant temperature
X_1 ≡ Power/P_0
X_2 ≡ (Precursor concentration in units of power)/P_0

X_3 ≡ (Fuel temperature)/T_{0c}
X_4 ≡ (Coolant temperature)/T_{0c}
ρ_0 ≡ $\alpha_F X_3(0) + \alpha_c X_4(0)$ initial feedback reactivity (reactivity)
α_{0F} ≡ Fuel temperature feedback coefficient (C^{-1})
α_{0C} ≡ Coolant temperature feedback coefficient (C^{-1})
α_F ≡ $\alpha_{0F}\ T_{0C}$ (dimensionless)
α_C ≡ $\alpha_{0C}\ T_{0C}$ (dimensionless)
β ≡ Delayed neutron fraction
Λ ≡ Neutron generation time (seconds)
λ_p ≡ Delayed neutron precusors decay constant (s^{-1})
λ_q ≡ Power to fuel heat transfer rate constant (s^{-1})
λ_F ≡ Rate constant for heat transport in the fuel (s^{-1})
λ_C ≡ Rate constant for heat transport in the coolant (s^{-1})
λ_R ≡ Inverse coolant transit time (s^{-1})
U_1 ≡ Excess control rod worth from equilibrium conditions (reactivity)
X_{CL} ≡ (Initial inlet coolant temperature)/T_{0c}
U_2 ≡ Coolant inlet temperature control in units of T_{0c} (dimensionless)

The sensor signals are: Y_1 power level, Y_2 fuel temperature, and Y_3 coolant temperature. For the present illustration it was assumed that the unit impulse response function for the power signal monitoring system was a δ-function (i.e., flat frequency response), hence we make $Y_1 = X_1$. The temperature signals were assumed to be thermocouple readings, related to the fuel and coolant temperatures by

$$\frac{dY_2}{dt} = \lambda_{DF}(X_3 - Y_2) = G_2[X,Y,b] \tag{V.5}$$

$$\frac{dY_3}{dt} = \lambda_{DC}(X_4 - Y_3) = G_3[X,Y_c b] \tag{V.6}$$

where Y_2 and Y_3 are dimensionless quantities, and

λ_{DF} = break frequency for the fuel thermocouple transfer function (s^{-1})
λ_{DC} = break frequency for the coolant thermocouple transfer function (s^{-1})

The design values for the plant parameters are listed in Table I. The time-varying parameters are:

$$b^T = (\alpha_c, \lambda_{DF}, \lambda_{DC}) \tag{V.7}$$

Table I. Design values used for the validation of the ACTOR algorithm[a]

Parameter	Value	Units
α_{0F}	-1.1E-5*TCO	Reactivity
α_{0C}	-2.0E-4*TCO	Reactivity
β	0.006898	Dimensionless
Λ	17.9E-6	Second
λ_p	0.08326	1/s
λ_q	248.066/TCO	1/s
λ_F	0.25322	1/s
λ_C	0.9699	1/s
λ_R	1.42696	1/s
λ_{DF}	1/15	1/s
λ_{DC}	1/15	1/s

[a] P_0 = 3436 MW.

TCLO = 283.6 (C)

and the demand vector is

$$D^T = (D_1, D_4) \tag{V.8}$$

where D_1 is the demand in power and D_4 the demand in coolant temperature. For simplicity, the weighting matrices $\underline{Q}$, $\underline{R}$, $\underline{M}$, and L are defined as diagonal matrices.

For the present model, the Hamiltonians, H_1 and H_2, Eqs. (IV.7) and (IV.8) respectively, become

$$H_1 = \frac{1}{2}\left\{(D_1-X_1)^2Q_1 + (D_4-X_4)^2Q_4 + U_1^2R_1 + U_2^2R_4\right\} + \sum_{i=1}^{4} W_iF_i \tag{V.9}$$

$$H_2 = \frac{1}{2}\left\{(Y_{p1} - X_1)^2M_1 + (Y_{p2} - Y_2)^2M_2 + (Y_{p3} - Y_3)^2M_3 + (\alpha_C - \alpha_{0C})^2L_1 + (\lambda_{DF}-\lambda_{0DF})^2L_2 + (\lambda_{CF}-\lambda_{0CF})^2L_3\right\} + \sum_{i=1}^{4} E_iF_i + Z_2G_2 + Z_3G_3 \tag{V.10}$$

with

$$F_1 = \frac{1}{\Lambda}(\rho_0 + U_1 - \alpha_FX_3 - \alpha_CX_4 - \beta)X_1 + \lambda_pX_2 \tag{V.11}$$

$$F_2 = B/\Lambda\ X_1 - \lambda_pX_2 \tag{V.12}$$

$$F_3 = \lambda_qX_1 - \lambda_FX_3 + \lambda_FX_4 \tag{V.13}$$

$$F_4 = \lambda_C X_3 - (\lambda_C + \lambda_R)X_4 + \lambda_R(X_{CL} + U_2) \tag{V.14}$$

$$G_2 = \lambda_{DF}(X_3 - Y_2) \tag{V.15}$$

$$G_3 = \lambda_{DC}(X_4 - Y_3) \tag{V.16}$$

and where we shall remind ourselves that we made $Y_1 = X_1$.

On the basis of the above Hamiltonians, one recovers from Eqs. (IV.10) and (IV.11), the model Eqs. (V.1) to (V.4), and Eqs. (V.5) and (V.6) for the model estimation of the plant signals. Use of Eqs. (IV.12) to (IV.16) yields equations for the adjoint vectors and relations for the calcultion of the optimal set of controls and plant parameters i.e.:

$$\frac{dW_1}{dt} = \frac{W_1}{\Lambda}(\rho_0 + U_1 - \alpha_F X_3 - \alpha_C X_4 - \beta) + \frac{\beta}{\Lambda} W_2 + \lambda_q W_3 - (D_1 - X_1)Q_1 \tag{V.17}$$

$$\frac{dW_2}{dt} = \lambda_p(W_1 - W_2) \tag{V.18}$$

$$\frac{dW_3}{dt} = -\frac{\alpha_F X_1}{\Lambda} W_1 - \lambda_F W_3 + \lambda_C W_4 \tag{V.19}$$

$$\frac{dW_4}{dt} = -\frac{\alpha_c}{\Lambda} X_1 W_1 + \lambda_F W_3 - (\lambda_C + \lambda_R)W_4 - (D_4 - X_4)Q_4 \tag{V.20}$$

$$\frac{dE_1}{dt} = \frac{1}{\Lambda}(\rho_0 + U_1 - \alpha_F X_3 - \alpha_c X_4 - \rho)E_1 + \beta/\Lambda\ E_2$$

$$+ \lambda_q E_3 - (Y_{p1} - X_1)M_1 \tag{V.21}$$

$$\frac{dE_2}{dt} = \lambda_p(E_1 - E_2) \tag{V.22}$$

$$\frac{dE_3}{dt} = -\frac{\alpha_F X_1}{\Lambda} E_1 - \lambda_F E_3 + \lambda_c E_4 + \lambda_{DF} Z_2 \tag{V.23}$$

$$\frac{dE_4}{dt} = -\frac{\alpha_C X_1}{\Lambda} E_1 + \lambda_F E_3 - (\lambda_C + \lambda_R) E_4 + \lambda_{DC} Z_3 \tag{V.24}$$

$$\frac{dZ_2}{dt} = -\lambda_{DF} Z_2 - (Y_{p2} - Y_2) M_2 \tag{V.25}$$

$$\frac{dZ_3}{dt} = -\lambda_{DC} Z_3 - (Y_{p3} - Y_3) M_3 \tag{V.26}$$

$$\frac{\partial H_1}{\partial U_1} = U_1 R_1 + W_1 \frac{X_1}{\Lambda} = 0 \tag{V.27}$$

$$\frac{\partial H_1}{\partial U_2} = U_2 R_2 + \lambda_R W_4 = 0 \tag{V.28}$$

$$\frac{\partial H_2}{\partial \alpha_C} = (\alpha_C - \alpha_{OC}) L_1 - \frac{X_4 X_1}{\Lambda} E_1 = 0 \tag{V.29}$$

$$\frac{\partial H_2}{\partial \lambda_{DF}} = (\lambda_{DF} - \lambda_{ODF}) L_2 + (X_3 - Y_2) Z_2 = 0 \tag{V.30}$$

$$\frac{\partial H_2}{\partial \lambda_{DC}} = (\lambda_{DC} - \lambda_{ODC}) L_3 + (X_4 - Y_3) Z_3 = 0 \tag{V.31}$$

The relations in Eqs. (V.27) to (V.31) are used to eliminate the controls and time-varying parameters from the model and adjoint equations, as well as from the equations for the sensor signals. This procedure results in 16 nonlinear coupled differential equations, which are easily solved numerically by the LSODE algorithm.[10] The controls and time-varying parameters are computed at each integration step from Eqs. (V.27) to (V.31). The calculations performed for the implementation of the present algorithm

are illustrated in the block diagram in Fig. 1. The plant was simulated by the model Eqs. (V.1) to (V.4), and plant signals were generated by means of Eqs. (V.5) and (V.6). The parameters for the plant equations were externally varied as a function of time. To illustrate the flexibility of the algorithm, the demand was made to be an increasing ramp function for the first 100 seconds of the transient, followed by a sinusoidal variation for a duration of 100 seconds, and ending with a decreasing ramp. Figures 2a and 2b show that ACTOR faithfully follows the imposed demand, in both power and coolant temperature. The optimal control motion is shown in Figs. 3a and 3b. Note that the reactivity control was constrained to $U_{max}=0.016$ units, hence it was out of operation several times during the transient. During these outings the coolant temperature control would perform double duty, keeping the reactor in the correct demand-following mode.

The parameter tracking capabilities of the present control algorithm are illustrated in Figs. 4a, 4b, and 4c. The ACTOR algorithm was able to track a sinusoidal time variation of the coolant feedback coefficient, α_C, and to detect changes in the time constants of the sensor's transfer functions. Figure 5 illustrates how ACTOR follows the sensor signals (coolant temperature and fuel temperature signals) which are delayed by the sensors response functions, and uses this information to follow demand in the time scale of the actual plant transient. Control flexibility is achieved through the values assigned to the weighting matrices $\underline{Q}$, $\underline{R}$, $\underline{L}$, and $\underline{M}$. Skillful use of these values allows emphasis or deemphasis of selected options of the control algorithm at any point during the transient. For instance [see Eq. (V.9)] one may deemphasize somewhat the demand following requirement of the coolant temperature by increasing the value of Q_1 with respect to Q_4. The ACTOR algorithm runs faster than real time (real time being defined in relation to the duration of the plant transient) and is a good candidate for on-line computer applications.

VI. SUMMARY AND CONCLUSIONS

A demand-following parameter tracking algorithm has been developed which utilizes the Pontryagin Maximum Principle. Starting from a variational principle, we have derived the methodology for the construction of a hamiltonian function, with the analytical properties required by the application of the PMP method to Free Terminal Time optimization problems. A crucial result has been the conversion of the two point boundary value problem, typical of the PMP method for non-linear systems, into a nonitera-tive initial value problem. The introduction of sensor signals information, as a set of differential equations complementing the model's equation, allows for the reformulation of parameter tracking as a control optimization problem, where the demands are the plant signals, and the controls are the time-varying plant parameters. The present algorithm does

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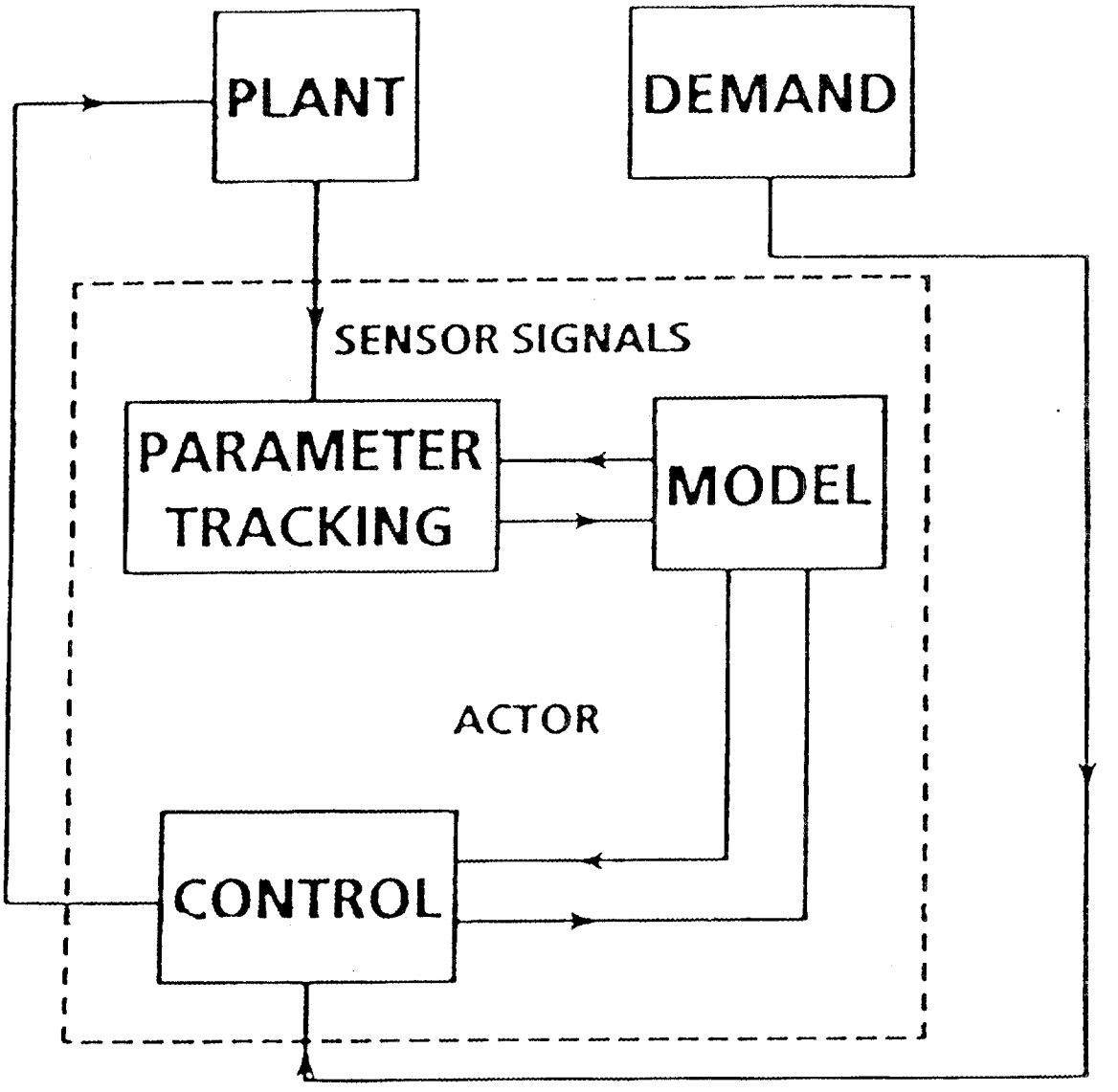

Figure 1. Block diagram for the implementation of the demand-following, parameter tracking ACTOR algorithm.

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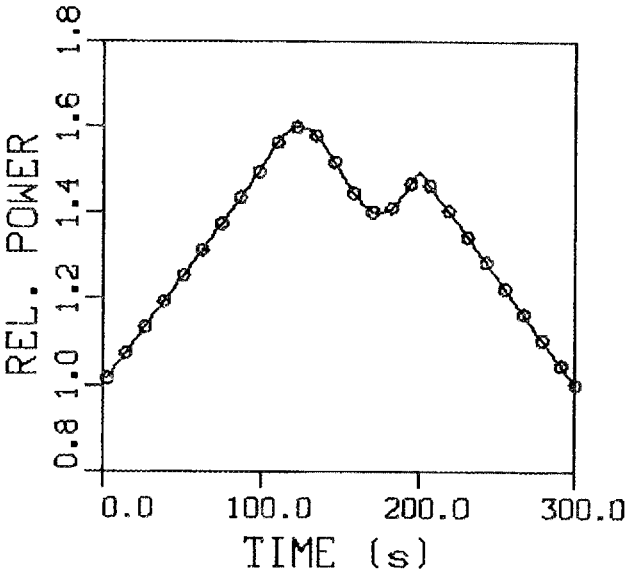

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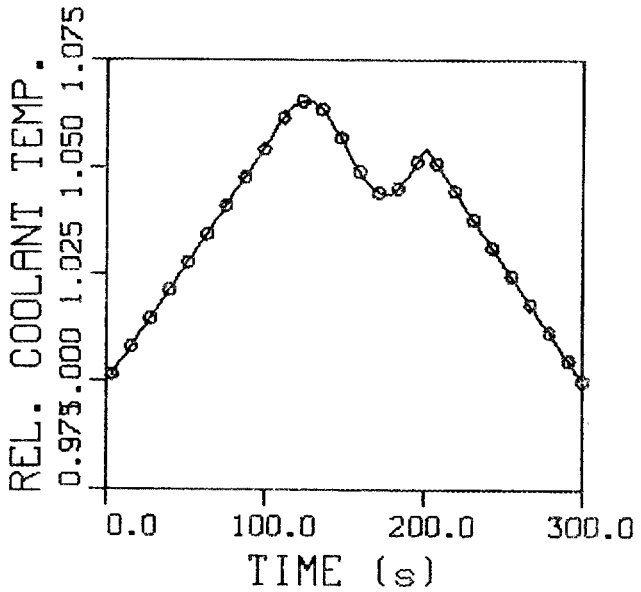

Figure 2. Demand-following by ACTOR algorithm: (a) the demand (continuous line) is a ramp increase in relative power for the first 100 seconds of the transient, followed by a sinusoidal variation for a duration of 100 seconds, and ending with a decreasing ramp in power. The symbols show the plants response; (b) the demand (continuous line) is a ramp increase in coolant temperature (relative units) for the first 100 seconds of the transient, followed by a sinusoidal variation for a duration of 100 seconds, and ending with a decreasing ramp in the coolant temperature. The symbols show the plant's response.

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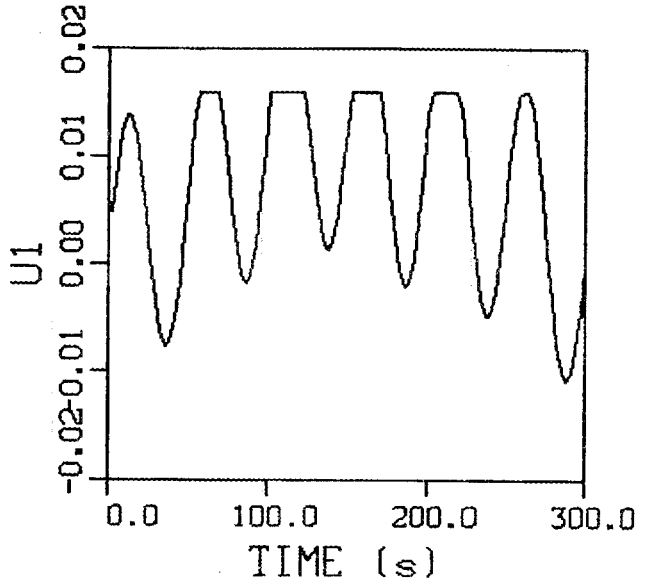

Figure 3a. Reactivity, U_1, versus time during the plant transient. The sinusoidal shape is due to the coolant feedback time dependence. Note that the reactivity control was constrained to U_{max} = 0.016 units, hence it was out of operation several times during the transient.

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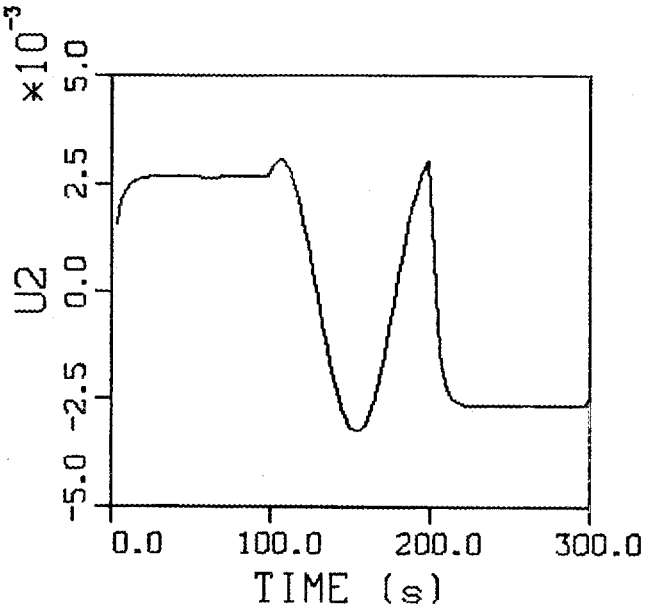

Figure 3b. Normalized inlet coolant temperature change, U_2, versus time during the plant transient.

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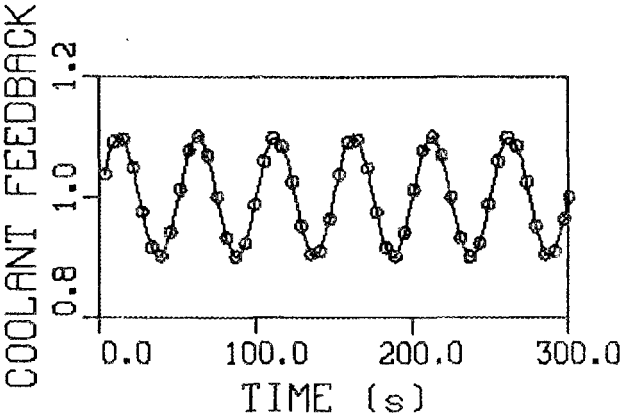

Figure 4a. Normalized coolant feedback coefficient versus time during the plant transient.

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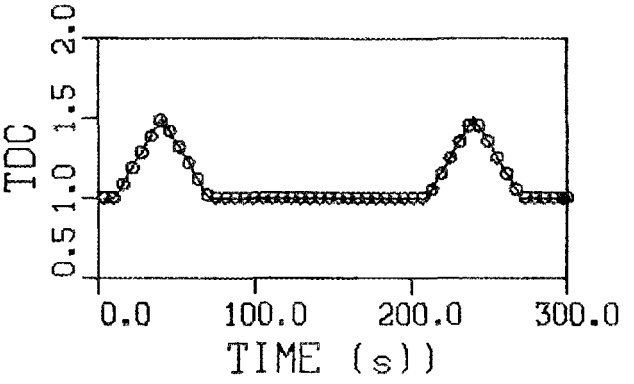

Figure 4b. Normalized time constant for the coolant temperature thermocouple versus time (TDC).

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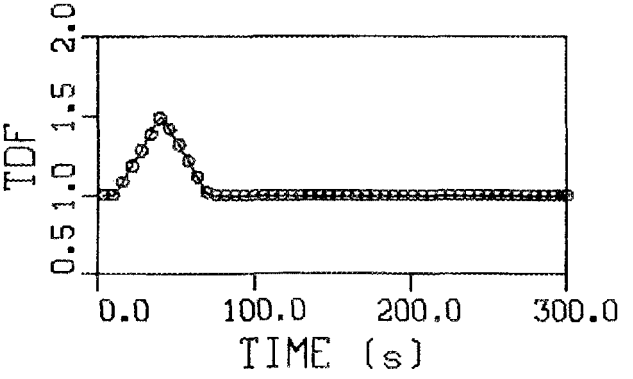

Figure 4c. Normalized time constant for the fuel temperature thermocouple versus time. Solid lines represent plant actual parameters, symbols the ACTOR algorithm estimates.

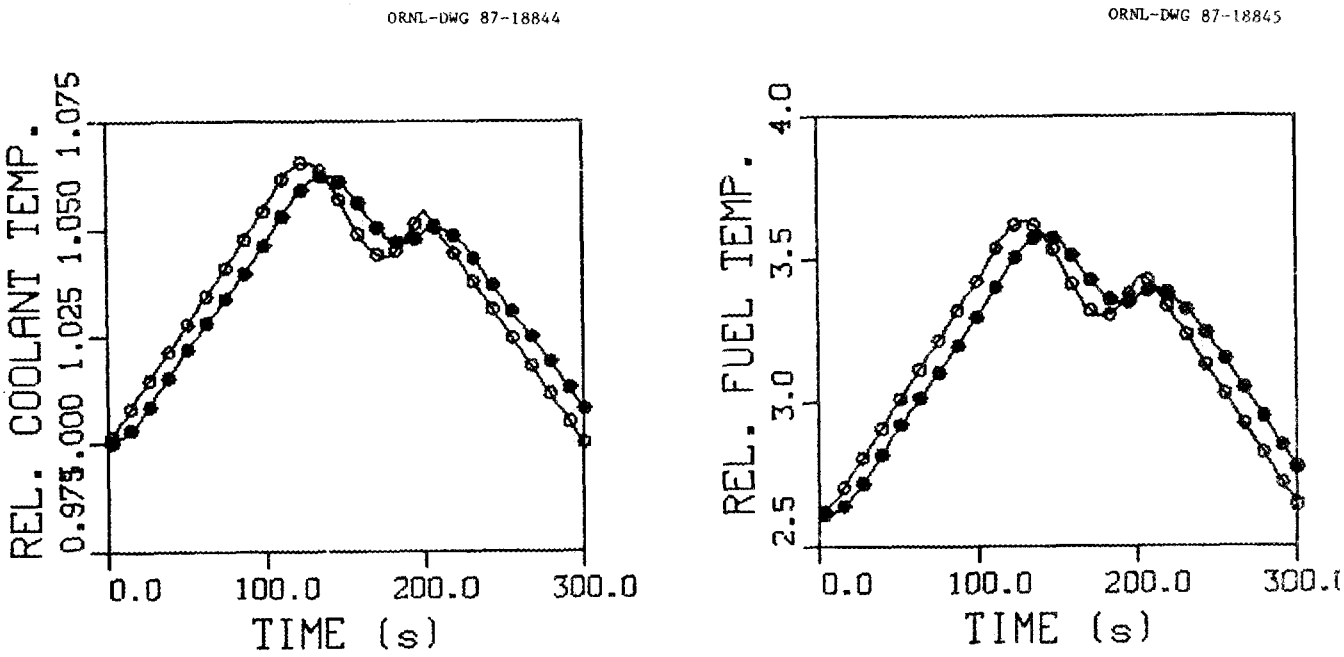

Figure 5. Plant signal-following by ACTOR. The continuous lines with superimposed solid circles are: (a) the coolant and (b) the fuel thermocouple signals, which are delayed with respect to the actual plant temperatures (continuous lines with open circles). Continuous lines are plant values, symbols are the demand-following and signal-following by ACTOR.

in fact correct for the time delays which affect the information flowing from the plant. The nonlinear character of the system's model does not pose any serious problem for on-line computer control applications. Despite the sometimes formidable appearance of the analytical developments, the technique shown in this work is in fact very easy to use by applying the methodology shown in Section IV to the particular plant model at hand.

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