ORNL/TM-10351

ornl

OAK RIDGE NATIONAL LABORATORY

MARTIN MARIETTA

Analysis of a Mechanically Simple External Combustion Engine with an Unusual Cycle

N. C. J. Chen
C. D. West

OPERATED BY
MARTIN MARIETTA ENERGY SYSTEMS, INC.
FOR THE UNITED STATES
DEPARTMENT OF ENERGY

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
NTIS price codes—Printed Copy A03; Microfiche A01

ORNL/TM-10351

Engineering Technology Division

ANALYSIS OF A MECHANICALLY SIMPLE EXTERNAL COMBUSTION ENGINE WITH AN UNUSUAL CYCLE

N. C. J. Chen C. D. West

Date Published - March 1987

Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37831
operated by
MARTIN MARIETTA ENERGY SYSTEMS, INC.
for the
U.S. DEPARTMENT OF ENERGY
under Contract No. DE-AC05-84OR21400

CONTENTS

ANALYSIS OF A MECHANICALLY SIMPLE EXTERNAL COMBUSTION ENGINE WITH AN UNUSUAL CYCLE

N. C. J. Chen   C. D. West

ABSTRACT

A single-cylinder, single-piston heat engine with no valves is described. Heat is supplied from an external source, so that a wide variety of heat sources or fuels could be used. The ideal performance of the thermodynamic cycle is calculated; a reasonable power output and indicated efficiency appears to be attainable from a machine of practicable size, operating under very modest conditions of temperature and pressure.

1. INTRODUCTION

In a single cylinder containing both an adiabatic volume that can be varied by a piston and a fixed isothermal volume (Fig. 1), cyclic movement of the piston is accompanied by a power loss.[1] The loss is caused by exchange of gas between the adiabatic space, in which the gas temperature varies according to the instantaneous pressure and the pressure history, and the isothermal space, which is at a fixed temperature; the temperature differences result in a heat flow and consequent irreversibilities. The effect is important in Stirling or hot gas engines.[2]

Based on a study of these mixed adiabatic/isothermal space losses, one of us proposed using a similar effect to generate power — that is, to make an engine.[3] The basic single-cylinder, single-piston engine is shown in Fig. 2: a piston moves in an open cylinder, in which a port communicates with the atmosphere or with a large reservoir of gas; the space above the piston's top dead center (t.d.c.) position is filled with wire screen, metal sponge, narrow tubes, or other means to make the gas within it behave isothermally. The isothermal region is maintained at a higher than ambient temperature by a heat source. The dimensions of the cylinder and the frequency of the piston movement are chosen so that the gas behavior in the open part of the cylinder is almost adiabatic; that

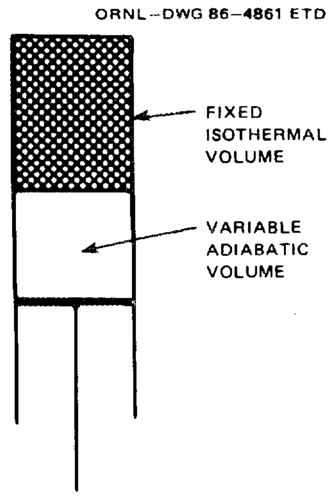

Fig. 1. Working space of isothermal and adiabatic volumes.

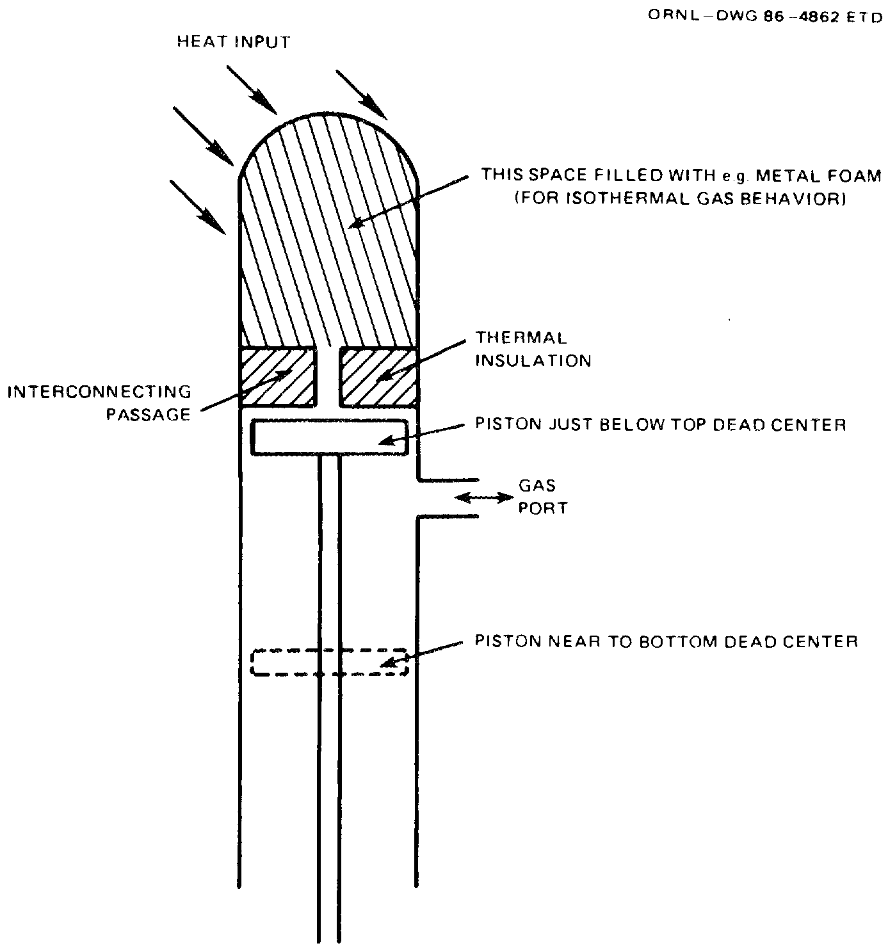

Fig. 2. Basic engine configuration.

is, little heat is exchanged between the gas and the piston crown, cylinder walls, or cylinder head.

The principle of operation is perhaps most easily seen by considering the sequence of events, shown in Fig. 3, that begins with the piston at bottom dead center (b.d.c.). Because the port is open, the pressure of the gas at this time must be equal to the atmosphere pressure, and the gas temperature in the adiabatic region is relatively low as a result of cool air drawn in through the port during the preceding downstroke. At first, raising the piston causes no change in gas temperature or pressure because the port remains open. At position 2, however, the piston covers

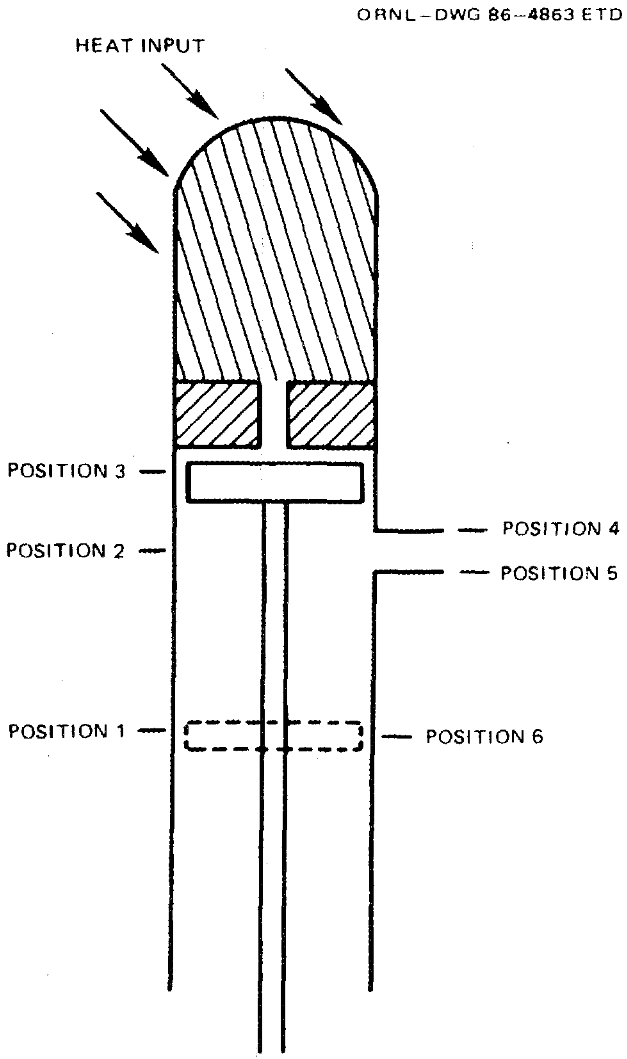

Fig. 3. Basic engine configuration showing piston position at various points during recycles.

the port; subsequent upward piston motion compresses the cool gas, raising its temperature while forcing some, and eventually all, of it into the isothermal space.

After t.d.c. the piston falls, drawing gas out of the isothermal space. During the downstroke to position 4 just before the port is uncovered, the gas already in the adiabatic space is being expanded, and its temperature is therefore falling. However, the gas temperature (and therefore pressure) is higher during the expansive downstroke than during the upstroke because of the extra heat carried into the open part of the cylinder by the hot gas that is continually being drawn from the isothermal space by the falling pressure.

At position 5, the port is uncovered, and gas exhales into the atmosphere, lowering the working-space pressure to atmospheric and cooling the gas remaining in the adiabatic part of the cylinder. Further downward movement to b.d.c. — position 6 — draws in cool air from outside and lowers the average temperature still further, finally returning to the beginning of the cycle at position 6. The pressure and volume (P-V) diagram associated with this sequence of events is shown in Fig. 4.

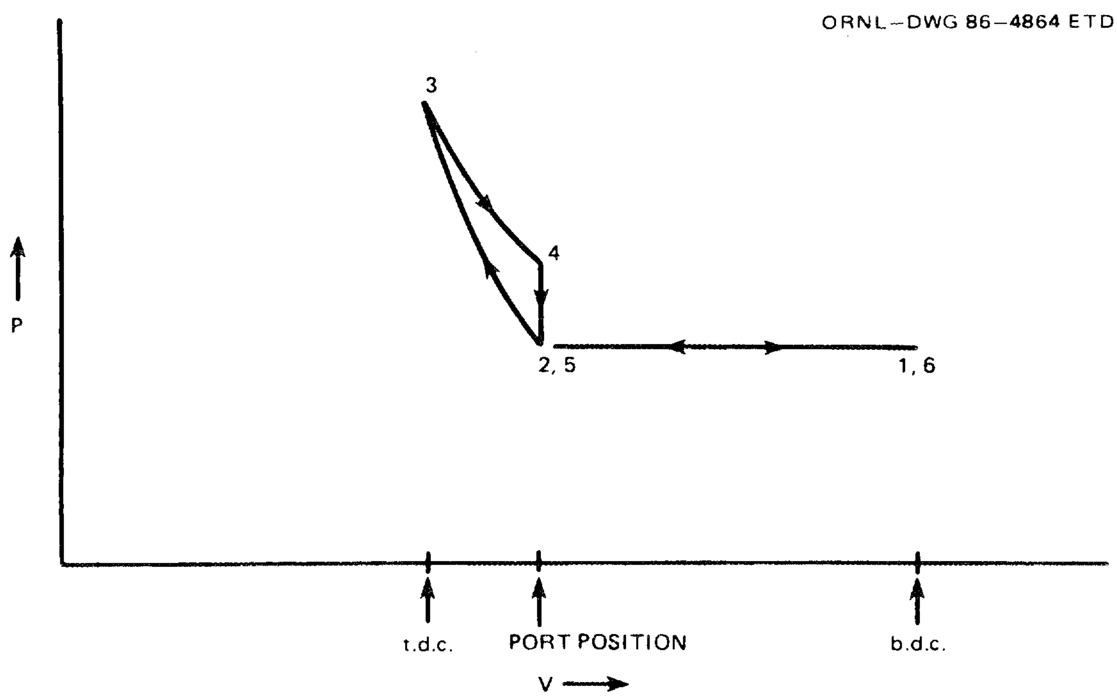

Fig. 4. Typical P-V diagram showing various points during the cycle.

It is believed that a related cycle may have been used for cryogenic refrigeration, but no published reference to such an engine has yet been identified.

2. ANALYSIS

Fortunately, some of the analytical expressions developed for the single-cylinder loss mechanism and reported in Ref. 1 are directly applicable to the engine described above.

Before beginning the analysis, it is necessary to establish an unambiguous nomenclature for the variables to be used and for the various points on the cycle.

State 1 is b.d.c. State 2 is the point at which the rising piston has just covered the port but not yet begun to compress the working fluid. State 3 is t.d.c. with all of the gas forced into the isothermal space, which is maintained at temperature T_I throughout. State 4 is the point on the downstroke just before the port is uncovered. Between states 2 and 4, therefore, the port is covered, and the mass of working fluid in the engine is constant.

Immediately after state 4, with negligible further movement by the piston, the port is uncovered. Gas is blown out of the port until the internal pressure has fallen to P_O. The expansion will cool the gas in the adiabatic space. Following this exhalation, the cooler, lower pressure gas in the cylinder is at state 5. The piston stroke continues to b.d.c., state 6, which for a repetitive cycle must be the same as state 1. Between states 4 and 6 (as between 1 and 2) the port is open, and therefore the gas pressure is constant and atmospheric.

Gas volume, temperature, and mass are indicated by subscripted letters V, T, and M, respectively. The first subscript indicates that the variable refers to the adiabatic space (subscript A), the isothermal space (subscript I), or the working space as a whole (no letter subscript). The numerical subscript indicates the state — 1, 2, 3, 4, 5, or 6 — that is represented. These variable names are summarized in Table 1.

At any moment, the pressure is assumed to be the same throughout the system.

Note that the port could open to a large constant-pressure, constant-temperature reservoir instead of to the atmosphere, thus making it possible to pressurize the system.

Table 1. Nomenclature

State No.	Adiabatic space			Isothermal space	Working space	
	Volume	Temperature	Mass	Mass	Pressure	Mass
1	V_{A1}	T_{A1}	M_{A1}	M_{I1}	P_1	M_1
2	V_{A2}	T_{A2}	M_{A2}	M_{I2}	P_2	M_2
3	V_{A3}	T_{A3}	M_{A3}	M_{I3}	P_3	M_3
4	V_{A4}	T_{A4}	M_{A4}	M_{I4}	P_4	M_4
5	V_{A5}	T_{A5}	M_{A5}	M_{I5}	P_5	M_5
6[a]	V_{A6}	T_{A6}	M_{A6}	M_{I6}	P_6	M_6

[a]For a repetitive cycle: $V_{A6} = V_{A1}$
$T_{A6} = T_{A1}$
$M_{A6} = M_{A1}$
$P_6 = P_1$

From the description of the states in the previous section, it is possible without further calculations to write down expressions for some of the variables at some of the states (see Table 2).

Those variables that cannot be obtained by inspection, beginning with state 1, will now be calculated.

2.1 States 1 and 2

Assume for the moment that the temperature T_{A1} of the gas in the adiabatic space at b.d.c. is known. Then from the ideal gas law,

$$M_{A1} = \frac{P_o V_B}{RT_{A1}} , \tag{1}$$

and

$$M_{I1} = \frac{P_o V_I}{RT_I} . \tag{2}$$

Table 2. Variable values obtained by inspection[a]

State No.	Adiabatic space			Isothermal space	Working space	
	Volume	Temperature	Mass	Mass	Mass	Pressure
1	V_B	T_{A1}		$(P_o V_I)/RT_I$		P_o
2	V_A	Same		Same	M_2	P_o
3	0	N.A.	0	M_2	Same	
4	V_A				Same	
5	V_A			$(P_o V_I)/RT_I$		P_o
6	V_B			Same		P_o

[a]Note fixed values: V_A = adiabatic volume above port
V_B = adiabatic volume at bottom dead center
V_I = isothermal volume
T_I = isothermal temperature
P_o = atmospheric (or reservoir) pressure
R = gas constant

Therefore, the total mass of gas is given by

$$M_1 = \frac{P_o}{R}\left(\frac{V_B}{T_{A1}} + \frac{V_I}{T_I}\right) . \tag{3}$$

In moving from state 1 to state 2, the port is open, and the gas pressure does not change as the adiabatic volume is reduced from V_B to V_A. Therefore, T_A does not change, and

$$M_{A2} = M_{A1} \times \frac{V_A}{V_B} = \frac{P_o V_A}{RT_{A1}} , \tag{4}$$

and

$$M_2 = \frac{P_o}{R}\left(\frac{V_A}{T_{A1}} + \frac{V_I}{T_I}\right) . \tag{5}$$

2.2 States 3 and 4

At the end of the compression, all of the gas (mass M_2) has been compressed into a volume V_I at temperature T_I.

$$\frac{P_3 V_I}{RT_I} = M_2 = \frac{P_o}{R} \left(\frac{V_A}{T_{A1}} + \frac{V_I}{T_I} \right) .$$

Therefore

$$P_3 = P_o \left(1 + \frac{V_A}{V_I} \frac{T_I}{T_{A1}} \right) . \tag{6}$$

Later it will be convenient to denote the quantity in parentheses, which is the pressure ratio (the ratio between the maximum and minimum of working fluid pressure), by a single variable:

$$\alpha = \left(1 + \frac{V_A}{V_I} \frac{T_I}{T_{A1}} \right) , \tag{7}$$

and then

$$P_3 = \alpha P_o . \tag{8}$$

The mass of gas in the system is unchanged from state 2, but in state 3 all of the gas is collected in the isothermal space.

The equations governing the gas behavior during the expansion in a combination of adiabatic and isothermal spaces were derived in Ref. 1 and can be copied — with the appropriate change of nomenclature — from Eq. (2) of that reference.

$$\frac{P_3}{P_4} = \left(1 + \frac{V_A}{\gamma V_I} \right)^{\gamma} . \tag{9}$$

Substituting the expression for P_3 from Eq. (8),

$$P_4 = P_o \frac{\alpha}{\left(1 + \frac{V_A}{\gamma V_I} \right)^{\gamma}} . \tag{10}$$

From Eq. (6) of Ref. 1, the temperature of the gas in the adiabatic space at the end of the expansion may be written as

$$T_{A4} = \frac{T_I\, V_A}{V_I \left[\left(1 + \frac{V_A}{\gamma V_I}\right)^{\gamma} - 1\right]} \; . \tag{11}$$

From the above expressions the mass of gas in each space may be found.

$$M_{A4} = \frac{P_4 V_A}{RT_{A4}} = \frac{P_o V_I}{RT_I} \propto \left[1 - \left(1 + \frac{V_A}{\gamma V_I}\right)^{-\gamma}\right] , \tag{12}$$

and

$$M_{I4} = \frac{P_4 V_I}{RT_I} = \frac{P_o V_I}{RT_I} \cdot \propto \left(1 + \frac{V_A}{\gamma V_I}\right)^{-\gamma} \; . \tag{13}$$

The total amount of gas in the system is, of course, unchanged [as may be verified by adding Eqs. (12) and (13) and comparing the result with Eq. (5)] because the port is closed during this phase of the cycle.

2.3 State 5

Between states 4 and 5, the pressure falls to P_o as gas leaves the cylinder. The temperature in the adiabatic space at the end of this blowdown can be obtained from Eq. (5) of Ref. 1:

$$T_{A5} = T_I \cdot \gamma \frac{(P_3/P_o)^{1/\gamma} - 1}{P_3/P_o - 1} \; . \tag{14}$$

Substituting for P_3 from Eq. (8) yields

$$T_{A5} = T_I \cdot \gamma \frac{\propto^{1/\gamma} - 1}{\propto - 1} \; . \tag{15}$$

It is now possible to calculate the mass of gas in the adiabatic space as well as in the isothermal space in state 5:

$$M_{I5} = \frac{P_o V_I}{RT_I} , \tag{16}$$

and

$$M_{A5} = \frac{P_o V_A}{RT_{A5}} = \frac{P_o V_A}{RT_I} \cdot \frac{\alpha - 1}{\gamma(\alpha^{1/\gamma} - 1)} .$$

Now substitute Eq. (7) for α in the numerator, giving

$$M_{A5} = \frac{P_o V_A}{RT_{A1}} \cdot \frac{V_A}{V_I} \cdot \frac{1}{\gamma(\alpha^{1/\gamma} - 1)} . \tag{17}$$

2.4 State 6

As the piston continues downward toward b.d.c. cool gas is drawn in through the port, mixing with the gas already in the adiabatic space. Because this is a constant-pressure process, the consequences are fairly easy to calculate.

Assuming that the specific heat, at constant pressure, of the gas is independent of temperature,

$$M_{A6}\ T_{A6} = M_{A5}\ T_{A5} + (M_{A6} - M_{A5})\ T_o ,$$

or

$$M_{A6} = M_{A5} \frac{T_{A5} - T_o}{T_{A6} - T_o} . \tag{18}$$

Also, from the ideal gas law,

$$M_{A5} = \frac{P_o V_A}{RT_{A5}} . \tag{19}$$

Combining Eqs. (18) and (19) to eliminate M_{A6} yields

$$\frac{T_o}{T_{A6}} = 1 - \frac{V_A}{V_B} \left(1 - \frac{T_o}{T_{A5}}\right) .$$

Now substitute for T_{A5} from Eq. (15), simplify, and rearrange to give

$$\frac{T_o}{T_{A6}} = 1 - \frac{V_A}{V_B} + \frac{V_A^{\,2}}{V_B V_I} \frac{T_o}{T_{A1}} \frac{1}{\gamma\ (\alpha^{1/\gamma} - 1)} . \tag{20}$$

2.5 Closing the Cycle

If the cycle has been running long enough to reach equilibrium and become repetitive, then $T_{A6} = T_{A1}$. Replacing T_{A6} on the left-hand side of Eq. (20) with T_{A1} gives, following some manipulation, an expression for T_{A1} (note that T_{A1} was earlier assumed to be known).

$$T_{A1} = \frac{T_o}{1 - V_A/V_B} \left[1 - \frac{V_A^{\,2}}{V_B V_I} \cdot \frac{1}{\gamma(\alpha^{1/\gamma} - 1)}\right] . \tag{21}$$

There is a catch to this equation: the variable α appearing on the right-hand side is a function of T_{A1}, as revealed by its definition in Eq. (7). The form of Eq. (21) is such that a numerical solution is required. The equation-solving routines built into many hand calculators can be used, but the simplest method is to guess a value for T_{A1} — the atmospheric temperature T_o is a good starting point — and evaluate α by using Eq. (7). Then an approximate value of T_{A1}, calculated directly from this value of α by using Eq. (21), is used to calculate a better value of α and so on. The answer converges after only a few iterations.

The important variables in each state are summarized in Table 3.

2.6 The P-V Loop

The expressions derived above can be used to evaluate the gas pressure and volume at each state so that a P-V diagram can be constructed. The loop in Fig. 5 corresponds to an engine with $V_A = V_I/2$, $V_I = V_B$ (i.e., the port is at midstroke of the piston), $T_I/T_o = 3$, and $\gamma = 1.4$. For reference, the value of T_{A1} calculated for this engine by iteration of Eq. (21) is equal to 1.43 T_o [155°C if the atmospheric temperature is 27°C (300 K)].

2.7 Indicated Power Output

P-V work is done on the piston as the gas expands from state 3 to state 4, and work is done by the piston in compressing the gas between state 2 and state 3. During all other parts of the cycle, the pressure is equalized above and below the piston because the port is open, and no

Table 3. State variables

State No.	Adiabatic space: Volume	Adiabatic space: Temperature	Adiabatic space: Mass	Isothermal space: Mass	Working space: Pressure	Working space: Mass
1	V_B	T_{A1}	$\frac{P_o V_B}{RT_{A1}}$	$\frac{P_o V_I}{RT_I}$	P_o	$\frac{P_o}{R}\left(\frac{V_B}{T_{A1}} + \frac{V_I}{T_I}\right)$
2	V_A	Same	$\frac{P_o V_A}{RT_{A1}}$	Same	Same	$\frac{P_o}{R}\left(\frac{V_A}{T_{A1}} + \frac{V_I}{T_I}\right)$
3	0	Not applicable	0	$\frac{P_o}{R} \quad \frac{V_A}{T_{A1}} + \frac{V_I}{T_I}$	$P_o \left(1 + \frac{V_A}{V_I}\frac{T_I}{T_{A1}}\right)$	Same
4	V_A	$T_I \quad \frac{V_A}{V_I} \quad \frac{1}{\left(1 + \frac{V_A}{V_I}\right)^{\gamma} - 1}$	$\frac{P_o V_I}{RT_I}\left(1 + \frac{V_A T_I}{V_I T_{A1}}\right)\left[1 - \left(1 + \frac{V_A}{\gamma V_I}\right)^{-\gamma}\right]$	$\frac{P_o V_I}{RT_I} \frac{\left(1 + \frac{V_A}{V_I}\frac{T_I}{T_{A1}}\right)}{\left(1 + \frac{V_A}{\gamma V_I}\right)^{\gamma}}$	$P_o \frac{\left(1 + \frac{V_A}{V_I}\frac{T_I}{T_{A1}}\right)}{\left(1 + \frac{V_A}{\gamma V_I}\right)^{\gamma}}$	Same
5	V_A	$T_I \gamma \frac{\left(1 + \frac{V_A}{V_I}\frac{T_I}{T_{A1}}\right)^{1/\gamma} - 1}{\frac{V_A}{V_I}\frac{T_I}{T_{A1}}}$	$\frac{P_o V_A}{RT_{A1}} \quad \frac{V_A}{\gamma V_I} \quad \frac{1}{\left[\left(1 + \frac{V_A}{V_I}\frac{T_I}{T_{A1}}\right)^{1/\gamma} - 1\right]}$	$\frac{P_o V_I}{RT_I}$	P_o	Not needed
6	V_B	$\frac{V_B}{\frac{V_B - V_A}{T_o} + \frac{V_A^2}{\gamma V_I T_{A1}} \frac{1}{\left(1 + \frac{V_A}{V_I}\frac{T_I}{T_{A1}}\right)^{1/\gamma} - 1}}$	Not needed	Same	Same	Not needed

ORNL–DWG 86–4865 ETD

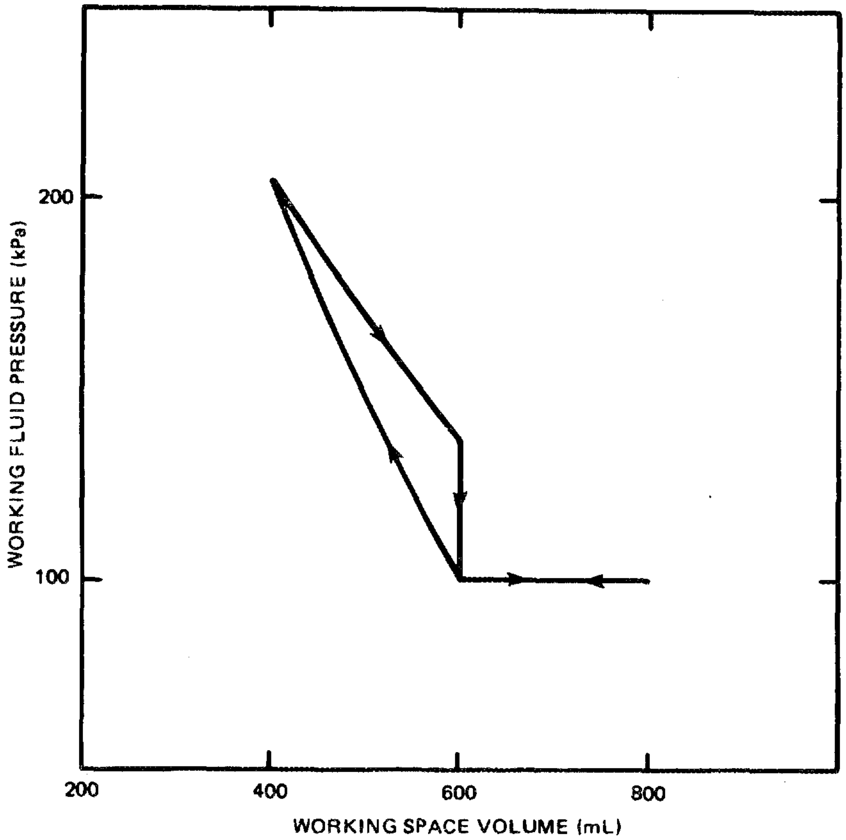

Fig. 5. P-V diagram for nominal engine.

work is done. The net work available from the piston is the difference between the expansive and compressive work.

The expansive work can be calculated from Eq. (3) of Ref. 1:

$$W_e = P_3 V_I \left(\frac{\gamma}{\gamma - 1}\right)\left[1 - \left(\frac{P_3}{P_4}\right)^{1/\gamma - 1}\right] . \tag{22}$$

Equations (8) and (10) above can be used to evaluate Eq. (22):

$$W_e = P_o V_I \frac{\alpha \gamma}{\gamma - 1}\left[1 - \left(1 + \frac{V_A}{\gamma V_I}\right)^{1 - \gamma}\right] . \tag{23}$$

The compressive work, that is, the work done by the piston in going between states 2 and 3, cannot be calculated from any of the equations given in Ref. 1 because that reference does not include the effect of the blowdown phase between the expansion and compression. However, the same analytical method can be adapted to the new configuration. The work is given by

$$W_c = \int_{P_o}^{P_3} PdV = \int_{P_o}^{P_3} P \frac{dV}{dP} dP . = P_o \int_{1}^{P_3/P_o} (P/P_o) \frac{dV}{d(P/P_o)} d(P/P_o) .$$

For convenience in carrying out the calculation, variables during the course of the compression phase (i.e., between states) will be denoted by lowercase subscripts. Remembering that $P_3/P_o = \alpha$ and writing, for convenience, $p = P/P_o$, the expression for compressive work becomes

$$W_c = P_o \int_1^{\alpha} p \frac{dV_a}{dp} dp . \tag{24}$$

Because the port is closed there is a fixed mass of gas during the compression and, therefore, at any time during this phase.

$$T_a = T_{A1} \left(\frac{P}{P_o}\right)^{1 - 1/\gamma} . \tag{25}$$

Also, from the ideal gas law for a fixed mass of gas,

$$P \left(\frac{V_a}{T_a} + \frac{V_I}{T_I}\right) = P_o \left(\frac{V_A}{T_{A1}} + \frac{V_I}{T_I}\right) = \frac{\alpha P_o V_I}{T_I} . \tag{26}$$

Substituting the expression for T_a from Eq. (25) into Eq. (26) and using the resulting relationships to rewrite the integral expression for the compressive work leads to

$$W_c = - P_o V_I \frac{T_{A1}}{T_I} \int_1^{\alpha} \left(\frac{\alpha}{\gamma} p^{-1/\gamma} + \frac{\gamma - 1}{\gamma} p^{1 - 1/\gamma}\right) dp$$

$$= - P_o V_I \frac{T_{A1}}{T_I} \left[\frac{\alpha}{\gamma - 1} (\alpha^{1-1/\gamma} - 1) + \frac{\gamma - 1}{2\gamma - 1} (\alpha^{2-1/\gamma} - 1)\right] . \tag{27}$$

The net work over the cycle W_o is obtained by adding Eqs. (23) and (27):

$$W_o = P_o V_I \left\{ \frac{\alpha\gamma}{\gamma - 1} \left[1 - \left(1 + \frac{V_A}{\gamma V_I} \right)^{1-\gamma} \right] - \frac{T_{A1}}{T_I} \left[\frac{\alpha^{2-1/\gamma} - \alpha}{\gamma - 1} + \frac{(\gamma - 1)\,(\alpha^{2-1/\gamma} - 1)}{2\gamma - 1} \right] \right\} . \quad (28)$$

To calculate the power output, the work per cycle W_o is multiplied by the operating frequency.

2.8 Indicated Heat Input and Efficiency

The indicated heat input is calculated by using the method introduced by Creswick[4] for Stirling engines. In general, for any space,

heat input = change of internal energy + PV work — enthalpy convected in.

For the isothermal space, the only one in which heat can be exchanged to or from gas in the cylinder between states 2 and 5, the P-V work term is zero because the volume is fixed.

Consider first the expansion phase. During the expansion, gas leaving the isothermal region does so at temperature T_I. Therefore, during this phase,

$$dQ_e = C_V T_I dm_I - h dm_I .$$

C_V is the specific heat at constant volume of the (ideal) working gas, and h is its specific enthalpy. The multiplicands of dm_I are not functions of dm_I, and, therefore,

$$\Delta Q_e = (C_V T_I - h)\, \Delta M_I .$$

During the expansion phase, $\Delta M_I = M_{I5} - M_{I3}$, and for an ideal gas, $h = C_p T_I$. Therefore,

$$Q_e = (C_V - C_p)\, T_I\, (M_{I5} - M_{I3}) .$$

For an ideal gas, $C_p - C_V = R$, so

$$Q_e = RT_I \,(M_{I3} - M_{I5}) \,. \tag{29}$$

Substituting the expressions in Table 3 for M_{I5} and M_{I3} yields

$$Q_e = RT_I \left[\frac{P_o}{R}\left(\frac{V_A}{T_{A1}} + \frac{V_I}{T_I}\right) - \frac{P_o V_I}{RT_I}\right] = \frac{P_o V_A T_I}{T_{A1}} \,.$$

Using the relationship between T_{A1} and α from Eq. (7) results in a further simplification:

$$Q_e = P_o V_I \,(\alpha - 1) \,. \tag{30}$$

During the compression phase, gas entering the isothermal space does so at the instantaneous temperature T_a of the adiabatic space. Therefore,

$$dQ_c = C_V\, T_I\, dm_I - C_p\, T_a\, dm_I \,.$$

From the ideal gas law,

$$dm_I = \frac{V_I}{RT_I}\, dP \,,$$

and, therefore,

$$dQ_c = \left(C_V T_I - C_p T_a\right) \frac{V_I}{RT_I}\, dP \,.$$

From Eq. (25), however,

$$T_a = T_{A1} \left(\frac{P}{P_o}\right)^{1 - 1/\gamma} \,;$$

therefore,

$$dQ_c = \frac{V_I}{RT_I} \left(C_V T_I - \frac{C_p\, T_{A1}}{P_o^{\,1 - 1/\gamma}}\, P^{1 - 1/\gamma}\right) dP \,.$$

The total heat input during the compression is

$$Q_c = \frac{V_I}{RT} \int_{P_o}^{P_3} \left(C_V\, T_I - \frac{C_p\, T_{A1}}{P_o^{\,1-1/\gamma}} \cdot P^{1-1/\gamma} \right) dP$$

$$= \frac{V_I}{RT_I} \left(C_V\, T_I P - \frac{C_p T_{A1}}{P_o^{\,1-1/\gamma}} \cdot \frac{P^{2-1/\gamma}}{2-1/\gamma} \right)_{P_o}^{P_3}$$

$$= \frac{V_I}{RT_I} \left[C_V T_I\, (P_3 - P_o) - \frac{C_p T_{A1}}{P_o^{\,1-1/\gamma}} \cdot \frac{\gamma}{2\gamma-1} \cdot (P_3^{\,2-1/\gamma} - P_o^{\,2-1/\gamma}) \right]$$

$$= P_o V_I \left[\frac{C_V}{R} (\alpha - 1) - \frac{C_p}{R} \frac{T_{A1}}{T_I} \frac{\gamma}{2\gamma - 1} (\alpha^{2-1/\gamma} - 1) \right] .$$

Substituting the relations between the specific heats and the gas constants gives

$$Q_c = P_o V_I \left[\frac{\alpha - 1}{\gamma - 1} - \frac{\gamma^2}{(\gamma - 1)(2\gamma - 1)} \frac{T_{A1}}{T_I} (\alpha^{2-1/\gamma} - 1) \right] . \qquad (31)$$

Q_c is generally negative, reflecting the fact that during compression in the isothermal space, heat is rejected by the gas. However, the heat rejected is usually much smaller than the heat absorbed during expansion. The net heat input is obtained by adding Q_e and Q_c.

$$Q_e + Q_c = P_o V_I \left[(\alpha - 1) + \frac{\alpha - 1}{\gamma - 1} - \frac{\gamma^2}{(\gamma - 1)(2\gamma - 1)} \frac{T_{A1}}{T_I} (\alpha^{2-1/\gamma} - 1) \right] ,$$

or

$$Q_o = P_o V_I \frac{\gamma}{\gamma - 1} \left[\alpha - 1 - \frac{\gamma}{2\gamma - 1} \frac{T_{A1}}{T_I} (\alpha^{2-1/\gamma} - 1) \right] . \qquad (32)$$

The indicated efficiency is calculated by dividing Eq. (32) into Eq. (28).

2.9 Examples

In this section, the indicated power output and efficiency are calculated (with the help of the simple computer program listed in the Appendix) for a machine that might be considered of practical size. The effect of varying certain parameters is also calculated. Note that the figures are for indicated power and efficiency — parasitic losses of mechanical power or heat are not included in the calculations. The results indicate that an engine of reasonable size might produce a useful amount of power at an indicated efficiency that, although much less than Carnot, is not too small to be interesting.

The baseline or nominal case is described in Table 4. The single cylinder has a piston swept volume of 400 mL, about the same as each cylinder of a 100-in.^{3}, four-cylinder automobile engine. The 400-mL isothermal space is heated to a very modest temperature of 650°C, and the atmospheric air is supposed to be at 25°C. The port is placed at midstroke of the piston, and the engine operates at 2400 rpm.

Table 5 shows the effect on indicated power and efficiency of increasing or decreasing the isothermal space temperature from the nominal value.

Table 6 shows the effect of increasing or decreasing the isothermal volume, and Table 7 shows the effect of placing the port above or below the piston midstroke position.

Table 4. Nominal engine configuration

Quantity	Value
Isothermal volume (V_I), mL	400
Adiabatic volume at bottom dead center (V_B), mL	400
Adiabatic volume above port (V_A), mL	200
Temperature in isothermal space (T_I), K	923
Air temperature (T_o), K	298
Atmospheric pressure (P_o), Pa	10^5
Gas specific heat ratio (air) (γ)	1.4
Operating speed, Hz	40

Table 5. Indicated power per cylinder and efficiency at various heater temperatures

	Isothermal space temperature, T_I (°C)		
	450	650[a]	850
Power, W	114	176	240
Efficiency, %	6.19	6.38	6.53

[a]Baseline case.

Table 6. Indicated power per cylinder and efficiency with various isothermal volumes

	Isothermal volume, V_I (mL)		
	200	400[a]	600
Power, W	296	176	124
Efficiency, %	11.33	6.38	4.43

[a]Baseline case.

Table 7. Indicated power per cylinder and efficiency with the exhaust port in various positions

	Adiabatic volume above port, V_A (mL)		
	100	200[a]	300
Power, W	73	176	172
Efficiency, %	3.46	6.38	8.43

[a]Baseline case.

From the parametric studies summarized in Tables 5–7, a good combination of parameters seems to be the one shown in the first column of Table 6: compared with the nominal case, the isothermal volume has been reduced, thus increasing the compression ratio. The indicated power is 296 W/cylinder with the engine vented to atmospheric pressure and at a heater temperature of only 650°C. The cycle efficiency is >11%.

The corresponding output for a four-cylinder machine with the port venting to a reservoir at 0.5 MPa (75 psig) is 5.9 kW (or 8 hp). The cyclic efficiency figure is comparable with other small engines.

REFERENCES

1. C. D. West, "An Analytical Solution for a Stirling Machine with an Adiabatic Cylinder," Paper 809 453, Proc. 15th IECEC, August 1980.

2. C. D. West, Principles and Applications of Stirling Engines, Van Nostrand Reinhold Co., New York, 1986.

3. C. D. West, "Mechanically Simple Heat Engine with Unusual Cycle," Martin Marietta Energy Systems, Inc., report of possible invention or discovery, April 9, 1986.

4. F. A. Creswick, "Thermal Design of Stirling-Cycle Machines," Paper 949C, International Automotive Engineering Congress, Detroit, January 1965.

Appendix

HTENG2 PROGRAM LISTING (BASIC)

Name	Unit	Definition
AN	kg	Gas mass in adiabatic space; N=1,2,...,6, index of state number
C	J/cycle	Compression work
CO	J/(kg•K)	Constant volume specific heat of gas
DN	kg	Gas mass in isothermal space; N=1,2,...,6, index of state number
E	J/cycle	Expansion work
E1	%	Efficiency
F	Hz	Frequency
G		Gas specific heat ratio
PO	Pa	Ambient pressure
PN	Pa	Gas pressure; N=1,2,...,6, index of state number
Q	J/cycle	Heat input
R	J/(kg•K)	Gas constant
To	K	Ambient temperature
TN	K	Gas temperature in adiabatic space; N=1,2,...,6, index of state number
T8	K	Temperature of isothermal space
VN	m^3	Gas volume in adiabatic space; N=1,2,...,6, index of state number
VO	m^3	Adiabatic volume above port
V7	m^3	Adiabatic volume at bottom dead center
V8	m^3	Isothermal volume
W	J/cycle	Net work
WN	kg	Gas mass in working space; N=1,2,...,6, index of state number
Z	W	Power output

```
00010 F=40.
00020 R=285.
00030 G=1.4
00040 C0=R/(G-1.)
00050 T1=298.
00060 T0=298.
00070 T8=923.
00080 P0=1.E5
00090 V0=2.E-4
00100 V7=4.E-4
00110 V8=4.E-4
00120 PRINT "   VOLUME   TEMPERATURE AMASS     IMASS      PRESSURE    TMASS"
00130 PRINT "    (M**3)      (K)        (KG)     (KG)        (PA)        (KG)"
00140 PRINT "  "
00150 FOR I=1 TO 10
00160 V1=V7
00170 T1=T1
00180 A1=P0*V7/R/T1
00190 D1=P0*V8/R/T8
00200 P1=P0
00210 W1=A1+D1
00220 PRINT USING 650,V1,T1,A1,D1,P1,W1
00230 V2=V0
00240 T2=T1
00250 A2=P0*V0/R/T1
00260 D2=P0*V8/R/T8
00270 P2=P0
00280 W2=A2+D2
00290 PRINT USING 650,V2,T2,A2,D2,P2,W2
00300 V3=0.
00310 A3=0.
00320 X1=1.+V0*T8/V8/T1
00330 P3=P0*X1
00340 D3=P3*V8/R/T8
00350 W3=A3+D3
00360 PRINT USING 650,V3,T3,A3,D3,P3,W3
00370 V4=V0
00380 X2=1.+V0/V8/G
00390 T4=V0*T8/V8/(X2**(G)-1.)
00400 X3=P0*V8/R/T8
00410 A4=X3*X1*(1.-X2**(-G))
00420 D4=X3*X1/X2**(G)
00430 P4=P0*X1/X2**(G)
00440 W4=A4+D4
00450 PRINT USING 650,V4,T4,A4,D4,P4,W4
00460 V5=V0
00470 T5=B2
00480 X4=P0*V0*V0/G/R/V8/T1
00490 A5=X4/(X1**(1./G)-1.)
00500 D5=P0*V8/R/T8
00510 P5=P0
00520 W5=A5+D5
```

```
00530 PRINT USING 650,V5,T5,A5,D5,P5,W5
00540 V6=V7
00550 Y1=(V7-V0)/T0
00560 Y2=V0*V0/(G*T1*V8*(X1**(1./G)-1.))
00570 T6=V7/(Y1+Y2)
00580 A6=P0*V7/R/T6
00590 D6=P0*V8/R/T8
00600 P6=P0
00610 W6=A6+D6
00620 PRINT USING 650,V6,T6,A6,D6,P6,W6
00630 T1=T6
00640 NEXT I
00650 :###.## ###.## ###.## ###.## ###.## ###.##
00660 G1=G/(G-1.)
00670 E=P0*V8*G1*X1*(1.-X2**(1.-G))
00680 C1=X1*(X1**(1.-1./G)-1.)
00690 C2=(G-1.)**2*(X1**(2.-1./G)-1.)/(2.*G-1.)
00700 C=-P0*V8*G1*T1*(C1+C2)/(G*T8)
00710 W=E+C
00720 Z=W*F
00730 Q1=P0*V8*(X1-1.)
00740 U1=(X1-1.)/(G-1.)
00750 U2=G**2*T1*(X1**(2.-1./G)-1.)
00760 U3=(G-1.)*(2.*G-1.)*T8
00770 Q2=P0*V8*(U1-U2/U3)
00780 Q=Q1+Q2
00790 E1=W*100./Q
00800 PRINT " "
00810 PRINT "POWER OUTPUT(W)= ",Z
00820 PRINT " "
00830 PRINT "COMP. WORK   EXP. WORK   NET WORK        HEAT INPUT     EFF."
00840 PRINT "   (J)          (J)         (J)              (J)          (%) "
00850 PRINT C,E,W,Q,E1
00860 END
```

Internal Distribution

1. D. W. Burton
2-4. N. C. J. Chen
5. F. A. Creswick
6. E. C. Fox
7. U. Gat
8. J. A. Getsi
9. R. L. Graves
10. F. P. Griffin
11. M. A. Kuliasha
12. J. A. Marasco
13. F. J. Sweeney
14. H. E. Trammell
15-17. C. D. West
18. A. Zucker
19. ORNL Patent Office
20. Central Research Library
21. Document Reference Section
22-23. Laboratory Records Department
24. Laboratory Records (RC)

External Distribution

25. Steve Baer, Zoneworks Corp., P.O. Box 25805, Albuquerque, NM 87125
26. William Beale, Sunpower, Inc., 6 Byard St., Athens, OH 45701
27. E. H. Cooke-Yarborough, Lincoln Lodge, Longworth, Near Abingdon, OX13 5DU, England
28. John Corey, Stirling Engine Systems Division, Mechanical Technology, Inc., 968 Albany-Shaker Rd., Latham, NY 12110
29. James G. Daley, Components Technology Division, Argonne National Laboratory, 9700 South Cass Ave., Argonne, IL 60439
30. O. R. Fauvel, The University of Calgary, Department of Mechanical Engineering, 2500 University Dr., NW, Calgary, Alberta, Canada T2N 1N4
31. T. Finkelstein, Stirling Associates, Inc., P.O. Box 643, Beverly Hills, CA 90213
32. Marvin Gunn, U.S. Department of Energy, Energy Conservation and Utilization Technologies, CE-142, Washington, DC 20585
33. Lt. Cdr. G. T. Reader, Royal Naval Engineering College, Manadon, Plymouth, Devon PL5 3AQ, England
34. Brad Ross, Battelle Pacific Northwest Laboratories, TRT4-5, P.O. Box 999, Richland, WA 99352
35. J. R. Senft, Department of Mathematics/Computer Systems, University of Wisconsin-River Falls, River Falls, WI 54022
36. Professor G. Walker, The University of Calgary, Mechanical Engineering Department, 2500 University Dr., NW, Calgary, Alberta, Canada T2N 1N4
37. M. A. White, University of Washington, Joint Center for Graduate Study, 100 Sprout Rd., Richland, WA 99352
38. Office of Assistant Manager for Energy Research and Development, Department of Energy, ORO, Oak Ridge, TN 37831
39-68. Technical Information Center, Department of Energy, Oak Ridge, TN 37831