ORNL-TM-3858

PRESSURE VESSEL RELIABILITY AS A FUNCTION OF ALLOWABLE STRESS

(Thesis)

H. G. Arnold

OAK RIDGE NATIONAL LABORATORY

OPERATED BY UNION CARBIDE CORPORATION • FOR THE U.S. ATOMIC ENERGY COMMISSION

ORNL TM-3858

Contract No. W-7405-eng-26

General Engineering Division

PRESSURE VESSEL RELIABILITY AS A FUNCTION OF ALLOWABLE STRESS

H. G. Arnold

The material contained herein was submitted to the Graduate Council of The University of Tennessee in partial fulfillment of the requirements for the Master of Science degree with a major in Mechanical Engineering

JUNE 1972

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
Nuclear Division
for the
U.S. ATOMIC ENERGY COMMISSION

ABSTRACT

The reliability of nuclear pressure vessels was investigated by probabilistic design methods to develop analytical expressions for evaluating the adequacy of existing allowable stress intensities. The change in reliability resulting from changes in allowable stress intensities was also investigated.

Analytical expressions were derived and combined with existing methods of reliability analysis and stress analysis to calculate failure probability as a function of general membrane stress, burst pressure, and cyclic stress amplitude. The results were applied to typical distributions of induced stress and material strength to demonstrate the method by which pressure vessel reliability can be estimated.

It was found that existing allowable stress intensities result in very high estimates of reliability when bursting or yielding under a steady-state pressure is defined as failure. However, when fatigue failure is considered, the probability of failure resulting from allowable cyclic stress amplitudes was found to be significantly higher. It was therefore concluded that consideration should be given to lowering the allowable cyclic stress amplitudes. It was also concluded that the allowable membrane stress intensities can be substantially increased without producing a significant change in the estimated failure probability.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

FIGURE PAGE

1. INTRODUCTION

The question of what margin of safety to assign in a design problem is basic to the design process. Historically, this safety margin has been based on experience and intuition. The improved technology and increased production costs that have arisen in recent years have led to the use of lower margins of safety, while the demands of the general public and the consumer for better quality and safety have increased. This set of circumstances has in turn led to the need to augment experience and intuition with a tractable analytical method that can be used to evaluate the reliability of a product as a function of the safety margin used in the design of that product. The investigation reported herein was undertaken to demonstrate the analytical methods of such an evaluation as applied to nuclear pressure vessels and to evaluate the adequacy of the allowable stress intensities now used in their design.

It is often difficult to see a need for reliability analysis since the word "reliability" has been used in a qualitative or subjective sense to imply absolute dependability; that is, something either is or is not reliable. The word "reliability" is not really defined in the dictionary (1)* nor is the word "reliable" except in terms of the word "rely", and the definitions given are basically in terms of human attributes. However, there is a large body of literature currently accepted by the "assurance sciences" in which reliability is defined as "the probability

---

*Numbers within parentheses in the text designate numbered references given in the List of References.

that an equipment will operate for a stated period of time under a specified set of conditions."(2) The three key words in this definition are probability, time, and conditions. When this definition is used, the concept of 99% reliability has no meaning unless the time span and conditions of operation are also stated. Thus, the equipment can have a reliability of 99% under normal operating conditions over a period of 10 years, but the reliability for a period of 1 year under the same conditions would be considerably higher in most cases.

There is also a large body of literature in which reliability is defined as the probability that the induced level of stress will remain below the instantaneous allowable stress of a component.(3) When loads or strengths are time dependent, this definition is essentially acceptable in the context of the preceding definition. However, the two definitions are not compatible for strengths and loads that cannot be assigned a time dependency, and care must be used when combining the analysis methods associated with these two definitions of reliability.

The history of quantitative reliability analysis is primarily related to large populations of equipment from which a 1% probability of failure can be translated into one of 100 parts failing during operation. If there is a foreknowledge that a part is likely to fail and cause the failure of a mission, the need for predicting this failure is easier to comprehend. Thus, the concept of predicting system failures by synthesizing component failure rates has been accepted. However, when a person speaks of a single, one-of-a-kind component having one chance in 100 of failing, the value of such knowledge is not as easily understood or explained. The skeptic is justifiably prone to respond "I want to know whether this part will fail, not how many out of 100 will fail." Although it is not possible to predict the failure of a single component, a reliability analysis can be used to estimate which component is more likely to fail, and in a relative sense, one chance in 100 has meaning when compared with one chance in 1,000. Thus, an estimated probability of failure of 0.001 is seen to be better than one of 0.01 regardless of whether or not failure can be predicted absolutely.

The majority of the reliability analyses currently performed are systems analyses. The weakest link in an unreliable system is usually made redundant to improve the chances of successful system operation. For example, of the millions of parts in a Saturn rocket, several thousand are expected to fail during a launch. However, these failures do not abort the mission because the systems are designed to tolerate component failures.(4) When performing a reliability analysis of a system, the analyst ordinarily makes a basic assumption that the components in the system will operate within a specified environment and range of loads. Without this assumption, neither the component failure rates nor the system reliability forecast will be valid. This in essence restricts a system reliability analysis to predicting random failure under known loads. Overloads, extreme environmental conditions, and extraneous factors are not part of this forecast except as they contribute to the "randomness" of failure.

Random failures are those which occur at a distributed rate over long periods of time. Wear-out failures are usually normally distributed near the end of component life, while unexpected failures are those which just happen during the useful life-span of equipment and may be exponentially distributed. The apparent cause of these unexpected failures is often not known. A resistor shorts out, a bearing begins to make noise, or a bracket cracks. Actually, these failures have causes, but generally they are randomly dispersed because they are the result of the chance occurrence of simultaneous events which result in failure. A certain set of environmental conditions, load fluctuations, part defects, and cyclic histories may combine at one instant to cause the failure of one component, but this may not happen to another component for several years or it may not happen at all.

The determination of component reliability by analytical methods is not a widely accepted practice, but it is the essential tie between system reliability based on known loads and system failure resulting from an unexpected load. Component reliability is typically determined by collecting failure rates on operating components, and these component failure rates are used to forecast a synthesized system failure rate. Thus, an underlying argument for reliability analysis of single components lies in the need for obtaining component failure rates to perform system analyses. This is particularly true for nuclear reactor systems, for which failure data are scarce and few collection programs exist to provide accurate data for component failure rates.

Nuclear pressure vessels are designed in accordance with rules which in essence establish the minimum safety factors to be used. There is increasing interest in lowering some of these safety factors and in raising others to lower costs and increase safety. Analytical methods now exist for estimating failure probability as a function of applied and allowable stresses. In the investigation reported herein, these methods are applied to the nuclear pressure vessel to develop analytical expressions for its failure probability. The terms used in these expressions and in their development are explained in the text, and a complete listing of these terms and definitions is presented in Appendix A. These analytical expressions are used to evaluate the minimum safety factors imposed on nuclear pressure vessels. Data from several sources are reviewed and used to estimate typical failure probabilities to be expected in vessel design. Conclusions relative to the adequacy of existing design rules are drawn, and recommendations for maintaining, lowering, or raising established safety margins are presented.

2. PROBABILISTIC DESIGN

Probabilistic design is the method by which the probability of a defined failure is predicted.(5) For expected frequency distributions of component strength and induced stress, this probability will be a function of the area of overlap of the two distribution functions.

Aside from the lack of adequate data upon which to base stress and strength distributions, the basic problem in probabilistic design is one of defining failure. Since the probability of failure has been stated in terms of induced stress and allowable stress (3) in Section 1, a definition of failure in these same terms will simplify the application of probabilistic design methods. However, this definition must be amplified to include the concept of cyclic failure. Failure is then defined as the occurrence of a single induced stress in excess of the allowable level for that stress or the occurrence of a number of induced stresses of a given amplitude in excess of the allowable number of stresses at that amplitude. The word "reliability" as used herein will therefore refer to the definition reported by Juran, (2) while the term "probability of failure" will be used in connection with the definition of failure just stated. Random failures can be thought of as those caused by the chance occurrence of a stress level which exceeds a simultaneous occurrence of component strength. The same strength at a different time would not result in failure if the stress level were lower at that instant.

Thus, the basic problem of deciding what to call a failure becomes one of defining a component strength above which a stress would be considered intolerable. There are several allowable stress intensities (minimum strengths) for a nuclear pressure vessel, and these allowable stresses are dependent upon the nature of the loads which produce the stresses and the extent of the stresses throughout the vessel.(6) The definition of failure employed herein permits failure to be treated analytically without tying it to a physical occurrence. Any induced stress greater than the value selected as the allowable stress is a failure by definition.

2.1 Analytical Model

For failure as defined herein, the probability of failure (Q) is the probability that an induced stress (S_i) will exceed the allowable stress (S_a) for any values of the induced stress and allowable stress that exist simultaneously.

$$Q = P(S_i \geq S_a) \quad . \tag{2.1}$$

When inequality operations are applied to Equation 2.1, the probability of failure can also be stated as

$$Q = P(S_a - S_i \leq 0) \quad . \tag{2.2}$$

Because there are uncertainties in the measurement of strength and there are variations in strength throughout a material, the allowable stress may take on a distributed set of values. In the simplest case, this may be thought of as a population of tensile specimens, each of which is subjected to a load that causes failure. The stresses at failure for each of the specimens become the allowable stress distribution of the material. Similarly, the material will experience an induced stress that takes on distributed values because of the uncertainties in forecasting loads and the operating transients that are likely to occur. This induced stress frequency distribution can be thought of as a single tensile specimen subjected to a population of loads until failure occurs.

If each tensile specimen has an equal chance of experiencing any of the possible loads, the probability of failure of any given specimen is the joint probability that a particular tensile specimen will be chosen and that a load greater than the strength of that specimen will occur. Stated in terms of stress, the probability of failure of any given specimen is the joint probability that an allowable stress exists and that an induced stress in excess of that allowable stress will occur.

If the probability density function of induced stress is

$$f(S_i) = f(I_1, I_2, \dots I_n) \tag{2.3}$$

and the probability density function of allowable stress is

$$g(S_a) = g(A_1, A_2, \dots A_n) , \tag{2.4}$$

the probability that any value of S_i exceeds a given value of S_a is given by the distribution function $F(S_i)$. (7)

$$F(S_i) = P(S_i \geq A_n) = \int_{-\infty}^{A_n} f(S_i)\, dS_i \ . \tag{2.5}$$

The probability that A_n exists is given by the expression (8)

$$P(S_a \leq A_n \leq S_a + dS_a) = g(S_a)\, dS_a \ . \tag{2.6}$$

The joint probability of A_n existing and of S_i exceeding A_n is given by the product of the individual probabilities.

$$P(S_a \leq A_n \leq S_a + dS_a \,\|\, S_i \geq A_n) = g(S_a) dS_a \int_{-\infty}^{A_n} f(S_i)\, dS_i \ . \tag{2.7}$$

For the probability that any values of allowable stress will be exceeded, Equation 2.7 must be integrated over the entire range of $g(S_a)$.(7) This will give the probability of failure Q.

$$Q = \int_{-\infty}^{\infty} g(S_a) \int_{-\infty}^{A_n} f(S_i)\, dS_i\, dS_a \;. \qquad (2.8)$$

Equation 2.8 can be thought of physically as the distribution function of the area of overlap of the two frequency distributions.(9)

The frequency distributions of induced stress values and allowable stress values are illustrated on the same axis in Figure 2.1. The shaded area represents the number of failure causing stresses. The description of the frequency distribution is of great importance in evaluating Equation 2.8. As written, Equation 2.8 applies to any frequency

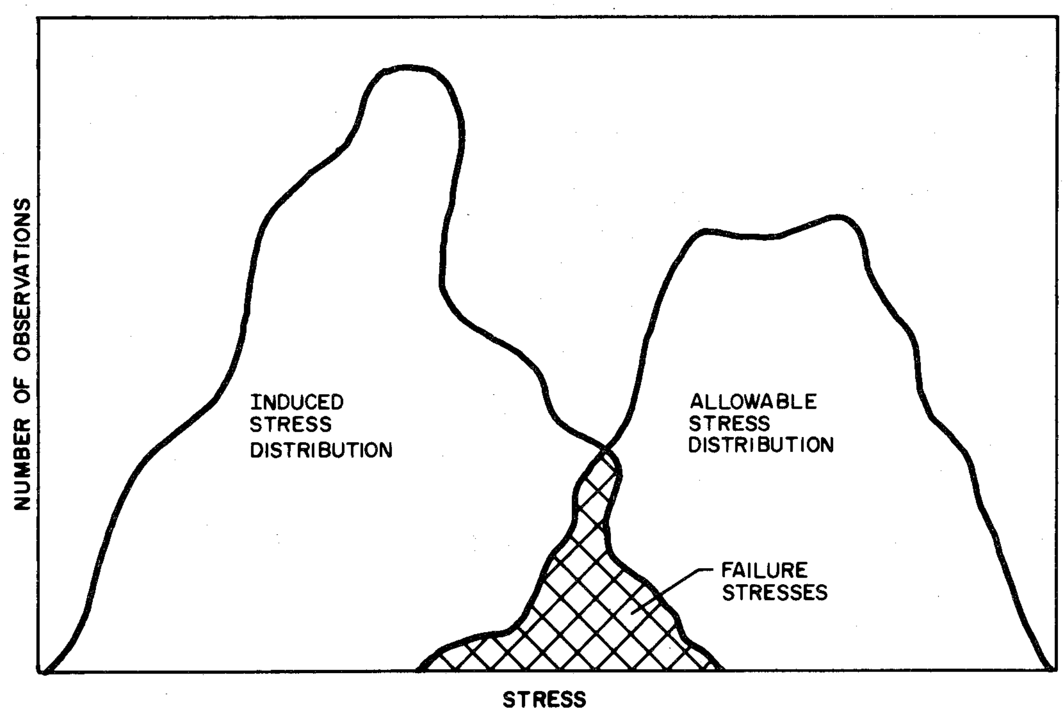

Figure 2.1. Failure Stresses Resulting From Overlap of Two Stress Distributions.

distribution, $f(S_i)$ or $g(S_a)$, that fulfills the requirements set forth in the development of the equation. However, in actual practice, the integral can be evaluated for only certain distributions such as exponential, normal, log normal, and Weibull. In later equations, certain relationships will hold for only one distribution function and care must be exercised in applying these equations to assure that the relationships are valid for the distribution function in use.

The distribution function, F(S), has a value of unity when the probability density function, f(S), is integrated over all possible values.

$$\int_{-\infty}^{\infty} f(S)\, dS = 1 \; . \qquad (2.9)$$

The integral can be broken up as follows.

$$\int_{-\infty}^{\infty} f(S)\, dS = \int_{-\infty}^{S_n} f(S)\, dS + \int_{S_n}^{\infty} f(S)\, dS = 1 \; . \qquad (2.10)$$

It can be seen from Equation 2.5 that the first part of the integral (for values below S_n) is the probability of failure (Q).

$$Q = \int_{-\infty}^{S_n} f(S)\, dS \; . \qquad (2.11)$$

The second part of the integral contains all values greater than S_n, which are all other possible values. Thus, the second part of the integral must be the probability of success (P).

$$P = \int_{S_n}^{\infty} f(S)\, dS \; . \qquad (2.12)$$

It follows from Equation 2.10 that

$$P + Q = 1 \ . \tag{2.13}$$

There are two parameters of the distribution function that will be needed in performing calculations of failure probability. These are the mean ($\overline{S}$) and the standard deviation (σ_s). By definition,(7)

$$\overline{S} = \int_{-\infty}^{\infty} Sf(S)\ dS \tag{2.14}$$

and

$$\sigma_s^2 = \int_{-\infty}^{\infty} (S - \overline{S})^2 f(S)\ dS \ , \tag{2.15}$$

where S is a distributed random variable.

2.2 Reliability and Failure

The probability of failure resulting from any two frequency distributions of induced and allowable stresses is expressed by Equation 2.8. This probability depends only on the distribution functions and their interference with one another. Conditions of the distribution that are not time dependent are referred to herein as steady-state conditions.

While the allowable stress distribution is not likely to change with time, the distribution of the applied stress may arise from a time-dependent situation, such as cyclic pressure fluctuations. As well as providing a basis for analyzing the fatigue life of the vessel, these cyclic pressures actually describe a frequency distribution for the steady-state design pressure. As long as certain values of these cyclic stresses do not cluster at a given time, the induced stress distribution will be independent of time and Equation 2.8 can be used to calculate the steady-state failure probability. When one or more of the distribution functions is a function of time, such as the allowable stress distribution for fatigue failure, Equation 2.8 represents the time-dependent failure probability from which reliability can be calculated by using Equation 2.13. Thus, Equation 2.8 can be used to calculate reliability only when the distribution functions are time dependent. In all other cases, Equation 2.8 is simply an expression for the probability that failure will occur at any time.

The probability of failure resulting from a steady-state stress distribution may have a time dependency that is not at first evident from the data of which the distribution is comprised. For example, the fact that a stress of any given level has an equal chance of occurring at any time, as was postulated for the steady-state distribution, will result in an exponential failure probability as a function of time.(7)

$$Q = 1 - e^{-\lambda T} , \tag{2.16}$$

where λ is the average rate at which failure stresses occur and T is the interval of time over which the failure probability is estimated.

A more general case is one in which it is assumed that the rate at which failure stresses occur can vary with time. Such a case can be expressed by the two-parameter Weibull distribution function.(10)

$$Q = 1 - e^{-(T^b/m)} , \tag{2.17}$$

where b and m are parameters determined from the distribution of failure causing stresses. If b = 1, the Weibull distribution becomes the exponential distribution of Equation 2.16 and m corresponds to the mean time between failures ($1/\lambda$). Equations 2.16 and 2.17 are statements of reliability because they express time-dependent probabilities. Thus, from Equation 2.13, reliability (R) can be expressed as

$$R = e^{-(T^b/m)} \quad . \tag{2.18}$$

The probability of failure can therefore be calculated as an interference of induced stress with allowable stress by using Equation 2.8, or it can be calculated as the reliability resulting from a specified level of stress (corresponding to the parametric values of b and m) for any time by using Equation 2.18.

2.3 Environmental Effects

The calculation of reliability as a time-dependent probability is usually done by measuring values of m and b in experimental or operational situations and using these values in equations similar to Equation 2.18 (depending upon the type of distribution R takes on). When this procedure is followed, the same environment observed while the parameters were measured must be postulated for the predicted operation, as stated by the definition of reliability used herein. If there is any deviation from the measured environment, the predicted reliability is theoretically invalid.

If the effects of environment (stress and strengths) are considered, a more accurate estimate of reliability is possible. For example, the measured failure rates might be used in Equation 2.16 to forecast component reliability and in Equation 2.8 to determine the allowable variation in environment necessary to maintain the validity of Equation 2.16.

The use of measured failure rates would not only forecast reliability but would also tie that reliability to acceptable limits of environmental variation.

3. STEADY-STATE DISTRIBUTION FUNCTIONS

In order to calculate a steady-state failure probability, the distribution functions must be determined. If data are available for a specific application, the distribution functions should be based on that data. However, there are many possible distributions, and some of them cannot be mathematically stated in a closed form. Hence, there will be many instances in which a design must be formulated before the distribution function is known. It is therefore important to determine whether pressure vessel data are likely to have typical distributions from which a generalized equation for failure probability can be developed.

3.1 Stress Distributions

There is much evidence that the material strength associated with a steady-state general membrane stress occurs as a normally distributed function. As a specific example, the distribution function was determined for 16 values of yield stress for A533 Grade B steel obtained under similar conditions.(11) These data were analyzed by computing the percentage of the total population of tensile test specimens expected to fail at each stress level, as outlined in Table 3.1. The symbol S_j in Table 3.1 represents the induced failure stress in kips (from the word kilo-pounds where 1 kip = 1,000 pounds) per square inch. The symbol N_j represents the population prior to application of the j-th stress, and r_j represents the number of failures resulting from the j-th stress. The percentage of the original population that survived application of the j-th stress level is represented by the symbol RS_j. The values of

Table 3.1. Parameters Used to Construct Distribution Function of Yield Strength

S_j	N_j	r_j	$N_j + 1$	$N_j + 1 - r_j$	$\frac{N_j + 1 - r_j}{N_j + 1}$	RS_j*
58.2	16	1	17	16	0.941	0.941
58.6	15	1	16	15	0.937	0.882
59.0	14	1	15	14	0.933	0.823
59.4	13	1	14	13	0.928	0.763
59.5	12	1	13	12	0.923	0.705
59.6	11	1	12	11	0.912	0.646
59.9	10	2	11	9	0.818	0.528
60.1	8	1	9	8	0.889	0.470
60.3	7	1	8	7	0.875	0.411
60.6	6	1	7	6	0.857	0.352
61.0	5	1	6	5	0.833	0.293
61.3	4	1	5	4	0.8	0.235
61.4	3	1	4	3	0.75	0.176
61.5	2	1	3	2	0.667	0.117
62.2	1	1	2	1	0.5	0.058

* $RS_j = \prod \frac{N_j + 1 - r_j}{N_j + 1}$

S_j are plotted as yield strengths and values of $1 - RS_j$ are plotted as percent failed in Figure 3.1. It can be seen that the resulting plot is nominally a straight line, thereby fulfilling the criterion for a normally distributed function.(7)

Figure 3.1 alone does not provide justification for assuming that a normal distribution is typical of allowable stresses. An extensive survey of test data reported by Lipson, Sheth, and Disney (9) revealed that the type of distribution changed significantly with changes in temperature. In that data, the distribution functions of yield and ultimate strengths for fully annealed low-carbon, low-alloy steels and for fully annealed low-carbon, high-alloy steels are approximately normal at

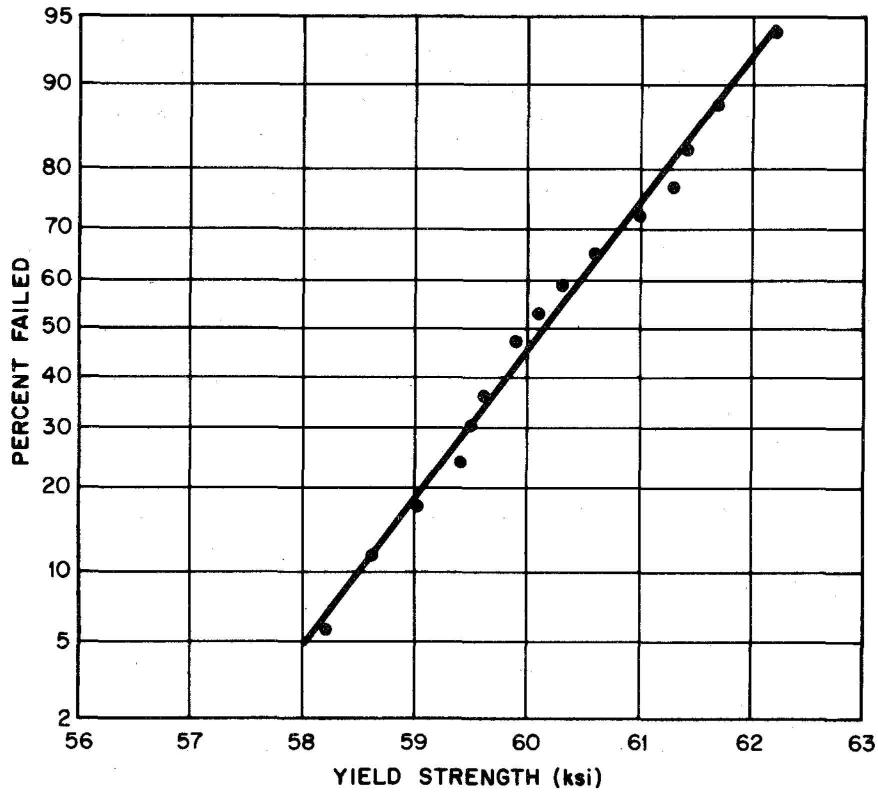

Figure 3.1. Distribution Function of Yield Strength Plotted on Normal Probability Scale.

temperatures between 250 and 750°F. Inasmuch as the data illustrated in Figure 3.1 were obtained within that temperature range, the assumption of normally distributed allowable stress functions for pressure vessels in light-water-cooled nuclear reactor systems would not be grossly inaccurate. However, this assumption might not be valid for very high temperature conditions, such as those experienced by vessels in liquid-metal fast breeder reactors, or for near ambient temperature conditions, such as those experienced by a containment vessel.

The induced stress distribution for steady-state conditions closely approximates a normal distribution when there is a very high incidence of peak pressures on either side of the design pressure. These pressures are generally assumed to occur as a factor in fatigue analysis, and a suitable example might be 10^6 cycles of ± 100 psi, 200 cycles of ± 120 psi, and 5 cycles of $\pm$ 500 psi. The induced stress will be normally distributed by specification in such a case, and it may be assumed as normally distributed in practice when large numbers of pressure fluctuations are assumed on either side of a mean. It may therefore be assumed that normally distributed induced and allowable stresses are typical for many steady-state stress conditions. Equation 2.8 will be developed for these specific distribution functions as a typical case for steady-state failure probability.

3.2 Failure Probability

When S is a normally distributed random variable, the frequency distribution is given by the following equation.(8)

$$f(S) = \frac{1}{\sigma_s (2\pi)^{1/2}} \left[e^{-\frac{1}{2}\left(\frac{S - \overline{S}}{\sigma_s}\right)^2} \right] , \tag{3.1}$$

where $\overline{S}$ is the mean, as defined in Equation 2.14, and σ_s is the standard deviation, as defined in Equation 2.15. The normal distribution function is obtained by integrating Equation 3.1 over all values of S.

$$F(S) = \frac{1}{\sigma_s (2\pi)^{1/2}} \int_{-\infty}^{S} e^{-\frac{1}{2}\left(\frac{S - \overline{S}}{\sigma_s}\right)^2} dS . \tag{3.2}$$

The probability of failure resulting from the interference of normal distributions of induced and allowable stress is obtained by using Equation 3.2 to calculate the probability that $S_a - S_i \leq 0$. When $S = S_i - S_a$ is taken as the difference between two normally distributed independent variables, it can be shown by the method of maximum likelihood estimators (5) that

$$\overline{S} = \overline{S}_i - \overline{S}_a \tag{3.3}$$

and

$$\sigma_s = \left(\sigma_{S_i}^2 + \sigma_{S_a}^2\right)^{1/2} , \tag{3.4}$$

where $\overline{S}$, $\overline{S}_i$, $\overline{S}_a$, σ_s, σ_{S_i}, and σ_{S_a} are the respective means and standard deviations of S, S_i, and S_a.

When S is the difference between the induced stress and the allowable stress, the probability of failure is obtained by evaluating Equation 3.2 over positive values of S, as shown in Equation 3.5.

$$Q = \frac{1}{\sigma_s (2\pi)^{1/2}} \int_o^{\infty} e^{-\frac{1}{2}\left(\frac{S - \overline{S}}{\sigma_s}\right)^2} dS \; . \tag{3.5}$$

If

$$u = \frac{S - \overline{S}_i + \overline{S}_a}{\left(\sigma_{S_i}^2 + \sigma_{S_a}^2\right)^{1/2}} = \frac{S - \overline{S}}{\sigma_s} \tag{3.6}$$

and if appropriate changes in the variables of integration are made, Equation 3.5 can be expressed as

$$Q = \frac{1}{(2\pi)^{1/2}} \int_m^{\infty} e^{-u^2/2} \, du \; , \tag{3.7}$$

where

$$m = \frac{-\overline{S}_i + \overline{S}_a}{\left(\sigma_{S_i}^2 + \sigma_{S_a}^2\right)^{1/2}} \; . \tag{3.8}$$

Equation 3.8 is the "coupling" equation whose value is used to evaluate Equation 3.7 with standard tables of probability functions.(9) The probabilities of failure, determined by using Equation 3.7, for several values of m are given in Table 3.2.

Table 3.2. Values of Normal Integral

m	Q = (1 - P)
5.0	2.87×10^{-7}
4.5	3.40×10^{-6}
4.0	3.12×10^{-5}
3.5	2.33×10^{-4}
3.0	1.35×10^{-3}
2.8	2.56×10^{-3}
2.6	4.66×10^{-3}
2.4	8.20×10^{-3}
2.2	0.0139
2.0	0.0227
1.8	0.0359
1.6	0.0548
1.4	0.0807
1.2	0.1151
1.0	0.1586
0.5	0.3085
0.0	0.50

The method of maximum likelihood estimators (5) used to develop Equations 3.3 and 3.4 can also be used to show other mathematical operations with normally distributed variables. Several of these operations that are of use in performing stress analyses of pressure vessels are given, without proof, in Table 3.3. These operations are valid for values of the coefficient of variation (V_s) less than 0.1, where the coefficient of variation is defined as

$$V_s = \frac{\sigma_s}{\bar{S}} . \tag{3.9}$$

When substituted into Equation 3.8, the coefficients of variation of allowable and induced stresses (V_{S_a} and V_{S_i}) and the ratio of allowable to induced stress (M) reduce the number of variables, as is shown in Equation 3.10.

$$m = \frac{M - 1}{\left(V_{S_a}^2 M^2 + V_{S_i}^2\right)^{1/2}} , \tag{3.10}$$

where

$$M = \frac{\bar{S}_a}{\bar{S}_i} . \tag{3.11}$$

The number of variables in Equation 3.8 is further reduced by introduction of the coefficient of variation of failure probability (C), (8) where

$$C = \left(V_{S_a}^2 M^2 + V_{S_i}^2\right)^{1/2} . \tag{3.12}$$

Because each value of m in Equation 3.10 corresponds to a failure probability in Equation 3.7, the probability of failure is a function of the two variables M and C. The failure probabilities for several values of M and C reported by Kececioglu and Haugen (8) are illustrated in

Table 3.3. Operations With Normal Functions

Item	Operation
1.	$w = x + y$
	$\bar{w} = \bar{x} + \bar{y}$
	$\sigma_w^2 = \sigma_x^2 + \sigma_y^2$
2.	$w = x - y$
	$\bar{w} = \bar{x} - \bar{y}$
	$\sigma_w^2 = \sigma_x^2 + \sigma_y^2$
3.	$w = xy$
	$\bar{w} = \bar{x}\bar{y}$
	$\sigma_w^2 = \sigma_x^2\sigma_y^2 + \bar{x}^2\sigma_y^2 + \bar{y}^2\sigma_x^2$
4.	$w = \frac{x}{y}$
	$w = \frac{\bar{x}}{\bar{y}} + \frac{\sigma_y^2\bar{x}}{\bar{y}^3}$
	$\sigma_w^2 = \frac{1}{\bar{y}^2}\left(\frac{\bar{x}^2\sigma_y^2 + \bar{y}^2\sigma_x^2}{\bar{y}^2 + \sigma_y^2}\right)$

Figure 3.2. The data illustrated in Figure 3.2 can be used in lieu of Equation 3.7 and Table 3.2 (page 20) to determine the failure probability for normally distributed induced and allowable stresses. The parameter M can be thought of physically as the "safety factor" corresponding to a desired probability of failure (Q) and coefficient of variation of failure probability (C).

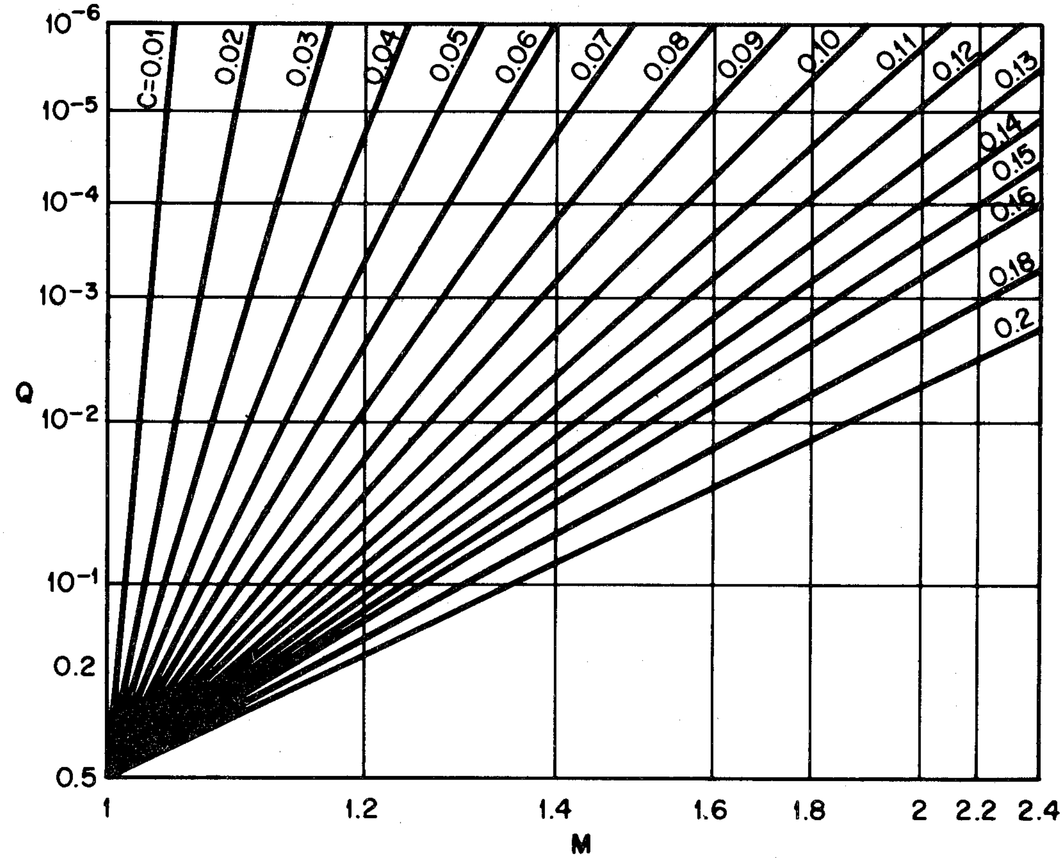

Figure 3.2. Probability of Failure as a Function of Dimensionless Parameters of Normal Functions.

Source: D. Kececioglu and E. B. Haugen, "A Unified Look at Design Safety Factors, Safety Margins, and Measures of Reliability," pp. 520-530 in 1968 Annals of Assurance Sciences, Seventh Reliability and Maintainability Conference, The American Society of Mechanical Engineers, New York, 1968.

4. STEADY-STATE INDUCED STRESSES

The probability of failure is stated in Section 3 in terms of allowable stress distributions and induced stress distributions. When these two stress distributions are normal, the failure probability can be obtained either directly from Figure 3.2 (page 24) or from Equation 3.7 and Table 3.2 (page 20). While the allowable stress distribution can be measured or estimated as stress, the induced stress distribution usually cannot be obtained as a directly measured stress. The induced stress distribution is usually the result of a distributed set of loads acting on a component with a distributed set of dimensions, and it must be calculated by using stress analysis methods in conjunction with probability calculus. Such analyses are often referred to as "probabilistic design."(5)

The rules for the stress analysis of nuclear pressure vessels set forth in Section III of the ASME Boiler and Pressure Vessel Code (12) are based on the maximum shear stress theory of failure.(13) This theory stipulates that failure occurs when the maximum shear stress at a point exceeds the shear stress corresponding to the yield point in a uniaxial tension test specimen. Stated mathematically, for principal stresses $S_1 > S_2 > S_3$ and a yield stress S_y, failure results when

$$\frac{S_1 - S_3}{2} \geq \frac{S_y}{2} \; . \tag{4.1}$$

The probabilistic design methods by which the principal stresses in a vessel are determined can be used to determine the induced stress distribution required to calculate the probability of failure when the allowable stress level is defined by the maximum shear stress theory. Since the principal stresses in a pressure vessel away from discontinuities generally consist of the hoop stress, the meridional stress, and the internal pressure, a generalized derivation of probabilistic design formulas for pressure vessels is possible.

4.1 Hoop Stress

If the hoop stress (S_h) in a pressure vessel subjected to an internal pressure (p) is given by the equation

$$S_h = pY \ , \tag{4.2}$$

where Y is a function of the radius of the vessel and thickness of the shell, the mean value of the hoop stress resulting from a normally distributed p and Y is determined from Table 3.3 (page 22) to be

$$\bar{S}_h = \bar{p}\bar{Y} \ . \tag{4.3}$$

The standard deviation of hoop stress (σ_{S_h}) expressed in terms of Equation 4.3 is

$$\sigma_{S_h} = \left(\bar{p}^2\sigma_Y^{\ 2} + \bar{Y}^2\sigma_p^{\ 2} + \sigma_Y^{\ 2}\sigma_p^{\ 2}\right)^{1/2} \ . \tag{4.4}$$

The mean values of the two distributions can be eliminated by substituting the coefficients of variation of S_h, p, and Y into Equation 4.4.

$$V_{S_h} = \left(V_p^{\ 2} + V_Y^{\ 2} + V_p^{\ 2}V_Y^{\ 2}\right)^{1/2} \ . \tag{4.5}$$

Values of V_{S_h} for several values of V_p and V_Y that were calculated by using Equation 4.5 are illustrated in Figure 4.1. Figure 4.1 can be used as a graphical aid in determining the coefficient of variation of hoop stress when the mean value of the hoop stress is expressed by

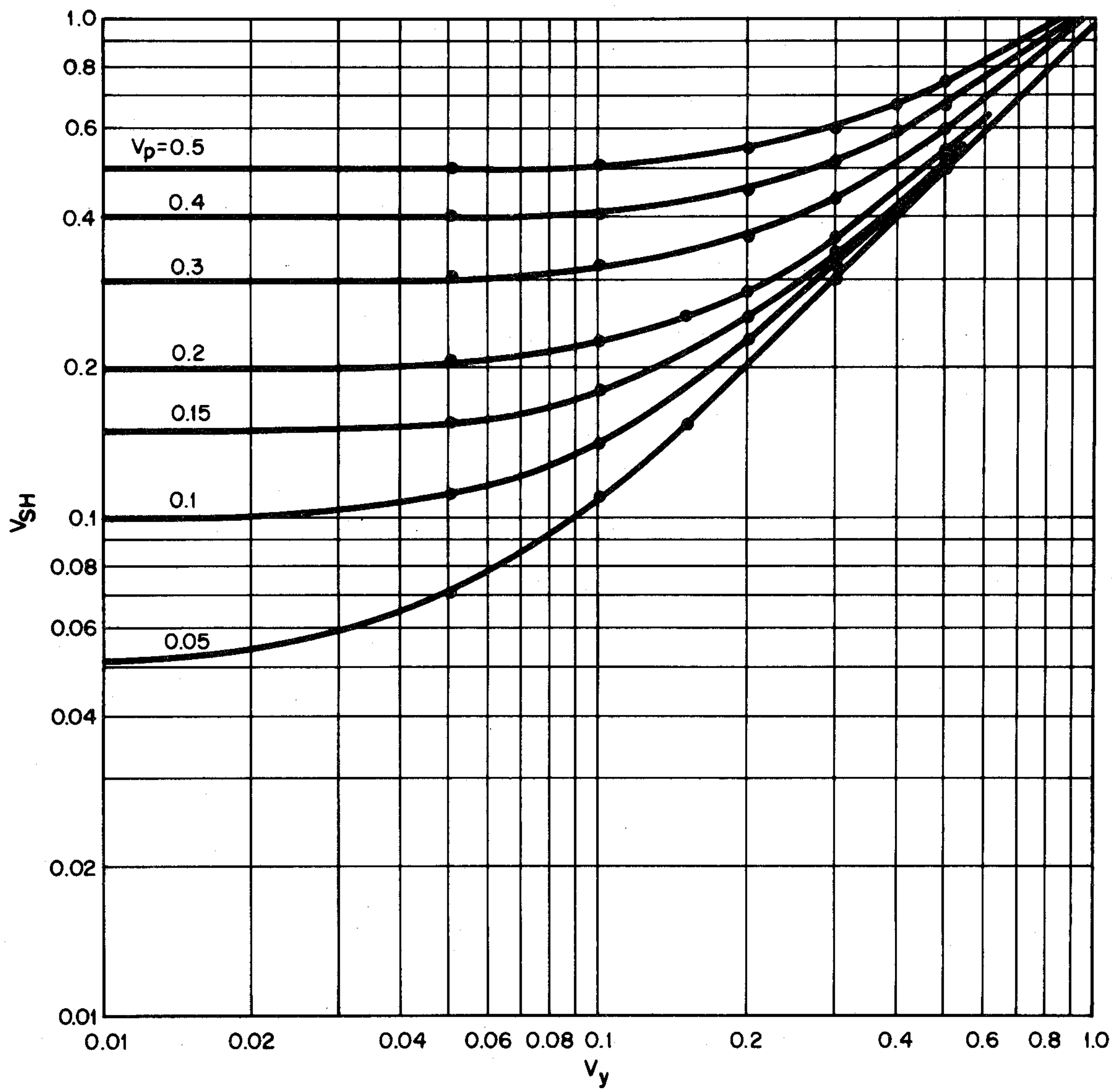

Figure 4.1. Coefficient of Variation of Hoop Stress as a Function of Coefficients of Variation of the Product of Two Normally Distributed Functions.

Equation 4.3. The standard deviation is then obtained as

$$\sigma_{S_h} = V_{S_h}\bar{S}_h \; . \tag{4.6}$$

The quantity Y in Equation 4.2 is a function of the radius (r) and shell thickness (t) of a cylinder or sphere subjected to internal pressure. If r and t are invariant, Y is invariant and the standard deviation of hoop stress is simply the product of Y and the standard deviation of pressure. If r and t are normally distributed, Y will also be normally distributed. For a cylinder,

$$\bar{Y} = \bar{r}/\bar{t} \; . \tag{4.7}$$

From Table 3.3 (page 22), the standard deviation of Y for values of V_t less than 0.1 is

$$\sigma_Y{}^2 = \frac{1}{\bar{t}^2}\left(\frac{\bar{t}^2\sigma_r{}^2 + \bar{r}^2\sigma_t{}^2}{\bar{t}^2 + \sigma_t{}^2}\right) . \tag{4.8}$$

The coefficient of variation of Y is determined by substituting the coefficients of variation of $\bar{r}$ and $\bar{t}$ into Equation 4.8.

$$V_Y = \left(\frac{V_r{}^2 + V_t{}^2}{1 + V_t{}^2}\right)^{1/2} \; . \tag{4.9}$$

Values of V_Y for several values of V_r and V_t that were calculated by using Equation 4.9 are illustrated in Figure 4.2. Figure 4.2 can be used to graphically determine values of V_Y to be used in Figure 4.1 (page 27) or in Equation 4.5 when r and t are normally distributed variables.

Although Equation 4.9 was derived for hoop stress in a cylinder, it is valid for any Y = ar/t, where a is a constant or invariant, because the coefficient of variation of Y is calculated as

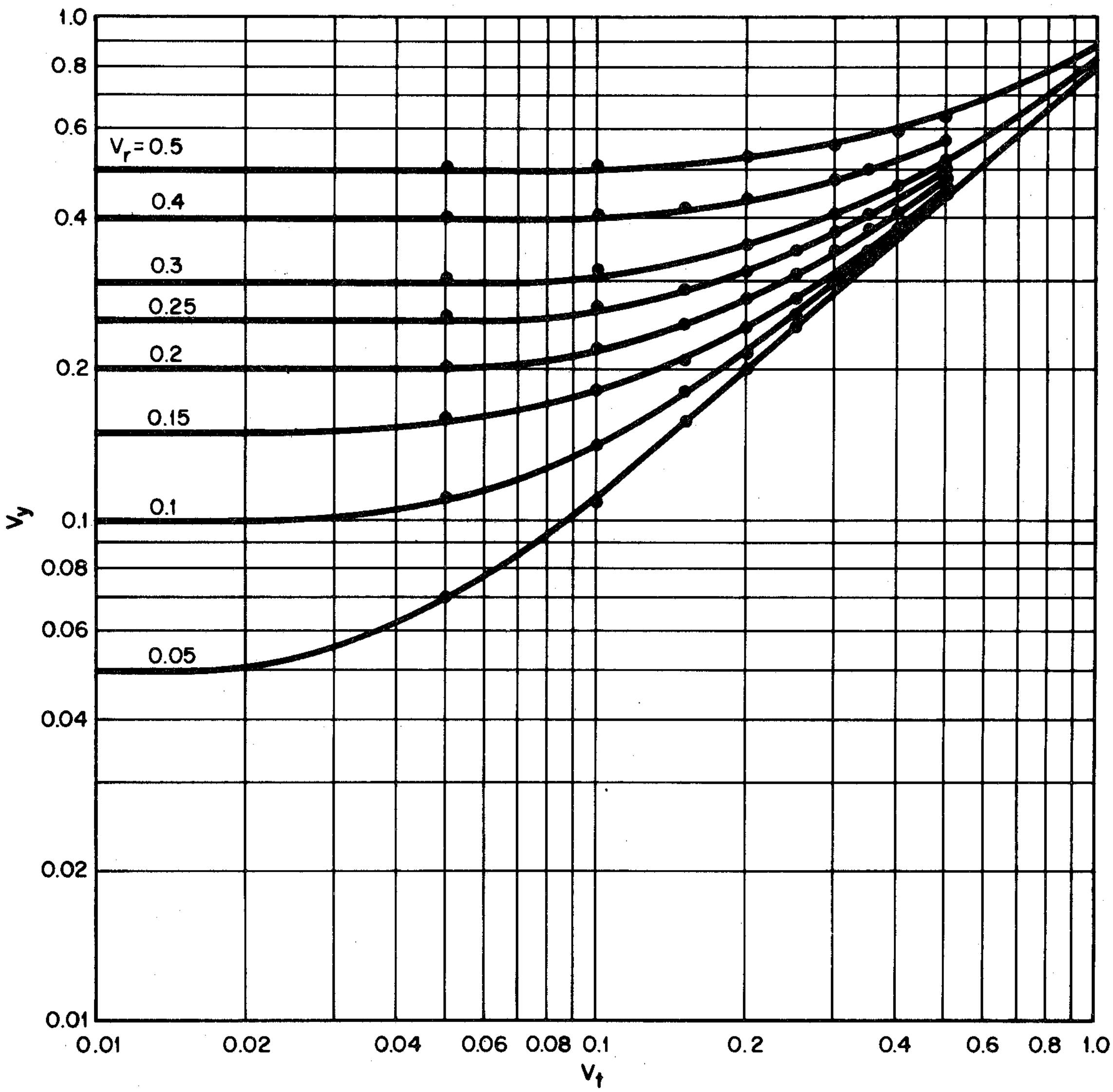

Figure 4.2. Coefficient of Variation of the Quotient (Y) of Two Normally Distributed Functions.

$$V_Y = \frac{\sigma_Y}{\bar{Y}} = \frac{a\sigma_{r/t}}{a\,\frac{\bar{r}}{t}}$$

and the constant (a) does not affect the coefficient of variation. Thus, Equation 4.9 and Figure 4.2 apply to a sphere or any other shape for which Y is expressed in terms of a constant multiplied by the radius-to-thickness ratio. Equation 4.5 and Figure 4.1 (page 27) are also applicable to these same geometries for the same reasons.

The normal distribution function of hoop stress in pressure vessels of several shapes can therefore be determined as a function of pressure and geometry by using Equations 4.3 and 4.5. This stress is then used to determine the induced stress distribution in accordance with the maximum shear stress theory of failure.

4.2 Induced Membrane Stress

The maximum principal stress in a cylinder or sphere under internal pressure is the hoop stress, and the minimum principal stress is the negative of the internal pressure.(13) The mean value of induced stress is therefore

$$\bar{S}_i = \frac{\bar{S}_h + \bar{p}}{2}\,, \tag{4.10}$$

and the standard deviation of induced stress, from Table 3.3 (page 22), is expressed as

$$\sigma_{S_i} = \frac{1}{2}\left(\sigma_{S_h}^{\;2} + \sigma_p^{\;2}\right)^{1/2}\,. \tag{4.11}$$

Equation 4.11 is not simplified by substitution of the coefficients of variation of S_h and p. However, the mean and standard deviation of induced stress required in Equation 3.8 for determining the probability of failure are stated directly in Equations 4.10 and 4.11. Therefore, the probabilistic design calculations for determining the induced stress for the maximum shear stress theory of failure are culminated in these two equations, with intermediate steps represented by Figures 4.1 (page 27) and 4.2 (page 29) or Equations 4.5 and 4.9.

When local effects are considered, the hoop stress and pressure are not always the maximum and minimum principal stresses. Such local effects have not been considered here, but they can be calculated by probabilistic methods if necessary. The simpler stresses, such as bending, tension, and torsion, have been treated probabilistically by Haugen. (5) These stresses in combination with the hoop stress can be evaluated by using Table 3.3 (page 22) and following a procedure similar to that followed to derive the probabilistic statement for hoop stress.

The induced stress calculations are greatly simplified if Y is invariant. In practice, the radius and shell thickness of a pressure vessel are held to rather close tolerances (about 2%) when compared with the expected variation in loads and material strengths (about 15%). It is therefore possible in many cases to neglect the dimensional distribution functions and calculate the induced stress as a function of a distributed load only.

The failure probability resulting from the induced stress distribution is calculated for a specified allowable stress distribution by using Equation 3.7 or Figure 3.2 (page 24). However, care must be exercised in doing this because the induced stress calculated by using Equation 4.10 must be compared with an allowable stress defined in accordance with the maximum shear stress theory of failure; that is, an allowable stress corresponding to one-half of the yield strength determined in an uniaxial tensile test. Because both the induced stress and the yield strength are divided by two, "stress intensity" is defined in Section III of the ASME Boiler and Pressure Vessel Code (12) as a stress equal to twice the induced shear stress, and "allowable stress intensity" (S_m) is defined (12) as an allowable design stress equal to twice the shear stress allowed by Code rules. The term "allowable stress" used herein is a defined failure stress, and its values do not correspond to the "allowable stress intensity" values specified in Section III of the ASME Boiler and Pressure Vessel Code inasmuch as those values have built-in "factors of safety."

4.3 Burst Pressure

The failure defined by Equation 3.7 is a mode of failure for which the probability is calculated by using induced stress. On the other hand, gross rupture or bursting is not easily expressed in terms of an induced stress because strain rather than stress plays the dominant role above the yield point of a material. The concept of "burst pressure" was presented by Langer (14) to correlate Section III ASME Code (12) allowable stress intensity values with data from rupture tests performed by the Pressure Vessel Research Committee. As reported by Langer,(14) the burst pressure (p_b) corresponding to the ultimate tensile strength (S_u) of a material can be calculated by using Equation 4.12.

$$p_b = S_u B \ln W \ , \qquad (4.12)$$

where B is a strain-hardening factor and

$$W = \left(1 + \frac{t}{r}\right) , \tag{4.13}$$

where

t = thickness of vessel shell and

r = inside radius of vessel.

Values of B were determined in the tests performed by the Pressure Vessel Research Committee,(14) and these values can be approximated by using Equations 4.14 through 4.19 in which B_c denotes the strain-hardening factor for a cylinder, B_s denotes the strain-hardening factor for a sphere, and n is the strain-hardening exponent of the material measured in tensile tests.

$$B_c = 1.16 - 0.9n \quad \text{for} \quad 0 \leq n < 0.1 . \tag{4.14}$$

$$B_c = 1.14 - 0.75n \quad \text{for} \quad 0.1 \leq n < 0.3 . \tag{4.15}$$

$$B_c = 1.08 - 0.51n \quad \text{for} \quad 0.3 \leq n < 1.0 . \tag{4.16}$$

$$B_s = 2.0 - 1.2n \quad \text{for} \quad 0 \leq n < 0.1 . \tag{4.17}$$

$$B_s = 1.98 - 1.06n \quad \text{for} \quad 0.1 \leq n < 0.3 . \tag{4.18}$$

$$B_s = 1.90 - 0.76n \quad \text{for} \quad 0.3 \leq n < 1.0 . \tag{4.19}$$

While Equation 4.10 can be used to estimate an induced stress to be compared with a measured allowable stress, Equation 4.12 can be used to compute a burst pressure to be compared with a measured induced pressure. The probability of failure by bursting can therefore be derived in a manner similar to that used to derive the probability of failure defined by maximum shear stress.

For normally distributed S_u, B, and p, Equation 4.12 can be written as

$$\overline{p_b} = \overline{S_u}\,\overline{B} \ln W , \tag{4.20}$$

where r and t are assumed to be invariant. From Table 3.3 (page 22), the standard deviation of burst pressure (σ_{p_b}) is expressed as

$$\sigma_{p_b} = \ln W\left(\bar{S}_u^{\,2}\sigma_B^{\,2} + \bar{B}^2\sigma_{S_u}^{\,2} + \sigma_{S_u}^{\,2}\sigma_B^{\,2}\right)^{1/2} . \qquad (4.21)$$

The coefficient of variation of burst pressure is expressed in Equation 4.22.

$$V_{p_b} = \left(V_B^{\,2} + V_{S_u}^{\,2} + V_B^{\,2}V_{S_u}^{\,2}\right)^{1/2} , \qquad (4.22)$$

where

$V_{p_b} = \sigma_{p_b} / \bar{p}_b$,

$V_B = \sigma_B/\bar{B}$, and

$V_{S_u} = \sigma_{S_u} / \bar{S}_u$.

Since the coefficient of variation of burst pressure is the coefficient of a product, Figure 4.1 (page 27) can be used to obtain values of V_{p_b} when the values of V_{S_u} and V_B are known and normally distributed.

The probability of failure is obtained by computing a value of m consistent with Equation 4.20 and $M = \bar{p}_b/\bar{p}$.

$$m = \frac{\bar{p}_b - \bar{p}}{\left(\sigma_{p_b}^{\,2} + \sigma_p^{\,2}\right)^{1/2}} , \qquad (4.23)$$

or

$$m = \frac{M - 1}{\left(V_{p_b}^{\,2}M^2 + V_p^{\,2}\right)^{1/2}} . \qquad (4.24)$$

The computed value of m can be used to determine the probability of failure by using Table 3.2 (page 20). To use Figure 3.2 (page 24) to determine failure probability, the value of the coefficient of variation of failure probability (C) must be calculated.

$$C = \left(V_{p_b}^{2} M^2 + V_p^{2}\right)^{1/2} . \tag{4.25}$$

Equations 4.24 and 4.25 together with Table 3.2 (page 20) or Figure 3.2 (page 24) provide a means of estimating the probability of failure by bursting based on the probability that an induced pressure will exceed a calculated burst pressure.

4.4 Numerical Example

A numerical example is presented here to demonstrate the method of calculating failure probability discussed in the preceding subsections. A cylinder with specified design conditions typical of those for a pressure vessel in a pressurized-water nuclear reactor is considered in this example, and the radius and shell thickness of this vessel are treated as invariant in this example wherein σ_p and σ_{S_a} are large as compared with σ_r and σ_t. The specified design conditions for this vessel are as follows.

$\bar{p}$ = mean or average internal pressure = 2,250 psi,

σ_p = standard deviation of internal pressure = 86 psi,

$\bar{S}_y$ = mean yield strength of the vessel material = 57,500 psi,

σ_{S_y} = standard deviation of the yield point = 3,068 psi,

$\bar{S}_u$ = mean ultimate tensile strength of vessel material = 83,000 psi,

σ_{S_u} = standard deviation of ultimate tensile strength = 4,650 psi,

S_m = allowable stress intensity as specified in Section III of the ASME Boiler and Pressure Vessel Code (12) = 26,700 psi,

r = radius of cylinder = 91 inches,

$\bar{B}_c$ = mean strain-hardening factor for cylinder = 0.97, and

σ_{B_c} = standard deviation of strain-hardening factor = 0.0116.

The rules of Section III of the ASME Boiler and Pressure Vessel Code (12) require that a minimum shell thickness (t_{min}) be established for the vessel by using the prescribed value of S_m for S in Equation 4.10. Equation 4.10 then becomes

$$t_{min} = \frac{\bar{p}r}{S_m - \bar{p}} = \frac{2,250(91)}{26,700 - 2,250} = 8.37 \text{ inches} . \tag{4.26}$$

The probability of failure defined by the maximum shear stress theory can be estimated by using Equation 3.8. The allowable stress is $\bar{S}_y/2$, and the induced stress must be calculated. For the minimum shell thickness, the mean hoop stress

$$\bar{S}_h = \frac{\bar{p}r}{t} = \frac{2,250(91)}{8.37} = 24,462 \text{ psi} , \tag{4.27}$$

and the standard deviation of the hoop stress

$$\sigma_{S_h} = \frac{\sigma_p r}{t} = \frac{86(91)}{8.37} = 935 \text{ psi} . \tag{4.28}$$

The mean induced stress

$$\bar{S}_i = \frac{\bar{S}_h + \bar{p}}{2} = \frac{24,462 + 2,250}{2} = 13,356 \text{ psi} , \tag{4.10}$$

and the standard deviation of induced stress

$$\sigma_{S_i} = \frac{1}{2}\left(\sigma_{S_h}^2 + \sigma_p^2\right)^{1/2} = \frac{940}{2} = 470 \text{ psi} . \tag{4.11}$$

The value of m to be used in Table 3.2 (page 20) is determined from Equation 3.8 as

$$m = \frac{-13,356 + 57,500}{\left[(470)^2 + \left(\frac{3,068}{2}\right)^2\right]^{1/2}} = 9.59 . \tag{3.8}$$

This value in Table 3.2 (page 20) corresponds to a failure probability of essentially zero.

Different shell thicknesses were selected for the vessel and design conditions specified in this example, and the same computational procedures were used to obtain the data given in Table 4.1. Examination of the data reveals that the probability of failure is very low for shell thicknesses greater than 5.0 inches, indicating that the required (12) minimum shell thickness of 8.37 inches given by Equation 4.10 for the stipulated design conditions provides more than adequate assurance against failure.

Table 4.1. Maximum Shear Stress Failure Probability as a Function of Shell Thickness for a Cylinder

t (in.)	m	Q
4.00	0.3	0.3
4.25	2.0	2.3×10^{-3}
4.50	2.8	2.5×10^{-3}
4.75	3.5	2.3×10^{-4}
5.0	4.8	2.0×10^{-7}
8.37	9.6	~0

If the minimum thickness of 8.37 inches were not mandatory (12) and a thickness corresponding to a very low failure probability were desired, the value of 5 inches would be a desirable thickness for the stated conditions of this example. Although there would still be uncertainty about how closely the specified conditions correspond to actual conditions, much of this uncertainty has been removed by the design methods used.

The probability of failure by rupture can be calculated in a similar manner by using Equations 4.20 and 4.24 to obtain the value of m corresponding to the stated design conditions. The mean burst pressure

$$\bar{p}_b = \bar{S}_u \bar{B} \ln W \ , \tag{4.20}$$

where for the minimum shell thickness of 8.37 inches

$$\ln W = \ln \left(1 + \frac{8.37}{91}\right) = 0.088 \ .$$

Therefore, $\bar{p}_b = 83{,}000(0.97)(0.088) = 7{,}120$ psi ,

and from Equation 4.21, the standard deviation of burst pressure

$$\sigma_{p_b} = 0.088(10{,}040) = 890 \text{ psi} \ .$$

The value of m to be used in Table 3.2 (page 20) is determined as

$$m = \frac{7{,}120 - 2{,}250}{[(890)^2 + (86)^2]^{1/2}} = 5.45 \ . \tag{4.23}$$

The corresponding probability of failure given in Table 3.2 (page 20) is less than 3×10^{-7}.

Different shell thicknesses were selected for the vessel, and the preceding computational procedures were used to obtain the data given in Table 4.2. Examination of the data given in Table 4.2 reveals that the probability of failure is significantly decreased for shell thicknesses between 3 and 4 inches, but the value for a 4-inch thickness still represents credible failure (about one chance in 1,000). The selection of a design value of shell thickness greater than 4 inches would therefore be arbitrary; the ideal value being perhaps 6 inches, corresponding to a failure probability of about four chances in 1,000,000.

Table 4.2. Burst Pressure Failure Probability as a Function of Shell Thickness for a Cylinder

t (in.)	m	Q
3.0	1.18	0.115
4.0	3.07	1.3×10^{-3}
5.0	4.04	3.1×10^{-5}
6.0	4.61	3.4×10^{-6}
7.0	5.05	2.0×10^{-7}
8.37	5.45	2.0×10^{-8}

The failure probability for rupture by bursting is higher than for the maximum shear stress theory of failure, and rupture is likely to be the dominant mode of failure in this case. The allowable stresses corresponding to each failure mode are compared in more detail in Section 5.

5. ASME ALLOWABLE STRESS INTENSITIES

The examples of Subsection 4.4 show a correlation between shell thickness and failure probability, and a very low failure probability was computed for the minimum shell thickness required by Section III of the ASME Boiler and Pressure Vessel Code.(12) The shell thickness is an indirect statement of induced stress for a specific geometry; thus, it says little of the adequacy of the Code (12) allowable stress intensities for other geometries. A generalized prediction of failure probability can be made if the allowable stress intensities specified in Section III of the ASME Boiler and Pressure Vessel Code (that is, the design stresses which are permitted to exist in a vessel) are compared with the material strengths likely to exist in the vessel pressure boundary (that is, the actual distribution of yield or ultimate strengths).

Since the ASME Code (12) allowable stress intensity (S_m) is equal to either two-thirds of the minimum specified yield strength (S_y) or one-third of the minimum specified ultimate tensile strength (S_u), the allowable stress intensity provides a "safety factor" of approximately three against rupture (based on classical definitions of the term "safety factor"). The failure probability resulting from Code (12) allowable stress intensities is obtained from Equation 3.10 by using an induced stress equal to the allowable stress intensity and an allowable stress equal to the yield strength of the material.

Data collected from three sources (11, 15, 16) indicate that the mean value of yield stress for pressure vessel steels will be 1.35 times higher than the minimum specified yield strength. Conformance with the stated criterion of keeping the allowable stress intensity less than two-thirds of the minimum specified yield stress therefore requires that

$$\bar{S}_y = 1.35(1.5)S_m \ . \tag{5.1}$$

This corresponds to a value of M determined from Equation 3.11 of

$$M = \bar{S}_a / \bar{S}_i = 2.02 \tag{5.2}$$

when $\bar{S}_i = S_m$ and $\bar{S}_a = \bar{S}_y$. The data analysis illustrated in Figure 3.1 (page 17) and the results of the literature survey reported by Lipson et al. (9) indicate a coefficient of variation of allowable stress (V_{S_a}) near 0.05 for a specific material and establish the likely range for all steels as being between 0.01 and 0.1. Similarly, the available data on pressure variations indicate that a coefficient of variation of induced stress (V_{S_i}) of 0.05 would be common. Substitution of these values into Equation 3.12 yields a coefficient of variation of failure probability of

$$C = \left(V_{S_a}^2 M^2 + V_{S_i}^2\right)^{1/2} = 0.144 \ . \tag{5.3}$$

From Figure 3.2 (page 24), the value of C = 0.144 and the value of M = 2.02 result in a failure probability of roughly 10^{-4} or one chance in 10,000. The approximate value of Q should be considered sufficiently accurate in analyses such as this since there is no accepted minimum level of failure probability for pressure vessels.

The probability of failure by shear stress based on an allowable stress intensity of two-thirds of the minimum specified yield strength is somewhat higher than might be desired. Should the designer prefer greater assurance against failure, he might select a lower value of allowable stress intensity such as one-half of the minimum specified yield strength, which corresponds to a failure probability of 10^{-6} when calculated in the foregoing manner. Since the yield strength of a material is the governing factor in the maximum shear stress theory of failure, a more acceptable failure probability would result from an allowable stress intensity equal to or less than one-half of the minimum specified yield strength if the same variance in loads and material strengths assumed herein is expected.

To evaluate the probability of failure resulting from the criterion that the allowable stress intensity shall not exceed one-third of the ultimate tensile strength, the burst pressure discussed in Subsection 4.3 will be used. The failure probability is obtained from Equation 4.23 when the burst pressure corresponding to the ultimate tensile strength (computed by using Equation 4.20) is compared with the induced pressure corresponding to the allowable stress intensity (computed by using Equation 4.10). The data collected from three sources (11, 15, 16) indicate that the mean value of ultimate tensile strength for pressure vessel steels will be only 1.06 times higher than the minimum specified ASME Code (12) value. Thus,

$$\bar{S}_u = 1.06 S_u \ . \tag{5.4}$$

Designation of the ratio of the minimum specified ultimate tensile strength to the allowable stress intensity as N

$$N = \frac{S_u}{S_m} \tag{5.5}$$

and substitution of the value of $\bar{S}_u$ obtained in Equation 5.4 into Equation 5.5 yields

$$S_m = \frac{\bar{S}_u}{1.06N} . \tag{5.6}$$

The induced pressure corresponding to the allowable stress intensity is obtained by substituting Equation 5.6 into Equation 4.10 (where $\bar{S}_h = \bar{p}Y$) and solving for $\bar{p}$.

$$\bar{p} = \frac{\bar{S}_u}{1.06N}\left(\frac{1}{1 + Y}\right) . \tag{5.7}$$

The value of M to be used in Figure 3.2 (page 24) or Equation 4.24 is

$$M = \frac{\bar{p}_b}{\bar{p}} = \frac{\bar{S}_u\bar{B} \ln W}{\frac{\bar{S}_u}{1.06N}\left(\frac{1}{1 + Y}\right)} . \tag{5.8}$$

Substitution of the coefficient of variation of burst pressure (V_{p_b}) defined in Equation 4.22 into Equation 4.25 yields a coefficient of variation of failure probability (C) of

$$C = \left[\left(V_{S_u}^2 + V_B^2 + V_{S_u}^2 V_B^2\right)M^2 + V_p^2\right]^{1/2} . \tag{5.9}$$

The value of m given by Equation 4.24 is therefore expressed by substitution of Equations 5.8 and 5.9.

$$m = \frac{\left[1.06N\bar{B}(1 + Y) \ln \left(1 + \frac{1}{Y}\right)\right] - 1}{\left[\left(V_{S_u}^2 + V_B^2 + V_{S_u}^2 V_B^2\right)M^2 + V_p^2\right]^{1/2}} . \tag{5.10}$$

The probability of failure by bursting was calculated for ASME Code (12) allowable stress intensities by assuming a value of N = 3 for the ratio of minimum specified ultimate tensile strength (S_u) to allowable stress intensity (S_m). Available data on strain hardening (7) were used to obtain the values $\bar{B} = 0.97$ and $V_B = 0.01$. The value of V_{S_u} was found to be the same as that for V_{S_y} or 0.05. (9, 11) Since $\bar{p}$ is the same pressure used to evaluate the maximum shear stress, the value of V_p is again assumed as 0.05. These values were used in Equation 5.10 to calculate the failure probability for several values of Y by using Table 3.2 (page 20). The calculations were then repeated for other values of N. The results of these calculations are illustrated in Figure 5.1.

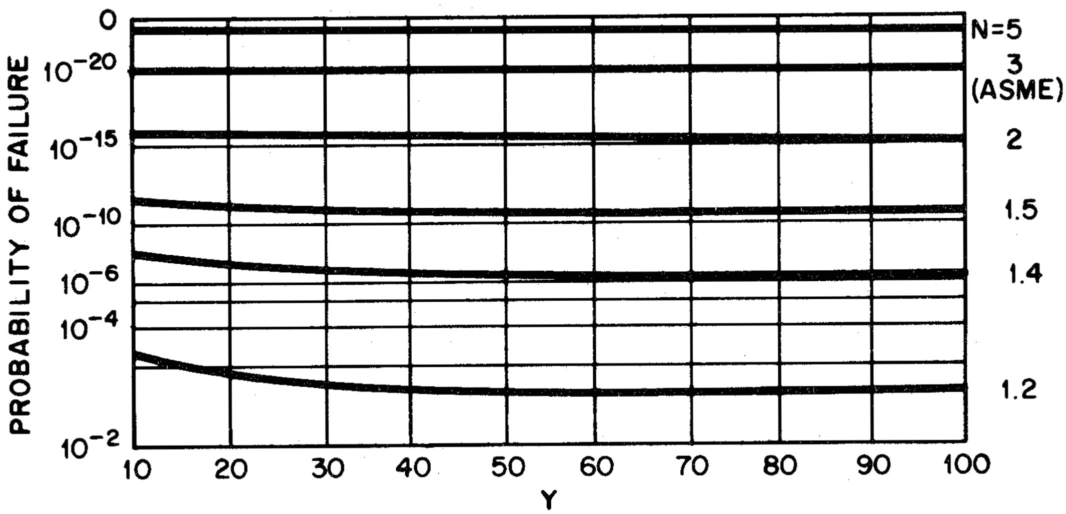

Figure 5.1. Probability of Failure as a Function of the Ratios of Vessel Radius to Thickness (Y) and Ultimate Strength to Allowable Stress Intensity (N).

The curve for N = 3 illustrated in Figure 5.1 represents the failure probability by bursting resulting from an allowable stress intensity of one-third of the ultimate tensile strength. The probability of failure is so low that it can be considered as zero probability of failure. The probability of failure would likely be considered acceptable for values of N as low as 1.4, corresponding to less than one chance in 1,000,000. However, the failure probability is sensitive to changes in N for values of N less than approximately 2.0. This sensitivity makes it advisable to select values of N for which errors in data analysis will not result in gross errors in failure probability. In this case, the value of N = 2 appears to offer adequate assurance against failure by bursting.

On the basis of this analysis, an allowable stress intensity equal to one-half of the minimum specified ultimate tensile strength of the vessel material, as opposed to the presently accepted value (12) of one-third of the minimum specified ultimate tensile strength, would be acceptable. The use of this value should be based on other considerations in addition to probabilistic design since considerable uncertainty can still exist even though an extensive probability analysis has been performed. On the other hand, the results obtained for yield strength suggest a change from the presently accepted value of an allowable stress intensity of two-thirds of the minimum specified yield strength to one-half of the minimum specified yield strength would be justified on the strength of the probabilistic analysis by itself because a potentially non-conservative criterion was discovered.

6. CYCLIC FAILURE PROBABILITY

The equations for steady-state stresses presented in Section 4 were used in Section 5 to develop a generalized prediction of the steady-state failure probabilities for the allowable stress intensities stipulated in Section III of the ASME Boiler and Pressure Vessel Code (12) as applied to pressure vessel steels. However, an additional mode of failure arises from the repeated application of loads of a cyclic nature. The probability of failure resulting from the repeated application of cyclic stresses was defined in Section 2 (page 6) as the probability that the number of stress amplitudes of a specified level exceeds the allowable number of stress amplitudes for that stress level. The probability of failure so defined is a function of the number of stress cycles as well as the amplitude of the stress. The distribution function must therefore include the effects of both the stress level and the number of stresses. One method of including both would be to develop the probability that a number of stresses exceed an allowable number for a specified stress level. Another method would be to develop the probability that an induced stress exceeds an allowable stress for a given number of stress cycles. This latter method was selected for use herein to be consistent with the preceding development of a steady-state failure probability in terms of an induced stress exceeding an allowable steady-state stress.

In the development of the steady-state probability in Sections 3 and 4, the induced and allowable stresses were assigned a distribution invariant with time (or the number of stresses). In the cyclic analysis, an induced stress distribution and an allowable stress distribution must be specified for each number of cycles of interest in the evaluation. These stress distributions may not be the same for each cyclic life. For example, a stress may be log normally distributed for one life and exponentially distributed for another. This increases the complexity of the statement of failure probability.

Equation 3.1 can be solved for cyclic failure probability when a distribution of stress differences can be expressed for the cyclic life of interest. The nature of the cyclic stress distribution must be determined before the integral of Equation 3.1 can be evaluated.

6.1 Cyclic Distribution Functions

The distribution function for induced stress is generally not known. Induced stresses are usually specified in terms of a number of cycles of a stress of an exact value and significant variations from that exact stress value are treated as a number of stresses of a different value. The distribution of induced stress for any specific cyclic life can therefore usually be treated as invariant unless extreme accuracy is required in the estimate of failure probability.

The fatigue life (allowable stress) of materials subjected to cyclic stress is a different situation in which there are significant variations with respect to both cyclic life and stress level. To avoid the complexity involved in the determination of several specific functions for the allowable cyclic stress distribution, the versatility of the Weibull distribution function will be used to generalize the statement of the distribution of allowable stress for all numbers of cycles and stress levels.

The two-parameter Weibull distribution function was given in Equation 2.17 (page 12) with time as the independent variable. The probability that a stress will exceed a specified value in a Weibull distribution is obtained by using induced stress (S_i) as the independent variable in lieu of time.

$$Q = 1 - e^{-S_i^{\,b}/m} \quad . \tag{6.1}$$

The parameters b and m in Equation 6.1 are determined by analysis of the allowable stress distribution for cyclic failure. Equation 6.1 therefore states the probability that an invariant induced stress will exceed an allowable stress with a Weibull distribution that is characterized by different values of the parameters b and m for each cyclic life.

6.2 Determination of Weibull Parameters

The manner in which cyclic failure data are obtained is important to the determination of Weibull parameters for cyclic allowable stresses. The data are generally presented as the number of cycles of a given stress amplitude that resulted in failure, with the stress amplitude assumed invariant and the cyclic life distributed about some mean value. The allowable stress distribution corresponding to an invariant cyclic life must therefore be obtained by converting life data to strength data since it is not practical to test at a specified number of cycles to obtain a failure stress distribution.(9)

6.2.1 Cyclic Rupture

Data on the number of stress amplitude cycles that resulted in the propagation of a crack through the shell (hereinafter referred to as cyclic rupture) of pressure vessels were obtained by the Pressure Vessel Research Committee and reported by The American Society of Mechanical Engineers.(13) These data are illustrated in Figure 6.1. To convert these data from a life distribution to a stress distribution, a least-squares line was fitted through the data points and parallel lines were drawn, as described in Appendix B, to construct a family of S-N curves. A vertical line was drawn through this family of S-N curves at representative values of cyclic life. The number of data points on an S-N curve was then assumed to represent the number of failures resulting from the stress amplitude (S_A) corresponding to the intersection of the S-N curve with the vertical line denoting cyclic life. The failure distribution

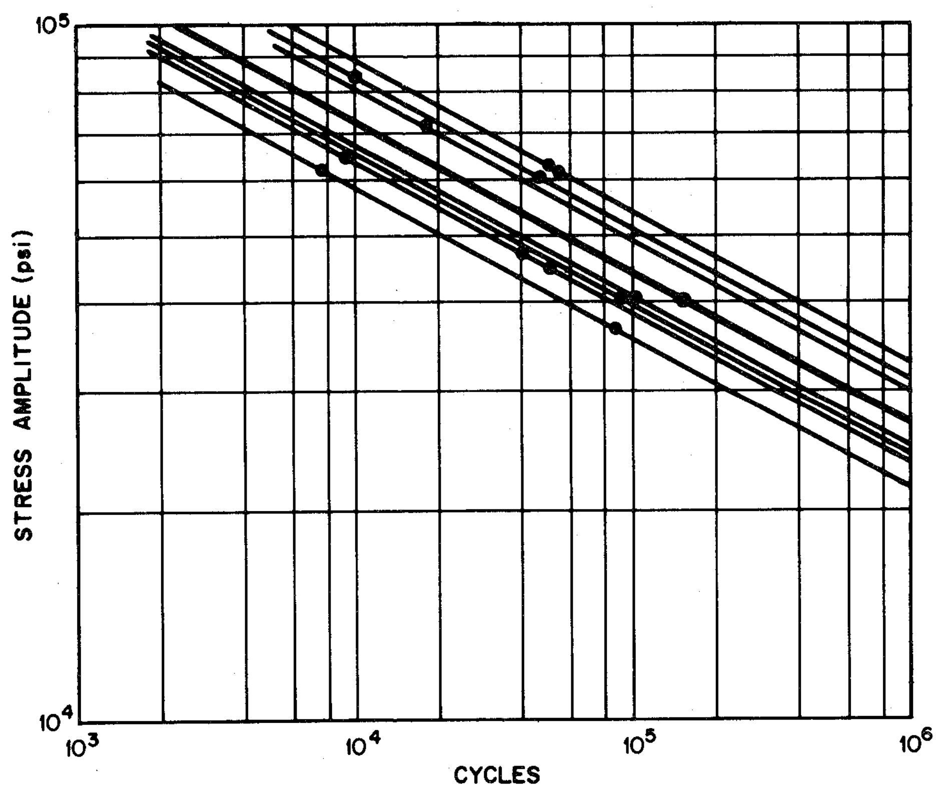

Figure 6.1. Family of S-N Curves Constructed for Pressure Vessels That Failed by Rupture as a Result of Cyclic Internal Pressure.

function for each cyclic life was determined from this data in the manner discussed in Subsection 3.1 and outlined in Table 3.1 (page 16). The cyclic stress amplitudes corresponding to the estimated number of failures by cyclic rupture for three values of cyclic life are given in Table 6.1.

Table 6.1. Cyclic Stress Amplitudes Corresponding to Estimated Number of Cyclic Ruptures for Three Values of Cyclic Life

Number of Data Points	Percent Failed	Stress Amplitude at 10^4 Cycles (psi)	Stress Amplitude at 10^5 Cycles (psi)	Stress Amplitude at 10^6 Cycles (psi)
2	12.7	59,000	35,000	21,000
3	35.1	64,000	39,000	23,000
1	42.5	66,000	40,000	24,000
1	50.0	68,000	41,000	25,000
1	57.5	73,000	44,000	27,000
1	64.9	81,000	49,000	30,000
2	79.9	85,000	52,000	32,000
2	94.8	90,000	54,000	35,000

The distribution functions for cyclic rupture given in Table 6.1 were plotted on a specially constructed Weibull probability paper, (9) as illustrated in Figure 6.2, to graphically determine the Weibull parameters for each cyclic life. The scales of this paper were drawn to show Weibull parameters that simplify the use of Equation 6.1 when performing numerical computations since very large numerical quantities for S_i^b would be involved for positive values of the parameter b greater than unity. If a constant equal to the b-th root of the parameter m in Equation 6.1 is determined, the numerical values in the intermediate

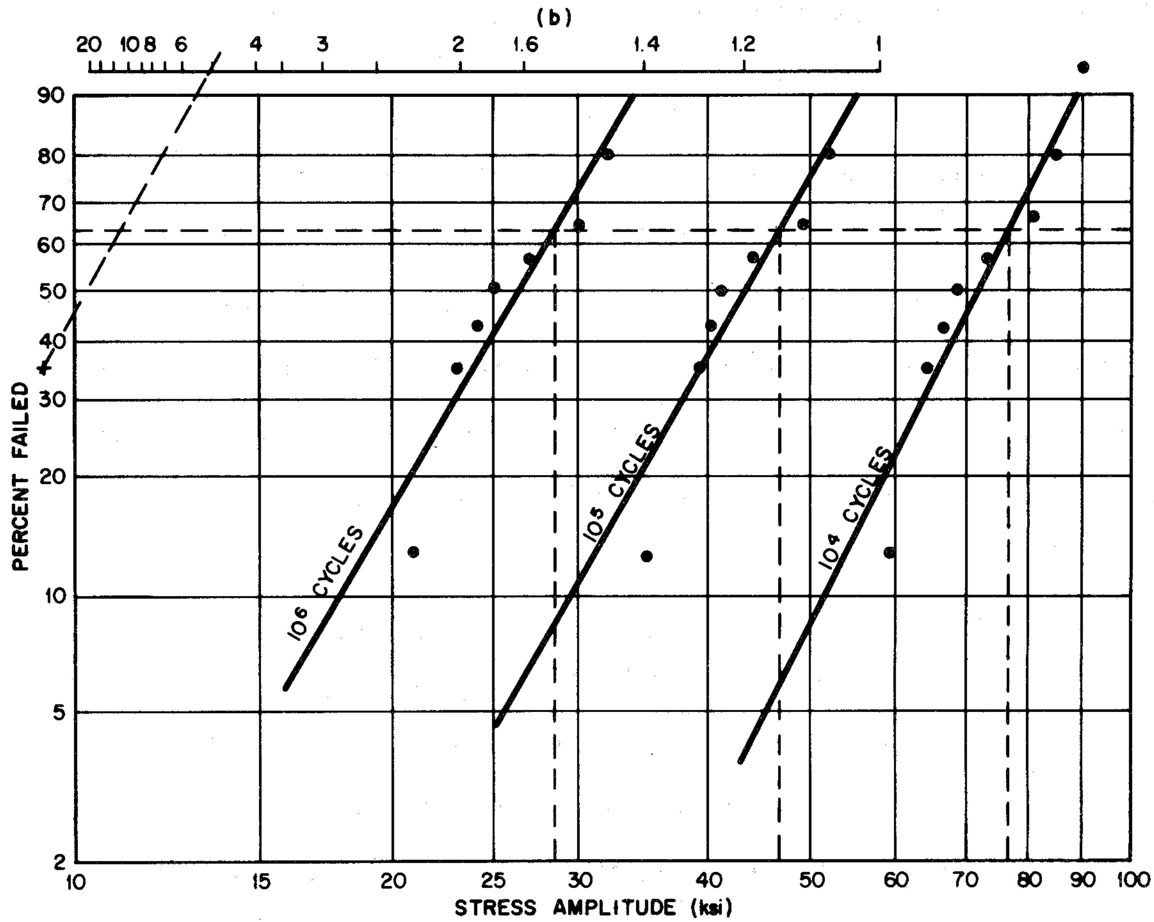

Figure 6.2. Distribution Functions of Cyclic Rupture Data for Pressure Vessels Plotted on a Weibull Probability Scale.

computations will be smaller. When

$$m = \theta^{b} , \tag{6.2}$$

Equation 6.1 can be written as

$$Q = 1 - e^{-(S_i/\theta)^{b}} . \tag{6.3}$$

Hence, the Weibull probability paper used herein will yield a value of θ to be used in Equation 6.3, but it cannot be used to directly determine the value of m to be used in Equation 6.1. The use of Equation 6.3 also requires that values of stress amplitude rather than the total stress be used with the parameters because the S-N curves correlate stress amplitude and cyclic life. Thus, Equation 6.3 is more properly written as

$$Q = 1 - e^{-(S_A/\theta)^{b}} . \tag{6.4}$$

Since this Weibull probability paper has the property that Weibull distribution functions plot as straight lines,(7) a straight line was fitted through each distribution to graphically determine values of the Weibull parameters. For example, the value of the parameter b for 10^4 cycles was obtained by drawing a line through the central point (+) parallel to the line representing the distribution function for 10^4 cycles, as is shown in Figure 6.2. The value was then read from the "b" scale at the top of the graph. The value of θ was read from the abscissa corresponding to the stress amplitude producing 63.2% of the failures at 10^4 cycles. The Weibull parameters obtained in this manner are shown in Figure 6.3 as a function of cyclic life. These parameters result in the Weibull distribution of allowable stress for a specific cyclic life when substituted into Equation 6.4. The probability that a specified number of cycles of induced stress amplitude will cause rupture is calculated

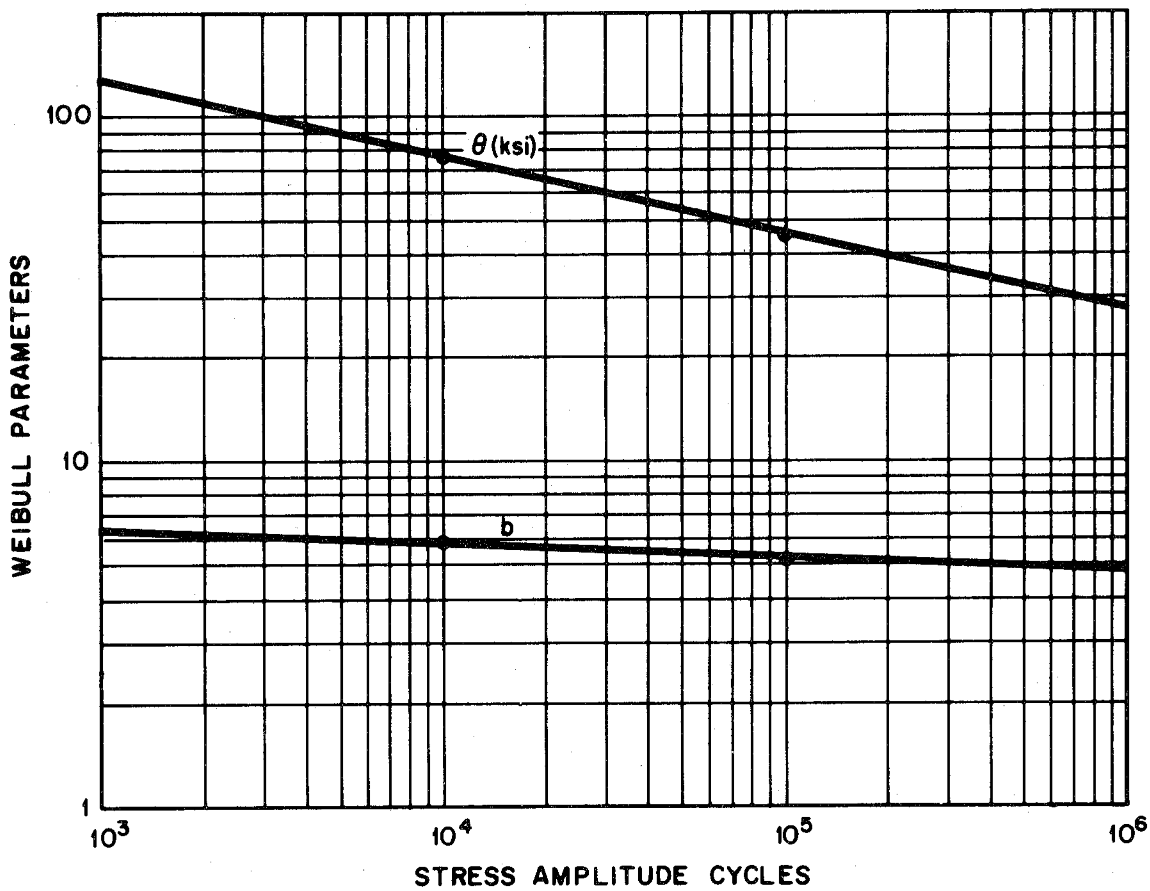

Figure 6.3. Cyclic Rupture Weibull Parameters for Pressure Vessels as a Function of Number of Cycles.

by substituting the induced stress amplitude and the Weibull parameters for that number of cycles into Equation 6.4.

Several values of rupture probability were calculated as a function of stress amplitude and cyclic life by using Equation 6.4 and the Weibull parameters shown in Figure 6.3. These values of failure probability are illustrated in Figure 6.4 as S-N curves of a constant failure probability. The S-N curve for low-carbon alloy steel given in Section III of the ASME Boiler and Pressure Vessel Code (12) is shown as a dashed line in Figure 6.4, and it corresponds to a probability of failure by cyclic rupture greater than one chance in 100. Such a result should be expected since, as stated in the ASME Criteria,(13) the S-N curves given in Section III of the ASME Boiler and Pressure Vessel Code do not necessarily result in a factor of safety for cyclic life inasmuch as they were only corrected to compensate for the difference between test data and operating conditions.

It can be concluded that rupture resulting from cyclic loads is a credible event (greater than one chance in 100) if the design stress amplitudes are permitted to reach the values allowed by the S-N curve in Section III of the ASME Boiler and Pressure Vessel Code. A factor of safety for cyclic life would substantially decrease this failure probability. For example, Figure 6.4 shows that designing for the ASME Code (12) allowable stress amplitude at 10^6 cycles would result in a failure probability of 10^{-3} at 10^5 cycles, which would represent a considerable improvement in the failure probability.

Attention should also be directed to the typical practice of assuming that 10^6 cycles is equivalent to infinite life.(6) The cyclic life

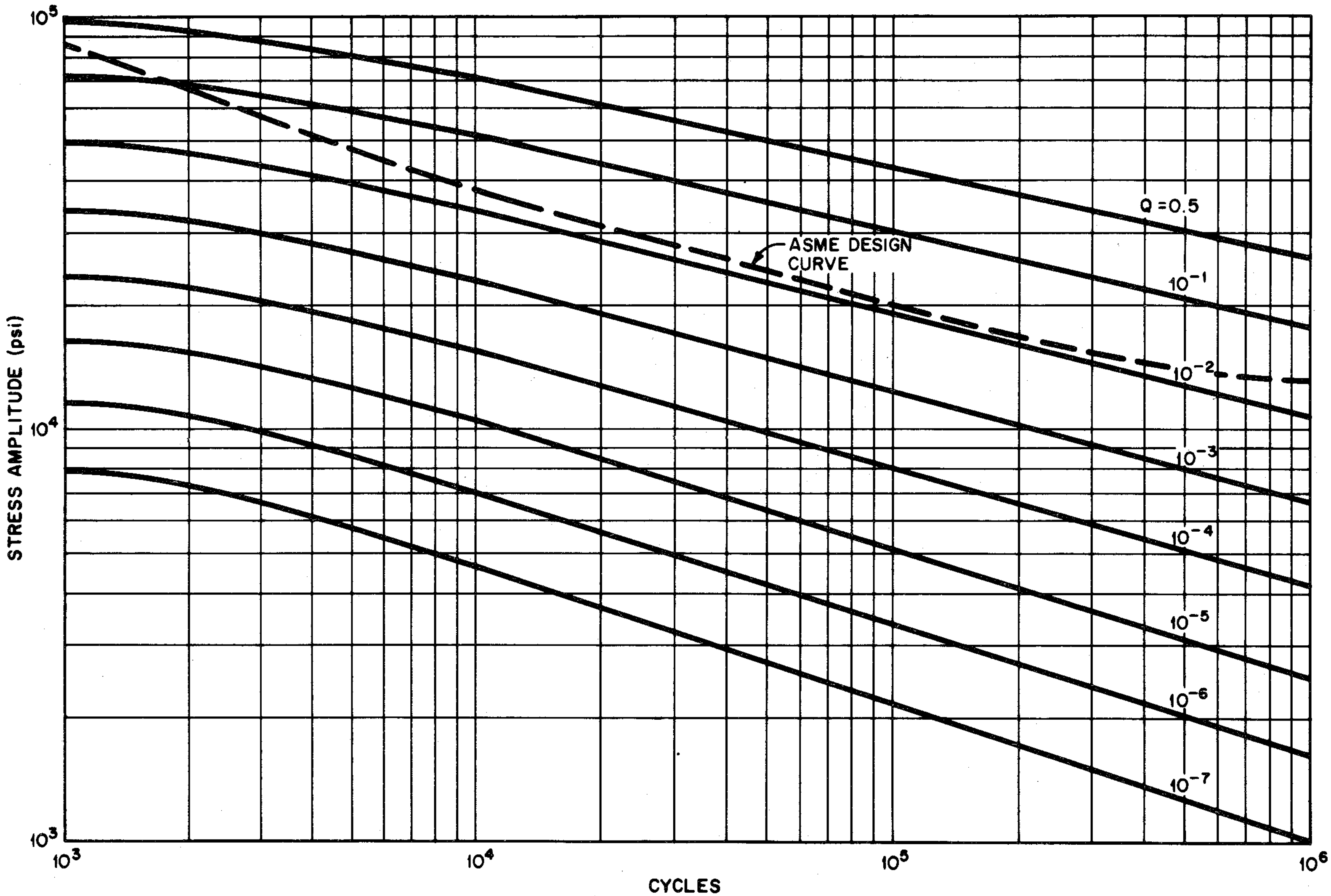

Figure 6.4. Family of S-N Curves Constructed for the Probabability of Rupture in Pressure Vessels Subjected to Cyclic Internal Pressure.

for the rupture data from which the failure probability was determined did not exceed 10^5 cycles. However, at 10^5 cycles, there was no indication that stress amplitude would asymptotically approach some value at 10^6 cycles. Therefore, care should be exercised in extrapolating the cyclic failure probability data in Figure 6.4 to lives greater than 10^6 cycles.

The cyclic failure data shown in Figure 6.1 (page 49) are a combination of data from two research installations on two kinds of material. All of the data points were considered together in the foregoing data analysis, and a very close fit to the straight line required for a Weibull distribution was not achieved, as is shown in Figure 6.2 (page 51). (However, the parameters determined are sufficiently accurate to substantiate the conclusions drawn herein.) This lack of a close fit resulted from the possibility that there may be as many as four different distributions involved: one for each of the two materials tested and one for each of the two research installations performing the tests. The data as analyzed therefore result in a probability that failure by rupture will occur irrespective of the material and data source.

6.2.2 Cyclic Crack Initiation

Data to document the onset of cracking in cyclically loaded pressure vessels were also obtained by the Pressure Vessel Research Committee, and these data are cited (13) as justification for the design curve for allowable stresses in Section III of the ASME Boiler and Pressure Vessel Code.(12) These data are illustrated in Figure 6.5, and they were converted from a cyclic life distribution to a stress distribution in the same manner used to convert the data shown in Figure 6.1 (page

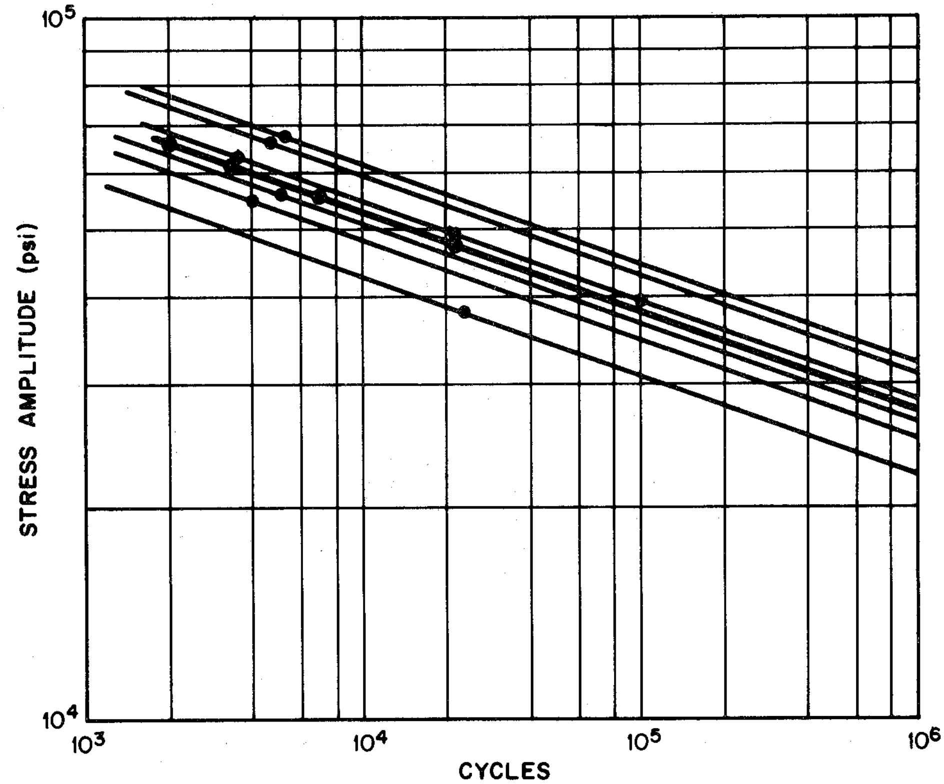

Figure 6.5. Family of S-N Curves Constructed for Pressure Vessels That Failed by Crack Initiation as a Result of Cyclic Internal Pressure.

49). The failure distribution function for each cyclic life was determined, and the cyclic stress amplitudes corresponding to the estimated number of failures by crack initiation for two values of cyclic life are given in Table 6.2.

The distribution functions for cyclic crack initiation given in Table 6.2 were plotted on Weibull probability paper, as shown in Figure 6.6, to graphically determine the Weibull parameters for each cyclic life. The Weibull parameters determined in this manner are shown as a function of cyclic life in Figure 6.7. The probability that a specified number of cycles of induced stress amplitude will cause failure by crack initiation is calculated by substituting the induced stress

Table 6.2. Cyclic Stress Amplitudes Corresponding to Estimated Number of Crack Initiations for Two Values of Cyclic Life

Number of Data Points	Percent Failed	Stress Amplitude at 10^4 Cycles (psi)	Stress Amplitude at 10^5 Cycles (psi)
1	7.7	44,000	32,000
1	15.4	49,000	36,000
3	38.5	52,000	38,000
4	69.2	54,000	40,000
1	76.9	56,000	41,000
1	84.6	60,000	44,000
1	93.2	62,000	45,000

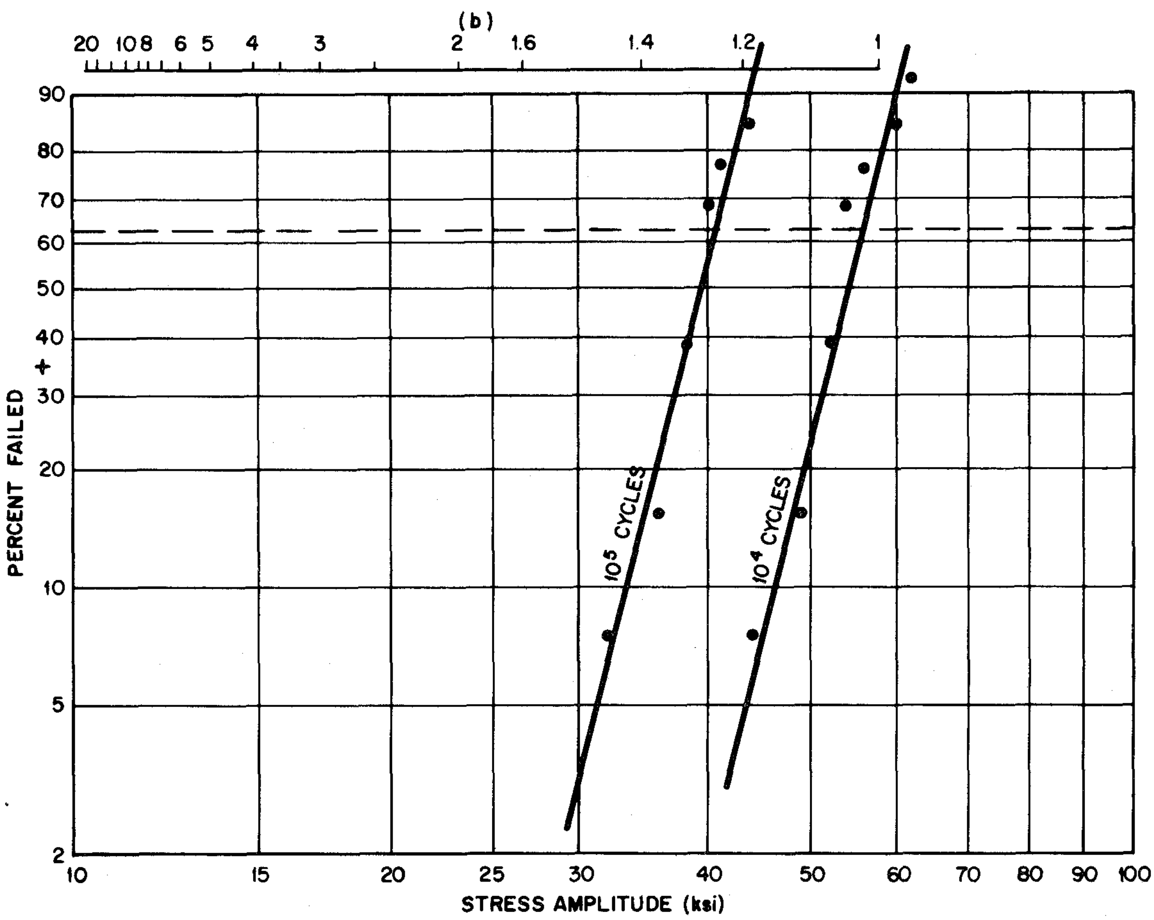

Figure 6.6. Distribution Functions of Cyclic Crack Initiation Data for Pressure Vessels Plotted on a Weibull Probability Scale.

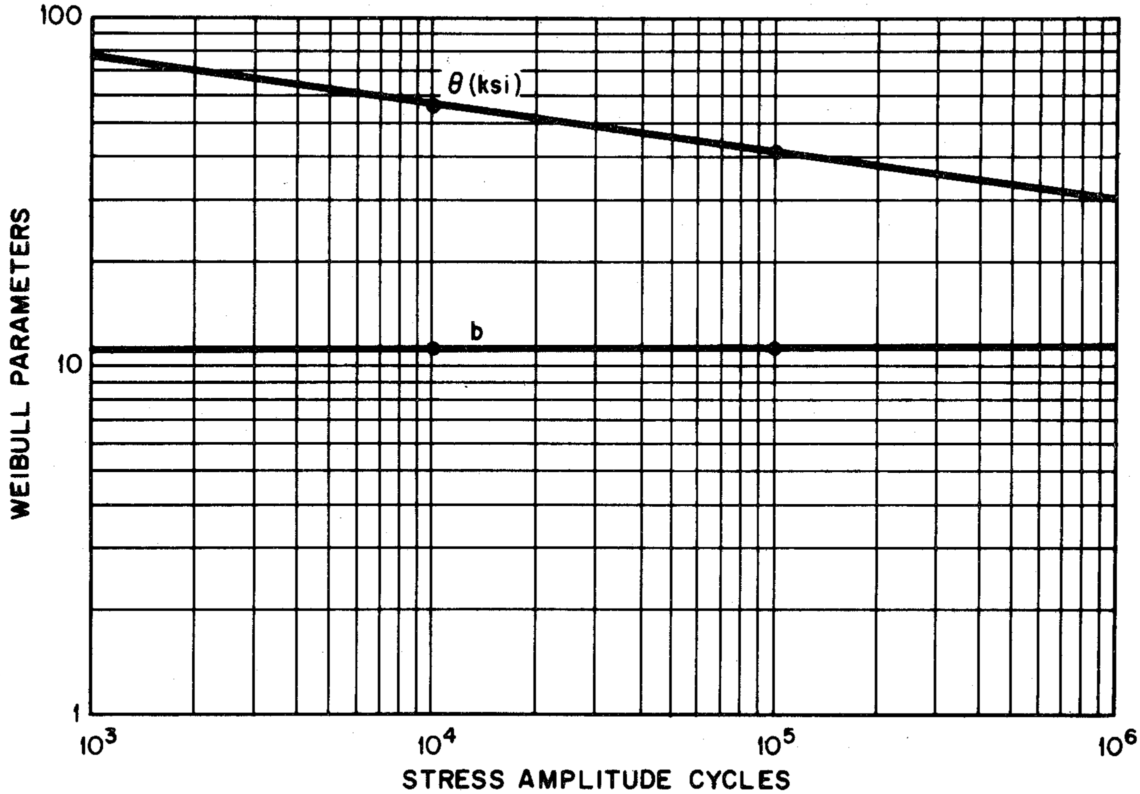

Figure 6.7. Cyclic Crack Initiation Weibull Parameters for Pressure Vessels as a Function of Number of Cycles.

amplitude and the Weibull parameters for that number of cycles into Equation 6.4. Several values of the probability of failure by crack initiation were calculated in this manner, and these values of failure probability are shown in Figure 6.8 as S-N curves of a constant failure probability.

A comparison of the curves for the probability of vessel failure by crack initiation shown in Figure 6.8 with the curves for the probability of failure by rupture shown in Figure 6.4 (page 55) shows that there is a much higher probability for vessel rupture than for crack initiation above 10^4 cycles. This anomally results from the nature of the two sets of data. It can be observed that the slope of the S-N curve for rupture shown in Figure 6.4 (page 55) is much steeper than that of the S-N curve for crack initiation shown in Figure 6.8. Since the slope of each curve is more closely associated with the data source than the material or other known factors, it can be assumed that the probability of failure by crack initiation is not comparable to that by rupture in this case. Since the curves representing the probability of failure by rupture developed herein include data from two sources and result in conservative estimates of failure, they will be used as the basis of further discussion.

However, it can be noted from the failure probability curves for either failure mode that the design will not be conservative below 10^4 cycles if the design stress amplitudes are permitted to reach the values allowed by the S-N design curve in Section III of the ASME Boiler and Pressure Vessel Code.(12) There is certainly no indication that an increase in the allowable cyclic stresses would be desirable.

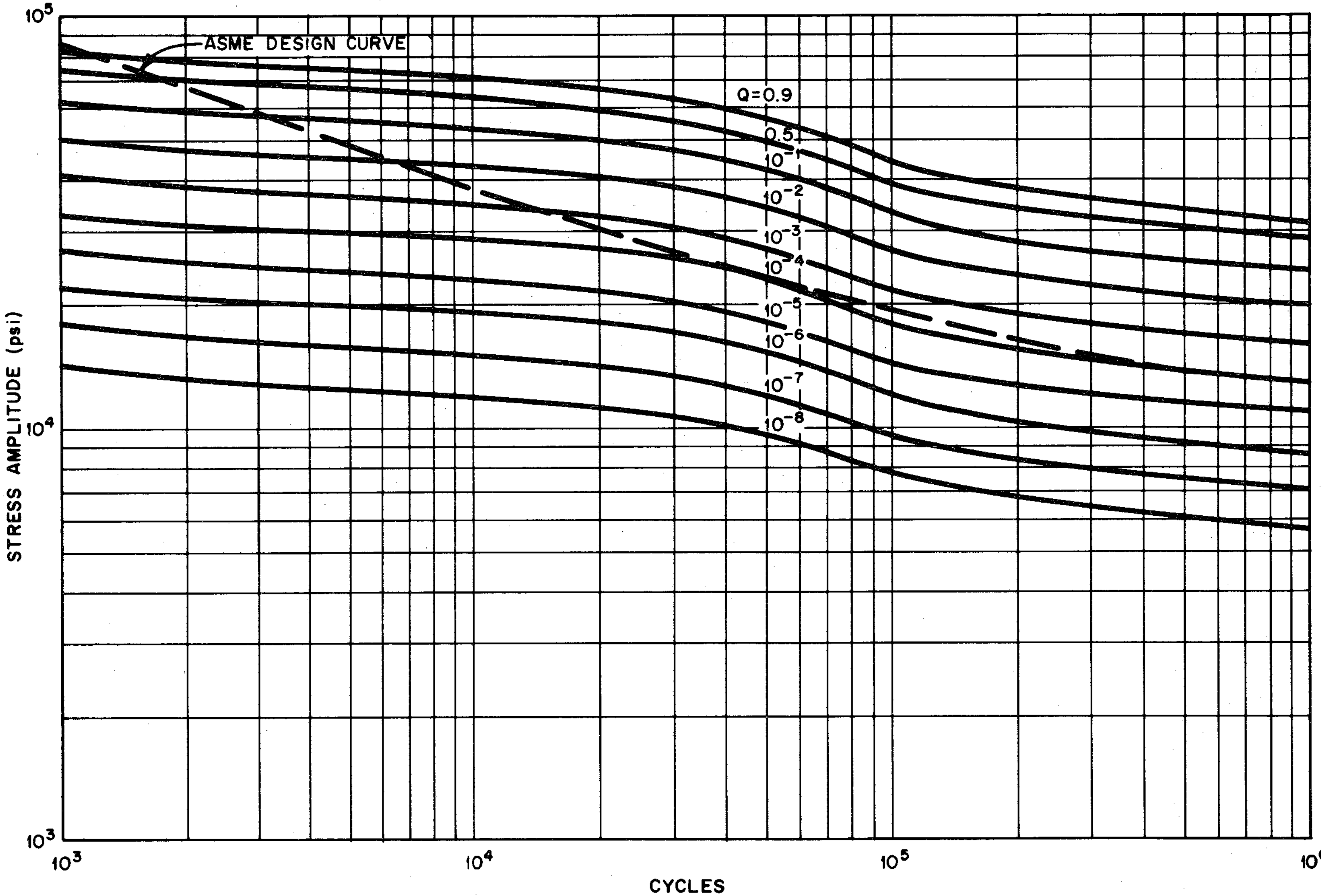

Figure 6.8. Family of S-N Curves Constructed for the Probability of Crack Initiation in Pressure Vessels Subjected to Cyclic Internal Pressure.

6.3 Numerical Example

If the cylindrical pressure vessel of the numerical example in Subsection 4.4 (page 35) had a discontinuity corresponding to a stress concentration factor of 3 on the hoop stress, the cyclic failure probability would be estimated as presented here. Assume that the mean internal pressure ($\bar{p}$ = 2,250 psi) and its standard deviation (σ_p = 86 psi) were calculated in Subsection 4.4 from the following cyclic pressure specifications.

Type-1 cycle: 100,000 cycles of 2,250 ± 100 psi .

Type-2 cycle: 200 cycles of 2,250 ± 120 psi .

Type-3 cycle: 5 cycles of 2,250 ± 210 psi .

Cycles of pressure corresponding to start-up and shutdown of the system were not considered in the example of Subsection 4.4, but they will be added here for the purposes of this investigation.

Type-4 cycle: 40 cycles of 0 to 2,250 psi .

The analysis methods presented in Subsection 6.2.1 will be used to calculate the fatigue failure probability for the minimum required (12) shell thickness.

The stress range is determined from the equation (12)

$$S_r = S_{max} - S_{min} , \tag{6.5}$$

where S_{max} is the stress value corresponding to the maximum amplitude of pressure and S_{min} is the value corresponding to the minimum amplitude of pressure. The alternating stress intensity (S_A) is given by the equation (12)

$$S_A = 0.5S_r . \tag{6.6}$$

In the example of Subsection 4.4, the minimum required (12) shell thickness for the cylinder was determined to be 8.37 inches. Assuming a stress concentration factor of 3 applied to hoop stress, the following cyclic analysis results from that minimum shell thickness and the four types of cyclic load. For the Type-1 cycle,

$$S_{max} = \frac{2,350(91)(3)}{8.37} + 2,350 = 79,000 \text{ psi} ,$$

$$S_{min} = \frac{2,150(91)(3)}{8.37} + 2,150 = 72,275 \text{ psi} ,$$

$$S_r = 79,000 - 72,275 = 6,725 \text{ psi} ,$$

and $$S_A = \frac{6,725}{2} = 3,362 \text{ psi for } 100,000 \text{ cycles} .$$

For the Type-2 cycle,

$$S_{max} = \frac{2,370(91)(3)}{8.37} + 2,370 = 79,671 \text{ psi} ,$$

$$S_{min} = \frac{2,130(91)(3)}{8.37} + 2,130 = 69,473 \text{ psi} ,$$

$$S_r = 79,671 - 69,473 = 10,198 \text{ psi} ,$$

and $$S_A = \frac{10,198}{2} = 5,100 \text{ psi for } 200 \text{ cycles} .$$

For the Type-3 cycle,

$$S_{max} = \frac{2,460(91)(3)}{8.37} + 2,460 = 82,696 \text{ psi} ,$$

$$S_{min} = \frac{2,040(91)(3)}{8.37} + 2,040 = 68,578 \text{ psi} ,$$

$$S_r = 82,696 - 68,578 = 14,118 \text{ psi} ,$$

and $$S_A = \frac{14,118}{2} = 7,057 \text{ psi for } 5 \text{ cycles} .$$

For the Type-4 cycle,

$$S_{max} = \frac{2,250(91)(3)}{8.37} + 2,250 = 75,636 \text{ psi} ,$$

$$S_{min} = 0 ,$$

$$S_r = 75,636 - 0 = 75,636 \text{ psi} ,$$

and
$$S_A = \frac{75,636}{2} = 37,818 \text{ psi for 40 cycles} .$$

The allowable number of cycles for each amplitude can be found from Figure I-9.1 in Section III of the ASME Boiler and Pressure Vessel Code (12) or from the ASME Design Curve shown as a dashed line in Figure 6.4 (page 55). These values are tabulated below.

S_A (psi)	Number of Allowable Cycles	Number of Actual Cycles	Ratio of Actual to Allowable Cycles
3,362	$> 10^6$	10^5	~ 0.05
5,100	$> 10^6$	200	~ 0
7,057	$> 10^6$	5	~ 0
37,818	10^4	40	~ 0.004

The sum of the ratio of actual cycles to allowable cycles must be less than unity to satisfy the cumulative damage requirements of Section III of the ASME Boiler and Pressure Vessel Code. In this example, the sum is approximately 0.054, which is an acceptable value by ASME Code (12) rules.

When each type of cyclic loading is considered as independent of the other types, the probability of non-failure is given by the following equation. (7)

$$(1 - Q) = (1 - Q_1)(1 - Q_2)(1 - Q_3)(1 - Q_4) , \tag{6.7}$$

where the subscripts 1, 2, 3, and 4 refer to the respective types of cyclic loading. Substitution of the values shown in Figure 6.4 (page 55) into Equation 6.7 yields

$$(1 - Q) = (1 - 10^{-6})(1 - 0)(1 - 0)(1 - 10^{-3}) ,$$

where the value for the probability of failure for cycles lower than 10^3 in Figure 6.4 was assumed to be the value at 10^3 because of the shape of the curves, and

$$Q = \sim 10^{-3} .$$

If the 40 start-up and shutdown cycles are not included, Equation 6.7 is

$$(1 - Q) = (1 - 10^{-6})(1 - 0)(1 - 0) ,$$

and the probability of failure is

$$Q = 10^{-6} .$$

7. VESSEL RELIABILITY

The probability of non-failure, implicit in the probability of failure, is an estimate of reliability only if there is a time dependency associated with the failure distribution function. The cyclic failure probability, discussed in Section 6, constitutes a reliability forecast for a specified number of cycles in that the cyclic failure probability is a function of the number of cycles (time) a stress amplitude is to occur. On the other hand, the steady-state failure probability, discussed in Section 4, is independent of time since it is based on the assumption that the stress distributions exist at any instant and do not change from one instant to another.

The probability of failure for steady-state stress distributions is difficult to visualize physically for a single component because there is intuitive knowledge that the induced stress distribution must occur over a specified period of time to exist (as opposed to 100 vessels of random strength, each of which is subjected to a random pressure at the same time). The correlation between steady-state failure probability and reliability is therefore important for the case in which a small population of similar components is subjected to a large population of stresses. This correlation is based on the non-mathematical events surrounding the derivation of the probability that an induced steady-state stress will exceed an allowable steady-state stress.

One event which changes the reliability forecast is inspection. If a component is examined periodically for evidence of incipient failure and no evidence is found, the original reliability forecast is invalid and a new forecast is required. Similarly, if the system of which the component is a part is examined periodically and restored to a new condition, the probability of unexpected stresses being placed on the component by the system is affected by each examination. Under such conditions, it would be reasonable to assume that the steady-state stress distributions discussed in Section 4 exist for the interval of time between such inspections and restorations. The probability of failure given by Equation 3.7 (page 19) is therefore the same as the reliability given by Equation 2.18 (page 13) for the period of time (T_I) during which the stress distributions are not disturbed by the inspection or repair of their source of origin (either material properties or system loads).

$$Q = 1 - e^{-T_I^{\,b}/m} \; . \tag{7.1}$$

If it can be further assumed that any pressure above a specified value has an equal chance of occurring at any time, Equation 7.1 becomes the exponential expression of reliability originally stated in Equation 2.16 (page 12) as the time-dependent probability of failure.

$$Q = 1 - e^{-\lambda T_I} \; . \tag{7.2}$$

After k inspections, the reliability with inspection (R_T) is given by the equation (7)

$$R_T = e^{-\lambda(kT_I + T)} \; , \tag{7.3}$$

where T_I is the interval between inspections and T is the length of time beyond the last inspection. The failure rate (λ) in Equation 7.3 is obtained from Equation 7.2 as

$$\lambda = -\frac{\ln(1 - Q)}{T_I} , \qquad (7.4)$$

where Q is the probability of failure resulting from the steady-state stress distribution of Equation 3.7 (page 19). Thus, Equations 7.3 and 7.4 state the relationship between reliability and the probability of failure under steady-state stress distributions when these stress distributions require a finite time to occur and that time can be identified by the inspection interval for the vessel and when the additional restriction is imposed that induced stresses above a specified level must have an equal probability of occurring at any time.

If it is assumed that the inspection interval for a nuclear pressure vessel is 2 years, the reliability of the vessel for 40 years is calculated as follows. For failure defined by the maximum shear stress theory and an allowable stress intensity equal to two-thirds of the minimum specified yield strength of the vessel material, the probability of failure (Q) was approximately 10^{-4} (page 41). From Equation 7.4, the failure rate corresponding to this probability of failure for an inspection interval of 2 years is

$$\lambda = -\frac{\ln 0.9999}{2} = 5 \times 10^{-5} \text{ failures/year} .$$

The desired 40-year reliability (R_{40}) corresponds to 20 inspection intervals, and from Equation 7.3, the resulting vessel reliability is

$$R_{40} = e^{-20(5 \times 10^{-5})} = 0.998 .$$

The probability of failure by bursting based on an allowable stress intensity of one-third of the minimum specified ultimate tensile strength of the vessel material was found to be about 10^{-20} (Figure 5.1, page 44).

This corresponds to a failure rate of essentially zero with a reliability of unity for a 40-year life. Similar results are obtained for an allowable stress intensity of one-half of the minimum specified ultimate tensile strength of the vessel material ($Q \simeq 10^{-15}$ and $R_{40} \simeq 1.0$). However, a ratio of ultimate strength to allowable stress intensity as low as 1.4, corresponding to a probability of failure of about 10^{-6}, will result in a failure rate of 5×10^{-7} failures per year and a 40-year reliability of 0.99998.

Thus, the import of Equations 7.3 and 7.4 is that the distributed steady-state failure probability given by Equation 3.7 over a known period of time represents a failure rate (λ) from which reliability can be calculated for any time period.

The cyclic rupture probability discussed in Section 6 is a direct statement of reliability when the parameters b and θ, shown in Figure 6.3 (page 53), are expressed as a function of cycles (time). For example, when N represents the number of cycles, the equations of the lines shown in Figure 6.3 were found to be

$$b = 12.44N^{-0.0792} \tag{7.5}$$

and

$$\theta = 551{,}000N^{-0.2151} \; . \tag{7.6}$$

Substitution of these values into Equation 6.4 (page 52) results in a statement of cyclic rupture reliability as a function of induced stress and the number of cycles.

$$R = e^{-\left(\frac{S_A}{551{,}000N^{-0.2151}}\right)^{12.44N^{-0.0792}}} \; . \tag{7.7}$$

Since Equation 7.7 does not clearly define a failure rate independent of life (N), the reliability with respect to cyclic rupture cannot be assigned a failure rate. The effects of inspection and repair are not clearly defined by Equation 7.7, but since some of the data points from which this equation was derived represented failures that occurred after repairs had been made, it may be assumed that Equation 7.7 includes the effects of some inspection and repair.

8. CONCLUSIONS AND RECOMMENDATIONS

The probability of failure was developed for failure defined by the maximum shear stress theory, for rupture from steady-state stress, and for rupture from cyclic stress. Normal distribution functions of induced stresses and allowable stresses were used for the steady-state stresses. For the cyclic stresses, the two-parameter Weibull distribution was used to account for the change in the distribution function of allowable stress with cyclic life, while the induced cyclic stress amplitude was treated as invariant for a specified value.

The allowable stress intensities and the S-N curves for low-carbon alloy steel were evaluated by using the analysis methods developed for the three failure modes. It was found that an allowable stress intensity equal to two-thirds of the minimum specified yield strength of the material corresponds to a failure probability of about 10^{-4} for expected loads and material variations when failure was defined by the maximum shear stress theory. The probability of failure corresponding to an allowable stress intensity equal to one-third of the minimum specified ultimate tensile strength of the material was found to be essentially zero when failure was defined in terms of an allowable burst pressure for the vessel. The probability of failure resulting from the stress amplitudes allowed for specified numbers of cycles was found to be greater than 10^{-2} for low-carbon alloy steel.

Numerical examples were presented for a cylinder under internal pressure to demonstrate the use of the probabilistic design method developed herein for pressure vessels. The minimum required shell thickness produced low probabilities of failure for the steady-state stresses. The cyclic failure probability was also judged to be adequate.

The effect of the steady-state failure probability on vessel life was discussed, and an analytical expression was derived for the relationship. It was concluded that the probability of failure defined as the probability that a steady-state induced stress will exceed a steady-state allowable stress can be converted to time-dependent reliability by considering the time intervals during which the stress distributions are not perturbed by inspection and maintenance. A method of estimating the failure rate for a pressure vessel that is based on this conclusion was presented.

On the basis of these results, it is recommended for adequate assurance against failure that the allowable stress intensity should not be higher than one-half of the minimum specified yield strength of the material, as opposed to the presently accepted value of two-thirds of the minimum specified yield strength of the material, but it can be as high as one-half of the minimum specified ultimate tensile strength of the material, as opposed to the presently accepted value of one-third of the minimum specified ultimate tensile strength of the material. No direct recommendation relative to the allowable cyclic stress amplitude can be made as a result of this investigation, but S-N curves corresponding to different probabilities of failure were superimposed on the allowable S-N curve to aid in an evaluation of the adequacy of existing allowable cyclic stresses.

Significant discrepancies in cyclic failure data were observed, and they apparently were a function of the data sources. These discrepancies resulted in a calculated probability of failure by cyclic rupture that was higher than the calculated probability of failure by crack initiation. These results are not consistent if it is assumed that cyclic rupture is a result of crack growth. It is therefore recommended that additional data analyses be performed on the probability of cyclic crack initiation and cyclic rupture. This is particularly advisable in view of the relatively high failure probability which resulted for the data analyzed herein.

The reader should consider the significance of the numbers resulting from the analyses presented herein. References to numerical quantities of reliability or failure probability have been presented throughout this document as approximate values. Despite the very precise values that could result from some of the analytical techniques used, the resulting values were termed approximate for two reasons. One reason is that the data used in the equations are from small populations and therefore represent low levels of confidence in calculated values of probability. The other reason is that critics of engineering practices and of reliability analyses are prone to question whether a value of 10^{-7}, for example, is adequate assurance against a catastrophic event as a moral judgment. The analysis method presented herein should therefore be looked upon as a tool to be used to compare alternative designs rather than as a tool to be used to calculate a value of merit and as a tool to be used in conjunction with other well developed methods of pressure vessel technology.

LIST OF REFERENCES

1. P. B. Gove, Ed., Webster's Third New International Dictionary of the English Language, Unabridged, G. and C. Merrimam Company, Springfield, Massachusetts, 1964.

2. J. M. Juran, Ed., Quality Control Handbook, McGraw-Hill Book Company, New York, 2nd ed., 1962.

3. C. Mischke, "A Method of Relating Factor of Safety and Reliability," pp. 537-542 in Journal of Engineering for Industry, August 1970.

4. F. L. Hale and W. B. Dalrymple, "Management of Systems Effectiveness Aspects of Saturn V Contracting and Procurement," pp. 250-257 in 1968 Annals of Assurance Sciences, Seventh Reliability and Maintainability Conference, The American Society of Mechanical Engineers, New York, 1968.

5. E. B. Haugen, Probabilistic Approaches to Design, John Wiley and Sons, Inc., New York, 1968.

6. J. F. Harvey, Pressure Vessel Design: Nuclear and Chemical Applications, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1967.

7. W. H. Von Alven, Ed., Reliability Engineering, Prentice-Hall, Inc., Inglewood Cliffs, New Jersey, 1964.

8. D. Kececioglu and E. B. Haugen, "A Unified Look at Design Safety Factors, Safety Margins, and Measures of Reliability," pp. 520-530 in 1968 Annals of Assurance Sciences, Seventh Reliability and Maintainability Conference, The American Society of Mechanical Engineers, New York, 1968.

9. C. Lipson, N. J. Sheth, and R. L. Disney, "Reliability Prediction, Mechanical Stress/Strength Interference," Technical Report No. RADC-TR-66-710, Rome Air Development Center, Griffiss Air Force Base, New York, 1967.

10. W. Weibull, "A Statistical Distribution Function of Wide Applicability," pp. 293-297 in Journal of Applied Mechanics, September 1951.

11. D. A. Canonico and R. G. Berggren, "Tensile and Impact Properties of Thick-Section Plate and Weldments," USAEC Report ORNL-TM-3211, Oak Ridge National Laboratory, Oak Ridge, Tennessee, January 1971.

12. "ASME Boiler and Pressure Vessel Code, Section III, Nuclear Power Plant Components," The American Society of Mechanical Engineers, New York, 1971.

13. "Criteria of the ASME Boiler and Pressure Vessel Code for Design by Analysis in Sections III and VIII, Division 2," The American Society of Mechanical Engineers, New York, 1969.

14. B. F. Langer, "Design-Stress Basis for Pressure Vessels," Report WCAP-5370, Westinghouse Electric Corporation, Nuclear Energy Systems, Pittsburgh, Pennsylvania, June 1970.

15. R. D. Wylie and W. J. McGonnagle, Quality Control in Fabrication of Nuclear Pressure Vessels, Rowman and Littlefield, Inc., New York, 1964.

16. J. N. Robinson, Inspection Engineering Department, Oak Ridge National Laboratory, personal communication relative to unpublished statistical survey performed on properties of as-received type 304 stainless steel to H. G. Arnold, Oak Ridge National Laboratory, January 1972.

APPENDIXES

APPENDIX A

LIST OF SYMBOLS

a = constant or invariant multiplier

B = strain-hardening factor

B_c = strain-hardening factor for cylindrical vessel

B_s = strain-hardening factor for spherical vessel

b = slope parameter of Weibull distribution function

C = coefficient of variation of failure probability resulting for normally distributed allowable and induced stresses

k = number of inspections

M = ratio of allowable to induced stress or of burst pressure to induced pressure

m = coupling parameter of normal distribution function

m = shape parameter of Weibull distribution function

m = mean time between failures for exponential reliability function

N = ratio of minimum specified ultimate tensile strength to the allowable stress intensity

N = number of cycles of stress amplitude in cyclic analysis

n = strain-hardening exponent of a material as measured in tensile tests

P = probability of success

p = internal pressure

p_b = burst pressure

Q = probability of failure

R = reliability

R_T = reliability with inspection

r = radius

S = a distributed random variable

$\overline{S}$ = mean of the frequency distribution of S

S_A = stress amplitude

S_a = allowable stress

S_h = hoop stress

S_i = induced stress

S_m = allowable stress intensity as specified in Section III of the ASME Boiler and Pressure Vessel Code

S_r = stress range

S_u = ultimate tensile strength

S_y = yield stress of a material

T = time

T_I = time span between inspections

t = thickness

V_p = coefficient of variation of pressure

V_s = coefficient of variation of the frequency distribution of S

V_{S_a} = coefficient of variation of allowable stress

V_{S_h} = coefficient of variation of hoop stress

V_{S_i} = coefficient of variation of induced stress

V_Y = coefficient of variation of Y

Y = function of radius of vessel and thickness of shell

θ = b-th root of shape parameter of Weibull distribution function where b is the Weibull slope parameter

λ = average rate at which failure stresses occur or the failure rate for the exponential reliability function

σ_s = standard deviation of the frequency distribution of S

σ_{S_h} = standard deviation of hoop stress

σ_Y = standard deviation of Y

APPENDIX B

DETERMINATION OF CYCLIC ALLOWABLE STRESSES

The manner in which fatigue testing is performed usually involves the subjection of a number of specimens to a repeated stress of a specified value until failure occurs and repetition of the process on other specimens at a different stress level until failure occurs at that stress level. The scatter obtained in such testing is a scatter of cyclic life for a given stress amplitude. Theoretically, it is necessary to fatigue test all specimens with different stress amplitudes until failure occurs at a given number of cycles if one wishes to estimate the probability that a given stress amplitude will cause failure in a specified number of cycles, but such testing is impractical. The life scatter obtained by conventional testing methods was converted to a strength distribution for use in Section 6 as follows.

The fatigue data obtained for cyclic vessel failure were plotted in a conventional S-N diagram (Figure 6.1, page 49, or Figure 6.5, page 57). It was assumed that each specimen (data point) corresponded to an individual S-N curve and that, for fixed test conditions, there would be a family of nonintersecting S-N curves, each of which would correspond to a different probability for the occurrence of failure. The average S-N curve was fitted through the points by assuming a straight line and making a least squares fit of that line to the data points. An S-N curve, parallel to the average S-N curve, was then drawn through each data point. The families of S-N curves corresponding to cyclic rupture and cyclic crack initiation are shown in Figures 6.1 (page 49) and 6.5 (page 57), respectively.

The frequency distribution of allowable cyclic stress was obtained for a life of 10^5 cycles by drawing a vertical line at $N = 10^5$ that intersected the family of S-N curves. The points of intersection represent a sample of the strength distribution at a desired life. These data were then plotted on the Weibull probability paper as a cumulative distribution function of allowable cyclic stress, as illustrated in Figures 6.2 (page 51) and 6.6 (page 58).

ORNL TM-3858

Internal Distribution

1. J. W. Anderson
2-5. H. G. Arnold
6. S. E. Beall
7. M. Bender
8. W. A. Bush
9. D. D. Cannon
10. C. W. Collins
11. W. B. Cottrell
12. F. L. Culler, Jr.
13. R. W. Dehoney
14. H. G. Duggan
15. W. F. Ferguson
16. F. C. Fitzpatrick
17. W. R. Gall
18. D. L. Gray (K-25)
19. B. L. Greenstreet
20. H. A. Nelms
21. T. W. Pickel, Jr.
22. J. N. Robinson
23. P. Rubel
24. R. A. Schmidt
25. W. C. T. Stoddart
26. D. B. Trauger
27. V. R. R. Uppuluri
28. H. K. Walker
29. G. D. Whitman
30. W. R. Winsbro
31. F. J. Witt
32-33. Central Research Library
34. Document Reference Section
35. GE Division Library
36-49. Laboratory Records Department
50. Laboratory Records, ORNL R.C.
51. ORNL Patent Office

External Distribution

52. L. E. Alsager, U. S. Atomic Energy Commission, Division of Reactor Development and Technology, Washington, D. C. 20545
53. L. J. Chockie, General Electric Company, 175 Curtner Avenue, M/C 510, San Jose, California 95125
54. R. W. Cochran, U. S. Atomic Energy Commission, Division of Reactor Development and Technology, Washington, D. C. 20545
55. W. E. Cooper, Teledyne Materials Research, 303 Bear Hill Road, Waltham, Mass. 02154
56. S. A. Davis, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60440
57. A. J. Edmondson, Department of Mechanical Engineering, The University of Tennessee, Knoxville, Tennessee 37916
58. R. J. Gimera, Liquid Metal Engineering Center, Post Office Box 1449, Canoga Park, California 91304

59. J. F. Harvey, The Babcock and Wilcox Company, Nuclear and Special Products Engineering, Barberton, Ohio 44203
60. J. W. Hodgson, Department of Mechanical Engineering, The University of Tennessee, Knoxville, Tennessee 37916
61. D. H. Hutchinson, Department of Industrial Engineering, The University of Tennessee, Knoxville, Tennessee 37916
62. B. F. Langer, Westinghouse Atomic Power Division, Post Office Box 355, Pittsburgh, Pa. 15230
63. R. R. Maccary, U. S. Atomic Energy Commission, Assistant Director for Reactor Safety, Directorate of Licensing, Washington, D. C. 20545
64. J. L. Mershon, U. S. Atomic Energy Commission, Division of Reactor Development and Technology, Washington, D. C. 20545
65. W. Mikesell, Chicago Bridge and Iron Works, Post Office Box 13308, Memphis, Tennessee 38113
66. R. W. Schneider, Bonney Forge and Foundry, Inc., Bonney Forge Division, Cedar and Meadow Streets, Allentown, Pa. 18105
67. D. W. Sher, Combustion Engineering, Inc., 911 West Main Street, Chattanooga, Tennessee 37401
68. J. Smith, Department of Mathematics, The University of Tennessee, Knoxville, Tennessee 37916
69. W. R. Smith, Sr., Bechtel Corporation, Post Office Box 3965, San Francisco, California 94119
70. F. H. Speckhart, Department of Mechanical Engineering, The University of Tennessee, Knoxville, Tennessee 37916
71. S. A. Weber, Hanford Engineering Development, Post Office Box 1970, Richland, Washington 99352
72. L. P. Zick, Jr., Chicago Bridge and Iron Works, 901 West 22nd Street, Oak Brook, Illinois 60521

73-74. Technical Information Center, U. S. Atomic Energy Commission, Oak Ridge Operations

75. Research and Technical Support Division, U. S. Atomic Energy Commission, Oak Ridge Operations