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A PROBABILISTIC MODEL FOR ESTIMATING THE OPERATING COST OF AN ELECTRIC POWER GENERATING SYSTEM

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D. S. Joy and R. T. Jenkins*

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*Division of Power Resource Planning, Tennessee Valley Authority, Chattanooga, Tenn.

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ABSTRACT

The concept of using a probabilistic simulation for estimating the operating cost of generating plants within an electrical utility was first introduced in 1967, and has recently been gaining acceptance in the United States. Basically, the probabilistic simulation technique develops the cost of operating a utility system over an extended period of time by forecasting the power to be generated by each plant. The basic model for this technique requires the following information: system load duration curve, loading order of units, generating unit characteristics, and energy supplied by energy-limited units such as hydroelectric units. The major advantage of the technique is its capability for simulating the effects of random events such as unit forced-outages.

This report describes the basic probabilistic simulation model and modifications that have been made to represent more effectively the operation of large thermal units during low load periods and to take into consideration the effect of hydroelectric and pumped-storage units. The model is versatile and can be modified easily. Computation times have averaged between 0.001 and 0.01 sec/unit on an IBM 360/91 computer with a memory requirement of approximately 80 K bytes. The probabilistic simulation model is used as a subroutine for estimating operating costs in conjunction with optimization techniques for studying utility planning problems.

1. INTRODUCTION

The prediction of system operating expenses for a wide range of conditions is an important aspect of utility planning. Some of the long-range planning problems include those for system expansion, maintenance scheduling, nuclear refueling, and hydroelectric utilization. In applying classical optimization techniques to these problems, it is necessary to simulate the operation of the utility system for different values of the decision variables. This report describes an improved technique that is capable of simulating the effects of random events such as unit forced-outages.

Programs are available for determining the optimal dispatching of generation units.[1,2] These programs consider load changes on an hourly basis (Fig. 1) and are generally applied to short-term problems (i.e., problems extending from one day to one week) because the uncertainty of unit availability and system load is relatively small. For long-term studies, the uncertainties of unit availabilities and load forecast errors are significant and these factors become more important. Since these factors involve random events, a Monte-Carlo technique[3] would have to be used in conjunction with the hourly loading models in order to estimate a statistically significant range and average for the operating costs over extended time periods. The resulting computational requirements make this approach unattractive, particularly in multiyear studies.

Prior to 1960, a load duration curve, which is constructed by rearranging the hourly loads in decreasing order of magnitude, was utilized for estimating long-term utility operation. Horizontal lines were drawn at the capacities of the various units, and the area between these lines represented an approximation of the generating requirements for each unit (Fig. 2). Outages were approximated by reducing the capacities of the units by a few percent. This method tends to underestimate the system operating cost since it cannot reflect realistically the effects of unit outages and load forecast errors. In such a case, the expected operation of the base loaded units will generally be overestimated while the operation of the peaking units will be underestimated. The magnitude of this bias depends on the particular power system and becomes more important in systems featuring:

(1) a small number of units;
(2) a wide range of unit production costs;
(3) high outage rates;
(4) large variability in loads;
(5) large variability of energy availability.

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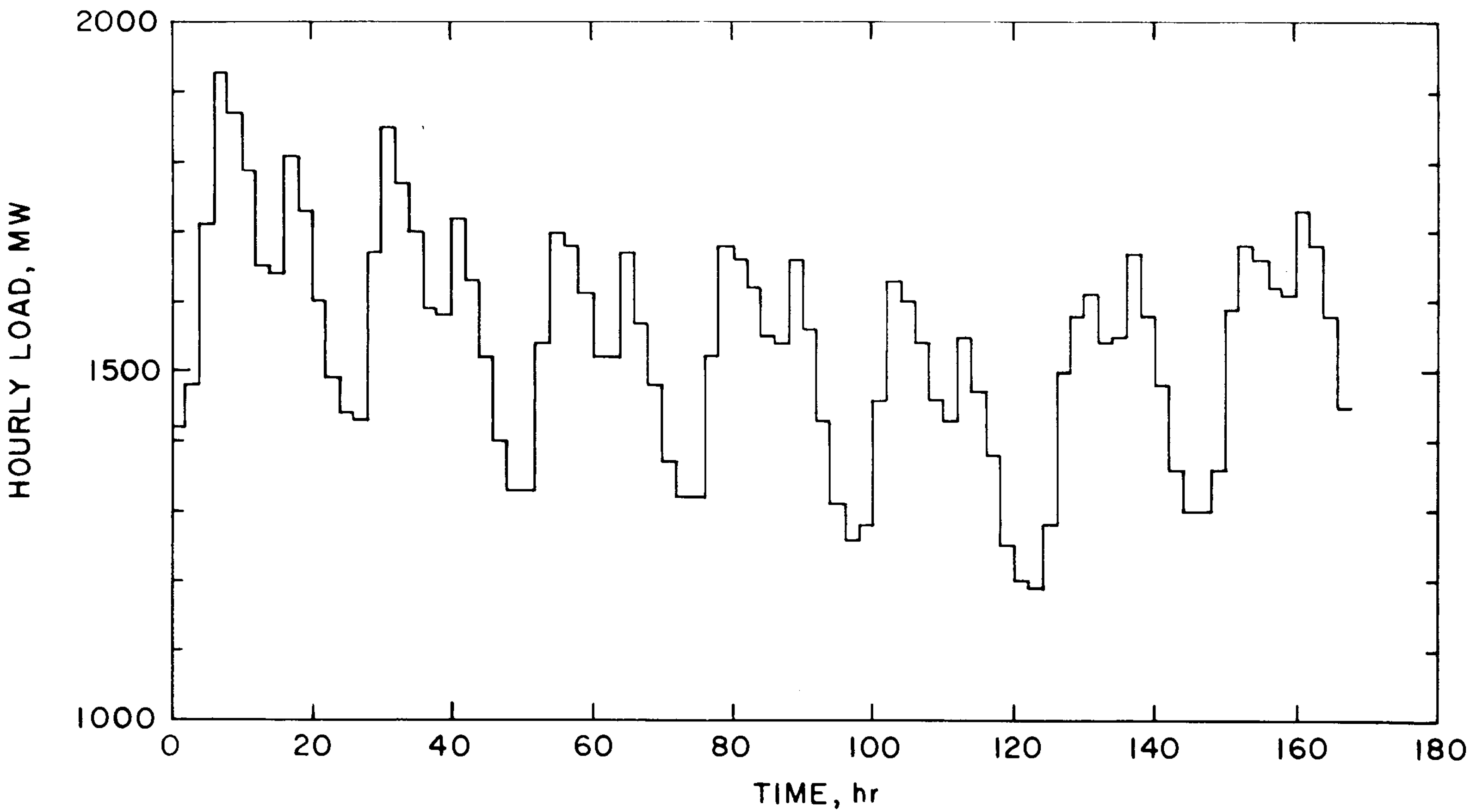

Fig. 1. Hourly Load Representation.

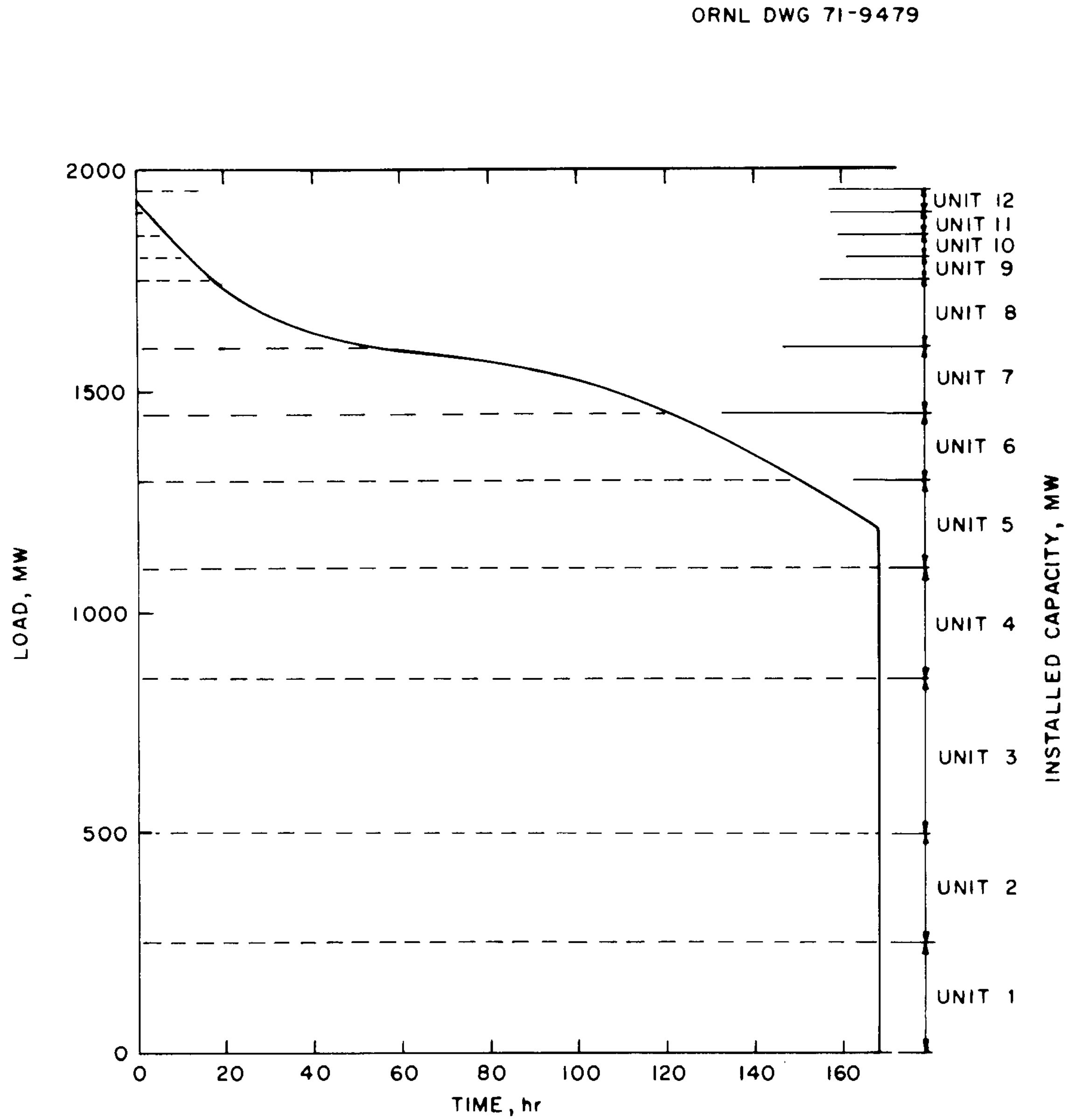

Fig. 2. Estimating Unit Loadings via a Load Duration Curve.

The need for a probabilistic simulation of utility operation has developed because many of the modern utility systems are acquiring the above-mentioned characteristics. The current trend toward the construction of larger units and the subsequent retirement of old, smaller units is concentrating a significant amount of the generating capacity in a smaller number of units (item 1). In recent years, a large number of gas turbine peaking units have been installed to help carry the peak load. With the increased operating cost associated with these units, item 2 begins to be satisfied. The new, larger units are experiencing a higher forced-outage rate than the older units (item 3). Item 4 has emerged in the past decade with the advent of electrical heating and air conditioning, thus making the daily peak load dependent on weather changes, particularly during the summer and winter months. Hydroelectric energy is dependent on rainfall, and nuclear units will probably be classified under item 5 as these units approach their refueling date.

During the past few years a stochastic approach to utility simulation has been developed by Baleriaux[4] and Booth.[5] This approach statistically incorporates the random effect of unit outages into the load duration curve. A plant loading order is required since the expected operation of a given plant is influenced by the forced-outage rates of the plants listed below it in the loading order. The algorithm calculates the probability of loss of load, the expected generation requirement, and hours of operation for each plant.

The basic algorithm is presented in this report. The simulation of energy-limited plants such as hydroelectric units or pumped-storage units is also discussed.

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2. THE BASIC BALERIAUX METHOD OF COMPUTING ENERGY CONTRIBUTIONS OF THERMAL UNITS TO A GIVEN LOAD PATTERN

2.1 Load Representation

When the hourly system loads are plotted as a function of time, the resulting curve (Fig. 1) gives a chronological representation of the transient generating power (load) requirement of a utility system. The area under the curve is the total energy requirement to be delivered by the system over the time period being considered. If these same hourly loads are rearranged in decreasing order of magnitude, the resulting curve (Fig. 2) is called a load duration curve. The area under the load duration curve is the total energy requirement; however, the chronological sequence of the loads has been lost. For the load duration curve, the abscissa represents the number of hours during which the system load equals or exceeds the value of the associated power on the ordinate. By normalizing the time variable, the value at any point on the abscissa becomes the fraction of the entire period for which the load equals or exceeds the associated power. Carrying this logic a step further, the abscissa can be considered to represent the probability that a particular value of the system load will be equaled or exceeded.

By reversing the ordinate and abscissa (Fig. 3), the load duration can be considered as a cumulative probability distribution. The load density function (Fig. 4) can be derived from the load duration curve by using Eq. (1):

$$\ell(x) = - \frac{dL(x)}{dx} , \qquad (1)$$

where

$\ell(x)$ = load density function,

$L(x)$ = cumulative load distribution function,

x = load, MW.

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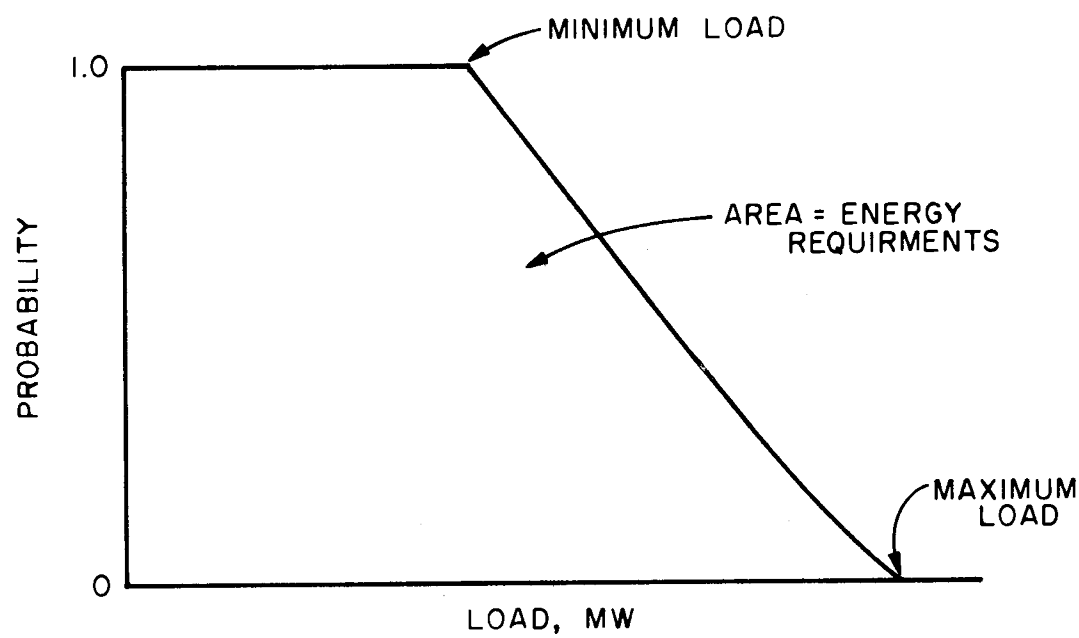

Fig. 3. Load Duration Curve.

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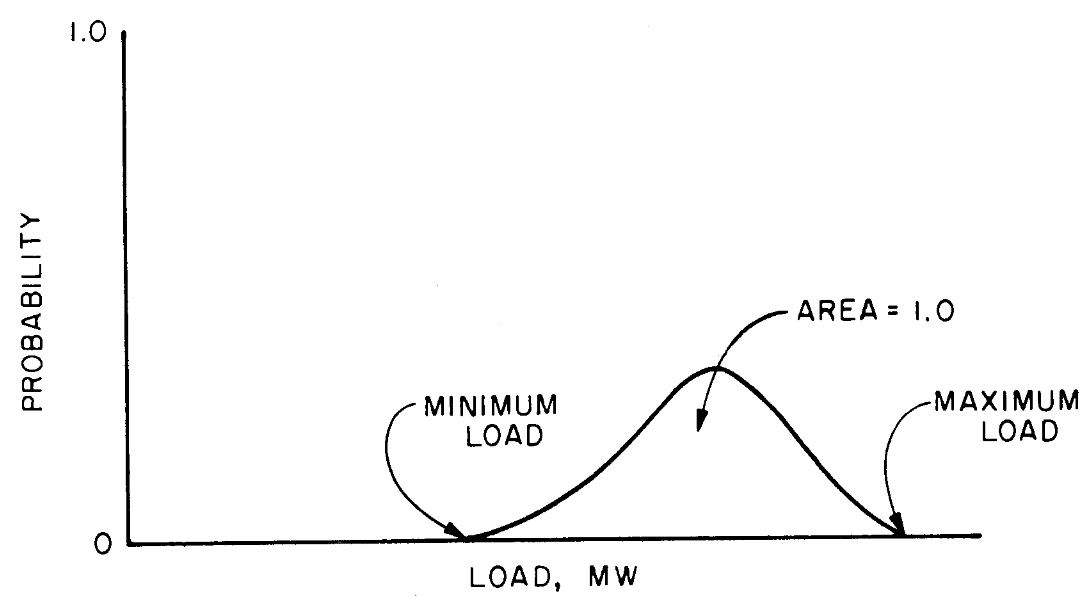

Fig. 4. Load Density Function.

2.2 Unit Forced-Outages

All generating units are subject to random outages caused by various types of equipment malfunction. The occurrence and duration of these forced-outages are unpredictable. The simplest stochastic method for treating unit reliability is to define two possible states for each unit. The unit is either available and capable of full power generation, or the unit is not available and is unable to deliver any power. More-complicated "state definitions" can be used to represent unit deratings.

Associated with each state is the probability of a unit being in that state. The probability of a unit being available is a_i, while the probability of not being available is q_i. Since the unit must be in one of the two states,

$$a_i + q_i = 1.0 \ . \tag{2}$$

The probability q_i is normally called the expected forced-outage rate.

The probability of having various amounts of capacity out of service due to forced-outages can be determined for any set of generating units once the availabilities (a_i's) are defined. By assuming that the units are totally independent of one another, a forced-outage distribution function can be calculated using a recursive relationship similar to that used for a binomial distribution. If all the generating units had the same capacity and forced-outage rate, the forced-outage distribution would be a binomial distribution. However, since generating units have various capacities and different forced-outage rates, the forced-outage distribution is built up sequentially by considering one unit at a time. The following example will illustrate how the forced-outage distribution function is generated. Consider the three generating units described in Table 1. For a system consisting of only the first unit, the distribution function of forced-outages can be written by inspection and is shown in column 2 of Table 2. When the second unit is added, it is necessary to calculate the probabilities of having various amounts of capacity out of service. This is done by using the previously calculated distribution function and the availability of unit 2. When both

Table 1. Sample Generating Units

Unit No.	Capacity (MW)	Unit Probability: Available	Unit Probability: Not Available
1	200	0.80	0.20
2	200	0.80	0.20
3	100	0.90	0.10

Table 2. Distribution of Forced-Outages

Power Outage, x (MW)	Probability of Occurrence		
	$P_1(x)$	$P_2(x)$	$P_3(x)$
0	0.80	0.64	0.576
100	0.0	0.0	0.064
200	0.20	0.32	0.288
300	0.0	0.0	0.032
400	0.0	0.04	0.036
500	0.0	0.0	0.004

unit 1 and unit 2 are available the outage, x (in MW), would be zero; and, by multiplying the probabilities of these events, one obtains:

$$P_2(0) = P_1(0) \cdot a_2 \qquad (3)$$
$$= (0.80)(0.80) = 0.64,$$

where

$P_2(x)$ = the probability of having x MW out of service in a two-plant system,

$P_1(x)$ = the probability of having x MW out of service in a one-plant system,

a_2 = the availability of unit 2.

The following two possible events would result in 200 MW being out of service: unit 1 down, with unit 2 available; and unit 1 available, with unit 2 down. The probability of this event is:

$$P_2(200) = P_1(200) \cdot a_2 + P_1(0) \cdot q_2 \qquad (4)$$
$$= (0.2)(0.8) + (0.8)(0.2) = 0.32.$$

The probability of both unit 1 and unit 2 being out of service simultaneously (a total of 400 MW out of service) is:

$$P_2(400) = P_1(200) \cdot q_2 \qquad (5)$$
$$= (0.2)(0.2) = 0.04.$$

The forced-outage distribution for the two units is shown in column 3 of Table 2. The third unit is added to the distribution in a manner similar to that just described, and the results of this calculation are summarized in column 4 of Table 2.

The general form of the recursion equation for adding any number of units is:

$$P_n(y) = P_{n-1}(y) \cdot a_n + P_{n-1}(z) \cdot q_n \text{ for } z \geq 0, \qquad (6)$$
$$P_n(y) = P_{n-1}(y) \cdot a_n \quad \text{for } z < 0 \text{ ,}$$

where

x = capacity of nth unit,

y = MW out of service,

z = y - x.

In Eq. (6), y is assumed to take on all possible values of the power outages. For computational purposes, it is convenient to divide the possible outages into even steps of 50 or 100 MW with the probability of an outage not falling exactly on these steps being divided proportionally between the two adjacent steps.

2.3 Equivalent Load

After the load and the outages have been developed in separate distribution functions, it is necessary to consider the combined effects of the two sets of stochastic events. One generally considers that the customer-imposed load on a power system is served by the total installed capacity, minus the capacity which is out of service. An alternative approach is to assume that the units that are down due to forced-outages contribute their rated capacity to the total system capacity but, at the same time, impose a load exactly equal to their rated capacity. This approach leads to the following definition of equivalent load:

$$E = L + O \;, \tag{7}$$

where

E = equivalent load, MW,

L = system demand, MW

O = load caused by forced-outages, MW.

Both terms on the right-hand side of Eq. (7) are random variables, and an equivalent load distribution function can be generated by convolving the load density function with the forced-outage density function. This technique is equivalent to determining the probability of observing all possible combinations of system load and unit outages which equal the same equivalent load and assigning the sum of these probabilities as the probability of observing the equivalent load, as follows:

$$P_E(x) = \sum P_L\overbrace{(z)}^{x-y} \cdot P_O(y) \;, \text{ subject} \tag{8}$$

$$\text{to } x = y + z \;,$$

$z = x - y$

where

$P_E(x)$ = equivalent load distribution function,
$P_L(z)$ = system load distribution function,
$P_O(y)$ = forced-outage distribution function,
x = equivalent load, MW,
y = total system outage, MW,
z = system load, MW.

For continuous density functions, the equivalent load distribution function would be calculated by

$$P_E(x) = \int_0^{\infty} P_L(z)\, P_O(y)\, dy \; . \tag{9}$$

This technique is best illustrated by an example. Consider the outage distribution for a three-unit system, shown in Table 2, and the discrete load density function, shown in Table 3. The calculation of the probability of obtaining various levels of equivalent load is shown in Table 4.

Table 3. Discrete Load Density Function

Load (MW)	Probability
0	0.0
100	0.2
200	0.4
300	0.3
400	0.1
500	0.0

Table 4. Calculation of the Probability of Obtaining Various Levels of Equivalent Load

Equivalent Load, E (MW)	System Load, L (MW)	Power Outage, O (MW)	Probabilities For: Load	Outage	Combined Probability	Probability of E
0	0	0	0.0	0.576	0.0	0.0
100	100	0	0.2	0.576	0.1152	0.1152
	0	100	0.0	0.064	0.0	
200	200	0	0.4	0.576	0.2304	0.2432
	100	100	0.20	0.064	0.0128	
	0	200	0.00	0.288	0.0	
300	300	0	0.3	0.576	0.1728	0.2560
	200	100	0.4	0.064	0.0256	
	100	200	0.2	0.288	0.0576	
	0	300	0.0	0.032	0.0	
400	400	0	0.1	0.576	0.0576	0.1984
	300	100	0.3	0.064	0.0192	
	200	200	0.4	0.288	0.1152	
	100	300	0.2	0.032	0.0064	
	0	400	0.0	0.036	0.0	
500	500	0	0.0	0.576	0.0	0.1128
	400	100	0.1	0.064	0.0064	
	300	200	0.3	0.288	0.0864	
	200	300	0.4	0.032	0.0128	
	100	400	0.2	0.036	0.0072	
	0	500	0.0	0.004	0.0	
600	400	200	0.1	0.288	0.0288	0.0536
	300	300	0.3	0.032	0.0096	
	200	400	0.4	0.036	0.0144	
	100	500	0.2	0.004	0.0008	
700	400	300	0.1	0.032	0.0032	0.0156
	300	400	0.3	0.036	0.0108	
	200	500	0.4	0.004	0.0016	
800	400	400	0.1	0.036	0.0036	0.0048
	300	500	0.3	0.004	0.0012	
900	400	500	0.1	0.004	0.0004	0.0004
					TOTAL	1.0000

The load density function and the equivalent load distribution have the same general shape. As expected, the sum of each distribution is equal to unity. Both distributions have the same minimum load; however, in general, the maximum equivalent load is equal to the sum of the peak system load and the total system capacity. The maximum equivalent load would be less than this sum only if some of the generating units were 100% reliable ($a_i = 1.0$).

A cumulative equivalent load distribution (Fig. 5) can be generated from the equivalent load density function by Eq. (10):

$$F(x) = 1.0 - \int_0^x P_E(y)\, dy \ , \tag{10}$$

where

$F(x)$ = the cumulative equivalent load distribution function.

As will be shown later, the cumulative distribution function is a convenient way to represent the equivalent load data, and the curve generated by using these data will be called the equivalent load curve. As shown in Fig. 5, the equivalent load curve has the same general shape as the load duration curve. As mentioned earlier, the area under the load duration curve equals the total energy requirement for the system over the time period being considered. As a result of the contribution of unit outages to the equivalent load, the area under the equivalent load curve is greater than the area under the load duration curve and represents equivalent energy.

2.4 Loss-of-Load Probability, and Unserved Energy

Much information can be derived from the equivalent load curve and its subsequent modifications. In Fig. 6, the equivalent load curve is shown for a system of N generating units and is labeled $F_N(x)$. That is, the availabilities of N plants were used to generate the forced-outage distribution that comprises a part of the equivalent load curve. The system capacity, $\overline{X}_N$, has also been plotted on the abscissa. Referring to the definition of the equivalent load curve [Eq. (10)], it is evident

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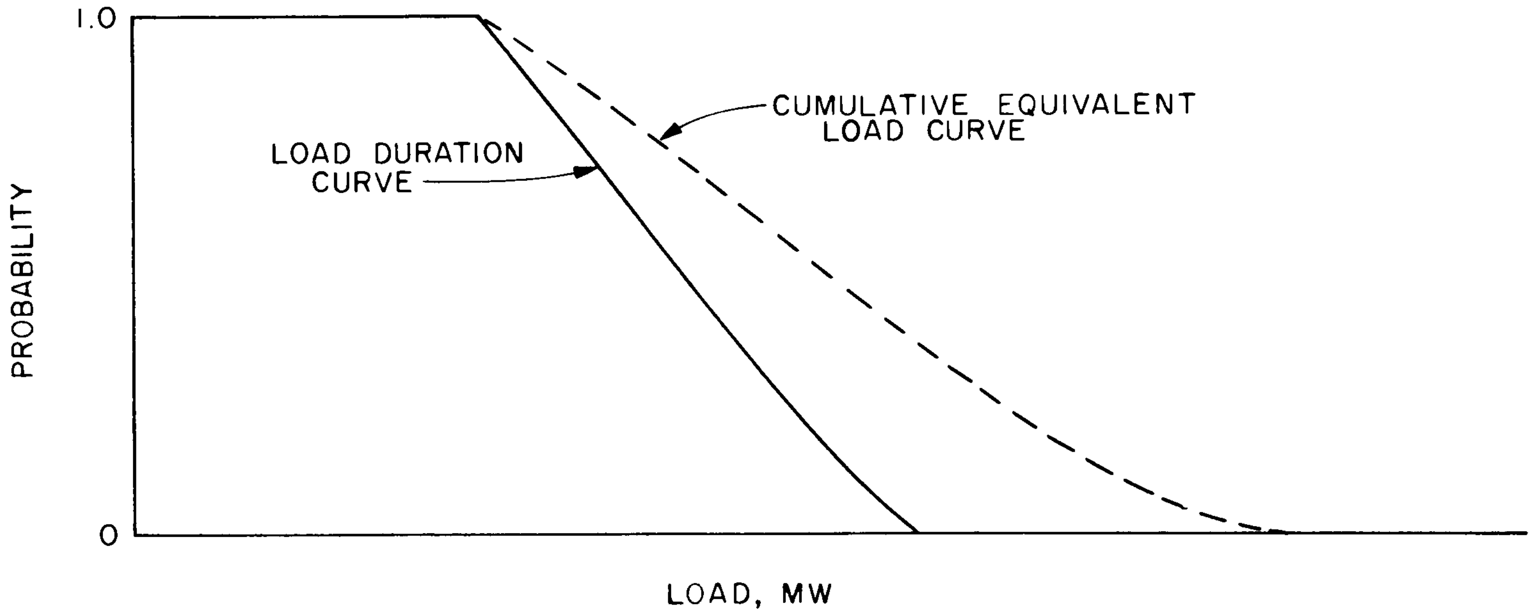

Fig. 5. Comparison of Load Duration Curve and Cumulative Equivalent Load Curve.

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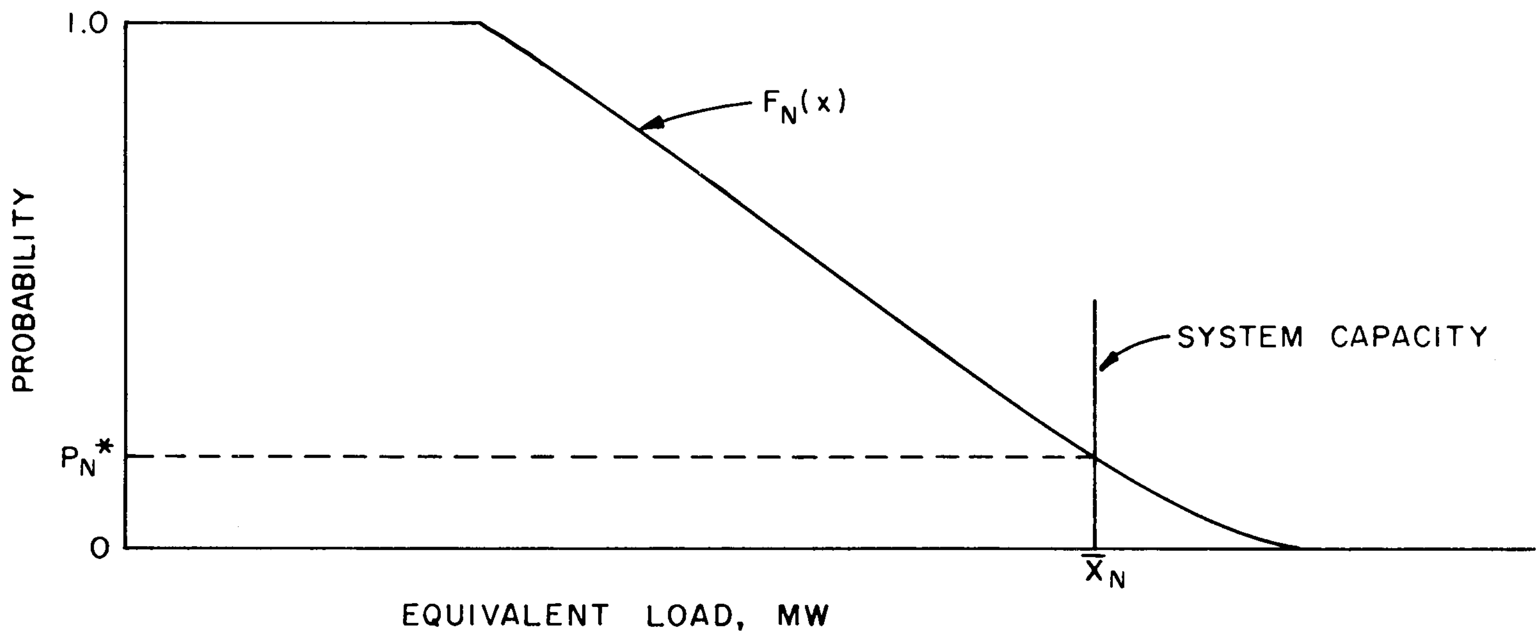

Fig. 6. Use of Cumulative Equivalent Load Curves to Determine Loss of Load Probability and Unserved Energy.

that P_N^* is the probability of having an equivalent load equal to or greater than the system capacity. Since the generating system would not be able to supply loads greater than the system capacity, P_N^* is the probability of loss of load, and the area under the curve to the right of $\overline{X}_N$ is proportional to the expected energy demand that could not be served. It is important to note that, whereas the area under the equivalent load curve is not system energy, the portion of the area that lies to the right of the system capacity, $\overline{X}_N$, is <u>true</u> energy. Hence

$$u = \tau \int_{\overline{X}}^{\infty} F_N(x)\ dx \ , \tag{11}$$

where

u = expected amount of unserved energy, MWhr,

τ = length of time period, hr.

2.5 Contribution of Individual Generating Units to Serving Load

The procedure described above can be modified to calculate the expected generation requirement for each unit in a power system. The discussion in this section will consider only conventional thermal units. The simulation of hydroelectric and pumped-storage units will be discussed in later sections.

When a power system is simulated, the order in which the plants are loaded must be specified. In practice, the loading order will be based on unit economics and geographic considerations that could include transmission constraints. In the basic algorithm, a simplified interpretation which assumes sequential loading of the generating units is made concerning the loading order.

Consider a subset of the original system containing (N - 1) units, where the last unit in the loading order, unit N, has been removed from the system. Following the procedures previously outlined, an equivalent load curve can be generated for this subsystem. This new curve is labeled $F_{N-1}(x)$ in Fig. 7. Notice that the curve $F_{N-1}(x)$ lies to the left and below the original equivalent load curve, $F_N(x)$. This shift

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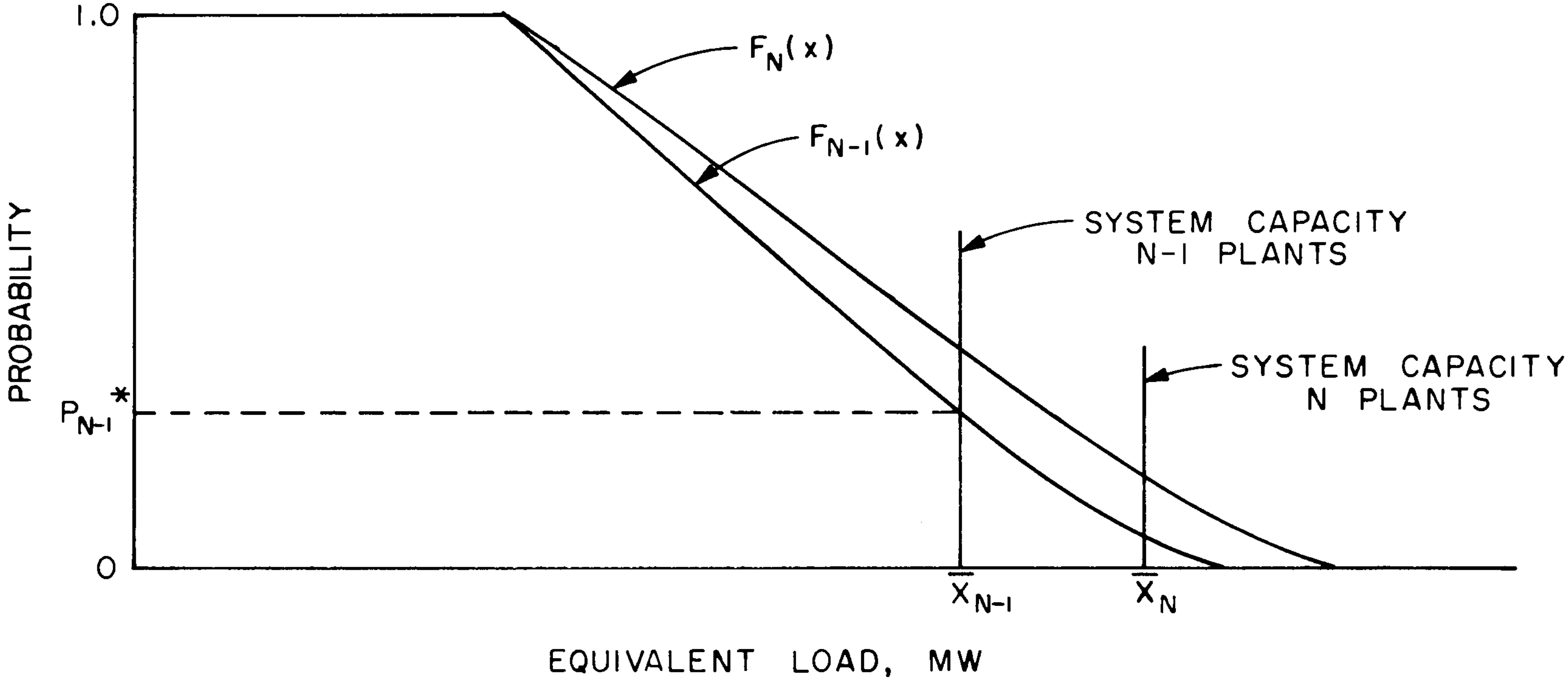

Fig. 7. Calculation of Plant Loadings.

is caused by removing the effect of the forced-outages of unit N from the outage distribution before the latter is convolved with the load distribution. The probability P^*_{N-1} can be interpreted as the loss-of-load probability of the subset of (N - 1) plants. The area under the curve $F_{N-1}(x)$ and located to the right of the new system capacity, $\overline{X}_{N-1}$, represents the expected energy requirement that cannot be served with the set of (N - 1) units.

If an additional unit (unit N) of capacity Y, which is defined as

$$Y = \overline{X}_N - \overline{X}_{N-1} \ , \tag{12}$$

is available, it would be used to generate the energy under the equivalent load curve $F_{N-1}(x)$ between the capacities $\overline{X}_{N-1}$ and $\overline{X}_N$. However, unit N is also subject to random forced-outages and is not expected to be available 100% of the time. Hence the expected generation of this unit would be:

$$ENG_N = a_N \ \tau \int_{\overline{X}_{N-1}}^{\overline{X}_N} F_{N-1}(x) \ dx \ , \tag{13}$$

where

ENG_N = expected generation of unit N, MWhr.

The probability P^*_{N-1} represents the fraction of time that unit N will be called upon to carry some load. Hence the hours of operation of unit N (i.e., H_N) can be calculated by Eq. (14):

$$H_N = P^*_{N-1} \cdot \tau \cdot a_N \ . \tag{14}$$

Also, the hours of operating unit N between any two-capacity limits c and b can be determined from the equivalent load curve $F_{N-1}(x)$ and Eq. (15):

$$\theta = \tau a_N \left[F_{N-1}(c) - F_{N-1}(d) \right] \tag{15}$$

$$\text{for } \overline{X}_{N-1} \leq c \leq d < \overline{X}_N \ .$$

The expected loadings of the next unit in the loading order, unit (N - 1), can be calculated by considering a subset containing (N - 2) plants. Iterations of this type are repeated until the expected generation of all units in the system have been calculated. When the expected loading of the first plant in the loading order is calculated, the subset used to develop the equivalent load curve is a null set; and, from Eq. (7), the equivalent load curve, F_0, must be identical to the original load duration curve. Hence the original equivalent load curve $F_N(x)$ will gradually reduce to the load duration curve as the effects of unit forced-outages are removed.

Once the expected generations have been determined for each unit, the expected operating costs may be estimated. The simplest method would be to use an average incremental cost for each unit and simply multiply this cost by the expected generation. More complicated cost functions could be used since the hours of operating between levels of various capacities can be determined from Eq. (15).

2.6 Sample Results

The results of applying the basic probabilistic model to a sample generating system containing only thermal units are shown in Table 5. The load duration curves are represented by a fourth-order polynomial, Eq. (16):

$$y = ax^4 + bx^3 + cx^2 + dx + e \ , \tag{16}$$

where

y = fraction of peak load,

x = fraction of time increment.

Three cases are shown in Table 5. The coefficients in Eq. (16) are the same in each case for peak loads varying from 1872 MW to 2209 MW. Each case covers a time period of 672 hr.

The basic probabilistic method predicts that each unit in the system will be operated to some extent. There is also an expected load that the system would not be able to serve. Note that the sum of the

Table 5. Results of Applying the Basic Probabilistic Model to a Sample Generating System

Plant	No. of Units	Capacity/Unit (MW)	Availability	Position in Loading Order	Case I[a] Expected Generation (GWhr)	Case I[a] Capacity Factor	Case II Expected Generation (GWhr)	Case II Capacity Factor	Case III Expected Generation (GWhr)	Case III Capacity Factor
BH	1	75	1.00	1	50.40	1.000	50.40	1.000	50.40	1.000
N1	1	300	0.92	2	185.47	0.920	185.47	0.920	185.47	0.920
BA	1	250	0.85	3	142.80	0.850	142.80	0.850	142.80	0.850
B1	1	250	0.90	4	151.20	0.900	151.20	0.900	151.20	0.900
B2	1	350	0.94	5	219.97	0.935	221.09	0.940	221.09	0.940
B3	1	250	0.92	6	132.37	0.788	142.93	0.851	152.32	0.907
B4	1	200	0.95	7	67.97	0.506	91.70	0.682	112.57	0.838
B5	1	150	0.96	8	29.43	0.292	41.64	0.413	69.23	0.687
PK	3	50	0.98	9	14.00	0.139	25.40	0.252	46.23	0.459
PS	2	125	0.99	10	8.19	0.049	15.66	0.093	39.34	0.234
H	1	400	0.99	11	1.50	0.006	3.51	0.013	12.50	0.046
			Unserved energy, GWhr		0.05		0.16		0.83	
			Total, GWhr		1003.35		1071.96		1183.98	
			Peak load, MW		1872		2000		2209	
			Energy required, GWhr		1003.35		1071.95		1183.97	
			Loss-of-load probability		0.00079		0.00208		0.01034	

[a]Load duration curve for each case:

$$y = 1.437186\ x^4 - 3.818328\ x^3 + 3.218145\ x^2 - 1.223198\ x + 1.003612,$$

where y = fraction of peak load

x = fraction of 672-hr time period.

expected generation for each plant plus the unserved load equals the area under the load duration curve.

For comparison, the same three cases are repeated in Table 6 by assuming that all generating units are 100% reliable. This is equivalent to using the load duration curve to predict the unit loading. Under this assumption, some of the units in the upper positions of the loading order are not expected to generate any energy. These units (e.g., gas turbines) are generally the most expensive units to operate, and the system operating costs would be underestimated, in this case, by ignoring the effect of forced-outages.

2.7 Two-Block Representation of Units

The simulation of the loading of the generating units used in the basic method is somewhat unrealistic. This method essentially assumes that a particular plant will be operated at its rated capacity before the next unit in the loading order will be required to carry any load. In actual utility operations, the units are not loaded in this manner. Instead, several units will be operated at levels below their maximum capacities in order to supply sufficient spinning reserve. During periods of low load, many units might be operated near their minimum capacity instead of forcing a short-term shutdown.

Booth[6] has suggested that the simulation of unit loadings could be improved by dividing the total capacity into two capacity blocks, which are then placed in nonadjacent positions in the loading order. The cross-hatched areas in Fig. 8 show how a single unit might be represented. The sum of the line segments ab and ef equals the total capacity of the unit. It would be unrealistic to have the segment ab represent a capacity less than the minimum capacity; however, this segment could represent a larger portion of the total capacity. The segment ab will be referred to as the "base block" of the unit, and the segment ef will be referred to as the "load following block".

The area abcd in Fig. 8 is proportional to the expected generation of the base block, and the length ad is proportional to the expected

Table 6. Sample Results, Assuming That All Plants Are 100% Available

Plant	No. of Units	Capacity/Unit (MW)	Availability	Position in Loading Order	Case I[a] Expected Generation (GWhr)	Case I[a] Capacity Factor	Case II Expected Generation (GWhr)	Case II Capacity Factor	Case III Expected Generation (GWhr)	Case III Capacity Factor
BH	1	75	1.00	1	50.40	1.000	50.40	1.000	50.40	1.000
N1	1	300	1.00	2	201.60	1.000	201.60	1.000	201.60	1.000
BA	1	250	1.00	3	168.00	1.000	168.00	1.000	168.00	1.000
B1	1	250	1.00	4	168.00	1.000	168.00	1.000	168.00	1.000
B2	1	350	1.00	5	233.50	0.993	235.20	1.000	235.20	1.000
B3	1	250	1.00	6	132.04	0.786	148.90	0.886	164.31	0.978
B4	1	200	1.00	7	42.35	0.315	77.35	0.576	108.70	0.809
B5	1	150	1.00	8	7.01	0.070	17.15	0.170	57.52	0.571
PK	3	50	1.00	9	0.44	0.004	5.20	0.052	21.32	0.212
PS	2	125	1.00	10	0.0	0.000	0.15	0.001	8.92	0.053
H	1	400	1.00	11	0.0	0.000	0.0	0.000	0.00	0.000
			Unserved energy, GWhr		0.0		0.0		0.0	
			Total, GWhr		1003.34		1071.95		1183.97	
			Peak load, MW		1872		2000		2209	
			Energy required, GWhr		1003.35		1071.95		1183.97	
			Loss-of-load probability		0.0		0.0		0.0	

[a]Load duration curve for each case:

$$y = 1.437186\, x^4 - 3.818328\, x^3 + 3.218145\, x^2 - 1.223198\, x + 1.003612,$$

where y = fraction of peak load
x = fraction of 672-hr time period.

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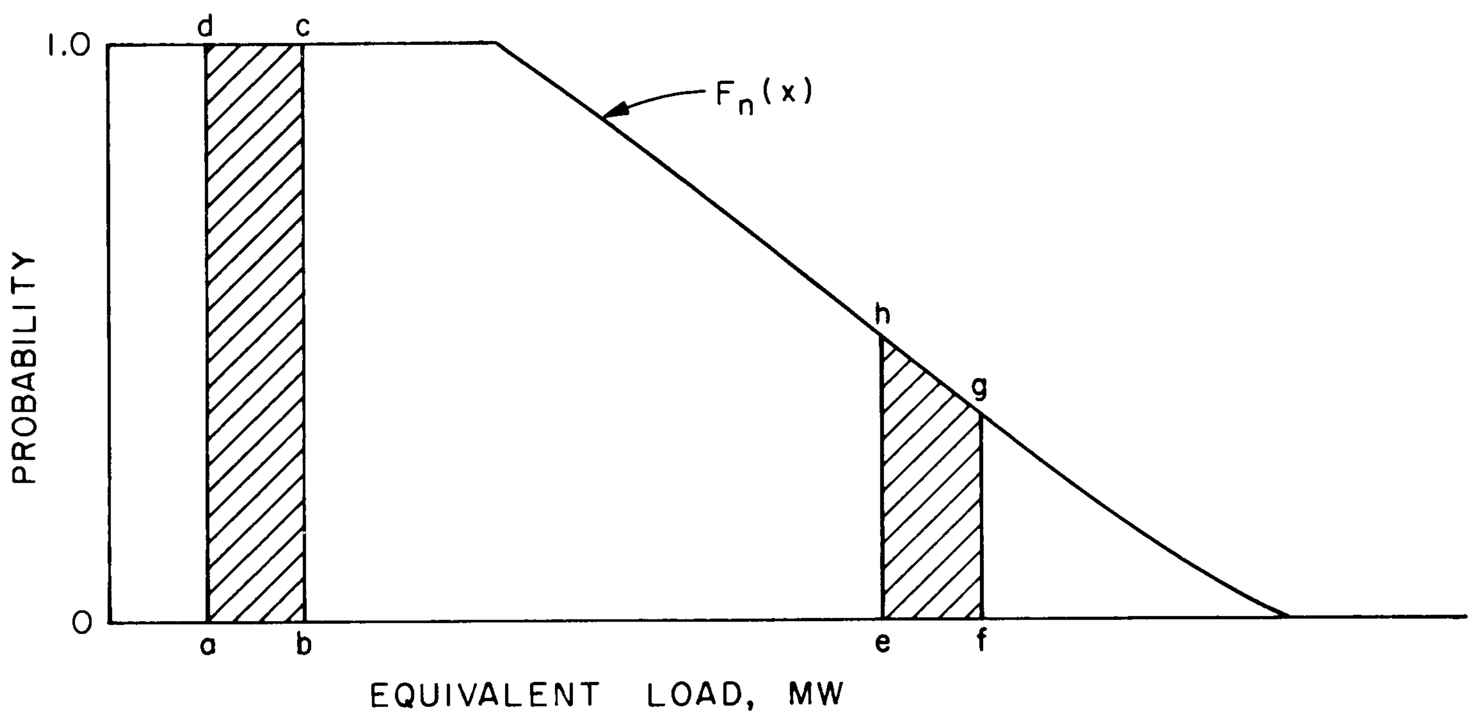

Fig. 8. Two-Block Representation of Generation Units.

number of hours of operation. For the load following block, the expected generation is proportional to the area efgh, and the expected hours of operation is proportional to the length eh.

In the two-block representation, the development of the original equivalent load curve $F_N(x)$ and the calculation of the loss-of-load probability and the associated energy that is not served are the same as described for the basic model. Only the total capacities of the generating units are considered in these steps. The logic used to modify the equivalent load curve for the calculation of the energy contributions of each plant is somewhat different. To demonstrate the logic of the two-block technique, consider a system containing four generating units (P1, P2, P3, and P4), each of which will be represented by two capacity blocks. The nomenclature P1B and P1L will be used to distinguish the base block and the load following block of the various units. The loading order for this example is: P1B, P2B, P3B, P1L, P2L, P4B, P3L, P4L. As discussed previously, the energy requirements of the units are calculated in the reverse order. Hence the first block to be considered is P4L (load following block of unit 4). The equivalent load curve is modified by removing the forced-outage effects of unit 4, thus generating a new curve $F_3(x)$. The entire unit is removed in this step. If this was not done, forced-outages of the base block would be partially covered by the load following block. Such an arrangement is not physically possible and also violates the two-state assumption, which specifies that a unit is either 100% available or not available at all. As in the basic method, the area efhg is multiplied by the availability of the unit and the number of hours in the time period to determine the energy contribution of the load following block [see Eq. (13)]. The expected operating time of this block is calculated by Eq. (14). Next, the forced-outage rate of the base block of unit 4 must be added to the equivalent load curve used to calculate the energy requirements of block P4L. This step is necessary since any units above P4B in the loading order (unit P3L in this case) will help cover forced-outages of the base block.

The next capacity block to be considered is P3L, and the calculational procedure used above is repeated. Moving further down the loading order, block P4B, the base block of unit 4, is encountered. The equivalent load curve is again modified; however, only the forced-outage effects of the base part of unit 4 are removed. The forced-outage effects of the load following block were removed when the energy requirement of P4L was calculated. The energy requirement and the hours of operation of this block are again calculated by Eqs. (13) and (14). Since this is a base block of a unit, it is not necessary to remodify the equivalent load curve before the calculations for the next capacity block are started.

In general, when a two-block representation of a generating unit is used, the initial modification of the equivalent load curve consists of removing the forced-outage effects of the block under consideration and all blocks of the unit occupying lower positions in the loading order. After the expected energy requirements have been calculated, the equivalent load curve must be remodified by adding the forced-outage effects of only those blocks of the unit that occupy a lower position in the loading order. Note that, during these modifications, the unit is treated as a single entity with a capacity equal to the sum of the blocks being removed or added.

Consistent with the two-state representation, both blocks of a generating unit are assumed to have the same availability. However, different fuel costs may be assigned to each block to more accurately reflect actual operating costs.

The use of the two-block representation gives the user two degrees of freedom in simulating a utility system. The capacities associated with the base block and the load following block are variables. The only constraint placed on these variables is that the sum of the block capacities must equal the total unit capacity. The position of each of the two blocks in the loading order is also a variable. Additional capacity blocks could be defined for any unit without any loss in generality; however, experience has indicated that two blocks are sufficient.

A sample utility system containing only thermal units is described in Table 7, where the division of the major units into two capacity blocks and the assumed loading order are shown. The expected energy of each unit was calculated for the system, using the same load data reported in Table 5; the results are shown in Table 8. This is the same system that was used for the sample calculations of Sect. 2.6. Note that the loss-of-load probability and expected unserved energy calculated with the two-block model are identical to the results shown in Table 5. Also, the last three units in the loading order (PK, PS and H) are represented by a single capacity block; and, as expected, the calculated expected energies for these units are identical with the results of Table 5.

2.8 Simulation of Hydroelectric Units

Thermal generating units can be operated at their maximum capacity for extended periods of time if this type of operation is required. Because of reservoir constraints, hydroelectric units may only be able to generate a fixed amount of energy. Hence it is desirable to utilize the hydroelectric energy in the most economical manner. These units are generally used to replace or off-load the more expensive thermal units (i.e., the units in the upper positions of the loading order). In order to simulate the effect of hydroelectric units, the amount of energy to be generated by each unit must be specified.

The hydroelectric units are assigned to the top positions of the loading order. As the result of various combinations of system load and unit outages, a hydroelectric unit would be expected to generate some energy to prevent excessive loss of load. At this point, the model does not distinguish between a hydroelectric unit and a thermal unit. The expected generation calculated for the hydroelectric unit would be identical to the expected generation for a thermal unit with the same capacity, availability, and position in the loading order. The expected generation of the hydroelectric plant can be interpreted as a minimum generation requirement. The expected generation of plant H (see Table 8) would represent the expected minimum hydroelectric requirement.

Table 7. Description of Sample Utility System, Using Two-Block Representation Technique

Plant	No. of Units	Total Capacity/Unit (MW)	Availability	Base Block		Peak Block	
				Capacity (MW)	Position in Loading Order	Capacity (MW)	Position in Loading Order
BH	1	75	1.00	75	1	0	-
N1	1	300	0.92	75	2	225	7
BA	1	250	0.85	90	3	160	8
B1	1	250	0.90	90	4	160	9
B2	1	350	0.94	110	5	240	10
B3	1	250	0.92	100	6	150	13
B4	1	200	0.95	65	11	135	14
B5	1	150	0.96	50	12	100	15
PK	3	50	0.98	50	16	0	-
PS	2	125	0.99	125	17	0	-
H	1	400	0.99	400	18	0	-

Table 8. Expected Unit Generations Using Two-Block Representation

Plant	Case I				Case II				Case III			
	Expected Generation (GWhr)				Expected Generation (GWhr)				Expected Generation (GWhr)			
	Base Block	Load Following Block	Total	Capacity Factor	Base Block	Load Following Block	Total	Capacity Factor	Base Block	Load Following Block	Total	Capacity Factor
BH	50.40	-	50.40	1.000	50.40	-	50.40	1.000	50.40	-	50.40	1.000
N1	46.37	139.10	185.47	0.920	46.37	139.10	185.47	0.920	46.37	139.10	185.47	0.920
BA	51.41	91.39	142.80	0.850	51.41	91.39	142.80	0.850	51.41	91.39	142.80	0.850
B1	54.43	96.77	151.20	0.900	54.43	96.77	151.20	0.900	54.43	96.77	151.20	0.900
B2	69.48	145.41	214.89	0.914	69.48	149.97	219.45	0.933	69.48	151.61	221.09	0.940
B3	61.82	58.89	120.71	0.718	61.82	74.06	135.88	0.809	61.82	84.81	146.63	0.873
B4	35.78	34.54	70.32	0.523	38.40	49.05	87.45	0.651	41.35	70.91	112.26	0.835
B5	26.27	17.55	43.82	0.435	28.71	25.87	54.58	0.541	31.40	43.83	75.23	0.746
PK	13.99	-	13.99	0.139	25.40	-	25.40	0.252	46.22	-	46.22	0.458
PS	8.19	-	8.19	0.049	15.66	-	15.66	0.093	39.34	-	39.34	0.234
H	1.50	-	1.50	0.006	3.51	-	3.51	0.013	12.50	-	12.50	0.046
	Unserved energy, GWhr		0.05				0.16				0.83	
		Total, GWhr	1003.34				1071.96				1183.97	
	Peak load, MW		1872				2000				2209	
	Energy required, GWhr		1003.35				1071.95				1183.97	
	Loss-of-load probability		0.00079				0.00208				0.01034	

If the minimum energy requirement is greater than the energy allocated for the hydroelectric unit, the system is deficient in energy, and this deficiency should be added to the unserved energy. However, if the allocation of hydroelectric energy is greater than the minimum requirement, the excess energy is available for reducing the generation requirements of the units that lie below the hydroelectric unit in the loading order.

After the minimum generation requirement of the hydroelectric unit has been calculated, the expected generation of the next unit in the loading order (unit z) is calculated. Following the procedure outlined in the basic method, the expected generation of this unit would be proportional to area A in Fig. 9(a).

$$ENG_z = a_z \tau \int_a^b F_n(x)\, dx \qquad (17)$$

The result obtained from Eq. (17) assumes that the hydroelectric unit is loaded after unit z. However, if the hydroelectric unit has sufficient available energy, it would be loaded in preference to unit z. When the hydroelectric unit is available 100% of the time, the equivalent load would be reduced by the capacity of the hydroelectric unit; this would shift the equivalent load curve to the left by the capacity of the hydroelectric unit [see Fig. 9(b)]. In this case, the expected generation of unit z would be proportional to area B in Fig. 9(b).

$$ENG'_z = a_z \tau \int_a^b F'_n(x)\, dx \ , \qquad (18)$$

where

ENG'_z = expected generation of unit z if it is loaded after the hydroelectric unit,

$F'_n(x)$ = shifted equivalent load curve.

Since the hydroelectric unit is being loaded before unit z, unit z would be expected to help cover any forced-outages of the hydroelectric unit. Therefore, the forced-outage rate of the hydroelectric unit must be included in the shifted equivalent load curve shown in Fig. 9(b).

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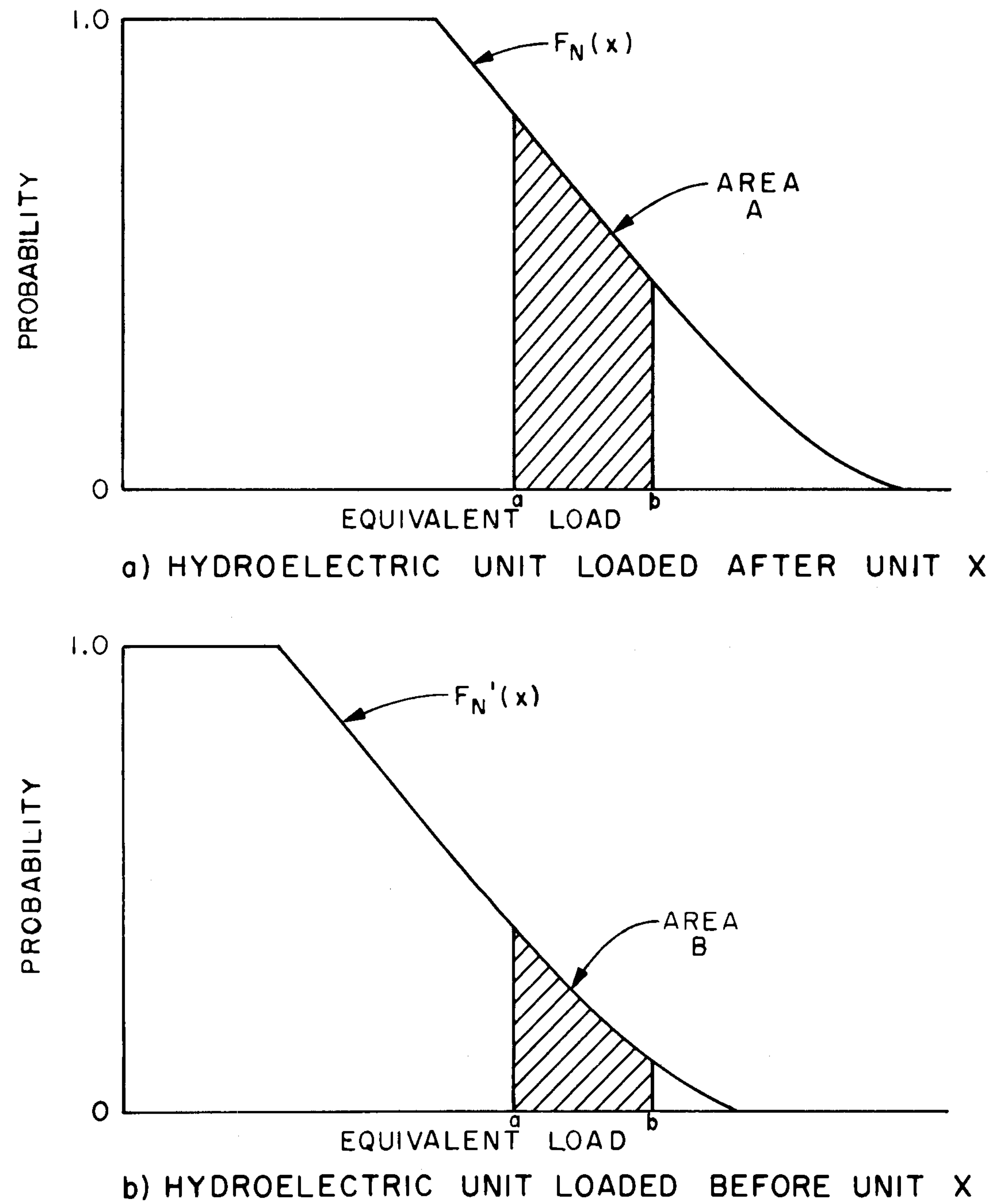

Fig. 9. Effect of Hydroelectric Unit on Expected Generation of Unit X.

The energy generated by the hydroelectric unit is calculated from the difference between the expected generations of unit z,

$$ENG_H = ENG_z - ENG'_z \ , \tag{19}$$

and this amount of energy is then subtracted from the remaining hydroelectric allocation.

This procedure of shifting the equivalent load curve to calculate the expected generations of the various units is repeated until the entire hydroelectric energy allocation is utilized. Generally, the remaining hydroelectric energy allocation will not permit the complete off-loading of the last thermal unit. In this case, the fraction of the energy that could be off-loaded is calculated by dividing the remaining hydroelectric allocation by the value of ENG_H calculated from Eq. (19). The equivalent load curve used to calculate the expected energy of the thermal unit is then constructed from a linear interpolation of the curves in Figs. 9(a) and 9(b) (see Fig. 10).

This procedure can be easily extended to include multiple hydroelectric units. No assumptions were made as to what type of plant was represented by unit z in the above discussion. This unit could be another hydroelectric unit, with the area B in Fig. 9(b) being proportional to its minimum generation requirement. In order to calculate the expected energy of the next unit in the loading order (i.e., unit y), the appropriate equivalent load curve must be shifted twice, once for each hydroelectric unit. This case is illustrated in Fig. 11(a), where the area A is proportional to the basic energy requirements of unit y if both hydroelectric units are loaded after unit y. In Fig. 11(b), the equivalent load curve has been shifted to the left by the capacity of the first hydroelectric unit, and the area B is proportional to the expected energy of unit y if unit y is loaded after the first hydroelectric unit but before the second hydroelectric unit. The difference between areas A and B is proportional to the energy generated by the first hydroelectric unit. To calculate the effect of the second hydroelectric unit, the equivalent load curve in Fig. 11(b) must be shifted to the left by the capacity of this unit [Fig. 11(c)]. The area C is proportional to the expected

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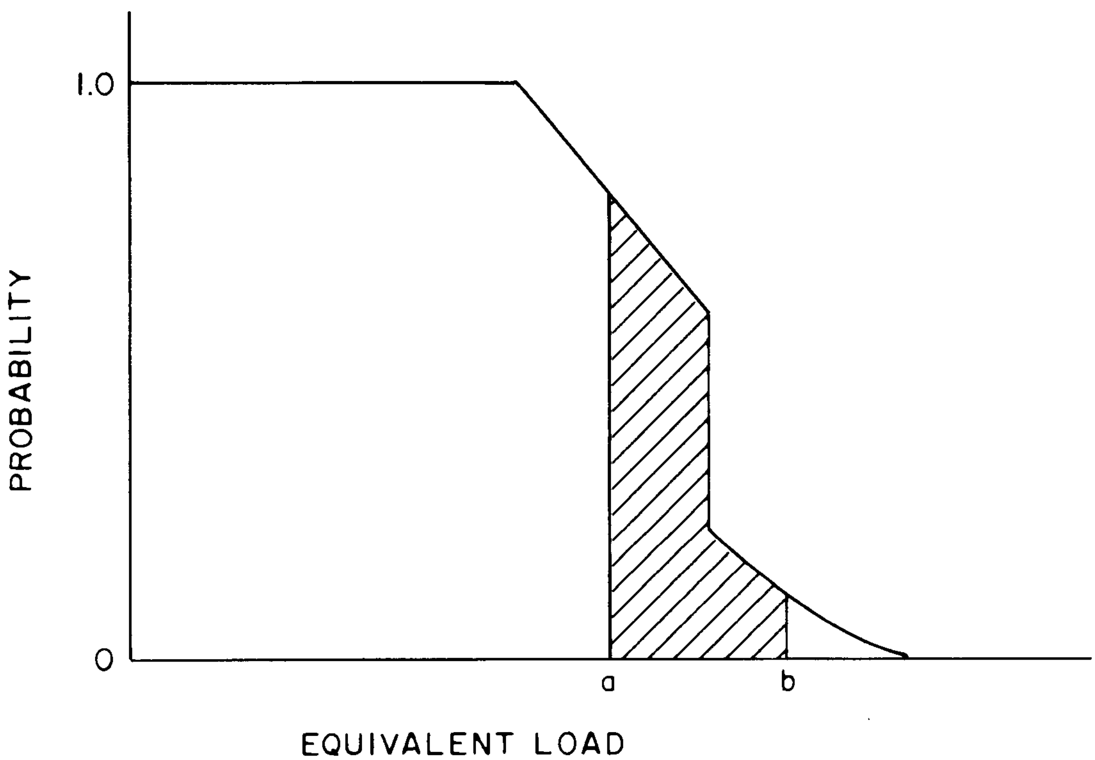

Fig. 10. Interpolation of Equivalent Load Curves.

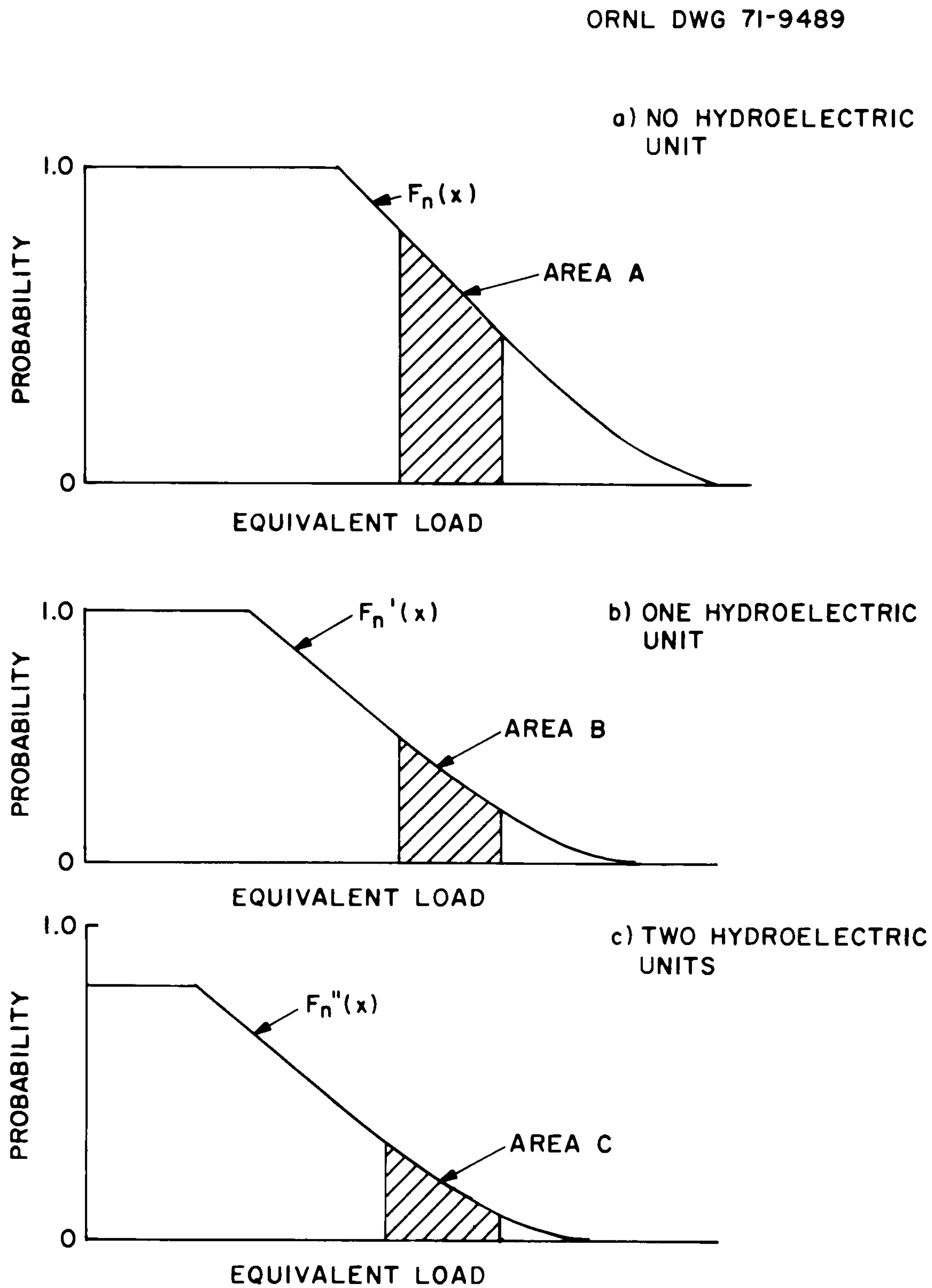

Fig. 11. Calculational Procedure for Multiple Hydroelectric Units.

generation of unit y if unit y is to be loaded after both hydroelectric units, and the area (B - C) is proportional to the amount of energy being generated by the second hydroelectric unit.

This procedure can be extended to include as many hydroelectric units as desired, and is compatible with the two-block representation of thermal units.

2.9 Simulation of Pumped-Storage Units

Pumped-storage units are simulated in a manner quite similar to that used for simulating hydroelectric units; however, it is not necessary to specify an energy allocation in the case of pumped-storage units. During periods of reduced load, energy of low incremental cost is employed to pump water into the pumped-storage reservoir; this water is later used during periods of high system load to replace high-cost thermal generation. Hence the pumped-storage unit should be placed above the thermal units in the loading order so that it is able to off-load the units below it. Since hydroelectric energy is less expensive than pumped-storage energy, the hydroelectric units would be used to off-load both the pumped-storage and the thermal units. Therefore, the pumped-storage units are placed in the loading order between the thermal units and the hydroelectric units.

The calculation for determining the amount of energy that a pumped-storage unit could generate is the same as that described for hydroelectric units (Fig. 12). Since the pumped-storage unit would not be used to replace thermal energy unless there is sufficient pumping energy available at an economic price, the expected generation of unit y is proportional to area B in Fig. 12(b). If the pumped-storage unit is operated, the expected generation of unit y would be proportional to area C in Fig. 12(c), and the area (B - C) would be proportional to the expected generation of the pumped-storage unit. The information concerning possible pumped-storage generation is retained until the expected pumping energy has been calculated for each thermal unit.

When water is pumped continuously into the pumped-storage reservoir, the capacity of the pumps will be added to the load at all times. Hence

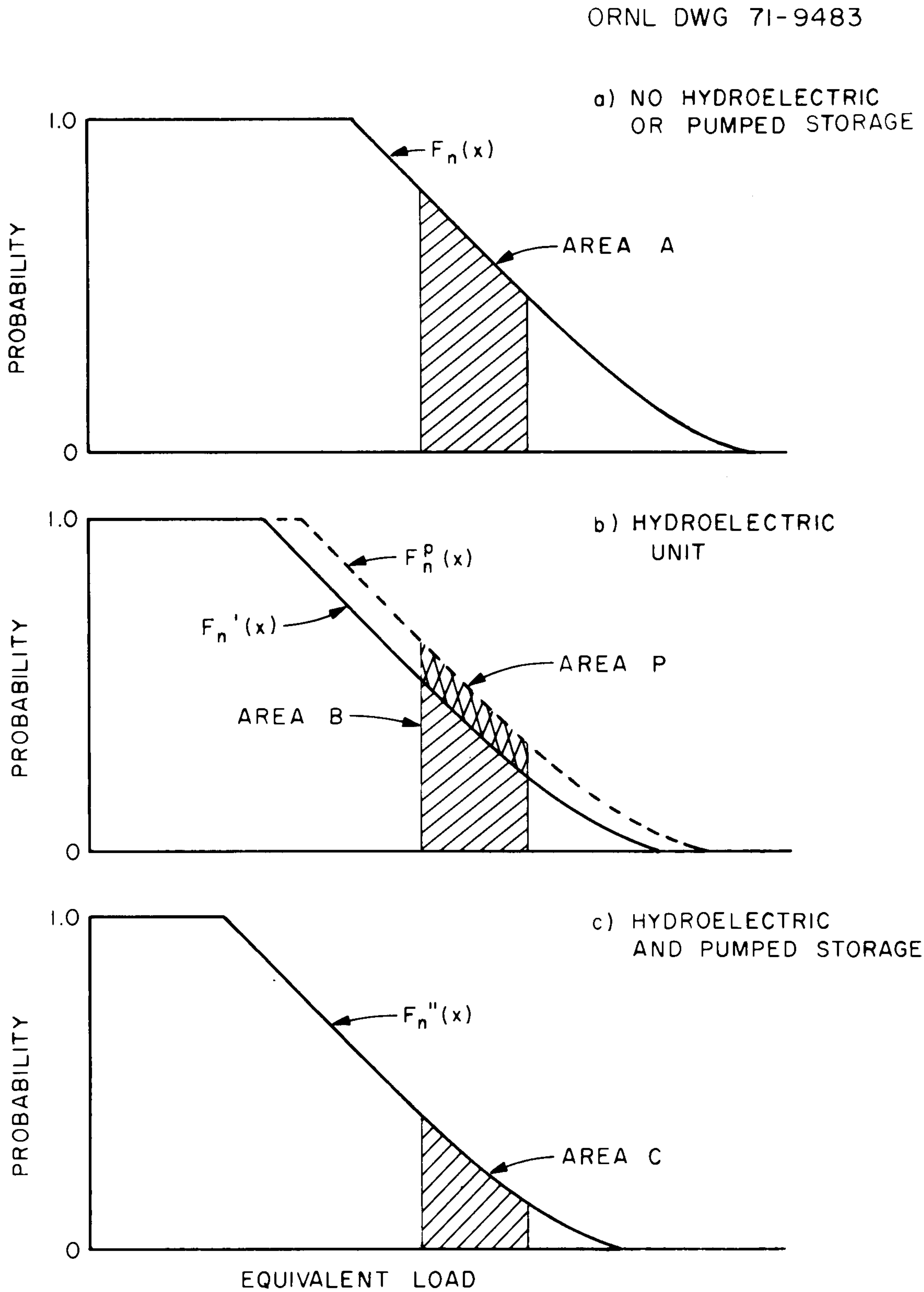

Fig. 12. Calculational Procedure for Hydroelectric and Pumped Storage Units.

the equivalent load curve will be shifted to the right by the pump capacity, as shown by the curve $F_n^p(x)$ in Fig. 12(b). The equivalent load curve shown in Fig. 12(b) is applicable because this is the curve used to calculate the expected generation of the thermal unit. Area P, which is shown in Fig. 12(b), is proportional to the expected pumping energy that could be obtained from the thermal unit. It would be calculated by Eq. (20):

$$PUMP_y = a_y a_p \tau \int_a^b F_n^p(x)\,dx - \int_a^b F_n(x)\,dx \;, \tag{20}$$

where

a_p = availability of pumps.

After the expected pumping duty and the expected pumped-storage generation have been calculated for each thermal unit, an economic check is made to determine the action of the pumped-storage unit. In order to utilize the pumped-storage energy to off-load a thermal plant, sufficient pumping energy must be available to satisfy the following constraint:

$$\frac{\lambda_p}{\lambda_g} < eff_p \cdot eff_g \;, \tag{21}$$

where

λ_p = cost of pumping energy,
λ_g = cost of energy displaced by pumped storage,
eff_p = efficiency of pumps,
eff_g = pumped-storage generating efficiency.

Once the final action of the pumped-storage unit has been determined, the expected energies of the thermal generating units must be adjusted to reflect the energy displaced by the pumped-storage unit and additional output required to supply the necessary pumping duty.

The expected generations for each unit of the sample system described in Table 7 are shown in Table 9. In this example, plant H is a hydroelectric unit, and the energy allocations specified for each case are 30.60, 45.20, and 105.30 GWhr respectively. Plant PS is a pumped-storage

Table 9. Probabilistic Simulation of a Sample Utility System Containing Thermal, Hydroelectric, and Pumped-Storage Generating Units

		Case I				Case II				Case III			
		Expected Generation (GWhr)				Expected Generation (GWhr)				Expected Generation (GWhr)			
Plant	Type	Base Block	Load Following Block	Total	Capacity Factor	Base Block	Load Following Block	Total	Capacity Factor	Base Block	Load Following Block	Total	Capacity Factor
BH	Thermal	50.40	-	50.40	1.000	50.40	-	50.40	1.000	50.40	-	50.40	1.000
N1	Thermal	46.37	139.10	185.47	0.920	46.37	139.10	185.47	0.920	46.37	139.10	185.47	0.920
BA	Thermal	51.41	91.39	142.80	0.850	51.41	91.39	142.80	0.850	51.41	91.39	142.80	0.850
B1	Thermal	54.43	96.77	151.20	0.900	54.43	96.77	151.20	0.900	54.43	96.77	151.20	0.900
B2	Thermal	69.48	151.55	221.03	0.940	69.48	151.59	221.07	0.940	69.48	151.61	221.09	0.940
B3	Thermal	61.82	58.89	120.71	0.718	61.82	74.06	135.88	0.809	61.82	92.66	154.48	0.920
B4	Thermal	35.78	34.54	70.32	0.523	41.34	49.05	90.39	0.673	41.49	77.43	118.92	0.885
B5	Thermal	26.27	6.04	32.31	0.320	28.71	21.51	50.22	0.498	32.25	23.76	56.01	0.556
PK	Thermal	0.16	-	0.16	0.002	0.42	-	0.42	0.004	1.73	-	1.73	0.017
PS	Pumped-storage	4.44	-	4.44	0.026	3.29	-	3.29	0.020	11.10	-	11.10	0.066
H	Hydro-electric	30.60	-	30.60	0.114	45.20	-	45.20	0.168	105.30	-	105.30	0.3917
	Unserved energy, GWhr			0.05				0.16				0.83	
	Total, GWhr			1009.49				1076.50				1199.33	
	Pumping requirements for pumped-storage, GWhr			6.14				4.55				15.36	
	Net energy produced, GWhr			1003.34				1071.95				1183.97	
	Peak load, MW			1872				2000				2209	
	Energy required, GWhr			1003.35				1071.95				1183.97	
	Loss-of-load probability			0.00079				0.00208				0.01034	

unit with a generating and pumping efficiency equal to 85%. The load data used in this example are the same as those used in Tables 5, 6, and 8.

3. CONCLUSIONS

The probabilistic simulation model is designed to incorporate the effect of random events in estimating the operation of a series of thermal generating units. The capacity, forced-outage rate, operating cost, and position in the loading order must be specified for each unit in the system. The capacity of any thermal unit may be divided into blocks, which can be placed in nonadjacent positions in the loading order. The amount of energy to be generated by the hydroelectric units, the pumping capacity and efficiency of the pumped-storage facility, and the load duration curve are additional items required as input information. The model will calculate an expected generation of each thermal unit and the pumped-storage unit. The expected hours of operation for each unit, the expected operating costs, the probability of loss of load, and the expected unserved energy are also calculated.

The probabilistic simulation model is versatile and can be easily modified to fit the user's needs. To date, the model has been used as a costing subroutine in a variety of ways, for example, (1) in forming the basis of a production costing model for estimating future operating costs, fuel requirements, and interchange requirements; (2) in a hydroelectric optimization program for determining the optimal usage of hydroelectric resources to minimize expected operating costs while taking hydroelectric reservoir constraints into consideration; and (3) in a program for developing an optimal maintenance schedule. A system expansion program[6] using the probabilistic simulation model to evaluate future operating costs has been developed.

Computation times required have averaged between 0.001 and 0.01 sec/unit on an IBM 360/91 with a storage requirement of approximately 80 K bytes.

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194. S. R. Fields
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196. J. A. Swartout, Union Carbide Corporation, 270 Park Avenue, New York, New York 10017.

197. R. R. Booth, State Electricity Commission of Victoria, Melbourne, Australia.

198. Patent Office, AEC, ORO.

199. Laboratory and University Division, AEC, ORO.

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