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Abstract

High-energy muon transport calculations are presented and the "shape" of the muon backstop required for a 200- and a 500-GeV proton accelerator is determined. The shield material considered is heavy concrete. Results obtained with an energy-dependent and an energy-independent stopping power are presented and compared. The error introduced by using a constant stopping power is clearly evident, but it may be tolerable for some design purposes.

I. INTRODUCTION

In a previous report,[1] hereinafter referred to as paper 1, muon transport calculations were carried out and it was shown that approximate semianalytic calculations were in reasonable agreement with results obtained using Monte Carlo methods. The approximate calculations can be simplified considerably if it is assumed that the muon stopping power is independent of the muon energy.[2,3] In this report the validity of this additional approximation is tested by comparing transport calculations carried out using a constant stopping power with calculations carried out using an energy-dependent stopping power.

In all of the calculations, the semianalytic method as formulated in paper 1 is used. There the "shape" of an iron backstop for a 200-GeV proton accelerator was determined. Here the "shape" of a heavy concrete backstop for a 200-GeV and a 500-GeV proton accelerator is considered.

In Section II the details of the calculations are described. In Section III the results are presented and discussed.

II. CALCULATIONAL DETAILS

The geometry considered in this report is the same as that used in paper 1--i.e., a point source of muons located a distance d in front of a semiinfinite slab shield. The point source, of course, represents the interaction of a high-energy proton beam with a target. After the beam interacts with the target it enters the backstop. Here, as in paper 1, the muons produced by the strongly interacting particles in the backstop will be neglected, and relatively small depths in the backstop where the strongly interacting particles will determine the shielding will not be treated. The energy and angular distribution of the muons from the source for both 200- and 500-GeV protons is calculated in the same manner as in paper 1. Muons from kaon decay have again been neglected. In the case of the 200-GeV protons, the source distribution is the same as that shown in Fig. 2 of paper 1.

In paper 1 it was shown that

$$I_\mu(E_\mu,\cos\theta,z,\rho) = \frac{\cos\theta}{4\pi\ A_2(E_\mu,z/\cos\theta)}$$

$$\times\ I_o\left[\frac{2(z+d)\ \rho\ \sin\theta\ \cos\theta}{4A_2(E_\mu,z/\cos\theta)}\right] \qquad \text{(II.1)}$$

$$\times\ \exp\left[-\frac{1}{4A_2(E_\mu,z/\cos\theta)}\left[\rho^2\ \cos^2\theta + (z+d)^2\ \sin^2\theta\right]\right]$$

$$A_2(E_\mu,z/\cos\theta) = \frac{m_e^2(4\pi\alpha)}{x_o}\int_o^{z/\cos\theta}\frac{(z/\cos\theta-\eta)^2\ d\eta}{\left[\frac{E'_\mu(E'_\mu + 2m_\mu)}{E'_\mu + m_\mu}\right]^2} \qquad \text{(II.2)}$$

$$\int_{E'_\mu}^{E_\mu} \frac{dE'}{T(E')} = \eta \qquad \text{(II.3)}$$

where

$I_\mu(E_\mu,\cos\theta,z,\rho)$ = the number of muons per unit area at depth z and radius ρ when one muon with energy E_μ traveling in the direction θ and uniformly spread over all azimuthal angles is incident on the shield,

ρ, z = the cylindrical coordinates defining a point in the shield,

$E_\mu, \cos\theta$ = the kinetic energy and cosine of the angle of emission with respect to the z axis of a muon from the source,

d = the distance from the point source to the front of the slab shield,

I_o = the modified Bessel function of order zero,

x_o = the radiation length,

α = the fine structure constant,

m_e = the electron rest energy,

m_μ = the muon rest energy,

T = the muon stopping power.

In terms of the function I_μ, the current density $I(z,\rho)$ from the source $S(E_\mu,\cos\theta)$ may be written

$$I(z,\rho) = 2\pi \int_{E_{MIN}}^{E_{MAX}} d\,E_\mu \int_o^1 d(\cos\theta)\; S(E_\mu,\cos\theta)\; I_\mu(E_\mu,\cos\theta,z,\rho) \;, \qquad \text{(II.4)}$$

where

E_{MAX} = the maximum kinetic energy of a muon emitted by the source,

E_{MIN} = the minimum muon kinetic energy which can reach a depth z,

and isocurrent density contours in the shield may be obtained from the equation

$$I(z,\rho) = \text{constant} \;. \qquad \text{(II.5)}$$

Equations II.1-II.5 were used in paper 1 and were shown to give results in reasonable agreement with more exact results obtained using Monte Carlo methods. However, since Eqs. II.1-II.5 contain four integrations, the calculations are by no means trivial. If the stopping power T is assumed to be independent of energy, computations may be simplified considerably since two of the integrations may be carried out analytically. In this case, Eq. II.3 becomes

$$E_\mu - E'_\mu = T\,\eta \;, \qquad \text{(II.6)}$$

and Eq. II.2 may be integrated to yield

$$A_2(E_\mu, z/\cos\theta) = \frac{m_e^2(4\pi\ \alpha)}{x_o}\,\frac{1}{T^2}\left\{\eta - 2C_1\ \log\left(\frac{E_\mu}{T} - \eta\right)\right.$$
$$- C_2\ \log\left(\frac{E_\mu + 2m_\mu}{T} - \eta\right) \qquad \text{(II.7)}$$
$$+ \frac{C_1\ C_2\ T}{m_\mu}\ \log\left[\frac{\dfrac{E_\mu + 2m_\mu}{T} - \eta}{\dfrac{E_\mu}{T} - \eta}\right]$$
$$\left. + \frac{C_1^2}{\dfrac{E_\mu}{T} - \eta} + \frac{C_2^2}{\dfrac{E_\mu + 2m_\mu}{T} - \eta}\right\}_o^{z/\cos\theta} \;,$$

where

$$C_1 = \frac{z}{2\cos\theta} - \frac{E_\mu}{2T} \qquad \text{(II.8)}$$

$$C_2 = \frac{z}{2\cos\theta} - \frac{E_\mu + 2m_\mu}{2T} \ . \qquad \text{(II.9)}$$

To test the validity of assuming T constant, calculations have been carried out using both Eqs. II.2 and II.7 for a shield of heavy concrete.

The heavy concrete considered is composed of 48.48% Fe, 38.56 % O, 7.25% Ca, and 5.71% Al by weight and has a density of 4 g/cm^2.* The stopping power of this material for muons was calculated using the work of Barkas and Berger,[4] Sternheimer,[5,6] and Hayman et al.[7] The stopping power as a function of energy is shown in Fig. 1. There is considerable variation with energy and it is not clear how to best approximate this stopping power by a constant. In this report, the value of 2.5 MeV g^{-1} cm^2 has been employed in all calculations that utilize Eq. II.7.

The radiation length for heavy concrete was calculated from the equation

$$\frac{1}{x_o} = \sum_i \frac{F_i}{x_{oi}} \ , \qquad \text{(II.10)}$$

where

F_i = the fraction by weight of the ith element,

x_{oi} = the radiation length of the ith element,

and x_o = 19.2 g/cm^2 was obtained.

*This composition was supplied by M. Awschalom of the National Accelerator Laboratory at Batavia, Illinois.

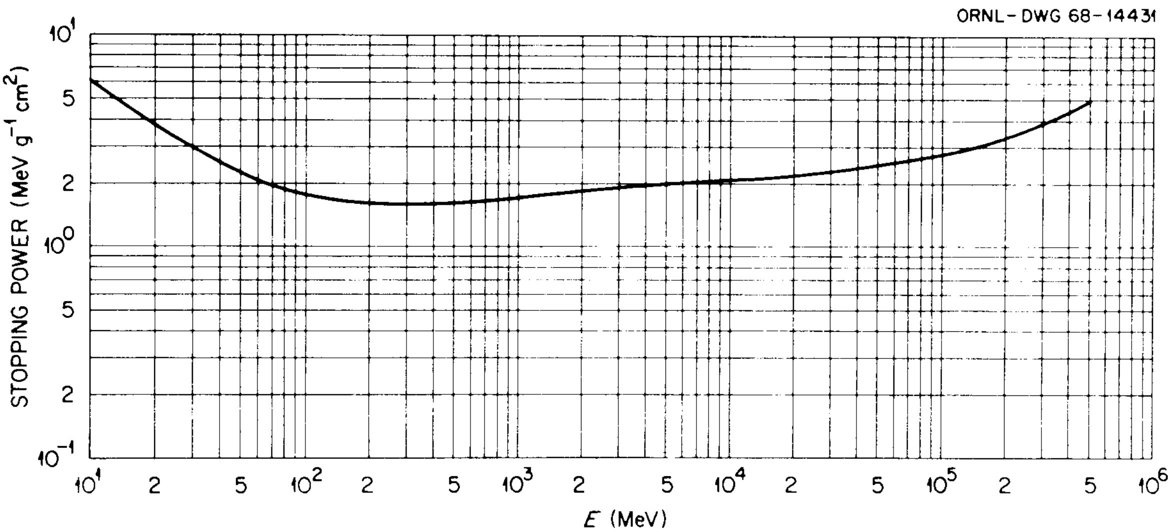

Fig. 1. Muon Stopping Power in Heavy Concrete vs Energy.

III. RESULTS AND DISCUSSION

Isocurrent-density contours have been calculated for the cases of 200- and 500-GeV protons using both a constant and a variable stopping power. The results are shown in Figs. 2 and 3.

In the 200-GeV case (Fig. 2), the contours are similar in shape to those given in paper 1, but the magnitude of the shield radius and the shield depth required is approximately a factor of two larger than in paper 1 because of the smaller density of heavy concrete.

The contours in Fig. 2 obtained with a constant stopping power of 2.5 MeV g^{-1} cm^2 underestimate the shield radius at small depths and overestimate the shield radius at large depths. This is easily understood on the basis of the stopping-power curve shown in Fig. 1. At relatively small depths and large radii muons with incident energy of less than 50 GeV are the major contributors to the current-density contours since the high-energy muons are not emitted from the source at large angles. For these particles the constant stopping power used is larger than the actual stopping power over a large portion of the energy range (see Fig. 1), and thus in the approximate calculation these muons do not travel as far as they do when the variable stopping power is used. On the other hand, at the very large depths and relatively small radii only the higher energy muons contribute to the current density, and for these muons the constant stopping power used is smaller than the actual stopping power over a portion of the energy range shown, so these particles travel farther than they would if the variable stopping power were used.

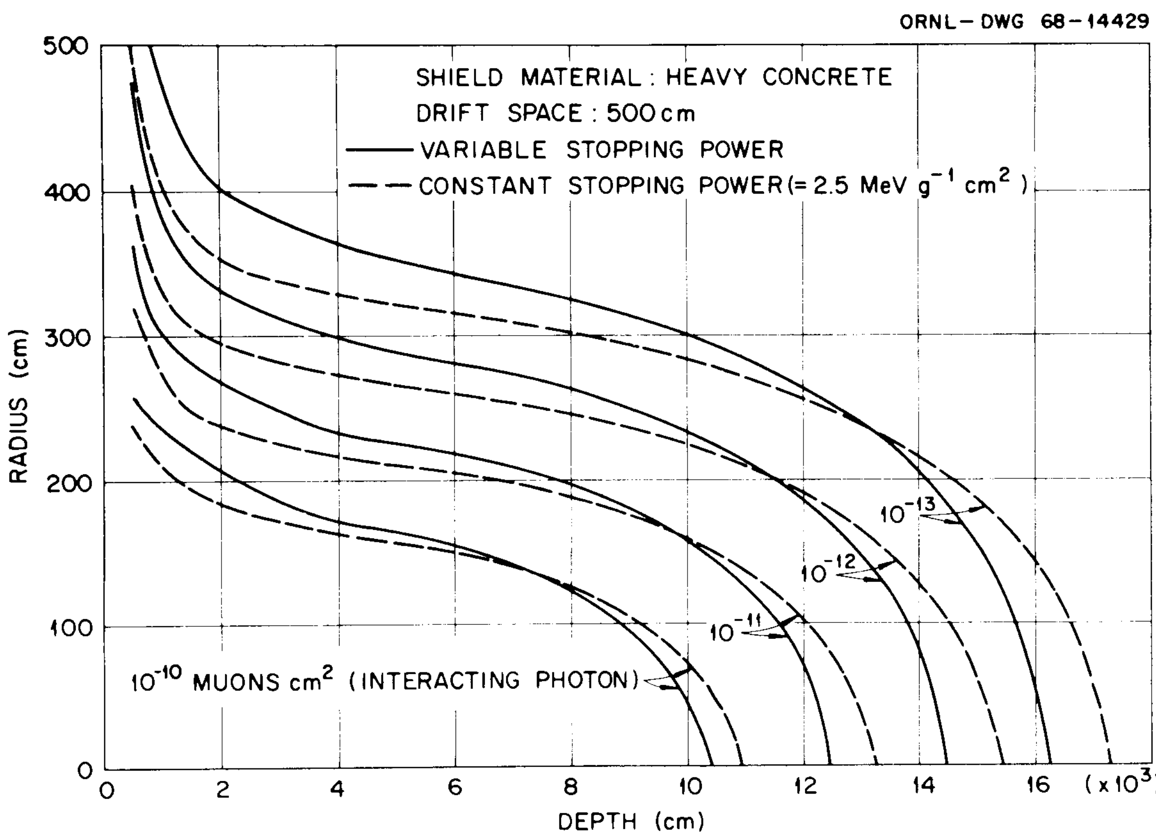

Fig. 2. Isocurrent Density Contours for 200-GeV Protons.

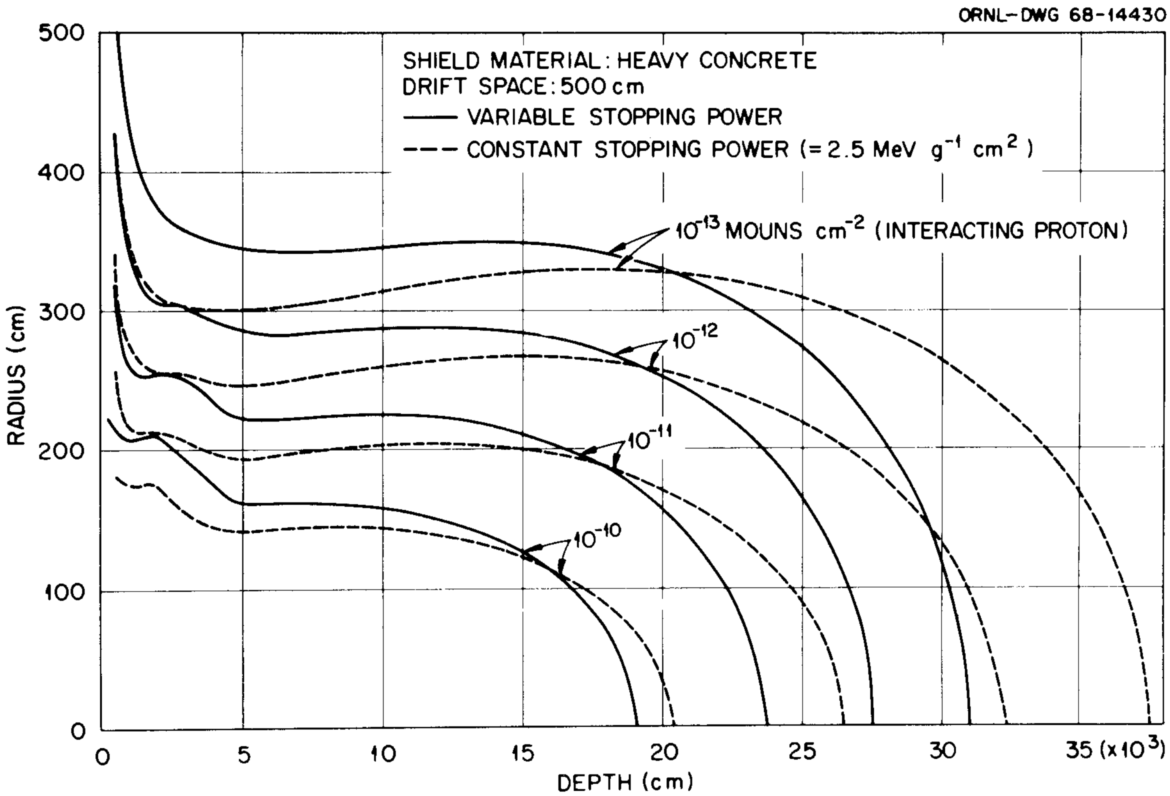

Fig. 3. Isocurrent Density Contours for 500-GeV Protons.

The isocurrent density contours for the 500-GeV proton case (Fig. 3) extend to much larger depths because of the larger range of the higher energy muons. The slight maxima and minima in some of the contours at small depths arise from the shape of the pion-production spectrum given by Trilling.[9] At small angles the Trilling production spectrum and consequently the calculated muon spectrum exhibit a maxima and a minima in the vicinity of 25 GeV (for example, see Fig. 2 of paper 1). The relation between the contours obtained with a variable stopping power and those obtained with a constant stopping power is similar to that obtained in the 200-GeV case, and the explanation given previously applies equally well to this case.

The error introduced by using a constant stopping power is clearly evident but it may be tolerable for some design purposes. In considering the results, it must be remembered that the constant value of the stopping power used was somewhat arbitrary. By choosing the constant value judiciously, it is probably possible to reduce over a limited portion of the contour the error caused by the approximation. For example, in the 500-GeV case (Fig. 3), a stopping power somewhat larger than 2.5 MeV g^{-1} cm^2 would surely reduce the error at large depths.

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