OAK RIDGE NATIONAL LABORATORY
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COMPUTER PROGRAMS FOR SOME EDDY CURRENT PROBLEMS

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ABSTRACT

This report contains computer programs for solving some eddy-current problems frequently encountered in nondestructive testing. Operating instructions and examples are given for each program. These programs are written in BASIC language and may be used on a time-sharing computer. Various eddy-current tests can be quickly and accurately designed by utilizing these programs.

INTRODUCTION

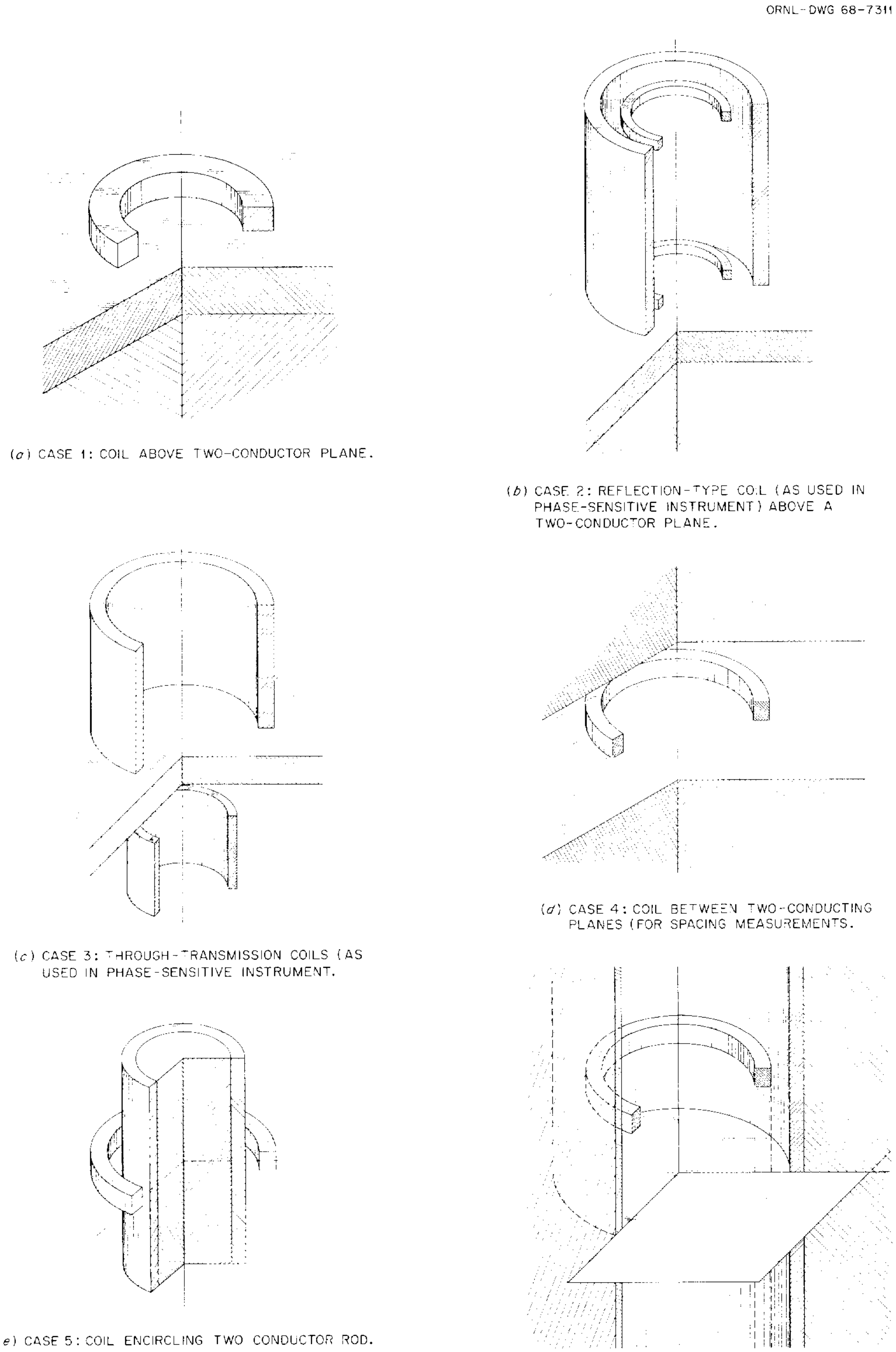
This report contains computer programs developed to calculate the solutions to the equations derived in a previous report,[2] which discusses the applicability of the equations to actual test problems. The programs are written in BASIC language, which is usually used with a time-sharing computer system. While these programs will operate, with a few modifications, on many different computer systems, they were written specifically for the Data Network,[3] SDS 940 BASIC. Instructions for running these programs and examples of their use are included with each program. Figure 1 illustrates those cases for which solutions have been obtained. Table 1 lists the various programs written for the specified geometrical configurations.

Despite the apparent repetition, each of these programs and its respective discussion has been made essentially self-contained, although in some instances more than one program must be run to obtain the

[1]Consultant from the University of Tennessee, Knoxville.

[2]C. V. Dodd, W. E. Deeds, J. W. Luquire, and W. G. Spoeri, Some Eddy-Current Problems and Their Integral Solutions, ORNL-4384 (April 1969).

[3]Data Network Corporation, 460 Twelfth Avenue, New York, N. Y. 10018.



(a) CASE 1: COIL ABOVE TWO-CONDUCTOR PLANE.

(b) CASE 2: REFLECTION-TYPE COIL (AS USED IN PHASE-SENSITIVE INSTRUMENT) ABOVE A TWO-CONDUCTOR PLANE.

(c) CASE 3: THROUGH-TRANSMISSION COILS (AS USED IN PHASE-SENSITIVE INSTRUMENT.

(d) CASE 4: COIL BETWEEN TWO-CONDUCTING PLANES (FOR SPACING MEASUREMENTS.

(e) CASE 5: COIL ENCIRCLING TWO CONDUCTOR ROD.

(f) CASE 6: COIL INSIDE TWO CONDUCTOR TUBE.

Fig. 1. Eddy-Current Testing Cases for Which Solutions Have Been Obtained in the Form of Integral Equations.

Table 1. Computer Programs for Solution of Equations for Eddy-Current Test Configurations

Configuration	Program Calculation	Program Name	Page
Coil in air	Coil inductance	AIRCO5	6
Coil above a two-conductor plane	Normalized coil impedance	CLADT5	10
	Defect sensitivity factor for a defect in the cladding material	DEFEC5	15
	Defect sensitivity factor for a lattice of defects in the cladding material	DELAT5	21
	Defect sensitivity factor for a defect in the base material	DEFECB	28
	Net eddy-current force exerted on a conductor	FORCE5	33
Reflection-type coil above a two-conductor plane	Magnitude and phase of induced voltage	RFLCT	37
	Phase shift of induced voltage with "lift-off" set	DISC	43
	Phase of induced voltage for a range of cladding thickness values with "lift-off" automatically set	RFLCYC	46
	Change in magnitude and phase of induced voltage due to a defect in the cladding material	RFDFT	53
	Change in magnitude and phase of induced voltage due to lattice of defects in the cladding material	RFDFL2	59
	Change in phase of induced voltage due to a lattice of defects in the cladding material with the "lift-off" set	BASREAD	67
	Change in magnitude and phase of induced voltage due to a defect in the base material	RFDFTB	73
Through-transmission coils	Magnitude and phase of the induced voltage	THRU5	80
	Change in magnitude and phase of induced voltage due to a defect in the plate between the coils	THRUDT	85

Table 1 (continued)

Configuration	Program Calculation	Program Name	Page
Coil between two conducting planes	Normalized coil impedance	BTNCO	91
Coil encircling a two-conductor rod	Normalized coil impedance	ENCCO5	96
	Defect sensitivity factor for a defect in the outer material	ENDFT5	105
	Defect sensitivity factor for a lattice of defects in the outer material	ENDFTL	114
	Defect sensitivity factor for a differential coil system for a lattice of defects	READIN	124
	Defect sensitivity factor for a defect in the inner material	ENDFB5	129
Coil inside a two-conductor tube	Normalized coil impedance	INNCO5	138
	Defect sensitivity factor for a defect in the inner material	INDFT5	147
	Defect sensitivity factor for a lattice of defects in the inner material	INDFTL	155
	Defect sensitivity factor for defect in the outer material	INDFB5	165

desired answer. For example, note that all the coil impedance values are given in terms of normalized impedances. The normalized values of impedances must be multiplied by the magnitude of the coil impedance in air, ωL, to obtain the actual coil impedance. The inductance, L, may be calculated from AIRCO5.

The dimensions used in these programs are also normalized with respect to the mean coil radius, $\bar{r}$, and therefore have no units; M is defined as $\bar{r}^2\omega\mu\sigma$ and is also dimensionless, since $2/\omega\mu\sigma$ is the square of the eddy-current skin depth in mks units. The value of M may also be calculated by the formula $M = 5.09397 \times 10^{-7} \left(\frac{\bar{r}^2 f}{\rho}\right)$, where $\bar{r}$ is now expressed in inches, f is the frequency in cycles per second (Hertz), and ρ is the resistivity in microhm-centimeters.

The discussion and example that precede each of the detailed programs presume that the specific program has already been entered into the active computer memory. The accuracy of these programs is discussed in a later section.

INDUCTANCE OF A COIL IN AIR

We here consider the calculation of the inductance in air of a coil of rectangular cross section, as shown in Fig. 2. This is the type of coil that is used in many eddy-current tests. The impedance such a coil will have in air is:

$$Z_{air} = j \cdot 2 \cdot \pi \cdot f \cdot L \ ,$$

where Z is the impedance in ohms, j is the square root of minus one, f is the frequency in Hertz, and L is the inductance in henries. The impedance the coil will have when placed on a metal is $Z = |Z_{air}| \cdot Z_n$,

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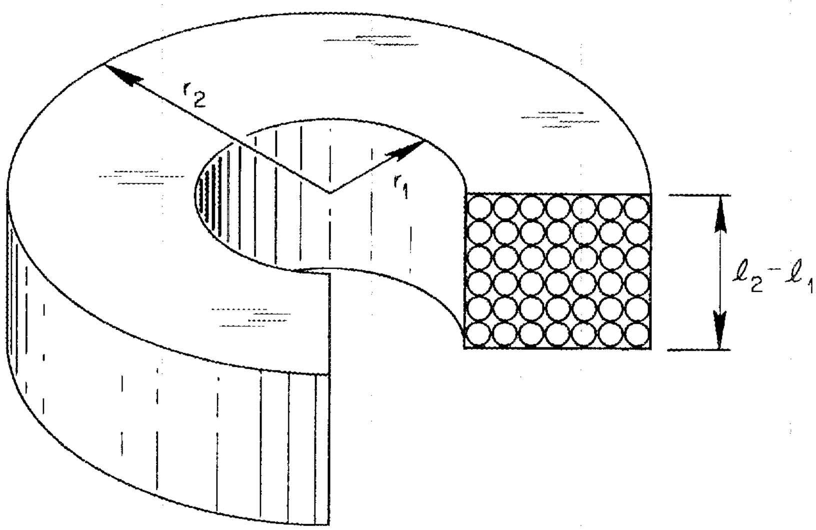

Fig. 2. A Rectangular Cross-Section Coil.

where Z_n is the normalized impedance, which is calculated from one of the impedance programs. In addition, the integral calculated in the air inductance program is also calculated (along with several other integrals) in all the impedance programs. Since this integral is one of the slowest to converge, it can be calculated separately for a coil in air and used in any of the impedance programs for cases with the same relative coil dimensions. The program AIRCO5 calculates the coil inductance and a "normalization factor" used in impedance calculations. The first normalization factor is used with planar conductors and the second is used with cylindrical conductors.

Discussion of AIRCO5

This program is designed to calculate the inductance in air of a coil with rectangular cross section. Such a coil is shown in Fig. 2. In addition, this program will calculate two normalization factors that are used in other programs appearing in this report. To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized."

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses should not be typed.)]

```
20 LET R1 = (numerical value of normalized inner coil radius)
30 LET R2 = (numerical value of normalized outer coil radius)
40 LET L = (numerical value of coil length).
```

The program may now be run.

The print-out by the computer will have the following format:

```
COIL LENGTH IS (L)              R1=(R1)           R2=(R2)
          INDUCTANCE/N↑2*R(BAR) = ------

                    NORMALIZATION FACTOR
PLANAR CONDUCTOR ------      CYLINDRICAL CONDUCTOR ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The actual inductance of the coil is obtained by multiplying the second line by the square of the number of turns and the mean radius of the coil. The remaining print-out is self-explanatory.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of AIRCO5

Suppose that we wish to know the inductance of a coil 0.250 in. long with inner and outer radii of 0.125 and 0.375 in., respectively. We shall assume that the coil consists of 200 turns of wire.

First, we determine the mean coil radius to be 0.250 in., so that

Normalized inner coil radius = 0.125/0.250 = 0.5

Normalized outer coil radius = 0.375/0.250 = 1.5

Normalized coil length = 0.250/0.250 = 1

This information is now typed into the computer in the following format:

```
20 LET R1 = 0.5
30 LET R2 = 1.5
40 LET L = 1
```

The program is now run with the following results:

```
COIL LENGTH IS 1                R1=0.5          R2=1.5
          INDUCTANCE/N↑2*R(BAR) = 0.1266473E-05

                    NORMALIZATION FACTOR
PLANAR CONDUCTOR 0.16040079   CYLINDRICAL CONDUCTOR 0.50391397
```

From this we obtain the inductance in henries simply by multiplying the second line by the square of the number of turns on the coil and the mean radius of the coil in meters.

For our problem, N = 200 turns of wire and the mean coil radius is 0.250 in. To determine the inductance in mks units, we must convert from inches to meters by the conversion factor 1 m = 39.37 in. We then have

$$L = (200)^2 \times 0.250 \text{ in.} \left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right) \times 1.266473 \times 10^{-6} \text{ h}$$

$$= 3.216848 \times 10^{-4} \text{ h} .$$

Thus, the inductance of our coil is 3.216848×10^{-4} h or about 322 μh.

AIRC05 Program

```
10 REM          AIRC05        VERSIØN    2/17/69
20 LET R1=1-.001
30 LET R2=1+.001
40 LET L=.2E-2
50 PRINT"CØIL LENGTH IS";L,"R1=";R1,"R2=";R2
60 LETS1=1E-2
70 LETS2=1
80 LETE1=.1
90 LET I6=0
100 LET B1=0
110 LET B2 =S2
120 LETI9=I6
130 FØRX=B1+S1/2 TØ B2 STEP S1
140 LET Z=R2*X
150 LET G1=R2
160 GØSUB 380
170 LET I2=F2
180 LET Z=R1*X
190 LET G1=R1
200 GØSUB 380
210 LET I1=F2
220 IF X*L>.005 THEN 250
230 LET A1=.5*X*L↑2-X↑2*L↑3/6
240 GØ TØ 260
250 LET A1=L+(EXP(-X*L)-1)/X
260 LET I6=I6+S1*(I2-I1)↑2*A1/X
270 NEXT X
280 IF(I6-I9)/I6>E1 THEN 600
290 READS1,S2,E1
300 IF E1>0THEN600
310 DATA2E-2,2,1E-2,5E-2,5,1E-3,1E-1,10,1E-4,.5,50,1E-5,2,200,1E-6,1,1,+
320 LET G6=8E-7*PI↑2*I6/(L*(R2-R1))↑2
330 PRINT"            INDUCTANCE/N↑2*R(BAR)=";G6
340 PRINT
350 PRINT" ","          NØRMALIZATIØN FACTØR"
360 PRINT"PLANAR CØNDUCTØR";I6;"CYLINDRICAL CØNDUCTØR";PI*I6
370 GØ TØ 630
380 IFZ>5THEN 470
390 LETL5=INT(2*Z)+3
400 LETF1=.5*G1*G1*Z
410 LET F2=F1/3
420 FØRN=1TØL5
430 LETF1=-F1*.250*Z*Z/(N*N+N)
```

```
440 LETF2=F2+F1/(2*N+3)
450 NEXTN
460 GØTØ 590
470 IF Z>30 THEN 540
480 LET Q1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
490 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
500 LET Q2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
510 LET Q2=(Q2-.187344E-2)/Z+.7979095
520 LET F2=(1-SQR(Z)*(Q2*CØS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X)
530 GØ TØ 590
540 LET F3=1/(Z*Z)
550 LETF1=Z*(-1+F3*(-.5546875+2.48062114*F3))
560 LETF2=.875+F3*(-.93457031+8.98975114*F3)
570 LETF2=1+.79788456*(F1*CØS(Z-.78539816)+F2*SIN(Z-.78539816))/SQR(Z)
580 LETF2=F2/(X*X)
590 RETURN
600 LETB1=B2
610 LET B2=B2+S2
620 GØ TØ 120
630 END
```

COIL ABOVE A TWO-CONDUCTOR PLANE

The case of a single coil above one conductor clad on another, as shown in Fig. 3, is typical of many eddy-current tests. Many impedance-bridge eddy-current instruments use this type of coil. In addition, this general case will reduce to a number of simpler cases. The programs in this section will calculate the normalized coil impedance (CLADT5), the sensitivity of a single defect in the cladding material (DEFEC5), the sensitivity of a defect at any point on a lattice in the cladding material (DELAT5), the sensitivity factor for a defect in the base material (DEFECB), and the net eddy-current force exerted on the conductor (FORCE5).

The impedance program can be used to study the effect on normalized coil impedance of variations in the thickness of the cladding and the conductivity and permeability of the cladding and base materials. In addition, we may take the conductivity of the base material to be zero and obtain the case of a coil above a plate.

The defect programs can be used to study the change in normalized coil impedance due to the presence of a defect. We shall call any discontinuity in the electromagnetic properties of the material a defect.

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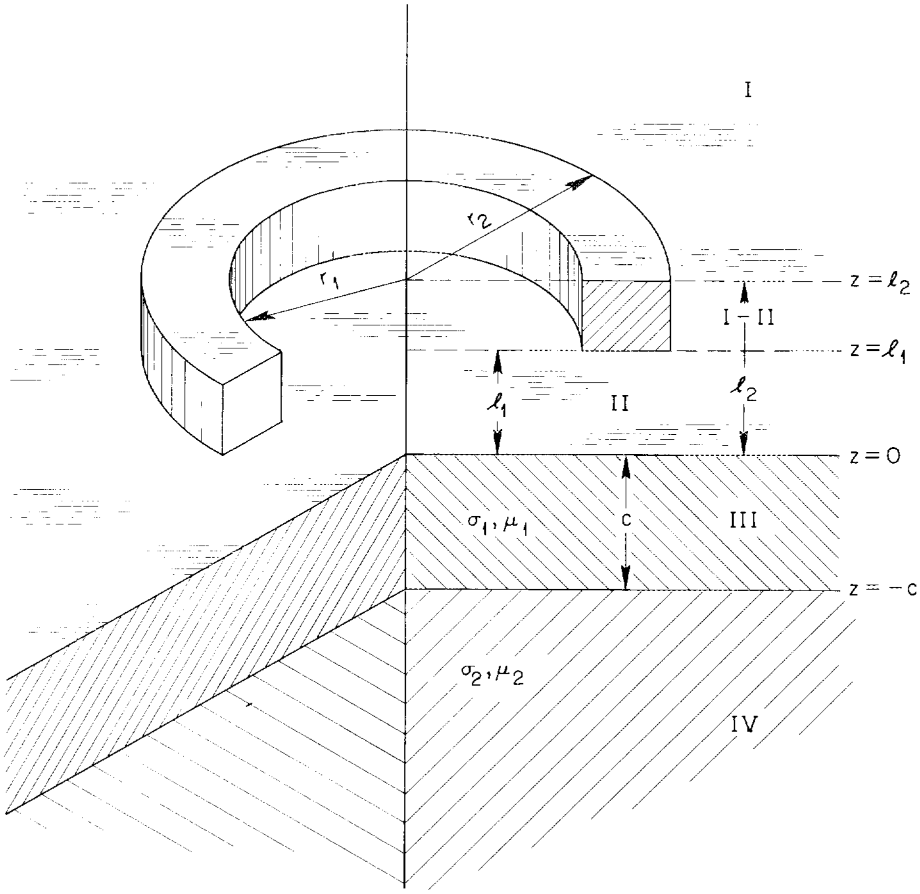

Fig. 3. A Coil of Rectangular Cross Section above a Two-Conductor Plane.

In addition, the lattice program can be used to study the effects of a defect located at any point in a lattice. From these programs, we can optimize the effectiveness of a coil in detecting defects.

Normalized Coil Impedance

Discussion of CLADT5

This program is designed to calculate the normalized impedance of a coil with rectangular cross section above a conductor consisting of two layers, which may be of different types of material (see Fig. 3).

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower layer.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

20 LET R1 = (numerical value of normalized inner coil radius)

30 LET R2 = (numerical value of normalized outer coil radius)

40 LET L1 = (numerical value of normalized lift-off)

50 LET L2 = (numerical value of lift-off plus normalized coil length)

60 LET M1 = (numerical value of $\omega\mu_1\sigma_1\bar{r}^2$)

Note: If the conductivity of the upper conductor is zero, one must type two additional lines:

320 LET X1 = X

330 LET Y1 = 0

70 LET U1 = (numerical value of relative permeability of upper conductor)

80 LET M2 = (numerical value of $\omega\mu_2\sigma_2\bar{r}^2$)

Note: If the conductivity of the lower conductor is zero, one must type two additional lines:

350 LET X2 = X

360 LET Y2 = 0

90 LET U2 = (numerical value of relative permeability of lower conductor)

100 LET C = (numerical value of normalized thickness of upper conductor)

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)           R2 = (R2)           L1 = (L1)    L2 = (L2)
CLAD = (C),  M1 = (M1),  U1 = (U1),  M2 = (M2),  U2 = (U2)
X                   AIR VALUE           REAL PART         IMAG PART
5.                  ------              ------            ------
10                  ------              ------            ------
15                  ------              ------            ------
20                  ------              ------            ------
40                  ------              ------            ------
NORMALIZED IMAG PART  ------  NORMALIZED REAL PART  ------
```

The various symbols enclosed in parentheses are used to indicate that the numerical value of the symbol will be printed. The four columns, headed "X," "AIR VALUE," "REAL PART," and "IMAG PART," are the upper limit of the numerical integration being performed by the computer, the value of the integral in the absence of the conductors, the real part of the integral in the presence of the conductors, and the imaginary part of the integral in the presence of the conductors, respectively. These appear principally to allow one to inspect the convergence of the integration. The last line is the normalized impedance of the coil in the presence of the conductors.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of CLADT5

Suppose that we wish to know the normalized impedance of a coil 0.250 in. long and with inner and outer radii of 0.125 and 0.375 in., respectively, due to the presence of 0.020 in. of copper clad on an aluminum conductor of effectively infinite thickness (at least four-coil diameters or skin depths, whichever is smaller). We shall assume the driving frequency to be 5 kHz and the lift-off to be 0.005 in.

First, we determine the mean coil radius to be 0.250 in., so that

Normalized radius of inner coil = 0.125/0.250 = 0.5
Normalized radius of outer coil = 0.375/0.250 = 1.5
Normalized lift-off = 0.005/0.250 = 0.02
Normalized coil length = 0.250/0.250 = 1
Normalized thickness of copper = 0.020/0.250 = 0.08

Furthermore,

Angular frequency of driving current = $2\pi \times 5000 \text{ sec}^{-1}$

Permeability of both conductors = $4\pi \times 10^{-7}$ h/m

Relative permeability of both conductors = 1

Conductivity of copper = 5.77×10^7 mhos/m

Conductivity of aluminum = 2.86×10^7 mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

This information is now typed into the program; that is, one types

```
20 LET R1 = 0.5
30 LET R2 = 1.5
40 LET L1 = 0.02
50 LET L2 = 1.02
60 LET M1 = 91.91
70 LET U1 = 1
80 LET M2 = 45.48
90 LET U2 = 1
100 LET C = 0.08
```

The program is now run with the following results:

```
R1 = 0.5          R2 = 1.5            L1 = 0.2E-01     L2 = 1.02
CLAD = 0.08   ,M1 = 91.91      ,U1 = 1    ,M2 = 45.48      ,U2 = 1
X                  AIR VALUE           REAL PART        IMAG PART
 5                 0.1592323           0.107332       -0.118106E-01
 10                0.1602305           0.1083125      -0.118325E-01
 15                0.1603371           0.108419       -0.11833E-01
 20                0.1603754           0.1084573      -0.11833E-01
 40                0.1604008           0.1084826      -0.11833E-01
NORMALIZED IMAG PART 0.6763225   NORMALIZED REAL PART 0.0737719
```

From this, we see that the normalized impedance of the coil is

$$Z_n = 0.0737719 + j\ 0.6763225\ .$$

CLADT5 Program

```
10 REM            CLADT               VERSIØN           07/08/68
20 LET K1=.8333
30 LET K2=1.1667
40 LET L1=.0476
50 LET L2=.3809
60 LET M1=77.05
70 LET U1=1
80 LET M2=40
90 LET U2=1
100 LET C=.05
110 PRINT "K1=";K1,"K2=";K2,"L1=";L1,"L2=";L2
120PRINT"CLAD=";C;",";"M1=";M1;",";"U1=";U1;",";"M2=";M2;",";"U2=";U2
130 PRINT "X", "AIR VALUE", "REAL PART", "IMAG PART"
140 LET S1=1E-2
150 LET S2=5
160 LET I6=0
170 LET I7=0
180 LET I8=0
190 LET I9 =0
200 LET B1=0
210 LET B2 =S2
220 FØR X = B1 +S1/2 TØ B2 STEP S1
230 LET Z=K2*X
240 LET Q1=K2
250 GØSUB 780
260 LET I2=F2
270 LET Z=K1*X
280 LET Q1=K1
290 GØSUB 780
300 LET I1=F2
310 LET S3=S1*(I2-I1)↑2/X
320 LET X1=.707107*(SQR(SQR(X*X*X*X+M1*M1)+X*X))/U1
330 LET Y1=.707107*(SQR(SQR(X*X*X*X+M1*M1)-X*X))/U1
340 IF X1*C>15 THEN 490
350 LET X2=.707107*(SQR(SQR(X*X*X*X+M2*M2)+X*X))/U2
360 LET Y2=.707107*(SQR(SQR(X*X*X*X+M2*M2)-X*X))/U2
370 LET X3=EXP(2*X1*C*U1)
380 LET Y3=CØS(2*Y1*C*U1)
390 LET Y4=SIN(2*Y1*C*U1)
400 LET A6=(X-X1)*(X1+X2)+Y1*(Y1+Y2)
410 LET A7=(X-X1)*(Y1+Y2)-Y1*(X1+X2)
420 LETA5=(X+X1)*(X1-X2)-Y1*(Y1-Y2)+(A6*Y3-A7*Y4)*X3
430 LET B5=Y1*(X1-X2)+(X+X1)*(Y1-Y2)+(A7*Y3+A6*Y4)*X3
440 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
450 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
460 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
470 LET D5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
480 GØ TØ 530
490 LET A5 =X-X1
500 LET B5=-Y1
510 LET C5=X+X1
520 LET D5=Y1
530 LET K1=(A5*C5+B5*D5)/(C5*C5+D5*D5)
540 LET K2=(C5*B5-A5*D5)/(C5*C5+D5*D5)
550 IF X*L2>30 THEN 590
```

```
560 LET G=EXP(-2*X*L1)+EXP(-2*X*L2)-2*EXP(-X*(L1+L2))
570 LETG3=EXP(-X*(L2-L1))-1
580 GØ TØ 610
590 LET G=EXP(-X*L1)
600 LET G3=-1
610 LET I6=I6+S3*G*K1/(2*X)
620 LET I8=I8+S3*G*K2/(2*X)
630 LET I9=I9+S3*(G3/X+L2-L1)
640 NEXT X
650 LETB1=B2
660 LET B2=B2+S2
670 LET Q3=X+S1/2
680 LET I7=I9+I6
690 PRINT Q3,I9,I7,I8
700 IF X < 3 THEN 220
710 LET S1=5E-2
720 IF X<10 THEN 220
730 LET S1=.1
740LETS2=20
750 IF X<35 THEN 220
760 PRINT "NØRMALIZED IMAG PART";I7/I9,"NØRMALIZED REAL PART";-I8/I9
770 GØTØ 930
780 IF Z>5 THEN870
790 LETL5=INT(2*Z)+3
800 LETF1=.5*Q1*Q1*Z
810 LET F2=F1/3
820 FØRN=1TØL5
830 LETF1=-F1*.250*Z*Z/(N*N+N)
840 LETF2=F2+F1/(2*N+3)
850 NEXTN
860 GØTØ 920
870 LET Q1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
880 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
890 LET Q2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
900 LET Q2=(Q2-.137344E-2)/Z+.7979095
910 LET F2=(1-SQR(Z)*(Q2*CØS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X)
920 RETURN
930 END
```

Defect in Cladding Material

Discussion of DEFEC5

This program is designed to calculate the sensitivity for a coil with rectangular cross section above a conductor consisting of two layers, which may be of different types of material. For this program, the defect must be located in the upper layer, which is labeled III in Fig. 3.

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses should not be typed.)]

```
20 LET R1 = (numerical value of normalized inner coil radius)
30 LET R2 = (numerical value of normalized outer coil radius)
40 LET L1 = (numerical value of normalized lift-off)
50 LET L2 = (numerical value of normalized lift-off plus normalized
             coil length)
60 LET M1 = (numerical value of ωμ1σ1r̄²)
70 LET M2 = (numerical value of ωμ2σ2r̄²
```

Note: If the conductivity of the lower conductor is zero, one must type two additional lines:

```
380 LET X2 = X
390 LET Y2 = 0
80 LET C = (numerical value of normalized thickness of upper
           conductor)
90 LET R = (numerical value of normalized R position coordinate
           of defect)
100 LET Z1 = (numerical value of normalized Z position coordinate
             of defect)
110 LET I9 = (numerical value of normalization factor)
```

Note: This value may be obtained with the aid of AIRCO5 simply by placing the appropriate coil dimensions in that program.

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)       R2 = (R2)       L1 = (L1)          L2 = (L2)
CLAD THICKNESS IS (C)           M1 = (M1)          M2 = (M2)
DEFECT IS LOCATED AT (R,Z) = (R), (Z1)
X               MAGNITUDE       PHASE
5               ------          ------
10              ------          ------
15              ------          ------
20              ------          ------
25              ------          ------
30              ------          ------
35              ------          ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The three columns, headed "X," "MAGNITUDE," and "PHASE" are the upper limit of the numerical integration being performed by the computer, the magnitude of the defect sensitivity factor, and the phase of the defect sensitivity factor, respectively. These appear principally to allow one to inspect the convergence of the integration. The magnitude and phase values for X = 35 are considered to be correct, since the integrations in most cases converge sufficiently well for this value of X.

The example below is presented for additional aid to those who wish to use this program.

<u>Sample Calculation of DEFEC5</u>

Suppose that we wish to know the defect sensitivity factor for a coil 0.250 in. long with inner and outer radii of 0.125 in. and 0.375 in., respectively, above 0.020 in. of copper clad on an aluminum conductor of effectively infinite thickness (at least four-coil diameters or skin depths, whichever is smaller). Let us further suppose that we are interested in the defect sensitivity factor for the case in which the defect is located 0.010 in. below the surface of the copper, 0.250 in. from the coil axis. We shall assume the driving frequency to be 5 kHz and the lift-off to be 0.005 in.

First, we determine the mean coil radius to be 0.250 in., so that

Normalized inner coil radius = 0.125/0.250 = 0.5

Normalized outer coil radius = 0.375/0.250 = 1.5

Normalized lift-off = 0.005/0.250 = 0.02

Normalized coil length = 0.250/0.250 = 1

Normalized thickness of copper = 0.020/0.250 = 0.08

Normalized R position of defect = 0.250/0.250 = 1

Normalized Z position of defect = 0.010/0.250 = 0.04

Furthermore,

Angular frequency of driving current = $2\pi \times 5000\ \text{sec}^{-1}$

Permeability of both conductors = $4\pi \times 10^{-7}$ h/m

Conductivity of copper = 5.77×10^{7} mhos/m

Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$

$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

At this point, one determines the normalization factor by placing the normalized coil dimensions in AIRCO5 and running that program. It is worth noting that the normalization factor is independent of all variables except the coil dimensions. This factor is the first normalization factor in the print-out of AIRCO5 and for the case in point is found to be 0.1604008. The remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
20 LET R1 = 0.5
30 LET R2 = 1.5
40 LET L1 = 0.02
50 LET L2 = 1.02
60 LET M1 = 91.91
70 LET M2 = 45.48
80 LET C = 0.08
90 LET R = 1
100 LET Z1 = -0.04
110 LET I9 = 0.1604008
```

The program is now run with the following results:

```
R1 = 0.5          R2 = 1.5      L1 = 0.2E-01      L2 = 1.02
CLAD THICKNESS IS 0.08          M1 = 91.91        M2 = 45.48
DEFECT IS LOCATED AT (R,Z) = 1        ,-0.4E-01
X                 MAGNITUDE          PHASE
 4.9999999         0.14757756        -1.9638444
 9.9999999         0.14903352        -1.9711088
 15                0.14840038        -1.9742207
 20                0.14882443        -1.9713002
 25                0.14864662        -1.9726833
 30                0.14869698        -1.9722641
 35                0.14868914        -1.9723336
```

From this, we see that the defect sensitivity factor is 0.14868914 exp (-j 1.9723336). To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

DEFEC5 Program

```
10 REM            DEFEC5            VERSION            03/03/69
20 LET R1=.5
30 LET R2=1.5
40 LET L1=.02
50 LET L2=1.02
60 LET M1=91.91
70 LET M2=45.48
80 LET C=.08
90 LET R=1
100 LET Z1=-.04
110 LET I9=.1604008
120 PRINT "R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
130 PRINT "CLAD THICKNESS IS ";C,"M1=";M1,"M2=";M2
140 PRINT "DEFECT IS LOCATED AT (R,Z)=";R;",";Z1
150 PRINT "X", "MAGNITUDE" , "PHASE"
160 LET S1=1E-2
170 LET S2=5
180 LET I6=0
190 LET I8=0
200 LET B1=0
210 LET B2=S2
220 FOR X=B1 +S1/2 TO B2 STEP S1
230 LET Z=R2*X
240 LET G1=R2
250 GOSUB 780
260 LET I2=F2
270 LET Z=R1*X
```

```
280 LET Q1=K1
290 GØSUB 780
300 LET I1=F2
310 LET I3=I2-I1
320 LET Z=X*R
330 GØSUB 930
340 LETS3=(I2-I1)*J1*S1*(EXP(-X*L1)-EXP(-X*L2))
350 LET X1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
360 LET Y1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
370 IFX1*C>15THEN560
380 LET X2=.707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
390 LET Y2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
400 LET X3=EXP(2*X1*C)
410 LET Y3=CØS(2*Y1*C)
420 LET Y4=SIN(2*Y1*C)
430 LET X5=X1+X2
440 LET X6=X1-X2
450 LET Y5=Y1+Y2
460 LET Y6=Y1-Y2
470 LETA5=(X5*CØS(Y1*(2*C+Z1))-Y5*SIN(Y1*(2*C+Z1)))*EXP(X1*(2*C+Z1))
480 LET A5=A5+(X6*CØS(-Y1*Z1)-Y6*SIN(-Y1*Z1))*EXP(-X1*Z1)
490 LETB5=(Y5*CØS(Y1*(2*C+Z1))+X5*SIN(Y1*(2*C+Z1)))*EXP(X1*(2*C+Z1))
500 LETB5=B5+(Y6*CØS(Y1*Z1)+X6*SIN(Y1*Z1))*EXP(-X1*Z1)
510 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
520 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
530 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
540 LET D5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
550 GØTØ600
560 LET A5=EXP(X1*Z1)*CØS(Y1*Z1)
570 LET B5=EXP(X1*Z1)*SIN(Y1*Z1)
580 LET C5=X+X1
590 LET D5=Y1
600 LETK1=(A5*C5+B5*D5)/(C5*C5+D5*D5)
610 LETK2=(C5*B5-A5*D5)/(C5*C5+D5*D5)
620 LETI6=I6+S3*K1
630 LETI8=I8+S3*K2
640 NEXT X
650 LET B1=B1+S2
660 LET B2=B2+S2
670 LET Q3=X+S1/2
680 LETQ2=ATN(I8/I6)
690 IFI6>0THEN710
700 LET Q2=Q2+3.1415927
710 PRINTQ3,3*M1*(I6*I6+I8*I8)/(4*PI*I9),2*Q2
720 IF X < 3 THEN 220
730 LET S1=5E-2
740 IF X<10 THEN 220
750 LETS1=.1
760 IFX<30THEN220
770 GØTØ 1040
780 IFZ>5THEN 870
790 LETL5=INT(2*Z)+3
800 LETF1=.5*Q1*Q1*Z
810 LET F2=F1/3
820 FØRN=1TØL5
830 LETF1=-F1*.250*Z*Z/(N*N+N)
840 LETF2=F2+F1/(2*N+3)
850 NEXTN
860 GØTØ 920
870 LET Q1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
880 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
```

```
890 LET Q2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
900 LET Q2=(Q2-.187344E-2)/Z+.7979095
910 LET F2=(1-SQR(Z)*(Q2*COS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X)
920 RETURN
930 IFZ>3THEN 980
940 LET Q1=(((2.1E-11*Z↑2-5.38E-9)*Z↑2+6.757E-7)*Z↑2-5.42443E-5)*Z↑2
950 LET Q1=((Q1+2.60415E-3)*Z↑2-6.25E-2)*Z↑2+.5
960 LET J1=Z*Q1
970 GOTO 1030
980 LET Q3=(((-.14604057/Z+.27617679)/Z-.2021039 1)/Z+4.61835E-3)/Z
990 LET Q3=((Q3+.14937)/Z+4.68E-6)/Z+.79788456
1000 LET Q4=(((-.21262014/Z+.1939 7232)/Z+6.022188E-2)/Z-1.7222733E-1)/Z
1010 LET Q4=((Q4+5.085E-4)/Z+.37498836)/Z-2.35619449+Z
1020 LET J1=Q3*COS(Q4)/SQR(Z)
1030 RETURN
1040 END
```

Lattice of Defects in Cladding Material

Discussion of DELAT5

This program is designed to calculate the defect sensitivity factor for a coil with rectangular cross section above a conductor consisting of two layers, which may be of different types of material. For this program, the defect may be located at any one of a number of points on a lattice in the upper layer, which is labeled III in Fig. 3, p. 10.

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses should not be typed.)]

12 LET R1 = (numerical value of normalized inner coil radius)
14 LET R2 = (numerical value of normalized outer coil radius)
16 LET L1 = (numerical value of normalized lift-off)
18 LET L2 = (numerical value of normalized lift-off plus normalized coil length)
20 LET M1 = (numerical value of $\omega\mu_1\sigma_1\bar{r}^2$)
22 LET M2 = (numerical value of $\omega\mu_2\sigma_2\bar{r}^2$)

Note: If the conductivity of the lower conductor is zero, one must type two additional lines:

```
86 LET X2 = X
88 LET Y2 = 0
24 LET C = (numerical value of normalized thickness of upper
           conductor)
26 LET I9 = (normalization factor)
```

Note: This value may be obtained with the aid of AIRCO5 simply by placing the appropriate coil dimensions in that program.

```
28 LET M9 = (numerical value of number of lattice spacings in
            R direction)
30 LET N9 = (numerical value of number of lattice spacings in
            Z direction)
32 LET R9 = (numerical value of maximum normalized radial position
            of defect)
```

Note: Neither M9 nor N9 may exceed 15. The lattice spacing in the R direction is R9/M9, while it is C/N9 in the Z direction.

The program may now be run.

The print-out by the computer will have the following format:

```
R1=(R1)           R2=(R2)       L1=(L1)        L2=(L2)
CLAD THICKNESS    IS (C)        M1=(M1)        M2=(M2)
Z,R               (R(1))        (R(2))        **********     (R(M9))
(Z(1))            ------        ------        **********     ------
                  ------        ------        **********     ------
(Z(2))            ------        ------        **********     ------
                  ------        ------        **********     ------
  *                 *             *               *             *
  *                 *             *               *             *
  *                 *             *               *             *
  *                 *             *               *             *
(Z(N9))           ------        ------        **********     ------
                  ------        ------        **********     ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The upper value at each lattice point is the magnitude of the defect sensitivity factor, while the lower one is the phase of the defect sensitivity factor for the case in which the defect is located at the given lattice point. It should be noted that the lattice points for which

$M9 > 4$ will be printed below the last line — that is, the line containing Z(N9).

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of DELAT5

Suppose that we wish to know the defect sensitivity factor for a coil 0.250 in. long with inner and outer radii of 0.125 and 0.375 in., respectively, above 0.020 in. of copper clad on an aluminum conductor of effectively infinite thickness (at least four-coil diameters or skin depths, whichever is smaller). Let us further suppose that we are interested in the defect sensitivity factor for the cases in which the defect is located at any one of the points which lie within 0.500 in. of the coil axis on a 4×2 lattice in the copper conductor. We shall assume the driving frequency to be 5 kHz and the lift-off to be 0.005 in.

First, we determine the mean coil radius to be 0.250 in. so that

Normalized inner coil radius = 0.125/0.250 = 0.5
Normalized outer coil radius = 0.375/0.250 = 1.5
Normalized lift-off = 0.005/0.250 = 0.02
Normalized coil length = 0.250/0.250 = 1
Normalized thickness of copper = 0.020/0.250 = 0.08
Maximum radial position of defect = 0.500/0.250 = 2
Number of R lattice spacings = 4
Number of Z lattice spacings = 2

Furthermore,

Angular frequency of driving current = $2\pi \times 5000$ sec^{-1}
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

At this point one determines the normalization factor by placing the normalized coil dimensions in AIRCO5 and running that program. It is worth noting that the normalization factor is independent of all variables except for the coil dimensions. This factor is the first normalization factor in the print-out of AIRCO5 and for the case in point is found to be 0.1604008. The remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.5
14 LET R2 = 1.5
16 LET L1 = 0.02
18 LET L2 = 1.02
20 LET M1 = 91.91
22 LET M2 = 45.48
24 LET C = 0.08
26 LET I9 = 0.1604008
28 LET M9 = 4
30 LET N9 = 2
32 LET R9 = 2
```

The program is now run with the following results:

R1 = 0.5	R2 = 1.5	L1 = 0.2E-01	L2 = 1.02	
CLAD = 0.08	M1 = 91.91	M2 = 45.48		
Z,R	0.25	0.75	1.25	1.75
-0.2E-01	0.1584851E-01	0.16428266	0.1572279	0.2153014E-01
	-1.9775757	-1.7060864	-1.707901	-2.0792097
-0.6E-01	0.0985171E-01	0.09716863	0.0927353	0.132156E-01
	-2.5026601	-2.2132172	-2.2138198	-2.6091414

From this we see that for the case in which the defect is located at R = 1.25, Z = -0.06 the defect sensitivity factor is 0.0925047 exp (-j2.21414).

To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

DELAT5 Program

```
10 REM              DELAT5          VERSIØN             02/24/69
12 LET R1=.5
14 LET R2=1.5
16 LET L1=.02
18 LET L2=1.02
20 LET M1=91.91
22 LET M2=45.48
24 LET C=.08
26 LET I9=.1604008
28 LET M9=4
30 LET N9=2
32 LET R9=2
34 DIM R(18,15), I(18,15), F(18,15)
35 ØPEN /Ø1/, ØUTPUT
36 PRINT FILE R1,R2,L1,L2
38 PRINT FILE C,M1,M2,I9
39 PRINT FILE M9,N9,R9
40 LET S1=1E-2
42 LET S2=5
44 FØRJ=1TØN9
46 FØRI=1TØM9
48 LETR(I,J)=0
50 LETI(I,J)=0
52 NEXTI
54 NEXTJ
56 LET B1=0
58 LET B2 =S2
60 FØRX=B1+S1/2TØB2STEPS1
62 LET Z=R2*X
64 LET Q1=R2
66 GØSUB 246
68 LETI2=F2
70 LET Z=R1*X
72 LET Q1=R1
74 GØSUB246
76 LET I1=F2
78 LET S3=(I2-I1)*S1*(EXP(-X*L1)-EXP(-X*L2))
80 LET X1 = .707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
82 LET Y1 = .707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
84 IFX1*C>15THEN114
86 LET X2 = .707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
88 LET Y2 =.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
90 LET X3=EXP(2*X1*C)
92 LET Y3=CØS(2*Y1*C)
94 LET Y4=SIN(2*Y1*C)
96 LET X5=X1+X2
98 LET X6=X1-X2
100 LET Y5=Y1+Y2
```

```
102 LET Y6=Y1-Y2
104 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
106 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
108 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
110 LETD5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
112 GØ TØ 118
114 LETC5=X+X1
116 LETD5=Y1
118 FØRJ=1TØN9
120 LETZ1=(.5-J)*C/N9
122 IFX1*C>15THEN136
124 LETZ2=2*C+Z1
126 LETA5=(X5*CØS(Y1*Z2)-Y5*SIN(Y1*Z2))*EXP(X1*Z2)
128 LETA5=A5+(X6*CØS(-Y1*Z1)-Y6*SIN(-Y1*Z1))*EXP(-X1*Z1)
130 LET B5=(Y5*CØS(Y1*Z2)+X5*SIN(Y1*Z2))*EXP(X1*Z2)
132 LETB5=B5+(Y6*CØS(Y1*Z1)+X6*SIN(Y1*Z1))*EXP(-X1*Z1)
134 GØ TØ 140
136 LETA5=EXP(X1*Z1)*CØS(Y1*Z1)
138 LETB5=EXP(X1*Z1)*SIN(Y1*Z1)
140 LETK1=(A5*C5+B5*D5)/(C5*C5+D5*D5)
142 LET K2=(C5*B5-A5*D5)/(C5*C5+D5*D5)
144 FØRI=1TØM9
146 LETR=(I-.5)*R9/M9
148 LETZ=X*R
150 GØSUB276
152 LETR(I,J)=R(I,J)+S3*K1*J1
154 LETI(I,J)=I(I,J)+S3*K2*J1
156 NEXT I
158 NEXTJ
160 NEXTX
162 LETB1=B1+S2
164 LETB2=B2+S2
166 LETS1=5E-2
168 IF X<15 THEN 60
170 LET S1=.1
172 IF X<35 THEN 60
174 FØRJ=1TØN9
176 FØRI=1TØM9
178 LET P(I,J)=2*ATN(I(I,J)/R(I,J))
180 IF R(I,J)>0 THEN 184
182 LETP(I,J)=P(I,J)+6.2831854
184 LET R(I,J)=3*M1*(R(I,J)*R(I,J)+I(I,J)*I(I,J))/(4*P1*L9)
186 NEXTI
188 NEXTJ
190 PRINT FILE"Z,R",.5*R9/M9,1.5*R9/M9,2.5*R9/M9,3.5*R9/M9
192 FØRJ=1TØN9
194 PRINT FILE (.5-J)*C/N9,R(1,J),R(2,J),R(3,J),R(4,J)
196 PRINT FILE" ",P(1,J),P(2,J),P(3,J),P(4,J)
198 PRINT FILE
200 NEXTJ
202 IFM9<4.5THEN238
204 PRINT FILE4.5*R9/M9,5.5*R9/M9,6.5*R9/M9,7.5*R9/M9,8.5*R9/M9
206 FØRJ=1TØN9
```

```
208 PRINT FILE R(5,J), R(6,J),R(7,J),R(8,J),R(9,J)
210 PRINT FILEP(5,J),P(6,J),P(7,J),P(8,J),P(9,J)
212 PRINT FILE
214 NEXTJ
216 IFM9<9.5THEN298
218 PRINT FILE9.5*R9/M9,10.5*R9/M9,11.5*R9/M9,12.5*R9/M9,13.5*R9/M9
220 FØRJ=1TØN9
222 PRINT FILE R(10,J),R(11,J),R(12,J),R(13,J),R(14,J)
224 PRINT FILE P(10,J),P(11,J),P(12,J),P(13,J),P(14,J)
226 PRINT FILE
228 NEXTJ
230 IFM9<14.5THEN298
232 PRINT FILE14.5*R9/M9,15.5*R9/M9,16.5*R9/M9,17.5*R9/M9,18.5*R9/M9
234 FØRJ=1TØN9
236 PRINT FILE R(15,J),R(16,J),R(17,J),R(18,J)
238 PRINT FILEP(15,J),P(16,J),P(17,J),P(18,J)
240 PRINT FILE
242 NEXTJ
244 GØTØ 298
246 IFZ>5THEN 264
248 LETL5=INT(2*Z)+3
250 LETF1=.5*Q1*Q1*Z
252 LET F2=F1/3
254 FØRN=1TØL5
256 LETF1=-F1*.250*Z*Z/(N*N+N)
258 LETF2=F2+F1/(2*N+3)
260 NEXTN
262 GØTØ 274
264 LET Q1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
266 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
268 LET Q2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
270 LET Q2=(Q2-.187344E-2)/Z+.7979095
272 LET F2=(1-SQR(Z)*(Q2*CØS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X)
274 RETURN
276 IFZ>3THEN 286
278 LET Q1=(((2.1E-11*Z↑2-5.38E-9)*Z↑2+6.757E-7)*Z↑2-5.42443E-5)*Z↑2
280 LET Q1=((Q1+2.60415E-3)*Z↑2-6.25E-2)*Z↑2+.5
282 LET J1=Z*Q1
284 GØTØ 296
286 LET Q3=(((-.14604057/Z+.27617679)/Z-.20210391)/Z+4.618353E-3)/Z
288 LET Q3=((Q3+.14937)/Z+4.68E-6)/Z+.79788456
290 LET Q4=(((-.21262014/Z+.19397232)/Z+6.022188E-2)/Z-1.7222733E-1)/Z
292 LET Q4=((Q4+5.085E-4)/Z+.37498836)/Z-2.35619449+Z
294 LET J1=Q3*CØS(Q4)/SQR(Z)
296 RETURN
298 END
```

Defect in Base Material

Discussion of DEFECB

This program is designed to calculate the defect sensitivity factor for a coil with rectangular cross section above a conductor consisting of two layers, which may be of different types of material. For this program, the defect must be located in the lower layer, which is labeled IV in Fig. 3, p. 10.

To use this program, one must divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses should not be typed.)]

```
20 LET R1 = (numerical value of normalized inner coil radius)
30 LET R2 = (numerical value of normalized outer coil radius)
40 LET L1 = (numerical value of normalized lift-off)
50 LET L2 = (numerical value of normalized lift-off plus
             normalized coil length)
60 LET M1 = (numerical value of ωμ1σ1r̄²)
```

Note: If the conductivity of the upper conductor is zero, one must type two additional lines:

```
350 LET X1 = X
360 LET Y1 = 0
```

```
70 LET M2 = (numerical value of ωμ2σ2r̄²)
80 LET C = (numerical value of normalized thickness of upper
           conductor)
90 LET R = (numerical value of normalized R position coordinate of
           defect)
100 LET Z1 = (numerical value of normalized Z position coordinate
             of defect)
110 LET I9 = (normalization factor)
```

Note: This value may be obtained with the aid of AIRCO5 simply by placing the appropriate coil dimensions in that program.

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)          R2 = (R2)     L1 = (L1)     L2 = (L2)
CLAD THICKNESS IS (C)            M1 = (M1)     M2 = (M2)
DEFECT POSITION IS               R = (R)       Z = (Z1)
X                  MAGNITUDE      PHASE
5                  ------        ------
10                 ------        ------
15                 ------        ------
20                 ------        ------
25                 ------        ------
30                 ------        ------
35                 ------        ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The three columns, headed "X," "MAGNITUDE," and "PHASE" are the upper limit of the numerical integration being performed by the computer, the magnitude of the defect sensitivity factor, and the phase of the defect sensitivity factor, respectively. These appear principally to allow one to inspect the convergence of the integration. The magnitude and phase values for X = 35 are the ones for which convergence can be considered complete to within 0.01%.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of DEFEC5

Suppose that we wish to know the defect sensitivity factor for a coil 0.250 in. long with inner and outer radii of 0.125 and 0.375 in., respectively, above 0.020 in. of copper clad on an aluminum conductor of effectively infinite thickness (at least four-coil diameters or skin depths, whichever is smaller). Let us further suppose that we are interested in the defect sensitivity factor for the case in which the defect is located 0.030 in. below the surface of the copper, 0.250 in. from the coil axis. We shall assume the driving frequency to be 5 kHz and the lift-off to be 0.005 in.

First, we determine the mean coil radius to be 0.250 in., so that

Normalized inner coil radius = 0.125/0.250 = 0.5

Normalized outer coil radius = 0.375/0.250 = 1.5

Normalized lift-off = 0.005/0.250 = 0.02

Normalized coil length = 0.250/0.250 = 1

Normalized thickness of copper = 0.020/0.250 = 0.08

Normalized R position of defect = 0.250/0.250 = 1

Normalized Z position of defect = 0.030/0.250 = 0.12

Furthermore,

Angular frequency of driving current = $2\pi \times 5000\ \text{sec}^{-1}$

Permeability of both conductors = $4\pi \times 10^{-7}$ h/m

Conductivity of copper = 5.77×10^{7} mhos/m

Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

At this point one determines the normalization factor by placing the normalized coil dimensions in AIRC05 and running that program. It is worth noting that the normalization factor is independent of all variables excluding the coil dimensions. This factor is the first normalization factor in the print-out of AIRC05 and for the case in point is found to be 0.1604008. The remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
20 LET R1 = 0.5
30 LET R2 = 1.5
40 LET L1 = 0.02
50 LET L2 = 1.02
60 LET M1 = 91.91
70 LET M2 = 45.48
80 LET C = 0.08
90 LET R = 1
100 LET Z1 = -0.12
110 LET I9 = 0.1604008
```

The program is now run with the following results:

```
R1 = 0.5          R2 = 1.5          L1 = 0.2E-01          L2 = 1.02
CLAD THICKNESS IS 0.08              M1 = 91.91            M2 = 45.48
DEFECT POSITION IS                  R = 1                 Z = -0.12
X                 MAGNITUDE         PHASE
 5                0.1452237         -2.7559514
 10               0.14763532        -2.7590948
 15               0.14728876        -2.761718
 20               0.14740717        -2.760311
 25               0.14737916        -2.7607304
 30               0.14738376        -2.7606505
 35               0.1473835         -2.7606562
```

From this we see that the defect sensitivity factor is 0.1471948 exp(−j2.76095). To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

DEFECB Program

```
10 REM          DEFECB          VERSIØN          03/05/69
20 LET R1=.5
30 LET R2=1.5
40 LET L1=.02
50 LET L2=1.02
60 LET M1=91.91
70 LET M2=45.48
80 LET C=.08
90 LETR=1
100 LET Z1=-.12
110 LET I9=.1604008
120 PRINT "R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
130 PRINT "CLAD THICKNESS IS";C,"M1=";M1,"M2=";M2
140 PRINT "DEFECT PØSITIØN IS","R=";R,"Z=";Z1
150 PRINT "X", "MAGNITUDE" , "PHASE"
160 LET S1=1E-2
170 LET S2=5
180 LET I6=0
190 LET I8=0
200 LET B1=0
210 LET B2 =S2
220 FØR X = B1 +S1/2 TØ B2 STEP S1
230 LET Z=R2*X
240 LET Q1=R2
250 GØSUB 680
260 LET I2=F2
270 LET Z=R1*X
280 LET Q1=R1
290 GØSUB 680
```

```
300 LET I1=F2
310 LET I3=I2-I1
320 LET Z=X*R
330 GØSUB 830
340 LETS3=(I2-I1)*J1*S1*(EXP(-X*L1)-EXP(-X*L2))
350 LET X1 = .707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
360 LET Y1 = .707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
370 LET X2 = .707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
380 LET Y2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
390 LET X3=EXP(2*X1*C)
400 LET Y3=CØS(2*Y1*C)
410 LET Y4=SIN(2*Y1*C)
420 LET X5=CØS((Y1+Y2)*C+Y2*Z1)
430 LET Y5=SIN((Y1+Y2)*C+Y2*Z1)
440 LET X6=2*EXP((X1+X2)*C+X2*Z1)
450 LET A5=X1*X5-Y1*Y5
460 LET B5=Y1*X5+X1*Y5
470 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
480 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
490 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
500 LET D5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
510 LETK1=X6*(A5*C5+B5*D5)/(C5*C5+D5*D5)
520 LETK2=X6*(C5*B5-A5*D5)/(C5*C5+D5*D5)
530 LETI6=I6+S3*K1
540 LETI8=I8+S3*K2
550 NEXT X
560 LET B1=B1+S2
570 LET B2=B2+S2
580 LETQ2=ATN(I8/I6)
590 IFI6>0THEN610
600 LET Q2=Q2+3.1415927
610 PRINT X+S1/2,M2*(I6*I6+I8*I8)/I9,2*Q2
620 IF X < 3 THEN 220
630 LET S1=5E-2
640 IF X<10 THEN 220
650 LETS1=.1
660 IFX<30THEN220
670 GØTØ 940
680 IFZ>5THEN 770
690 LETL5=INT(2*Z)+3
700 LETF1=.5*Q1*Q1*Z
710 LET F2=F1/3
720 FØRN=1TØL5
730 LETF1=-F1*.250*Z*Z/(N*N+N)
740 LETF2=F2+F1/(2*N+3)
750 NEXTN
760 GØTØ 820
770 LET Q1=(((-138.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
780 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
790 LET Q2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
800 LET Q2=(Q2-.187344E-2)/Z+.7979095
810 LET F2=(1-SQR(Z)*(Q2*CØS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X)
820 RETURN
830 IFZ>3THEN 880
840 LET Q1=(((2.1E-11*Z↑2-5.38E-9)*Z↑2+6.757E-7)*Z↑2-5.42443E-5)*Z↑2
850 LET Q1=((Q1+2.60415E-3)*Z↑2-6.25E-2)*Z↑2+.5
860 LET J1=Z*Q1
870 GØTØ 930
880 LET Q3=(((-.14604057/Z+.27617679)/Z-.20210391)/Z+4.61835E-3)/Z
```

```
890 LET Q3=((Q3+.14937)/Z+4.68E-6)/Z+.79788456
900 LET Q4=(((-.21262014/Z+.19397232)/Z+6.022188E-2)/Z-1.7222733E-1)/Z
910 LET Q4=((Q4+5.085E-4)/Z+.37498836)/Z-2.35619449+Z
920 LET J1=Q3*COS(Q4)/SQR(Z)
930 RETURN
940 END
```

Force on a Conductor

Discussion of FORCE5

This program is designed to calculate the force on a conducting plate of effectively infinite thickness due to the current flowing in a coil of rectangular cross section placed above it. The physical situation is depicted by Fig. 3, p. 10, assuming regions III and IV to be identical.

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for the conductor. Let this product be denoted by $\omega\mu\sigma\bar{r}^2$.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses should not be typed.)]

```
20 LET R1 = (numerical value of normalized inner coil radius)
30 LET R2 = (numerical value of normalized outer coil radius)
40 LET L = (numerical value of normalized lift-off)
50 LET L1 = (numerical value of normalized coil length)
60 LET M1 = (numerical value of ωμσr̄²)
```

The program may now be run.

The print-out by the computer will have the following format:

COIL LENGTH IS (L1)	R1 = (R1)	R2 = (R2)
LIFT-OFF IS (L)	M1 = (M1)	
X	FORCE/I↑2	
5	------	
10	------	
15	------	
20	------	

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the numerical integration being performed by the computer, while the second column is the force divided by the square of the ampere-turns of the coil. These appear principally to allow one to inspect the convergence of the integration. The force value for X = 20 is the one for which convergence can be considered to be complete.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of FORCE5

Suppose that we wish to know the force on a copper conductor of effectively infinite thickness (at least four-coil diameters or skin depths, whichever is smaller) due to the current in a 0.250-in.-long coil with inner and outer radii of 0.125 and 0.375 in., respectively. The driving frequency is assumed to be 50 kHz and the lift-off to be 0.005 in.

First, we determine the mean coil radius to be 0.250 in., so that

Normalized inner coil radius = 0.125/0.250 = 0.5
Normalized outer coil radius = 0.375/0.250 = 1.5
Normalized lift-off = 0.005/0.250 = 0.02
Normalized coil length = 0.250/0.250 = 1

Furthermore,

Angular frequency of driving current = $2\pi \times 50{,}000\ \text{sec}^{-1}$
Conductivity of copper = 5.77×10^{7} mhos/m
Permeability of copper = $4\pi \times 10^{-7}$ h/m

Thus,

$$\omega\mu\sigma\bar{r}^2 = 919.1.$$

The above information is now typed into the program as follows:

```
20 LET R1 = 0.5
30 LET R2 = 1.5
40 LET L = 0.02
50 LET L1 = 1
60 LET M1 = 919.1
```

The program is now run with the following results:

COIL LENGTH IS 1	R1 = 0.5	R2 = 1.5
LIFT-OFF IS 0.2E-01	M1 = 919.1	
X	FORCE/I↑2	
5	0.322771E-06	
10	0.324026E-06	
15	0.324096E-06	
20	0.324111E-06	

From this we see that the force per ampere-turn squared is 0.324111×10^{-6} N/(amp-turn)^2.

FORCE5 Program

```
10 REM          FØRCE5          VERSIØN          07/26/68
20 LET R1=.8333
30 LET R2=1.1667
40 LET L=.0952
50 LET L1=.3333
60 LET M1=862.8
70 PRINT"CØIL LENGTH IS";L1,"R1=";R1,"R2=";R2
80 PRINT"LIFTØFF IS";L,"M1=";M1
90 PRINT"X"," ","FØRCE/I↑2"
100 LET S1=1E-2
110 LET S2=5
120 LET I6=0
130 LET B1=0
140 LET B2 =S2
150 FØR X=B1+S1/2 TØ B2 STEP S1
160 LET Z=R2*X
170 LET Q1=R2
180 GØ SUB 390
190 LET I2=F2
200 LET Z=R1*X
210 LET Q1=R1
220 GØ SUB 390
230 LET I1=F2
240 LET I3=I2-I1
250 LET S3=S1*I3*I3
260 LET X1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
270 LET Y1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
280 LET K1=Y1/(X1*X*X+2*X*X1*X1+X1*X1*X1+Y1*Y1*X1)
290 LET I6=I6+S3*K1*X*(EXP(-X*L)-EXP(-X*(L+L1)))↑2
300 NEXT X
310 LET B1=B1+S2
320 LET B2=B2+S2
330 LET F1=1.97392E-6*I6*M1/(L1*(R2-R1))↑2
```

```
340 PRINTX+S1/2," ",F1
350 IF X<3THEN 150
360 LET S1=5E-2
370 IF X<15 THEN 150
380 GO TO 540
390 IFZ>5THEN 480
400 LETL5=INT(2*Z)+3
410 LETF1=.5*G1*G1*G1
420 LET F2=F1/3
430 FORN=1TOL5
440 LETF1=-F1*.250*Z*Z/(N*N+N)
450 LETF2=F2+F1/(2*N+3)
460 NEXTN
470 GOTO 530
480 LET G1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
490 LET G1=((G1-.1730503)/Z+.7034845)/Z-.064109E-3
500 LET G2=(((-5.817517/Z+2.105374)/Z-.6896196)/Z+.4952024)/Z
510 LET G2=(G2-.187344E-2)/Z+.7979095
520 LET F2=(1-SGN(Z)*(G2*COS(Z-PI/4)-G1*SIN(Z-PI/4)))/(X*X*X)
530 RETURN
540 END
```

REFLECTION-TYPE COIL ABOVE A TWO-CONDUCTOR PLANE

This type of coil system is used with the phase-sensitive eddy-current instrument[4,5] and consists of a large driver coil surrounding small pickup coils at each end, wound in opposition to each other.

The programs in this section calculate the magnitude and phase of the induced voltage (RFLCT), the phase shift of the induced voltage with "lift-off" set (DISC), the phase shift over a range of cladding thickness values with "lift-off" automatically set (RFLCYC), the change in the magnitude and phase of the induced voltage due to a defect in the cladding material (RFDFT), the change in the magnitude and phase of the induced voltage due to a defect located at any point in a lattice in the cladding material (RFDFL2), the change in phase of the induced voltage due to a defect located at any point in a lattice in the cladding material with the "lift-off" set (BASREAD), and the change in the magnitude and phase of the induced voltage due to a defect in the base material (RFDFTB).

---

[4] C. V. Dodd, Mater. Evaluation **22**(6), 260–263 (1964).

[5] C. V. Dodd, Mater. Evaluation **26**(3), 33–36 (1968).

These programs are useful in optimizing the coil dimensions and the operating frequency for reflection-type coils.

Magnitude and Phase of Induced Voltage

Discussion of RFLCT

This program is designed to calculate the differences in magnitude and phase of the "voltages" induced in two oppositely wound pickup coils of the same size by the current in a large driver coil above a two-layered conductor. The physical situation is shown in Fig. 4. The pickup coils must be identical, equidistant from the respective ends of the driver coil, and be within its length. They must also be coaxial with the driver coil. The two conducting layers may have different permeabilities and electrical conductivities.

To use this program, one must first divide all dimensions by the mean radius of the driver coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the driver coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

20 LET R1 = (numerical value of normalized inner radius of driver)

30 LET R2 = (numerical value of normalized outer radius of driver)

40 LET R3 = (numerical value of normalized inner radius of pickups)

50 LET R4 = (numerical value of normalized outer radius of pickups)

60 LET L = (numerical value of normalized lift-off)

70 LET L2 = (numerical value of normalized length of driver)

80 LET L6 = (numerical value of normalized length of pickups)

90 LET L5 = (numerical value of normalized distance of pickups from ends of driver)

100 LET M1 = (numerical value of $\omega\mu_1\sigma_1\bar{r}^2$)

Note: If the conductivity of the upper conductor is zero, one must type two additional lines:

ORNL-DWG 68-7312

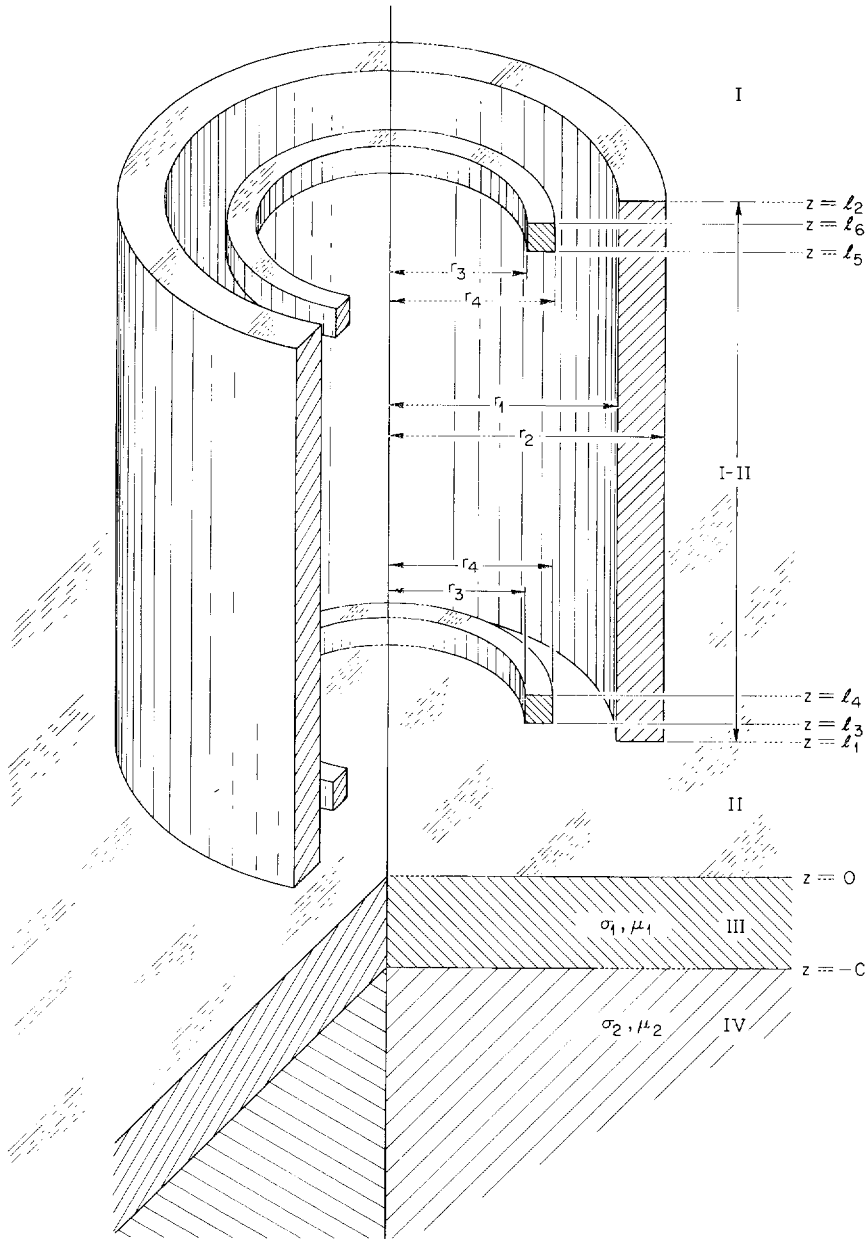

Fig. 4. A Reflection-Type Coil above a Two-Conductor Plane.

```
440 LET X1 = X
450 LET Y1 = 0
```

110 LET U1 = (numerical value of relative permeability of upper conductor)

120 LET M2 = (numerical value of $\omega\mu_2\sigma_2\overline{r}^2$)

Note: If the conductivity of the lower conductor is zero, one must type two additional lines:

```
470 LET X2 = X
480 LET Y2 = 0
```

```
130 LET U2 = (numerical value of relative permeability of lower
              conductor)

140 LET C = (numerical value of normalized thickness of upper
             conductor)
```

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)          R2 = (R2)        DRIVER LENGTH IS (L2)
R3 = (R3)          R4 = (R4)        PICKUP LENGTH IS (L6)
PICKUPS ARE RECESSED (L5)           CLAD THICKNESS IS [C]
LIFT-OFF = (L) , M1 = (M1) , U1 = (U1) , M2 = (M2) , U2 = (U2)
X                 MAGNITUDE              PHASE
5.                ------                 ------
10.               ------                 ------
15.               ------                 ------
20.               ------                 ------
    VOLTAGE/FINN'R(BAR) = ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The other two columns have self-explanatory headings. The last line printed is the magnitude of the peak (or root mean square) voltage difference divided by the product of the frequency of the driving current, the magnitude of the peak (or root mean square) current, the number of turns on the driver coil, the number of turns on a pickup coil, and the mean radius of the driver coil in meters.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of RFLCT

Let us suppose that we wish to know the difference in the voltages induced in two pickup coils positioned inside the driver coil and flush with its ends. The driver coil is assumed to be 2 in. long with inner and outer radii of 0.200 and 0.300 in., respectively, while the pickup cores are assumed to be 0.200 in. long with inner and outer radii of 0.100 and 0.150 in., respectively. The conductors are taken to be 0.010 in. of copper clad on an effectively infinite thickness of aluminum (at least four-coil diameters or skin depths, whichever is smaller). The driving current frequency is assumed to be 5 kHz; the lift-off 0.005 in.

First, we determine the mean coil radius to be 0.250 in., so that

Normalized inner radius of driver coil = 0.200/0.250 = 0.8
Normalized outer radius of driver coil = 0.300/0.250 = 1.2
Normalized inner radius of pickup coil = 0.100/0.250 = 0.4
Normalized outer radius of pickup coil = 0.150/0.250 = 0.6
Normalized length of driver coil = 2.000/0.250 = 8
Normalized length of pickup coil = 0.200/0.250 = 0.8
Normalized lift-off = 0.005/0.250 = 0.02
Normalized distance of pickup from ends of driver coil = 0
Normalized thickness of copper = 0.010/0.250 = 0.04

Furthermore,

Angular frequency of driving current = $2\pi \times 5000\ \text{sec}^{-1}$
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Relative permeability of both conductors = 1
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48.$$

This information is now typed into the program using the following format:

```
20 LET R1 = 0.8
30 LET R2 = 1.2
40 LET R3 = 0.4
50 LET R4 = 0.6
60 LET L = 0.02
70 LET L2 = 8
80 LET L6 = 0.8
90 LET L5 = 0
100 LET M1 = 91.91
110 LET U1 = 1
120 LET M2 = 45.48
130 LET U2 = 1
140 LET C = 0.04
```

The program is now run with the following results:

```
R1 = 0.8          R2 = 1.2          DRIVER LENGTH IS 8
R3 = 0.4          R4 = 0.6          PICKUP LENGTH IS 0.8
PICKUPS ARE RECESSED 0              CLAD THICKNESS IS 0.4E-01
LIFT-OFF = 0.2E-01    ,M1 = 91.91   ,U1 = 1    ,M2 = 45.48   , U2 = 1
X                     MAGNITUDE         PHASE
5                     0.3806977E-02       1.8412638
10                    0.3764764E-02       1.8319976
15                    0.3765228E-02       1.8322294
20                    0.3765232E-02       1.8322326
   VOLTAGE/FINN'R (BAR) = 0.1824152E-06
```

from which we see that the phase of the voltage difference is 1.8322326 (i.e., the phase value for X = 20). The actual magnitude of the voltage difference is obtained by multiplying the last line by the frequency of the driving current (5000), the magnitude of the current, the number of turns on the driver coil, the number of turns on a pickup coil, and the mean radius of the driver coil in meters.

<u>RFLCT Program</u>

```
10 REM          RFLCT          VERSION          02/24/69
20 LET R1=.8
30 LET R2=1.2
40 LET R3=.4
50 LET R4=.6
60 LET L=.02
70 LET L2=8
80 LET L6=.8
90 LET L5=0
100 LET M1=91.91
110 LET U1=1
120 LET M2=45.48
130 LET U2=1
140 LET C=.04
150 PRINT "R1=";R1, "R2=";R2, "DRIVER LENGTH IS";L2
160 PRINT "R3=";R3, "R4=";R4, "PICKUP LENGTH IS";L6
170 PRINT "PICKUPS ARE RECESSED";L5, "CLAD THICKNESS IS";C
180 PRINT"LIFT ØFF=";L",M1=";M1",U1=";U1",M2=";M2",U2=";U2
190 PRINT "X", "MAGNITUDE", "PHASE"
200 LET S1=1E-2
210 LET S2=5
220 LET I6=0
230 LET I8=0
240 LET B1=0
250 LET B2 =S2
260 FØR X = B1 +S1/2 TØ B2 STEP S1
270 LET Z=R2*X
280 LET Q1=R2
290 GØSUB 920
```

```
300 LET I2=F2
310 LET Z=K1*X
320 LET U1=K1
330 GOSUB 920
340 LET I1=F2
350 LET Z=K4*X
360 LET U1=K4
370 GOSUB 920
380 LET J2=F2
390 LET Z=K3*X
400 LET U1=K3
410 GOSUB 920
420 LET J1=F2
430 LET S3=S1*(I2-I1)*(J2-J1)
440 LETX1=.70710678 1*SQR(SQR(X*X*X*X+M1*M1)+X*X)/U1
450 LETY1=.70710678 1*SQR(SQR(X*X*X*X+M1*M1)-X*X)/U1
460 IFX1*C>15THEN610
470 LETX2=.70710678 1*SQR(SQR(X*X*X*X+M2*M2)+X*X)/U2
480 LETY2=.70710678 1*SQR(SQR(X*X*X*X+M2*M2)-X*X)/U2
490 LET X3=EXP(2*X1*C*U1)
500 LET Y3=COS(2*Y1*C*U1)
510 LET Y4=SIN(2*Y1*C*U1)
520 LET A6=(X-X1)*(X1+X2)+Y1*(Y1+Y2)
530 LET A7=(X-X1)*(Y1+Y2)-Y1*(X1+X2)
540 LETA5=(X+X1)*(X1-X2)-Y1*(Y1-Y2)+(A6*Y3-A7*Y4)*X3
550 LET B5=Y1*(X1-X2)+(X+X1)*(Y1-Y2)+(A7*Y3+A6*Y4)*X3
560 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
570 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
580 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
590 LET D5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
600 GO TO 650
610 LET A5 =X-X1
620 LET B5=-Y1
630 LET C5=X+X1
640 LET D5=Y1
650 LET K1=(A5*C5+B5*D5)/(C5*C5+D5*D5)
660 LET K2=(C5*B5-A5*D5)/(C5*C5+D5*D5)
670 IF X*L2>50 THEN 710
680 LET G=EXP(-X*L5)+EXP(-X*(L2-L5))-EXP(-X*(L2-L5-L6))-EXP(-X*(L5+L6))
690 LET G5=EXP(-X*L2)-1
700 GO TO 730
710 LET G=EXP(-X*L5)*(1-EXP(-X*L6))
720 LET G5=-1
730 LET G6=G*G5*EXP(-2*X*L)
740 LET I6=I6+G6*K1*S3
750 LET I8=I8+G6*K2*S3
760 NEXT X
770 LET B1=B2
780 LET B2=B2+S2
790 LET Q1=SQR(I6*I6+I8*I8)
800 LET Q2=ATN(-I6/I8)
810 IF I8<0 THEN 830
820 LET Q2=Q2+3.14159265
830 PRINT X+S1/2, Q1, Q2
840 IF X < 3 THEN 260
850 LET S1=5E-2
860 IF X<9 THEN 260
870 LET S1=.1
880 IF X<19 THEN 260
890 PRINT "     VOLTAGE/FINN'R(BAR)=";
```

```
900 PRINT 2.4805E-5*Q1/((R4-R3)*(R2-R1)*L2*L6)
910 GØ TØ 1070
920 IF Z>5 THEN 1010
930 LET L9=INT(2*Z)+3
940 LET F1=.5*Q1*Q1*Q1
950 LET F2=F1/3
960 FØR N=1TØL9
970 LETF1=-F1*.250*Z*Z/(N*N+N)
980 LETF2=F2+F1/(2*N+3)
990 NEXTN
1000 GØTØ 1060
1010 LET Q1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
1020 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
1030 LET Q2=(((-5.897517/Z+2.105874)/Z-.6896196)/Z+.4NZ2024)/Z
1040 LET Q2=(Q2-.187344E-2)/Z+.7979095
1050 LET F2=(1-SQR(Z)*(Q2*CØS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X*X)
1060 RETURN
1070 END
```

Magnitude and Phase of Induced Voltage with "Lift-Off" Set

Discussion of DISC

This program is designed to simulate the lift-off adjustment in the phase-sensitive eddy-current instrument. The voltage from either a reflection coil or a through transmission-type coil may vary as shown in Fig. 5. The discriminator in the phase-sensitive eddy-current instrument may be set to trigger at a voltage level, V_1. With the discriminator set at this level, there is relatively little variation in phase shift with lift-off. This program first calculates the voltage level V_1 from the amplitude and phase of the voltage for maximum and minimum lift-off.

ORNL-DWG 68-10310

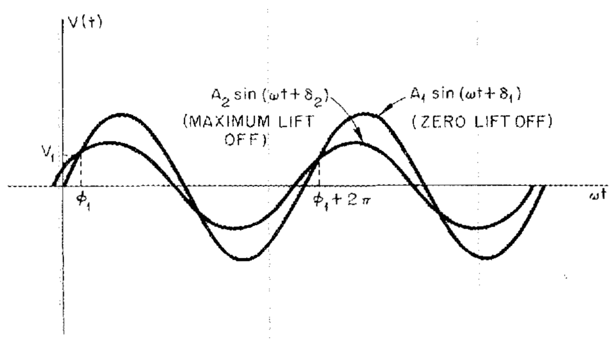

Fig. 5. Voltage Wave Form for Zero Lift-Off and Maximum Lift-Off.

It then calculates the phase shift for any other signal with the discriminator set to trigger at voltage V_2.

To use this program, one must execute the preceding program (RFLCT) for maximum and minimum lift-off and then for various clad thicknesses, conductivities, or lift-off values. In each instance, the data of interest will be the magnitude and phase values for X = 20 since they comprise the input data for the program at hand.

Once this information has been obtained, one need only type the following line into the program. [Note: (Parentheses are not typed.)]

10 DATA (Magnitude and phase values for minimum and maximum lift-off)

The program may now be run.

The print-out by the computer will have the following format:

```
ZERO PHASE IS ------            DISC. VOLT. IS ------
          WHAT ARE AI AND DI?
```

At this point, one types the magnitude and phase values for the thickness, conductivity, or lift-off of interest. The computer will then print

```
PHASE SHIFT IS ------
WHAT ARE AI AND DI?
```

and the user again types magnitude and phase values of interest. The phase shift is printed, and new magnitude and phase values are requested. This sequence continues until the user exhausts all data obtained from the preceding program. The second quantity on the first line is the voltage at which the discriminator of the Phase-Sensitive Eddy-Current Instrument must be set in order that lift-off effects be minimized.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of DISC

Let us suppose that we wish to know the variation with clad thickness, with lift-off set, of the difference in phase of the voltages induced in two pickup coil positions coaxially inside and flush with the ends of the driver coil. The driver coil is assumed to be 2 in. long with inner and outer radii of 0.200 and 0.300 in., respectively, while the pickup coils are assumed to be 0.200 in. long with inner and outer radii of 0.100 and 0.150 in., respectively. The conductors are assumed to be copper with a nominal thickness of 0.010 in. clad on an effectively infinite thickness of aluminum (at least four-coil diameters or skin depths, whichever is smaller). Let us further suppose that we are interested in a thickness range of copper of ±10% about the nominal thickness. The frequency of the driving current is assumed to be 5 kHz, and the maximum and minimum lift-off values to be 0.050 and 0 in., respectively.

First we determine the mean driver coil radius to be 0.250 in., so that

Normalized inner radius of driver coil = 0.200/0.250 = 0.8
Normalized outer radius of driver coil = 0.300/0.250 = 1.2
Normalized inner radius of pickup coil = 0.100/0.250 = 0.4
Normalized outer radius of pickup coil = 0.150/0.250 = 0.6
Normalized minimum lift-off = 0/0.250 = 0
Normalized maximum lift-off = 0.050/0.250 = 0.2
Normalized length of driver coil = 2.000/0.250 = 8
Normalized length of pickup coil = 0.200/0.250 = 0.8
Normalized distance of pickup coil from ends of driver coil = 0
Normalized thicknesses of copper = 0.036, 0.040, 0.044

Furthermore,

Angular frequency of driving current = $2\pi \times 5000\ \text{sec}^{-1}$
Permeability of <u>both</u> conductors = $4\pi \times 10^{-7}$ h/m
Relative permeability of both conductors = 1
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

This information is now typed into the preceding program which is then executed. The following information results:

Lift-Off	Copper Thickness	Magnitude	Phase
0	0.04	0.3978011E-02	1.8335539
0.2	0.04	0.2343262E-02	1.8107041
0	0.036	0.39597E-02	1.8366082
0	0.044	0.3994483E-02	1.8304675

The magnitude and phase values for minimum and maximum lift-off are now typed into the program — that is, one types

10 DATA 0.3978011E-2,   1.8335539, 0.2343262E-2,   1.8107041

At this point the program is run, and the following results are obtained:

```
ZERO PHASE IS 1.2753124          DISC. VOLT. IS 0.1301623E-03
          WHAT ARE AI AND DI?   0.39597E-2,   1.8366082

                    PHASE SHIFT IS 3.0790429

          WHAT ARE AI AND DI?  0.3994483E-2,   1.8304675

                    PHASE SHIFT IS 3.0731886
```

From this we see that for zero lift-off and a copper thickness of 0.036 the phase shift, with lift-off set, is 3.0790429 while for the same lift-off and a copper thickness of 0.044 it is 3.0731886.

DISC Program

```
10 DATA 1.51288E-3,.656582,7.36538E-4,.535609
20 READ A1,D1,A2,D2
30 LET Ø1=ATN((A2*SIN(D2)-A1*SIN(D1))/(A1*CØS(D1)-A2*CØS(D2)))
40 LET V1=A1*SIN(Ø1+D1)
50 PRINT"ZERØ PHASE IS ";Ø1,"DISC. VØLT. IS ";V1
60 PRINT"               WHAT ARE AI AND DI";
70 INPUT A3,D3
80 PRINT
90 PRINT" ","PHASE SHIFT IS";Ø1-ATN(V1/SQR(A3*A3-V1*V1))+D3
100 PRINT
110 GØ TØ 60
120 END
```

Phase of Induced Voltage for a Range of Cladding Thickness Values with "Lift-Off" Automatically Set

Discussion of RFLCYC

This program is designed to calculate the variation with cladding thickness, with lift-off set, of the difference in phase of the voltages induced in two oppositely wound pickup coils of the same size by the current in a large driver coil above a two-layered conductor. The physical situation is shown in Fig. 4, p. 38. The pickup coils must be identical, equidistant from the respective ends of the driver coil, and entirely within its length. They must also be coaxial with the driver coil. The two conducting layers may have different permeabilities and electrical conductivities; in fact, the program performs the calculations for any number of permeability and conductivity values for the upper conductor.

To use this program, one must first divide all dimensions by the mean radius of the driver coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the driver coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

```
20 LET R1 = (Numerical value of normalized inner radius of driver
             coil)
30 LET R2 = (Numerical value of normalized outer radius of driver
             coil)
40 LET R3 = (Numerical value of normalized inner radius of pickup
             coil)
50 LET R4 = (Numerical value of normalized outer radius of pickup
             coil)
60 LET L1 = (Numerical value of normalized minimum lift-off)
70 LET L2 = (Numerical value of normalized maximum lift-off)
80 LET L3 = (Numerical value of normalized length of driver coil)
90 LET L4 = (Numerical value of normalized length of pickup coil)
100 LET L5 = (Numerical value of normalized distance of pickup
              coil from ends of driver coil)
120 LET U1 = (Numerical value of normalized relative permeability
              of upper conductor)
130 LET M2 = (Numerical value of ωμ2σ2r̄²)
```

Note: If the conductivity of the lower conductor is zero, one must type two additional lines:

```
500 LET X2 = X
510 LET Y2 = 0
```

```
140 LET U2 = (Numerical value of relative permeability of lower
              conductor)
150 LET C = (Numerical value of normalized thickness of upper
             conductor)
160 LET S = (Numerical value of fractional thickness range)
1300 DATA (Values of ωμ1σ1r̄² separated by commas)
```

The program may now be run.

For each value of $\omega\mu_1\sigma_1\bar{r}^2$, the print-out by the computer will have the following format:

```
R1 = (R1)        R2 = (R2)         DRIVER LENGTH IS (L3)
R3 = (R3)        R4 = (R4)         PICKUP LENGTH IS (L4)
M1 = (M1)        U1 = (U1)         PICKUPS RECESSED (L5)
                 M2 = (M2)         U2 = (U2)
 5                10                15              20

DISC. VOLT ------                  ZERO PHASE ------

CLAD THICKNESS IS (C)              LIFT-OFF IS (L1 + L2)/2
          PHASE SHIFT IS ------

CLAD THICKNESS IS (C - S/2)        LIFT-OFF IS (L1)
          PHASE SHIFT IS ------
CLAD THICKNESS IS (C + S/2)        LIFT-OFF IS (L1)
          PHASE SHIFT IS ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The numbers 5, 10, 15, and 20 appear during the execution of the program in order that one may ascertain whether the program is running properly. The next line contains the voltage level at which the discriminator of the Phase-Sensitive Eddy-Current Instrument should be set to minimize lift-off effects and the phase value from which the phase shifts are measured. The remaining lines are self-explanatory.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of RFLCYC

Suppose that we wish to know the variation with cladding thickness, with lift-off set, of the difference in phase of the voltages induced in two pickup coils positioned inside and flush with the end of the driver coil. The driver coil is assumed to be 2 in. long with inner and outer radii 0.200 and 0.300 in., respectively, while the pickup coils are assumed to be 0.200 in. long with inner and outer radii of 0.100 and 0.150 in., respectively. The conductors are assumed to be copper with a nominal thickness of 0.010 in. clad on an effectively infinite thickness of aluminum (at least four-coil diameters or skin depths, whichever is smaller). Let us further suppose that we are interested in a thickness range of copper of ±10% about the nominal thickness. The frequency of the driving current is assumed to be 5 kHz, and the maximum and minimum values of lift-off are assumed to be 0.050 and 0 in., respectively.

First we determine the mean driver coil radius to be 0.250 in., so that

Normalized inner radius of driver coil = 0.200/0.250 = 0.8
Normalized outer radius of driver coil = 0.300/0.250 = 1.2
Normalized inner radius of pickup coils = 0.100/0.250 = 0.4
Normalized outer radius of pickup coils = 0.150/0.250 = 0.6
Normalized minimum lift-off = 0/0.250 = 0
Normalized maximum lift-off = 0.050/0.250 = 0.2
Normalized length of driver coil = 2.000/0.250 = 8
Normalized length of pickup coils = 0.200/0.250 = 0.8
Normalized distance of pickup coils from ends of driver = 0
Normalized nominal thickness of copper = 0.010/0.250 = 0.04
Fractional thickness range (±10%) = 0.2

Furthermore,

Angular frequency of driving current = $2\pi \times 5000$ sec^{-1}
Permeability of <u>both</u> conductors = $4\pi \times 10^{-7}$ h/m
Relative permeability of both conductors = 1
Conductivity of copper = 5.77×10^7 mhos/m
Conductivity of aluminum = 2.86×10^7 mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

This information is now typed into the program with the following format:

```
20 LET R1 = 0.8
30 LET R2 = 1.2
40 LET R3 = 0.4
50 LET R4 = 0.6
60 LET L1 = 0
70 LET L2 = 0.2
80 LET L3 = 8
90 LET L4 = 0.8
100 LET L5 = 0
120 LET U1 = 1
130 LET M2 = 45.48
140 LET U2 = 1
150 LET C = 0.4
160 LET S = 0.2
1300 DATA 91.91
```

The program is now run with the following results:

```
R1 = 0.8              R2 = 1.2           DRIVER LENGTH IS 8
R3 = 0.4              R4 = 0.6           PICKUP LENGTH IS 0.8
M1 = 91.91            U1 = 1             PICKUP RECESSED 0
                      M2 = 45.48         U2 = 1
 5                     10                 15               20

DISC. VOLT - 0.06306E-07                 ZERO PHASE - 1.86628

CLAD THICKNESS IS 0.4E-01                LIFT-OFF IS 0.1
          PHASE SHIFT IS 0.081373E-02

CLAD THICKNESS IS 0.36E-01               LIFT-OFF IS 0
          PHASE SHIFT IS 0.320568E-02

CLAD THICKNESS IS 0.43999E-01            LIFT-OFF IS 0
          PHASE SHIFT IS -0.322143E-02
```

from which we see that for a clad thickness of 3.6% of the mean coil radius the phase shift, with lift-off set, is 0.320568×10^{-2} while for a thickness of 4.4% of the mean coil radius the phase shift is -0.322143×10^{-2}.

<u>RFLCYC Program</u>

```
10 REM             RFLCYC          VERSION          08/21/68
20 LET R1=.8
30 LET R2=1.2
40 LET R3=.4
50 LET R4=.6
60 LET L1=0
70 LET L2=.2
80 LET L3=8
90 LET L4=.8
100 LET L5=0
```

```
110 READ M1
120 LET U1=1
130 LET M2=45.48
140 LET U2=1
150 LET C=.04
160 LET S=.2
170 PRINT "R1=";R1, "R2=";R2, "DRIVER LENGTH IS";L3
180 PRINT "R3=";R3, "R4=";R4, "PICKUP LENGTH IS";L4
190 PRINT"M1=";M1,"U1=";U1,"PICKUPS RECESSED";L5
200 PRINT" ","M2=";M2,"U2=";U2
210 LET S1=1E-2
220 LET S2=5
230 FØR J=1 TØ 5
240 LET R(J)=0
250 LET I(J)=0
260 NEXT J
270 LET B1=0
280 LET B2 =S2
290 FØR X = B1 +S1/2 TØ B2 STEP S1
300 LET Z=R2*X
310 LET Q1=R2
320 GØSUB 1320
330 LET I2=F2
340 LET Z=R1*X
350 LET Q1=R1
360 GØSUB 1320
370 LET I1=F2
380 LET Z=R4*X
390 LET Q1=R4
400 GØSUB 1320
410 LET J2=F2
420 LET Z=R3*X
430 LET Q1=R3
440 GØSUB 1320
450 LET J1=F2
460 LET S3=S1*(I2-I1)*(J2-J1)
470 LETX1=.707106781*SQR(SQR(X*X*X*X+M1*M1)+X*X)/U1
480 LETY1=.707106781*SQR(SQR(X*X*X*X+M1*M1)-X*X)/U1
490 IFX1*C>15THEN570
500 LETX2=.707106781*SQR(SQR(X*X*X*X+M2*M2)+X*X)/U2
510 LETY2=.707106781*SQR(SQR(X*X*X*X+M2*M2)-X*X)/U2
520 LET A6=(X-X1)*(X1+X2)+Y1*(Y1+Y2)
530 LET A7=(X-X1)*(Y1+Y2)-Y1*(X1+X2)
540 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
550 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
560 GØ TØ 610
570 LET A5 =X-X1
580 LET B5=-Y1
590 LET C5=X+X1
600 LET D5=Y1
610 IF X*L3>50 THEN 650
620 LET G=EXP(-X*L5)+EXP(-X*(L3-L5))-EXP(-X*(L3-L4-L5))-EXP(-X*(L5+L4))
630 LET G5=EXP(-X*L3)-1
640 GØ TØ 670
650 LET G=EXP(-X*L5)*(1-EXP(-X*L4))
660 LET G5=-1
670 FØR J=1 TØ 3
680 LET L(J)=L1+(J-1)*(L2-L1)/2
690 LET G(J)=G*G5*EXP(-2*X*L(J))
700 IF X1*C>15 THEN 720
```

```
710 GO SUB 1470
720 LET K1=(A5*C5+B5*D5)/(C5*C5+D5*D5)
730 LET K2=(C5*B5-A5*D5)/(C5*C5+D5*D5)
740 LET R(J)=R(J)+S3*K1*G(J)
750 LET I(J)=I(J)+S3*K2*G(J)
760 IF J>1 THEN 920
770 LET C=(1-S/2)*C
780 IF X1*C>15 THEN 800
790 GO SUB 1470
800 LET K3=(A5*C5+B5*D5)/(C5*C5+D5*D5)
810 LET K4=(C5*B5-A5*D5)/(C5*C5+D5*D5)
820 LET R(J+3)=R(J+3)+S3*K3*G(J)
830 LET I(J+3)=I(J+3)+S3*K4*G(J)
840 LET C=(1+S/2)*C/(1-S/2)
850 IF X1*C>15 THEN 870
860 GO SUB 1470
870 LET K5=(A5*C5+B5*D5)/(C5*C5+D5*D5)
880 LET K6=(C5*B5-A5*D5)/(C5*C5+D5*D5)
890 LET R(J+4)=R(J+4)+S3*K5*G(J)
900 LET I(J+4)=I(J+4)+S3*K6*G(J)
910 LET C=C/(1+S/2)
920 NEXT J
930 NEXT X
940 LET B1=B2
950 LET B2=B2+S2
960 PRINT X+S1/2,
970 IF X < 3 THEN 290
980 LET S1=5E-2
990 IF X<9 THEN 290
1000 LET S1=.1
1010 IF X<19 THEN 290
1020 PRINT
1030 PRINT
1040 FOR J=1 TO 5
1050 LET M(J)=SQR(R(J)↑2+I(J)↑2)*2.4805E-5/((R2-R1)*(R4-R3)*L3*L4)
1060 LET P(J)=ATN(-R(J)/I(J))
1070 IF I(J)<0 THEN 1090
1080 LET P(J)=P(J)+PI
1090 NEXT J
1100 LET H1=M(1)
1110 LET H2=M(3)
1120 LET H5=P(1)
1130 LET H6=P(3)
1140 LET G1=M(2)
1150 LET G2=P(2)
1160 LET L=L(2)
1170 GO SUB 1550
1180 LET G1=M(4)
1190 LET G2=P(4)
1200 LET C=(1-S/2)*C
1210 LET L=L(1)
1220 GO SUB 1550
1230 LET G1=M(5)
1240 LET G2=P(5)
1250 LET C=(1+S/2)*C/(1-S/2)
1260 GO SUB 1550
1270 PRINT
1280 PRINT
1290 PRINT
1300 DATA 91.91
```

```
1310 GØ TØ 20
1320 IF Z>5THEN1410
1330 LET L9=INT(2*Z)+3
1340 LET F1=.5*Q1*Q1*Q1
1350 LET F2=F1/3
1360 FØR N=1TØL9
1370 LETF1=-F1*.250*Z*Z/(N*N+N)
1380 LETF2=F2+F1/(2*N+3)
1390 NEXTN
1400 GØTØ 1460
1410 LET Q1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
1420 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
1430 LET Q2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
1440 LET Q2=(Q2-.187344E-2)/Z+.7979095
1450 LETF2=(1-SQR(Z)*(Q2*CØS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X*X)
1460 RETURN
1470 LET X3=EXP(2*X1*C*U1)
1480 LET Y3=CØS(2*Y1*C*U1)
1490 LET Y4=SIN(2*Y1*C*U1)
1500 LETA5=(X+X1)*(X1-X2)-Y1*(Y1-Y2)+(A6*Y3-A7*Y4)*X3
1510 LET B5=Y1*(X1-X2)+(X+X1)*(Y1-Y2)+(A7*Y3+A6*Y4)*X3
1520 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
1530 LET D5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
1540 RETURN
1550 LET Ø1=ATN((H2*SIN(H6)-H1*SIN(H5))/(H1*CØS(H5)-H2*CØS(H6)))
1560 IFH1*CØS(H5)-H2*CØS(H6)>0THEN1580
1570 LETØ1=Ø1-3.14159265
1580 LET V1= H1*SIN(Ø1+H5)
1590 LET P=Ø1-ATN(V1/SQR(Q1*Q1-V1*V1))+Q2
1600 IF L<L(2) THEN 1620
1610 PRINT"DISC. VØLT";V1,"ZERØ PHASE";Ø1
1620 PRINT
1630 PRINT"CLAD THICKNESS IS";C,"LIFT ØFF IS";L
1640 PRINT"                PHASE SHIFT IS";P
1650 RETURN
1660 END
```

Change in Magnitude and Phase of Induced Voltage Due to a Defect in the Cladding Material

Discussion of RFDFT

This program is designed to calculate the defect sensitivity factor for a reflection-type coil above a two-layered conductor (see Fig. 4, p. 38). For this program, the defect must be in the upper layer. The pickup coils must be identical, equidistant from the respective ends of the driver coil, and entirely within its length. They must also be coaxial with the driver coil. The two conducting layers may have different electrical conductivities.

To use this program, one must first divide all dimensions by the mean radius of the driver coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the driver coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

```
12 LET R1 = (Numerical value of normalized inner radius of driver
             coil)
14 LET R2 = (Numerical value of normalized outer radius of driver
             coil)
16 LET R3 = (Numerical value of normalized inner radius of pickup
             coil)
18 LET R4 = (Numerical value of normalized outer radius of pickup
             coil)
20 LET L = (Numerical value of normalized lift-off)
22 LET L1 = (Numerical value of normalized length of driver coil)
24 LET L2 = (Numerical value of normalized length of pickup coil)
26 LET L3 = (Numerical value of normalized distance of pickup
             coil from ends of driver coil)
28 LET M1 = (Numerical value of ωμ1σ1r̄²)
30 LET M2 = (Numerical value of ωμ2σ2r̄²)
```

Note: If the conductivity of the lower conductor is zero, one must type:

```
118 LET X2 = X
120 LET Y2 = 0
```

```
32 LET C = (Numerical value of normalized thickness of upper
            conductor)
34 LET R = (Numerical value of normalized r position coordinate
            of defect)
36 LET Z1 = (Numerical value of normalized z position coordinate
             of defect)
```

The program may now be run.

For each lift-off value, the print-out by the computer will have the following format:

DEFECT PROGRAM FOR PHASE-SENSITIVE EDDY-
CURRENT INSTRUMENT

DRIVER LENGTH IS (L1)		R1 = (R1)	R2 = (R2)	
PICKUP LENGTH IS (L2)		R3 = (R3)	R4 = (R4)	
CLAD THICKNESS IS (C)		M1 = (M1)	M2 = (M2)	
DEFECT LOCATION IS		R = (R)	Z = (Z1)	
PICKUP IS RECESSED (L3)		LIFT-OFF IS (L)		
X	REAL #1	IMAG #1	REAL #2	IMAG #2
5	------	------	------	------
10	------	------	------	------
15	------	------	------	------
20	------	------	------	------
25	------	------	------	------
30	------	------	------	------
VOLTAGE/FINN'R(BAR) = ------			PHASE = ------	

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer, while the other four columns are the real and imaginary parts of the two integrals which appear in the expression for the defect voltage. These appear primarily to allow one to inspect the convergence of the integration. In addition to the phase of the defect sensitivity factor, the last line contains the magnitude of the defect sensitivity factor divided by the product of the frequency of the driving current, the numbers of turns on the driver and pickup coils, and the mean radius of the driver coil.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of RFDFT

Suppose that we wish to know the defect sensitivity factor for the case of two pickup coils positioned inside and flush with the ends of the driver coil. The driver coil is assumed to be 2 in. long, with inner and outer radii of 0.200 and 0.300 in., respectively, while the pickup coils are assumed to be 0.200 in. long, with inner and outer radii 0.100 and 0.150 in., respectively. The conductor we assumed to be 0.010 in. of copper clad on an effectively infinite thickness of aluminum (at least four-coil diameters or skin depths, whichever is smaller). Let us further suppose that we are interested in the defect sensitivity factor for the case in which the defect is located 0.005 in. below the surface of the copper 0.250 in. from the coil axis. We shall assume the driving frequency to be 5 kHz and the lift-off to be zero.

First, we determine the mean driver coil radius to be 0.250 in., so that

Normalized inner radius of driver coil = 0.200/0.250 = 0.8
Normalized outer radius of driver coil = 0.300/0.250 = 1.2
Normalized inner radius of pickup coils = 0.100/0.250 = 0.4
Normalized outer radius of pickup coils = 0.150/0.250 = 0.6
Normalized lift-off = 0/0.250 = 0
Normalized length of driver = 2.000/0.250 = 8
Normalized length of pickup = 0.200/0.250 = 0.8
Normalized distance of pickups from ends of driver = 0
Normalized thickness of copper = 0.010/0.250 = 0.04
Normalized r position of defect = 0.250/0.250 = 1
Normalized z position of defect = 0.005/0.250 = 0.02

Furthermore,

Angular frequency of driving current = $2\pi \times 5000$ sec^{-1}
Permeability of <u>both</u> conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$

$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

This information is now typed into the program as follows:

```
12 LET R1 = 0.8
14 LET R2 = 1.2
16 LET R3 = 0.4
18 LET R4 = 0.6
20 LET L = 0
22 LET L1 = 8
24 LET L2 = 0.8
26 LET L3 = 0
28 LET M1 = 91.91
30 LET M2 = 45.48
32 LET C = 0.04
34 LET R = 1
36 LET Z1 = -0.02
```

The program is now run with the following results:

DEFECT PROGRAM FOR PHASE-SENSITIVE EDDY-
CURRENT PROGRAM

```
DRIVER LENGTH IS 8              R1 = 0.8        R2 = 1.2
PICKUP LENGTH IS 0.8            R3 = 0.4        R4 = 0.6
CLAD THICKNESS IS 0.4E-01       M1 = 91.91      M2 = 45.48
DEFECT LOCATION IS              R = 1           Z = -0.2E-01
PICKUP IS RECESSED 0            LIFT-OFF IS 0
```

X	REAL #1	IMAG #1	REAL #2	IMAG #2
5	0.1696E-01	−0.1824701E-01	0.2077798E-02	−0.2385628E-02
10	0.20692E-01	−0.1991495E-01	0.0790374E-02	−0.1800469E-02
15	0.2108679E-01	−0.1999053E-01	0.1154164E-02	−0.187069E-02
20	0.2103699E-01	−0.1998544E-01	0.1120786E-02	−0.1868363E-02
25	0.2094097E-01	−0.1997792E-01	0.1116701E-02	−0.1867812E-02
30	0.209154E-01	−0.1997639E-01	0.1123086E-02	−0.1868201E-02

VOLTAGE/FINN'R(BAR) → 0.1340369E-06   PHASE = −1.7919639

from which we see that the phase of the defect sensitivity factor is −1.7908883, while its magnitude is the product of 1.340401×10^{-7}, the frequency of the driving current, the numbers of turns on the driver coil and pickup coils, and the mean radius of the driver coil.

<u>RFDFT Program</u>

```
10 REM          RFDFT          VERSIØN          03/03/69
12 LET R1=.8
14 LET R2=1.2
16 LET R3=.4
18 LET R4=.6
20 LET L=0
22 LET L1=8
24 LET L2=.8
26 LET L3=0
28 LET M1=91.91
30 LET M2=45.48
32 LET C=.04
34 LET R=1
36 LET Z1=-.02
38PRINT"                 DEFECT PRØGRAM FØR PHASE SENSITIVE EDDY"
40PRINT"                              CURRENT INSTRUMENT"
42PRINT
44PRINT
46PRINT"DRIVER LENGTH IS";L1,"R1=";R1,"R2=";R2
48PRINT"PICKUP LENGTH IS";L2,"R3=";R3,"R4=";R4
50PRINT"CLAD THICKNESS IS";C,"M1=";M1,"M2=";M2
52PRINT"DEFECT LØCATIØN IS","R=";R,"Z=";Z1
54PRINT "PICKUP IS RECESSED";L3,"LIFT ØFF IS";L
56PRINT"X","REAL #1","IMAG #1","REAL #2","IMAG #2"
58 LET S1=1E-2
60 LET S2=5
62 FØR I=1 TØ 2
64LET R(I)=0
```

```
66LETI(I)=0
68 NEXT I
70 LET B1=0
72 LET B2 =S2
74FØRX=B1+S1/2TØB2STEPS1
76 LET Z=R2*X
78 LET G1=R2
80 GØSUB 218
82LETI2=F2
84 LET Z=R1*X
86 LET G1=R1
88 GØ SUB 218
90 LET I1=F2
92 LET S3=(I2-I1)*S1*(EXP(-X*L)-EXP(-X*(L+L1)))
94 LET Z=R4*X
96 LETG1=R4
98 GØ SUB 218
100 LET I4=F2
102 LET Z=R3*X
104 LET G1=R3
106 GØ SUB 218
108 LET I3=F2
110 LET S4=(I4-I3)*S1*(EXP(-X*L)-EXP(-X*(L+L2)))*EXP(-X*L3)
112 LET X1 = .707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
114 LET Y1 = .707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
116IFX1*C>15THEN146
118 LET X2 = .707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
120 LET Y2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
122 LET X3=EXP(2*X1*C)
124 LET Y3=CØS(2*Y1*C)
126 LET Y4=SIN(2*Y1*C)
128 LET X5=X1+X2
130 LET X6=X1-X2
132 LET Y5=Y1+Y2
134 LET Y6=Y1-Y2
136 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
138 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
140 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
142LETD5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
144 GØ TØ 150
146 LETC5=X+X1
148LETD5=Y1
150IFX1*C>15THEN164
152LETZ2=2*C+Z1
154LETA5=(X5*CØS(Y1*Z2)-Y5*SIN(Y1*Z2))*EXP(X1*Z2)
156LETA5=A5+(X6*CØS(-Y1*Z1)-Y6*SIN(-Y1*Z1))*EXP(-X1*Z1)
158 LET B5=(Y5*CØS(Y1*Z2)+X5*SIN(Y1*Z2))*EXP(X1*Z2)
160LETB5=B5+(Y6*CØS(Y1*Z1)+X6*SIN(Y1*Z1))*EXP(-X1*Z1)
162GØ TØ 168
164LETA5=EXP(X1*Z1)*CØS(Y1*Z1)
166LETB5=EXP(X1*Z1)*SIN(Y1*Z1)
168LETK1=(A5*C5+B5*D5)/(C5*C5+D5*D5)
170 LET K2=(C5*B5-A5*D5)/(C5*C5+D5*D5)
172LETZ=X*R
174GØSUB248
176LETR(1)=R(1)+S3*K1*J1
178 LET I(1)=I(1)+S3*K2*J1
180 LET R(2)=R(2)+S4*K1*J1
182 LET I(2)=I(2)+S4*K2*J1
184 NEXT X
```

```
186 PRINT X+S1/2,R(1),I(1),R(2),I(2)
188LETB1=B1+S2
190LETB2=B2+S2
192LETS1=5E-2
194 IF X<15 THEN 74
196 LET S1=.1
198 IF X<25 THEN 74
200 LET R(3)=R(1)*R(2)-I(1)*I(2)
202 LET I(3)=R(1)*I(2)+R(2)*I(1)
204 LET P=ATN(I(3)/R(3))
206 IF R(3)>0 THEN 210
208 LET P=P-PI
210 LET M=SQR(R(3)*R(3)+I(3)*I(3))
212 LET G=3*2.4805E-5*M1/(2*PI*(R2-R1)*(R4-R3)*L1*L2)
214PRINT"VØLTAGE/FINN'R(BAR)=";G*M,"PHASE=";P
216 GØ TØ 270
218 IF Z>5 THEN 236
220 LETL5=INT(2*Z)+3
222 LETF1=.5*G1*G1*Z
224 LET F2=F1/3
226 FØRN=1TØL5
228 LETF1=-F1*.250*Z*Z/(N*N+N)
230 LETF2=F2+F1/(2*N+3)
232 NEXTN
234 GØTØ 246
236 LET F1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
238 LET F1=((F1-.1730503)/Z+.7034845)/Z-.064109E-3
240 LET F2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
242 LET F2=(F2-.187344E-2)/Z+.7979095
244 LET F2=(1-SQR(Z)*(F2*CØS(Z-PI/4)-F1*SIN(Z-PI/4)))/(X*X)
246 RETURN
248 IFZ>3THEN 258
250 LET G1=(((2.1E-11*Z↑2-5.38E-9)*Z↑2+6.757E-7)*Z↑2-5.42443E-5)*Z↑2
252 LET G1=((G1+2.60415E-3)*Z↑2-6.25E-2)*Z↑2+.5
254 LET J1=Z*G1
256 GØTØ 268
258 LET G3=(((-.14604057/Z+.27617679)/Z-.20210391)/Z+4.61835E-3)/Z
260 LET G3=((G3+.14937)/Z+4.68E-6)/Z+.79788456
262 LET G4=(((-.21262014/Z+.19397232)/Z+6.022188E-2)/Z-1.7222733E-1)/Z
264 LET G4=((G4+5.085E-4)/Z+.37498836)/Z-2.35619449+Z
266 LET J1=G3*CØS(G4)/SQR(Z)
268 RETURN
270 LET L=L+.1
272 PRINT
274 IF L<.25 THEN 46
276 END
```

Change in Magnitude and Phase of Induced Voltage Due to a Lattice of Defects in the Cladding Material

Discussion of RFDFL2

This program is designed to calculate the defect sensitivity factor for a reflection-type coil above a two-layered conductor. For this program, the defect may be located at any one of a number of points on a lattice in the upper layer, which is labeled III in Fig. 4, p. 38. The pickup coils must be identical, equidistant from the respective ends of the driver coil, and entirely within its length. They must also be coaxial with the driver coil. The two conducting layers may have different electrical conductivities.

Before proceeding further, it is worth noting that in the execution of this program the data generated by the computer are stored in a data block which is used as input data for another program. Since there exist BASIC compilers which do not have this capability, this program may or may not be executable, depending upon the compiler used. If the latter situation arises, one may delete line number 42 and the word "FILE" wherever it appears in the program, thus causing the data to be printed at the console; otherwise, one may access the data by using the name /@1/.

To use this program, one must first divide all dimensions by the mean radius of the driver coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean driver coil radius must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made, one must type the following lines into the program. [Note: (Parentheses are not typed.)]

```
12 LET R1 = (Numerical value of normalized inner radius of driver
             coil)
14 LET R2 = (Numerical value of normalized outer radius of driver
             coil)
16 LET R3 = (Numerical value of normalized inner radius of pickup
             coils)
18 LET R4 = (Numerical value of normalized outer radius of pickup
             coils)
20 LET L = (Numerical value of normalized lift-off)
22 LET L1 = (Numerical value of normalized length of driver coil)
24 LET L2 = (Numerical value of normalized length of pickup coils)
26 LET L3 = (Numerical value of normalized distance of pickup
             coils from ends of driver coil)
28 LET M1 = (Numerical value of ωμ1σ1r̄²)
30 LET M2 = (Numerical value of ωμ2σ2r̄²)
```

Note: If the conductivity of the lower conductor is zero, one must type two additional lines:

```
124 LET X2 = X
126 LET Y2 = 0
```

```
32 LET C = (Numerical value of normalized thickness of upper
            conductor)
34 LET R9 = (Numerical value of normalized maximum radial position
             of defect)
36 LET M9 = (Numerical value of number of lattice spacings in
             r direction)
38 LET N9 = (Numerical value of number of lattice spacings in
             z direction)
```

Note: Neither M9 nor N9 may exceed 15. The lattice spacing in the r direction is R9/M9, while it is C/N9 in the z direction.

After typing these lines, one need only create an empty data file using the name /@1/. The program may then be run.

During the execution of this program, integral multiples of 5 which are less than or equal to 30 will be printed at the console. These appear as indications of how far the integration has proceeded. The data resulting from this program will have the following format:

```
(L1)        (R1)        (R2)
(L2)        (R3)        (R4)
(C)         (M1)        (M2)
(L3)        (L)
(M9)        (N9)
            (R(1))      (R(2))      **********      (R(M9))
(Z)1))      ------      ------      **********      ------
            ------      ------      **********      ------

(Z(2))      ------      ------      **********      ------
            ------      ------      **********      ------

  *           *           *             *              *
  *           *           *             *              *
  *           *           *             *              *
  *           *           *             *              *

(Z(N9))     ------      ------      **********      ------
            ------      ------      **********      ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The upper value at each lattice point is the magnitude of the defect sensitivity factor, while the lower one is the phase of the defect sensitivity factor for the case in which the defect is located at the given lattice point. It should be noted that the lattice points for which $M9 > 4$ will be printed below the last line – that is, the line containing Z(N9).

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of RFDFL2

Suppose that we wish to know the defect sensitivity factor for the case of two pickup coils positioned inside and flush with the ends of the driver coil. The driver coil is assumed to be 2 in. long with inner and outer radii of 0.200 and 0.300 in., respectively, while the pickup coils are assumed to be 0.200 in. in length with inner and outer radii of 0.100 and 0.150 in., respectively. The conductors are assumed to be 0.010 in. of copper clad on an effectively infinite thickness of aluminum (at least four-coil diameters or skin depths, whichever is smaller). Let us further suppose that we are interested in the defect sensitivity factor for the cases in which the defect is located at any one of the points which lie within 0.500 in. of the coil axis on a 4×2 lattice in the copper conductor. We shall assume the driving frequency to be 5 kHz and the lift-off to be zero.

First, we determine the mean radius of the driver coil to be 0.250 in., so that

Normalized inner radius of driver coil = 0.200/0.250 = 0.8
Normalized outer radius of driver coil = 0.300/0.250 = 1.2
Normalized inner radius of pickups = 0.100/0.250 = 0.4
Normalized outer radius of pickups = 0.150/0.250 = 0.6
Normalized lift-off = 0/0.250 = 0
Normalized length of driver coil = 2.000/0.250 = 8
Normalized length of pickups = 0.200/0.250 = 0.8
Normalized distance of pickups from ends of driver = 0/0.250 = 0
Normalized thickness of copper = 0.010/0.250 = 0.04
Maximum radial position of defect = 0.500/0.250 = 2
Number of r lattice spacings = 4
Number of z lattice spacings = 2

Furthermore,

Angular frequency of driving current = $2\pi \times 5000$ sec^{-1}
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

This information is now typed into the program using the following format:

```
12 LET R1 = 0.8
14 LET R2 = 1.2
16 LET R3 = 0.4
18 LET R4 = 0.6
20 LET L = 0
22 LET L1 = 8
24 LET L2 = 0.8
26 LET L3 = 0
28 LET M1 = 91.91
30 LET M2 = 45.48
32 LET C = 0.04
34 LET R9 = 2
36 LET M9 = 4
38 LET N9 = 2
```

An empty data file is now created, and the program is run with the following results:

```
8            0.8            1.2
0.8          0.4            0.6
-4E-01       91.91          45.48
0            0
4            2

             0.25           0.75           1.25           1.75
-0.1E-01     0.0775839E-06  0.2755224E-06  0.5087424E-07  0.6095831E-08
             -1.620616      -1.5616094     -1.8510375     -2.0433756

-0.3E-01     0.6201136E-07  0.2192486E-06  0.4020022E-07  0.4800129E-08
             -1.8620754     -1.7993881     -2.1067626     -2.3014832
```

from which we see that for the case in which the defect is located at R = 1.25, z = −0.03, the defect sensitivity factor is

$$0.4031929 \times 10^{-7} \exp(-j2.103904)$$

RFDFL2 Program

```
10 REM            RFDFL2            VERSIØN            03/03/69
12 LET R1=.8
14 LET R2=1.2
16 LET R3=.4
18 LET R4=.6
20 LET L=0
22 LET L1=8
24 LET L2=.8
26 LET L3=0
28 LET M1=91.91
30 LET M2=45.48
32 LET C=.04
34 LET R9=2
36 LET M9=4
38 LET N9=2
40 DIM M(18,18),I(18,18),S(18,18),J(18,18),N(18,18),P(18,18),R(18),Z(18)
42  ØPEN /61/, ØUTPUT
44PRINT FILE L1,R1,R2
46PRINT FILE L2,R3,R4
48PRINT FILE C,M1,M2
50PRINT FILE L3,L
52 PRINT FILE M9,N9
54PRINT FILE
56 LET S1=1E-2
58 LET S2=5
60 FØR I=1 TØ N9
62 FØR J=1 TØ M9
64LETM(I,J)=0
66LETI(I,J)=0
68 LET S(I,J)=0
70 LET J(I,J)=0
72 NEXT J
74 NEXT I
76 LET B1=0
78 LET B2 =S2
80FØRX=B1+S1/2TØB2STEPS1
82 LET Z=R2*X
84 LET G1=R2
86 GØSUB 326
88LETI2=P2
90 LET Z=R1*X
92 LET G1=R1
94 GØ SUB 326
96 LET I1=P2
98 LET S3=(I2-I1)*S1*(EXP(-X*L)-EXP(-X*(L+L1)))
100 LET Z=R4*X
102 LETG1=R4
104 GØ SUB 326
106 LET I4=P2
108 LET Z=R3*X
110 LET G1=R3
112 GØ SUB 326
114 LET I3=P2
116 LET S4=(I4-I3)*S1*(EXP(-X*L)-EXP(-X*(L+L2)))*EXP(-X*L3)
118 LET X1 = .707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
120 LET Y1 = .707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
122IFX1*C>15THEN152
124 LET X2 = .707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
126 LET Y2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
128 LET X3=EXP(2*X1*C)
```

```
130 LET Y3=COS(2*Y1*C)
132 LET Y4=SIN(2*Y1*C)
134 LET X5=X1+X2
136 LET X6=X1-X2
138 LET Y5=Y1+Y2
140 LET Y6=Y1-Y2
142 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
144 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
146 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
148LETD5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
150 GØ TØ 156
152 LETC5=X+X1
154LETD5=Y1
156 FØR I=1 TØ N9
158 LET Z(I)=(.5-I)*C/N9
160 LET Z1=Z(I)
162IFX1*C>15THEN176
164LETZ2=2*C+Z1
166LETA5=(X5*CØS(Y1*Z2)-Y5*SIN(Y1*Z2))*EXP(X1*Z2)
168LETA5=A5+(X6*CØS(-Y1*Z1)-Y6*SIN(-Y1*Z1))*EXP(-X1*Z1)
170 LET B5=(Y5*CØS(Y1*Z2)+X5*SIN(Y1*Z2))*EXP(X1*Z2)
172LETB5=B5+(Y6*CØS(Y1*Z1)+X6*SIN(Y1*Z1))*EXP(-X1*Z1)
174GØ TØ 180
176LETA5=EXP(X1*Z1)*CØS(Y1*Z1)
178LETB5=EXP(X1*Z1)*SIN(Y1*Z1)
180LETK1=(A5*C5+B5*D5)/(C5*C5+D5*D5)
182 LET K2=(C5*B5-A5*D5)/(C5*C5+D5*D5)
184 FØR J=1 TØ M9
186 LET R(J)=(J-.5)*R9/M9
188LETZ=X*R(J)
190GØSUB356
192LETM(I,J)=M(I,J)+S3*K1*J1
194 LET I(I,J)=I(I,J)+S3*K2*J1
196 LET S(I,J)=S(I,J)+S4*K1*J1
198 LET J(I,J)=J(I,J)+S4*K2*J1
200 NEXT J
202 NEXT I
204 NEXT X
206 PRINT X+S1/2,
208LETB1=B1+S2
210LETB2=B2+S2
212LETS1=5E-2
214 IF X<15 THEN 80
216 LET S1=.1
218 IF X<25 THEN 80
220 LET G=3*2.4805E-5*M1/(2*PI*(R2-R1)*(R4-R3)*L1*L2)
222 FØR I=1 TØ N9
224 FØR J=1 TØ M9
226 LET N(I,J)=M(I,J)*S(I,J)-I(I,J)*J(I,J)
228 LET I(I,J)=M(I,J)*J(I,J)+S(I,J)*I(I,J)
230 LET P(I,J)=ATN(I(I,J)/N(I,J))
232 IF N(I,J)>0 THEN 236
234 LET P(I,J)=P(I,J)-PI
236 LET M(I,J)=SQR(N(I,J)*N(I,J)+I(I,J)*I(I,J))
238 NEXT J
240 NEXT I
242 PRINT FILE " ",
244 FØR J=1 TØ 4
246 PRINT FILE R(J),
248 NEXT J
```

```
250 FØR I=1 TØ N9
252 PRINT FILE Z(I),
254 FØR J=1 TØ 4
256 PRINT FILE G*M(I,J).
258 NEXT J
260 PRINT FILE " ",
262 FØR J=1 TØ 4
264 PRINT FILE F(I,J),
266 NEXT J
268 PRINT FILE
270 NEXT I
272 IF M9<4.5 THEN 378
274 FØR J=5 TØ 9
276 PRINT FILE R(J),
278 NEXT J
280 FØR I=1 TØ N9
282 FØR J=5 TØ 9
284 PRINT FILE G*M(I,J),
286 NEXT J
288 FØR J=5 TØ 9
290 PRINT FILE F(I,J),
292 NEXT J
294 PRINT FILE
296 NEXT I
298 IF M9<9.5 THEN 378
300 FØR J=10 TØ 14
302 PRINT FILE R(J),
304 NEXT J
306 FØR I=1 TØ N9
308 FØR J=10 TØ 14
310 PRINT FILE G*M(I,J),
312 NEXT J
314 FØR J=10 TØ 14
316 PRINT FILE F(I,J),
318 NEXT J
320 PRINT FILE
322 NEXT I
324 IF M9<14.5 THEN 378
326 IF Z>5 THEN 344
328 LETL5=INT(2*Z)+3
330 LETF1=.5*G1*G1*Z
332 LET F2=F1/3
334 FØRN=1TØL5
336 LETF1=-F1*.250*Z*Z/(N*N+N)
338 LETF2=F2+F1/(2*N+3)
340 NEXTN
342 GØTØ 354
344 LET F1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
346 LET F1=((F1-.1730503)/Z+.7034845)/Z-.06410 9E-3
348 LET F2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
350 LET F2=(F2-.187344E-2)/Z+.7979095
352 LET F2=(1-SQR(Z)*(F2*CØS(Z-PI/4)-F1*SIN(Z-PI/4)))/(X*X)
354 RETURN
356 IFZ>3THEN 366
358 LET G1=(((2.1E-11*Z↑2-5.38E-9)*Z↑2+6.757E-7)*Z↑2-5.42443E-5)*Z↑2
360 LET G1=((G1+2.60415E-3)*Z↑2-6.25E-2)*Z↑2+.5
362 LET J1=Z*G1
364 GØTØ 376
366 LET G3=(((-.14604057/Z+.27617679)/Z-.20210391)/Z+4.61835E-3)/Z
368 LET G3=((G3+.14937)/Z+4.68E-6)/Z+.79788456
```

```
370 LET G4=(((-.21262014/Z+.19397232)/Z+6.022188E-2)/Z-1.7222733E-1)/Z
372 LET G4=((G4+5.085E-4)/Z+.37498836)/Z-2.35619449+Z
374 LET J1=G3*CØS(G4)/SQR(Z)
376 RETURN
378 LET L=L+.1
380PRINT FILE
382 IF L<.25 THEN 44
384 END
```

Change in Phase of Induced Voltage Due to a Lattice of Defects in the Cladding Material with the "Lift-Off" Set

Discussion of BASREAD

This program is designed to calculate, with lift-off set, the variation with defect parameters of the phase difference of the voltages induced in two identical, oppositely wound pickup coils by the current in a large driver coil above a two-layered conductor (see Fig. 4, p. 38). For this program, the defect may be located at any one of a number of points on a lattice in the upper layer; it may also have any size, shape, and orientation. The pickup coils must be identical, equidistant from the respective ends of the driver coil, and be within its length. They must also be coaxial with the driver coil. The two conducting layers may have different electrical conductivities.

To use this program, one must first execute RFLCYC and then RFDFL2. The output data from the latter program comprises the input data for this program. It is worth noting that the data generated by the computer in the execution of RFDFL2 are stored in a data block, which is then read as input for this program. Since there exist BASIC compilers which do not have this capability, this program may or may not be executable, depending upon the compiler used.

Once the execution of RFLCYC and RFDFL2 is complete, one need only type the following lines into the program [Note: (Parentheses are not typed)].

```
12 LET M = (number of different size defects)
14 LET N = (number of shape and orientation factors)
28 LET V1 = (numerical value of discriminator voltage setting)
```

Note: This value may be obtained from the print-out of RFLCYC.

30 LET O1 = (numerical value of zero phase)

Note: This value may also be obtained from the print-out of RFLCYC.

290 DATA = (numerical value of normalized defect volumes)
292 DATA = ("N" shape and orientation factors[6])
294 DATA = (magnitude and phase of voltage for each lift-off in absence of defect)

Note: These values may be obtained from RFLCYC by inserting the lines 1090 PRINT L(J), M(J), P(J), and 1091 NEXT J. Of the three columns printed, the first contains lift-off values while the second and third contain the corresponding magnitude and phase values. The first three magnitude and phase values are the ones used here.

The program may now be run.

The print-out by the computer will have the following format:

```
DRIVER LENGTH IS (L1)      R1 = (R1)                R2 = (R2)
PICKUP LENGTH IS (L2)      R3 = (R3)                R4 = (R4)
CLAD THICKNESS IS (C)      M1 = (M1)                M2 = (M2)
PICKUP IS RECESSED (L3)    LIFT-OFF IS (L)
DISC VOLTAGE IS (V1)       ZERO PHASE IS (O1)
DEFECT VOLUME IS (V(D))    S AND O FACTOR IS (S(O))

    Z    ,   R   (R(1))     (R(2))     **********    (R(M9))
 (Z(1))          ------     ------     **********    ------
                 ------     ------     **********    ------
                 ------     ------     **********    ------

 (Z(2))          ------     ------     **********    ------
                 ------     ------     **********    ------
                 ------     ------     **********    ------

      *            *          *            *            *
      *            *          *            *            *
      *            *          *            *            *
      *            *          *            *            *

 (Z(N9))         ------     ------     **********    ------
                 ------     ------     **********    ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. All symbols not defined above are defined in the explanatory material for RFDFL2.

---

[6]C. V. Dodd, W. E. Deeds, J. W. Luquire, and W. G. Spoeri, Some Eddy-Current Problems and Their Integral Solutions, ORNL-4384 (April 1969).

The first value at each lattice point is the magnitude of the defect sensitivity factor, while the second one is the phase of the defect sensitivity factor for the case in which the defect is located at the given lattice point. The third value at each lattice point is the phase shift, with lift-off set, in the presence of the defect. It should be noted that the lattice points for which M9 > 4 will be printed below the last line [i.e., the line containing Z(N9)]. The entire format is repeated for each lift-off, defect volume, and shape and orientation factor.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of BASREAD

Suppose that we are interested in the case for which the input data is just that obtained from the sample calculation of RFDFL2. Furthermore, let us assume a spherical defect with a radius of 0.0125 in. in the upper layer.

First, the appropriate values are placed in RFLCYC and that program is executed, which yields the following information:

LIFT-OFF	MAGNITUDE	PHASE
0	0.1927237E-6	1.8335539
0.1	0.1469769E-6	1.824176
0.2	0.1135246E-6	1.8107041

V1 = −0.06306E-7   01 = −1.86628

Furthermore,

Normalized defect volume = 5.235988E-4
Shape and orientation factor = 1

This information is now typed into the program using the following format:

```
12 LET M = 1
14 LET N = 1
28 LET V1 = -0.06306E-7
30 LET 01 = -1.86628
290 DATA 5.235988E-4
292 DATA 1
294 DATA.1927237E-6,1.8335539,.1469769E-6,1.824176,.1135246E-6,1.8107041
```

Execution of the program yields the results that follow:

```
DRIVER LENGTH IS 8
PICKUP LENGTH IS 0.8
CLAD THICKNESS IS 0.4E-01
PICKUP IS RECESSED 0
DISC. VOLTAGE IS -0.0630599E-07  ZERO PHASE IS -1.86628
DEFECT VOLUME IS 0.5235987E-03   S AND O FACTOR IS 1
```

Z ,	R 0.25	0.75	1.25	1.75
-0.1E-01	0.077631E-06	0.2754239E-06	0.5103326E-07	0.6173647E-08
	-1.6201885	-1.5614521	-1.8482656	-2.0370686
	0.715163E-03	0.2116282E-03	0.0753722E-03	0.1174203E-04
-0.3E-01	0.6204239E-07	0.2191606E-06	0.4031928E-07	0.486054E-08
	-1.8616698	-1.7992387	-2.103904	-2.2950456
	0.0934912E-03	0.2983547E-03	0.080938E-03	0.1141581E-04

from which we see that a spherical defect with a radius of 0.0125 in. located at r = 1.25, z = -0.03 yields a phase difference, with lift-off set, equal to 0.080938E-03. The results for other values of lift-off have been omitted for the sake of brevity.

BASREAD Program

```
10 DIM M(18,18),P(18,18),R(18),Z(18),V(18),S(18)
12 LET M=5
14 LET N=1
16 FØR D=1 TØ M
18 READ V(D)
20 NEXT D
22 FØR Ø=1 TØ N
24 READ S(Ø)
26 NEXT Ø
28 LET V1=-.40926E-8
30 LET Ø1=-3.394969
32PRINT"                    DEFECT PRØGRAM FØR PHASE SENSITIVE EDDY"
34PRINT"                               CURRENT INSTRUMENT"
36PRINT"                                    (LATTICE)"
38PRINT
40PRINT
42 ØPEN /@1/,INPUT
44 READ V,P
46 INPUT FILE L1,R1,R2
48 INPUT FILE L2,R3,R4
50 INPUT FILE C,M1,M2
52 INPUT FILE L3,L
54 INPUT FILE M9,N9
56 FØR J=1 TØ 4
58 INPUT FILE R(J)
60 NEXT J
62 FØR I=1 TØ N9
```

```
64 INPUT FILE Z(I)
66 FØR J=1 TØ 4
68 INPUT FILE M(I,J)
70 NEXT J
72 FØR J=1 TØ 4
74 INPUT FILE P(I,J)
76 NEXT J
78 NEXT I
80 IF M9<4.5 THEN 130
82 FØR J=5 TØ 9
84 INPUT FILE R(J)
86 NEXT J
88 FØR I=1 TØ N9
90 FØR J=5 TØ 9
92 INPUT FILE M(I,J)
94 NEXT J
96 FØR J=5 TØ 9
98 INPUT FILE P(I,J)
100 NEXT J
102 NEXT I
104 IF M9<9.5 THEN 130
106 FØR J=10 TØ 14
108 INPUT FILE R(J)
110 NEXT J
112 FØR I=1 TØ N9
114 FØR J=10 TØ 14
116 INPUT FILE M(I,J)
118 NEXT J
120 FØR J=10 TØ 14
122 INPUT FILE P(I,J)
124 NEXT J
126 NEXT I
128 IF M9<14.5 THEN 130
130 FØR Ø=1 TØ N
132 FØR D=1 TØ M
134 PRINT"DRIVER LENGTH IS";L1,"R1=";R1,"R2=";R2
136 PRINT"PICKUP LENGTH IS";L2,"R3=";R3,"R4=";R4
138 PRINT"CLAD THICKNESS IS";C,"M1=";M1,"M2=";M2
140 PRINT "PICKUP IS RECESSED";L3,"LIFT ØFF IS";L
142 PRINT"DISC. VØLTAGE IS "V1,"ZERØ PHASE IS ";Ø1
144 PRINT"DEFECT VØLUME IS";V(D),"S & Ø FACTØR IS";S(Ø)
146 PRINT
148 PRINT "  Z  ,  R",
150 FØR J=1 TØ 4
152 PRINT R(J),
154 NEXT J
156 FØR I=1 TØ N9
158 PRINT Z(I),
160 FØR J=1 TØ 4
162 PRINT M(I,J),
164 NEXT J
166 PRINT " ",
168 FØR J=1 TØ 4
170 PRINT P(I,J),
172 NEXT J
174 PRINT" ",
```

```
176 FØR J=1 TØ 4
178 LET W1=V(D)*S(Ø)*M(I,J)*CØS(F(I,J))+V*CØS(P)
180 LET W2=V(D)*S(Ø)*M(I,J)*SIN(F(I,J))+V*SIN(P)
182 LET W3=ATN(W2/W1)
184 IF W1>0 THEN188
186 LET W3=W3+PI
188 PRINT Ø1-ATN(V1/SQR(W1↑2+W2↑2-V1↑2))+W3,
190 NEXT J
192 PRINT
194 NEXT I
196 IF M9<4.5 THEN 282
198 FØR J=5 TØ 9
200 PRINT R(J),
202 NEXT J
204 FØR I=1 TØ N9
206 FØR J=5 TØ 9
208 PRINT M(I,J),
210 NEXT J
212 FØR J=5 TØ 9
214 PRINT F(I,J).
216 NEXT J
218 FØR J=5 TØ 9
220 LET W1=V(D)*S(Ø)*M(I,J)*CØS(F(I,J))+V*CØS(P)
222 LET W2=V(D)*S(Ø)*M(I,J)*SIN(F(I,J))+V*SIN(P)
224 LET W3=ATN(W2/W1)
226 IF W1>0 THEN 230
228 LET W3=W3+PI
230 PRINT Ø1-ATN(V1/SQR(W1↑2+W2↑2-V1↑2))+W3,
232 NEXT J
234 PRINT
236 NEXT I
238 IF M9<9.5 THEN 282
240 FØR J=10 TØ 14
242 PRINT R(J),
244 NEXT J
246 FØR I=1 TØ N9
248 FØR J=10 TØ 14
250 PRINT M(I,J),
252 NEXT J
254 FØR J=10 TØ 14
256 PRINT F(I,J),
258 NEXT J
260 FØR J=10 TØ 14
262 LET W1=V(D)*S(Ø)*M(I,J)*CØS(F(I,J))+V*CØS(P)
264 LET W2=V(D)*S(Ø)*M(I,J)*SIN(F(I,J))+V*SIN(P)
266 LET W3=ATN(W2/W1)
268 IF W1>0 THEN 272
270 LET W3=W3+PI
272 PRINT Ø1-ATN(V1/SQR(W1↑2+W2↑2-V1↑2))+W3,
274 NEXT J
276 PRINT
278 NEXT I
280 IF M9<14.5 THEN 282
282 NEXT D
284 NEXT Ø
286 PRINT
```

```
288 IF L<.25 THEN 44
290 DATA .143675E-2,.214466E-2,.305362E-2,.418879E-2,.557527E-2
292 DATA 1
294DATA.136045E-6,3.364882,.103404E-6,3.35574,.079711E-6,3.343604
296 END
```

Change in Magnitude and Phase of Induced Voltage Due to a Defect in the Base Material

Discussion of RFDFTB

This program is designed to calculate the defect sensitivity factor for a reflection-type coil above a two-layered conductor (see Fig. 4, p. 38). For this program the defect must be in the lower layer. The pickup coil must be identical, equidistant from the respective ends of the driver coil, and be within its length. They must also be coaxial with the driver coil. In addition, the two conductor layers may have different electrical conductivities.

To use this program, one must first divide all dimensions by the mean radius of the driver coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, and the square of the mean driver coil radius must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper layer and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

12 LET R1 = (numerical value of normalized inner radius of driver coil)
14 LET R2 = (numerical value of normalized outer radius of driver coil)
16 LET R3 = (numerical value of normalized inner radius of pickup coil)
18 LET R4 = (numerical value of normalized outer radius of pickup coil)
20 LET L = (numerical value of normalized lift-off)
22 LET L1 = (normalized length of driver coil)
24 LET L2 = (normalized length of pickup coil)
26 LET L3 = (normalized distance of pickup coil from ends of driver coil)
28 LET M1 = ($\omega\mu_1\sigma_1\bar{r}^2$)
30 LET M2 = ($\omega\mu_2\sigma_2\bar{r}^2$)

Note: If the conductivity of the upper conductor is zero, one must type two additional lines:

```
112 LET X1 = X
114 LET Y1 = 0
```

```
32 LET C = (numerical value of normalized thickness of upper
            conductor)
34 LET R = (numerical value of normalized r position coordinate of
            defect)
36 LET Z1 = (numerical value of normalized z position coordinate
             of defect)
```

The program may now be run.

For each lift-off value, the print-out by the computer will have the following format:

```
              DEFECT PROGRAM FOR PHASE-SENSITIVE EDDY-
                         CURRENT INSTRUMENT

DRIVER LENGTH IS (L1)      R1 = (R1)       R2 = (R2)
PICKUP LENGTH IS (L2)      R3 = (R3)       R4 = (R4)
CLAD THICKNESS IS (C)      M1 = (M1)       M2 = (M2)
DEFECT LOCATION IS         R = (R)         Z = (Z1)
PICKUP IS RECESSED (L3)    LIFT-OFF IS (L)
X          REAL #1         IMAG #1         REAL #2        IMAG #2
 5         ------          ------          ------         ------
 10        ------          ------          ------         ------
 15        ------          ------          ------         ------
 20        ------          ------          ------         ------
 25        ------          ------          ------         ------
 30        ------          ------          ------         ------
VOLTAGE/FINN'R(BAR) = ------               PHASE = ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer, while the other four columns are the real and imaginary parts of the two integrals which appear in the expression for the defect voltage. These appear primarily to allow one to inspect the convergence of the integration. In addition to the phase of the defect sensitivity factor, the last line contains the magnitude of the defect sensitivity factor divided by the product of the frequency of the driving current, the number of turns on the driver and pickup coils, and the mean radius of the driver coil.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of RFDFTB

Let us suppose that we wish to know the defect sensitivity factor for the case of two pickup coils positioned inside the driver coil and flush with its ends. The driver coil is assumed to be 2 in. long with inner and outer radii of 0.200 and 0.300 in., respectively, while the pickup coils are assumed to be 0.200 in. long with inner and outer radii of 0.100 and 0.150 in., respectively. The conductors are assumed to be 0.010 in. of copper clad on an effectively infinite thickness of aluminum (at least four-coil diameters or skin depths, whichever is smaller). Let us further suppose that we are interested in the defect sensitivity factor for the case in which the defect is located 0.015 in. below the surface of the copper, 0.250 in. from the coil axis. We shall assume the driving frequency to be 5 kHz and the lift-off to be 0.

First, we determine the mean radius of the driver coil to be 0.250 in., so that

Normalized inner radius of driver coil = 0.200/0.250 = 0.8
Normalized outer radius of driver coil = 0.300/0.250 = 1.2
Normalized inner radius of pickup coils = 0.100/0.250 = 0.4
Normalized outer radius of pickup coils = 0.150/0.250 = 0.6
Normalized lift-off = 0/0.250 = 0
Normalized length of driver coil = 2.000/0.250 = 8
Normalized length of pickup coils = 0.200/0.250 = 0.8
Normalized distance of pickup coils from ends of driver coil = 0
Normalized thickness of copper = 0.010/0.250 = 0.04
Normalized r position of defect = 0.250/0.250 = 1
Normalized z position of defect = 0.015/0.250 = 0.06

Furthermore,

Angular frequency of driving current = 10,000 π sec^{-1}
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

This information is now typed into the program as follows:

```
12 LET R1 = 0.8
14 LET R2 = 1.2
16 LET R3 = 0.4
18 LET R4 = 0.6
20 LET L = 0
22 LET L1 = 8
24 LET L2 = 0.8
26 LET L3 = 0
28 LET M1 = 91.91
30 LET M2 = 45.48
32 LET C = 0.04
34 LET R = 1
36 LET Z1 = -0.06
```

The program is now run with the following results:

```
                 DEFECT PROGRAM FOR PHASE-SENSITIVE EDDY-
                           CURRENT INSTRUMENT

DRIVER LENGTH IS 8           R1 = 0.8         R2 = 1.2
PICKUP LENGTH IS 0.8         R3 = 0.4         R4 = 0.6
CLAD THICKNESS IS 0.4E-01    M1 = 91.91       M2 = 45.48
DEFECT LOCATION IS           R = 1            Z = -0.6E-01
PICKUP IS RECESSED 0         LIFT-OFF IS 0
X                            REAL # 1         IMAG # 1          REAL # 2         IMAG # 2
 5                           0.1113324E-01    -0.1749774E-01    0.135291E-02     -0.229157E-02
 10                          0.1374891E-01    -0.1911757E-01    0.450548E-03     -0.1722902E-02
 15                          0.1398878E-01    -0.1918473E-01    0.0672111E-02    -0.1785403E-02
 20                          0.1396504E-01    -0.1918094E-01    0.0658329E-02    -0.1783994E-02
 25                          0.1392592E-01    -0.1917596E-01    0.0655865E-02    -0.1783552E-02
 30                          0.1391729E-01    -0.1917509E-01    0.0658044E-02    -0.1783775E-02
VOLTAGE/FINN'R(BAR) = 0.4739195E-07                          PHASE = -2.1603427
```

from which we see that the phase of the defect sensitivity factor is −2.1603427, while its magnitude is the product of 0.4739195×10^{-7}, the frequency of the driving current, the number of turns on the driver coil and pickup coils, and the mean radius of the driver coil.

RFDFTB Program

```
10 REM          RFDFTB          VERSION          03/03/69
12 LET R1=.8
14 LET R2=1.2
16 LET R3=.4
18 LET R4=.6
20 LET L=0
22 LET L1=8
24 LET L2=.8
26 LET L3=0
28 LET M1=91.91
30 LET M2=45.48
```

```
32 LET C=.04
34 LET R=1
36 LET Z1=-.06
38 PRINT"                    DEFECT PRØGRAM FØR PHASE SENSITIVE EDDY"
40 PRINT"                              CURRENT INSTRUMENT"
42 PRINT
44 PRINT
46 PRINT"DRIVER LENGTH IS";L1,"R1=";R1,"R2=";R2
48 PRINT"PICKUP LENGTH IS";L2,"R3=";R3,"R4=";R4
50 PRINT"CLAD THICKNESS IS";C,"M1=";M1,"M2=";M2
52 PRINT"DEFECT LØCATIØN IS","R=";R,"Z=";Z1
54 PRINT "PICKUP IS RECESSED";L3,"LIFT ØFF IS";L
56 PRINT"X","REAL #1","IMAG #1","REAL #2","IMAG #2"
58 LET S1=1E-2
60 LET S2=5
62 FØR I=1 TØ 2
64 LETR(I)=0
66 LETI(I)=0
68 NEXT I
70 LET B1=0
72 LET B2 =S2
74 FØRX=B1+S1/2TØB2STEPS1
76 LET Z=R2*X
78 LET Q1=R2
80 GØSUB 194
82 LETI2=F2
84 LET Z=R1*X
86 LET Q1=R1
88 GØ SUB 194
90 LET I1=F2
92 LET S3=(I2-I1)*S1*(EXP(-X*L)-EXP(-X*(L+L1)))
94 LET Z=R4*X
96 LETQ1=R4
98 GØ SUB 194
100 LET I4=F2
102 LET Z=R3*X
104 LET Q1=R3
106 GØ SUB 194
108 LET I3=F2
110 LET S4=(I4-I3)*S1*(EXP(-X*L)-EXP(-X*(L+L2)))*EXP(-X*L3)
112 LET X1 = .707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
114 LET Y1 = .707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
116 LET X2 = .707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
118 LET Y2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
120 LET X3=EXP(2*X1*C)
122 LET Y3=CØS(2*Y1*C)
124 LET Y4=SIN(2*Y1*C)
126 LET X5=CØS((Y1+Y2)*C+Y2*Z1)
128 LET Y5=SIN((Y1+Y2)*C+Y2*Z1)
130 LET X6=2*EXP((X1+X2)*C+X2*Z1)
132 LET A5=X1*X5-Y1*Y5
134 LET B5=Y1*X5+X1*Y5
136 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
138 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
140 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
142 LETD5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
144 LETK1=X6*(A5*C5+B5*D5)/(C5*C5+D5*D5)
146 LET K2=X6*(C5*B5-A5*D5)/(C5*C5+D5*D5)
148 LETZ=X*R
150 GØSUB224
```

```
152 LETR(1)=R(1)+S3*K1*J1
154 LET I(1)=I(1)+S3*K2*J1
156 LET R(2)=R(2)+S4*K1*J1
158 LET I(2)=I(2)+S4*K2*J1
160 NEXT K
162 PRINT X+S1/2,R(1),I(1),R(2),I(2)
164 LETB1=B1+S2
166 LETB2=B2+S2
168 LETS1=5E-2
170 IF X<15 THEN 74
172 LET S1=.1
174 IF X<25 THEN 74
176 LET R(3)=R(1)*R(2)-I(1)*I(2)
178 LET I(3)=R(1)*I(2)+R(2)*I(1)
180 LET P=ATN(I(3)/R(3))
182 IF R(3)>0 THEN 186
184 LET P=P-PI
186 LET M=SQR(R(3)*R(3)+I(3)*I(3))
188 LET Q=3*2.4805E-5*M2/(2*PI*(R2-R1)*(R4-R3)*L1*L2)
190 PRINT"VØLTAGE/FINN'R(BAR)=";Q*M,"PHASE=";P
192 GØ TØ 246
194 IF Z>5 THEN 212
196 LETL5=INT(2*Z)+3
198 LETF1=.5*Q1*Q1*Z
200 LET F2=F1/3
202 FØRN=1TØL5
204 LETF1=-F1*.250*Z*Z/(N*N+N)
206 LETF2=F2+F1/(2*N+3)
208 NEXTN
210 GØTØ 222
212 LET F1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
214 LET F1=((F1-.1730503)/Z+.7034845)/Z-.064109E-3
216 LET F2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
218 LET F2=(F2-.187344E-2)/Z+.7979095
220 LET F2=(1-SQR(Z)*(F2*CØS(Z-PI/4)-F1*SIN(Z-PI/4)))/(X*X)
222 RETURN
224 IFZ>3THEN 234
226 LET Q1=(((2.1E-11*Z↑2-5.38E-9)*Z↑2+6.757E-7)*Z↑2-5.42443E-5)*Z↑2
228 LET Q1=((Q1+2.60415E-3)*Z↑2-6.25E-2)*Z↑2+.5
230 LET J1=Z*Q1
232 GØTØ 244
234 LET Q3=(((-.14604057/Z+.27617679)/Z-.20210391)/Z+4.61835E-3)/Z
236 LET Q3=((Q3+.14937)/Z+4.68E-6)/Z+.79788456
238 LET Q4=(((-.21262014/Z+.19397232)/Z+6.022188E-2)/Z-1.7222733E-1)/Z
240 LET Q4=((Q4+5.085E-4)/Z+.37498836)/Z-2.35619449+Z
242 LET J1=Q3*CØS(Q4)/SQR(Z)
244 RETURN
246 LET L=L+.1
248 PRINT
250 IF L<.25 THEN 46
252 END
```

THROUGH-TRANSMISSION COILS

This type of coil and conductor configuration, shown in Fig. 6, is also used with the phase-sensitive eddy-current instrument. It consists of a large driver coil and a pickup coil on opposite sides of a conductor. The signal transmitted by the driver coil passes through the conductor and is then detected by the pickup coil.

In this section there are programs that calculate the magnitude and phase of the induced voltage (THRU5) and the change in the magnitude and phase of the induced voltage due to a defect (THRUDF). In addition, the program, DISC, in the preceding section can be used to calculate the phase shift of the induced voltage with the "lift-off" set.

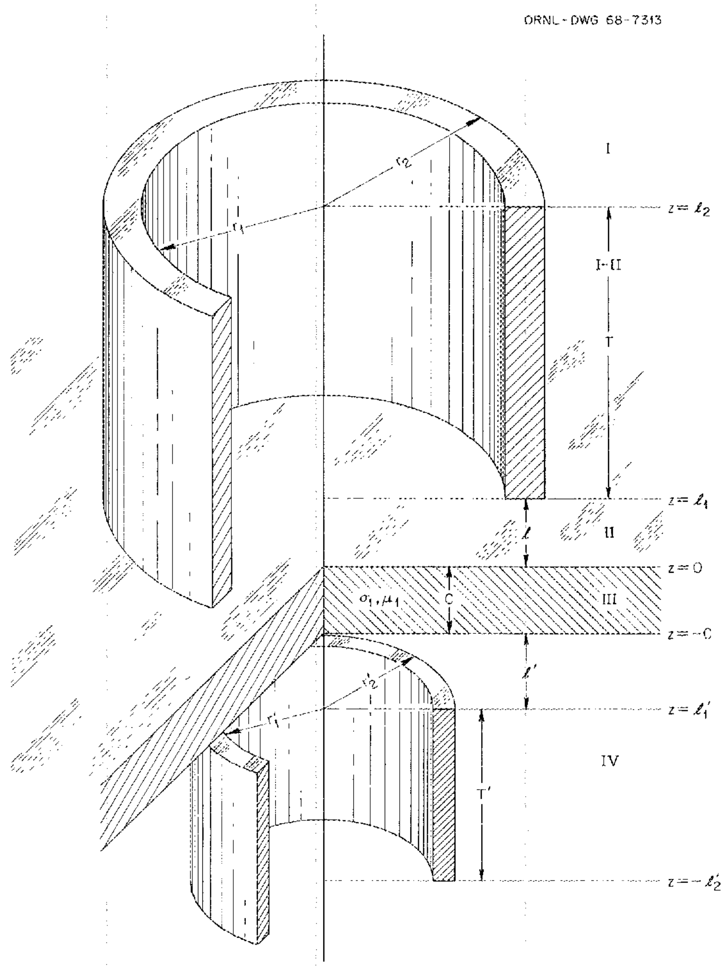

Fig. 6. A Through-Transmission Coil Arrangement.

Magnitude and Phase of Induced Voltage

Discussion of THRU5

This program is designed to calculate the magnitude and phase of the "voltage" induced in a pickup coil of rectangular cross section by the current in a driver coil of rectangular cross section positioned on the opposite side of a conducting plate. The coils must be coaxial but may have different lengths and radii. The physical situation is shown in Fig. 6.

To use this program, one must first divide all dimensions by the mean radius of the driver coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean driver coil radius must be calculated for the conducting plate. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$.

Once these calculations have been made one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

```
20 LET R1 = (numerical value of normalized inner radius of driver
             coil)
30 LET R2 = (numerical value of normalized outer radius of driver
             coil)
40 LET R3 = (numerical value of normalized inner radius of pickup
             coil)
50 LET R4 = (numerical value of normalized outer radius of pickup
             coil)
60 LET L = (normalized lift-off of driver coil plus normalized
            lift-off of pickup coil)
70 LET L2 = (normalized length of driver coil)
80 LET L4 = (normalized length of pickup coil)
90 LET M1 = (numerical value of ωμ1σ1r̄²)
```

Note: If the conductivity of the conducting plate is zero, one must type two additional lines:

```
400 LET X1 = X
410 LET Y1 = 0
```

```
100 LET U1 = (numerical value of relative permeability of conducting
              plate)
110 LET C = (numerical value of normalized thickness of conducting
             plate)
```

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)          R2 = (R2)      R3 = (R3)      R4 = (R4)
L2 = (L2)          L4 = (L4)      LIFT-OFF = (L1)
THICKNESS = (C)    M1 = (M1)      U1 = (U1)
X                  REAL OF PICKUP IMAG OF PICKUP
1.                 ------         ------
2.                 ------         ------
3.                 ------         ------
4.                 ------         ------
5.                 ------         ------
10.                ------         ------
15.                ------         ------
25.                ------         ------
35.                ------         ------
MAGNITUDE IS ------               PHASE IS ------
        VOLTAGE/FINN'R(BAR) = ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The other two columns are the real and imaginary parts of the integral. The next to the last line contains the magnitude and phase of the integral. The last line printed is the magnitude of the peak (or rms) induced voltage divided by the product of the frequency of the driving current, the magnitude of the peak (or rms) current, the number of turns on the driver coil, the number of turns on the pickup coil, and the mean radius of the driver coil in meters.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of THRU5

Suppose that we wish to know the voltage induced in a pickup coil 0.025 in. long with inner and outer radii of 0.4875 and 0.5125 in., respectively, by the current in a driver coil positioned on the opposite side of a 0.250-in.-thick copper plate. The driver is assumed to be 0.800 in. long with inner and outer radii of 0.100 and 0.900 in., respectively. The frequency of the driving current is assumed to be 1.25 kHz, the lift-off of the driver coil to be 0.0125 in., and the lift-off of the pickup coil to be 0.0125 in.

First, we determine the mean radius of the driver coil to be 0.500 in., so that

Normalized inner radius of driver coil = 0.100/0.500 = 0.2
Normalized outer radius of driver coil = 0.900/0.500 = 1.8
Normalized inner radius of pickup coil = 0.4875/0.500 = 0.0975
Normalized outer radius of pickup coil = 0.5125/0.500 = 1.025
Normalized lift-off of driver coil = 0.0125/0.500 = 0.025
Normalized lift-off of pickup coil = 0.0125/0.500 = 0.025
Normalized length of driver coil = 0.800/0.500 = 1.6
Normalized length of pickup coil = 0.025/0.500 = 0.05
Normalized thickness of copper plate = 0.250/0.500 = 0.5

Furthermore,

Angular frequency of driving current = $2\pi \times 1250\ \mathrm{sec}^{-1}$
Permeability of copper = $4\pi \times 10^{-7}$ h/m
Relative permeability of copper = 1
Conductivity of copper = 5.77×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$

This information is now typed into the program in the following format:

```
20 LET R1 = 0.2
30 LET R2 = 1.6
40 LET R3 = 0.0975
50 LET R4 = 1.025
60 LET L = 0.05
70 LET L2 = 1.6
80 LET L4 = 0.05
90 LET M1 = 91.91
100 LET U1 = 1
110 LET C = 0.5
```

The program is now run with the following results:

```
R1 = 0.2            R2 = 1.6          R3 = 0.0975   R4 = 1.025
L2 = 1.6            L4 = 0.5B-01      LIFT-OFF = 0.5E-01
THICKNESS = 0.5     M1 = 91.91        U1 = 1
X                   REAL OF PICKUP    IMAG OF PICKUP
1                   -0.2095097E-05     0.2713761E-05
2                   -0.1267416E-04     0.1313368E-04
3                   -0.2058863E-04     0.1865904E-04
4                   -0.2141352E-04     0.1906794E-04
5                   -0.211496E-04      0.1901738E-04
10                  -0.2112395E-04     0.1902734E-04
15                  -0.2112471E-04     0.190259E-04
25                  -0.2112472E-04     0.1902593E-04
35                  -0.2112472E-04     0.1902593E-04
MAGNITUDE IS 0.2842956E-04         PHASE IS -3.979216
        VOLTAGE/FINN'R (BAR) = 0.2715425E-07
```

from which we see that the phase of the induced voltage is −3.979216. The actual magnitude of the induced voltage is obtained by multiplying the last line by the frequency of the driving current (1250), the magnitude of the current, the number of turns on the driver coil, the number of turns on the pickup coil, and the mean radius of the driver coil in meters.

<u>THRU5 Program</u>

```
10 REM           THRU5           VERSION           07/17/68
20 LET R1=0
30 LET R2=2
40 LET R3=.975
50 LET R4=1.025
60 LET L=0
70 LET L2=2
80 LET L4=.05
90 LET M1=77.05
100 LET U1=100
110 LET C=.5
120 PRINT "R1=";R1,"R2=";R2,"R3=";R3,"R4=";R4
130 PRINT"L2=";L2,"L4=";L4,"LIFTOFF=";L
140PRINT"THICKNESS=";C,"M1=";M1,"U1=";U1
150 PRINT"X","REAL OF PICKUP","IMAG OF PICKUP"
160 LET S1=1E-2
170 LET S2=1
180 LET I6=0
190 LET I8=0
200 LET B1=0
210 LET B2 =S2
220 FOR X=B1+(S1)/2 TO B2 STEP S1
230 LET Z=R2*X
240 LET Q1=R2
250 GOSUB 760
260 LET I2=F2
270 LET Z=R1*X
280 LET Q1=R1
290 GOSUB 760
300 LET I1=F2
310 LET Z=R4*X
320 LET Q1=R4
330 GOSUB 760
340 LET J2=F2
350 LET Z=R3*X
360 LET Q1=R3
370 GOSUB 760
380 LET J1=F2
390 LET S3=X*S1*(I2-I1)*(J2-J1)
```

```
400LETX1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)/U1
410LETY1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)/U1
420 LET X3=EXP(2*X1*C*U1)
430 LET Y3=CØS(2*Y1*C*U1)
440 LET Y4=SIN(2*Y1*C*U1)
450 LETA3=X1*CØS(Y1*C)-Y1*SIN(Y1*C)
460 LETB3=Y1*CØS(Y1*C)+X1*SIN(Y1*C)
470 LETC1=((X+X1)*(X+X1)-Y1*Y1)*Y3-2*Y1*(X+X1)*Y4
480 LETC2=Y1*Y1-(X-X1)*(X-X1)+X3*C1
490 LETD1=Y4*((X+X1)*(X+X1)-Y1*Y1)+2*Y1*(X+X1)*Y3
500 LETD2=2*Y1*(X-X1)+X3*D1
510 LETK1=(A3*C2+B3*D2)/(C2*C2+D2*D2)
520 LETK2=(B3*C2-A3*D2)/(C2*C2+D2*D2)
530 LETG=(1-EXP(-X*L2))*(1-EXP(-X*L4))*EXP(-L*X+X1*C)
540LETI6=I6+S3*K1*G
550LETI8=I8+S3*K2*G
560 NEXT X
570LETB1=B2
580 LETB2=B2+S2
590 PRINT X+S1/2,I6,I8
600 IFX<3.5THEN220
610 LET S1=5E-2
620LETS2=5
630IFX<9THEN220
640LETS2=10
650LETS1=.1
660IFX<25THEN220
670 LETQ1=SQR(I6*I6+I8*I8)
680 LET Q2=ATN(-I6/I8)
690 IF I8<0 THEN 710
700LETQ2=Q2+3.1415927
710 PRINT"MAGNITUDE IS";Q1,"PHASE IS";-Q2
720 PRINT "            VØLTAGE/FINN'K(BAR)=";
730 PRINT 9.92201E-5*Q1/((K4-K3)*(K2-K1)*L2*L4)
740PRINT
750 GØ TØ 910
760 IFZ>5 THEN 850
770 LET L9=INT(2*Z)+3
780 LET F1=.5*Q1*Q1*Q1
790 LET F2=F1/3
800 FØR N=1TØL9
810 LETF1=-F1*.250*Z*Z/(N*N+N)
820 LETF2=F2+F1/(2*N+3)
830 NEXTN
840 GØTØ 900
850 LET Q1=(((-183.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
860 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
870 LET Q2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
880 LET Q2=(Q2-.187344E-2)/Z+.7979095
890 LET F2=(1-SQR(Z)*(Q2*CØS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X*X)
900 RETURN
910 END
```

Change in Magnitude and Phase of Induced Voltage Due to a Defect in the Plate Between the Coils

Discussion of THRUDF

This program is designed to calculate the defect sensitivity factor for a pickup coil of rectangular cross section due to the current in a driver coil of rectangular cross section positioned on the opposite side of the conducting plate containing the defect. The coils must be coaxial but may have different lengths and radii. The physical situation is shown in Fig. 6, p. 79.

To use the program, one must first divide all dimensions by the mean driver coil radius. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity and the square of the mean driver coil radius must be calculated for the conducting plate. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

```
12 LET R1 = (numerical value of normalized inner radius of driver
             coil)
14 LET R2 = (numerical value of normalized outer radius of driver
             coil)
16 LET R3 = (numerical value of normalized inner radius of pickup
             coil)
18 LET R4 = (numerical value of normalized outer radius of pickup
             coil)
20 LET L1 = (numerical value of normalized lift-off of driver coil)
22 LET T1 = (numerical value of normalized length of driver coil)
24 LET L2 = (numerical value of normalized lift-off of pickup coil)
26 LET T2 = (numerical value of normalized length of pickup coil)
28 LET M1 = (normalized ωμ1σ1r̄²)
30 LET C = (normalized thickness of conducting plate)
32 LET R = (normalized r position coordinate of defect)
34 LET Z1 = (normalized z position coordinate of defect)
```

The program may now be run.

The print-out by the computer will have the following format:

DRIVER LENGTH IS (T1)		R1 = (R1)	R2 = (R2)	
PICKUP LENGTH IS (T2)		R3 = (R3)	R4 = (R4)	
DRIVER LIFT-OFF IS (L1)		PICKUP LIFT-OFF IS (L2)		
PLATE THICKNESS IS (C)		M1 = (M1)		
DEFECT POSITION IS		R = (R)	Z = (Z1)	
X	REAL #1	IMAG #1	REAL #2	IMAG #2
1	------	------	------	------
2	------	------	------	------
3	------	------	------	------
4	------	------	------	------
5	------	------	------	------
10	------	------	------	------
15	------	------	------	------
20	------	------	------	------
25	------	------	------	------
30	------	------	------	------
35	------	------	------	------
VOLTAGE/FINN R'(BAR) = ------			PHASE = ------	

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer, while the other four columns are the real and imaginary parts of the two integrals which appear in the expression for the defect induced voltage. These appear primarily to allow one to inspect the convergence of the integration. In addition to the phase of the defect sensitivity factor, the last line contains the magnitude of the defect sensitivity factor divided by the product of the frequency of the driving current, the number of turns on the driver and pickup coils, and the mean radius of the driver coil.

The example below is presented for additional aid to those who wish to use the program.

Sample Calculation of THRUDF

Suppose that we wish to know the defect sensitivity factor for a pickup coil 0.025 in. long with inner and outer radii of 0.4875 and 0.5125 in., respectively, positioned on the opposite side of a 0.250-in.-thick copper plate from a driver coil that is 0.800 in. long with inner and outer radii of 0.100 and 0.900 in., respectively. Let us further suppose that we are interested in the defect sensitivity factor for the coil in which the defect is located 0.125 in. below the surface of the copper 0.500 in. from the coil axis. The frequency of the driving current is assumed to be 1.25 kHz, the lift-off of the driver coil to be 0.0125 in. and the lift-off of the pickup coil to be 0.0125 in.

First, we determine the mean radius of the driver coil to be 0.500 in., so that

Normalized inner radius of driver coil = 0.100/0.500 = 0.2
Normalized outer radius of driver coil = 0.900/0.500 = 1.8
Normalized inner radius of pickup coil = 0.4875/0.500 = 0.0975
Normalized outer radius of pickup coil = 0.5125/0.500 = 1.025
Normalized lift-off of driver coil = 0.0125/0.500 = 0.025
Normalized lift-off of pickup coil = 0.0125/0.500 = 0.025
Normalized length of driver coil = 0.800/0.500 = 1.6
Normalized length of pickup coil = 0.0125/0.500 = 0.025
Normalized thickness of copper plate = 0.250/0.500 = 0.5
Normalized r position of defect = 0.500/0.500 = 1
Normalized z position of defect = 0.125/0.500 = 0.25

Furthermore,

Angular frequency of driving current = $2\pi \times 1250$ sec^{-1}
Permeability of copper = $4\pi \times 10^{-7}$ mhos/m
Relative permeability of copper = 1
Conductivity of copper = 5.77×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$

This information is now typed into the program in the following format:

```
12 LET R1 = 0.2
14 LET R2 = 1.8
16 LET R3 = 0.0975
18 LET R4 = 1.025
20 LET L1 = 0.025
22 LET T1 = 1.6
24 LET L2 = 0.025
26 LET T2 = 0.05
28 LET M1 = 91.91
30 LET C = 0.5
32 LET R = 1
34 LET Z1 = -0.25
```

The program is now run with the following results:

DRIVER LENGTH IS 1.6 R1 = 0.2 R2 = 1.8
PICKUP LENGTH IS 0.5E-01 R3 = 0.0975 R4 = 1.025
DRIVER LIFT-OFF IS 0.25E-01 PICKUP LIFT-OFF IS 0.25E-01
PLATE THICKNESS IS 0.5 M1 = 91.91
DEFECT POSITION IS R = 1 Z = -0.25

X	REAL #1	IMAG #1	REAL #2	IMAG #2
1	0.0629491E-01	0.0632373E-01	0.0691648E-03	0.0694502E-0
2	0.3331064E-01	0.328999E-01	0.0712987E-02	0.0700937E-0
3	0.464029E-01	0.453173E-01	0.1760296E-02	0.1686355E-0
4	0.4647456E-01	0.4538658E-01	0.2099335E-02	0.1990084E-0
5	0.4626008E-01	0.4522971E-01	0.1793536E-02	0.1744791E-0
10	0.4658471E-01	0.4536996E-01	0.1921084E-02	0.17991E-02
15	0.4640574E-01	0.4533249E-01	0.1934439E-02	0.1801759E-0
25	0.4642996E-01	0.4533432E-01	0.1946801E-02	0.1802697E-0
35	0.4643259E-01	0.4533443E-01	0.1949332E-02	0.1802795E-0

VOLTAGE/FINN'R (BAR) = 0.0789925E-05 PHASE = 1.5197954

From this we see that the phase of the defect sensitivity factor is 1.5197954, while its actual magnitude is obtained by multiplying 7.89925×10^{-7} by the frequency of the driving current (1250), the number of turns on the driver coil, the number of turns on the pickup coil, and the mean radius of the driver coil in meters.

THRUDF Program

```
10 REM          THRUDF          VERSION          03/03/69
12 LET R1=.2
14 LET R2=1.8
16 LET R3=.0975
18 LET R4=1.025
20 LET L1=.025
22 LET T1=1.6
24 LET L2=.025
26 LET T2=.05
28 LET M1=91.91
30 LET C=.5
32 LET R=1
34 LET Z1=-.25
36 PRINT"DRIVER LENGTH IS";T1,"R1=";R1,"R2=";R2
38 PRINT"PICKUP LENGTH IS";T2,"R3=";R3,"R4=";R4
40 PRINT"DRIVER LIFTØFF IS";L1,"PICKUP LIFTØFF IS";L2
42 PRINT"PLATE THICKNESS IS";C,"M1=";M1
44 PRINT"DEFECT PØSITIØN IS","R=";R,"Z=";Z1
46 PRINT"X","REAL #1","IMAG #1","REAL #2","IMAG #2"
48 LET S1=1E-2
50 LET S2=1
52 FØR J=1 TØ 2
54 LET R(J)=0
56 LET I(J)=0
58 NEXT J
60 LET B1=0
```

```
62 LET B2=S2
64 FØR X=B1+S1/2 TØ B2 STEP S1
66 LET Z=R2*X
68 LET Q1=R2
70 GØ SUB 210
72 LET I2=F2
74 LET Z=R1*X
76 LET Q1=R1
78 GØ SUB 210
80 LET I1=F2
82 LET Z=R4*X
84 LET Q1=R4
86 GØ SUB 210
88 LET J3=F2
90 LET Z=R3*X
92 LET Q1=R3
94 GØ SUB 210
96 LET J2=F2
98 LET Z=R*X
100 GØ SUB 240
102 LET S3=S1*(I2-I1)*J1
104 LET S4=S1*(J3-J2)*J1
106 LET X1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
108 LET Y1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
110 LET X3=EXP(2*X1*C)
112 LET Y3=CØS(2*Y1*C)
114 LET Y4=SIN(2*Y1*C)
116 LET C6=(X+X1)*(X1+X2)-Y1*(Y1+Y2)
118 LET C7=Y1*(X1+X2)+(X+X1)*(Y1+Y2)
120 LET C5=(X-X1)*(X1-X2)+Y1*(Y1-Y2)+(C6*Y3-C7*Y4)*X3
122 LET D5=(X-X1)*(Y1-Y2)-Y1*(X1-X2)+(C7*Y3+C6*Y4)*X3
124 LET K=C5*C5+D5*D5
126 LET A1=X3*((X+X1)*Y3-Y1*Y4)
128 LET C1=X3*((X+X1)*Y4+Y1*Y3)
130 LET L=L1
132 LET T=T1
134 LET Y=Z1
136 GØ SUB 262
138 LET A4=A2
140 LET B4=C2
142 LET L=L2
144 LET T=T2
146 LET Y=-(Z1+C)
148 GØ SUB 262
150 LET A5=A2
152 LET B5=C2
154 LET K1=(A4*C5+B4*D5)/K
156 LET K2=(A4*D5-B4*C5)/K
158 LET K3=(A5*C5+B5*D5)/K
160 LET K4=(A5*D5-B5*C5)/K
162 LET R(1)=R(1)+S3*K1
164 LET I(1)=I(1)+S3*K2
166 LET R(2)=R(2)+S4*K3
168 LET I(2)=I(2)+S4*K4
```

```
170 NEXT X
172 PRINT X+S1/2,R(1),I(1),R(2),I(2)
174 LET B1=B1+S2
176 LET B2=B2+S2
178 IF X<3.5 THEN 64
180 LET S1=5E-2
182 LET S2=5
184 IF X<9 THEN 64
186 LET S1=.1
188 LET S2=10
190 IF X<25 THEN 64
192 LET R(3)=R(1)*R(2)-I(1)*I(2)
194 LET I(3)=R(1)*I(2)+R(2)*I(1)
196 LET M=SQR(R(3)*R(3)+I(3)*I(3))
198 LET P=ATN(I(3)/R(3))
200 IF R(3)>0 THEN 204
202 LET P=P+PI
204 LET G=3*2.4805E-5*M1/(4*PI*T1*T2*(R2-R1)*(R4-R3))
206 PRINT"VOLTAGE/FINN'R(BAR)=";G*M,"PHASE=";P
208 GO TO 276
210 IFZ>5THEN 228
212 LETL5=INT(2*Z)+3
214 LETF1=.5*Q1*Q1*Z
216 LET F2=F1/3
218 FORN=1TOL5
220 LETF1=-F1*.250*Z*Z/(N*N+N)
222 LETF2=F2+F1/(2*N+3)
224 NEXTN
226 GOTO 238
228 LET Q1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
230 LET Q1=((Q1-.1730503)/Z+.7034845)/Z-.064109E-3
232 LET Q2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
234 LET Q2=(Q2-.187344E-2)/Z+.7979095
236 LET F2=(1-SQR(Z)*(Q2*COS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(X*X)
238 RETURN
240 IFZ>3THEN 250
242 LET Q1=(((2.1E-11*Z↑2-5.38E-9)*Z↑2+6.757E-7)*Z↑2-5.42443E-5)*Z↑2
244 LET Q1=((Q1+2.60415E-3)*Z↑2-6.25E-2)*Z↑2+.5
246 LET J1=Z*Q1
248 GOTO 260
250 LET Q3=(((-.14604057/Z+.27617679)/Z-.20210391)/Z+4.61835E-5)/Z
252 LET Q3=((Q3+.14937)/Z+4.68E-6)/Z+.79788456
254 LET Q4=(((-.21262014/Z+.19397232)/Z+6.022188E-2)/Z-1.7222733E-1)/Z
256 LET Q4=((Q4+5.085E-4)/Z+.37498836)/Z-2.35619449+Z
258 LET J1=Q3*COS(Q4)/SQR(Z)
260 RETURN
262 LET X4=EXP(-X1*Y)
264 LET X5=COS(Y1*Y)
266 LET Y5=SIN(Y1*Y)
268 LET X6=EXP(-X*L)*(1-EXP(-X*T))
270 LET A2=X6*(A1+X4*((X-X1)*X5-Y1*Y5))
272 LET C2=X6*(C1-X4*((X-X1)*Y5+Y1*X5))
274 RETURN
276 END
```

COIL BETWEEN TWO CONDUCTING PLANES

A coil between two conducting planes, shown in Fig. 7, is normally used to measure the spacing between the two planes.[7] The program (BTNCO) in this section calculates the normalized coil impedance as a function of the various coil and conductor variables, including the spacing between the planes.

Discussion of BTNCO

This program is designed to calculate the normalized impedance of a coil of rectangular cross section between two conducting plates of

[7] C. V. Dodd, Microtecnic 18(5,6), 1–7 (1964).

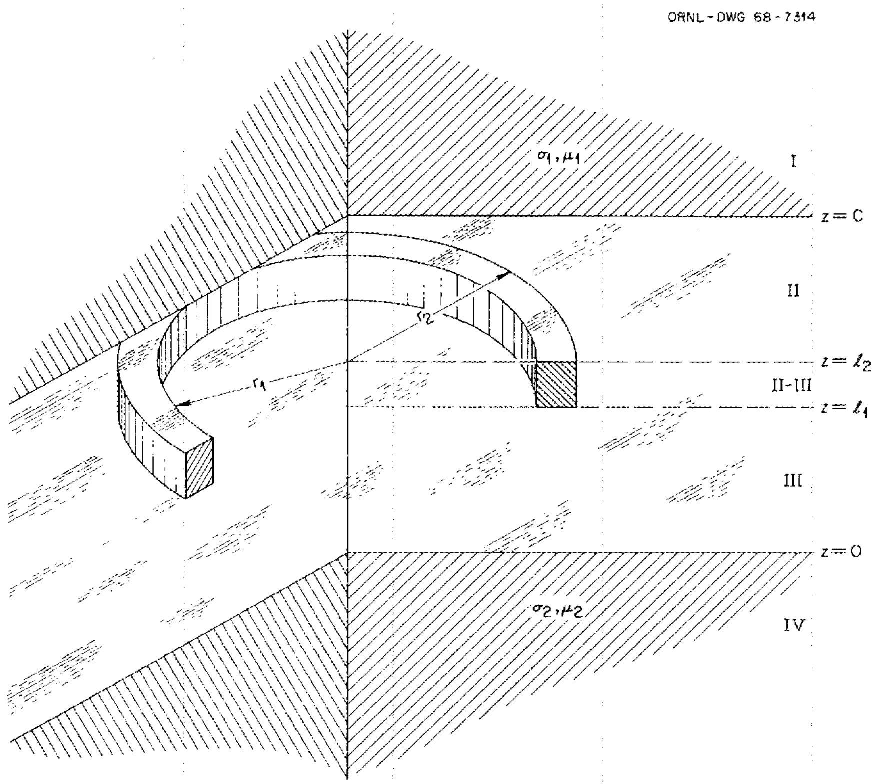

Fig. 7. A Coil of Rectangular Cross Section Between Two Conducting Plates.

effectively infinite thicknesses (at least four-coil diameters or skin depths, whichever is smaller) (see Fig. 7). The plates may have different permeabilities and electrical conductivities.

To use this program, one must divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the upper plate and by $\omega\mu_2\sigma_2\bar{r}^2$ for the lower one.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

```
20 LET R1 = (Numerical value of normalized inner coil radius)
30 LET R2 = (Numerical value of normalized outer coil radius)
40 LET L1 = (Numerical value of normalized lift-off from lower
             conductor)
50 LET L2 = (Numerical value of normalized lift-off from lower
             conductor plus normalized coil length)
60 LET M1 = (Numerical value of ωμ1σ1r̄²)
```

Note: If the conductivity of the upper plate is zero, one must type two additional lines:

```
320 LET X1 = X
330 LET Y1 = 0
```

```
70 LET U1 = (Numerical value of relative permeability of upper
             conductor)
80 LET M2 = (Numerical value of ωμ2σ2r̄²)
```

Note: If the conductivity of the lower plate is zero, one must type two additional lines:

```
340 LET X2 = X
350 LET Y2 = 0
```

```
90 LET U2 = (Numerical value of relative permeability of lower
             conductor)
100 LET C = (Numerical value of normalized separation of conductors)
```

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)          R2 = (R2)       L1 = (L1)       L2 = (L2)
M1 = (M1)          U1 = (U1)       M2 = (M2)       U2 = (U2)
                   CONDUCTOR SEPARATION IS (C)
X                  AIR VALUE       REAL PART       IMAG PART
 5                 ------          ------          ------
 10                ------          ------          ------
 15                ------          ------          ------
 20                ------          ------          ------
 40                ------          ------          ------
NORMALIZED IMAG PART ------  NORMALIZED REAL PART ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The second column, headed "AIR VALUE," is the value of the integral in the absence of the two conductors, while the remaining two columns are the real and imaginary parts of the integral in the presence of the conductors. These appear chiefly to allow one to inspect the convergence of the integration. The last line is the normalized impedance of the coil in the presence of the conductors.

The example below is presented for additional aid to those who wish to use the program.

Sample Calculation of BTNCO

Let us suppose that we wish to know the normalized impedance of a coil of rectangular cross section 0.100 in. long with inner and outer radii of 0.125 and 0.375 in., respectively, positioned between two conducting plates which are separated by a distance of 0.125 in. The upper conductor is assumed to be copper, while the lower is assumed to be aluminum. The bottom of the coil is taken to be 0.005 in. from the aluminum conductor, and the frequency of the driving current to be 5 kHz.

First, we determine the mean coil radius to be 0.250 in., so that

Normalized inner coil radius = 0.125/0.250 = 0.5
Normalized outer coil radius = 0.375/0.250 = 1.5
Normalized lift-off = 0.005/0.250 = 0.02
Normalized coil length = 0.100/0.250 = 0.4
Normalized separation of conductors = 0.125/0.250 = 0.5

Furthermore,

Angular frequency of driving current = $2\pi \times 5000\ \text{sec}^{-1}$
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Relative permeability of both conductors = 1
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\overline{r}^2 = 91.91$$

$$\omega\mu_2\sigma_2\overline{r}^2 = 45.48$$

This information is now typed into the program using the following format:

```
20 LET R1 = 0.5
30 LET R2 = 1.5
40 LET L1 = 0.02
50 LET L2 = 0.42
60 LET M1 = 91.91
70 LET U1 = 1
80 LET M2 = 45.48
90 LET U2 = 1
100 LET C = 0.5
```

The program is now run with the following results:

```
R1 = 0.5          R2 = 1.5      L1 = 0.2E-01    L2 = 0.42
M1 = 91.91        U1 = 1        M2 = 45.48      U2 = 1
                  CONDUCTOR SEPARATION IS 0.5
X                 AIR VALUE     REAL PART       IMAG PART
 5                0.06617157    0.2512494E-01   -0.0852233E-01
 10               0.0557785     0.2569981E-01   -0.0856341E-01
 15               0.06684212    0.2577326E-01   -0.0856415E-01
 20               0.06687971    0.2580084E-01   -0.0856423E-01
 40               0.06689865    0.2581979E-01   -0.0856424E-01
NORMALIZED IMAG PART  0.38595379  NORMALIZED REAL PART 0.12801815
```

From this, we see that the normalized impedance is

$$Z_n = 0.12801815 + j\ 0.38595379\ .$$

BTNCO Program

```
10 REM          BTNCO               VERSION          07/03/68
20 LET R1=.8333
30 LET R2=1.1667
40 LET L1=.0476
50 LET L2=.3809
60 LET M1=77.05
70 LET U1=1
80 LET M2=40
90 LET U2=1
```

```
100 LET C=.5
110 PRINT "R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
120PRINT"M1=";M1,"U1=";U1,"M2=";M2,"U2=";U2
130 PRINT" ","CØNDUCTØR SEPARATIØN IS";C
140 PRINT "X", "AIR VALUE", "REAL PART", "IMAG PART"
150 LET S1=1E-2
160 LET S2=5
170 LET I6=0
180 LET I7=0
190 LET I8=0
200 LET B1=0
210 LET B2 =S2
220 FØR X = B1 +S1/2 TØ B2 STEP S1
230 LET Z=R2*X
240 LET Q1=R2
250 GØSUB 720
260 LET I2=F2
270 LET Z=R1*X
280 LET Q1=R1
290 GØSUB 720
300 LET I1=F2
310 LET S3=S1*(I2-I1)↑2/X
320 LET X1=.707107*(SQR(SQR(X*X*X*X+M1*M1)+X*X))/U1
330 LET Y1=.707107*(SQR(SQR(X*X*X*X+M1*M1)-X*X))/U1
340 LET X2=.707107*(SQR(SQR(X*X*X*X+M2*M2)+X*X))/U2
350 LET Y2=.707107*(SQR(SQR(X*X*X*X+M2*M2)-X*X))/U2
360 LET Y=X
370 GØ SUB 600
380 LET K3=K1
390 LET K4=K2
400 LET Y=-X
410 GØ SUB 600
420 LET K5=K1+K3
430 LET K6=K2+K4
440 LET I6=I6+2*S3*((L2-L1)+(EXP(-X*(L2-L1))-1)/X)
450 LET I7=I7+S3*(2*(L2-L1)+K5)
460 LET I8=I8+S3*K6
470 NEXT X
480 LETB1=B2
490 LET B2=B2+S2
500 LET Q3=X+S1/2
510 PRINT Q3,I6,I7,I8
520 IF X < 3 THEN 220
530 LET S1=5E-2
540 IF X<10 THEN 220
550 LET S1=.1
560LETS2=20
570 IF X<35 THEN 220
580 PRINT "NØRMALIZED IMAG PART";I7/I6,"NØRMALIZED REAL PART";-I8/I6
590 GØTØ 370
600 LET G1=2*(EXP(-Y*(L2-L1))-1)
610 LET G2=(EXP(-Y*L2)-EXP(-Y*L1))↑2
620 LET A1=G1*(Y+X2)+G2*(Y-X2)
630 LET C1=(Y+X1)*A1-Y1*Y2*(G1-G2)
640 LET C2=Y2*(Y+X1)*(G1-G2)+Y1*A1
650 LET A2=(Y+X1)*(Y+X2)-Y1*Y2
660 LET A3=(Y+X1)*Y2+(Y+X2)*Y1
670 LET C5=A2*Y-Y*((Y-X1)*(Y-X2)-Y1*Y2)*EXP(-2*Y*C)
680 LET D5=A3*Y+Y*((Y-X1)*Y2+Y1*(Y-X2))*EXP(-2*Y*C)
```

```
690 LET K1=(C1*C5+C2*D5)/(C5*C5+D5*D5)
700 LET K2=(C2*C5-C1*D5)/(C5*C5+D5*D5)
710 RETURN
720 IF Z>5 THEN810
730 LETL5=INT(2*Z)+3
740 LETF1=.5*Q1*Q1*Z
750 LET F2=F1/3
760 FØRN=1TØL5
770 LETF1=-F1*.250*Z*Z/(N*N+N)
780 LETF2=F2+F1/(2*N+3)
790 NEXTN
800 GØTØ 860
810 LET Q1=(((-138.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
820 LET Q1=((Q1-.1730503)/Z+.7034345)/Z-.364109E-3
830 LET Q2=(((-5.817517/Z+2.105874)/Z-.6396196)/Z+.4952024)/Z
840 LET Q2=(Q2-.137344E-2)/Z+.7979095
850 LET F2=(1-SQR(Z)*(Q2*CØS(Z-PI/4)-Q1*SIN(Z-PI/4)))/(K*K)
860 RETURN
870 END
```

COIL ENCIRCLING A TWO-CONDUCTOR ROD

The case of a coil encircling a two-conductor rod, shown in Fig. 8, is typical of many eddy-current tests. If we let the inner conductor have zero conductivity, this general case reduces to the case of a coil encircling a tube. This type of coil is used with impedance bridge instruments.

In this section, we have programs that calculate the normalized coil impedance (ENCCO5), the defect sensitivity factor for a defect in the outer material (ENDFT5), the defect sensitivity factor for a defect located at any point in a lattice in the outer material (ENDFTL), the defect sensitivity factor for a differential coil system and a defect located at any point in a lattice (READIN), and the defect sensitivity factor for a defect in the base material (ENDFTB).

Normalized Coil Impedance

Discussion of ENCCO5

This program is designed to calculate the normalized impedance of a coil with rectangular cross section coaxial with and encircling a two-conductor rod of effectively infinite (three- or four-coil diameters) length (see Fig. 8). The conductors may have different permeabilities and electrical conductivities.

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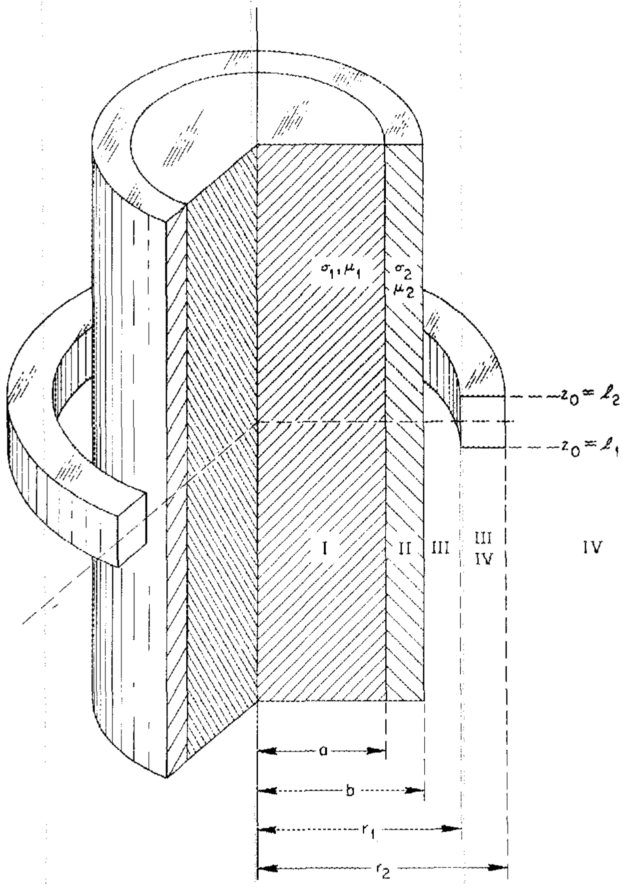

Fig. 8. A Coil of Rectangular Cross Section Encircling a Two-Conductor Rod.

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the inner conductor and by $\omega\mu_2\sigma_2\bar{r}^2$ for the outer one.

Once these calculations have been made, one need only type the following lines into the program. (Note: Parentheses are not typed.)

```
12 LET R1 = (Numerical value of normalized inner coil radius)
14 LET R2 = (Numerical value of normalized outer coil radius)
16 LET L1 = (Numerical value of normalized distance between bottom
             of coil and z = 0 plane)
```

18 LET L2 = (Numerical value of normalized distance between top of coil and z = 0 plane)
20 LET M1 = (Numerical value of $\omega\mu_1\sigma_1\bar{r}^2$)

Note: If the conductivity of the inner conductor is zero, one must type two additional lines.

60 LET X1 = X
62 LET Y1 = 0

22 LET U1 = (Numerical value of relative permeability of inner conductor)
24 LET M2 = (Numerical value of $\omega\mu_2\sigma_2\bar{r}^2$)

Note: If the conductivity of the outer conductor is zero, one must type two additional lines.

56 LET X2 = X
58 LET Y2 = 0

26 LET U2 = (Numerical value of relative permeability of outer conductor)
28 LET A = (Numerical value of normalized radius of inner conductor)
30 LET B = (Numerical value of normalized radius of outer conductor)
32 LET A9 = (Normalization factor)

Note: This value may be obtained with the aid of AIRCO5 simply by placing the appropriate coil dimensions in that program.

The program may now be run.

The print-out by the computer will have the following format:

R1 = (R1)	R2 = (R2)	L1 = (L1)	L2 = (L2)
INN RAD = (A)	OUT RAD = (B)	AIR VALUE = (A9)	
M1 = (M1)	U1 = (U1)	M2 = (M2)	U2 = (U2)
X	AIR VALUE	REAL PART	IMAG PART
1	------	------	------
2	------	------	------
3	------	------	------
4	------	------	------
5	------	------	------
6	------	------	------
7	------	------	------
8	------	------	------
9	------	------	------
10	------	------	------
11	------	------	------
12	------	------	------
13	------	------	------
14	------	------	------
15	------	------	------
16	------	------	------

NORMALIZED IMAG PART ------ NORMALIZED REAL PART ------

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The second column, headed "AIR VALUE," is the value of the integral in the absence of the two-conductor rod, while the remaining two columns are the real and imaginary parts of the integral in the presence of the conductors. These appear chiefly to allow one to inspect the convergence of the integration. The last line is the normalized impedance of the coil in the presence of the two-conductor rod.

The example below is presented for additional aid to those who wish to use the program.

Sample Calculation of ENCC05

Let us suppose that we wish to know the normalized impedance of a coil 0.155 in. long with inner and outer radii 0.420 and 0.580 in., respectively, due to the presence of a coaxial two-conductor rod of effectively infinite (three- or four-coil diameters) length. The inner conductor is assumed to be aluminum with a radius of 0.300 in., while the outer conductor is assumed to be copper with a radius of 0.400 in. We shall assume the frequency of the driving current to be 1.25 kHz and the distance of the bottom of the coil from the $z = 0$ plane to be 0.005 in.

First, we determine the mean coil radius to be 0.500 in., so that

Normalized inner coil radius = 0.420/0.500 = 0.84
Normalized outer coil radius = 0.580/0.500 = 1.16
Normalized distance from bottom of coil to $z = 0$ plane = 0.01
Normalized distance from top of coil to $z = 0$ plane = 0.32
Normalized radius of inner conductor = 0.300/0.500 = 0.6
Normalized radius of outer conductor = 0.400/0.500 = 0.8

Furthermore,

Angular frequency of driving current = $2\pi \times 1250$ sec^{-1}
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Relative permeability of both conductors = 1
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$\omega\mu_1\sigma_1\bar{r}^2 = 45.48$

$\omega\mu_2\sigma_2\bar{r}^2 = 91.91$

At this point one determines the normalization factor by placing the normalized coil dimensions in AIRCO5 and running that program. It is worth noting that the normalization factor is independent of all parameters, except the coil dimensions. This factor is normalization factor 2 in the print-out of AIRCO5 and for the case in point is found to be 0.01011632. With the normalization factor in hand, the remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.84
14 LET R2 = 1.16
16 LET L1 = 0.01
18 LET L2 = 0.32
20 LET M1 = 45.48
22 LET U1 = 1
24 LET M2 = 91.91
26 LET U2 = 1
28 LET A = 0.6
30 LET B = 0.8
32 LET A9 = 0.1011632E-1
```

Execution of the program yields the following results:

```
R1 = 0.84         R2 = 1.16        L1 = 0.1E-01     L2 = 0.32
INN RAD = 0.6     OUT RAD = 0.8    AIR VALUE = 0.1011632E-01
M1 = 45.48        U1 = 1           M2 = 91.91       U2 = 1
X                 AIR VALUE        REAL PART        IMAG PART
 1                0.1040337E-02    -0.0965715E-02   -0.4482236E-03
 2                0.4916285E-02     0.2001791E-02   -0.0728955E-02
 3                0.0769353E-01     0.4400818E-02   -0.0892771E-02
 4                0.0802257E-01     0.4571806E-02   -0.0985552E-02
 5                0.084957E-01      0.4978293E-02   -0.1036855E-02
 6                0.0928377E-01     0.573816E-02    -0.1064592E-02
 7                0.0945268E-01     0.589514E-02    -0.1079286E-02
 8                0.0956387E-01     0.6001279E-02   -0.1086941E-02
 9                0.0983787E-01     0.0627314E-01   -0.1090878E-02
 10               0.0991995E-01     0.0635431E-01   -0.1092884E-02
 11               0.0994553E-01     0.063795E-01    -0.1093895E-02
 12               0.1003168E-01     0.0646549E-01   -0.1094399E-02
 13               0.100638E-01      0.0649754E-01   -0.1094646E-02
 14               0.100687E-01      0.0650241E-01   -0.1094765E-02
 15               0.1008879E-01     0.0652249E-01   -0.109482E-02
 16               0.1009676E-01     0.0653045E-01   -0.109844E-02
NORMALIZED IMAG PART  0.6474704   NORMALIZED REAL PART  0.10822558
```

From this we see that the normalized impedance is

$$Z_n = 0.10822558 + j\,0.6474704$$

ENCC05 Program

```
10 REM        ENCC05      VERSIØN 02/10/69
12 LET R1=.8333
14 LET R2=1.1667
16 LET L1=.0952
18 LET L2=.4285
20 LETM1=27
22 LET U1=1
24LETM2=81
26 LET U2=1
28 LETA=.6
30 LETB=.8
32 LETA9=1.23559E-2
34PRINT"R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
36PRINT "INN RAD=";A;"ØUT RAD=";B;"AIR VALUE=";A9
38PRINT"M1=";M1,"U1=";U1,"M2=";M2,"U2=";U2
40 PRINT "X","AIR VALUE","REAL PART","IMAG PART"
42 LET M7=0
43 LET M8=0
44 LETM9=0
46 LETS1=1E-2
48 LETS2=1
50 LET B1=0
52 LET B2 =S2
54 FØR X = B1 +S1/2 TØ B2 STEP S1
56 LET X2 = .707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
58 LET Y2 = .707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
60 LET X1 = .707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
62 LET Y1 = .707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
64 LET Z1 = SQR(X1*X1+Y1*Y1)
66 LET Ø1 = ATN(Y1/X1)
68 LET Z2 = SQR(X2*X2+Y2*Y2)
70 LET  Ø2= ATN(Y2/X2)
72LETZ=A*Z1
74LETØ=Ø1
76GØSUB 316
78LETI1=Q1
80LETJ1=Q2
82LETI2=Q5
84LETJ2=Q6
86LETZ=A*Z2
88LETØ=Ø2
90GØSUB 316
92LETI3=Q1
94LETJ3=Q2
96LETK3=Q3
98LETN3=Q4
100LETI4=Q5
```

```
102LETJ4=Q6
104LETK4=Q7
106LETN4=Q8
108LETZ=X*B
110LETØ=0
112GØSUB 316
114LETK5=-Q3
116LETI6=Q5
118LETK6=Q7
120LETZ=B*Z2
122LETØ=Ø2
124GØSUB 316
126LETI7=Q1
128LETJ7=Q2
130LETK7=Q3
132LETN7=Q4
134LETI8=Q5
136LETJ8=Q6
138LETK8=Q7
140LETN8=Q8
142LETQ1=X*K5*K8*CØS(N8)-Z2*K7*K6*CØS(Ø2+N7)/U2
144LETQ2=X*K5*K8*SIN(N8)-Z2*K7*K6*SIN(Ø2+N7)/U2
146 LET D1 = SQR(Q1*Q1+Q2*Q2)
148 GØSUB514
150LETE1=Q3
152LETQ1=Z1*I4*I1*CØS(Ø1+J4+J1)/U1-Z2*I2*I3*CØS(Ø2+J2+J3)/U2
154LETQ2=Z1*I4*I1*SIN(Ø1+J4+J1)/U1-Z2*I2*I3*SIN(Ø2+J2+J3)/U2
156 LET D2 = SQR(Q1*Q1+Q2*Q2)
158 GØSUB514
160LETE2=Q3
162LETQ1=Z1*K4*I1*CØS(Ø1+N4+J1)/U1-Z2*K3*I2*CØS(Ø2+N3+J2)/U2
164LETQ2=Z1*K4*I1*SIN(Ø1+N4+J1)/U1-Z2*K3*I2*SIN(Ø2+N3+J2)/U2
166 LET D3 = SQR(Q1*Q1+Q2*Q2)
168 GØSUB514
170LETE3=Q3
172LETQ1=Z2*I7*K6*CØS(Ø2+J7)/U2-X*I8*K5*CØS(J8)
174LETQ2=Z2*I7*K6*SIN(Ø2+J7)/U2-X*I8*K5*SIN(J8)
176 LET D4 = SQR(Q1*Q1+Q2*Q2)
178 GØSUB514
180LETE4=Q3
182 LET Q1 = D1*D2*CØS(E1+E2)+D3*D4*CØS(E3+E4)
184 LET Q2 = D1*D2*SIN(E1+E2)+D3*D4*SIN(E3+E4)
186 LET D5 = SQR (Q1*Q1+Q2*Q2)
188 GØSUB514
190LETE5=Q3
192LETG1=(I8*D3*CØS(J8+E3-E5)-K8*D2*CØS(N8+E2-E5))/(B*K6*D5)
194LETG1=G1-I6/K6
196LETG2=(I8*D3*SIN(J8+E3-E5)-K8*D2*SIN(N8+E2-E5))/(B*K6*D5)
198 LET Z = K2*X
200 LET Q1 = K2
202GØSUB 248
204 LET Q7 = K7
206LETI2=F8
208 LET Z = K1*X
210 LET Q1 = K1
212GØSUB 248
```

```
214 LET G8 = K7
216LETI1=F8
218 LET S5=4*S1*((G7-G8)*SIN(X*(L2-L1)/2))↑2
220 LET S3=3.141593*X*S1*(I2-I1)↑2
222 LET A3 = S3*((EXP(-X*(I2-I1))-1)/X+L2-L1)
224 LET M7=M7+A3+G1*S5
226 LET M8=M8+G2*S5
228LETM9=M9+A3
230 NEXT X
232 PRINT X+S1/2,M9,M7,M8
234 LET S1=5E-2
236 LETB1=B1+S2
238 LETB2=B2+S2
240 IFX<15.5 THEN 54
242PRINT"NØRMALIZED IMAG PART";(A9-M9+M7)/A9,
244PRINT"NØRMALIZED REAL PART";-M8/A9
246GØTØ522
248 IF Z>5 THEN 296
250 LET L5=INT(20*(1-EXP(-Z/10))+4)
252 LETF1=0
254 LET F2=1
256LETF6=1
258LETT5=Q1*Q1*Q1
260 LETF8=T5/6
262 LET T6=1/2
264 LET K7 = Q1/(X*X)+(.577215665-1/2-1/3+LØG(Z/2))*T5/6
266 FØR K=1 TØ L5
268 LET F6=-1*F6
270 LET F1=F1+1/K
272 LET F2=F2+1/(K+1)
274 LET T5=T5*Z*Z
276 LETT6=T6/(4*K*(K+1))
278 LETC2=T6/(3+2*K)
280 LETF7=F6*T5*C2
282 LET C1=(.577215665-(F1+F2)/2-1/(3+2*K))*C2
284 LET F3=C1*T5+C2*LØG(Z/2)*T5
286 LET K7=K7+F3
288LETF8=F8+F7
290 NEXT K
292 LET K7=K7-3.1415927/(2*X*X*X)
294 GØ TØ 314
296 LET P1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
298 LET P1=((P1-.1730503)/Z+.7034845)/Z-.064109E-3
300 LET P2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
302 LET P2=(P2-.187344E-2)/Z+.7979095
304 LET P4=3.1415927
306 LET F8=(1-SQR(Z)*(P2*CØS(Z-P4/4)-P1*SIN(Z-P4/4)))/(X*X*X)
308 LET P3=(((.7989839/Z-1.1768576)/Z+.91571421)/Z-.67491295)/Z
310 LET P3=(P3+1.0958276)/Z+1.2533263
312 LET K7=-P3*SQR(Z)*EXP(-Z)/(X*X*X)
314 RETURN
316 IFZ>8THEN390
318 LETL5=20*(1-EXP(-Z/10))+4
320 T1=.5
322 LETT5=Z/2
324 LETV1=T1
```

```
326 LETV2=0
328 V3=-.25
330 V4=0
332 LETV5=T5*CØS(Ø)
334 LETV6=T5*SIN(Ø)
336 LETV7=V5
338 LETV8=V6
340 LETF1=0
342 LETF2=1
344 FØR K=1 TØ L5
346 LET Z3=CØS(2*K*Ø)
348 LET Z4=SIN(2*K*Ø)
350 LET Z5=CØS((2*K+1)*Ø)
352 LET Z6=SIN((2*K+1)*Ø)
354 LETF1=F1+1/K
356 LETF2=F2+1/(K+1)
358 LETT1=T1*Z*Z/(4*K*(K+1))
360 LET T5=T5*Z*Z/(4*K*(K+1))
362 V1=V1+T1*Z3*(2*K+1)
364 V2=V2+T1*Z4*(2*K+1)
366 V3=V3+T1*Z3*((2*K+1)*(F1+F2)/2-1)
368 V4=V4+T1*Z4*((2*K+1)*(F1+F2)/2-1)
370 LET V5=V5+T5*Z5
372 LET V7=V7+T5*(F1+F2)*Z5
374 LET V6=V6+T5*Z6
376 LET V8 =V8+T5*(F1+F2)*Z6
378 NEXTK
380 V3=-V3-CØS(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V1-Ø*V2
382 V4=-V4+SIN(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V2+Ø*V1
384 LETV7=-V7/2+CØS(Ø)/Z+(.577215665+LØG(Z/2))*V5-Ø*V6
386 LETV8=-V8/2-SIN(Ø)/Z+(.577215665+LØG(Z/2))*V6+Ø*V5
388 GØTØ480
390 LETF1=1
392 LETW7=1
394 LETW5=1
396 W3=.5
398 W1=.5
400 LETT2=1
402 LETT1=1
404 LET W2=0
406 LETW4=0
408 LETW6=0
410 LETW8=0
412 L5=3+20/Z
414 FØRK=1TØL5
416 LETF1=-F1
418 LET T2 =T2*(4-(2*K-1)*(2*K-1))/(8*K*Z)
420 LETT1=T2*(2*K+1)/2
422 LET Z3=CØS(K*Ø)
424 LET Z4=SIN(K*Ø)
426 LET W1=W1+F1*T1*Z3
428 LET W2=W2-F1*T1*Z4
430 LET W3=W3+T1*Z3
432 LET W4=W4-T1*Z4
434 LET W5=W5+F1*T2*Z3
436 LET W6=W6-F1*T2*Z4
```

```
438 LET W7 =W7+T2*Z3
440 LET W8=W8-T2*Z4
442 NEXT K
444 LETG9=.3989422804*EXP(Z*CØS(Ø))
446 LETG5=G9*(CØS(Z*SIN(Ø)-.5*Ø))/SQR(Z)
448 LETG6=G9*(SIN(Z*SIN(Ø)-.5*Ø))/SQR(Z)
450 LETG1=G9*(CØS(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
452 LETG2=G9*(SIN(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
454 LETG9=1.253314137*EXP(-Z*CØS(Ø))
456 LETG7=G9*(CØS(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
458 LETG8=G9*(SIN(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
460 LETG3=G9*(CØS(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
462 LETG4=G9*(SIN(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
464 LETV5=W5*G5-W6*G6
466 LETV6=W6*G5+W5*G6
468 LETV7=W7*G7-W8*G8
470 LETV8=W8*G7+W7*G8
472 LETV1=V5-W1*G1+W2*G2
474 LETV2=V6-W1*G2-W2*G1
476 LETV3=-V7-W3*G3+W4*G4
478 LETV4=-V8-W3*G4-W4*G3
480 LETG1=SQR(V1*V1+V2*V2)
482 LETG2=ATN(V2/V1)
484 IFV1>0THEN488
486 LETG2=G2+3.1415927
488 LETG3=SQR(V3*V3+V4*V4)
490 LETG4=ATN(V4/V3)
492 IFV3>0THEN496
494 LETG4=G4+3.1415927
496 LETG5=SQR(V5*V5+V6*V6)
498 LETG6=ATN(V6/V5)
500 IFV5>0THEN504
502 LETG6=G6+3.1415927
504 LETG7=SQR(V7*V7+V8*V8)
506 LETG8=ATN(V8/V7)
508 IFV7>0THEN512
510 LETG8=G8+3.1415927
512 RETURN
514 LETG3=ATN(G2/G1)
516 IFG1>0THEN520
518 LETG3=G3+3.1415927
520 RETURN
522END
```

Defect Sensitivity Factor for a Defect in the Outer Material

Discussion of ENDFT5

This program is designed to calculate the defect sensitivity factor for a coil with rectangular cross section, coaxial with and encircling a two-conductor rod of effectively infinite (three- or four-coil diameters) length. For this program, the defect must be in the outer conductor which is labeled II in Fig. 8, p. 97. The conductors may have different electrical conductivities.

To use this program, one must divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the inner conductor and by $\omega\mu_2\sigma_2\bar{r}^2$ for the outer one.

Once these calculations have been made, one need only type the following lines into the program. (Note: Parentheses are not typed.)

```
12 LET R1 = (Numerical value of normalized inner coil radius)
14 LET R2 = (Numerical value of normalized outer coil radius)
16 LET L1 = (Numerical value of normalized distance between bottom
             of coil and z = 0 plane)
18 LET L2 = (Numerical value of normalized distance between top
             of coil and z = 0 plane)
20 LET M1 = (Numerical value of ωμ1σ1r̄²)
```

Note: If the conductivity of the inner conductor is zero, one must type two additional lines.

```
60 LET X1 = X
62 LET Y1 = 0

22 LET M2 = (Numerical value of ωμ2σ2r̄²)
24 LET A = (Numerical value of normalized radius of inner conductor)
26 LET B = (Numerical value of normalized radius of outer conductor)
28 LET R9 = (Numerical value of normalized r position coordinate of
             defect)
30 LET Z9 = (Numerical value of normalized z position coordinate of
             defect)
32 LET A9 = (Normalization factor).
```

Note: This value may be obtained with the aid of AIRC05 simply by placing the appropriate coil dimensions in that program.

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)           R2 = (R2)          L1 = (L1)           L2 = (L2)
INN RAD = (A)       OUT RAD = (B)      AIR VALUE = (A9)
M1 = (M1)                              M2 = (M2)
DEFECT POSITION IS                     R = (R9)            Z = (Z9)
X                  MAGNITUDE           PHASE
 1                  ------             ------
 5                  ------             ------
 10                 ------             ------
 15                 ------             ------
 20                 ------             ------
 25                 ------             ------
 30                 ------             ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The other two columns have self-explanatory headings and appear chiefly to allow one to inspect the convergence of the integration. The magnitude and phase values for X = 30 are considered to be correct since the integration in most cases converges sufficiently well for this value of X.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of ENDFT5

Let us suppose that we wish to know the defect sensitivity factor for a coil 0.155 in. long with inner and outer radii of 0.420 and 0.580 in., respectively, coaxial with and encircling a two-conductor rod of effectively infinite (three- or four-coil diameters) length. The inner conductor is assumed to be aluminum with a radius of 0.300 in., while the outer conductor is assumed to be copper with a radius of 0.400 in. Let us further suppose that we are interested in the defect sensitivity factor for the case in which the defect is located 0.350 in. from the axis and 0.0825 in. above the Z = 0 plane. We shall assume the frequency of the driving current to be 1.25 kHz and the distance of the bottom of the coil from the Z = 0 plane to be 0.005 in.

Normalized inner coil radius = 0.420/0.500 = 0.84
Normalized outer coil radius = 0.580/0.500 = 1.16
Normalized distance from bottom of coil to Z = 0 plane = 0.01
Normalized distance from top of coil to Z = 0 plane = 0.32
Normalized radius of inner conductor = 0.300/0.500 = 0.6
Normalized radius of outer conductor = 0.400/0.500 = 0.8
Normalized r position of defect = 0.350/0.500 = 0.7
Normalized z position of defect = 0.0825/0.500 = 0.165

Furthermore,

Angular frequency of driving current = $2\pi \times 1250\ \text{sec}^{-1}$
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 45.48$$

$$\omega\mu_2\sigma_2\bar{r}^2 = 91.91$$

At this point, one determines the normalization factor by placing the normalized coil dimensions in AIRC05 and running that program. It is worth noting that the normalization factor is independent of all parameters excluding the coil dimensions. This factor is normalization factor No. 2 in the print-out of AIRC05 and for the case in point is found to be 0.01011632. With the normalization factor in hand, the remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.84
14 LET R2 = 1.16
16 LET L1 = 0.01
18 LET L2 = 0.32
20 LET M1 = 45.48
22 LET M2 = 91.91
24 LET A = 0.6
26 LET B = 0.8
28 LET R9 = 0.7
30 LET Z9 = 0.165
32 LET A9 = 0.1011632E-1
```

Execution of the program yields the following results:

```
R1 = 0.84          R2 = 1.16           L1 = 0.1E-01        L2 = 0.32
INN RAD = 0.6      OUT RAD = 0.8       AIR VALUE = 0.1011632E-01
M1 = 45.48                             M2 = 91.91
DEFECT POSITION IS                     R = 0.7             Z = 0.165
X                  MAGNITUDE           PHASE
 0.99999999        0.1693847E-01       -2.8708385
 4.99999999        0.16850739          -2.6409193
 9.999999999       0.24065815          -2.4869539
 15                0.25056687          -2.4526001
 20                0.25138255          -2.448787
 25                0.25130007          -2.4492792
 30                0.25123016          -2.4497367
```

From this, we see that the defect sensitivity factor is

$$0.25123016\ e^{-j2.4497367}$$

To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

<u>ENDFT5 Program</u>

```
10 REM          ENDFT5          VERSIØN          2/24/69
12 LET R1=.84
14 LET R2=1.16
16 LET L1=.01
18 LET L2=.32
20 LET M1=45.48
22 LET M2=91.91
24 LET A=.6
26 LET B=.8
28 LET R9=.7
30 LET Z9=.165
32 LET A9=.1011632E-1
34 PRINT"R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
36 PRINT"INN RAD=";A,"ØUT RAD=";B,"AIR VALUE=";A9
38 PRINT"M1=";M1," ","M2=";M2
40 PRINT"DEFECT PØSITIØN IS","R=";R9,"Z=";Z9
42 PRINT "X","MAGNITUDE","PHASE"
44 LET M9=0
46 LET M7=0
48 LET M8=0
50 LETS1=1E-2
52 LETS2=1
54 LET B1=0
56 LET B2 =S2
58 FØR X = B1 +S1/2 TØ B2 STEP S1
```

```
60 LETX2=.707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
62 LETY2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
64 LETX1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
66 LETY1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
68 LET Z1 = SQR(X1*X1+Y1*Y1)
70 LET Ø1 = ATN(Y1/X1)
72 LET Z2 = SQR(X2*X2+Y2*Y2)
74 LET  Ø2= ATN(Y2/X2)
76 LETZ=A*Z1
78 LETØ=Ø1
80 GØSUB 316
82 LETI1=G1
84 LETJ1=G2
86 LETI2=G5
88 LETJ2=G6
90 LETZ=A*Z2
92 LETØ=Ø2
94 GØSUB 316
96 LETI3=G1
98 LETJ3=G2
100 LETK3=G3
102 LETN3=G4
104 LETI4=G5
106 LETJ4=G6
108 LETK4=G7
110 LETN4=G8
112 LETZ=X*B
114 LETØ=0
116 GØSUB 316
118 LETK5=-G3
120 LETI6=G5
122 LETK6=G7
124 LETZ=B*Z2
126 LETØ=Ø2
128 GØSUB 316
130 LETI7=G1
132 LETJ7=G2
134 LETK7=G3
136 LETN7=G4
138 LETI8=G5
140 LETJ8=G6
142 LETK8=G7
144 LETN8=G8
146 LETZ=K9*Z2
148 LETØ=Ø2
150 GØ SUB 316
152 LETI9=G5
154 LETJ9=G6
156 LETK9=G7
158 LETN9=G8
160 LETG1=X*K5*K8*CØS(N8)-Z2*K7*K6*CØS(Ø2+N7)
162 LETG2=X*K5*K8*SIN(N8)-Z2*K7*K6*SIN(Ø2+N7)
```

```
164 LET D1 = SQR(Q1*Q1+Q2*Q2)
166 GØSUB514
168 LETE1=Q3
170 LETQ1=Z1*I4*I1*CØS(Ø1+J4+J1)-Z2*I2*I3*CØS(Ø2+J2+J3)
172 LETQ2=Z1*I4*I1*SIN(Ø1+J4+J1)-Z2*I2*I3*SIN(Ø2+J2+J3)
174 LET D2 = SQR(Q1*Q1+Q2*Q2)
176 GØSUB514
178 LETE2=Q3
180 LETQ1=Z1*K4*I1*CØS(Ø1+N4+J1)-Z2*K3*I2*CØS(Ø2+N3+J2)
182 LETQ2=Z1*K4*I1*SIN(Ø1+N4+J1)-Z2*K3*I2*SIN(Ø2+N3+J2)
184 LET D3 = SQR(Q1*Q1+Q2*Q2)
186 GØSUB514
188 LETE3=Q3
190 LETQ1=Z2*I7*K6*CØS(Ø2+J7)-X*I8*K5*CØS(J8)
192 LETQ2=Z2*I7*K6*SIN(Ø2+J7)-X*I8*K5*SIN(J8)
194 LET D4 = SQR(Q1*Q1+Q2*Q2)
196 GØSUB514
198 LETE4=Q3
200 LET Q1 = D1*D2*CØS(E1+E2)+D3*D4*CØS(E3+E4)
202 LET Q2 = D1*D2*SIN(E1+E2)+D3*D4*SIN(E3+E4)
204 LET D5 = SQR (Q1*Q1+Q2*Q2)
206 GØSUB514
208 LETE5=Q3
210 LETG1=(I9*D3*CØS(J9+E3-E5)-K9*D2*CØS(N9+E2-E5))/(B*D5)
212 LETG2=(I9*D3*SIN(J9+E3-E5)-K9*D2*SIN(N9+E2-E5))/(B*D5)
214 LET Z = R2*X
216 LET Q1 = R2
218 GØSUB 268
220 LET Q7 = K7
222 LET Z = R1*X
224 LET Q1 = R1
226 GØSUB 268
228 LET Q8 = K7
230 LETS5=S1*(Q7-Q8)*(SIN(X*(Z9-L1))-SIN(X*(Z9-L2)))
232 LETM7=M7+G1*S5
234 LET M8=M8+G2*S5
236 NEXTX
238 LET M9=ATN(M8/M7)
240 IF M7>0THEN244
242 LETM9=M9+3.1415927
244 LETN9=(3*M2)/(4*9.8696044*A9)
246 PRINTX+S1/2,N9*(M7*M7+M8*M8),2*M9
248 LET S1=5E-2
250 IF X>4.5 THEN 258
252 LETB1=1
254 LET B2=5
256 GØ TØ 58
258 LET S2=5
260 LET B1=B2
262 LET B2=B2+S2
264 IFX<29.5THEN58
266 GØTØ 522
```

```
268 IFZ>3THEN308
270 LET L5=INT(20*(1-EXP(-Z/10))+4)
272 LETF1=0
274 LET F2=1
276 LETF6=1
278 LETT5=Q1*Q1*Q1
280 LET T6=1/2
282 LET K7 = Q1/(X*X)+(.577215665-1/2-1/3+LØG(Z/2))*T5/6
284 FØR K=1 TØ L5
286 LET F1=F1+1/K
288 LET F2=F2+1/(K+1)
290 LET T5=T5*Z*Z
292 LETT6=T6/(4*K*(K+1))
294 LETC2=T6/(3+2*K)
296 LET C1=(.577215665-(F1+F2)/2-1/(3+2*K))*C2
298 LET F3=C1*T5+C2*LØG(Z/2)*T5
300 LET K7=K7+F3
302 NEXT K
304 LET K7=K7-3.1415927/(2*X*X*X)
306 GØ TØ 314
308 LET F3=(((.7989839/Z-1.1768576)/Z+.91571421)/Z-.6749129 5)/Z
310 LET F3=(F3+1.0958276)/Z+1.2533263
312 LET K7=-F3*SQR(Z)*EXP(-Z)/(X*X*X)
314 RETURN
316 IFZ>8THEN390
318 LETL5=20*(1-EXP(-Z/10))+4
320 LETT1=.5
322 LETT5=Z/2
324 LETV1=T1
326 LETV2=0
328 LETV3=-.25
330 LETV4=0
332 LETV5=T5*CØS(Ø)
334 LETV6=T5*SIN(Ø)
336 LETV7=V5
338 LETV8=V6
340 LETF1=0
342 LETF2=1
344 FØR K=1 TØ L5
346 LET Z3=CØS(2*K*Ø)
348 LET Z4=SIN(2*K*Ø)
350 LET Z5=CØS((2*K+1)*Ø)
352 LET Z6=SIN((2*K+1)*Ø)
354 LETF1=F1+1/K
356 LETF2=F2+1/(K+1)
358 LETT1=T1*Z*Z/(4*K*(R+1))
360 LET T5=T5*Z*Z/(4*K*(R+1))
362 LETV1=V1+T1*Z3*(2*K+1)
364 LETV2=V2+T1*Z4*(2*K+1)
366 LETV3=V3+T1*Z3*((2*K+1)*(F1+F2)/2-1)
368 LETV4=V4+T1*Z4*((2*K+1)*(F1+F2)/2-1)
370 LET V5=V5+T5*Z5
```

```
372 LET V7=V7+T5*(F1+F2)*Z5
374 LET V6=V6+T5*Z6
376 LET V8 =V8+T5*(F1+F2)*Z6
378 NEXTR
380 LETV3=-V3-CØS(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V1-Ø*V2
382 LETV4=-V4+SIN(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V2+Ø*V1
384 LETV7=-V7/2+CØS(Ø)/Z+(.577215665+LØG(Z/2))*V5-Ø*V6
386 LETV8=-V8/2-SIN(Ø)/Z+(.577215665+LØG(Z/2)) T=;U=;XS6WØ=;NX
390 LETF1=1
392 LETW7=1
394 LETW5=1
396 LETW3=.5
398 LETW1=.5
400 LETT2=1
402 LETT1=1
404 LET W2=0
406 LETW4=0
408 LETW6=0
410 LETW8=0
412 LETL5=3+20/Z
414 FØRR=1TØL5
416 LETF1=-F1
418 LET T2 =T2*(4-(2*R-1)*(2*R-1))/(8*R*Z)
420 LETT1=T2*(2*R+1)/2
422 LET Z3=CØS(R*Ø)
424 LET Z4=SIN(R*Ø)
426 LET W1=W1+F1*T1*Z3
428 LET W2=W2-F1*T1*Z4
430 LET W3=W3+T1*Z3
432 LET W4=W4-T1*Z4
434 LET W5=W5+F1*T2*Z3
436 LET W6=W6-F1*T2*Z4
438 LET W7 =W7+T2*Z3
440 LET W8=W8-T2*Z4
442 NEXT R
444 LETQ9=.398942280 4*EXP(Z*CØS(Ø))
446 LETQ5=Q9*(CØS(Z*SIN(Ø)-.5*Ø))/SQR(Z)
448 LETQ6=Q9*(SIN(Z*SIN(Ø)-.5*Ø))/SQR(Z)
450 LETQ1=Q9*(CØS(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
452 LETQ2=Q9*(SIN(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
454 LETQ9=1.253314137*EXP(-Z*CØS(Ø))
456 LETQ7=Q9*(CØS(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
458 LETQ8=Q9*(SIN(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
460 LETQ3=Q9*(CØS(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
462 LETQ4=Q9*(SIN(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
464 LETV5=W5*Q5-W6*Q6
466 LETV6=W6*Q5+W5*Q6
468 LETV7=W7*Q7-W8*Q8
470 LETV8=W8*Q7+W7*Q8
472 LETV1=V5-W1*Q1+W2*Q2
474 LETV2=V6-W1*Q2-W2*Q1
476 LETV3=-V7-W3*Q3+W4*Q4
```

```
478 LETV4=-V8-W3*Q4-W4*Q3
480 LETQ1=SQR(V1*V1+V2*V2)
482 LETQ2=ATN(V2/V1)
484 IFV1>0THEN488
486 LETQ2=Q2+3.1415927
488 LETQ3=SQR(V3*V3+V4*V4)
490 LETQ4=ATN(V4/V3)
492 IFV3>0THEN496
494 LETQ4=Q4+3.1415927
496 LETQ5=SQR(V5*V5+V6*V6)
498 LETQ6=ATN(V6/V5)
500 IFV5>0THEN504
502 LETQ6=Q6+3.1415927
504 LETQ7=SQR(V7*V7+V8*V8)
506 LETQ8=ATN(V8/V7)
508 IFV7>0THEN512
510 LETQ8=Q8+3.1415927
512 RETURN
514 LETQ3=ATN(Q2/Q1)
516 IFQ1>0THEN520
518 LETQ3=Q3+3.1415927
520 RETURN
522 END
```

Defect Sensitivity Factor for a Lattice of Defects in the Outer Material

Discussion of ENDFTL

This program is designed to calculate the defect sensitivity factor for a coil with rectangular cross section coaxial with and encircling a two-conductor rod of effectively infinite (three- or four-coil diameters) length. For this program, the defect may be located at any one of a number of points on a lattice in the outer conductor, which is labeled II in Fig. 8, p. 97. The conductors may have different electrical conductivities.

Before proceeding further, it is worth noting that in the execution of this program the data generated by the computer are stored in a data block which is used as input data for another program. Since there exist BASIC compilers which do not have this capability, this program may or may not be executable, depending upon the compiler used. If the latter situation arises, one may delete line 38 and the word "FILE" wherever it appears in the program, thus causing the data to be printed at the console; otherwise, one may access the data by using the name /@3/.

To use this program, one must divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the inner conductor and by $\omega\mu_2\sigma_2\bar{r}^2$ for the outer one.

Once these calculations have been made, one must type the following lines into the program. (Note: Parentheses are not typed.)

```
12 LET R1 = (Numerical value of normalized inner coil radius)
14 LET R2 = (Numerical value of normalized outer coil radius)
16 LET L1 = (Numerical value of normalized distance between bottom
             of coil and z = 0 plane)
18 LET L2 = (Numerical value of normalized distance between top
             of coil and z = 0 plane)
20 LET M1 = (Numerical value of ωμ1σ1r̄²)
```

Note: If the conductivity of the inner conductor is zero, one must type two additional lines:

```
80 LET X1 = X
82 LET Y1 = 0
```

```
22 LET M2 = (Numerical value of ωμ2σ2r̄²)
24 LET A = (Numerical value of normalized radius of inner conductor)
26 LET B = (Numerical value of normalized radius of outer conductor)
28 LET A9 = (Normalization factor)
```

Note: This value may be obtained with the aid of AIRC05 simply by placing the appropriate coil dimensions in that program.

```
30 LET R9 = (Numerical value of number of lattice spacings in r
             direction)
32 LET Z9 = (Numerical value of number of lattice spacings in z
             direction)
34 LET Z8 = (Numerical value of maximum normalized z position
             coordinate of defect)
```

Note: Neither R9 nor Z9 may exceed 15. The lattice spacing in the r direction is (B-A)/R9, while it is Z8/Z9 in the z direction. In addition, Z8 is measured from the midplane of the coil.

After typing these lines, one need only create an empty data file using the name /@3/; and the program may then be run.

During the execution of this program, the integers which are less than or equal to 30 will be consecutively printed at the console. These appear as indications of how far the integration has proceeded. The data resulting from this program will have the following format:

```
(R1)          (R2)          (L1)          (L2)
(A)           (B)           (A9)
(M1)                        (M2)
(R9)          (Z9)          (Z8)
              (R(1))        (R(2))        **********      (R(R9))
(Z(1))        ------        ------        **********      ------
              ------        ------        **********      ------
(Z(2))        ------        ------        **********      ------
              ------        ------        **********      ------
  *             *             *               *              *
  *             *             *               *              *
  *             *             *               *              *
  *             *             *               *              *
(Z(Z9))       ------        ------        **********      ------
              ------        ------        **********      ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The upper value at each lattice point is the magnitude of the defect sensitivity factor, while the lower one is the phase of the defect sensitivity factor for the case in which the defect is located at the given lattice point. It should be noted that the lattice points for which R9 > 4 will be printed below the last line [i.e., the line containing Z(Z9)].

The example below is presented for additional aid to those who wish to use this program.

<u>Sample Calculation of ENDFTL</u>

Let us suppose that we wish to know the defect sensitivity factor for the case of a coil 0.155 in. long with inner and outer radii of 0.420 and 0.580 in., respectively, coaxial with and encircling a two-conductor rod of effectively infinite (three- or four-coil diameters) length. The inner conductor is assumed to be aluminum with a radius of 0.300 in., while the outer conductor is assumed to be copper with a radius of 0.400 in. Let us further suppose that we are interested in the defect sensitivity factor for the cases in which the defect is located at any one of the points which lie within 1.000 in. of the midplane of the coil on a 4 by 2 lattice in the copper. We shall assume the frequency of the driving current to be 1.25 kHz and the distance of the bottom of the coil from the Z = 0 plane to be 0.005 in.

First, we determine the mean coil radius to be 0.500 in., so that

Normalized inner coil radius = 0.420/0.500 = 0.84
Normalized outer coil radius = 0.580/0.500 = 1.16
Normalized distance from bottom of coil to Z = 0 plane = 0.01
Normalized distance from top of coil to Z = 0 plane = 0.32
Normalized radius of inner conductor = 0.300/0.500 = 0.6
Normalized radius of outer conductor = 0.400/0.500 = 0.8
Number of r lattice spacings = 4
Number of z lattice spacings = 2
Maximum normalized z coordinate of defect = 1.000/0.500 = 2

Furthermore,

Angular frequency of driving current = $2\pi \times 1250$ sec^{-1}
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\overline{r}^2 = 45.48$$
$$\omega\mu_2\sigma_2\overline{r}^2 = 91.91$$

At this point one determines the normalization factor by placing the normalized coil dimensions in AIRCO5 and running that program. It is worth noting that the normalization factor is independent of all parameters excluding the coil dimensions. This factor is normalization factor 1 in the print-out of AIRCO5 and for the case in point is found to be 0.01011632. With the normalization factor in hand, the remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.84
14 LET R2 = 1.16
16 LET L1 = 0.01
18 LET L2 = 0.32
20 LET M1 = 45.48
22 LET M2 = 91.91
24 LET A = 0.6
26 LET B = 0.8
28 LET A9 = 0.1011632E-1
30 LET R9 = 4
32 LET Z9 = 2
34 LET Z8 = 2
```

An empty data file is now created and the program run with the following results:

0.84	1.16	0.1E-01	0.32	
0.6	0.8	0.1011632E-01		
45.48		91.91		
4	2	2		
	0.625	0.675	0.725	0.775
−0.335	0.1148246E-01	0.1621343E-01	0.2498759E-01	0.4155218E-01
	8.2691402	8.9025646	−2.9333451	−2.1856129
−1.335	0.0977185E-03	011517243E-03	0.2567598E-03	0.4615816E-03
	8.1021059	8.7575633	−3.0851137	−2.3642342

From this, we see that for the case of a defect located at R = 0.725, Z = −1.335 the defect sensitivity factor is

$$2.567598 \times 10^{-4} \exp(-j3.0851137)$$

To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

ENDFTL Program

```
10 REM          ENDFTL          VERSION          02/24/69
12 LET R1=.84
14 LET R2=1.16
16 LET L1=.01
18 LET L2=.32
20 LET M1=45.48
22 LET M2=91.91
24 LET A=.6
26 LET B=.8
28 LET A9=.1011632E-1
30 LET R9=4
```

```
32 LET Z9=2
34 LET Z8=2
36 DIMM(18,18),I(18,18),P(18,18),R(18),Z(18)
38 ØPEN/84/,ØUTPUT
40 PRINTFILER1,R2,L1,L2
42 PRINTFILEA,B,A9
44 PRINTFILEM1," ",M2
46 PRINTFILER9,Z9,Z8
48 PRINTFILE
50 PRINT"          A PRØGRAM IS NØW RUNNING.  PLEASE DØ NØT DISCØNNECT!!"
52 PRINT
54 FØRI=1TØZ9
56 FØRJ=1TØR9
58 LETM(I,J)=0
60 LETI(I,J)=0
62 NEXTJ
64 NEXTI
66 LETS1=1E-2
68 LETS2=1
70 LETB1=0
72 LETB2=S2
74 FØRX=B1+S1/2TØB2STEPS1
76 LETX2=.707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
78 LETY2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
80 LETX1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
82 LETY1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
84 LETZ1=SQR(X1*X1+Y1*Y1)
86 LETØ1=ATN(Y1/X1)
88 LETZ2=SQR(X2*X2+Y2*Y2)
90 LET Ø2=ATN(Y2/X2)
92 LETZ=A*Z1
94 LETØ=Ø1
96 GØSUB426
98 LETI1=Q1
100 LETJ1=Q2
102 LETI2=Q5
104 LETJ2=Q6
106 LETZ=A*Z2
108 LETØ=Ø2
110 GØSUB426
112 LETI3=Q1
114 LETJ3=Q2
116 LETK3=Q3
118 LETN3=Q4
120 LETI4=Q5
122 LETJ4=Q6
124 LETK4=Q7
126 LETN4=Q8
128 LETZ=X*B
130 LETØ=0
132 GØSUB426
134 LETK5=-Q3
136 LETI6=Q5
138 LETK6=Q7
140 LETZ=B*Z2
142 LETØ=Ø2
144 GØSUB426
146 LETI7=Q1
148 LETJ7=Q2
150 LETK7=Q3
152 LETN7=Q4
```

```
154 LETI8=G5
156 LETJ8=G6
158 LETK8=G7
160 LETN8=G8
162 LETG1=X*K5*K8*COS(N8)-Z2*K7*K6*COS(O2+N7)
164 LETG2=X*K5*K8*SIN(N8)-Z2*K7*K6*SIN(O2+N7)
166 LETD1=SQR(G1*G1+G2*G2)
168 GOSUB624
170 LETE1=G3
172 LETG1=Z1*I4*I1*COS(O1+J4+J1)-Z2*I2*I3*COS(O2+J2+J3)
174 LETG2=Z1*I4*I1*SIN(O1+J4+J1)-Z2*I2*I3*SIN(O2+J2+J3)
176 LETD2=SQR(G1*G1+G2*G2)
178 GOSUB624
180 LETE2=G3
182 LETG1=Z1*K4*I1*COS(O1+N4+J1)-Z2*K3*I2*COS(O2+N3+J2)
184 LETG2=Z1*K4*I1*SIN(O1+N4+J1)-Z2*K3*I2*SIN(O2+N3+J2)
186 LETD3=SQR(G1*G1+G2*G2)
188 GOSUB624
190 LETE3=G3
192 LETG1=Z2*I7*K6*COS(O2+J7)-X*I8*K5*COS(J8)
194 LETG2=Z2*I7*K6*SIN(O2+J7)-X*I8*K5*SIN(J8)
196 LETD4=SQR(G1*G1+G2*G2)
198 GOSUB624
200 LETE4=G3
202 LETG1=D1*D2*COS(E1+E2)+D3*D4*COS(E3+E4)
204 LETG2=D1*D2*SIN(E1+E2)+D3*D4*SIN(E3+E4)
206 LETD5=SQR(G1*G1+G2*G2)
208 GOSUB624
210 LETE5=G3
212 LETZ=R2*X
214 LETG1=R2
216 GOSUB378
218 LETG7=K7
220 LETZ=R1*X
222 LETG1=R1
224 GOSUB378
226 LETS6=G7-K7
228 LETO=O2
230 FORJ=1TOR9
232 LETR(J)=A+(J-.5)*(B-A)/R9
234 LETZ=Z2*R(J)
236 GOSUB426
238 LETI9=G5
240 LETJ9=G6
242 LETK9=G7
244 LETN9=G8
246 LETG1=(I9*D3*COS(J9+E3-E5)-K9*D2*COS(N9+E2-E5))/(B*D5)
248 LETG2=(I9*D3*SIN(J9+E3-E5)-K9*D2*SIN(N9+E2-E5))/(B*D5)
250 FORI=1TOZ9
252 LETZ(I)=(L1+L2)/2-(I-.5)*Z8/Z9
254 LETS5=S1*S6*(SIN(X*(Z(I)-L1))-SIN(X*(Z(I)-L2)))
256 LETM(I,J)=M(I,J)+G1*S5
258 LETI(I,J)=I(I,J)+G2*S5
260 NEXTI
262 NEXTJ
264 NEXTX
266 PRINTX+S1/2,
268 LETS1=5E-2
270 LETB1=B1+S2
272 LETB2=B2+S2
```

```
274 IFX<29.5THEN74
276 FØRI=1TØZ9
278 FØRJ=1TØR9
280 LETP(I,J)=2*ATN(I(I,J)/M(I,J))
282 IFM(I,J)>0THEN286
284 LETP(I,J)=P(I,J)+2*PI
286 LETM(I,J)=M(I,J)*M(I,J)+I(I,J)*I(I,J)
288 NEXTJ
290 NEXTI
292 LETG=(3*M2)/(4*9.8696044*A9)
294 PRINTFILE"",
296 FØRJ=1TØ4
298 PRINTFILER(J),
300 NEXTJ
302 FØRI=1TØZ9
304 PRINTFILEZ(I),
306 FØRJ=1TØ4
308 PRINTFILEG*M(I,J),
310 NEXTJ
312 PRINTFILE"",
314 FØRJ=1TØ4
316 PRINTFILEP(I,J),
318 NEXTJ
320 PRINTFILE
322 NEXTI
324 IFR9<4.5THEN632
326 FØRJ=5TØ9
328 PRINTFILER(J),
330 NEXTJ
332 FØRI=1TØZ9
334 FØRJ=5TØ9
336 PRINTFILEG*M(I,J),
338 NEXTJ
340 FØRJ=5TØ9
342 PRINTFILEP(I,J),
344 NEXTJ
346 PRINTFILE
348 NEXTI
350 IFR9<9.5THEN632
352 FØRJ=10TØ14
354 PRINTFILER(J),
356 NEXTJ
358 FØRI=1TØZ9
360 FØRJ=10TØ14
362 PRINTFILEG*M(I,J),
364 NEXTJ
366 FØRJ=10TØ14
368 PRINTFILEP(I,J),
370 NEXTJ
372 PRINTFILE
374 NEXTI
376 IFR9<14.5THEN632
378 IFZ>3THEN418
380 LETL5=INT(20*(1-EXP(-Z/10))+4)
382 LETF1=0
384 LETF2=1
386 LETF6=1
388 LETT5=G1*G1*G1
390 LETT6=1/2
392 LETK7=G1/(X*X)+(.577215665-1/2-1/3+LØG(Z/2))*T5/6
394 FØRR=1TØL5
```

```
396 LETF1=F1+1/K
398 LETF2=F2+1/(K+1)
400 LETT5=T5*Z*Z
402 LETT6=T6/(4*K*(K+1))
404 LETC2=T6/(3+2*K)
406 LETC1=(.577215665-(F1+F2)/2-1/(3+2*K))*C2
408 LETF3=C1*T5+C2*LØG(Z/2)*T5
410 LETK7=K7+F3
412 NEXTK
414 LETK7=K7-3.1415927/(2*X*X*X)
416 GØTØ424
418 LETP3=(((.7989839/Z-1.1768576)/Z+.91571421)/Z-.67491295)/Z
420 LETP3=(P3+1.0958276)/Z+1.2533263
422 LETK7=-P3*SQR(Z)*EXP(-Z)/(X*X*X)
424 RETURN
426 IFZ>8THEN500
428 LETL5=20*(1-EXP(-Z/10))+4
430 LETT1=.5
432 LETT5=Z/2
434 LETV1=T1
436 LETV2=0
438 LETV3=-.25
440 LETV4=0
442 LETV5=T5*CØS(Ø)
444 LETV6=T5*SIN(Ø)
446 LETV7=V5
448 LETV8=V6
450 LETF1=0
452 LETF2=1
454 FØR K=1 TO L5
456 LET Z3=CØS(2*K*Ø)
458 LET Z4=SIN(2*K*Ø)
460 LET Z5=CØS((2*K+1)*Ø)
462 LET Z6=SIN((2*K+1)*Ø)
464 LETF1=F1+1/K
466 LETF2=F2+1/(K+1)
468 LETT1=T1*Z*Z/(4*K*(K+1))
470 LET T5=T5*Z*Z/(4*K*(K+1))
472 LETV1=V1+T1*Z3*(2*K+1)
474 LETV2=V2+T1*Z4*(2*K+1)
476 LETV3=V3+T1*Z3*((2*K+1)*(F1+F2)/2-1)
478 LETV4=V4+T1*Z4*((2*K+1)*(F1+F2)/2-1)
480 LET V5=V5+T5*Z5
482 LET V7=V7+T5*(F1+F2)*Z5
484 LET V6=V6+T5*Z6
486 LET V8 =V8+T5*(F1+F2)*Z6
488 NEXTK
490 LETV3=-V3-CØS(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V1-Ø*V2
492 LETV4=-V4+SIN(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V2+Ø*V1
494 LETV7=-V7/2+CØS(Ø)/Z+(.577215665+LØG(Z/2))*V5-Ø*V6
496 LETV8=-V8/2-SIN(Ø)/Z+(.577215665+LØG(Z/2))*V6+Ø*V5
498 GØTØ590
500 LETF1=1
502 LETW7=1
504 LETW5=1
506 LETW3=.5
508 LETW1=.5
510 LETT2=1
512 LETT1=1
514 LET W2=0
516 LETW4=0
```

```
518 LETW6=0
520 LETW8=0
522 LETL5=3+20/Z
524 FØRR=1TØL5
526 LETF1=-F1
528 LET T2 =T2*(4-(2*R-1)*(2*R-1))/(8*R*Z)
530 LETT1=T2*(2*R+1)/2
532 LET Z3=CØS(R*Ø)
534 LET Z4=SIN(R*Ø)
536 LET W1=W1+F1*T1*Z3
538 LET W2=W2-F1*T1*Z4
540 LET W3=W3+T1*Z3
542 LET W4=W4-T1*Z4
544 LET W5=W5+F1*T2*Z3
546 LET W6=W6-F1*T2*Z4
548 LET W7 =W7+T2*Z3
550 LET W8=W8-T2*Z4
552 NEXT R
554 LETQ9=.398942280 4*EXP(Z*CØS(Ø))
556 LETQ5=Q9*(CØS(Z*SIN(Ø)-.5*Ø))/SQR(Z)
558 LETQ6=Q9*(SIN(Z*SIN(Ø)-.5*Ø))/SQR(Z)
560 LETQ1=Q9*(CØS(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
562 LETQ2=Q9*(SIN(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
564 LETQ9=1.253314137*EXP(-Z*CØS(Ø))
566 LETQ7=Q9*(CØS(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
568 LETQ8=Q9*(SIN(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
570 LETQ3=Q9*(CØS(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
572 LETQ4=Q9*(SIN(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
574 LETV5=W5*Q5-W6*Q6
576 LETV6=W6*Q5+W5*Q6
578 LETV7=W7*Q7-W8*Q8
580 LETV8=W8*Q7+W7*Q8
582 LETV1=V5-W1*Q1+W2*Q2
584 LETV2=V6-W1*Q2-W2*Q1
586 LETV3=-V7-W3*Q3+W4*Q4
588 LETV4=-V8-W3*Q4-W4*Q3
590 LETQ1=SQR(V1*V1+V2*V2)
592 LETQ2=ATN(V2/V1)
594 IFV1>0THEN598
596 LETQ2=Q2+3.1415927
598 LETQ3=SQR(V3*V3+V4*V4)
600 LETQ4=ATN(V4/V3)
602 IFV3>0THEN606
604 LETQ4=Q4+3.1415927
606 LETQ5=SQR(V5*V5+V6*V6)
608 LETQ6=ATN(V6/V5)
610 IFV5>0THEN614
612 LETQ6=Q6+3.1415927
614 LETQ7=SQR(V7*V7+V8*V8)
616 LETQ8=ATN(V8/V7)
618 IFV7>0THEN622
620 LETQ8=Q8+3.1415927
622 RETURN
624 LETQ3=ATN(Q2/Q1)
626 IFQ1>0THEN630
628 LETQ3=Q3+3.1415927
630 RETURN
632END
```

Defect Sensitivity Factor for a Differential Coil System for a Lattice of Defects

Discussion of READIN

This program is designed to calculate the differential defect sensitivity for a differential coil system which is shown encircling a two-conductor rod in Fig. 9. One may also use this program for a differential coil system inside a two-conductor tube. The coils must have rectangular cross sections, identical dimensions, and a common axis. The two conductors need not have the same electrical conductivity.

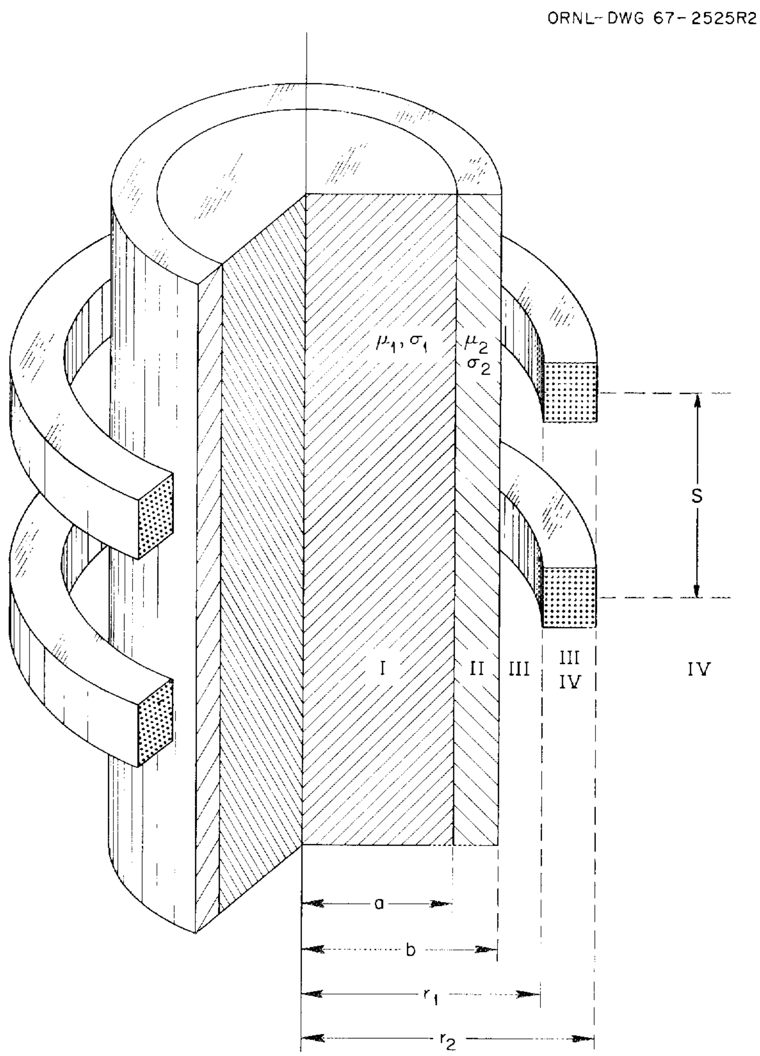

Fig. 9. A Differential Encircling Coil System.

To use this program, one must execute the program which calculates the defect sensitivity factor for a coil encircling a two-conductor rod (ENDFTL) or for a coil inside a two-conductor tube (INDFTL). The output data from either of these programs comprise the input data for the program at hand. It is worth noting that the data generated by the computer in the execution of either ENDFTL or INDFTL are stored in a data block, which is then read as input for this program. Since there exist BASIC compilers which do not have this capability, this program may or may not be executable, depending upon the compiler used.

Once the execution of either of the above-named programs is complete, one need only type the following lines into the program. (Note: Parentheses are not typed.)

12 LET D = (Numerical value of center-to-center coil spacing)

Note: This spacing is in units of z lattice spacings, which are equal to Z8/Z9 for either case of interest. The definitions of Z8 and Z9 can be found in the explanatory material for either ENDFTL or INDFTL.

18 OPEN/(Name of file containing data)/,INPUT

The program may now be run.

The print-out by the computer will have the following format:

```
RADII OF BOTH COILS ARE          INNER = (R1)   OUTER = (R2)
LENGTH OF BOTH COILS IS (L2-L1)    AIR VALUE IS (A9)
RADII OF CONDUCTORS ARE          INNER = (A)    OUTER = (B)
COILS ARE SEPARATED (D*Z8/Z9)    M1 = (M1)      M2 = (M2)

   Z  ,  R     (R(1))        (R(2))       **********     (R(R9))
(Z(1))         ------        ------       **********     ------
               ------        ------       **********     ------

(Z(2))         ------        ------       **********     ------
               ------        ------       **********     ------

   *             *             *              *             *
   *             *             *              *             *
   *             *             *              *             *
   *             *             *              *             *

(Z(Z9'))       ------        ------       **********     ------
               ------        ------       **********     ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. All lengths are normalized and the Z(I)'s are measured from the midplane of the coil system. The upper value at each lattice point is the magnitude of the differential defect sensitivity factor, while the lower one is the phase of the differential defect sensitivity factor for the case in which the defect is located at the given lattice point. It should be noted here that the lattice points for which R9 > 4 will be printed below the last line [i.e., the line containing Z(Z9′)]. Furthermore, Z9′ = Z9 − 1 − Int(D/2), where Z9 and all other symbols are defined in the explanatory material for both ENDFTL and INDFTL.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of READIN

Let us suppose that we are interested in the differential defect sensitivity factor for the case in which the input data are just those obtained from the example immediately preceding ENDFTL. Furthermore, let us suppose that the center-to-center separation of the coils is two z lattice spacings.

Since the output of ENDFTL is written on the file /@3/, the following lines are typed into the program:

```
10 LET D = 2
18 OPEN /@3/, INPUT
```

The program is then run with the following results:

```
RADII OF BOTH COILS ARE         INNER = 0.84   OUTER = 1.16
LENGTH OF BOTH COILS IS 0.81    AIR VALUE IS 0.101168E-01
RADII OF CONDUCTORS ARE         INNER = 0.6    OUTER = 0.8
COILS ARE SEPARATED 2           M1 = 45.48     M2 = 91.91

  Z  ,  R       0.625           0.675           0.725           0.775
-0.5            0.1188564E-01   0.1606826-01    0.2478397E-01   0.4109833E-01
                -4.2957535      -8.6623862      -2.9317212      -2.1835753
```

From this we see that for the case in which the defect is located at R = 0.725, Z = −0.5, the differential defect sensitivity factor is

$$0.2478397 \times 10^{-1}\ e^{-j2.9317212}$$

READIN Program

```
10 REM          READIN          VERSIØN          12/04/68
12 LETD=4
14 LET H=INT(D/2)
16 DIM M(18,18),P(18,18),S(18,18),I(18,18),Ø(18,18),R(18),Z(18)
18 ØPEN /@3/,INPUT
20 INPUT FILER1,R2,L1,L2
22 INPUT FILEA,B,A9
24 INPUT FILEM1,M2
26 INPUT FILER9,Z9,Z8
28 FØRJ=1TØ4
30 INPUT FILER(J)
32 NEXTJ
34 FØRI=1TØZ9
36 INPUT FILEZ(I)
38 FØRJ=1TØ4
40 INPUT FILE M(I,J)
42 NEXTJ
44 FØRJ=1TØ4
46 INPUT FILEP(I,J)
48 NEXTJ
50 NEXTI
52 IFR9<4.5THEN100
54 FØR J=5 TØ 9
56 INPUT FILER(J)
58 NEXTJ
60 FØRI=1TØZ9
62 FØRJ=5TØ9
64 INPUT FILE M(I,J)
66 NEXTJ
68 FØRJ=5TØ9
70 INPUT FILEP(I,J)
72 NEXTJ
74 NEXTI
76 IFR9<9.5THEN100
78 FØRJ=10TØ14
80 INPUT FILER(J)
82 NEXTJ
84 FØRI=1TØZ9
86 FØRJ=10TØ14
88 INPUT FILE M(I,J)
90 NEXTJ
92 FØRJ=10TØ14
94 INPUT FILEP(I,J)
96 NEXTJ
98 NEXTI
100 PRINT"RADII ØF BØTH CØILS ARE","INNER=";R1,"ØUTER=";R2
102 PRINT"LENGTH ØF BØTH CØILS IS";L2-L1,"AIR VALUE IS";A9
104 PRINT"RADII OF CØNDUCTØRS ARE","INNER=";A,"ØUTER=";B
106 PRINT"CØILS ARE SEPARATED";D*Z8/Z9,"M1=";M1,"M2=";M2
108 FØR I=H+1 TØ Z9
```

```
110 LET G=I-H
112 IF I>D THEN 118
114 LET K=D+I-1
116 GØ TØ 120
118 LET K=I-D
120 FØR J=1 TØ R9
122 LET S(G,J)=M(K,J)*CØS(P(K,J))-M(I,J)*CØS(P(I,J))
124 LET I(G,J)=M(K,J)*SIN(P(K,J))-M(I,J)*SIN(P(I,J))
126 LET Ø(G,J)=ATN(I(G,J)/S(G,J))
128 IF S(G,J)>0 THEN 132
130 LET Ø(G,J)=Ø(G,J)-PI
132 LET S(G,J)=SQR(S(G,J)*S(G,J)+I(G,J)*I(G,J))
134 NEXT J
136 NEXT I
138 FØR I=1 TØ Z9-(H+1)
140 LET Z(I)=Z(I)-(L1+L2)/2
142 NEXT I
144 IF D=2*H THEN 154
146 LET Z=Z(1)
148 FØR I=1 TØ Z9-(H+1)
150 LET Z(I)=Z(I)-Z
152 NEXT I
154 PRINT
156 PRINT"      Z    ,    R",
158 FØRJ=1TØ4
160 PRINTR(J),
162 NEXTJ
164 FØRI=1TØZ9-(H+1)
166 PRINTZ(I),
168 FØRJ=1TØ4
170 PRINT S(I,J),
172 NEXTJ
174 PRINT" ",
176 FØRJ=1TØ4
178 PRINTØ(I,J),
180 NEXTJ
182 PRINT
184 NEXTI
186 IFR9<4.5THEN238
188 FØRJ=5TØ9
190 PRINTR(J),
192 NEXTJ
194 FØRI=1TØZ9-(H+1)
196 FØRJ=5TØ9
198 PRINT S(I,J),
200 NEXTJ
202 FØRJ=5TØ9
204 PRINTØ(I,J),
206 NEXTJ
208 PRINT
210 NEXTI
212 IFR9<9.5THEN238
```

```
214 FØRJ=1TØ14
216 PRINTR(J),
218 NEXTJ
220 FØRI=1TØ29-(H+1)
222 FØRJ=1TØ14
224 PRINT S(I,J),
226 NEXTJ
228 FØRJ=1TØ14
230 PRINTØ(I,J),
232 NEXTJ
234 PRINT
236 NEXTI
238 PRINT
240 GØ TØ 12
242 END
```

Defect Sensitivity Factor for a Defect in the Base Material

Discussion of ENDFB5

This program is designed to calculate the defect sensitivity factor for a coil with rectangular cross section coaxial with and encircling a two-conductor rod of effectively infinite (three- or four-coil diameters) length. For this program, the defect must be in the inner conductor which is labeled II in Fig. 8, p. 97. The conductors may have different electrical conductivities.

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the inner conductor and by $\omega\mu_2\sigma_2\bar{r}^2$ for the outer one.

Once these calculations have been made, one need only type the following lines into the program. [Note: Parentheses are not typed.)]

12 LET R1 = (Numerical value of the normalized inner coil radius)
14 LET R2 = (Numerical value of the normalized outer coil radius)
16 LET L1 = (Numerical value of the normalized distance between bottom of coil and Z = 0 plane)
18 LET L2 = (Numerical value of the normalized distance between top of coil and Z = 0 plane)
20 LET M1 = (Numerical value of $\omega\mu_1\sigma_1\bar{r}^2$)
22 LET M2 = (Numerical value of $\omega\mu_2\sigma_2\bar{r}^2$)

Note: If the conductivity of the outer conductor is zero, one must type two additional lines:

```
60 LET X2 = X
62 LET Y2 = 0

24 LET A = (Numerical value of normalized radius of inner conductor)
26 LET B = (Numerical value of normalized radius of outer conductor)
28 LET R9 = (Numerical value of normalized r position coordinate
             of defect)
30 LET Z9 = (Numerical value of normalized z position coordinate
             of defect)
32 LET A9 = (Normalization factor)
```

Note: This value may be obtained with the aid of AIRCO5 simply by placing the appropriate coil dimensions in that program.

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)          R2 = (R2)          L1 = (L1)          L2 = (L2)
INN RAD = (A)      OUT RAD = (B)      AIR VALUE = (A9)
M1 = (M1)                             M2 = (M2)
DEFECT POSITION IS                    R = (R9)           Z = (Z9)
X                  MAGNITUDE          PHASE
 1                 ------             ------
 5                 ------             ------
 10                ------             ------
 15                ------             ------
 20                ------             ------
 25                ------             ------
 30                ------             ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The other two columns have self-explanatory headings and appear primarily to allow one to inspect the convergence of the integration. The magnitude and phase values for X = 30 are considered to be correct since the integration, in most cases, converges sufficiently well for this value of X.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of ENDFB5

Let us suppose that we wish to know the defect sensitivity factor for a coil 0.155 in. in length with inner and outer radii of 0.420 and 0.580 in., respectively, coaxial with and encircling a two-conductor rod of effectively infinite (three- or four-coil diameters) length. The inner conductor is assumed to be aluminum with a radius of 0.300 in., while the outer conductor is assumed to be copper with a radius of 0.400 in. Let us further suppose that we are interested in the defect sensitivity factor for the case in which the defect is located 0.250 in. from the axis and 0.0825 in. above the Z = 0 plane. We shall assume the frequency of the driving current to be 1.25 kHz and the distance of the bottom of the coil from the Z = 0 plane to be 0.005 in.

Normalized inner coil radius = 0.420/0.500 = 0.84
Normalized outer coil radius = 0.580/0.500 = 1.16
Normalized distance from bottom of coil to Z = 0 plane = 0.01
Normalized distance from top of coil to Z = 0 plane = 0.32
Normalized radius of inner conductor = 0.300/0.500 = 0.6
Normalized radius of outer conductor = 0.400/0.500 = 0.8
Normalized r position of defect = 0.200/0.500 = 0.5
Normalized z position of defect = 0.0825/0.500 = 0.165

Furthermore,

Angular frequency of driving current = $2\pi \times 1250\ \text{sec}^{-1}$
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\overline{r}^2 = 45.48$$

$$\omega\mu_2\sigma_2\overline{r}^2 = 91.91$$

At this point one determines the normalization factor by placing the normalized coil dimensions in AIRC05 and running that program. It is worth noting that the normalization factor is independent of all parameters excluding the coil dimensions. This factor is normalization factor 2 in the print-out of AIRC05 and for the case in point is found to be 0.01011632. With the normalization factor in hand, the remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.84
14 LET R2 = 1.16
16 LET L1 = 0.01
18 LET L2 = 0.32
20 LET M1 = 45.48
22 LET M2 = 91.91
24 LET A = 0.6
26 LET B = 0.8
28 LET R9 = 0.5
30 LET Z9 = 0.165
32 LET A9 = 0.1011632E-1
```

Execution of the program yields the following results:

```
R1 = 0.84          R2 = 1.16          L1 = 0.01             L2 = 0.32
INN RAD = 0.6      OUT RAD = 0.8      AIR VALUE = 1.01163E-2
M1 = 45.48                            M2 = 91.91
DEFECT POSITION IS                    R = 0.5               Z = 0.165
X                  MAGNITUDE          PHASE
 1                 1.53765E-3         7.54113
 5                 1.34157E-2         7.85115
 10                0.016374           8.00735
 15                1.64628E-2         8.02217
 20                1.64633E-2         8.02278
 25                1.64633E-2         8.02276
 30                1.64634E-2         8.02276
```

From this we see that the defect sensitivity factor is

$$0.0164634\ e^{j8.02276}$$

To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

<u>ENDFB5 Program</u>

```
10 REM          ENDFB5          VERSIØN          2/24/69
12 LET R1=.84
14 LET R2=1.16
16 LET L1=.01
18 LET L2=.32
20 LET M1=45.48
22 LET M2=91.91
24 LET A=.6
26 LET B=.8
```

```
28 LET R9=.5
30 LET Z9=.165
32 LET A9=.1011632E-1
34 PRINT"R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
36 PRINT"INN RAD=";A,"ØUT RAD=";B,"AIR VALUE=";A9
38 PRINT"M1=";M1," ","M2=";M2
40 PRINT"DEFECT PØSITIØN IS","R=";R9,"Z=";Z9
42 PRINT "X","MAGNITUDE","PHASE"
44 LET M9=0
46 LET M7=0
48 LET M8=0
50 LETS1=1E-2
52 LETS2=1
54 LET B1=0
56 LET B2 =S2
58 FØR X = B1 +S1/2 TØ B2 STEP S1
60 LET X2 = .707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
62 LET Y2 = .707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
64 LET X1 = .707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
66 LET Y1 = .707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
68 LET Z1 = SQR(X1*X1+Y1*Y1)
70 LET Ø1 = ATN(Y1/X1)
72 LET Z2 = SQR(X2*X2+Y2*Y2)
74 LET  Ø2= ATN(Y2/X2)
76 LETZ=A*Z1
78 LETØ=Ø1
80 GØSUB 312
82 LETI1=Q1
84 LETJ1=Q2
86 LETI2=Q5
88 LETJ2=Q6
90 LETZ=A*Z2
92 LETØ=Ø2
94 GØSUB 312
96 LETI3=Q1
98 LETJ3=Q2
100 LETK3=Q3
102 LETN3=Q4
104 LETI4=Q5
106 LETJ4=Q6
108 LETK4=Q7
110 LETN4=Q8
112 LETZ=X*B
114 LETØ=0
116 GØSUB 312
118 LETK5=-Q3
120 LETI6=Q5
122 LETK6=Q7
124 LETZ=B*Z2
126 LETØ=Ø2
128 GØSUB 312
130 LETI7=Q1
```

```
132 LETJ7=Q2
134 LETK7=Q3
136 LETN7=Q4
138 LETI8=Q5
140 LETJ8=Q6
142 LETK8=Q7
144 LETN8=Q8
146 LETZ=K9*Z1
148 LETØ=Ø1
150 GØ SUB 312
152 LETI9=Q5
154 LETJ9=Q6
156 LETQ1=X*K5*K8*CØS(N8)-Z2*K7*K6*CØS(Ø2+N7)
158 LETQ2=X*K5*K8*SIN(N8)-Z2*K7*K6*SIN(Ø2+N7)
160 LET D1 = SQR(Q1*Q1+Q2*Q2)
162 GØSUB510
164 LETE1=Q3
166 LETQ1=Z1*I4*I1*CØS(Ø1+J4+J1)-Z2*I2*I3*CØS(Ø2+J2+J3)
168 LETQ2=Z1*I4*I1*SIN(Ø1+J4+J1)-Z2*I2*I3*SIN(Ø2+J2+J3)
170 LET D2 = SQR(Q1*Q1+Q2*Q2)
172 GØSUB510
174 LETE2=Q3
176 LETQ1=Z1*K4*I1*CØS(Ø1+N4+J1)-Z2*K3*I2*CØS(Ø2+N3+J2)
178 LETQ2=Z1*K4*I1*SIN(Ø1+N4+J1)-Z2*K3*I2*SIN(Ø2+N3+J2)
180 LET D3 = SQR(Q1*Q1+Q2*Q2)
182 GØSUB510
184 LETE3=Q3
186 LETQ1=Z2*I7*K6*CØS(Ø2+J7)-X*I8*K5*CØS(J8)
188 LETQ2=Z2*I7*K6*SIN(Ø2+J7)-X*I8*K5*SIN(J8)
190 LET D4 = SQR(Q1*Q1+Q2*Q2)
192 GØSUB510
194 LETE4=Q3
196 LET Q1 = D1*D2*CØS(E1+E2%+D3*D4*CØS(E3+E4)
198 LET Q2 = D1*D2*SIN(E1+E2)+D3*D4*SIN(E3+E4)
200 LET D5 = SQR (Q1*Q1+Q2*Q2)
202 GØSUB510
204 LETE5=Q3
206 LETG1=I9*CØS(J9-E5)/(A*B*D5)
208 LETG2=I9*SIN(J9-E5)/(A*B*D5)
210 LET Z = K2*X
212 LET Q1 = K2
214 GØSUB 264
216 LET G7 = K7
218 LET Z = K1*X
220 LET Q1 = K1
222 GØSUB 264
224 LET G8 = K7
226 LETS5=S1*(G7-G8)*(SIN(X*(Z9-L1))-SIN(X*(Z9-L2)))
228 LETM7=M7+G1*S5
230 LET M8=M8+G2*S5
232 NEXTX
234 LET M9=ATN(M8/M7)
236 IF M7>0THEN240
```

```
238 LETM9=M9+3.1415927
240 LETN9=(3*M1)/(4*9.8696044*A9)
242 PRINTX+S1/2,N9*(M7*M7+M8*M8),2*M9
244 LET S1=5E-2
246 IF X>4.5 THEN 254
248 LETB1=1
250 LET B2=5
252 GØ TØ 58
254 LET S2=5
256 LET B1=B2
258 LET B2=B2+S2
260 IFX<29.5THEN58
262 GØ TØ 518
264 IFZ>3THEN304
266 LET L5=INT(20*(1-EXP(-Z/10))+4)
268 LETF1=0
270 LET F2=1
272 LETF6=1
274 LETT5=Q1*Q1*Q1
276 LET T6=1/2
278 LET K7 = Q1/(X*X)+(.577215665-1/2-1/3+LØG(Z/2))*T5/6
280 FØR R=1 TØ L5
282 LET F1=F1+1/R
284 LET F2=F2+1/(R+1)
286 LET T5=T5*Z*Z
288 LETT6=T6/(4*R*(R+1))
290 LETC2=T6/(3+2*R)
292 LET C1=(.577215665-(F1+F2)/2-1/(3+2*R))*C2
294 LET F3=C1*T5+C2*LØG(Z/2)*T5
296 LET K7=K7+F3
298 NEXT R
300 LET K7=K7-3.1415927/(2*X*X*X)
302 GØ TØ 310
304 LET F3=(((.7989839/Z-1.1763576)/Z+.91571421)/Z-.67491295)/Z
306 LET F3=(F3+1.0958276)/Z+1.2533263
308 LET K7=-F3*SQR(Z)*EXP(-Z)/(X*X*X)
310 RETURN
312 IFZ>8THEN386
314 LETL5=20*(1-EXP(-Z/10))+4
316 LETT1=.5
318 LETT5=Z/2
320 LETV1=T1
322 LETV2=0
324 LETV3=-.25
326 LETV4=0
328 LETV5=T5*CØS(Ø)
330 LETV6=T5*SIN(Ø)
332 LETV7=V5
334 LETV8=V6
336 LETF1=0
338 LETF2=1
340 FØR R=1 TØ L5
342 LET Z3=CØS(2*R*Ø)
```

```
344 LET Z4=SIN(2*K*Ø)
346 LET Z5=CØS((2*K+1)*Ø)
348 LET Z6=SIN((2*K+1)*Ø)
350 LETF1=F1+1/K
352 LETF2=F2+1/(K+1)
354 LETT1=T1*Z*Z/(4*K*(K+1))
356 LET T5=T5*Z*Z/(4*K*(K+1))
358 LETV1=V1+T1*Z3*(2*K+1)
360 LETV2=V2+T1*Z4*(2*K+1)
362 LETV3=V3+T1*Z3*((2*K+1)*(F1+F2)/2-1)
364 LETV4=V4+T1*Z4*((2*K+1)*(F1+F2)/2-1)
366 LET V5=V5+T5*Z5
368 LET V7=V7+T5*(F1+F2)*Z5
370 LET V6=V6+T5*Z6
372 LET V8 =V8+T5*(F1+F2)*Z6
374 NEXTK
376 LETV3=-V3-CØS(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V1-Ø*V2
378 LETV4=-V4+SIN(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V2+Ø*V1
380 LETV7=-V7/2+CØS(Ø)/Z+(.577215665+LØG(Z/2))*V5-Ø*V6
382 LETV8=-V8/2-SIN(Ø)/Z+(.577215665+LØG(Z/2))*V6+Ø*V5
384 GØTØ476
386 LETF1=1
388 LETW7=1
390 LETW5=1
392 LETW3=.5
394 LETW1=.5
396 LETT2=1
398 LETT1=1
400 LET W2=0
402 LETW4=0
404 LETW6=0
406 LETW8=0
408 LETL5=3+20/Z
410 FØRK=1TØL5
412 LETF1=-F1
414 LET T2 =T2*(4-(2*K-1)*(2*K-1))/(8*K*Z)
416 LETT1=T2*(2*K+1)/2
418 LET Z3=CØS(K*Ø)
420 LET Z4=SIN(K*Ø)
422 LET W1=W1+F1*T1*Z3
424 LET W2=W2-F1*T1*Z4
426 LET W3=W3+T1*Z3
428 LET W4=W4-T1*Z4
430 LET W5=W5+F1*T2*Z3
432 LET W6=W6-F1*T2*Z4
434 LET W7 =W7+T2*Z3
436 LET W8=W8-T2*Z4
438 NEXT K
440 LETG9=.3989422804*EXP(Z*CØS(Ø))
442 LETG5=G9*(CØS(Z*SIN(Ø)-.5*Ø))/SQR(Z)
444 LETG6=G9*(SIN(Z*SIN(Ø)-.5*Ø))/SQR(Z)
446 LETG1=G9*(CØS(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
448 LETG2=G9*(SIN(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
```

```
450 LETQ9=1.253314137*EXP(-Z*COS(O))
452 LETQ7=Q9*(COS(-Z*SIN(O)-.5*O))/SQR(Z)
454 LETQ8=Q9*(SIN(-Z*SIN(O)-.5*O))/SQR(Z)
456 LETQ3=Q9*(COS(-Z*SIN(O)-1.5*O))/SQR(Z*Z*Z)
458 LETQ4=Q9*(SIN(-Z*SIN(O)-1.5*O))/SQR(Z*Z*Z)
460 LETV5=W5*Q5-W6*Q6
462 LETV6=W6*Q5+W5*Q6
464 LETV7=W7*Q7-W8*Q8
466 LETV8=W8*Q7+W7*Q8
468 LETV1=V5-W1*Q1+W2*Q2
470 LETV2=V6-W1*Q2-W2*Q1
472 LETV3=-V7-W3*Q3+W4*Q4
474 LETV4=-V8-W3*Q4-W4*Q3
476 LETQ1=SQR(V1*V1+V2*V2)
478 LETQ2=ATN(V2/V1)
480 IFV1>0THEN484
482 LETQ2=Q2+3.1415927
484 LETQ3=SQR(V3*V3+V4*V4)
486 LETQ4=ATN(V4/V3)
488 IFV3>0THEN492
490 LETQ4=Q4+3.1415927
492 LETQ5=SQR(V5*V5+V6*V6)
494 LETQ6=ATN(V6/V5)
496 IFV5>0THEN500
498 LETQ6=Q6+3.1415927
500 LETQ7=SQR(V7*V7+V8*V8)
502 LETQ8=ATN(V8/V7)
504 IFV7>0THEN508
506 LETQ8=Q8+3.1415927
508 RETURN
510 LETQ3=ATN(Q2/Q1)
512 IFQ1>0THEN516
514 LETQ3=Q3+3.1415927
516 RETURN
518 END
```

COIL INSIDE A TWO-CONDUCTOR TUBE

The case of a coil inside a two-conductor tube, shown in Fig. 10, is typical of a number of tubing inspection problems. If we take the conductivity of the outer conductor to be zero, the problem reduces to a coil inside a single tube. This type of coil is used with impedance-bridge instruments for many eddy-current tubing testers.

In this section, there are programs which calculate the normalized coil impedance (INNCO5), the defect sensitivity factor for a defect in the inner material (INDFT5), the defect sensitivity factor for a defect

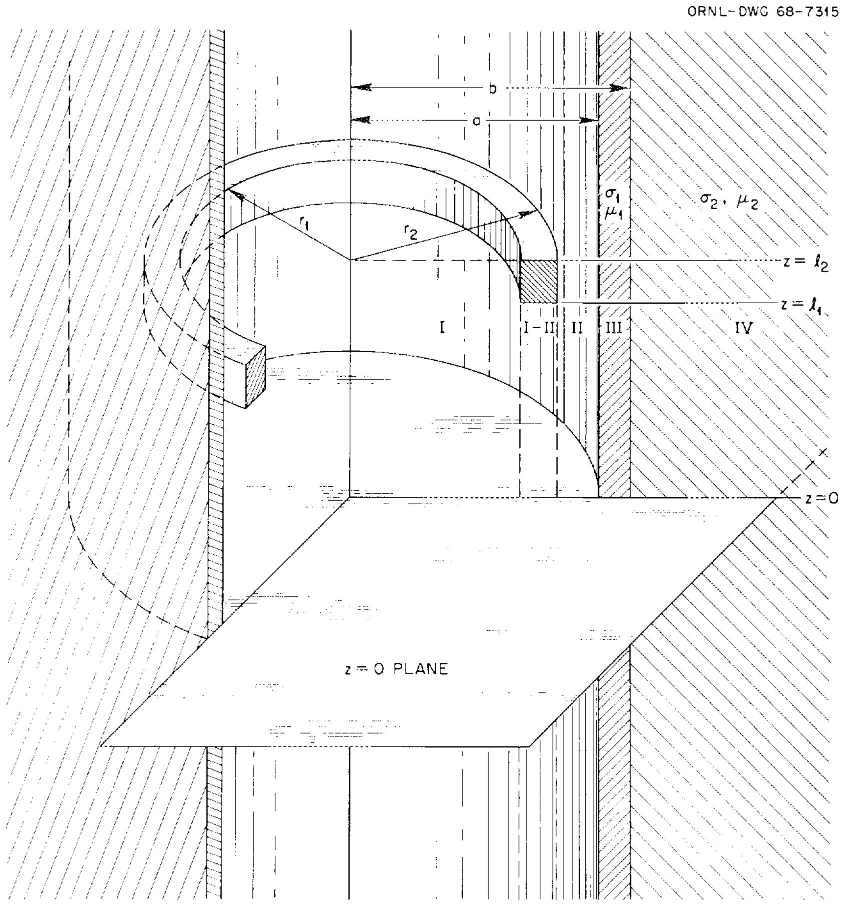

Fig. 10. A Coil of Rectangular Cross Section Inside a Two-Conductor Tube.

located at any point in a lattice in the inner material (INDFTL), and the defect sensitivity factor for a defect in the outer material (INDFB5). In addition the program, READIN, in the previous section can be applied to calculate the defect sensitivity factor for a differential coil system for a defect located at any point in a lattice.

Normalized Coil Impedance

Discussion of INNC05

This program is designed to calculate the normalized impedance of a coil with rectangular cross section coaxial with and inside a two-conductor tube of effectively infinite (three- or four-coil diameters) length. The wall thickness of the outer conductor is also effectively infinite as shown in Fig. 10. The conductors may have different permeabilities and electrical conductivities.

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the inner conductor and by $\omega\mu_2\sigma_2\bar{r}^2$ for the outer one.

Once these calculations have been made one need only type the following lines into the program. (Note: Parentheses are not typed.)

```
12 LET R1 = (Numerical value of normalized inner coil radius)
14 LET R2 = (Numerical value of normalized outer coil radius)
16 LET L1 = (Numerical value of normalized distance between bottom
             of coil and Z = 0 plane)
18 LET L2 = (Numerical value of normalized distance between top
             of coil and Z = 0 plane)
20 LET M1 = (Numerical value of ωμ1σ1r̄²)
```

Note: If the conductivity of the inner conductor is zero, one must type two additional lines:

```
62 LET X1 = X
64 LET Y1 = 0
```

```
22 LET U1 = (Numerical value of relative permeability of inner
             conductor)
24 LET M2 = (Numerical value of ωμ2σ2r̄²)
```

Note: If the conductivity of the outer conductor is zero, one must type two additional lines:

```
58 LET X2 = X
60 LET Y2 = 0
```

```
26 LET U2 = (Numerical value of relative permeability of outer
             conductor)
28 LET A = (Numerical value of normalized radius of inner conductor)
30 LET B = (Numerical value of normalized radius of outer conductor)
32 LET A9 = (Normalization factor)
```

Note: This value may be obtained with the aid of AIRCO5 simply by placing the appropriate coil dimensions in that program.

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)          R2 = (R2)          L1 = (L1)             L2 = (L2)
INN RAD = (A)      OUT RAD = (B)      AIR VALUE = (A9)
M1 = (M1)          U1 = (U1)          M2 = (M2)             U2 = (U2)
X                  AIR VALUE          REAL PART             IMAG PART
 1                 ------             ------                ------
 2                 ------             ------                ------
 3                 ------             ------                ------
 4                 ------             ------                ------
 5                 ------             ------                ------
 6                 ------             ------                ------
 7                 ------             ------                ------
 8                 ------             ------                ------
 9                 ------             ------                ------
 10                ------             ------                ------
 11                ------             ------                ------
 12                ------             ------                ------
 13                ------             ------                ------
 14                ------             ------                ------
 15                ------             ------                ------
NORMALIZED IMAG PART ------    NORMALIZED REAL PART ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The second column, headed "AIR VALUE," is the value of the integral in the absence of the two-conductor rod, while the remaining two columns are the real and imaginary parts of the integral in the presence of the conductors. These appear primarily to allow one to inspect the convergence of the integration. The last line is the normalized impedance of the coil in the presence of the two-conductor rod.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of INNC05

Let us suppose that we wish to know the normalized impedance of a coil 0.155 in. long with inner and outer radii of 0.420 and 0.580 in., respectively, due to the presence of a coaxial two-conductor tube of effectively infinite (three- or four-coil diameters) length. The inner conductor is assumed to be copper with a radius of 0.600 in., while the outer conductor is assumed to be aluminum with a radius of 0.700 in. We shall assume the frequency of the driving current to be 1.25 kHz and the distance of the bottom of the coil from the Z = 0 plane to be 0.005 in.

First, we determine the mean coil radius to be 0.500 in., so that

Normalized inner coil radius = 0.420/0.500 = 0.84
Normalized outer coil radius = 0.580/0.500 = 1.16
Normalized distance from bottom of coil to Z = 0 plane = 0.01
Normalized distance from top of coil to Z = 0 plane = 0.32
Normalized radius of inner conductor = 0.600/0.500 = 1.2
Normalized radius of outer conductor = 0.700/0.500 = 1.4

Furthermore,

Angular frequency of driving current = $2\pi \times 1250$ sec^{-1}
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Relative permeability of both conductors = 1
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$

$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

At this point, one determines the normalization factor by placing the normalized coil dimensions in AIRC05 and running that program. It is worth noting that the normalization factor is independent of all parameters excluding the coil dimensions. This factor is normalization factor 2 in the print-out of AIRC05 and for the case in point is found to be 0.01011632. With the normalization factor in hand, the remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.84
14 LET R2 = 1.16
16 LET L1 = 0.01
18 LET L2 = 0.32
20 LET M1 = 91.91
22 LET U1 = 1
24 LET M2 = 45.48
26 LET U2 = 1
28 LET A = 1.2
30 LET B = 1.4
32 LET A9 = 0.1011632E-1
```

Execution of the program yields the following results:

```
R1 = 0.84        R2 = 1.16       L1 = 0.01         L2 = 0.32
INN RAD = 1.2    OUT RAD = 1.4   AIR VALUE = 1.01163E-2
M1 = 91.91       U1 = 1          M2 = 45.48        U2 = 1
X                AIR VALUE       REAL PART         IMAG PART
 1               0.1040337E-02   -0.1322947E-02    -0.3355146E-03
 2               0.4916284E-02    0.1504308E-02    -0.5924591E-03
 3               0.0769353E-01    0.3852849E-02    -0.0758097E-02
 4               0.0802257E-01    0.4004045E-02    -0.0856239E-02
 5               0.084957E-01     0.4402016E-02    -0.091193E-02
 6               0.0928377E-01    0.5158041E-02    -0.0942619E-02
 7               0.0945268E-01    0.5313254E-02    -0.0959137E-02
 8               0.0956387E-01    0.5418599E-02    -0.0967861E-02
 9               0.0983787E-01    0.5690106E-02    -0.0972403E-02
 10              0.0991995E-01    0.5771114E-02    -0.097474E-02
 11              0.0994552E-01    0.5796231E-02    -0.0975931E-02
 12              0.1003168E-01    0.588219E-02     -0.097653E-02
 13              0.100638E-01     0.5914224E-02    -0.0976826E-02
 14              0.1006869E-01    0.5919087E-02    -0.0976969E-02
 15              0.1008879E-01    0.5939166E-02    -0.0977036E-02
NORMALIZED IMAG PART  0.58980875  NORMALIZED REAL PART  0.09658026
```

From this we see that the normalized impedance is

$$Z_n = 0.0965803 + j0.589809$$

<u>INNC05 Program</u>

```
10 REM            INNC05                VERSION        2/ 7/ 69
12 LETR1=.84
14 LETR2=1.16
16LETL1=.01
18LETL2=.32
20LETM1=91.91
22LETU1=1
24LETM2=45.48
26LETU2=1
28LETA=1.2
30LETB=1.4
32LETA9=.1011632E-1
34PRINT"R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
36PRINT"INN RAD=";A,"OUT RAD";B,"AIR VALUE=";A9
38PRINT"M1=";M1,"U1=";U1,"M2=";M2,"U2=";U2
40PRINT"X","AIR VALUE","REAL PART","IMAG PART"
42LETM7=0
44LETM8=0
46LETM9=0
48LETS1=1E-2
50LETS2=1
```

```
52LETB1=0
54LETB2=S2
56FØRX=B1+S1/2TØB2STEPS1
58LETX2=.707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
60LETY2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
62LETX1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
64LETY1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
66LETZ1=SQR(X1*X1+Y1*Y1)
68LETØ1=ATN(Y1/X1)
70LETZ2=SQR(X2*X2+Y2*Y2)
72LET Ø2=ATN(Y2/X2)
74LETZ=A*X
76LETØ=0
78 GØ SUB308
80LETI1=Q1
82LETI2=Q5
84LETK2=Q7
86LETZ=A*Z1
88LETØ=Ø1
90GØSUB308
92LETI3=Q1
94LETJ3=Q2
96LETK3=Q3
98LETN3=Q4
100LETI4=Q5
102LETJ4=Q6
104LETK4=Q7
106LETN4=Q8
108LETZ=B*Z1
110LETØ=Ø1
112GØSUB308
114LETI5=Q1
116LETJ5=Q2
118LETK5=Q3
120LETN5=Q4
122LETI6=Q5
124LETJ6=Q6
126LETK6=Q7
128LETN6=Q8
130LETZ=B*Z2
132LETØ=Ø2
134GØSUB308
136LETK7=Q3
138LETN7=Q4
140LETK8=Q7
142LETN8=Q8
144LETQ1=X*I4*I1*CØS(J4)-Z1*I3*I2*CØS(Ø1+J3)/U1
146LETQ2=X*I4*I1*SIN(J4)-Z1*I3*I2*SIN(Ø1+J3)/U1
148LETD1=SQR(Q1*Q1+Q2*Q2)
150GØSUB506
152LETE1=Q3
154LETQ1=Z1*K5*K8*CØS(Ø1+N5+N8)/U1-Z2*K7*K6*CØS(Ø2+N7+N6)/U2
156LETQ2=Z1*K5*K8*SIN(Ø1+N5+N8)/U1-Z2*K7*K6*SIN(Ø2+N7+N6)/U2
```

```
158LETD2=SQR(Q1*Q1+Q2*Q2)
160GØSUB506
162LETE2=Q3
164LETQ1=Z2*K7*I6*CØS(Ø2+N7+J6)/U1-Z1*I5*K8*CØS(Ø1+J5+N8)/U2
166LETQ2=Z2*K7*I6*SIN(Ø2+N7+J6)/U1-Z1*I5*K8*SIN(Ø1+J5+N8)/U2
168LETD3=SQR(Q1*Q1+Q2*Q2)
170GØSUB506
172LETE3=Q3
174LETQ1=X*K4*I1*CØS(N4)-Z1*K3*I2*CØS(Ø1+N3)/U1
176LETQ2=X*K4*I1*SIN(N4)-Z1*K3*I2*SIN(Ø1+N3)/U1
178LETD4=SQR(Q1*Q1+Q2*Q2)
180GØSUB506
182LETE4=Q3
184LETQ1=D1*D2*CØS(E1+E2)+D3*D4*CØS(E3+E4)
186LETQ2=D1*D2*SIN(E1+E2)+D3*D4*SIN(E3+E4)
188LETD5=SQR(Q1*Q1+Q2*Q2)
190GØSUB506
192LETE5=Q3
194LETG1=(I4*D2*CØS(J4+E2-E5)+K4*D3*CØS(N4+E3-E5))/(D5*A*I2)-K2/I2
196LETG2=(I4*D2*SIN(J4+E2-E5)+K4*D3*SIN(N4+E3-E5))/(D5*A*I2)
198LETZ=K2*X
200LETG1=K2
202GØSUB 260
204LETQ7=V1
206LETI2=V2
208LETZ=K1*X
210LETG1=K1
212GØSUB 260
214LETQ8=V1
216LETI1=V2
218LETQ9=Q7-Q8
220LETS5=4*S1*(Q9*SIN(X*(L2-L1)/2))↑2
222LETA1=G1*S5
224LETA2=G2*S5
226LETI3=I2-I1
228LETS3=3.141592*X*I3*I3*S1
230LETA3=S3*((EXP(-X*(L2-L1))-1)/X+L2-L1)
232LETM7=M7+A1+A3
234LETM8=M8+A2
236LETM9=M9+A3
238NEXTX
240LETG3=X+(S1)/2
242PRINTG3,M9,M7,M8
244LETS1=5E-2
246LETB1=B1+S2
248LETB2=B2+S2
250IFX<14THEN56
252PRINT"NØRMALIZED IMAG PART";(A9-M9+M7)/A9,
254PRINT"NØRMALIZED REAL PART";-M8/A9
256PRINT
258GØTØ514
260 IF Z>5 THEN 286
262LETL5=20*(1-EXP(-Z/10))+4
```

```
264LETF1=Q1*Q1*Q1/2
266LETF2=1
268LETV1=F1/3
270LETV2=V1
272FØRR=1TØL5
274LETF1=F1*Z*Z/(4*R*(R+1))
276LETF2=-F2
278LETV1=V1+F1/(3+2*R)
280LETV2=V2+F2*F1/(3+2*R)
282NEXTR
284 GØ TØ 306
286 LET P1=(((-188.1357/Z+109.1142)/Z-23.79333)/Z+2.050931)/Z
288 LET P1=((P1-.1730503)/Z+.7034845)/Z-.064109E-3
290 LET P2=(((-5.817517/Z+2.105874)/Z-.6896196)/Z+.4952024)/Z
292 LET P2=(P2-.187344E-2)/Z+.7979095
294 LET P4=3.1415927
296 LET V2=(1-SQR(Z)*(P2*CØS(Z-P4/4)-P1*SIN(Z-P4/4)))/(X*X*X)
298 LET P3=(((1660.794/Z-1737.556)/Z+543.6694)/Z+11.81804)/Z
300 LET P3=(((P3-33.78366)/Z+5.108402)/Z-.6130935)/Z
302 LET P3=(P3-.3360836)/Z+.3987795
304 LETV1=P3*SQR(Z)*EXP(Z)/(X*X*X)
306RETURN
308 IFZ>8THEN382
310 LETL5=20*(1-EXP(-Z/10))+4
312 T1=.5
314 LETT5=Z/2
316 LETV1=T1
318 LETV2=0
320 V3=-.25
322 V4=0
324 LETV5=T5*CØS(Ø)
326 LETV6=T5*SIN(Ø)
328 LETV7=V5
330 LETV8=V6
332 LETF1=0
334 LETF2=1
336 FØR R=1 TØ L5
338 LET Z3=CØS(2*R*Ø)
340 LET Z4=SIN(2*R*Ø)
342 LET Z5=CØS((2*R+1)*Ø)
344 LET Z6=SIN((2*R+1)*Ø)
346 LETF1=F1+1/R
348 LETF2=F2+1/(R+1)
350 LETT1=T1*Z*Z/(4*R*(R+1))
352 LET T5=T5*Z*Z/(4*R*(R+1))
354 V1=V1+T1*Z3*(2*R+1)
356 V2=V2+T1*Z4*(2*R+1)
358 V3=V3+T1*Z3*((2*R+1)*(F1+F2)/2-1)
360 V4=V4+T1*Z4*((2*R+1)*(F1+F2)/2-1)
362 LET V5=V5+T5*Z5
364 LET V7=V7+T5*(F1+F2)*Z5
366 LET V6=V6+T5*Z6
368 LET V8 =V8+T5*(F1+F2)*Z6
```

```
370 NEXTK
372 V3=-V3-C0S(2*0)/(Z*Z)+(.577215665+L0G(Z/2))*V1-0*V2
374 V4=-V4+SIN(2*0)/(Z*Z)+(.577215665+L0G(Z/2))*V2+0*V1
376 LETV7=-V7/2+C0S(0)/Z+(.577215665+L0G(Z/2))*V5-0*V6
378 LETV8=-V8/2-SIN(0)/Z+(.577215665+L0G(Z/2))*V6+0*V5
380 G0T0472
382 LETF1=1
384 LETW7=1
386 LETW5=1
388 W3=.5
390 W1=.5
392 LETT2=1
394 LETT1=1
396 LET W2=0
398 LETW4=0
400 LETW6=0
402 LETW8=0
404 L5=3+20/Z
406 F0RK=1T0L5
408 LETF1=-F1
410 LET T2 =T2*(4-(2*K-1)*(2*K-1))/(8*K*Z)
412 LETT1=T2*(2*K+1)/2
414 LET Z3=C0S(K*0)
416 LET Z4=SIN(K*0)
418 LET W1=W1+F1*T1*Z3
420 LET W2=W2-F1*T1*Z4
422 LET W3=W3+T1*Z3
424 LET W4=W4-T1*Z4
426 LET W5=W5+F1*T2*Z3
428 LET W6=W6-F1*T2*Z4
430 LET W7 =W7+T2*Z3
432 LET W8=W8-T2*Z4
434 NEXT K
436 LETQ9=.398942280 4*EXP(Z*C0S(0))
438 LETQ5=Q9*(C0S(Z*SIN(0)-.5*0))/SQR(Z)
440 LETQ6=Q9*(SIN(Z*SIN(0)-.5*0))/SQR(Z)
442 LETQ1=Q9*(C0S(Z*SIN(0)-1.5*0))/SQR(Z*Z*Z)
444 LETQ2=Q9*(SIN(Z*SIN(0)-1.5*0))/SQR(Z*Z*Z)
446 LETQ9=1.253314137*EXP(-Z*C0S(0))
448 LETQ7=Q9*(C0S(-Z*SIN(0)-.5*0))/SQR(Z)
450 LETQ8=Q9*(SIN(-Z*SIN(0)-.5*0))/SQR(Z)
452 LETQ3=Q9*(C0S(-Z*SIN(0)-1.5*0))/SQR(Z*Z*Z)
454 LETQ4=Q9*(SIN(-Z*SIN(0)-1.5*0))/SQR(Z*Z*Z)
456 LETV5=W5*Q5-W6*Q6
458 LETV6=W6*Q5+W5*Q6
460 LETV7=W7*Q7-W8*Q8
462 LETV8=W8*Q7+W7*Q8
464 LETV1=V5-W1*Q1+W2*Q2
466 LETV2=V6-W1*Q2-W2*Q1
468 LETV3=-V7-W3*Q3+W4*Q4
470 LETV4=-V8-W3*Q4-W4*Q3
472 LETG1=SQR(V1*V1+V2*V2)
474 LETG2=ATN(V2/V1)
```

```
476 IFV1>0THEN480
478 LETQ2=Q2+3.1415927
480 LETQ3=SQR(V3*V3+V4*V4)
482 LETQ4=ATN(V4/V3)
484 IFV3>0THEN488
486 LETQ4=Q4+3.1415927
488 LETQ5=SQR(V5*V5+V6*V6)
490 LETQ6=ATN(V6/V5)
492 IFV5>0THEN496
494 LETQ6=Q6+3.1415927
496 LETQ7=SQR(V7*V7+V8*V8)
498 LETQ8=ATN(V8/V7)
500 IFV7>0THEN504
502 LETQ8=Q8+3.1415927
504 RETURN
506 LETQ3=ATN(Q2/Q1)
508 IFQ1>0THEN512
510 LETQ3=Q3+3.1415927
512 RETURN
514END
```

Defect Sensitivity Factor for a Defect in the Inner Material

Discussion of INDFT5

This program is designed to calculate the defect sensitivity factor for a coil with rectangular cross section coaxial with and inside a two-conductor tube of effectively infinite (three- or four-coil diameters) length. The wall thickness of the outer conductor is also effectively infinite. For this program, the defect must be in the inner conductor which is labeled III in Fig. 10, p. 138. The conductors may have different electrical conductivities.

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the inner conductor and by $\omega\mu_2\sigma_2\bar{r}^2$ for the outer one.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

```
12 LET R1 = (Numerical value of normalized inner coil radius)
14 LET R2 = (Numerical value of normalized outer coil radius)
16 LET L1 = (Numerical value of normalized distance between
             bottom of coil and Z = 0 plane)
18 LET L2 = (Numerical value of normalized distance between
             top of coil and Z = 0 plane)
20 LET M1 = (Numerical value of ωμ1σ1r̄²)
22 LET M2 = (Numerical value of ωμ2σ2r̄²)
```

Note: If the conductivity of the outer conductor is zero, one must type two additional lines:

```
60 LET X2 = X
62 LET Y2 = 0

24 LET A = (Numerical value of normalized radius of inner conductor)
26 LET B = (Numerical value of normalized radius of outer conductor)
```

Note: This value may be obtained with the aid of AIRC05 simply by placing the appropriate coil dimensions in that program.

```
28 LET A9 = (Normalization factor)
30 LET R9 = (Numerical value of normalized radius of r position
             coordinate of defect)
32 LET Z9 = (Numerical value of normalized radius of z position
             coordinate of defect)
```

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)          R2 = (R2)          L1 = (L1)           L2 = (L2)
INN RAD = (A)      OUT RAD = (B)      AIR VALUE = (A9)
M1 = (M1)                             M2 = (M2)
DEFECT POSITION IS                    R = (R9)            Z = (Z9)
X                MAGNITUDE            PHASE
 1                 ------             ------
 5                 ------             ------
 10                ------             ------
 15                ------             ------
 20                ------             ------
 25                ------             ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The other two columns have self-explanatory headings and appear principally to allow one to inspect the convergence of the integration. The magnitude and phase values for X = 25 are considered to be correct since the integration in most cases converges sufficiently well for this value of X.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of INDFT5

Let us suppose that we wish to know the defect sensitivity factor for a coil 0.155 in. long with inner and outer radii of 0.420 and 0.580 in., respectively, coaxial with and inside a two-conductor tube of effectively infinite (three- or four-coil diameters) length. The inner conductor is assumed to be copper with a radius of 0.600 in., while the outer conductor is assumed to be aluminum with a radius of 0.700 in. Let us further suppose that we are interested in the defect sensitivity factor for the case in which the defect is located 0.650 in. from the axis and 0.0825 in. above the Z = 0 plane. We shall assume the frequency of the driving current to be 1.25 kHz and the distance of the bottom of the coil from the Z = 0 plane to be 0.005 in.

Normalized inner coil radius = 0.420/0.500 = 0.84
Normalized outer coil radius = 0.580/0.500 = 1.16
Normalized distance from bottom of coil to Z = 0 plane = 0.01
Normalized distance from top of coil to Z = 0 plane = 0.32
Normalized radius of inner conductor = 0.600/0.500 = 1.2
Normalized radius of outer conductor = 0.700/0.500 = 1.4
Normalized r position of defect = 0.650/0.500 = 1.3
Normalized z position of defect = 0.0825/0.500 = 0.165

Furthermore,

Angular frequency of driving current = $2\pi \times 1250\ \mathrm{sec}^{-1}$
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$

$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

At this point one determines the normalization factor by placing the normalized coil dimensions in AIRCO5 and running that program. It is worth noting that the normalization factor is independent of all parameters excluding the coil dimensions. This factor is normalization factor 2 in the print-out of AIRC05 and for the case in point is found to be 0.01011632. With the normalization factor in hand, the remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.84
14 LET R2 = 1.16
16 LET L1 = 0.01
18 LET L2 = 0.32
20 LET M1 = 91.91
22 LET M2 = 45.48
24 LET A = 1.2
26 LET B = 1.4
28 LET A9 = 0.1011632E-1
30 LET R9 = 1.3
32 LET Z9 = 0.165
```

Execution of the program yields the following results:

```
R1 = 0.84           R2 = 1.16          L1 = 0.01          L2 = 0.32
INN RAD = 1.2       OUT RAD = 1.4      AIR VALUE = 1.01163E-2
M1 = 91.91                             M2 = 45.48
DEFECT POSITION IS                     R = 1.3            Z = 0.165
X                   MAGNITUDE          PHASE
0.99999999          0.0684334E-01      -2.8303405
4.99999999          0.08325121         -2.6224884
9.99999999          0.12353465         -2.4600687
15                  0.12931823         -2.4223783
20                  0.12980529         -2.4181076
25                  0.12975508         -2.4186688
```

From this we see that the defect sensitivity factor is

$$0.129755\ e^{-j2.41867}$$

To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

<u>INDFT5 Program</u>

```
10 REM             INDFT5            VERSION          02/24/69
12 LET R1=.84
14 LET R2=1.16
16 LET L1=.01
18 LET L2=.32
```

```
20 LET M1=91.91
22 LET M2=45.48
24 LET A=1.2
26 LET B=1.4
28 LET A9=.1011632E-1
30 LET R9=1.3
32 LET Z9=.165
34PRINT"R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
36PRINT"INN RAD=";A,"ØUT RAD=";B,"AIR VALUE=";A9
38PRINT"M1=";M1," ","M2=";M2
40PRINT"DEFECT PØSITIØN IS","R=";R9,"Z=";Z9
42PRINT"X","MAGNITUDE","PHASE"
44LETM7=0
46LETM8=0
48LETM9=0
50LETS1=1E-2
52LETS2=1
54LETB1=0
56LETB2=S2
58FØRX=B1+S1/2TØB2STEPS1
60LETX2=.707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
62LETY2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
64LETX1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
66LETY1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
68LETZ1=SQR(X1*X1+Y1*Y1)
70LETØ1=ATN(Y1/X1)
72LETZ2=SQR(X2*X2+Y2*Y2)
74LET Ø2=ATN(Y2/X2)
76LETZ=A*X
78LETØ=0
80GØSUB300
82LETI1=Q1
84LETI2=Q5
86LETK2=Q7
88LETZ=A*Z1
90LETØ=Ø1
92GØSUB300
94LETI3=Q1
96LETJ3=Q2
98LETK3=Q3
100LETN3=Q4
102LETI4=Q5
104LETJ4=Q6
106LETK4=Q7
108LETN4=Q8
110LETZ=B*Z1
112LETØ=Ø1
114GØSUB300
116LETI5=Q1
118LETJ5=Q2
120LETK5=Q3
122LETN5=Q4
124LETI6=Q5
126LETJ6=Q6
```

```
128LETK6=Q7
130LETN6=Q8
132LETZ=B*Z2
134LETØ=Ø2
136GØSUB300
138LETK7=Q3
140LETN7=Q4
142LETK8=Q7
144LETN8=Q8
146LETZ=R9*Z1
148LETØ=Ø1
150GØSUB300
152LETI9=Q5
154LETJ9=Q6
156LETK9=Q7
158LETN9=Q8
160LETQ1=X*I4*I1*CØS(J4)-Z1*I3*I2*CØS(Ø1+J3)
162LETQ2=X*I4*I1*SIN(J4)-Z1*I3*I2*SIN(Ø1+J3)
164LETD1=SQR(Q1*Q1+Q2*Q2)
166 GØSUB498
168LETE1=Q3
170LETQ1=Z1*K5*K8*CØS(Ø1+N5+N8)-Z2*K6*K7*CØS(Ø2+N7+N6)
172LETQ2=Z1*K5*K8*SIN(Ø1+N5+N8)-Z2*K6*K7*SIN(Ø2+N7+N6)
174LETD2=SQR(Q1*Q1+Q2*Q2)
176 GØSUB498
178LETE2=Q3
180LETQ1=Z2*K7*I6*CØS(Ø2+N7+J6)-Z1*I5*K8*CØS(Ø1+J5+N8)
182LETQ2=Z2*K7*I6*SIN(Ø2+N7+J6)-Z1*I5*K8*SIN(Ø1+J5+N8)
184LETD3=SQR(Q1*Q1+Q2*Q2)
186 GØSUB498
188LETE3=Q3
190LETQ1=X*K4*I1*CØS(N4)-Z1*K3*I2*CØS(Ø1+N3)
192LETQ2=X*K4*I1*SIN(N4)-Z1*K3*I2*SIN(Ø1+N3)
194LETD4=SQR(Q1*Q1+Q2*Q2)
196 GØSUB498
198LETE4=Q3
200LETQ1=D1*D2*CØS(E1+E2)+D3*D4*CØS(E3+E4)
202LETQ2=D1*D2*SIN(E1+E2)+D3*D4*SIN(E3+E4)
204LETD5=SQR(Q1*Q1+Q2*Q2)
206GØSUB498
208LETE5=Q3
210LETQ1=(I9*D2*CØS(J9+E2-E5)+K9*D3*CØS(N9+E3-E5))/(A*D5)
212LETQ2=(I9*D2*SIN(J9+E2-E5)+K9*D3*SIN(N9+E3-E5))/(A*D5)
214LETZ=R2*X
216LETQ1=R2
218GØSUB 272
220LETQ7=V1
222LETZ=R1*X
224LETQ1=R1
226GØSUB 272
228LETQ8=V1
230LETQ9=Q7-Q8
232LETS5=S1*Q9*(SIN(X*(Z9-L1))-SIN(X*(Z9-L2)))
```

```
234LETM7=M7+G1*S5
236LETM8=M8+G2*S5
238NEXTX
240LETM9=ATN(M8/M7)
242IFM7>0THEN246
244LETM9=M9+3.1415927
246LETN9=(3*M1)/(4*9.8696044*A9)
248PRINTX+S1/2,N9*(M7*M7+M8*M8),2*M9
250LETS1=5E-2
252IFX>4.5THEN260
254LETB1=1
256LETB2=5
258 GØTØ58
260LETS2=5
262LETB1=B2
264LETB2=B2+S2
266IFX<24.5THEN58
268PRINT
270 GØTØ506
272IFZ>3THEN290
274LETL5=20*(1-EXP(-Z/10))+4
276LETF1=Q1*Q1*Q1/2
278LETV1=F1/3
280FØRK=1TØL5
282LETF1=F1*Z*Z/(4*K*(K+1))
284LETV1=V1+F1/(3+2*K)
286NEXTK
288 GØ TØ 298
290LETF3=(((1660.794/Z-1737.556)/Z+543.6694)/Z+11.81804)/Z
292LETF3=(((F3-33.78366)/Z+5.108402)/Z-.6130935)/Z
294LETF3=(F3-.3360836)/Z+.3987795
296 LETV1=F3*SQR(Z)*EXP(Z)/(X*X*X)
298RETURN
300 IFZ>8THEN374
302 LETL5=20*(1-EXP(-Z/10))+4
304 LETT1=.5
306 LETT5=Z/2
308 LETV1=T1
310 LETV2=0
312 LETV3=-.25
314 LETV4=0
316 LETV5=T5*CØS(Ø)
318 LETV6=T5*SIN(Ø)
320 LETV7=V5
322 LETV8=V6
324 LETF1=0
326 LETF2=1
328 FØR K=1 TØ L5
330 LET Z3=CØS(2*K*Ø)
332 LET Z4=SIN(2*K*Ø)
334 LET Z5=CØS((2*K+1)*Ø)
336 LET Z6=SIN((2*K+1)*Ø)
338 LETF1=F1+1/K
```

```
340 LETF2=F2+1/(K+1)
342 LETT1=T1*Z*Z/(4*K*(K+1))
344 LET T5=T5*Z*Z/(4*K*(K+1))
346 LETV1=V1+T1*Z3*(2*K+1)
348 LETV2=V2+T1*Z4*(2*K+1)
350 LETV3=V3+T1*Z3*((2*K+1)*(F1+F2)/2-1)
352 LETV4=V4+T1*Z4*((2*K+1)*(F1+F2)/2-1)
354 LET V5=V5+T5*Z5
356 LET V7=V7+T5*(F1+F2)*Z5
358 LET V6=V6+T5*Z6
360 LET V8 =V8+T5*(F1+F2)*Z6
362 NEXTK
364 LETV3=-V3-CØS(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V1-Ø*V2
366 LETV4=-V4+SIN(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V2+Ø*V1
368 LETV7=-V7/2+CØS(Ø)/Z+(.577215665+LØG(Z/2))*V5-Ø*V6
370 LETV8=-V8/2-SIN(Ø)/Z+(.577215665+LØG(Z/2))*V6+Ø*V5
372 GØTØ464
374 LETF1=1
376 LETW7=1
378 LETW5=1
380 LETW3=.5
382 LETW1=.5
384 LETT2=1
386 LETT1=1
388 LET W2=0
390 LETW4=0
392 LETW6=0
394 LETW8=0
396 LETL5=3+20/Z
398 FØRK=1TØL5
400 LETF1=-F1
402 LET T2 =T2*(4-(2*K-1)*(2*K-1))/(8*K*Z)
404 LETT1=T2*(2*K+1)/2
406 LET Z3=CØS(K*Ø)
408 LET Z4=SIN(K*Ø)
410 LET W1=W1+F1*T1*Z3
412 LET W2=W2-F1*T1*Z4
414 LET W3=W3+T1*Z3
416 LET W4=W4-T1*Z4
418 LET W5=W5+F1*T2*Z3
420 LET W6=W6-F1*T2*Z4
422 LET W7 =W7+T2*Z3
424 LET W8=W8-T2*Z4
426 NEXT K
428 LETG9=.398942280 4*EXP(Z*CØS(Ø))
430 LETQ5=G9*(CØS(Z*SIN(Ø)-.5*Ø))/SQR(Z)
432 LETQ6=G9*(SIN(Z*SIN(Ø)-.5*Ø))/SQR(Z)
434 LETQ1=G9*(CØS(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
436 LETQ2=G9*(SIN(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
438 LETG9=1.253314137*EXP(-Z*CØS(Ø))
440 LETQ7=G9*(CØS(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
442 LETQ8=G9*(SIN(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
444 LETQ3=G9*(CØS(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
```

```
446 LETQ4=Q9*(SIN(-Z*SIN(Ø)-1.5*B))/SQR(Z*Z*Z)
448 LETV5=W5*Q5-W6*Q6
450 LETV6=W6*Q5+W5*Q6
452 LETV7=W7*Q7-W8*Q8
454 LETV8=W8*Q7+W7*Q8
456 LETV1=V5-W1*Q1+W2*Q2
458 LETV2=V6-W1*Q2-W2*Q1
460 LETV3=-V7-W3*Q3+W4*Q4
462 LETV4=-V8-W3*Q4-W4*Q3
464 LETQ1=SQR(V1*V1+V2*V2)
466 LETQ2=ATN(V2/V1)
468 IFV1>0THEN472
470 LETQ2=Q2+3.1415927
472 LETQ3=SQR(V3*V3+V4*V4)
474 LETQ4=ATN(V4/V3)
476 IFV3>0THEN480
478 LETQ4=Q4+3.1415927
480 LETQ5=SQR(V5*V5+V6*V6)
482 LETQ6=ATN(V6/V5)
484 IFV5>0THEN488
486 LETQ6=Q6+3.1415927
488 LETQ7=SQR(V7*V7+V8*V8)
490 LETQ8=ATN(V8/V7)
492 IFV7>0THEN496
494 LETQ8=Q8+3.1415927
496 RETURN
498 LETQ3=ATN(Q2/Q1)
500 IFQ1>0THEN504
502 LETQ3=Q3+3.1415927
504 RETURN
506 END
```

Defect Sensitivity Factor for a Lattice of Defects in the Inner Material

Discussion of INDFTL

This program is designed to calculate the defect sensitivity factor for a coil with rectangular cross section coaxial with and inside a two-conductor tube of effectively infinite (three- or four-coil diameters) length. The wall thickness of the outer conductor is also effectively infinite (three or four skin depths). For this program, the defect may be located at any one of a number of points on a lattice in the inner conductor, which is labeled III in Fig. 10, p. 138. The conductors may have different electrical conductivities.

Before proceeding further, it is worth noting that in the execution of this program the data generated by the computer are stored in a data block which is used as input data for another program. Since there exist BASIC compilers which do not have this capability, this program may or may not be executable, depending upon the compiler used. If the latter situation arises, one may delete line 38 and the word "FILE" wherever it appears in the program, thus causing the data to be printed at the console; otherwise, one may assess the data by using the name /@2/.

To use this program, one must first divide all dimensions by the mean radius of the core. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\overline{r}^2$ for the inner conductor and by $\omega\mu_2\sigma_2\overline{r}^2$ for the outer one.

Once these calculations have been made, one must type the following lines into the program. (Note: Parentheses are not typed.)

```
12 LET R1 = (Numerical value of normalized inner coil radius)
14 LET R2 = (Numerical value of normalized outer coil radius)
16 LET L1 = (Numerical value of normalized distance between bottom
             of coil and Z = 0 plane)
18 LET L2 = (Numerical value of normalized distance between top
             of coil and Z = 0 plane)
20 LET M1 = (Numerical value of ωμ1σ1r̄²)
22 LET M2 = (Numerical value of ωμ2σ2r̄²)
```

Note: If the conductivity of the outer conductor is zero, one must type two additional lines:

```
76 LET X2 = X
78 LET Y2 = 0

24 LET A = (Numerical value of normalized radius of inner conductor)
26 LET B = (Numerical value of normalized radius of outer conductor)
28 LET A9 = (Normalization factor)
```

Note: This value may be obtained with the aid of AIRC05 simply by placing the appropriate coil dimensions in that program.

```
30 LET R9 = (Numerical value of number of lattice spacings in
             r direction)
32 LET Z9 = (Numerical value of number of lattice spacings in
             z direction)
34 LET Z8 = (Numerical value of maximum normalized z position
             coordinate of defect)
```

Note: Neither R9 nor Z9 may exceed 15. The lattice spacing in the r direction is (B-A)/R9, while it is Z8/Z9 in the z direction. In addition, Z8 is measured from the midplane of the coil.

After typing these lines, one need only create an empty data file using the name /@2/; the program may then be run.

During the execution of this program, the integers which are less than or equal to 20 will be consecutively printed at the console. These appear as indications of how far the integration has proceeded. The data resulting from this program will have the following format:

```
(R1)        (R2)        (L1)        (L2)
(A)         (B)         (A9)
(M1)                    (M2)
(R9)        (Z9)        (Z8)
            (R(1))      (R(2))      **********    (R(R9))
(Z(1))      ------      ------      **********    ------
            ------      ------      **********    ------

(Z(2))      ------      ------      **********    ------
            ------      ------      **********    ------

  *           *           *             *            *
  *           *           *             *            *
  *           *           *             *            *
  *           *           *             *            *

(Z(Z9))     ------      ------      **********    ------
            ------      ------      **********    ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The upper value at each lattice point is the magnitude of the defect sensitivity factor, while the lower one is the phase of the defect sensitivity factor for the case in which the defect is located at the given lattice point. It should be noted that the lattice points for which R9 > 4 will be printed below the last line (i.e., the line containing Z/R9).

The following example is presented for additional aid to those who wish to use this program.

Sample Calculation of INDFTL

Let us suppose that we wish to know the defect sensitivity factor for the case of a coil 0.155 in. long with inner and outer radii of 0.420 and 0.580 in., respectively, coaxial with and inside a two-conductor tube of effectively infinite (three- or four-coil diameters) length. The inner conductor is assumed to be copper with a radius of 0.600 in., while the outer is assumed to be aluminum with a radius of 0.700 in. Let us further suppose that we are interested in the defect sensitivity factor for the cases in which the defect is located at any one of the points which lie within 1.000 in. of the midplane of the coil on a 4 by 2 lattice in the copper. We shall assume the frequency of the driving current to be 1.25 kHz and the distance of the bottom of the coil from the Z = 0 plane to be 0.005 in.

First, we determine the mean coil radius to be 0.500 in., so that

Normalized inner coil radius = 0.420/0.500 = 0.84
Normalized outer coil radius = 0.580/0.500 = 1.16
Normalized distance from bottom of coil to Z = 0 plane = 0.01
Normalized distance from top of coil to Z = 0 plane = 0.32
Normalized radius of inner conductor = 0.600/0.500 = 1.2
Normalized radius of outer conductor = 0.400/0.500 = 1.4
Number of r lattice spacings = 4
Number of z lattice spacings = 2
Maximum normalized z coordinate of defect = 1.000/0.500 = 2

Furthermore,

Angular frequency of driving current = $2\pi \times 1250$ sec^{-1}
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$
$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

At this point one determines the normalization factor by placing the normalized coil dimensions in AIRC05 and running that program. It is worth noting that the normalization factor is independent of all parameters excluding the coil dimensions. This factor is normalization factor 2 in the print-out of AIRC05 and for the case in point is found to be 0.01011632. With the normalization factor in hand, the remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.84
14 LET R2 = 1.16
16 LET L1 = 0.01
18 LET L2 = 0.32
20 LET M1 = 91.91
22 LET M2 = 45.48
24 LET A = 1.2
26 LET B = 1.4
28 LET A9 = 0.1011632E-1
30 LET R9 = 4
32 LET Z9 = 2
34 LET Z8 = 2
```

An empty data file is now created and the program run with the following results:

0.84	1.16	0.01	0.32	
1.2	1.4	1.01163E-2		
91.91		45.48		
4	2	2		
	1.225	1.275	1.325	1.375
−0.335	0.1731473E-01	0.0957986E-01	0.5735169E-02	0.3734065E-02
	−2.2356292	−3.0025318	8.8192924	8.1769891
−1.335	0.1839122E-04	0.0943341E-04	0.5139131E-05	0.3048009E-06
	−2.7034711	9.1139215	8.3586866	7.6698976

From this, we see that for the case of a defect located at R = 1.325, Z = −1.335 the defect sensitivity factor is

$$5.13913 \times 10^{-6}\ e^{j8.35869}$$

To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

INDFTL Program

```
10 REM            INDFTL            VERSIØN            03/02/69
12 LET R1=.84
14 LET R2=1.16
16 LET L1=.01
18 LET L2=.32
20 LET M1=91.91
22 LET M2=45.48
24 LET A=1.2
26 LET B=1.4
28 LET A9=.1011632E-1
30 LET R9=4
32 LET Z9=2
34 LET Z8=2
36DIMM(18,18),I(18,18),P(18,18),R(18),Z(18)
38ØPEN/02/,ØUTPUT
40PRINTFILER1,R2,L1,L2
42PRINTFILEA,B,A9
44PRINTFILEM1," ",M2
46PRINTFILER9,Z9,Z8
48PRINTFILE
50PRINT"          A PRØGRAM IS NØW RUNNING.   PLEASE DØ NØT DISCØNNECT!!!"
52PRINT
54FØRI=1TØZ9
56FØRJ=1TØR9
58LETM(I,J)=0
60LETI(I,J)=0
62NEXTJ
64NEXTI
66LETS1=1E-2
68LETS2=1
70LETB1=0
72LETB2=S2
74FØRX=B1+S1/2TØB2STEPS1
76LETX2=.707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
78LETY2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
80LETX1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
82LETY1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
84LETZ1=SQR(X1*X1+Y1*Y1)
86LETØ1=ATN(Y1/X1)
88LETZ2=SQR(X2*X2+Y2*Y2)
90LETØ2=ATN(Y2/X2)
92LETZ=A*X
94LETØ=0
96GØSUB408
98LETI1=Q1
100LETI2=Q5
102LETK2=Q7
104LETZ=A*Z1
106LETØ=Ø1
108GØSUB408
110LETI3=Q1
112LETJ3=Q2
114LETK3=Q3
116LETN3=Q4
118LETI4=Q5
120LETJ4=Q6
122LETK4=Q7
124LETN4=Q8
```

```
126LETZ=B*Z1
128LETØ=Ø1
130GØSUB408
132LETI5=Q1
134LETJ5=Q2
136LETK5=Q3
138LETN5=Q4
140LETI6=Q5
142LETJ6=Q6
144LETK6=Q7
146LETN6=Q8
148LETZ=B*Z2
150LETØ=Ø2
152GØSUB408
154LETK7=Q3
156LETN7=Q4
158LETK8=Q7
160LETN8=Q8
162LETQ1=X*I4*I1*CØS(J4)-Z1*I3*I2*CØS(Ø1+J3)
164LETQ2=X*I4*I1*SIN(J4)-Z1*I3*I2*SIN(Ø1+J3)
166LETD1=SQR(Q1*Q1+Q2*Q2)
168GØSUB606
170LETE1=Q3
172LETQ1=Z1*K5*K8*CØS(Ø1+N5+N8)-Z2*K7*K6*CØS(Ø2+N7+N6)
174LETQ2=Z1*K5*K8*SIN(Ø1+N5+N8)-Z2*K7*K6*SIN(Ø2+N7+N6)
176LETD2=SQR(Q1*Q1+Q2*Q2)
178GØSUB606
180LETE2=Q3
182LETQ1=Z2*K7*I6*CØS(Ø2+N7+J6)-Z1*I5*K8*CØS(Ø1+J5+N8)
184LETQ2=Z2*K7*I6*SIN(Ø2+N7+J6)-Z1*I5*K8*SIN(Ø1+J5+N8)
186LETD3=SQR(Q1*Q1+Q2*Q2)
188GØSUB606
190LETE3=Q3
192LETQ1=X*K4*I1*CØS(N4)-Z1*K3*I2*CØS(Ø1+N3)
194LETQ2=X*K4*I1*SIN(N4)-Z1*K3*I2*SIN(Ø1+N3)
196LETD4=SQR(Q1*Q1+Q2*Q2)
198GØSUB606
200LETE4=Q3
202LETQ1=D1*D2*CØS(E1+E2)+D3*D4*CØS(E3+E4)
204LETQ2=D1*D2*SIN(E1+E2)+D3*D4*SIN(E3+E4)
206LETD5=SQR(Q1*Q1+Q2*Q2)
208GØSUB606
210LETE5=Q3
212LETZ=R2*X
214LETQ1=R2
216GØSUB380
218LETQ7=V1
220LETZ=R1*X
222LETQ1=R1
224GØSUB380
226LETQ8=V1
228LETS6=Q7-Q8
230LETØ=Ø1
232FØRJ=1TØR9
234LETR(J)=A+(B-A)*(J-.5)/R9
236LETZ=Z1*R(J)
238GØSUB408
240LETI9=Q5
242LETJ9=Q6
244LETK9=Q7
```

```
246LETN9=Q8
248LETG1=(I9*L2*COS(J9+E2-E5)+K9*D3*COS(N9+E3-E5))/(A*D5)
250LETG2=(I9*L2*SIN(J9+E2-E5)+K9*D3*SIN(N9+E3-E5))/(A*D5)
252FØRI=1TØZ9
254LETZ(I)=(L1+L2)/2-(I-.5)*Z8/Z9
256LETS5=S1*S6*(SIN(X*(Z(I)-L1))-SIN(X*(Z(I)-L2)))
258LETM(I,J)=M(I,J)+G1*S5
260LETI(I,J)=I(I,J)+G2*S5
262NEXTI
264NEXTJ
266NEXTX
268PRINTX+S1/2,
270LETS1=5E-2
272LETB1=B1+S2
274LETB2=B2+S2
276IFX<19.5THEN74
278FØRI=1TØZ9
280FØRJ=1TØK9
282LETP(I,J)=2*ATN(I(I,J)/M(I,J))
284IFM(I,J)>0THEN288
286LETP(I,J)=P(I,J)+2*PI
288LETM(I,J)=M(I,J)*M(I,J)+I(I,J)*I(I,J)
290NEXTJ
292NEXTI
294LETQ=3*M1/(4*9.8696044*A9)
296PRINTFILE"",
298FØRJ=1TØ4
300PRINTFILER(J),
302NEXTJ
304FØRI=1TØZ9
306PRINTFILEZ(I),
308FØRJ=1TØ4
310PRINTFILEQ*M(I,J),
312NEXTJ
314PRINTFILE"",
316FØRJ=1TØ4
318PRINTFILEP(I,J),
320NEXTJ
322PRINTFILE
324NEXTI
326IFK9<4.5THEN614
328FØRJ=5TØ9
330PRINTFILER(J),
332NEXTJ
334FØRI=1TØZ9
336FØRJ=5TØ9
338PRINTFILEQ*M(I,J),
340NEXTJ
342FØRJ=5TØ9
344PRINTFILEP(I,J),
346NEXTJ
348PRINTFILE
350NEXTI
352IFK9<9.5THEN614
354FØRJ=10TØ14
356PRINTFILER(J),
358NEXTJ
360FØRI=1TØZ9
362FØRJ=10TØ14
364PRINTFILEQ*M(I,J),
366NEXTJ
```

```
368FØRJ=1TØ14
370PRINTFILEP(I,J),
372NEXTJ
374PRINTFILE
376NEXTI
378IFR9<14.5THEN614
380IFZ>3THEN398
382LETL5=INT(20*(1-EXP(-Z/10))+4)
384LETF1=Q1*Q1*Q1/2
386LETV1=F1/3
388FØRR=1TØL5
390LETF1=F1*Z*Z/(4*R*(R+1))
392LETV1=V1+F1/(3+2*R)
394NEXTR
396GØTØ406
398LETP3=(((1660.794/Z-1737.556)/Z+543.6694)/Z+11.81804)/Z
400LETP3=(((P3-33.78366)/Z+5.108402)/Z-.6130935)/Z
402LETP3=(P3-.3360836)/Z+.3987795
404LETV1=P3*SQR(Z)*EXP(Z)/(X*X*X)
406RETURN
408 IFZ>8THEN482
410 LETL5=20*(1-EXP(-Z/10))+4
412 LETT1=.5
414 LETT5=Z/2
416 LETV1=T1
418 LETV2=0
420 LETV3=-.25
422 LETV4=0
424 LETV5=T5*CØS(Ø)
426 LETV6=T5*SIN(Ø)
428 LETV7=V5
430 LETV8=V6
432 LETF1=0
434 LETF2=1
436 FØR R=1 TØ L5
438 LET Z3=CØS(2*R*Ø)
440 LET Z4=SIN(2*R*Ø)
442 LET Z5=CØS((2*R+1)*Ø)
444 LET Z6=SIN((2*R+1)*Ø)
446 LETF1=F1+1/R
448 LETF2=F2+1/(R+1)
450 LETT1=T1*Z*Z/(4*R*(R+1))
452 LET T5=T5*Z*Z/(4*R*(R+1))
454 LETV1=V1+T1*Z3*(2*R+1)
456 LETV2=V2+T1*Z4*(2*R+1)
458 LETV3=V3+T1*Z3*((2*R+1)*(F1+F2)/2-1)
460 LETV4=V4+T1*Z4*((2*R+1)*(F1+F2)/2-1)
462 LET V5=V5+T5*Z5
464 LET V7=V7+T5*(F1+F2)*Z5
466 LET V6=V6+T5*Z6
468 LET V8 =V8+T5*(F1+F2)*Z6
470 NEXTR
472 LETV3=-V3-CØS(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V1-Ø*V2
474 LETV4=-V4+SIN(2*Ø)/(Z*Z)+(.577215665+LØG(Z/2))*V2+Ø*V1
476 LETV7=-V7/2+CØS(Ø)/Z+(.577215665+LØG(Z/2))*V5-Ø*V6
478 LETV8=-V8/2-SIN(Ø)/Z+(.577215665+LØG(Z/2))*V6+Ø*V5
480 GØTØ572
482 LETF1=1
484 LETW7=1
486 LETW5=1
488 LETW3=.5
```

```
490 LETW1=.5
492 LETT2=1
494 LETT1=1
496 LET W2=0
498 LETW4=0
500 LETW6=0
502 LETW8=0
504 LETL5=3+20/Z
506 FØRK=1TØL5
508 LETF1=-F1
510 LET T2 =T2*(4-(2*K-1)*(2*K-1))/(8*K*Z)
512 LETT1=T2*(2*K+1)/2
514 LET Z3=CØS(K*Ø)
516 LET Z4=SIN(K*Ø)
518 LET W1=W1+F1*T1*Z3
520 LET W2=W2-F1*T1*Z4
522 LET W3=W3+T1*Z3
524 LET W4=W4-T1*Z4
526 LET W5=W5+F1*T2*Z3
528 LET W6=W6-F1*T2*Z4
530 LET W7 =W7+T2*Z3
532 LET W8=W8-T2*Z4
534 NEXT K
536 LETQ9=.393942280 4*EXP(Z*CØS(Ø))
538 LETQ5=Q9*(CØS(Z*SIN(Ø)-.5*Ø))/SQR(Z)
540 LETQ6=Q9*(SIN(Z*SIN(Ø)-.5*Ø))/SQR(Z)
542 LETQ1=Q9*(CØS(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
544 LETQ2=Q9*(SIN(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
546 LETQ9=1.253314137*EXP(-Z*CØS(Ø))
548 LETQ7=Q9*(CØS(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
550 LETQ8=Q9*(SIN(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
552 LETQ3=Q9*(CØS(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
554 LETQ4=Q9*(SIN(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
556 LETV5=W5*Q5-W6*Q6
558 LETV6=W6*Q5+W5*Q6
560 LETV7=W7*Q7-W8*Q8
562 LETV8=W8*Q7+W7*Q8
564 LETV1=V5-W1*Q1+W2*Q2
566 LETV2=V6-W1*Q2-W2*Q1
568 LETV3=-V7-W3*Q3+W4*Q4
570 LETV4=-V8-W3*Q4-W4*Q3
572 LETQ1=SQR(V1*V1+V2*V2)
574 LETQ2=ATN(V2/V1)
576 IFV1>0THEN580
578 LETQ2=Q2+3.1415927
580 LETQ3=SQR(V3*V3+V4*V4)
582 LETQ4=ATN(V4/V3)
584 IFV3>0THEN588
586 LETQ4=Q4+3.1415927
588 LETQ5=SQR(V5*V5+V6*V6)
590 LETQ6=ATN(V6/V5)
592 IFV5>0THEN596
594 LETQ6=Q6+3.1415927
596 LETQ7=SQR(V7*V7+V8*V8)
598 LETQ8=ATN(V8/V7)
600 IFV7>0THEN604
602 LETQ8=Q8+3.1415927
604 RETURN
606 LETQ3=ATN(Q2/Q1)
608 IFQ1>0THEN612
610 LETQ3=Q3+3.1415927
612 RETURN
614END
```

Defect Sensitivity Factor for a Defect in the Outer Material

Discussion of INDFB5

This program is designed to calculate the defect sensitivity factor for a coil with rectangular cross section coaxial with and inside a two-conductor tube of effectively infinite (three- or four-coil diameters) length. The wall thickness of the outer conductor is also effectively infinite (three or four skin depths). For this program, the defect must be in the outer conductor which is labeled IV in Fig. 10, p. 138. The conductors may have different electrical conductivities.

To use this program, one must first divide all dimensions by the mean radius of the coil. The results will, of course, be dimensionless and will be referred to as being "normalized." The product of the angular frequency of the driving current, the permeability, the conductivity, and the square of the mean radius of the coil must be calculated for each conductor. Let this product be denoted by $\omega\mu_1\sigma_1\bar{r}^2$ for the inner conductor and by $\omega\mu_2\sigma_2\bar{r}^2$ for the outer one.

Once these calculations have been made, one need only type the following lines into the program. [Note: (Parentheses are not typed.)]

```
12 LET R1 = (Numerical value of normalized inner coil radius)
14 LET R2 = (Numerical value of normalized outer coil radius)
16 LET L1 = (Numerical value of normalized distance between bottom
             of coil and Z = 0 plane)
18 LET L2 = (Numerical value of normalized distance between top
             of coil and Z = 0 plane)
20 LET M1 = (Numerical value of ωμ1σ1r̄²)
22 LET M2 = (Numerical value of ωμ2σ2r̄²)
```

Note: If the conductivity of the inner conductor is zero, one must type two additional lines:

```
60 LET X1 = X
62 LET Y1 = 0
```

```
24 LET A = (Numerical value of normalized radius of inner conductor)
26 LET B = (Numerical value of normalized radius of outer conductor)
28 LET A9 = (Normalization factor)
```

Note: This value may be obtained with the aid of AIRCO5 simply by placing the appropriate coil dimensions in that program.

```
30 LET R9 = (Numerical value of normalized r position coordinate
            of defect)
32 LET Z9 = (Numerical value of normalized z position coordinate
            of defect)
```

The program may now be run.

The print-out by the computer will have the following format:

```
R1 = (R1)        R2 = (R2)        L1 = (L1)            L2 = (L2)
INN RAD = (A)    OUT RAD = (B)    AIR VALUE = (A9)
M1 = (M1)                         M2 = (M2)
DEFECT POSITION IS                R = (R9)             Z = (Z9)
X               MAGNITUDE         PHASE
 1               ------            ------
 5               ------            ------
 10              ------            ------
 15              ------            ------
 20              ------            ------
 25              ------            ------
```

The various symbols enclosed in parentheses are used here to indicate that the numerical value of the symbol will be printed. The first column, headed "X," is the upper limit of the integration being performed by the computer. The other two columns have self-explanatory headings and appear chiefly to allow one to inspect the convergence of the integration. The magnitude and phase values for X = 25 are considered to be correct since the integration in most cases converges sufficiently well for this value of X.

The example below is presented for additional aid to those who wish to use this program.

Sample Calculation of INDFB5

Let us suppose that we wish to know the defect sensitivity factor for a coil 0.155 in. long with inner and outer radii of 0.420 and 0.580 in., respectively, coaxial with and inside a two-conductor tube of effectively infinite (three- or four-coil diameters) length. The inner conductor is assumed to be copper with a radius of 0.600 in., while the outer conductor is assumed to be aluminum with a radius of 0.700 in. Let us further suppose that we are interested in the defect sensitivity factor for the case in which the defect is located 0.750 in. from the axis and 0.0825 in. above the Z = 0 plane. We shall assume the frequency of the driving current to be 1.25 kHz and the distance of the bottom of the coil from the Z = 0 plane to be 0.005 in.

Normalized inner coil radius = 0.420/0.500 = 0.84
Normalized outer coil radius = 0.580/0.500 = 1.16
Normalized distance from bottom of coil to Z = 0 plane = 0.01
Normalized distance from top of coil to Z = 0 plane = 0.32
Normalized radius of inner conductor = 0.600/0.500 = 1.2
Normalized radius of outer conductor = 0.700/0.500 = 1.4
Normalized r position of defect = 0.750/0.500 = 1.5
Normalized z position of defect = 0.0825/0.500 = 0.165

Furthermore,

Angular frequency of driving current = $2\pi \times 1250$ sec^{-1}
Permeability of both conductors = $4\pi \times 10^{-7}$ h/m
Conductivity of copper = 5.77×10^{7} mhos/m
Conductivity of aluminum = 2.86×10^{7} mhos/m

Thus,

$$\omega\mu_1\sigma_1\bar{r}^2 = 91.91$$

$$\omega\mu_2\sigma_2\bar{r}^2 = 45.48$$

At this point one determines the normalization factor by placing the normalized coil dimensions in AIRCO5 and running that program. It is worth noting that the normalization factor is independent of all parameters excluding the coil dimensions. This factor is normalization factor 2 in the print-out of AIRCO5 and for the case in point is found to be 0.01011632. With the normalization factor in hand, the remaining procedure is quite simple.

The above information is now typed into the program as follows:

```
12 LET R1 = 0.84
14 LET R2 = 1.16
16 LET L1 = 0.01
18 LET L2 = 0.32
20 LET M1 = 91.91
22 LET M2 = 45.48
24 LET A = 1.2
26 LET B = 1.4
28 LET A9 = 0.1011632E-1
30 LET R9 = 1.5
32 LET Z9 = 0.165
```

Execution of the program yields the following results:

R1 = 0.84	R2 = 1.16	L1 = 0.01	L2 = 0.32
INN RAD = 1.2	OUT RAD = 1.4	AIR VALUE = 1.01163E-2	
M1 = 91.91		M2 = 45.48	
DEFECT POSITION	IS	R = 1.5	Z = 0.165
X	MAGNITUDE	PHASE	
1	0.1058329E-03	7.5448504	
5	0.3230766E-02	7.848431	
10	0.4183083E-02	8.0364868	
15	0.4214363E-02	8.0554679	
20	0.4214578E-02	8.0562493	
25	0.4214595E-02	8.0562245	

From this we see that the defect sensitivity factor is

$$4.21459 \times 10^{-3}\ e^{j8.05622}$$

To obtain the change in normalized impedance, we multiply the defect sensitivity factor by the product of the defect volume and the shape and orientation factor. Note that the defect volume must be calculated from dimensions which are normalized by the mean radius of the coil.

INDFB5 Program

```
10 REM              INDFB5              VERSION              03/3/69
12 LET R1=.84
14 LET R2=1.16
16 LET L1=.01
18 LET L2=.32
20 LET M1=91.91
22 LET M2=45.48
24 LET A=1.2
26 LET B=1.4
28 LET A9=.1011632E-1
30 LET R9=1.5
32 LET Z9=.165
34PRINT"R1=";R1,"R2=";R2,"L1=";L1,"L2=";L2
36PRINT"INNRAD=";A,"OUT RAD=";B,"AIR VALUE=";A9
38PRINT"M1=";M1," ","M2=";M2
40PRINT"DEFECT POSITION IS","R=";R9,"Z=";Z9
42PRINT"X","MAGNITUDE","PHASE"
44LETM7=0
46LETM8=0
48LETM9=0
50LETS1=1E-2
52LETS2=1
54LETB1=0
56LETB2=S2
```

```
58FØRX=E1+S1/2TØ82STEPS1
60LETX2=.707107*SQR(SQR(X*X*X*X+M2*M2)+X*X)
62LETY2=.707107*SQR(SQR(X*X*X*X+M2*M2)-X*X)
64LETX1=.707107*SQR(SQR(X*X*X*X+M1*M1)+X*X)
66LETY1=.707107*SQR(SQR(X*X*X*X+M1*M1)-X*X)
68LETZ1=SQR(X1*X1+Y1*Y1)
70LETØ1=ATN(Y1/X1)
72LETZ2=SQR(X2*X2+Y2*Y2)
74LET Ø2=ATN(Y2/X2)
76LETZ=A*X
78LETØ=0
80GØSUB296
82LETI1=Q1
84LETI2=Q5
86LETK2=Q7
88LETZ=A*Z1
90LETØ=Ø1
92GØSUB296
94LETI3=Q1
96LETJ3=Q2
98LETK3=Q3
100LETN3=Q4
102LETI4=Q5
104LETJ4=Q6
106LETK4=Q7
108LETN4=Q8
110LETZ=B*Z1
112LETØ=Ø1
114GØSUB296
116LETI5=Q1
118LETJ5=Q2
120LETK5=Q3
122LETN5=Q4
124LETI6=Q5
126LETJ6=Q6
128LETK6=Q7
130LETN6=Q8
132LETZ=B*Z2
134LETØ=Ø2
136GØSUB296
138LETK7=Q3
140LETN7=Q4
142LETK8=Q7
144LETN8=Q8
146 LETZ=R9*Z2
148 LETØ=Ø2
150 GØSUB296
152 LETK9=Q7
154 LETN9=Q8
156LETQ1=X*I4*I1*CØS(J4)-Z1*I3*I2*CØS(Ø1+J3)
158LETQ2=X*I4*I1*SIN(J4)-Z1*I3*I2*SIN(Ø1+J3)
160LETD1=SQR(Q1*Q1+Q2*Q2)
```

```
162GØSUB494
164LETE1=G3
166LETG1=Z1*K5*K8*CØS(Ø1+N5+N8)-Z2*K7*K6*CØS(Ø2+N7+N6)
168LETG2=Z1*K5*K8*SIN(Ø1+N5+N8)-Z2*K7*K6*SIN(Ø2+N7+N6)
170LETD2=SQR(G1*G1+G2*G2)
172GØSUB494
174LETE2=G3
176LETG1=Z2*K7*I6*CØS(Ø2+N7+J6)-Z1*I5*K8*CØS(Ø1+J5+N8)
178LETG2=Z2*K7*I6*SIN(Ø2+N7+J6)-Z1*I5*K8*SIN(Ø1+J5+N8)
180LETD3=SQR(G1*G1+G2*G2)
182GØSUB494
184LETE3=G3
186LETG1=X*K4*I1*CØS(N4)-Z1*K3*I2*CØS(Ø1+N3)
188LETG2=X*K4*I1*SIN(N4)-Z1*K3*I1*SIN(Ø1+N3)
190LETD4=SQR(G1*G1+G2*G2)
192GØSUB494
194LETE4=G3
196LETG1=D1*D2*CØS(E1+E2)+D3*D4*CØS(E3+E4)
198LETG2=D1*D2*SIN(E1+E2)+D3*D4*SIN(E3+E4)
200LETD5=SQR(G1*G1+G2*G2)
202GØSUB494
204LETE5=G3
206LETG1=-(K9*CØS(N9-E5))/(A*B*D5)
208LETG2=-(K9*SIN(N9-E5))/(A*B*D5)
210LETZ=K2*X
212LETG1=K2
214GØSUB 268
216LETG7=V1
218LETZ=K1*X
220LETG1=K1
222GØSUB 268
224LETG8=V1
226LETG9=G7-G8
228LETS5=S1*G9*(SIN(X*(Z9-L1))-SIN(X*(Z9-L2)))
230LETM7=M7+G1*S5
232LETM8=M8+G2*S5
234NEXTX
236LETM9=ATN(M8/M7)
238IFM7>0THEN242
240LETM9=M9+3.1415927
242LETN9=(3*M2)/(4*9.869044*A9)
244PRINTX+S1/2,N9*(M7*M7+M8*M8),2*M9
246LETS1=5E-2
248IFX>4.5THEN256
250LETB1=1
252LETB2=5
254GØTØ58
256LETS2=5
258LETB1=B2
260LETB2=B2+S2
262IFX<24.5THEN58
264PRINT
```

```
266G0T0500
268IFZ>3THEN286
270LETL5=20*(1-EXP(-Z/10))+4
272LETF1=Q1*Q1*Q1/2
274LETV1=F1/3
276F0RK=1T0L5
278LETF1=F1*Z*Z/(4*R*(R+1))
280LETV1=V1+F1/(3+2*R)
282NEXTR
284 G0 T0 294
286LETP3=(((1660.794/Z-1737.556)/Z+543.6694)/Z+11.81804)/Z
288LETP3=(((P3-33.78366)/Z+5.108402)/Z-.6130935)/Z
290LETP3=(P3-.33608)/Z+.3987795
292 LETV1=P3*SQR(Z)*EXP(Z)/(X*X*X)
294RETURN
296 IFZ>8THEN370
298 LETL5=20*(1-EXP(-Z/10))+4
300 LETT1=.5
302 LETT5=Z/2
304 LETV1=T1
306 LETV2=0
308 LETV3=-.25
310 LETV4=0
312 LETV5=T5*C0S(0)
314 LETV6=T5*SIN(0)
316 LETV7=V5
318 LETV8=V6
320 LETF1=0
322 LETF2=1
324 F0R R=1 T0 L5
326 LET Z3=C0S(2*R*0)
328 LET Z4=SIN(2*R*0)
330 LET Z5=C0S((2*R+1)*0)
332 LET Z6=SIN((2*R+1)*0)
334 LETF1=F1+1/R
336 LETF2=F2+1/(R+1)
338 LETT1=T1*Z*Z/(4*R*(R+1))
340 LET T5=T5*Z*Z/(4*R*(R+1))
342 LETV1=V1+T1*Z3*(2*R+1)
344 LETV2=V2+T1*Z4*(2*R+1)
346 LETV3=V3+T1*Z3*((2*R+1)*(F1+F2)/2-1)
348 LETV4=V4+T1*Z4*((2*R+1)*(F1+F2)/2-1)
350 LET V5=V5+T5*Z5
352 LET V7=V7+T5*(F1+F2)*Z5
354 LET V6=V6+T5*Z6
356 LET V8 =V8+T5*(F1+F2)*Z6
358 NEXTR
360 LETV3=-V3-C0S(2*0)/(Z*Z)+(.577215665+L0G(Z/2))*V1-0*V2
362 LETV4=-V4+SIN(2*0)/(Z*Z)+(.577215665+L0G(Z/2))*V2+0*V1
364 LETV7=-V7/2+C0S(0)/Z+(.577215665+L0G(Z/2))*V5-0*V6
366 LETV8=-V8/2-SIN(0)/Z+(.577215665+L0G(Z/2))*V6+0*V5
368 G0T0460
```

```
370 LET F1=1
372 LET W7=1
374 LET W5=1
376 LET W3=.5
378 LET W1=.5
380 LET T2=1
382 LET T1=1
384 LET  W2=0
386 LET W4=0
388 LET W6=0
390 LET W8=0
392 LET L5=3+20/Z
394 FOR K=1 TO L5
396 LET F1=-F1
398 LET  T2 =T2*(4-(2*K-1)*(2*K-1))/(8*K*Z)
400 LET T1=T2*(2*K+1)/2
402 LET  Z3=COS(K*Ø)
404 LET  Z4=SIN(K*Ø)
406 LET  W1=W1+F1*T1*Z3
408 LET  W2=W2-F1*T1*Z4
410 LET  W3=W3+T1*Z3
412 LET  W4=W4-T1*Z4
414 LET  W5=W5+F1*T2*Z3
416 LET  W6=W6-F1*T2*Z4
418 LET  W7 =W7+T2*Z3
420 LET  W8=W8-T2*Z4
422 NEXT  K
424 LET Q9=.3989422804*EXP(Z*COS(Ø))
426 LET Q5=Q9*(COS(Z*SIN(Ø)-.5*Ø))/SQR(Z)
428 LET Q6=Q9*(SIN(Z*SIN(Ø)-.5*Ø))/SQR(Z)
430 LET Q1=Q9*(COS(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
432 LET Q2=Q9*(SIN(Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
434 LET Q9=1.253314137*EXP(-Z*COS(Ø))
436 LET Q7=Q9*(COS(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
438 LET Q8=Q9*(SIN(-Z*SIN(Ø)-.5*Ø))/SQR(Z)
440 LET Q3=Q9*(COS(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
442 LET Q4=Q9*(SIN(-Z*SIN(Ø)-1.5*Ø))/SQR(Z*Z*Z)
444 LET V5=W5*Q5-W6*Q6
446 LET V6=W6*Q5+W5*Q6
448 LET V7=W7*Q7-W8*Q8
450 LET V8=W8*Q7+W7*Q8
452 LET V1=V5-W1*Q1+W2*Q2
454 LET V2=V6-W1*Q2-W2*Q1
456 LET V3=-V7-W3*Q3+W4*Q4
458 LET V4=-V8-W3*Q4-W4*Q3
460 LET Q1=SQR(V1*V1+V2*V2)
462 LET Q2=ATN(V2/V1)
464 IF V1>0 THEN 468
466 LET Q2=Q2+3.1415927
468 LET Q3=SQR(V3*V3+V4*V4)
470 LET Q4=ATN(V4/V3)
472 IF V3>0 THEN 476
```

```
474 LETQ4=Q4+3.1415927
476 LETQ5=SQR(V5*V5+V6*V6)
478 LETQ6=ATN(V6/V5)
480 IFV5>0THEN484
482 LETQ6=Q6+3.1415927
484 LETQ7=SQR(V7*V7+V8*V8)
486 LETQ8=ATN(V8/V7)
488 IFV7>0THEN492
490 LETQ8=Q8+3.1415927
492 RETURN
494 LETQ3=ATN(Q2/Q1)
496 IFQ1>0THEN500
498 LETQ3=Q3+3.1415927
500 RETURN
502 END
```

ACCURACY OF CALCULATIONS

In this section we shall discuss the accuracy with which these programs calculate the integrals presented in a previous report.[8] There are three main sources of error in the numerical evaluation of these integrals: (1) the accuracy of the approximation of the various functions in the integrands, (2) the step size in the numerical evaluation of the integral, and (3) the convergence of the integrals.

Approximation of Functions

We approximated the following functions which were not standard functions in the BASIC language:

$$J_1(x)\ ,\quad \int xJ_1(x)dx\ ,\quad I_1(z)\ ,\quad \int xI_1(x)dx\ ,\quad \frac{\partial I_1(z)}{\partial z}$$

$$K_1(z)\ ,\quad \int xK_1(x)dx\ ,\quad \frac{\partial K_1(z)}{\partial z}$$

J_1 is a first-order Bessel function of the first kind, and I_1 and K_1 are first-order modified Bessel functions of the first and second kind, respectively. The argument x is a real number, and z is a complex number. We shall now discuss the accuracy of each of these approximations.

[8] C. V. Dodd, W. E. Deeds, J. W. Luquire, and W. G. Spoeri, Some Eddy-Current Problems and Their Integral Solutions, ORNL-4384 (1969).

Discussion of $J_1(x)$

The approximation for $J_1(x)$ is taken from the Handbook of Mathematical Functions,[9] Eqs. 9.4.4 and 9.4.6. For x = 0 to 3 we use the approximation:

$$J_1(x) = \frac{1}{2}x - (6.25E\text{-}2)x^3 + (2.60415E\text{-}3)x^5 - (5.42443E\text{-}5)x^7$$
$$+ (6.757E\text{-}7)x^9 - (5.38E\text{-}9)x^{11} + (2.1E\text{-}11)x^{13} .$$

The reported error is less than 1.3E-8. For x between 3 and ∞ we use the approximation:

$$J_1(x) = \frac{Q_3}{x^{\frac{1}{2}}}\cos(Q_4)$$

where

$$Q_3 = 0.79788456 + (4.68E\text{-}6)/x + 0.14937/x^2 + (4.61835E\text{-}3)/x^3$$
$$- 0.20210391/x^4 + 0.27617679/x^5 - 0.14604057/x^6$$

and

$$Q = x - 2.35619449 + 0.37498836/x + (5.085E\text{-}4)/x^2 - 0.17222733/x^3$$
$$+ (6.022188E\text{-}2)/x^4 + 0.19397232/x^5 - 0.21262014/x^6 .$$

The reported error is less than 9E-8.

Discussion of $\int_0^x xJ_1(x)dx$

This range of integration was divided into two regions. To approximate the integral from 0 to 5, we use the series representation (Eq. 9.1.10) for $J_1(x)$, multiply by x, and integrate term by term with respect to x.

$$\int_0^x xJ_1(x)dx = \sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{2}\right)^{2n+1}\frac{x^{2n+3}}{n!\,(n+1)!\,(2n+3)}$$

[9]National Bureau of Standards, Handbook of Mathematical Functions, p. 370, U.S. Government Printing Office, Washington, D.C., 1964.

We take 2x+3 terms in the approximation.

For the region between 5 and ∞ we use the approximation:

$$\int_0^x xJ_1(x)dx = 1 - \sqrt{x}\,[Q_2 \cos(x - \pi/4) - Q_1 \sin(x - \pi/4)] \;,$$

where the coefficients,

$$Q_2 = 0.7979095 - (0.187344E\text{-}2)/x + 0.4952024/x^2 - 0.6896196/x^3 + 2.105874/x^4 - 5.817517/x^5$$

and

$$Q_1 = -(0.064109E\text{-}3) + 0.7034845/x - 0.1730503/x^2 + 2.050931/x^3 - 23.79333/x^4 + 109.1142/x^5 - 188.1357/x^6 \;,$$

have been determined by a least-squares fit. The "actual value" was calculated by numerical integration of $xJ_1(x)$ and agrees with values calculated from National Bureau of Standards tables to within ±1E-8.

X	APPROXIMATION	ACTUAL VALUE	DIFFERENCE
0.5	0.204456E-01	0.204456E-01	0.3410605E-12
1	0.15453272	0.15453272	0.0909494E-11
1.5	0.473708	0.473708	0.1273292E-10
2	0.97798873	0.97798873	−0.3637978E-11
2.5	1.5889403	1.5889403	0.2910383E-10
3	2.1677231	2.1677231	0.2910383E-10
3.5	2.5537528	2.5537528	0.2910383E-10
4	2.6133333	2.6133333	0.1455191E-10
4.5	2.2843038	2.2843038	0.1455191E-10
5	1.6032957	1.6032957	0.0727595E-10
5.5	0.70655645	0.70655573	−0.0719424E-05
6	−0.19765044	−0.10765039	0.4333287E-07
6.5	−0.87913766	−0.87913832	−0.0661486E-05
7	−1.1459147	−1.1459147	0.1809094E-06
7.5	−0.89837738	−0.89837617	0.1211148E-05
8	−0.16246094	−0.16245976	0.1174163E-05
8.5	0.90846388	0.90846427	0.3920904E-06
9	2.065269	2.0652688	−0.1579173E-06
9.5	3.0217648	3.0217649	0.0818108E-06
10	3.526368	3.5263689	0.0933519E-05
10.5	3.4286108	3.4286126	0.1799518E-05
11	2.7228899	2.7228921	0.2154891E-05
11.5	1.5570277	1.5570296	0.1853659E-05

X	APPROXIMATION	ACTUAL VALUE	DIFFERENCE
12	0.20184917	0.2018503	0.1133443E-05
12.5	−1.0120725	−1.0120721	0.4056128E-06
13	−1.775617	−1.775617	0.0633735E-07
13.5	−1.8801856	−1.8801856	0.4095636E-07
14	−1.2743509	−1.2743509	0.3716413E-06
14.5	−0.0828132	−0.08281247	0.0730123E-05
15	1.4185281	1.4185289	0.0868487E-05
15.5	2.8665793	2.86658	0.0673477E-05
16	3.8992502	3.8992504	0.2041342E-06
16.5	4.2463153	4.2463149	−0.3473251E-06
17	3.8000754	3.8000746	−0.0846717E-05
17.5	2.647369	2.6473679	−0.1120075E-05
18	1.0537113	1.0537101	−0.115321E-05
18.5	−0.597843	−0.59784403	−0.1029275E-05
19	−1.8990298	−1.8990306	−0.0873602E-05
19.5	−2.5175747	−2.5175755	−0.0777552E-05
20	−2.282114	−2.2821147	−0.0747525E-05
20.5	−1.2291259	−1.2291267	−0.071802E-05
21	0.40084917	0.4008486	−0.5746023E-06
21.5	2.2180202	2.2180199	−0.2416927E-06
22	3.7767232	3.7767235	0.2741726E-06
22.5	4.6849525	4.6849534	0.0864063E-05
23	4.7030693	4.7030707	0.1347827E-05
23.5	3.8068092	3.8068108	0.1541149E-05
24	2.1980817	2.198083	0.1335603E-05
24.5	0.25973223	0.25973298	0.0753498E-05
25	−1.5356547	−1.5356547	−0.4289904E-07
25.5	−2.7411629	−2.7411637	−0.0804589E-05
26	−3.0469315	−3.0469327	−0.1279477E-05
26.5	−2.3597314	−2.3597327	−0.1319582E-05
27	−0.83031503	−0.83031598	−0.0948286E-05
27.5	1.1792652	1.1792648	−0.3639288E-06
28	3.1813856	3.1813857	0.1216831E-06
28.5	4.6813188	4.681319	0.2125743E-06
29	5.2996209	5.2996206	−0.2395245E-06
29.5	4.8681301	4.8681289	−0.1147825E-05
30	3.475291	3.4752888	−0.2178421E-05

The error in the approximation is about ±2E-6 or less between 0 and 30. For greater values of x, the error is slightly larger, although the integral approximations have usually converged before x reaches 30.

In the program AIRCO5, which is somewhat slower to converge, we use a third approximation between 30 and ∞. It has the same form as used before:

$$\int_0^x xJ_1(x)\,dx = 1 + \sqrt{\frac{2}{\pi x}}\,[P_1\cos(x - \pi/4) + P_2\sin(x - \pi/4)]$$

where the coefficients $P_1 = -x - 0.5546875/x + 2.48062114/x^3$ and $P_2 = 0.875 - 0.93457031/x^2 + 8.98975114/x^4$ have been determined by a least-squares fit. The error in this approximation is less than 1E-6.

Discussion of $\int_0^x xI_1(x)dx$

This integral was also divided into integrals over two regions. To approximate the integral from 0 to 5, we use the series representation (Eq. 9.6.10) for $I_1(x)$, multiply by x, and integrate term by term with respect to x:

$$\int_0^x xI_1(x)dx = \sum_{n=0}^{L5} \left(\frac{1}{2}\right)^{1+2n} \frac{x^{2n+3}}{n!\,(n+1)!\,(2n+3)} .$$

We take L5 terms in the series where $L5 = 20 + \left(1 - e^{-x/10}\right) + 4$. For the region between 5 and ∞, we use the following approximation:

$$\int_0^\infty xI_1(x)dx = P_3 x^{\frac{1}{2}}\, e^x$$

where the coefficient $P_3 = 0.3987795 - 0.3360836/x - 0.6130935/x^2 + 5.108402/x^3 - 33.78366/x^4 + 11.81804/x^5 + 543.6694/x^6 - 1737.556/x^7 + 1660.794/x^8$ has been determined by a least-squares fit.

The variation between the approximation and the "actual value" is shown in the following table. The "actual value" was calculated by numerical integration of $xI_1(x)$ and agrees with values calculated from seven-place tables.[10]

[10] L. N. Karmazine and E. A. Cistova, Tables of Bessel Functions of Imaginary Argument and Their Integrals (in Russian), Moscow, 1958.

X	"ACTUAL VALUE"	APPROXIMATION	DIFFERENCE	DIFFERENCE/ "ACTUAL VALUE"
0.5	0.2122687E-01	0.2122687E-01	-0.6232312E-09	-0.294E-07
1	0.17954477	0.17954478	-0.0702584E-07	-0.391E-07
1.5	0.66401539	0.66401541	-0.2136584E-07	-0.322E-07
2	1.7841686	1.7841686	-0.462096E-07	-0.259E-07
2.5	4.0764673	4.0764675	-0.1673761E-06	-0.411E-07
3	8.4814158	8.4814162	-0.399712E-06	-0.471E-07
3.5	16.644762	16.644763	-0.0825617E-05	-0.496E-07
4	31.432477	31.432478	-0.1491745E-05	-0.475E-07
4.5	57.811153	57.811157	-0.4028668E-05	-0.0697E-06
5	104.35068	104.35069	-0.0853184E-04	-0.0818E-06
5.5	185.78408	185.78496	-0.0883081E-02	-0.47533E-05
6	327.36	327.36114	-0.1143995E-02	-0.34946E-05
6.5	572.23069	572.23054	0.1523569E-03	0.2663E-06
7	993.97292	993.96971	0.3207929E-02	0.32274E-05
7.5	1717.7781	1717.7708	0.0734394E-01	0.42753E-05
8	2956.269	2956.2582	0.1085959E-01	0.36734E-05
8.5	5069.9682	5069.9573	0.1093128E-01	0.21561E-05
9	8669.2788	8669.2752	0.3615498E-02	0.417E-06
9.5	14786.309	14786.325	-0.1590186E-01	-0.10754E-05
10	25164.12	25164.173	-0.5256569E-01	-0.20889E-05
10.5	42743.237	42743.346	-0.10980105	-0.25689E-05
11	72479.343	72479.53	-0.18637943	-0.25715E-05
11.5	122716.63	122716.9	-0.27078056	-0.22066E-05
12	207492.91	207493.25	-0.33177948	-0.1599E-05
12.5	350406.21	350406.51	-0.30313491	-0.08651E-05
13	591096.98	591097.04	-0.5995178E-01	-0.1014E-06
13.5	996109.6	996108.98	0.61592102	0.6183E-06
14	1.6770822E+06	1.6770801E+06	2.0863723	0.1244E-05
14.5	2.8211956E+06	2.8211907E+06	4.9243621	0.17455E-05
15	4.74212E+06	4.74211E+06	9.9997863	0.21087E-05
15.5	7.9652235E+06	7.9652049E+06	18.574401	0.23319E-05
16	1.3370026E+07	1.336999rE+07	32.384155	0.24221E-05
16.5	2.2428269E+07	2.2428215E+07	53.514404	0.2386E-05
17	3.7601638E+07	3.7601554E+07	84.590332	0.22496E-05
17.5	6.3005982E+07	6.3005854E+07	127.91943	0.20303E-05
18	1.0552023E+08	1.0552004E+08	184.25976	0.17462E-05
18.5	1.7663717E+08	1.7663692E+08	250.09277	0.14159E-05
19	2.9555155E+08	2.9555124E+08	312.30273	0.10567E-05
19.5	4.9431117E+08	4.9431083E+08	338.29492	0.06844E-05
20	8.2640693E+08	8.2640667E+08	259.04687	0.3135E-06
20.5	1.3810941E+09	1.3810942E+09	-59.710937	-0.432E-07
21	2.307263E+09	2.3072638E+09	-864.45312	-0.3747E-06
21.5	3.8532169E+09	3.8532195E+09	-2587.9687	-0.06716E-05
22	6.432944E+09	6.4329499E+09	-5959	-0.09263E-05
22.5	1.0736505E+10	1.0736517E+10	-12154.562	-0.11321E-05
23	1.7913864E+10	1.7913887E+10	-22997.125	-0.12838E-05
23.5	2.9880974E+10	2.9881015E+10	-41173	-0.13779E-05
24	4.9829313E+10	4.9829383E+10	-70311.75	-0.14111E-05
24.5	8.3073949E+10	8.3074064E+10	-114742.5	-0.13812E-05

X	"ACTUAL VALUE"	APPROXIMATION	DIFFERENCE	DIFFERENCE/ "ACTUAL VALUE"
25	1.3846482E+11	1.38465E+11	−178234	−0.12872E-05
25.5	2.3073483E+11	2.3073509E+11	−260375	−0.11285E-05
26	3.8440606E+11	3.844064E+11	−347820	−0.09048E-05
26.5	6.4028688E+11	6.4028727E+11	−394904	−0.6168E-06
27	1.0662769E+12	1.0662772E+12	−282428	−0.2649E-06
27.5	1.7753336E+12	1.7753333E+12	264592	0.149E-06
28	2.955342E+12	2.9553402E+12	1.845152E+06	0.6243E-06
28.5	4.9187696E+12	4.9187639E+12	5.700768E+06	0.1159E-05
29	8.1851962E+12	8.1851818E+12	1.4332672E+07	0.1751E-05
29.5	1.3618472E+13	1.3618439E+13	3.26672E+07	0.23987E-05
30	2.2654628E+13	2.2654558E+13	7.0231808E+07	0.31001E-05

The error is equally small for values greater than 30, although most of the integrals have converged before 30.

Discussion of $\int_0^x xK_1(x)dx$

This integral was also divided into integrals over two regions. To approximate the integral from 0 to 5, we use the series representation (Eq. 9.6.11), multiply by x, and integrate each term with respect to x. We obtain:

$$\int_0^x xK_1(x)dx = x + \sum_{n=0}^{L5} \frac{x^{2n+3}}{2^{2n+1}\; n!\,(n+1)!\,(2n+3)} \left[\ln x/2 - \frac{1}{2n+3} + 0.577215665 - \left(1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n} + \frac{1}{2n+2}\right)\right] .$$

We take L5 terms in the series where $L5 = 20 \left(1 - e^{-x/10}\right) + 4$.

For the region between 5 and ∞ we use the following approximation:

$$\int_0^x xK_1(x)dx = \frac{\pi}{2} - \sqrt{x}\; e^{-x}\; P_4 \quad ,$$

where $P_4 = 1.2533263 + 1.0958276/x - 0.67491295/x^2 + 0.91571421/x^3 - 1.1768576/x^4 + 0.79898397/x^5$ has been determined by a least-squares fit.

Since the value of this integral converges to $\pi/2$ at ∞ and we actually use the difference of two such integrals, we have subtracted

$\pi/2$ from the approximation in the program. In the following table we give values of $\pi/2 - \int_0^X xK_1(x)dx$ as these are easier to compare.

X	"ACTUAL VALUE"	APPROXIMATION	DIFFERENCE	DIFFERENCE/ "ACTUAL VALUE"
0.5	1.1059033	1.1059033	0.218287E-10	0.2E-10
1	0.74931091	0.74931091	0.4365574E-10	0.6E-10
1.5	0.4969222	0.4969222	0.0727595E-09	0.15E-09
2	0.32490833	0.32490833	0.0873114E-09	0.27E-09
2.5	0.21029136	0.21029136	0.0800355E-09	0.38E-09
3	0.13506675	0.13506675	0.5820766E-10	0.43E-09
3.5	0.08623054	0.08623054	0.0960426E-08	0.1114E-07
4	0.5478545E-01	0.5478545E-01	0.5529727E-09	0.1009E-07
4.5	0.3466817E-01	0.3466817E-01	0.327418E-09	0.0944E-07
5	0.2186441E-01	0.2186441E-01	0.4816683E-08	0.2203E-06
5.5	0.1375002E-01	0.1374999E-01	0.2589445E-07	0.188323E-05
6	0.0862574E-01	0.0862573E-01	0.0805027E-07	0.093329E-05
6.5	0.5399539E-02	0.5399535E-02	0.3622602E-08	0.067091E-05
7	0.3373622E-02	0.337362E-02	0.1857699E-08	0.55065E-06
7.5	0.2104306E-02	0.2104305E-02	0.0942506E-08	0.44789E-06
8	0.1310607E-02	0.1310606E-02	0.4552447E-09	0.34735E-06
8.5	0.0815179E-02	0.0815179E-02	0.208935E-09	0.25631E-06
9	0.5064157E-03	0.5064156E-03	0.0923456E-09	0.18235E-06
9.5	0.3142559E-03	0.3142559E-03	0.4061284E-10	0.12923E-06
10	0.1948157E-03	0.1948157E-03	0.1897326E-10	0.09739E-06
10.5	0.1206607E-03	0.1206607E-03	0.102629E-10	0.08506E-06
11	0.0746691E-03	0.0746691E-03	0.0668354E-10	0.08951E-06
11.5	0.4617194E-04	0.461719E-04	0.4966693E-11	0.10757E-06
12	0.2853005E-04	0.2853004E-04	0.3883671E-11	0.13613E-06
12.5	0.1761719E-04	0.1761719E-04	0.3036126E-11	0.17234E-06
13	0.1087178E-04	0.1087178E-04	0.232353E-11	0.21372E-06
13.5	0.0670522E-04	0.0670522E-04	0.1731087E-11	0.25817E-06
14	0.4133235E-05	0.4133234E-05	0.1256023E-11	0.30388E-06
14.5	0.2546516E-05	0.2546515E-05	0.089001E-11	0.3495E-06
15	0.1568178E-05	0.1568177E-05	0.6176448E-12	0.39386E-06
15.5	0.0965271E-05	0.0965271E-05	0.4209618E-12	0.43611E-06
16	0.5939093E-06	0.5939091E-06	0.2824546E-12	0.47559E-06
16.5	0.3652727E-06	0.3652725E-06	0.1869459E-12	0.5118E-06
17	0.2245692E-06	0.2245691E-06	0.1222615E-12	0.54443E-06
17.5	0.1380155E-06	0.1380155E-06	0.0791129E-12	0.57322E-06
18	0.0847927E-06	0.0847926E-06	0.5071247E-13	0.59808E-06
18.5	0.5207738E-07	0.5207735E-07	0.3223202E-13	0.61893E-06
19	0.3197471E-07	0.3197469E-07	0.2033009E-13	0.063582E-05
19.5	0.1962625E-07	0.1962624E-07	0.1273265E-13	0.064876E-05
20	0.1204335E-07	0.1204335E-07	0.0792275E-13	0.065785E-05
20.5	0.0738826E-07	0.0738825E-07	0.4899943E-14	0.066321E-05
21	0.4531344E-08	0.4531341E-08	0.3013214E-14	0.066497E-05
21.5	0.2778475E-08	0.2778473E-08	0.1842953E-14	0.06633E-05
22	0.1703276E-08	0.1703275E-08	0.1121315E-14	0.065833E-05
22.5	0.1043919E-0	0.1043918E-08	0.0678791E-14	0.065023E-05

X	"ACTUAL VALUE"	APPROXIMATION	DIFFERENCE	DIFFERENCE/ "ACTUAL VALUE"
23	0.063967E-08	0.063967E-08	0.4088661E-15	0.063918E-05
23.5	0.3918833E-09	0.3918831E-09	0.2450567E-15	0.062533E-05
24	0.2400334E-09	0.2400333E-09	0.1461453E-15	0.60885E-06
24.5	0.1469957E-09	0.1469956E-09	0.0867124E-15	0.5899E-06
25	0.0900033E-09	0.0900032E-09	0.5117688E-16	0.56861E-06
25.5	0.5509805E-10	0.5509802E-10	0.3003811E-16	0.54518E-06
26	0.3372412E-10	0.337241E-10	0.175268E-16	0.51971E-06
26.5	0.2063832E-10	0.2063831E-10	0.1016153E-16	0.49236E-06
27	0.1262815E-10	0.1262815E-10	0.5850297E-17	0.46327E-06
27.5	0.0772573E-10	0.0772573E-10	0.3341809E-17	0.43256E-06
28	0.4725805E-11	0.4725803E-11	0.1891953E-17	0.40035E-06
28.5	0.289035E-11	0.2890349E-11	0.1059969E-17	0.36673E-06
29	0.1767525E-11	0.1767524E-11	0.586557E-18	0.33185E-06
29.5	0.1080745E-11	0.1080744E-11	0.3196755E-18	0.29579E-06
30	0.0660731E-11	0.0660731E-11	0.1708922E-18	0.25864E-06

The "actual value" agrees with values calculated from seven-place tables.[10] The error for values greater than 30 is similarly small.

Discussion of $I_1(z)$, $\partial I_1(z)/\partial z$, $K_1(z)$, and $\partial K_1(z)/\partial z$

These functions are all calculated in a single subroutine. The approximations between 0 and 8 are calculated from the following series.[11]

$$I_1(z) = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{2^{2n+1}\, n!\, (n+1)!}$$

$$\frac{\partial I_1(z)}{\partial z} = \sum_{n=0}^{\infty} (2n+1) \frac{z^{2n}}{2^{2n+1}\, n!\, (n+1)!}$$

$$K_1(z) = \frac{1}{z} + \sum_{n=0}^{\infty} \frac{z^{2n+1}}{2^{2n+1}\, n!\,(n+1)!} \left[\ln z/2 + 0.577215665 - \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{n} + \frac{1}{2n+2}\right)\right]$$

[11]N. W. McLachlan, Bessel Functions for Engineers, 2nd. ed., Oxford, 1955.

$$\frac{\partial K_1(z)}{\partial z} = -\frac{1}{z^2} + \sum_{n=0}^{\infty} \frac{(2n+1)\ z^{2n}}{2^{2n+1}\ n!\ (n+1)!} \left[\ln z/2 + 0.577215665 - \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{n} + \frac{1}{2n+2}\right) + \frac{1}{2n+1}\right]$$

We take $20\left(1 - e^{-|z/10|}\right) + 4$ terms in the series.

For the functions between 8 and ∞ we use Formulae 157, p. 204 of ref. 11.

$$I_1(z) = \frac{e^z}{\sqrt{2\pi z}} \left[1 - \frac{4-1}{1!8z} + \frac{(4-1)(4-9)}{2!(8z)^2} - \ldots + \frac{(-1)^{n-1}(4-1)\ \ldots\ [4-(2n-3)^2]}{(n-1)!(8z)^{n-1}}\right]$$

$$\frac{\partial I_1(z)}{\partial z} = \frac{e^z}{\sqrt{2\pi z}} \left[1 - \frac{4-1}{1!8z} + \frac{(4-1)(4-9)}{2!(8z)^2} - \ldots + \frac{(-1)^{n-1}(4-1)\ \ldots\ [4-(2n-3)^2]}{(n-1)!(8z)^{n-1}}\right]$$

$$- \frac{e^z}{\sqrt{2\pi}\ z^{3/2}} \left[\frac{1}{2} - \frac{3/2\ (4-1)}{1!8z} + \frac{5/2\ (4-1)(4-9)}{2!(8z)^2} - \ldots + \frac{(-1)^{n-1}(2n-1)(4-1)\ \ldots\ [4-(2n-3)^2]}{2(n-1)!(8z)^{n-1}}\right]$$

$$K_1(z) = \sqrt{\frac{\pi}{2z}}\ e^{-z} \left[1 + \frac{4-1}{1!8z} + \frac{(4-1)(4-9)}{2!(8z)^2} + \ldots + \frac{(4-1)\ \ldots\ [4-(2n-3)^2]}{(n-1)!(8z)^{n-1}}\right]$$

$$\frac{\partial K_1(z)}{\partial z} = -\sqrt{\frac{\pi}{2z}}\, e^{-z}\left[1 + \frac{4-1}{1!8z} + \frac{(4-1)(4-9)}{z!(8z)^2} + \dots + \frac{(4-1)\dots[4-(2n-3)^2]}{(n-1)!(8z)^{n-1}}\right]$$

$$-\sqrt{\frac{\pi}{2}}\,\frac{e^{-z}}{z^{3/2}}\left[\frac{1}{2} + \frac{3\,(4-1)}{2\;1!8z} + \frac{5\,(4-1)(4-9)}{2\;2!(8z)^2} + \dots + \frac{(2n-1)(4-1)\dots[4-(2n-3)^2]}{2\,(n-1)!\,(8z)^{n-1}}\right]$$

We take 3 + 20/x terms in each series. The "actual value" was calculated from seven-place tables from Table II of ref. 11 between 0 and 10 and interpolated from five-place reciprocal values (National Bureau of Standards Table 9.12) between 10 and 30. The following table gives the values of these functions for $z = xe^{i\pi/4}$.

I_1' / I_1	MAG / MAG	PHASE / PHASE	K_1' / K_1 MAG / MAG	PHASE / PHASE
Z = 1				
APPROXIMATION	0.52169785	0.36684869	1.1914094	1.515654
"ACTUAL VALUE"	0.521698	0.366848	1.19141	1.51565
APPROXIMATION	0.50130106	0.91028994	0.77887041	−1.2548656
"ACTUAL VALUE"	0.5013011	0.9102899	0.7788704	−1.2548657
Z = 2				
APPROXIMATION	0.78275606	1.1868214	0.29724698	1.035719
"ACTUAL VALUE"	0.782756	1.18682	0.297247	1.03572
APPROXIMATION	1.0411672	1.2787395	0.24429343	4.378545
"ACTUAL VALUE"	1.0411672	1.2787396	0.2442934	4.3785449
Z = 3				
APPROXIMATION	1.5051089	1.9199306	0.107145	0.42279896
"ACTUAL VALUE"	1.50511	1.91993	0.107145	0.422799
APPROXIMATION	1.7999076	1.8450429	0.09451534	3.6977643
"ACTUAL VALUE"	1.799908	1.8450429	0.09451534	3.6977643
Z = 4				
APPROXIMATION	2.8694681	2.5878306	0.4334207E-01	−0.23443368
"ACTUAL VALUE"	2.86947	2.58783	0.433421E-1	−0.234434
APPROXIMATION	3.1728963	2.5116109	0.3952989E-01	3.0058023
"ACTUAL VALUE"	3.172896	2.5116109	0.3952990E-1	3.0058023

Z = 5				
APPROXIMATION	5.3988943	3.2628465	0.1851214E-01	−0.91095732
"ACTUAL VALUE"	5.39889	3.26285	0.185121E-1	−0.910958
APPROXIMATION	5.8090597	3.203566	0.1721294E-01	2.3085384
"ACTUAL VALUE"	5.809060	3.2035660	0.1721295E-1	2.3085387
Z = 6				
APPROXIMATION	10.209239	3.9500556	0.0815801E-01	4.6857101
"ACTUAL VALUE"	10.2092	3.95006	0.0815789E-1	4.68572
APPROXIMATION	10.850182	3.899976	0.076815E-01	1.6083434
"ACTUAL VALUE"	10.850182	3.8999760	0.07681504E-1	1.6083434
Z = 7				
APPROXIMATION	19.462173	4.6434166	0.3668565E-02	3.9933861
"ACTUAL VALUE"	19.4622	4.64342	0.366854E-2	3.99338
APPROXIMATION	20.500302	4.5991161	0.3484967E-02	0.90635435
"ACTUAL VALUE"	20.500302	4.5991161	0.3484972E-2	0.9063566
Z = 8				
APPROXIMATION	37.345829	−0.94313404	0.1673075E-02	3.2974192
"ACTUAL VALUE"	37.3458	−0.943134	0.167314E-2	3.29739
APPROXIMATION	39.069657	−0.98274486	0.1599913E-02	0.20318745
"ACTUAL VALUE"	39.06966	−0.9827449	0.1599890E-2	0.2031949
Z = 9				
APPROXIMATION	72.041153	−0.24428312	0.0771052E-02	2.5989414
"ACTUAL VALUE"	72.0413	−0.244281	0.0771052E-2	2.59894
APPROXIMATION	74.97413	−0.27999139	0.0741043E-02	−0.50077776
"ACTUAL VALUE"	74.97404	−0.2799960	0.07410441E-2	−0.5007787
Z = 10				
APPROXIMATION	139.58315	0.45618449	0.3581752E-03	1.8987637
"ACTUAL VALUE"	139.583	0.456184	0.358175E-3	1.89876
APPROXIMATION	144.67067	0.42368813	0.3456271E-03	−1.2053366
"ACTUAL VALUE"	144.67053	0.4236874	0.3456272E-3	−1.2053372
Z = 11				
APPROXIMATION	271.46009	1.1578356	0.1674337E-03	1.1973264
"ACTUAL VALUE"	271.451	1.15784	0.167434E-3	1.19733
APPROXIMATION	280.41888	1.1280324	0.1620998E-03	4.3728538
"ACTUAL VALUE"	280.419	1.12802	0.162099E-3	4.3728
Z = 12				
APPROXIMATION	529.61909	1.8603796	0.0786692E-03	0.49494718
"ACTUAL VALUE"	529.621	1.86038	0.0786697E-3	0.494922
APPROXIMATION	545.59121	1.832863	0.0763713E-03	3.6675283
"ACTUAL VALUE"	545.600	1.83286	0.0763705E-3	3.6675
Z = 13				
APPROXIMATION	1036.13	2.5636147	0.3711913E-04	−0.20815962
"ACTUAL VALUE"	1036.12	2.56362	0.371193E-4	−0.208159

APPROXIMATION	1064.9019	2.5380632	0.3611797E-04	2.9619429
"ACTUAL VALUE"	1064.93	2.53803	0.361179E-4	2.9619
Z = 14				
APPROXIMATION	2031.9028	3.2673952	0.1757633E-04	–0.9118383
"ACTUAL VALUE"	2031.90	3.26740	0.175764E-4	–0.911839
APPROXIMATION	2084.1893	3.2435497	0.17136E-04	2.2561507
"ACTUAL VALUE"	2084.21	3.24347	0.171359E-4	2.2562
Z = 15				
APPROXIMATION	3993.0282	3.9716136	0.0834772E-04	4.6672107
"ACTUAL VALUE"	3992.99	3.97161	0.0834769E-4	4.66721
APPROXIMATION	4088.7682	3.9492626	0.0815248E-04	1.5501911
"ACTUAL VALUE"	4088.69	3.94927	0.0815249E-4	1.5502
Z = 16				
APPROXIMATION	7861.5276	4.6761889	0.3974996E-05	3.9627026
"ACTUAL VALUE"	7861.58	4.67619	0.397499E-5	3.9627
APPROXIMATION	8037.9905	4.6551576	0.3887812E-05	0.84409437
"ACTUAL VALUE"	8038.04	4.65517	0.388782E-5	0.8441
Z = 17				
APPROXIMATION	15503.442	–0.90212634	0.189709E-05	3.2578884
"ACTUAL VALUE"	15503.5	–0.902122	0.189710E-5	3.25789
APPROXIMATION	15830.572	–0.92198411	0.1857918E-05	0.13788352
"ACTUAL VALUE"	15830.6	–0.921995	0.185792E-5	0.1379
Z = 18				
APPROXIMATION	30619.007	–0.19701018	0.0907198E-05	2.5528189
"ACTUAL VALUE"	30619.6	–0.197023	0.0907197E-5	2.55280
APPROXIMATION	31228.555	–0.21581768	0.0889502E-05	–0.56842307
"ACTUAL VALUE"	31228.0	–0.215795	0.0889516E-5	–0.5685
Z = 19				
APPROXIMATION	60552.497	0.50831363	0.4345914E-06	1.8475346
"ACTUAL VALUE"	60552.6	0.508314	0.434593E-6	1.84753
APPROXIMATION	61693.467	0.49045147	0.4265585E-06	–1.2748108
"ACTUAL VALUE"	61693.4	0.490455	0.426549E-6	–1.2749
Z = 20				
APPROXIMATION	119893.64	1.2138142	0.2085169E-06	1.1420677
"ACTUAL VALUE"	119895.	1.21381	0.208516E-6	1.14207
APPROXIMATION	122038.09	1.1968074	0.2048546E-06	4.3019172
"ACTUAL VALUE"	122037.	1.19681	0.204854E-6	4.3019
Z = 21				
APPROXIMATION	237648.83	1.9194666	0.1001872E-06	0.43644444
"ACTUAL VALUE"	237647.	1.91948	0.100187E-6	0.436442
APPROXIMATION	241694.21	1.9032373	0.098511E-06	3.5954002
"ACTUAL VALUE"	241693	1.90324	0.0985107E-6	3.5954

Z = 22				
APPROXIMATION	471530.69	2.6252503	0.4819867E-07	-0.26931405
"ACTUAL VALUE"	571535.	2.62525	0.481982E-7	-0.269309
APPROXIMATION	479187.7	2.6097307	0.4742877E-07	2.8888312
"ACTUAL VALUE"	479179.	2.60976	0.474292E-7	2.8888
Z = 23				
APPROXIMATION	936445.43	3.3311482	0.2321441E-07	-0.97519003
"ACTUAL VALUE"	936454.	3.33115	0.232142E-7	-0.975190
APPROXIMATION	950982.73	3.3162792	0.2285965E-07	2.1822169
"ACTUAL VALUE"	950975.	3.31628	0.228597E-7	2.1822
Z = 24				
APPROXIMATION	1.8613214E+06	4.0371463	0.1119272E-07	4.6020165
"ACTUAL VALUE"	1.86131E-6	4.03715	0.111927E-7	4.60202
APPROXIMATION	1.8889985E+06	4.0228755	0.1102877E-07	1.4755626
"ACTUAL VALUE"	1.88899E6	4.02292	0.110287E-7	1.4755
Z = 25				
APPROXIMATION	3.7025287E+06	-1.5399528	0.5401696E-08	3.8959473
"ACTUAL VALUE"	3.70249E6	-1.53995	0.540168E-8	3.89595
APPROXIMATION	3.7553576E+06	-1.5536714	0.5325726E-08	0.76887328
"ACTUAL VALUE"	3.75539E6	-1.55367	0.532573E-8	0.7688
Z = 26				
APPROXIMATION	7.3703585E+06	-0.83378861	0.2609196E-08	3.1897982
"ACTUAL VALUE"	7.37037E6	-0.833774	0.260919E-9	3.18980
APPROXIMATION	7.4714347E+06	-0.84699607	0.2573906E-08	0.6215264E-1
"ACTUAL VALUE"	7.47149E6	-0.84697	0.257390E-8	0.621E-1
Z = 27				
APPROXIMATION	1.4681469E+07	-0.12755494	0.126135E-08	2.483578
"ACTUAL VALUE"	1.46815E7	-0.127543	0.126135E-8	2.48358
APPROXIMATION	1.4875278E+07	-0.14028789	0.1244919E-08	-0.64459586
"ACTUAL VALUE"	1.48754E7	-0.14027	0.124490E-8	-0.6446
Z = 28				
APPROXIMATION	2.9263146E+07	0.57874076	0.6102251E-09	1.7772943
"ACTUAL VALUE"	2.92629E7	0.578749	0.610227E-9	1.77729
APPROXIMATION	2.9635523E+07	0.56644949	0.6025588E-09	-1.3513693
"ACTUAL VALUE"	2.96360E7	0.5664	0.602557E-9	-1.3514
Z = 29				
APPROXIMATION	5.8361317E+07	1.2850921	0.2954242E-09	1.0709537
"ACTUAL VALUE"	5.83614E9	1.28509	0.295424E-9	1.07096
APPROXIMATION	5.9078134E+07	1.2732129	0.2918403E-09	4.2250201
"ACTUAL VALUE"	5.90777E7	1.2732	0.291843E-9	4.2250
Z = 30				
APPROXIMATION	1.1645694E+08	1.9914936	0.1431141E-09	0.36456199
"ACTUAL VALUE"	1.16457E8	1.99150	0.143116E-9	0.364563
APPROXIMATION	1.1783823E+08	1.9799999	0.1414356E-09	3.5182039
"ACTUAL VALUE"	1.17841E8	1.9800	0.141436E-9	3.5182

The error is similarly small for values greater than 30.

Step Size

The integrals are evaluated by dividing the range of integration into a large number of small increments and summing the product of integrand (evaluated at the midpoint) and the increment. (A number of different integration schemes were tried, including a twenty-point Gaussian quadrature, but this was the fastest and most accurate up to a point.) The smaller the step size, the greater the accuracy and the greater the execution time. For very small step sizes, the round-off error in the computer will become a contributing factor. The step size was chosen to have the fastest running time and still retain high accuracy. For the region between 0 and 5, most of the integrands are varying rapidly, and the integrals converge to 90% or more of their final value. Therefore, in most programs, we have taken the step size to be 0.01 between 0 and 5, 0.05 between 5 and 15, and 0.1 for x greater than 15. In the encircling coil and inner coil programs, we take the step size to be 0.01 between 0 and 1 and 0.05 for values greater than 1. The error due to the step size varies with the coil and conductor configuration and the value of $\bar{r}^2\omega\mu\sigma$, but is smaller than 0.01% in all cases tested. This error can be reduced by decreasing the value of the step size. (This is named S1 in all the programs.)

Convergence

The values of the integrands in the programs are zero at $x = 0$ and go to zero as x goes to infinity. The convergence of the integrals also depends on the coil and conductor configuration and the values of $\bar{r}^2\omega\mu\sigma$. The air value in the impedance calculations is the slowest to converge. The following table shows the percentage error between the value at $x = 40$ and the final value as a function of the coil dimensions. In some of the longer running programs, such as INNCO5 and ENCCO5, the air value is calculated in AIRCO5; this value is used to calculate the final normalized impedances. Similar techniques can be used in the

Percentage Error in Convergence Tests of Air Value

Length	1	0.5	0.1	0.01
R_2-R_1				
0.01	2.418	3.435	9.095	23.287
0.05	0.8364	1.194	3.183	7.199
0.1	0.1203	0.1734	0.4810	1.171
0.2	0.0457	0.0671	0.1978	0.4961
0.3	0.0133	0.0199	0.05281	0.1760
0.4	0.0095	0.0144	0.0460	0.1247
0.5	0.0072	0.0110	0.0355	0.0957

other impedance programs if the error indicated in the above table is too large. The value calculated by AIRC05 is accurate to about 1E-6.

In addition, the integration can be carried out farther by modifying the program. The programs, in general, have a statement IF $X < X_{MAX}$ THEN 110 where the value of X_{MAX} is usually 35. This statement will transfer the control back to a "FOR" loop. By increasing the value of X_{MAX} the program will carry the integration to $X_{MAX} + 5$ in some cases and to $X_{MAX} + 1$ in others.

Additional Errors

In addition to the three major sources of error mentioned, there is some error due to computer round-off. While this is very small in most cases, for extreme coil dimensions it can become quite large. For example, very small values of the coil length, L, produce considerable round-off error due to inaccuracies in calculating the expression

$$L + \left(e^{-xL} - 1\right) \Big/ x \ .$$

For values of L less than 0.001, this should be calculated by a polynomial expansion for small values of x L. (This is done in AIRC05.) For values of L = 0.001 and larger, the error is less than 0.004%.

Round-off errors are in general very small on the SDS 940 computer, which carries twelve places. On other machines, however, it may have a much larger effect.

CONCLUSIONS

The programs in this report can be used to accurately calculate properties utilized in many eddy-current tests. By using these programs, the design engineer can quickly optimize a particular eddy-current testing parameter such as test frequency or a coil dimension. Use of these programs should prove extremely valuable in development of eddy-current testing procedures and will result in the design of better and more economical eddy-current tests.

LIST OF SYMBOLS

Symbol	Name	MKS Unit	Dimensions
f	Frequency	Hertz	$1/T$
j	Square root of minus 1		
L	Inductance	henries	ML^2/Q^2
N	Turns of wire		
$\bar{r}$	Mean coil radius	meters	L
Z_n	Normalized impedance	ohms	ML^2/Q^2T
μ	Permeability	henries/meter	ML/Q^2
ρ	Resistivity	ohm meter	ML^3/Q^2T
σ	Conductivity	mho/meter	Q^2T/ML^3
ω	Angular frequency	radians/sec	$1/T$

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