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91

REACTOR OPERATOR STUDY HANDBOOK
(Programmed Instruction Version)

VOLUME I - ELEMENTARY MATHEMATICS

S. D. Sheppard

Editor

C. D. Cagle

NOTICE This document contains information of a preliminary nature and was prepared primarily for internal use at the Oak Ridge National Laboratory. It is subject to revision or correction and therefore does not represent a final report.

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OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee
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FOREWORD

It is suggested that this programmed text be used as an aid in the study of reactor technology. It is not the intent of the author and editor that the text be considered a finished product. While field testing of both the subject matter and continuity of thought has been limited, the need for study material in programmed form was a basic consideration in the decision to publish the text. Revisions may be made at any time to correct errors, to expand the subject matter coverage, or to update the reactor technology. If the text is used with these reservations, and in conjunction with other study helps, it can be the basis for very rewarding individual study on the part of the student.

CONTENTS

REACTOR OPERATOR STUDY HANDBOOK
(Programmed Instruction Version)

VOLUME I - ELEMENTARY MATHEMATICS

As a part of the Reactor Operator Training Program of Operations Division, Oak Ridge National Laboratory, five areas of instruction have been programmed for individual study. They are:

Volume I	-	Elementary Mathematics Review
Volume II	-	Radiation Safety and Control
Volume III	-	Reactor Physics
Volume IV	-	Heat Theory and Fluid Flow
Volume V	-	Instrumentation and Controls

These programmed studies are a part of a course in reactor operation that includes classwork, lectures, and on-the-job training. At the end of the course, the operator trainee is tested for competence in all areas of reactor operations before being certified to operate a particular reactor.

It is suggested that the programs be studied in the sequence given above; however, sequential dependence has been minimized so that they may be studied either individually or as an integrated group.

The author and editor would like to especially acknowledge the patience and assistance of the members of the Operations Division clerical staff who worked on this report; namely, Gladys Carpenter, Linda Comstock, Milinda Compton, Joanne Nelson, and Barbara Burns.

INSTRUCTIONS

The material contained in this manual has been prepared using a technique called "programmed instruction". This technique of instruction consists of:

1. Presenting ideas or information in small, easily digestible steps called "frames".
2. Allowing you to set your own pace.
3. Encouraging response in an active way so that you have a stronger impression of the idea presented.
4. Letting you know immediately if your answer is right, thus reinforcing your impression.
5. Presenting many clues at first to help you arrive at the correct answer. (As you progress, the number of clues is reduced.)

A few sample frames are found on the next page. These will be used to illustrate the proper use of "programmed instruction". Most frames will require you to respond by filling in a blank, or blanks, to complete a sentence. Other frames will give you a choice of several responses. A few frames are for informational purposes only and require no response. The correct response to a given frame is always found on the right side of the page adjacent to the following frame. When reading a frame, a sheet (or strip) of paper should be used to cover the area below the dotted line which follows the frame. After completely reading a frame, you should write your response on a piece of paper. Next, move the paper down the page until you reach the next dotted line or turn the page. This will uncover the next frame and the correct response for the frame you have just completed. Compare your response with the correct response. If they do not match, read that frame again before moving on to the next one; <u>do not proceed</u> until you understand the information in the frame you are reading. If the responses do match, proceed to the frame you have just uncovered.

At the end of each section, there are self-test questions for review. If you miss one of the self-test questions, repeat the pertinent frames. It is not enough to respond correctly as you proceed through the material; you must remember correctly at the end of the program and even later. You should attempt to complete each section once you have started.

SAMPLE FRAMES

i. Programmed instruction is a method of presenting information in short paragraphs called "frames". These ____________ usually contain only one or two concepts for the student to grasp.

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ii. By requiring you to think of the appropriate response and to write that ____________ on a piece of paper, you take an active part in the program, and thereby reinforce your learning.

frames

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iii. This method of instruction, called _______________ _______________, allows you to proceed with the material at a rate which you determine for yourself.

response

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iv. Programmed instruction provides the appropriate response immediately and thus should reinforce the student's ______________.

programmed instruction

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learning

VOLUME I. ELEMENTARY MATHEMATICS

Introduction

This program has been developed to allow reactor operator trainees to refresh their knowledge of the basic principles and concepts of mathematics. It is the aim of the program to discuss basic principles and then offer enough drill to give the operator some proficiency in the use of the principles.

SECTION I-1

FRACTIONS

1.1. Common Fractions

1. A fraction is a part of any number, quantity, or object. The digit 2 is a fraction of the digit 8. A bottle of soda is a fraction of a carton or case of soda. A quart of liquid is a fraction of a gallon of liquid.

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2. In arithmetic, the use of a fraction is also a means of indicating that one number is to be divided by another.

1 Numerator
\- Division Sign
3 Denominator

The fraction, one-third, can also be written 1/3 and 1 ÷ 3. In the latter case, the 1 is called the dividend; and the 3 is called the divisor.

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3. The upper number of a fraction such as $\frac{2}{5}$ is called the __________; the lower number is called __________.

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4. If the fraction is written 2 ÷ 5, the first number is called the __________; the second number is called the __________________.

numerator, denominator

- - - - - - - - -

5. In most of our work, fractions will be written in the form 3/4, where the upper numeral is called the ________________________.

dividend, divisor

- - - - - - - - -

6. A fundamental rule for fractions is that when both the numerator and the denominator of a fraction are multiplied or divided by the same number, the value of the fraction is not changed. Thus,

numerator,

$$\frac{2 \times 16}{2 \times 64} = \frac{32}{128} \qquad \frac{16 \div 8}{64 \div 8} = \frac{2}{8}$$

16/64, 32/128, and 2/8 are equivalent.

- - - - - - - - -

7. To show that 5/8 and 15/24 are equivalent, either:

Multiply both 5 and 8 by 3 --

$$\frac{3 \times 5}{3 \times 8} = \frac{15}{24}$$

or

Divide both 15 and 24 by 3 --

$$\frac{15 \div 3}{24 \div 3} = \frac{5}{8}$$

- - - - - - - - -

8. Show that the following are equivalent fractions:

$$\frac{3}{5} \text{ and } \frac{21}{35}; \ \frac{15}{21} \text{ and } \frac{5}{7}; \ \frac{64}{36} \text{ and } \frac{16}{9}.$$

- - - - - - - - -

9. Whole numbers such as 3, 4, 14, and 72 may be written as fractions. When written as fractions, the denominator is always 1 (one). Thus, when a number such as 72 is written 72/1, we mean that 72 is divided by 1; and the answer is 72. The numbers 7, 64, 9, written as fractions would be _____, ________, _______.

$\frac{3 \times 7}{5 \times 7} = \frac{21}{35}$; $\frac{15 \div 3}{21 \div 3} = \frac{5}{7}$; $\frac{64 \div 4}{36 \div 4} = \frac{16}{9}$

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10. In order to add or subtract fractions, the denominators must be the same.

$\frac{7}{1}$; $\frac{64}{1}$; $\frac{9}{1}$

- - - - - - - - -

11. To add fractions which have the same denominator, add the numerators and place the sum over the denominator that is common to all of the fractions.

$$\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{5}{4} = \frac{?}{4}$$

- - - - - - - - -

12. In order to add fractions with different denominators, such as 1/4 + 1/3 + 5/8, you must find a common denominator into which each of the denominators can be divided evenly.

11

- - - - - - - - -

13. In the above case, 24 is the smallest number that can be used as a common denominator. Multiply both numerator and denominator of each, 1/4 + 1/3 + 5/8, by a number which will make the denominator 24.

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14. For the fraction 1/4, the number is 6. So,

$$\frac{6 \times 1}{6 \times 4} = \frac{6}{24} .$$

For the fraction 1/3, the number is ________.

For the fraction 5/8, the number is 3, giving:

$$\frac{3 \times 5}{3 \times 8} = \frac{\_\_\_}{\_\_\_} .$$

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15. The problem can now be changed from 1/4 + 1/3 + 5/8 = ? ; to 6/24 + 8/24 + 15/24 = ____________ . 8, $\frac{15}{24}$

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16. Add 1/6 + 3/8 + 5/12 . $\frac{29}{24}$

The smallest number into which 6, 8, and 12 can be divided evenly is usually determined by inspection. If this cannot be done, you can always use the product of the denominators. In this case, inspection should show that ________ is the smallest common denominator.

- - - - - - - - -

17. Now the problem may be written 4/24 + 9/24 + 10/24 = ________. 24

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18. If, in the above problem, the common denominator 72 was picked, the answer would have been 12/72 + 27/72 + 30/72 = 69/72. This fraction could then have been simplified by dividing both numerator and denominator by 3 to give the answer in its simplest form. $\frac{23}{24}$

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19. A fraction may be simplified as long as both numerator and denominator can be divided by the same number as:

$$\frac{56 \div 2}{70 \div 2} = \frac{28}{35} \text{ and } \frac{28 \div 7}{35 \div 7} = \frac{4}{5}.$$

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20. Add: 1/4 + 1/3 + 1/6 = ______.
This answer can be simplified to ________.
Add: 1/8 + 3/5 + 3/20 = ______.
This answer (can, cannot) be simplified.

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21. To subtract fractions, make sure that the denominators are the same and then subtract the numerators as indicated. For example, subtract 1/4 from 5/12. Write as: 9/12, 3/4, 35/40, can

5/12 - 1/4 = ? .

Rewrite with the common denominator 12:

5/12 - 3/12 = 2/12

which, when simplified, is ____________.

- - - - - - - - -

22. Subtract as follows: 1/6

$$7/8 - 1/4 = \_\_\_\_$$
$$21/40 - 3/8 = \_\_\_\_$$
$$5/16 - 1/5 = \_\_\_\_ .$$

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23. To multiply a fraction by a whole number, multiply the numerator by the whole number and place the product over the old denominator. Thus,

5/8,
6/40 or 3/20,
9/80

$$5 \times 3/8 = 15/8$$
$$3 \times 3/16 = 9/16$$

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24. To multiply a fraction by a fraction, the product of the numerators is the new numerator; the product of the denominators is the new denominator.

$$1/6 \times 5/8 = 5/48$$
$$3/4 \times 1/8 \times 6/2 = 18/64 .$$

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25. Often in writing problems, the word "of" is used, as: 1/4 of 16. The word "of" in this sense means to multiply 1/4 by 16.

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26. $1/8 \times 1/2 = \_\_\_\_$
1/3 of 1/2 = ____
1/2 of 6/2 = ____ or ____ .

- - - - - - - - -

27. Multiply: 1/16 , 1/6 , 6/4 or 3/2 .

$$5 \times 3/8 \times 1/2 = \_\_\_\_ .$$

$$2/3 \times 8/3 \times 3 = \_\_\_\_ .$$

This can be simplified to ___________.

$$7/8 \times 3/5 \times 5/6 = \_\_\_\_ .$$

This can be simplified to ___________.

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28. To divide a fraction by another fraction, invert the divisor and multiply it times the dividend (the other fraction). Thus, if you have the problem $3/8 \div 1/2$, the divisor is 1/2. Rewrite the problem as $3/8 \times 2/1$ and perform the indicated multiplication. The answer is 6/8 or ______.

15/16, 48/9, 16/3, 105/240, 7/16

- - - - - - - - -

29. Now try: 3/4

$$25/4 \div 5/16 ;$$

rewrite as:

$$25/4 \times \_\_\_\_ = 20$$

and:

$$\frac{3/8}{5/16} ;$$

rewrite as:

$$3/8 \times \_\_\_\_ = 6/5 .$$

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30. Perform the following divisions: 16/5, 16/5

$$3/8 \div 1/6 = \_\_\_\_\_ .$$
$$7/16 \div 3/8 = \_\_\_\_ .$$
$$12/5 \div 6/15 = \_\_\_\_ .$$

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31. In all cases where the numerator of a fraction is larger than the denominator, the denominator may be divided into the numerator to produce a mixed number (a whole number and a fraction). 18/8 or 9/4, 7/6 or 56/48, 180/30 or 6

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32. For example, the fraction 27/4 may be written as a mixed number by dividing 27 by 4. The answer is the whole number 6, and there is a remainder 3 which is placed over the divisor to make the fraction 3/4. The answer, then, is 6 3/4.

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33. Rewrite as mixed numbers:

18/8 = ____ ; 7/6 = ____ ; 180/30 = ____ .

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34. Also, when a number in a problem is given as a mixed number, such as 2 5/16, it can be changed to fraction form by multiplying the whole number by the denominator and adding the numerator to this product. Thus, 2 5/16 may be changed to a fraction by: 2 1/4; 1 1/6; 6

$$16 \times 2 = 32 \; ; \; 32 + 5 = 37$$

37/16 is the fraction.

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35. Change the following mixed numbers to fractions:

3 5/8 = ____

7 1/3 = ____

2 2/7 = ____ .

- - - - - - - - -

36. Often the simplification of fractions, as mentioned earlier, may be carried a bit further. Since we can divide both the numerator and denominator of a fraction by the same number without changing its value, we can do the same with two fractions that are being multiplied or divided. 29/8 ; 22/3 ; 16/7

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37. For example: With the problem 2/3 × 15/16, divide the numerator of the first by 2 and the denominator of the second by 2 to give

$$\frac{\overset{1}{\cancel{2}}}{3} \times \frac{15}{\underset{8}{\cancel{16}}} .$$

Then, divide the denominator of the first and the numerator of the second by 3 to give

$$\frac{1}{\underset{1}{\cancel{3}}} \times \frac{\overset{5}{\cancel{15}}}{8} .$$

Multiply as usual to give 5/8.

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38. The act of doing both the division by 2 and the division by 3 is called "cancellation" or "dividing out". It is a process used to simplify problems before they reach the final multiplication or division step.

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39. Now, take the problem $3/8 \div 14/16$. Write the problem with the divisor inverted:

$$3/8 \times 16/14 .$$

Now "cancel" an 8,

$$\frac{3}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{14} .$$

Before you multiply, note that the 2 can be cancelled into the 14 to give $3/1 \times 1/7$. Now multiply to give the answer, $3/7$.

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40. A note of warning--cancellation may be used only in problems that involve <u>multiplied</u> fractions.

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41. Reduce the following to simpler terms by cancellation:

$6/15 \times 5/6 =$ ____ $\times$ ____ $=$

$3/8 \div 15/16 =$ ____ $\times$ ____ $=$

$4/7 \times 3/8 \div 9/16 =$ ____ $\times$ ____ $\times$ ____ $=$

Put your work in the frame below. Compare your answers with the solutions in Frame 43.

- - - - - - - - -

42.

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43.

$$\frac{\overset{1}{\cancel{6}}}{\underset{3}{\cancel{15}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}} = \frac{1 \times 1}{3 \times 1} = \frac{1}{3}$$

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{5}{\cancel{15}}} = \frac{1 \times 2}{1 \times 5} = \frac{2}{5}$$

$$\frac{4}{7} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{9}}} = \frac{4 \times 1 \times 2}{7 \times 1 \times 3} = \frac{8}{21}$$

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44. Do the following examples:

a. $\frac{3}{7} \times \frac{3}{5} =$

b. $\frac{3}{8} \times \frac{2}{3} =$

c. $\frac{2}{15} \times \frac{4}{21} \times \frac{7}{8} \times 5 =$

d. $\frac{14}{15} \times \frac{9}{28} =$

e. $\frac{3}{8}$ of $12 =$

f. $\frac{15}{16}$ of $\frac{7}{90} =$

g. $\frac{5}{6}$ of $\frac{5}{9} =$

h. $\frac{7}{27}$ of $\frac{9}{14} =$

i. $18 \div \frac{1}{2} =$

j. $\frac{3}{5} \div \frac{1}{15} =$

k. $\frac{2}{3} \div \frac{1}{2} =$

l. $\frac{4}{7} \div \frac{1}{8} =$

m. $1\frac{2}{3} \times \frac{3}{4} =$

(continued)

44. (continued)

n. $2\frac{1}{2} \times 1\frac{3}{4} =$

o. $3\frac{1}{2} \div \frac{1}{4} =$

p. $1\frac{1}{8} \div \frac{3}{16} =$

q. $\dfrac{\frac{2}{3} + \frac{1}{4} + \frac{1}{2}}{\frac{5}{8} - \frac{1}{6} - \frac{1}{4}} =$

r. $\dfrac{\frac{2}{3} \div \frac{1}{6}}{\frac{1}{4} \times \frac{1}{3}} =$

s. $\dfrac{2\frac{1}{2} + 3\frac{1}{3}}{4 + \frac{2}{3}} =$

t. $\dfrac{\frac{1}{4} \text{ of } 8}{1\frac{1}{2} \text{ of } 3} =$

u. $\dfrac{\frac{1}{5} + \frac{8}{14} + \frac{3}{7}}{\frac{3}{7} \div 10} =$

v. $\dfrac{\frac{1}{6} + \frac{2}{5} + \frac{3}{4} + \frac{1}{3}}{\frac{7}{12} - \frac{1}{6} + \frac{1}{3}} =$

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45. Answers to Frame 44 are:

a. 9/35
b. 1/4
c. 1/9
d. 3/10
e. 9/2 or 4 1/2
f. 7/96
g. 25/54
h. 1/6
i. 36
j. 9
k. 4/3 or 1 1/3
l. 32/7 or 4 4/7
m. 5/4 or 1 1/4
n. 35/8 or 4 3/8
o. 14
p. 6
q. 34/5 or 6 4/5
r. 48
s. 5/4 or 1 1/4
t. 4/9
u. 28
v. 11/5 or 2 1/5

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46. Practice Problems:

a. The following amounts of grain are poured together: 1/2 bu wheat, 3/4 bu oats, and 1 5/8 bu of rice. There will be _____ bushels of combined grains.

b. A fuel element has the following dimensions: — 2 7/8
length of lower end piece -- 6 7/8 in.
length of fuel region -- 16 3/4 in.
length of upper end piece -- 4 3/16 in.
What is the total length of the element?

c. If three 3 5/8-in.-long p-tube rabbits and two 1 7/8-in.-long p-tube rabbits are placed end to end, the overall length of the rabbits will be _________. — 27 13/16 in.

d. A man wishes to run water to an outlet in a field which is 230 1/2 ft from an available water line. If the water pipe comes in 20 1/4 ft lengths, he will need to buy _______ full lengths and a short piece of pipe _______ long. — 14 5/8 in.

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1.2. Decimal Fractions

47. The word fraction means a part of a whole. So, a fraction that actually is less than a whole is a proper fraction. One-fourth of a pie is a proper fraction. — 11, 7 3/4 ft

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48. While the number 15/2 may be written in a fraction form, it is not a real fraction because its numerator is larger than its denominator. It is actually a whole number and a fraction. Thus, it is called an improper fraction.

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49. Decimal fractions are proper fractions that are written in a special way. The word "decimal" means related to 10, and a decimal fraction is always read as if it has a denominator that is some multiple of 10.

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50. Thus, the fractions 1/10, 2/100, 3/1000, 4/10,000 would be written 0.1, 0.02, 0.003, and 0.0004 because we are actually dividing 1 by 10 and 2 by 100, etc. More of the principles of division will be discussed later.

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51. The proper fractions 2/10, 25/100, and 46/1000 would be written 0.2, 0.25, and ________.

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52. The period is called the decimal point, and the numbers are placed a certain number of digits to the right of the decimal point. 0.046

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53. Given the decimal fraction 0.025, the zero is said to be in the tenths place, the 2 is in the hundredths place, and the 5 is in the thousandths place. This fraction is read 25 thousandths because thousandths is the last decimal place given.

\- - - - - - - - -

54. In the following fractions, give the decimal place for each digit number:

0.37; the 3 is in the ______ place
and the 7 is in the ______ place.
0.005; the 5 is in the ______ place.

- - - - - - - - -

55. The fractions 0.78, 0.01, 0.235, and 0.6 would be read:

tenths,
hundredths,
thousandths

0.78 would be ______________________________,
0.01 would be ______________________________,
0.235 would be _____________________________,
0.6 would be _______________________________.

- - - - - - - - -

56. As the decimal places to the right of the decimal point increase, the fourth place (0.0004) would be 4 ten thousandths and 0.00235 would be read 235 hundred thousandths.

78 hundredths,
1 hundredth,
235 thousandths,
6 tenths

- - - - - - - - -

57. You will note that zeros between the decimal point and the number serve to determine the size of the number; 0.05 shows that the number is 5 hundredths and not 5 tenths or 5 thousandths. However, a zero after the number does not change its value: 0.050 could be read 50 thousandths, but 50/1000 can be simplified to 5/100; so the zero after the number has no significance.

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58. The decimal fractions 0.32, 0.00045, and 0.0023 would be read:

0.32 would be ____________________;
0.00045 would be ________________;
0.0023 would be _________________.

- - - - - - - - -

59. While the fraction 0.135 is commonly read 135 thousandths, for brevity and convenience it is also read "point one three five". By this method, 0.023 would be read point "_____ _____ _____"; and a number such as 4.18 would be read "four point one eight".

32 hundredths,
45 hundred thousandths,
23 ten thousandths

- - - - - - - - -

60. For practice, write the following numbers as they would be read by this method:

0.01 = ____________________;
0.5 = ____________________;
2.10 = ____________________;
23.45 = ____________________;
0.0608 = ____________________;
3.125 = ____________________.

zero-two-three

- - - - - - - - -

61. Using the same fractions, write them using the tens, hundreds, thousands system:

0.01 = ____________________;
0.5 = ____________________;
2.10 = ____________________;
23.45 = ____________________;
0.0608 = ____________________;
3.125 = ____________________.

point zero one;
point five;
two point one zero;
twenty-three point four five;
point zero six zero eight;
three point one two five.

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62. Decimal fractions may be added or subtracted as if they were whole numbers with only one provision; the decimal points must be aligned correctly.

one hundredth;
5 tenths;
2 and 10 hundredths;
23 and 45 hundredths;
608 ten thousandths;
3 and 125 thousandths

- - - - - - - - -

63. For example, you wish to add 2.54 + 75.3 + 121.67 + 14. Place the numbers as for normal addition, but with the decimal points in a vertical line:

$$\begin{array}{r@{.}l} 2 & 54 \\ 75 & 3 \\ 121 & 67 \\ \underline{14} & \underline{} \end{array}$$

- - - - - - - - -

64. Since it has already been determined that a zero to the right of the decimal point does not change the number, we could also write the problem

$$\begin{array}{r} 2.54 \\ 75.30 \\ 121.67 \\ \underline{14.00} \end{array}$$

This form is preferred by many people because it is easier to keep the numbers in each column straight.

- - - - - - - - -

65. Now, add the numbers in each column and place the decimal point in the answer in the same vertical line. The answer is ______________________.

- - - - - - - - -

66. Add the following:

213.51

4.15 + 67.3 + 2.639 + 0.5 = ___________.

173.1 + 17.31 + 1.731 + 0.1731 = ______.

- - - - - - - - -

67. When subtracting decimals, it is necessary to keep decimal points aligned as when adding. For example, if you subtract 14.3 from 146.73, write — 74.589, 192.3141

$$\begin{array}{r} 146.73 \\ \underline{14.30} \end{array}$$

Subtract, as with whole numbers, to give the answer ______________.

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68. It is suggested that, as in the illustration above, you use zeros to make the same number of digits to the right of the decimal point in all cases. This form is especially advantageous if you are subtracting, for example, a number with 3 digits to the right from a number with only one or two digits to the right. In the problem, 146.1 - 27.462, it is usually less confusing to write — 132.43

$$\begin{array}{r} 146.100 \\ \underline{27.462} \end{array}$$

and subtract as usual to give the answer __________.

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69. Practice the following problems: — 118.638

65.321 - 17.4 = ________;

561.2 - 540.67 = ________;

0.275 - 0.05 = ________.

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70. Now, let us turn to the multiplication of decimal numbers. If you can multiply 25/100 times 6 and simplify it to tenths, you have 15/10. As a mixed number, it is read 1 5/10. 47.921; 20.53; 0.225.

- - - - - - - - -

71. Since 5/10, written as a decimal, is 0.5, we could write 1 5/10 as 1.5.

- - - - - - - - -

72. If we had written the original 25/100 as 0.25, we could have written the problem 0.25 × 6. Multiplying as if both numbers were whole numbers,

$$\begin{array}{r} 0.25 \\ \times\ 6 \\ \hline 1.50 \end{array} .$$

Since there are 2 decimal places in the problem, there must be the same number of places in the answer, giving 1.50. Since 0.50 can be read as either 50 hundredths or 5 tenths, the zero may be ignored and the answer is 1.5 as before.

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73. The rule for multiplying decimals is:

> To multiply a decimal by any number, multiply as with whole numbers. Count the places to the right of the decimal in both the multiplier and the multiplicand, and place that many places to the right of the decimal point in the product.

- - - - - - - - -

74. In the problem 0.31 × 5.3,

$$\begin{array}{rl} 0.31 & \text{multiplicand} \\ \underline{5.3} & \text{multiplier} \\ 93 & \\ \underline{155} & \\ 1.643 & \text{product} \end{array}$$

There are three places to the right of the decimal; two in 0.31 and one in 5.3. So, the answer is 1.643.

\- - - - - - - - - - - -

75. The answer in the above frame may be read either one and 643 thousandths or _____ point _____ _____ _____.

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76. Practice solving the following problems:

one,
six four three

18.5 × 4 = ________________ ;

3.9 × 2.4 = ________________ ;

0.56 × 0.74 = ______________ ;

0.0069 × 10,000 = __________ ;

7.49 × 10 = ________________ ;

0.021 × 1000 = _____________ ;

13.2 × 0.006 = _____________ .

\- - - - - - - - - -

77. Let us begin our division of fractions by restating a rule that was stated regarding common fractions. This rule is: When both dividend and divisor (numerator and denominator) are multiplied by the same number, the answer is not changed.

74.0;
9.36;
0.4144;
69;
74.9;
21;
0.0792.

\- - - - - - - - - -

78. Thus 1/5 could become 2/10, which could be written as the decimal fraction 0.2; and, $\frac{2 \times 4}{25 \times 4}$ would be 8/100 and could be written ______________.

- - - - - - - - -

79. The changing of a common fraction to a fraction with a denominator which is a multiple of 10 is not necessary in this procedure, as will be shown in the following frames. We repeated the rule for multiplying dividend and divisor by the same number for another reason. .08

- - - - - - - - -

80. This rule allows us to multiply a dividend and divisor by a multiple of 10 so that the divisor is a whole number, and the placing of the decimal point is simplified.

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81. Take the problem 16.42 ÷ 4.1: 16.42 × 10 = 164.2. The divisor, 4.1, × 10 = 41.0 (a whole number)

$$\begin{array}{r|l} & 4. \\ \hline 41 & 164.2 \\ & \underline{164} \end{array}$$

After the new decimal point is placed in the dividend (164.2), the decimal point in the quotient (quotient is the correct name for the answer in a division problem) is in line with it.

- - - - - - - - -

82. With the problem 135.4 ÷ 0.46, we would multiply both by (10, 100, 1000) in order to make the ________________ a whole number. Now the problem is to divide 13540 by 46.

- - - - - - - - -

83. We might simplify this in the following manner: Indicate the division as in the original-- 100, divisor

$$.46_{\wedge}\overline{)\,135.40_{\wedge}}$$

Move the decimal point in both the dividend and divisor enough places to the right to make the divisor a whole number.

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84. The problem is now

$$46.\ \overline{)\,13540.}$$

The decimal point in the quotient will be in line with the decimal point in the (divisor, dividend).

\- - - - - - - - -

85. Thus, in dividing 6.48 by 0.4, we multiply both 6.48 and 0.4 by 10 and do the division. dividend

$$\begin{array}{r} 16.2 \\ 0.4_{\wedge}\overline{)\,6.4_{\wedge}8} \\ \underline{4} \\ 2\ 4 \\ \underline{2\ 4} \\ 8 \\ \underline{8} \end{array}$$

The quotient is _________.

\- - - - - - - - -

86. To check that 16.2 is correct, multiply 16.2 × 0.4, and the product should be 6.48. It (is, is not) correct. 16.2

\- - - - - - - - -

87. Again, in order to divide decimals, multiply both dividend and divisor by a number which will make the <u>divisor</u> (the number you are dividing by) a whole number. — is

- - - - - - - - - -

88. If you are dividing 63.8 by 4.75, the number ____________________ is the divider.

- - - - - - - - - -

89. The first step, then, is to multiply the divisor, 4.75, by ____________, which will make it ________. — 4.75

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90. Then multiply the dividend, 63.8, by 100; and it becomes ____________________. — 100, 475

- - - - - - - - - -

91. Now divide as usual: — 6380

```
           13.4
475 ) 6380.
      475
      1630
      1425
       2050
       1900
        150
```

The answer, 13.4, is called the ________________.

- - - - - - - - - -

92. Now try 175.6 divided by 0.032. Place the numbers in the right form for division. — quotient

____.) ______.

- - - - - - - - - -

93. In the above problem, the quotient will be ____________________. | $32.\overline{)175600.}$

- - - - - - - - -

94. Work the following problems: | 5487.5

a. $167.25 \div 0.5 =$ ____________________;
b. $23.4 \div 1.4 =$ ____________________;
c. $78.3 \div 90.67 =$ ____________________;
d. $24.64 \div 0.48 =$ ____________________.

- - - - - - - - -

95. Now check your answers. To do this, multiply the divisor by the quotient. Thus using the problems in Frame 94:

a. $0.5 \times 334.5 =$ ____________________;
b. ____ $\times 16.7 =$ ____________________;
c. $90.67 \times$ ____ $=$ ____________________;
d. ____ $\times$ ____ $=$ ____________________.

a. 334.5;
b. 16.7;
c. 0.863 or 0.864;
d. 51.3.

- - - - - - - - -

96. Note that answers b, c, and d do not check exactly. You will remember that in the original problem there was a remainder in each of these cases. When solving actual problems, the number of digits to the right of the decimal point is usually left to the discretion of the person solving the problem.

a. 167.25;
b. 1.4, 23.38;
c. 0.863, 78.25 or 0.864, 78.34;
d. $.48 \times 51.3 = 24.624$.

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97. For example, if you are buying fencing for a yard, you will probably not measure a length or width any more precisely than tenths of a foot or one digit to the right of the decimal point. In such a case, it is not practical to measure to the hundredths place or two digits to the right of the decimal point.

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98. At the beginning of this section, we noted that it was relatively easy to change a fraction whose denominator was a multiple of 10, as 35/100, to a decimal fraction. Now let us change <u>any</u> fraction to a decimal.

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99. Proper fractions may be changed to decimal fractions by dividing the numerator by the denominator.

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100. You can change the fraction 5/8 to a decimal fraction by dividing the numerator, the number _______, by the denominator, the number _______.

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101. In effect, you are making the denominator unity (equal to one) as: 5, 8

$$\frac{5 \div 8}{8 \div 8} = \frac{0.625}{1}$$

and any number divided by one is just the number.

\- - - - - - - - -

102. To shorten the process, divide as below, and you find that 5/8 is the decimal fraction 0.625:

$$\begin{array}{r@{\,}l}
 & \,0.625 \\
8 & \overline{)\,5.0} \\
 & \,\underline{4\,8} \\
 & \,20 \\
 & \,\underline{16} \\
 & 40 \\
 & \underline{40}
\end{array}$$

\- - - - - - - - -

103. In the same way, show that 3/4 is 0.75 and that 1/32 is __________.

- - - - - - - - -

104. Change the following fractions to decimal numbers: 0.03125

a. 7/8 = ____________________;
b. 14/32 = ____________________;
c. 64/12 = ____________________;
d. 164/8 = ____________________.

- - - - - - - - -

105. To change a mixed number to a decimal number, write the whole number as it is. Then change the fraction part to a decimal and add to the whole number.

a. 0.875;
b. 0.4375;
c. 5.33;
d. 20.5.

- - - - - - - - -

106. For example, to change 12 3/4 to a decimal, change 3/4 to 0.75 and add to 12 as:

$$\begin{array}{r} 12.00 \\ \underline{.75} \\ 12.75 \end{array}$$

- - - - - - - - -

107. Change the following mixed numbers to decimals:

a. 23 1/8 = ____________________;
b. 14 2/3 = ____________________;
c. 123 14/16 = ____________________.

- - - - - - - - -

108. The method for changing decimal fractions to common fractions is quite simple. Just place all numbers to the right of the decimal point over the number 10, 100, 1000, etc., in whose place the last digit is found. For example, 0.235 can be read 235 thousandths. So, write the fraction 235/1000 and simplify by cancellation to 13/40.

a. 23.125;
b. 14.667 or 14.67;
c. 123.875.

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109. Write the following decimal fractions as the smallest common fractions possible:

a. 0.75 = ____________________;
b. 0.125 = ____________________;
c. 0.0625 = ____________________.

- - - - - - - - -

110. Practice exercises:

a. 3/4;
b. 1/8;
c. 1/16.

a. 18.5 × 4 =	k. 534.79 ÷ 100 =
b. 3.9 × 2.4 =	l. 534.79 × 100 =
c. 0.56 × 0.74 =	m. 534.79 ÷ 1000 =
d. 0.021 × 0.204 =	n. 642 ÷ 2.4 =
e. 0.601 × 0.003 =	o. 58.6 ÷ 2.93 =
f. 0.236 × 12.13 =	p. 0.46 ÷ 20 =
g. 7.43 × 0.132 =	q. 6.88 ÷ 320 =
h. 126 × 0.0064 =	r. 688 ÷ 3.20 =
i. 1200 × 0.003 =	s. 12 ÷ 0.03 =
j. 65.1 × 1.4 =	t. 0.03 ÷ 12 =

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111. Answers to the above practice exercises:

a. 74	k. 5.3479
b. 9.36	l. 53,479
c. 0.4144	m. 0.53479
d. 0.004284	n. 267.5
e. 0.001803	o. 20
f. 2.86268	p. 0.023
g. 0.98076	q. 0.0215
h. 0.8064	r. 215
i. 3.6	s. 400
j. 91.14	t. 0.0025

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112. Practical problems:

a. If one gallon of water weighs 8.25 pounds, 16,000 gallons will weigh ________ pounds.

b. If the primary water flow at the HFIR is 16,000 gpm, how many pounds of water move through the reactor per hour? per second?

- - - - - - - - -

1.3. Percentage

113. Percentage is another method used to express a fraction. In this method, the fraction is always expressed as a fraction whose denominator is 100, as a certain number of parts of 100.

132,000;
7,920,000;
2,200

- - - - - - - - -

114. For example, the fraction 1/4 may also be written as 0.25 or 25/100. Instead of reading this fraction as 25 hundredths, we use the word percent and read it 25 percent. The symbol for percent is %; the above number could be written 25%.

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115. Eight percent means 8 parts of 100. It can thus be written as either 8%, as a common fraction 8/100, or as a decimal fraction 0.08.

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116. If a tank holds 100 gallons of water and you put only 20 gallons of water in it, you are using only 20 parts out of 100 parts of its total capacity. This is 20/100 or 20 _________ of the tank's capacity.

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117. Since percent means parts of one hundred, it is easy to change a percent to a decimal fraction; just divide the percent by 100. Usually, this is done by moving the decimal point two digits to the left.

percent

- - - - - - - - -

118. Thus 63%, which is really 63 parts of 100, can be written 63/100. Divide 63 by 100, and you have 0.63. You have moved the decimal point from 63.% to 0.63, which is _________ digits to the ________.

- - - - - - - - -

119. Change the following percents to decimal fractions: | two, left

a. 2% e. 3.4%
b. 25% f. 365%
c. 179% g. 0.46%
d. 38.5%

- - - - - - - - - -

120. In order to work problems involving percents, it is advisable to change the percents to decimal fractions and then do the arithmetic according to the rules for fractions.

a. 0.02
b. 0.25
c. 1.79
d. 0.385
e. 0.034
f. 3.65
g. 0.0046

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121. In decimal fraction form, percents can be added, subtracted, multiplied, and divided according to the rules for decimals.

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122. You will recall that when decimals are added or subtracted, the decimal points must be correctly aligned. Thus, to add 14.7%, 64%, 5%, and 1.4%, change each percent to a ____________________.

- - - - - - - - - -

123. Now as decimals, you can align them as: | decimal

$$\begin{array}{r} 0.147 \\ 0.64 \\ 0.05 \\ \underline{0.014} \\ 0.851 \end{array}$$

The sum of these is 0.851 and can be written as a percent -- _________________ -- by moving the decimal point back two places to the right.

- - - - - - - - - -

124. Actually, percents may be added or subtracted without changing them to decimals. However, to avoid confusion, it is often advisable to change them to decimals. 85.1%

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125. Multiplication and division of percents is merely a matter of changing the percents to decimals and doing the necessary arithmetic.

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126. Practice exercises:

a. $5 \times 40\% =$ ______

b. $2 \times 15\% =$ ______

c. $10\% + 18.5\% + 30\% =$ ______

d. $14\% - 6\% =$ ______

e. $17\% \times 3 =$ ______

f. 1/2 of 96% = ______

g. 2/3 of 96% = ______

h. $51\% \div 3 =$ ______

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127. You will recall that in the language of mathematics, "of" means to multiply. Thus, in the practice exercises above, 2/3 of 96% was $2/3 \times 0.96$, to give the answer 0.64 or 64%.

a. 200%
b. 30%
c. 58.5%
d. 8%
e. 51%
f. 48%
g. 64%
h. 17%

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128. Practice problems:

a. Here in Tennessee, we pay a 3% sales tax. On a \$10 purchase, the sales tax is 3% of \$10, which we calculate by multiplying 0.03×10. to get \$______ tax.

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b. Three percent of $10 is $0.30 dollars. In terms of money, we write the above answer $0.30, but we read it 30 cents. This amount, as a percent, is __________ of $1 and only ________% of $100. — 0.30

- - - - - - - - -

c. If you get a 6% discount when buying a refrigerator that normally sells for $450, you actually pay $ _______ less than the normal price. — 30%, 0.3

- - - - - - - - -

d. You are asked to make 100 gallons of 5% caustic solution. Five percent of 100 gallons is _______ gallons, so this is the amount of caustic that you will need. — 27

- - - - - - - - -

e. One hundred gallons of solution minus 5 gal of caustic leaves _______ gal of water. So, you put 95 gal of water in the mix tank, add the 5 gal of caustic, mix thoroughly, and you have the 100 gal of 5% solution. — 5

- - - - - - - - -

f. If the No. 3 primary coolant pump handles 35% of the 16,000 gpm coolant flow, it pumps _________ gal per min. — 95

- - - - - - - - -

g. What percent of the 16,000 gpm coolant flow is handled by the No. 2 pump if it pumps 5,280 gpm? — 5,600

- - - - - - - - -

h. If you put 38 gal of water in a tank and add 2 gal of concentrated nitric acid, you have 40 gal of a _______% nitric acid solution. 33%

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5

SECTION I-2

ANALYSIS OF PROBLEMS

2.1. Numbers with Dimensions

1. Let us begin our analysis of problems with a discussion of the dimensions that are most often a part of the problem. Numbers that have dimensions are called dimensional numbers and are the results of a measurement.

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2. The numbers 25 miles, 5 gallons, and 15 seconds represent measures and are called ________________ numbers.

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3. The number 10 ft represents a measure of something that is 10 ft long; and the dimension, feet, is a very important part of the total number, 10 ft. dimensional

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4. There are two systems of measurement that are in common use today. The traditional British system is the one most familiar to people in the United States. Some of the most used dimensions are found in the following table.

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Table 2.1. British Units of Measure

Length	Volume
12 in. = 1 ft	2 pints = 1 quart
3 ft = 1 yd	4 quarts = 1 gallon
36 in. = 1 yd	
5280 ft = 1 mi	**Cubic Measure**
	1728 in.^{3} = 1 ft^3
Weight	27 ft^3 = 1 yd^3
16 oz = 1 lb	
2000 lb = 1 ton	**Time**
	60 sec = 1 min
	24 hrs = 1 day

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6. The other system, the metric system, is in common use in most non-English-speaking countries of the world. It is also preferred by most scientists. The following table gives a few of the most often used units.

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Table 2.2. Metric Units of Measure

Prefix	Meaning	Length	Mass	Volume
mega	1,000,000 ×			
kilo	1,000 ×	kilometer (km)	kilogram (kg)	kiloliter (kl)
	1 ×	meter (m)	gram (g)	liter (l)
centi	0.01 ×	centimeter (cm)		
milli	0.001 ×	millimeter (mm)	milligram (mg)	milliliter (ml)
micro	0.000001 ×	micron (μ)*	microgram (μg)	

*The symbol μ is the Greek letter Mu.

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7. Since most of our work will be with the British system, we will concentrate on its use.

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8. In adding and subtracting dimensioned numbers, the dimensions must be the same. Thus:

60 mi - 5 mi = ______________

1/2 hr + 3/8 hr = ____________

0.63 gal + 0.025 gal = _______

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9. When multiplying or dividing dimensioned numbers by nondimensioned numbers, the answer retains the dimensions. Thus:

55 mi,
7/8 hr,
0.655 gal

36 ft $\div 2 =$ ____________ ft

24 hrs $\div 3 =$ ____________ hrs

5×5 gal $=$ ____________ gal

6×50 lbs $=$ ____________ lbs

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10. However, when multiplying and dividing numbers, both of which have dimensions, care must be exercised to insure that the answer has the proper dimensions.

18,
8,
25,
300

- - - - - - - - -

11. For example, when finding the area of a plot of ground 20 ft wide by 60 ft long, we multiply 20×60 to get the number 1200 and also multiply ft $\times$ ft to get square feet (ft^2). The answer is 1200 ft^2; and we read the answer 1200 square feet.

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12. If we wanted the volume of a box 3 ft wide by 10 ft long by 2 ft deep, we would multiply:

$$3 \text{ ft} \times 10 \text{ ft} \times 2 \text{ ft} = 60 \text{ ft}^3$$

- - - - - - - - -

13. A number is squared when it is multiplied by itself. Thus:

$$2 \times 2 = 2^2 = 4$$
$$12 \times 12 = 12^2 = 144$$
$$\text{ft} \times \text{ft} = \text{ft}^2$$
$$\text{miles} \times \text{miles} = \underline{\hspace{3cm}}$$

- - - - - - - - -

14. A number multiplied by itself twice, that is, $3 \times 3 \times 3$, is said to be cubed. Thus: mi^2 or square miles

$$3 \times 3 \times 3 = 3^3 \text{ or } 27$$
$$4 \times 4 \times 4 = 4^3 \text{ or } 64$$
$$2 \text{ ft} \times 3 \text{ ft} \times 4 \text{ ft} = 24 \underline{\hspace{3cm}}$$

- - - - - - - - -

15. If the numbers multiplied have dimensions which are not the same, the dimensions are still multiplied. For example, the work performed when a 400-lb weight is lifted 2 ft = 800 ft lb. This is read 800 foot pounds. The foot pound is a unit that represents work, a measure of the work done. ft^3 or cubic feet

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16. Solve the following problems:

a. $5\ ft \times 6\ ft \times 20\ ft =$ ____________

b. $20\ lb \times 60\ ft =$ ________________

c. $3\ in. \times 5\ in. =$ ________________

d. $20\ kw \times 10\ hrs =$ ______________

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a. $600\ ft^3$
b. 1200 ft lb
c. $15\ in.^2$
d. 200 kwh

17. When dividing one dimensioned number by another of the same dimension, the answer is a nondimensioned number because the dimensions cancel in the same way that numbers are cancelled:

$$\frac{\overset{8}{\cancel{24}}\ \cancel{ft}}{\underset{1}{\cancel{3}}\ \cancel{ft}} = 8$$

or

$$\frac{24\ ft^2}{3\ ft} = \frac{\overset{8}{\cancel{24}} \times ft \times \cancel{ft}}{\underset{1}{\cancel{3}}\ \cancel{ft}} = 8\ ft.$$

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18. Solve the following problems:

a. 16 lb ÷ 32 lb =

b. 80 ft ÷ 2 ft =

c. 1000 gal ÷ 20 gal =

d. 240 hours ÷ 8 hours =

e. $500\ ft^3 \div 10\ ft =$

f. $200\ ft^2 \div 8\ ft =$

g. 900 Megawatt days ÷ 30 days =

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19. When dividing a dimensioned number by a number having another dimension, the answer's dimension is the numerator's dimension divided by the denominator's dimension. Thus, 510 miles ÷ 30 gallons of gasoline would be

a. 1/2 or 0.5
b. 40
c. 50
d. 30
e. 50 ft^2
f. 25 ft
g. 30 Mw

$$\frac{510\ \text{mi}}{30\ \text{gal}} = 17\ \text{mi/gal}\ .$$

This is read "17 miles per gallon".

- - - - - - - - -

20. Work the following problems:

a. 700 tons ÷ 20 days = ______________________________

b. 630 miles ÷ 15 hours = ___________________________

c. 8 hours × 3 dollars/hr = __________________________

d. 24 payments at 50 dollars/payment = _____________ .

- - - - - - - - -

21. These problems often involve converting from one size unit to another, such as feet to yards or minutes to hours. To change 3000 gallons per minute (gpm or gal/min) to gph, multiply 3000 gpm × 60 min/hr. The minute dimension cancels, leaving gph. The answer is ______________________________.

a. 35 tons/day
b. 42 mi/hr
c. $24
d. $1200

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22. Tennessee Ernie Ford shovels 16 tons of coal in an 8-hour day. How many tons does he shovel (a) in an hour? (b) in a minute?

a. 16 T ÷ 8 hr = _____________

b. 16 T/8 hr ÷ 60 min/hr -- for clarity, write:

$$\frac{\overset{2}{\cancel{16}}\ \text{T}}{\underset{1}{\cancel{8}}\ \text{h}\cancel{\text{r}}} \times \frac{\cancel{\text{hr}}}{60\ \text{min}} = \frac{\overset{2}{\cancel{16}}\ \text{T}}{\underset{60}{4\cancel{8}0}\ \text{min}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

180,000 gph

- - - - - - - - -

23. How many pounds per minute is 1/30 ton/min?

a. 2 T/hr
b. 1/30 T/min

$$\frac{2000\ \text{lb}}{\text{t}\cancel{\text{o}}\text{n}} \times \frac{1\ \text{t}\cancel{\text{o}}\text{n}}{30\ \text{min}} = \underline{\hspace{6em}}$$

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24. When working such problems, make sure that your work produces the desired dimensions for the answer.

66.7 lb/min

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25. If a pump will pump 100 gal of water per minute, how many hours will it take to empty an 18,000-gal tank? Our problem is twofold. First, how many minutes of water do we have at 100 gpm; then, how many hours of water in the tank? So, divide 18,000 gal by 100 gpm. Write it:

$$18{,}000\ \text{gal} \div \frac{100\ \text{gal}}{\text{min}}$$

and multiply as:

$$\frac{\overset{180}{180{,}000\ \cancel{\text{gal}}}}{1} \times \frac{1\ \text{min}}{1\cancel{0}0\ \text{g}\cancel{\text{a}}\text{l}} = 180\ \text{min}\ .$$

Now, change 180 min to hours.

180 min ÷ 60 min/hr is written:

$$\overset{3}{1\cancel{8}0}\ \text{m}\cancel{\text{i}}\text{n} \times \frac{1\ \text{hr}}{\underset{1}{6\cancel{0}\ \text{m}\cancel{\text{i}}\text{n}}} = 3\ \text{hours}\ .$$

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26. In the above problem, if in the latter part we had, without thinking, multiplied 180 min by 60 min/hr, the answer would have been much larger than we should have expected (10,800); and the dimensions would have been $\frac{\text{min}^2}{\text{hr}}$. There are no such dimensions, so it is obvious that we should not have ____________, but should have ____________ 180 min by 60 min/hr.

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27. A man uses 15 gallons of gas in driving 270 miles in six hours. How many gph does he use? Arrange the dimensions so that the answer is gph. Write ______ $\frac{\text{gal}}{\text{mi}} \times$ ________ $\frac{\text{mi}}{\text{hr}}$; the miles will cancel and give the answer ______________ gph.

multiplied, divided

- - - - - - - - -

28. A man's wages are \$2.50 per hour. Assuming that he works 248 eight-hour days per year, his yearly income (before taxes) is ______________.

$\frac{15 \text{ gal}}{270 \text{ mi}} \times \frac{270 \text{ mi}}{6 \text{ hr}}$, 2.5

- - - - - - - - -

29. A 60-watt lamp is left on when a family leaves the house for a week of vacation. When the family returns, 7 days and 2 hours later, they turn the light off. The watt hours of electricity used were ______________.

\$4,960

- - - - - - - - -

30. Since 1000 watts = 1 kilowatt, the lamp burned __________ kwh of electricity.

10,200

- - - - - - - - -

31. At \$0.02/kwh, the lamp used __________ cents worth of electricity.

10.2

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2.2. Ratio and Proportion

32. The term ratio means that two numbers are related, as in a fraction. For example, the numbers 1 and 2 can be related as the fractions 1/2 and 2/1. The first is read 1 is to 2 and the second is 2 is to 1. — 20.4

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33. The above ratios can also be written 1:2 and 2:1, but the common fraction is the form most often used.

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34. The ratio 16:64 is usually written as the fraction ____________.

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35. Whatever the manner of writing a ratio, its arithmetic value is always the same. — 16/64

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36. Since ratios may be written as fractions, the rule that both terms of a fraction may be multiplied or divided by the same number without affecting its value is true also for ratios.

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37. Thus, in the above illustration, the ratio 16/64 may be simplified by dividing both numerator and denominator by 16. The new ratio is ____________.

\- - - - - - - - -

38. Simplify the following ratios: 1/4

a. 16 to 4 = ________________
b. 13 to 39 = ________________
c. 7/56 = ________________
d. 49/56 = ________________
e. 21:36 = ________________ .

- - - - - - - - - -

39. When a man drives 512 miles and uses 32 gallons of gas, the ratio of miles traveled to gallons of gas used is 512:32. Normally, we simplify this ratio to ________________.

a. 4 to 1
b. 1 to 3
c. 1/8
d. 7/8
e. 7:12

- - - - - - - - - -

40. A proportion is a method of expressing equality between two ratios. This equality is usually written "=" as in 16/32 = 1/2; but it may be written as a double colon, "::". The above proportion is then written 16:32 :: 1:2 and is read "16 is to 32 as 1 is to 2".

16 mi/gal

- - - - - - - - - -

41. In the form 16:32 :: 1:2, the numbers at each end, 16 and 2, are called "extremes" of the proportion. The numbers on each side of the colon are called the "means" of the proportion, presumably because they are in the middle.

- - - - - - - - - -

42. Write, in the following blank spaces, the proportions below as they should be read:

a. 2/9 = 16/72 ________________________;
b. 10:50 :: 2:10 ________________________;
c. 6:30 :: 7:35 ________________________;
d. 3/8 = 24/64 ________________________.

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43. When writing proportions involving dimensions, it is customary to use the same dimension on the same side of the equality sign. Thus,

$$\frac{10\ \text{ft}}{30\ \text{ft}} = \frac{4\ \text{in.}}{12\ \text{in.}}$$

a. 2 is to 9 as 16 is to 72;
b. 10 is to 50 as 2 is to 10;
c. 6 is to 30 as 7 is to 35;
d. 3 is to 8 as 24 is to 64.

\- - - - - - - - - -

44. Write the following proportions as two fractions separated by an = (equal) sign.

a. 1 ft is to 10 ft as 6 mi is to 60 mi.
b. 5 gpm is to 30 gpm as 300 gal/hr is to 1800 gal/hr.
c. 20 Mw is to 100 Mw as 1 is to 5.

\- - - - - - - - - -

45. When two numbers are separated by an equal sign, as:

$$22 = 2 \times 11$$

$$3/4 = 0.75\ ,$$

we use the word "equation" to represent the arithmetic form.

a. $\frac{1\ \text{ft}}{10\ \text{ft}} = \frac{6\ \text{mi}}{60\ \text{mi}}$
b. $\frac{5\ \text{gpm}}{30\ \text{gpm}} = \frac{300\ \text{gal/hr}}{1800\ \text{gal/hr}}$
c. $\frac{20\ \text{Mw}}{100\ \text{Mw}} = \frac{1}{5}$

\- - - - - - - - - -

46. It is easy to see that a proportion written as 16/48 = 4/12 is an ______________. It is not quite so obvious that the form 16:48 :: 4:12 also represents an equality.

- - - - - - - - -

47. Proportions written in equation form often are used as methods for solving problems where three of the values are known and you need to know the other value. equation

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48. You will recall that the numerators and denominators of fractions can be multiplied by or divided by the same number without changing their value. We used this method to simplify fractions.

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49 There is a similar rule for equations. You can multiply both sides of an equation or divide both sides of an equation by the same number without destroying the equality.

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50. When the fractions are in an equation, as in a proportion, we use this method to simplify the equation. That is, we "clear" the equation of fractions.

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51. Let us take the proportion

$$3/16 = 15/80 .$$

A common denominator for these two fractions would be 80. Multiply the numerators by 80 as:

$$\frac{80 \times 3}{16} = \frac{80 \times 15}{80} ;$$

cancel common terms to give the equation the form:

$$5 \times 3 = 1 \times 15 .$$

- - - - - - - - -

52. Clear the following equations of fractions:

a. $3/8 = 12/32$;
b. $3 + 5/8 = 116/32$;
c. $2 + 3/7 = N/14$.

- - - - - - - - -

53. Note that the simplification process involves two steps. First, find the common denominator; second, multiply both sides, cancel, and combine terms. Take the above equation $3/8 = 12/32$. The common denominator is 32.

a. $3 \times 4 = 1 \times 12$
b. $29 \times 4 = 1 \times 116$
c. $17 \times 2 = 1 \times N$

$$\frac{\overset{4}{\cancel{32}} \times 3}{\underset{1}{\cancel{8}}} = \frac{\overset{1}{\cancel{32}} \times 12}{\underset{1}{\cancel{32}}} \text{ to give } 4 \times 3 = 1 \times 12 .$$

- - - - - - - - -

54. This can be shortened by a process called "cross-multiplication", which is another term for what we have been doing.

- - - - - - - - -

55. When the proportion is written as a fraction, such as 3/12 = 9/36, you cross-multiply. You multiply the numerator on one side by the denominator on the opposite side of the equal sign. Thus:

$$3 \times 36 = 9 \times 12 .$$

- - - - - - - - -

56. At times, as in the above example, this can be shortened by doing only one cross-multiplication.

$$3/12 = 9/36$$

Cross multiply the 36 and 3 to give

$$\frac{3 \times 36}{12} = \frac{9}{1} .$$

Now cancel the 12 into 36 to give

$$3 \times 3 = 9 .$$

- - - - - - - - -

57. Now a problem: 3 is to 12 as what number is to 72? Write the problem in fraction form and write the letter N (for the number) where the number you are looking for should be. As: 3/12 = N/72. Now cross multiply the 3 and 72 to get the fraction

$$\frac{3 \times 72}{12} =$$

on one side and _____ on the other side of the equal sign.

- - - - - - - - -

58. The number you are looking for is found by doing the arithmetic indicated by the $\frac{3 \times 72}{12}$. The number, then, is ______________. N

\- - - - - - - - -

59. In solving problems by the ratio and proportion method, it is first necessary to recognize that a proportion exists and, second, whether the proportion is direct or inverse. 18

\- - - - - - - - -

60. A proportion is direct when two quantities are so related that an increase (or decrease) in one causes a corresponding increase (or decrease) in the other.

\- - - - - - - - -

61. "The taller the object, the longer the shadow" can be stated thus: The length of the shadow is directly proportional to the height of the object. Written as a proportion, it would be

$$\frac{\text{shadow length 1}}{\text{shadow length 2}} = \frac{\text{height 1}}{\text{height 2}}$$

\- - - - - - - - -

62. For example, if a 6-ft man has a shadow 12 ft long, his 4-ft son will have a shadow _______ ft long.

\- - - - - - - - -

63. If it rains at a constant rate of 0.5 in. per hour, in 5 hours a rain gauge will show that the total rainfall has been ___________ in. 8

\- - - - - - - - -

64. If a pump is rated at 300 gpm, in 24 hours it will pump ____________ gallons of liquid. — 2.5

\- - - - - - - - -

65. The above problems are examples of situations where the relationship between the quantities is a ____________________ proportion. — 432,000

\- - - - - - - - -

66. An inverse proportion is one in which as one quantity increases, the other quantity shows a corresponding decrease. — direct

\- - - - - - - - -

67. In a situation such that as speed increases, the time to go a certain distance decreases, the proportion is ______________________.

\- - - - - - - - -

68. If a 100-gpm pump fills a 1,000-gal tank in 10 min, a 250-gpm pump will fill the same tank in T min. The form used in the inverse proportion is: — inverse

$$\frac{\text{Pump 1}}{\text{Pump 2}} = \frac{\text{Time 2}}{\text{Time 1}}$$

\- - - - - - - - -

69. Using this form on the above problem, we have:

$$\frac{100 \text{ gpm}}{250 \text{ gpm}} = \frac{T_2}{10 \text{ min}}\ .$$

Solving for T_2, then, the second pump will fill the tank in ____________ min.

\- - - - - - - - - -

70. If one tractor plows 10 acres in 8 hrs, 5 of the same size tractors can plow the same area in __________ hrs. — 4

- - - - - - - - - -

71. An army camp has provisions for 240 men for 28 days. If only 112 men are sent to the camp, the provisions should last ______ days. — 1.6 hrs

- - - - - - - - - -

72. A restaurant can feed 120 people in 30 minutes and can feed 452 people in 2 hours. The above relationship: — 60

a. Is an inverse proportion
b. Is a direct proportion
c. Is not a proportion

Circle the letter indicating the correct answer.

- - - - - - - - - -

73. In order for the ratio of people-fed-to-time to be a proportion, either the number fed in 30 min should be changed to __________ people, or the number fed in 2 hrs should be changed to _______ people. — c.

- - - - - - - - - -

2.3. Graphs

74. Graphs are diagrams in which two or more related bits of information are presented so that the relationship between the two kinds of information may be more easily understood and trends more readily diagnosed. — 113, 480

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75. For example, let us say you wish to show the relationship between the temperature of the air and the time of day. On some scaled paper, draw two lines perpendicular to each other as shown in Fig. 2.1 below.

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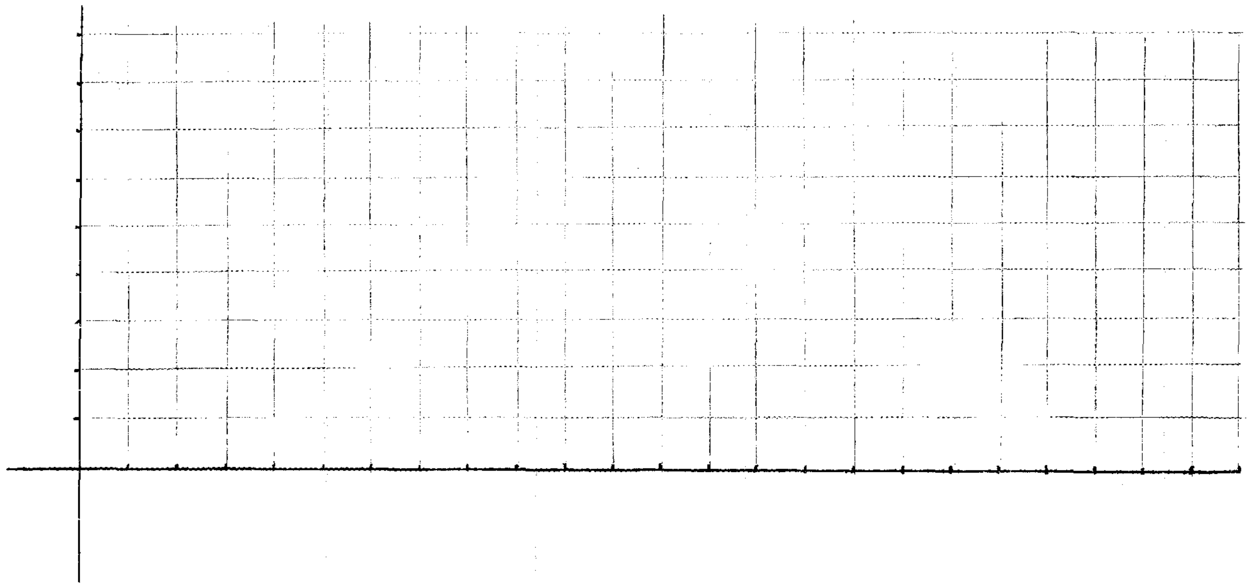

Fig. 2.1. Lines for Rectangular Coordinates

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76. Using the above form for your diagram, mark horizontal lines to represent temperatures from the coolest that you expect to have (at the bottom) to the hottest that you expect (at the top). In the summer the night temperature would seldom go below 50°F or the day temperature above 110°F, so mark the lowest line 50°F and the highest 110°F.

\- - - - - - - - -

77. Below the bottom horizontal line, mark the vertical lines to represent each hour beginning at the left with midnight; and mark the 24th line to the right midnight again. Your diagram should resemble Fig. 2.2 below.

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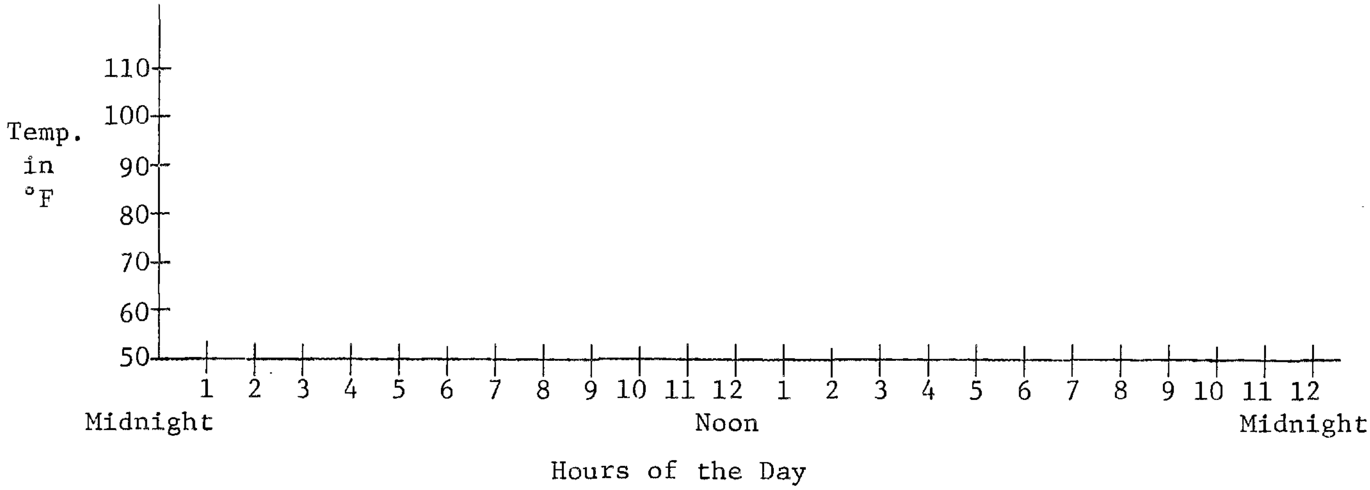

Fig. 2.2. Rectangular Coordinates for a Temperature vs Time Plot

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78. Let us assume that you have already taken temperature readings for 24 hours and they are as follows:

Time	Temp.	Time	Temp.	Time	Temp.
Midnight	70°	8:00	73°	4:00	89°
1:00	69°	9:00	76°	5:00	89°
2:00	68°	10:00	80°	6:00	87°
3:00	65°	11:00	82°	7:00	84°
4:00	61°	12:00-N	83°	8:00	80°
5:00	62°	1:00	85°	9:00	75°
6:00	64°	2:00	87°	10:00	74°
7:00	69°	3:00	88°	11:00	73°
				Midnight	72°

- - - - - - - - -

79. The numbers given (such as 1:00 and 69°) are called coordinates. The number 1:00 represents a vertical line on the chart and 69° represents a horizontal line on the chart. Where these two lines cross is a point on the diagram.

- - - - - - - - -

80. Each point on the diagram represents the intersection of two ________________.

- - - - - - - - -

81. Make a point on your diagram for each of the two ________________ given, time and temperature. Connect the points with a smoothly curved line because the temperature does not change sharply at a point (this is called a line graph).

coordinates

- - - - - - - - -

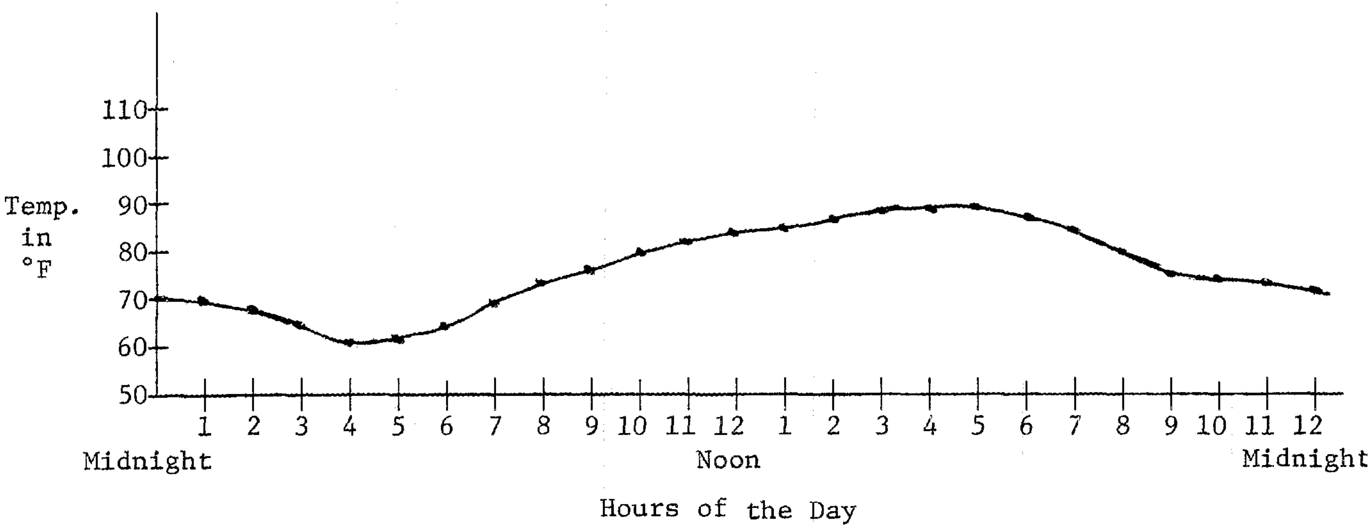

Fig. 2.3. A Graph of Temperature vs Time

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82. Now we can call the finished diagram a graph because it depicts how two kinds of information are related.

coordinates

- - - - - - - - -

83. As in our graph above, the most common graph has two scales. One type of information is plotted on a vertical scale, and the related information is on a horizontal scale.

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84. A similar graph is shown below:

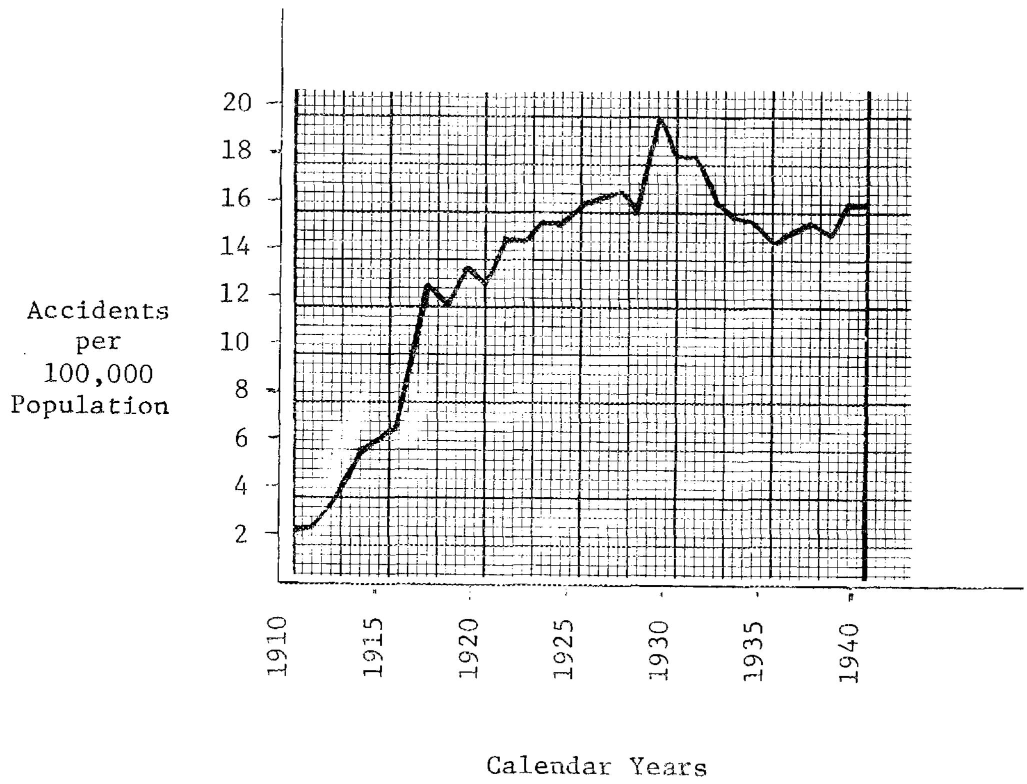


Fig. 2.4. Automobile Accidents per 100,000 Population

- - - - - - - - -

85. Points on the graph which relate information on the vertical scale to that on the horizontal scale have values called coordinates.

- - - - - - - - -

86. The preceding graph was plotted by making a dot in the appropriate place for each year's number of accidents per 100,000 population. Each dot or point would represent two values or numbers called __________________. The dots or ______________ were then connected by lines to produce the graph.

- - - - - - - - -

87. The coordinates for the year 1920 would be the number of accidents per 100,000 population and the year 1920. There were about 13 accidents per 100,000 population in 1920, so a point is placed on the graph on the line for 1920 and opposite the number ________.

coordinates, points

- - - - - - - - -

88. Information is obtained from a graph by picking one coordinate and reading the information from the scale for the other coordinate.

13

- - - - - - - - -

89. For example, which year had the highest accident record? From the graph, the greatest number of accidents on record was 20 per 100,000 population. The number 20 and the year 1929 are the coordinates for this highest point. Thus, the year that recorded the highest accident record was the year ________.

- - - - - - - - -

90. Find the following information:

1929

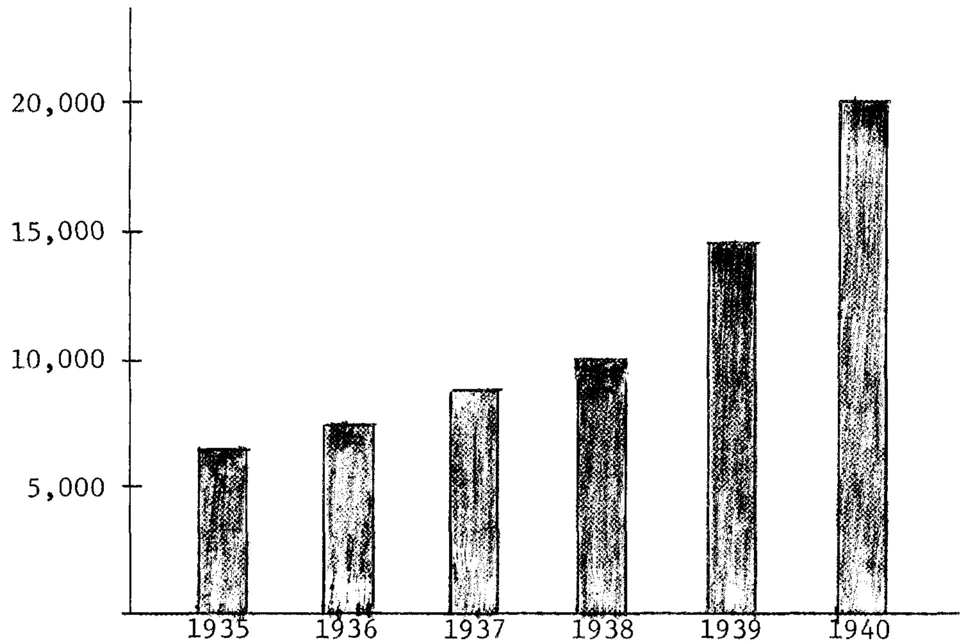
a. The number of accidents per 100,000 population in the year 1935.
b. The year in which there were almost six accidents per 100,000 population.
c. Between which years was the increase the greatest?
d. Between which years was the decrease the greatest?

- - - - - - - - -

91. Other types of graphs which are used to display certain types of information are bar graphs, as illustrated below, and circle graphs, as illustrated in Fig. 2.6.

a. 15
b. 1913
c. 1916 and 1917
d. 1931 and 1932

- - - - - - - - -



Cases Examined by a Medical Advisory Board

Fig. 2.5. Bar Graph

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92. If, in the above graph, we could have information for the years 1940 to 1945 added to this graph, we might expect that (more, fewer) cases would be examined by the medical advisory service.

- - - - - - - - -

93. In the years from 1936 to 1938, the increase in the number of cases examined was from about 7,000 to about _________--an increase of about ________ percent. more

- - - - - - - - -

94. Then in the years from 1938 to 1940, the increase in the number of cases examined was from about 10,000 to almost _______. This was an increase of ________ percent. 10,000, 43

- - - - - - - - -

95. The increase in the number of cases examined between 1939 and 1940 was almost as great as the total number of cases examined in the year ___________. 20,000, 100

- - - - - - - - - -

96. Figure 2.6 is a circle or pie graph. 1935

- - - - - - - - - -

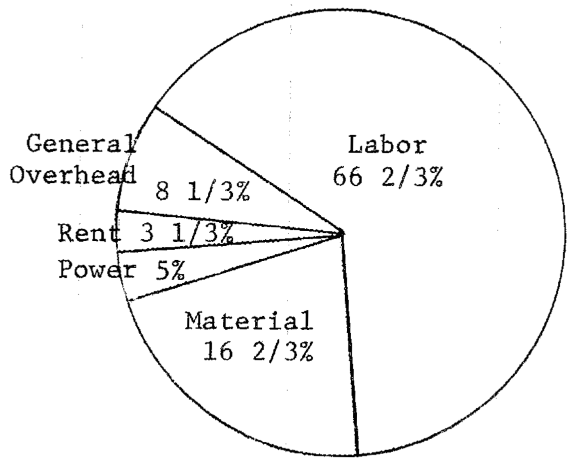

Fig. 2.6. How a Manufacturer's Expenses Are Apportioned

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97. The circle graph above is good for display purposes but cannot be used for predicting trends.

- - - - - - - - - -

98. A type of graph which is always helpful is similar to the line graph but is usually a presentation of quantities that change together according to a formula or an equation, as in Fig. 2.7.

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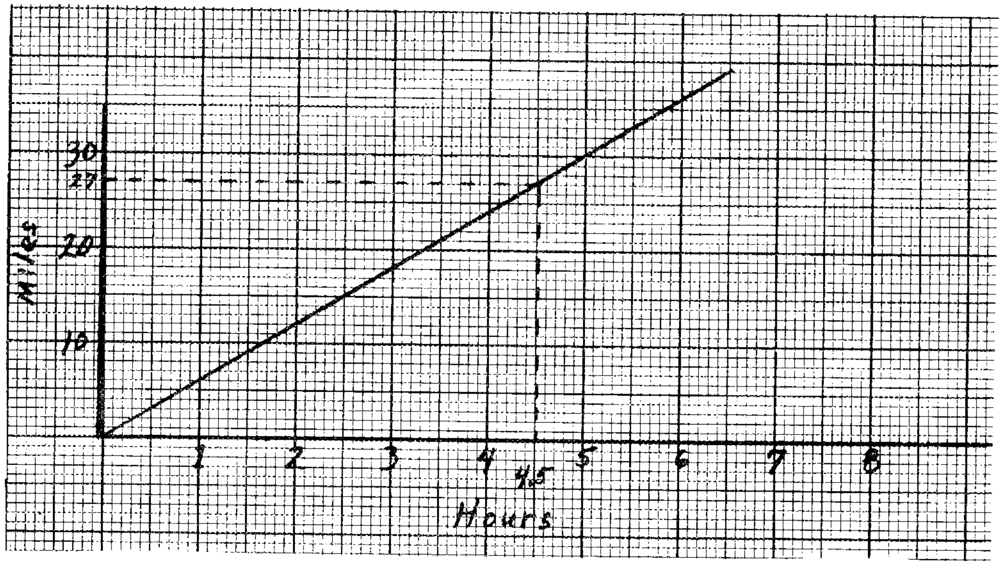

Fig. 2.7. Relations between Distance Walked and Time

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99. In Fig. 2.7., the graph represents the equation: distance walked = miles per hour × hours walked.

- - - - - - - - -

100. This graph is called a "linear" or straight-line graph because a given change in time produces a corresponding change in miles. We might say that the distance traveled in miles is directly proportional to the ____________ walked.

- - - - - - - - -

101. Since the line does not change direction, we can say that the man is walking with a constant speed. We can check that the rate is constant by noting that he is 12 miles from the start in 2 hours; and in 5 hours, he is ________ miles from the start.

time or hours

- - - - - - - - -

102. If the line on the above graph should reach the top of the graph in 4 hours, we would say that the line has a steeper slope. This steeper slope is produced by a (higher, lower) speed. 30

- - - - - - - - -

103. Using the graph below, answer the following questions: higher

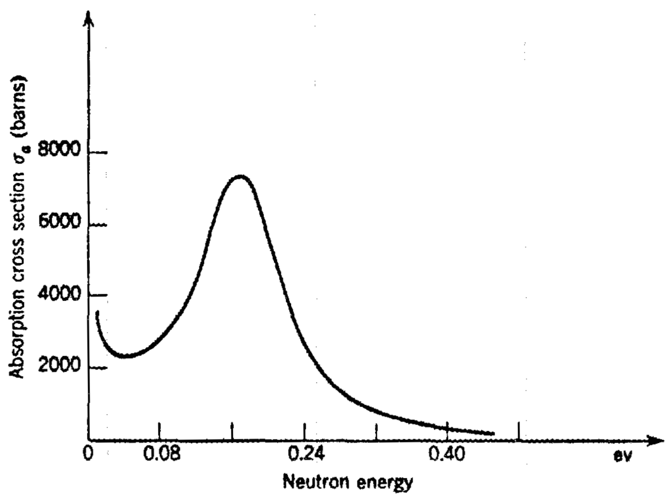

Fig. 2.8. An Absorption Cross Section Peak

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104. a. This (is, is not) a linear graph.

b. The absorption cross section for 0.24 ev neutrons is ______________.

c. A σ_a of 6,000 barns occurs for neutrons of two energies. These energies are ____ ev and ____ ev.

- - - - - - - - -

a. is not
b. 2,700 barns
c. 0.14,
 0.20

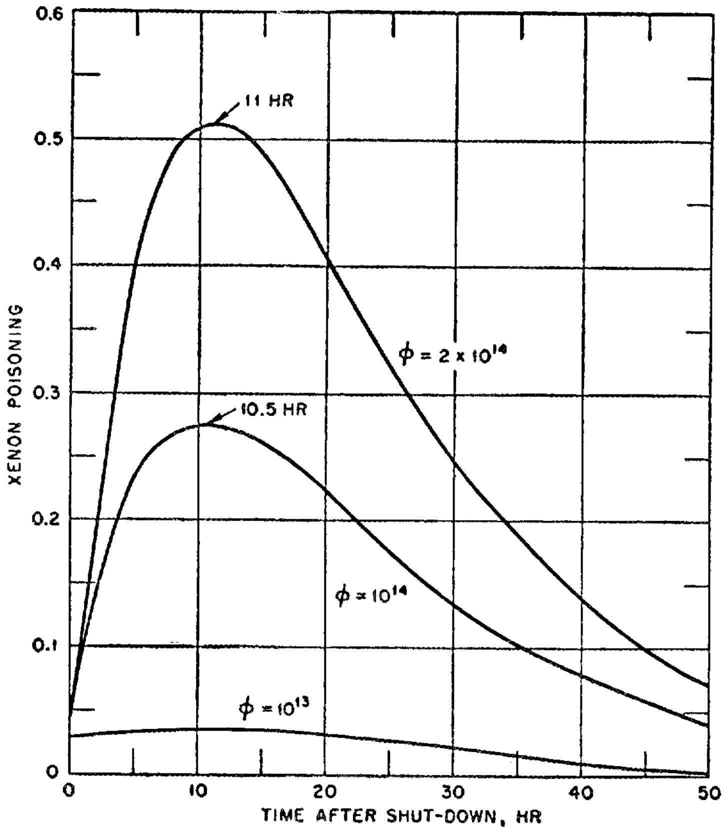

Fig. 2.9. Rate of ^{135}Xe Buildup after Shutdown

- - - - - - - - - -

105. Answer the following questions about the graph in Fig. 2.9.

a. This graph (is not, is) a linear graph.

b. If the flux (ϕ) is 10^{14}, the xenon poisoning 3 hours after shutdown is ________.

c. If the flux (ϕ) is 2×10^{14}, in 3 hours after shutdown the poisoning is ________.

- - - - - - - - - -

106. There is another type of graph with which we should become familiar. At first glance, this graph, Fig. 2.10, looks like the others we have studied. However, on this graph, note that both the vertical and horizontal scales increase by decades (factors or multiples of 10) rather than by even units.

a. is not
b. 0.19
c. 0.3

- - - - - - - - -

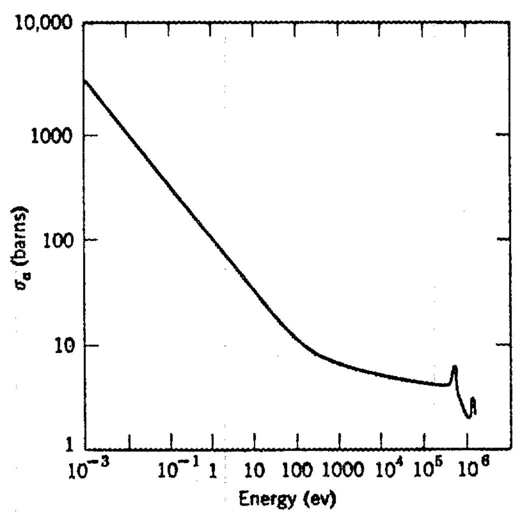

Fig. 2.10. Absorption Cross Section versus Neutron Energy

- - - - - - - - -

107. This type graph is most often used when the spread of information is too great to be placed on a graph with linear lines.

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108. Answer the following questions about the graph in Fig. 2.10:

a. As energy increases from 10^{-3} ev to 10^{5} ev the absorption cross section (increases, decreases).

b. One of these particles with an energy of 10 ev has a σ_a of about ________ barns.

c. A 1-ev particle has a much (better, poorer) chance of being absorbed than a 100-ev particle.

d. The absorption cross section decreases as the energy increases. Is this relationship linear? (Yes, No)

e. Is it linear between 10^{-3} ev and 10 ev?

f. Why did you answer e as you did?

- - - - - - - - - -

Answer to f:

If it were linear, a change from 10^{-3} ev to 10^{-2} ev (ratio of 1/10) and a change from 1 ev to 10 ev (ratio of 1/10) would give corresponding changes in σ_a. They do not. Between 10^{-3} ev and 10^{-2} ev, σ_a changes from 4,000 to 1,000 (ratio of 4/1); from 1 ev to 10 ev, σ_a changes from 100 to 40 (ratio of 2.5/1). Actually, the curve follows the form:

$$\sigma_a = \frac{1}{(ev)^2} .$$

a. decreases
b. 50
c. better
d. no
e. no

- - - - - - - - - -

2.4. Signed Numbers

109. Our discussion thus far has been concerned with positive numbers. However, there are many situations which can best be analyzed by considering negative numbers. We tentatively define negative numbers as those which represent measures of quantities opposite in direction to positive numbers.

\- - - - - - - - -

110. Temperatures below zero are called ______________ temperatures.

\- - - - - - - - -

111. If we define west as a positive direction, _______ would be considered a negative direction. negative

\- - - - - - - - -

112. When using signed numbers in arithmetic, there are a number of special rules to be remembered. The first rule is that if a number is unsigned, it is understood to be positive. east

\- - - - - - - - -

113. When adding numbers of like sign, add the numbers as usual and give the answer the common sign. When adding numbers of unlike sign, combine all positive and negative quantities, subtract the smaller from the larger, and give the result the sign of the larger.

\- - - - - - - - -

114. For example:

Add: 5 + 3 + 16 = 24

Add: 16 - 4 + 7 + 1 - 8 - 3 = ?

as below:

Add: 16 + 7 + 1 = 24

Add: -4 - 8 - 3 = $\underline{-15}$

?

- - - - - - - - -

115. Add the following:

a. 3 + 8 - 6 = ______

b. -4 - 7 - 9 + 3 = ______

c. 3 + 19 + 4 - 45 = ______

d. A man walks 5 blocks east, then turns and walks 8 blocks west. How far is he from where he started?

+9, +9

- - - - - - - - -

116. To subtract signed numbers, change the sign of the number to be subtracted and add according to the rules for addition of signed numbers. For example: Subtract 20 from 32; change 20 to -20 and add 32 -20. The answer is 12.

a. 5
b. -17
c. -19
d. -3 blocks or 3 blocks west

- - - - - - - - -

117. Subtract the following:

a. $\begin{array}{r} +47 \\ \underline{+19} \end{array}$ b. $\begin{array}{r} +54 \\ \underline{-12} \end{array}$ c. $\begin{array}{r} -26 \\ \underline{-27} \end{array}$ d. $\begin{array}{r} -80 \\ \underline{-50} \end{array}$

e. -5 minus -8 =

f. -20 minus +14 =

g. -9 minus -16 =

h. from 65°F to -6°F is ____ °F.

- - - - - - - - -

118. When multiplying signed numbers, the product is positive if the signs are alike; the product is negative if the signs are unlike. As:

$-8 \times -4 = 32$ $\quad$ $-8 \times 4 = -32$

$8 \times 4 = 32$ $\quad$ $8 \times -4 = -32$

a. 28
b. 66
c. 1
d. -30
e. 3
f. -34
g. 7
h. 71

- - - - - - - - -

119. Multiply:

-43 times +60 = ______
25 times -6 = ______
-14 times -5 = ______

-2580, -150, 70

- - - - - - - - -

120. The temperature change from -5°C to 20°C is ______ degrees. Three times as much change would raise the temperature from -15°C to ______°C.

25, 60

- - - - - - - - -

121. The mechanics of dividing signed numbers is no different from ordinary arithmetic division. However, the sign of the quotient is positive if the divisor and dividend signs are alike and negative if the divisor and dividend signs are unlike.

- - - - - - - - -

122. Divide:

$-16 \div -2 =$ ______
$-4 \div -16 =$ ______
$-32.4 \div 9 =$ ______
$-24 \times 4 \div 8 =$ ______

- - - - - - - - -

123. Often it is advisable to do a number of arithmetic manipulations in the same problem, as: $4 + 5 \times 8 - 4 \times 6 \div 12$. The accepted rule is to do the multiplication and division first and then the addition and subtraction. This would give an answer of $4 + 40 - 2 =$ _______.

8, 1/4 or 0.25, -3.6 -12

- - - - - - - - - -

124. However, one who does not know the rule might do the addition and subtraction first to give a quite different answer, as:

42

$$9 \times 4 \times \frac{1}{2} = 18 .$$

- - - - - - - - - -

125. In order to insure that the arithmetic is done in the intended order, parentheses are often used to enclose those items that need to be grouped together, such as:

$$2 \times (6 + 3) \times (8 - 6) \div (15 - 6) = \underline{\qquad}$$

- - - - - - - - - -

126. Work the following:

4

$$(3 + 8) \times (5 - 11) \div 22 = \underline{\qquad}$$

$$3 + (8 \times 5) - (11 \div 22) = \underline{\qquad}$$

$$\frac{(16 + 4) - (16 - 20)}{2} = \underline{\qquad}$$

- - - - - - - - - -

-3, 42.5, 12

SECTION I-3

ALGEBRA REVIEW

3.1. Algebraic Expressions

1. Algebraic expressions are those in which a letter or other symbol is used to represent a number.

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2. The symbol A, for area, could be used in algebraic expressions, such as $A = \pi r^2$ or A = length × width. In each of these expressions, the symbols are used in place of ______________, which will vary as measurements vary.

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3. When symbols such as letters of the alphabet are used to represent numbers, the multiplication symbol "x" can be confusing. So, when two symbols such as "a" and "b" or "x" and "y" are to be multiplied, the multiplication can be shown as: a × b; a · b; (a) (b); or ab. Usually the form is ab. numbers

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4. When two symbols in an algebraic expression are placed together, such as BC or WT, it is assumed that one is to be ______________ by the other.

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5. The symbols for addition, subtraction, and division remain unchanged because they cause no conflict. multiplied

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6. a ÷ b means b is ____________ ___ a.

a − b means b is ____________ ___ a.

a × b means a is multiplied by b. Write other forms which indicate a is multiplied by b:

____

____

____

$\frac{a}{b}$ means a is __________ ____ b.

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7. Let us use, for example, the letter "B" to represent one bushel of apples. We do not write 1B because B is only one. However, 5 bushels of apples is 5 times one bushel and is usually written 5B with the number (called the "coefficient" of the letter) always first.

added to;
subtracted from;
ab, a • b, (a)(b);
divided by

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8. Thirty-five bushels of apples would be written ______________.

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9. When adding or subtracting, we can add or subtract only symbols which are alike. Thus:

35B

2a + 3a = 5a, but

2 a + 3 cannot be added; it can only be indicated as shown.

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10. Thus:

$$2a + 4a - 5 = 6a - 5$$

and

$$3x + 2y - 5x = 2y - 2x \ .$$

Notice that in the second problem we placed the number that had the plus sign first and the number with the minus sign next. This is customary but not necessary; it could have been written:

$$-2x + 2y \ .$$

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11. You can add 6 ft and 12 ft to get 18 ft, but you cannot add 3 ft and 16 min.; you can only indicate the addition as ____________ + ______________.

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12. Addition and subtraction:

3 ft,
16 min.

$$3a + 2b + 5b - 2a = \underline{\hspace{3cm}} \ ;$$
$$x + 3y - 4x - y = \underline{\hspace{3cm}} \ ;$$
$$4r + 3s + 12s + 6r = \underline{\hspace{3cm}} \ .$$

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13. When two letters with coefficients are multiplied, as $2a \times 5a$, the coefficients and letters are multiplied separately and then are placed in the correct form. Thus:

$a + 7b$;
$2y - 3x$ or
$-3x + 2y$;
$10r + 15s$.

$$2 \times 5 = 10$$

and

$$a \times a = a^2 \ ,$$

so the complete answer is $10a^2$.

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14. Even when the letters are different, the procedure is the same. If you multiply $6y \times 3z$, $6 \times 3 = 18$, and $y \times z$ is yz. The answer then is ____________.

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15. Multiply the following: 18yz

$3a \times 4a =$ ____________ ;

$6b \times 2a =$ ____________ ;

$b \times 3a \times 6b =$ ________ ;

$ab \times 2a \times 3b =$ _______ .

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16. You will note that we are treating the letters in the above frames exactly as we treated dimensions when we were talking about dimensional numbers. The letter "A" is just as much a part of the number 5A as the dimension, feet, is part of the measurement 6 ft.

$12a^2$;
$12ab$;
$18ab^2$;
$6a^2b^2$.

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17. Division of numbers represented by letters is also performed in the same manner as division of dimensioned numbers. Thus, when you divide 12A by 4A, the A's cancel and 4 divides into 12 to give the quotient ____________.

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18. To check that the answer is correct, multiply the quotient, 3, times the divisor, 4A, to give the dividend ____________. 3

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19. Divide the following:

a. $144B \div 12 =$ ______ ;

b. $144B \div 12B =$ ______ ;

c. $144B \div 12A =$ ______ ;

d. $144 \div 12B =$ ______ .

12A

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20. Practice problems:

Use cancellation where possible.

a. $\frac{5a \times 6a \times 3b}{15a} =$ __________ ;

b. $\frac{5 \times 6a \times 3b}{15ab} =$ __________ ;

c. $\frac{15xyz}{5xz} =$ ______________ .

a. 12B;
b. 12;
c. $\frac{12B}{A}$;
d. $\frac{12}{B}$.

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21. You will recall that the sum of two numbers that are different can only be indicated, as: the sum of 2a and 3b can only be written as 2a + 3b. This number is called a "<u>bi</u>nomial"; "bi" means 2 and "nomial" means number, so we have a two-part number. A three-part number, such as 5a + 4b + c would be a <u>tri</u>nomial (tri is for three).

a. 6ab;
b. 6;
c. 3y

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22. If two such numbers are added or subtracted, they are usually enclosed in parentheses so that they are identified. Thus (a + b) + (a - b) is the sum and (a + b) - (a - b) is the difference of the two ____________ nomials.

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23. In adding or subtracting such numbers, there are two methods of procedure. The first is to place them in vertical columns, as bi

$$\begin{array}{r} a + b \\ \underline{a - b} \end{array}$$

and add or subtract as indicated. Thus, (a + b) + (a - b) would be

$$\begin{array}{r} a + b \\ a - b \\ \hline 2a + 0 \end{array}$$

or just 2a.

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24. And (a + b) - (a - b) would be

$$\begin{array}{r} a + b \\ a - b \\ \hline 0 + 2b \end{array} .$$

Recall that when you subtract signed numbers, you change the sign of the subtracted number and add.

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25. The second method is to remove the parentheses from the original form and add according to the sign. The rule to follow in this case is: if the parentheses are preceded by a +, remove the parentheses with no sign change; if the parentheses are preceded by a -, reverse the signs of all numbers in the parentheses and add the terms as usual.

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26. In the problem (a + b) + (a - b), the parentheses may be removed without changes to give a + b + a - b. When you add, you have ____________ for an answer.

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27. In the problem (a + b) - (a - b), the signs of the letters in the second binomial must be changed to give the problem the form a + b - a + b. Now, the answer is ____________.

2a

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28. Practice problems:

2b

a. (2a + 3b) - (a + 5b) = ________________ ;
b. (6x - 4y) + (x + y) = ________________ ;
c. (4r + w) - (6r - 4w) = ________________ ;
d. (a + 3b + 4c) + (2a - 3b - c) = ________ .

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29. A pack rat has a collection of 5 marbles, 2 pennies, and 6 Pepsi-Cola caps. While his collection is unguarded, another pack rat finds his collection and takes 3 marbles and 3 Pepsi-Cola caps but leaves 2 more pennies. The first pack rat now has _______ marbles, _________ pennies, and _________ Pepsi-Cola caps.

a. a - 2b;
b. 7x - 3y;
c. -2r + 5w;
d. 3a + 3c.

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30. The multiplication of a binomial by another number is usually indicated by writing the multiplier followed by the binomial in parentheses as:

2;
4;
3.

2a(5a - 3b)

or

2(3x + 4y) .

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31. Thus the problem, 5x times 3x + 4y would be written as ______________.

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32. The multiplier is multiplied times each number of the binomial in turn, and the sign given is determined by the rule for multiplying signed numbers (plus if the signs are alike; minus if they are different).

$5x(3x + 4y)$

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33. Thus, the problem in Frame 31 would be $5x(3x + 4y) = 15x^2 + 20xy$.

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34. Multiply:

a. $5(2x - 3y) =$ ________ ;
b. $5x(2 - 3y) =$ ________ ;
c. $5y(2x - 3) =$ ________ ;
d. $-5y(-2x + 3) =$ ________ ;
e. $-5x(-2x + 3y) =$ ________ ;
f. $5x^2(-2x + y) =$ ________ .

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35. Now, let us multiply two binomials such as $(a + b)$ and $(a - b)$. To do so, it is convenient to place one number above the other as in the multiplication of arithmetic numbers:

$$\begin{array}{c} a + b \\ \underline{a - b} \end{array}$$

a. $10x - 15y$;
b. $10x - 15xy$;
c. $10xy - 15y$;
d. $10xy - 15y$;
e. $10x^2 - 15xy$;
f. $-10x^3 + 5x^2y$.

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36. With the problem in the form shown in Frame 35, multiply the upper binomial by each of the lower numbers in turn, starting with the number on the left as:

$$\begin{array}{rll} & a + b & \\ & \underline{a - b} & \\ & a^2 + ab & \text{This is } a(a + b) \\ & \underline{ - ab - b^2} & \text{This is } -b(a + b) \\ \text{add to give:} & a^2 + 0 - b^2 & \text{or just } a^2 - b^2. \end{array}$$

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37. Note that the above problem could also be written in the form:

$$a(a + b) + (-b)(a + b)$$

to give

$$a^2 + ab + (-ab - b^2) .$$

Remove the parentheses and add to give:

$$a^2 + ab - ab - b^2 \text{ or } a^2 - b^2 .$$

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38. Multiply $(2a + 3b)(a - b)$. Write the correct form here ____________________. Multiply, beginning at the left $a(2a + 3b) =$ ______________. Now, the second multiplication: ________________. Add these according to sign to give _______________.

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39. Now, multiply:

a. $(a + b)(b + c) =$

b. $(2a + 3b)(a - c) =$

c. $(a + b)(a - b) =$

d. $(a + b)(a + b) =$

e. $(s + t)(r + h) =$

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$a(2a + 3b) + (-b)(2a + 3b)$

or

$$\begin{array}{r} 2a + 3b \\ \underline{a - b} \\ 2a^2 + 3ab \\ \underline{- 2ab - 3b^2} \\ 2a^2 + ab - 3b^2 \end{array}$$

40. The same rules apply for multiplying numbers of any size. Multiply

$(a + b + c)(b + c) =$

$(x + y + z)(x - y) =$

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a. $ab + b^2 + ac + bc$
b. $2a^2 + 3ab - 2ac - 3bc$
c. $a^2 - b^2$
d. $a^2 + 2ab + b^2$
e. $rs + rt + hs + ht$

41. When algebraic expressions are divided, the same rules are used as for dividing dimensioned numbers. Also, when a binomial is divided by a number, each part of the binomial is divided in turn.

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$ab + b^2 + 2bc + ac + c^2$;
$x^2 + xz - y^2 - yz$

42. For example, with the problem $(45x + 18y) \div 3$, divide 45x by 3 to give 15x and 18y by 3 to give 6y. The quotient, then, is $15x + 6y$.

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43. If the divisor and dividend have like signs, the rule for signs says that the sign of the quotient is (plus, minus). If the divisor and dividend have unlike signs, the sign of the quotient is ____________.

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44. Thus, if you divide $(15x - 25y)$ by $-5x$, the answer is ______________ 3 ______________ $\frac{5y}{x}$.

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plus, minus

45. Divide:

a. $5x$ by x = ______________ ;

b. $125y$ by 25 = ______________ ;

c. $(3a - 18b)$ by 3 = ________ ;

d. $(3a - 18b)$ by $6a$ = ________ ;

e. $(3a - 18b)$ by $-6b$ = ________ .

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minus, plus

46. Check your answers to the above problems by multiplying the quotient times the divisor.

a. $5 \cdot$ ______ = ______

b. $5y \cdot$ ______ = ______

c. $(a - 6b)$(______) = ______

d. $(0.5 - 3b/a)$(______) = ______

e. $(-0.5a/b + 3)$(______) = ______

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a. 5;
b. $5y$;
c. $a - 6b$;
d. $0.5 - \frac{3b}{a}$;
e. $\frac{-0.5a}{b} + 3$.

47. Division of a binomial, trinomial, or larger number by a binomial is very similar to arithmetic long division. For example, to divide $(2x^2 + 5xy + 3y^2)$ by $(x + y)$ use the form:

$$x + y \overline{\smash{)}\, 2x^2 + 5xy + 3y^2}$$

What number can you multiply x by to get $2x^2$?
$2x$, of course.
Place $2x$ in the quotient, multiply $2x(x + y)$ and place the product as below:

(continued)

a. x, $5x$
b. 25, $125y$
c. 3, $(3a - 18b)$
d. $6a$, $(3a - 18b)$
e. $-6b$, $(3a - 18b)$

47. (continued)

Subtract and bring down the next number

$$
\begin{array}{r@{\,}l}
 & 2x \\ \cline{2-2}
x + y \big) & 2x^2 + 5xy + 3y^2 \\
 & \underline{2x^2 + 2xy} \\
 & \phantom{2x^2 + {}} 3xy + 3y^2
\end{array}
$$

Repeat the process with +3y in the quotient.

$$
\begin{array}{r@{\,}l}
 & 2x \quad + 3y \\ \cline{2-2}
x + y \big) & 2x^2 + 5xy + 3y^2 \\
 & \underline{2x^2 + 2xy} \\
 & \phantom{2x^2 + {}} 3xy + 3y^2 \\
 & \phantom{2x^2 + {}} \underline{3xy + 3y^2}
\end{array}
$$

The quotient is 2x + 3y with no remainder.

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48. If there should happen to be a remainder, it is handled in the same manner as the remainder in long division of ordinary numbers.

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49. For example, if you divide 100 by 3, you have:

$$
\begin{array}{r@{\,}r}
 & 33 \\ \cline{2-2}
3 \big) & 100 \\
 & \underline{9} \\
 & 10 \\
 & \underline{9} \\
 & 1 \text{ Remainder}
\end{array}
$$

Place the remainder over the divisor as a fraction, in this case 1/3, and add it to the quotient to make the total answer 33 1/3.

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50. If we had the algebraic problem, $2x^2 + 6xy + 5y^2$ divided by $x + 2y$, it would be worked as follows:

$$
\begin{array}{r l}
 & \underline{\;2x \quad + 2y\qquad\qquad\;} \\
x + 2y \,\big) & 2x^2 + 6xy + 5y^2 \\
 & \underline{2x^2 + 4xy} \\
 & \qquad\quad 2xy + 5y^2 \\
 & \qquad\quad \underline{2xy + 4y^2} \\
 & \qquad\qquad\qquad y^2 \quad \text{Remainder}
\end{array}
$$

Place the remainder, y^2, over the divisor $x + 2y$ to make the fraction

$$\frac{y^2}{x + 2y} .$$

Add this fraction to the $2x + 2y$ to make the correct quotient

$$2x + 2y + \frac{y^2}{x + 2y} .$$

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51. Work the following:

$$(a^2 - b^2) \div (a + b) =$$
$$(x^2 + 2xy + y^2) \div (x + y) =$$
$$(3c^2 + cd - 4d^2) \div (c - d) =$$

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52. Review and practice problems:

$a - b$;
$x + y$;
$3c + 4d$.

a. $5a(2a + 3b - 4a) =$ ______________ ;

b. $\dfrac{3(5x + 15)}{5} =$ ______________ ;

c. $2ab(3a - 4b) - 5b(a^2 - ab) =$ ________ ;

d. $\dfrac{3xy(x + 5y)}{y} + \dfrac{2x^2(3x - 12y)}{2x} =$ ________ ;

e. $7(x + y) - (-5x + 6y) =$ ______________ .

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3.2. Algebraic Equations

53. In our discussion of proportions, we identified the word "equation" with the form used for setting two proportions equal to each other. Let us enlarge that concept by saying that any two mathematical equalities may be separated by an equal sign and be called an equation.

a. $15ab - 10a^2$
b. $3x + 9$
c. $a^2b - 3ab^2$
d. $6x^2 + 3xy$
e. $12x + y$

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54. Many algebraic equations are statements of fact written in the mathematical shorthand of algebra. For example, the statement, the area of a rectangle is the product of its length times its width, is written $A = Lw$.

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55. Write the following statements as equations:

a. The perimeter (p) of a rectangle equals twice its length (L) added to twice its width (w).
b. The distance (d) traveled by an object that moves at a constant speed (v) for a given time (t) equals the speed times the time.
c. The distance (d) in feet that any object will fall in a given time (t) is equal to the square of the time multiplied by 16.
d. To get the input horsepower (HP in) to an electric motor, multiply the number of volts (V) by the number of amperes (I) and divide by 746.
e. The output of an electric motor (HP out) is the input horsepower multiplied by the percent of efficiency of the motor (% Eff).

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56. Now that we can take problems and write them in equation form, let us review the rules for solving equations.

The equality (we call it an equation) remains an equality when:

a. The same number is added to both sides.

b. The same number is subtracted from both sides.

c. Both sides are multiplied by the same number.

d. Both sides are divided by the same number.

a. $p = 2L + 2w$ or $p = 2(L + w)$
b. $d = vt$
c. $d = 16t^2$
d. $HP\ in = \frac{VI}{746}$
e. $HP_{out} = HP_{in} \times \%Eff$

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57. When solving equations, we first try to get the equation into a form such that the unknown (the measurement for which we substitute a letter) is on one side of the equal sign and known values (numbers) are on the other side. For this manipulation, we use the above rules.

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58. For example, if:

$$5x + 4 - 3x = 16 \ ,$$

simplify to

$$2x + 4 = 16 \ ;$$

subtract 4 from both sides:

$$2x + 4 - 4 = 16 - 4$$
$$2x = 12 \ ;$$

divide both sides by 2:

$$\frac{2x}{2} = \frac{12}{2}$$
$$x = 6 \ .$$

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59. Thus, if:

a. $p + 3 = 8$, $p =$ ________

b. $2n = 25$, $n =$ __________

c. $1/2x = 14$, $x =$ ________

d. $5c - 3 = 27$, $c =$ ______

e. $18 = 5y - 2$, $y =$ ______

f. $\frac{2N}{3} = 24$, $N =$ __________

g. $\frac{a}{2} + \frac{a}{4} = 36$, $a =$ ______

h. $W = \frac{b}{c}$, $b =$ ___________

i. $V = \frac{W}{A}$, $A =$ ___________

j. $H = \frac{P}{AW}$, $W =$ __________

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60. Problems:

a. 5 g. 48
b. 12.5 h. cW
c. 28 i. $\frac{W}{V}$
d. 6
e. 4 j. $\frac{P}{HA}$
f. 36

a. If watts = volts × amperes, how many amperes of current will flow through a 60-watt lamp used in a 120-volt circuit? Let W = watts, V = volts, and a = amperes in your equation.

b. If centigrade temperature is equal to 5/9 times (°F - 32), the equation is ____________. Let °C = degrees centigrade; °F = degrees fahrenheit.

.5a

c. The centigrade equivalent of a temperature of 77°F is __________ °C.

°C = 5/9(°F - 32)

d. If the distance an object falls, neglecting air friction, is equal to 16 times the square of the time in the air, the equation is d = _______, where d = distance and t = time.

25°C

e. An object that falls for 10 sec will fall _____________ ft.

$16t^2$

f. A car has a speed of 30 miles/hr. In 2 sec it will move a distance of _________ ft. A mile is 5,280 ft.

1600

g. A tank is 6 ft in diameter (diameter = 2 × radius, r) and 20 ft long. If the volume is found by the equation $V = \pi r^2 L$ ($\pi = 3.14$), how many cubic feet of water will this tank hold?

88

60. (continued)

h. If a cubic foot of water weighs 62.4 lbs, the weight of water in the above tank would be ________________. — 565

i. If a gallon of water weighs 8.25 lbs, the above tank would hold __________ gallons of water. — 35,256 lbs

j. If fuel oil weighs only 7.5 lbs per gallon, the above tank would hold (more, fewer, the same number of) gallons of fuel oil (than/as) water) — 4273 gal

k. I answered the above question by (guessing, reasoning, arithmetic calculation) and I was (right, wrong). — the same number of, as

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3.3. Powers and Powers-of-Ten

61. The act of multiplying a number by itself is usually called squaring the number. It may also be called raising the number to the second power. — This answer depends on the individual.

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62. For example, 4^2 is read "4 squared" and means 4×4. The number 4^2 may also be read "4 to the second power".

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63. If you multiply 3 by itself twice to get $3 \times 3 \times 3$, you could write it 3^3 and read it either "3 cubed" or "3 raised to the third ____________".

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64. The small 3 to the upper right of the large 3 is called the exponent. — power

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65. The exponent determines the __________ to which a number is raised.

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66. If you write two to the fourth power as a number with an exponent, it is written __________. If you write it as a multiplication problem, it is written ______________. — power

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67. To raise a number to a given power, multiply the number by itself the number of times indicated by the ________. If the number is written B^5, the multiplication is $B \times B \times B \times B \times B$, or B to the __________ power. — 2^4, $2 \times 2 \times 2 \times 2$

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68. Practice problems: — exponent, fifth

a. $N \times N \times N =$ ______;
b. $6^3 =$ __________;
c. $(1/2)^2 =$ ________;
d. $4^5 =$ __________;
e. $(3/5)^3 =$ ________;
f. $16^2 =$ _________.

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69. When a number has a negative exponent, as 4^{-3}, it may be written $\frac{1}{4^3}$, the reciprocal of 4^3. (The reciprocal of a number is the fraction formed when you put one over the number.)

a. N^3
b. 216
c. 1/4 or 0.25
d. 1024
e. 0.216
f. 256

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70. Thus:

a. $2^{-3} = \frac{1}{2^3} =$ ______ ;

b. $4^{-2} = \frac{1}{4^2} =$ ______ ;

c. $3^{-4} = \frac{1}{3^4} =$ ______ .

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71. You will note that we have not mentioned numbers which have exponents of zero or one. A number with the exponent, 1, is just the number; so the exponent is not written.

a. 1/8;
b. 1/16;
c. 1/81.

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72. Before we discuss numbers with the exponent zero, let us learn the laws of exponents.

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73. The first law may be stated thus:

When a number of one power is multiplied by the same number of a different power, the number stays the same and the exponents are added.

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74. If 2^2 is multiplied by 2^3, the answer is 2^5. To check that this is true, multiply 2^2, which is __________, by 2^3, which is _________, to give 32, which is 2^5.

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75. In the above frame, the number 2, which had the exponents 2 and 3, was called the "base". The use of this word allows us to state the first law:

4, 8

To <u>multiply</u> powers of the same base, add their exponents.

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76. If 4^3 is multiplied by 4^{-1}, the exponents add to make the number ________. If we do the final multiplication, the answer is ________.

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77. In the above problem, the number 4 is called the _________. The numbers 3 and -1 are called the _________________.

4^2, 16

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78. Problems:

base, exponents

a. $2^3 \times 2^1 =$ ____________ ;

b. $N^3 \times N^4 \times N^{-4} =$ ______ ;

c. $A^2 \times A^{-1} \times A^4 =$ ______ ;

d. $3^3 \times 3^1 \times 3^{-2} =$ ______ .

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79. To <u>divide</u> numbers of the same base but with different exponents, subtract the exponent of the denominator from the exponent of the numerator.

a. 16;
b. N^3;
c. A^5;
d. 9.

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80. For example, if you divide 3^8 by 3^5, you have 3^{8-5} or 3^3. Note that $\frac{3^8}{3^5}$ could be written $3^8 \times 3^{-5} = 3^3$.

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81. A number with the exponent zero is always equal to one. This is easy to see if you multiply $4^2 \times 4^0$. $4^2 \times 4^0 = 4^2$ according to the law of multiplication; and since any number multiplied by 1 is the number, 4^0 must be equal to ________.

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82. Now, let us consider the number 10 as a base.

one

$$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1 \qquad 10^1 = 10$$

$$10^{-2} = 0.01 \qquad 10^2 = 100$$

$$10^{-3} = 0.001 \qquad 10^3 = 1000$$

$$10^{-4} = 0.0001 \qquad 10^4 = 10{,}000$$

$$10^{-6} = 0.000001 \qquad 10^6 = 1{,}000{,}000$$

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83. From the above frame we could conclude that 10 with a negative exponent is the number 1 placed as many digits to the right of the decimal point as the quantity of the exponent. Thus, 10^{-5} could be written as a one placed ________ digits to the right of a decimal point. 10^{-5} is ________.

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84. The number 10 with a positive exponent may be written as the number 1 followed by as many zeros as the quantity of the exponent; 10^5 is 1 followed by _________ zeros, or __________.

5, 0.00001

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85. Thus, very large or very small numbers may be written in shorter form as a small number multiplied by a power of 10.

5, 100,000

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86. For example: 32,000 may be written as 32×10^3 or 3.2×10^4; $10^4 = 10,000$; and $3.2 \times 10,000 = 32,000$.

$$6,900,000 = 6.9 \times 10^6$$
$$0.0000069 = 6.9 \times 10^{-6}$$
$$200 = 2 \times 10^2$$
$$0.002 = 2 \times 10^{-3}$$

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87. Normally, numbers changed to the power of 10 notation are written as a number between 1 and 10 times a power of 10. As $231,000 = 2.31 \times 10^5$.

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88. Write the following as powers of 10:

a. 23,000,000 = _____________;
b. 12,500 = ________________;
c. 320 = ___________________;
d. 0.0056 = ________________;
e. 0.000000089 = ____________.

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89. The reason for writing large numbers as powers of 10 is to make computation easier. Since the base is always 10, the rules for multiplication and division, by adding and subtracting exponents, hold true for the powers of 10. The numbers which are raised to powers of 10 are multiplied or divided as usual.

a. 2.3×10^7;
b. 1.25×10^4;
c. 3.2×10^2;
d. 5.6×10^{-3};
e. 8.9×10^{-8}.

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90. For example, let us multiply 32,000,000 by 140,000. Rewrite as powers of 10; $3.2 \times 10^7 \times 1.4 \times 10^5$.

$$3.2 \times 1.4 = 4.48$$

and

$$10^7 \times 10^5 = 10^{12} ,$$

so the answer is 4.48×10^{12}.

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91. If the numbers had been larger, such as 4.6×10^7 and 6×10^5, the product of 4.6×6 would be 27.6 and $10^7 \times 10^5 = 10^{12}$. So, a correct answer is 27.6×10^{12}. However, in order to hold to our general rule of writing numbers between 1 and 10 times a power of 10, we want to write 2.76 times some power of 10.

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92. To do this correctly, observe the rule:

> If you move a decimal point one digit to the left, change (multiply) the power of ten by 10^1. If you move the decimal point a digit to the right, change (multiply) the power of ten by 10^{-1}. Thus, 6.4×10^5 could be written $0.64 \times$ _______; or, it could be written $64 \times$ ____________.

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93. To divide: 10^6, 10^4

$$3.2 \times 10^7 \div 1.6 \times 10^5$$

$$\frac{3.2 \times 10^7}{1.6 \times 10^5} = 2 \times 10^{7-5} \text{ or } 2 \times 10^2 .$$

With as small a number as 2×10^2, you probably would rather write it __________________.

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94. Work the following, using power-of-10 notation: 200

a. $3600 \times 5500 =$ ____________________ ;

b. $6 \times 10^5 \times 4.3 \times 10^8 =$ ____________ ;

c. $6 \times 10^{-5} \times 4.3 \times 10^8 =$ ___________ ;

d. $6.3 \times 10^5 \div 4.2 \times 10^8 =$ __________ ;

e. $6.3 \times 10^5 \div 4.2 \times 10^{-8} =$ _________ ;

f. $\frac{7 \times 10^4 \times 6 \times 10^2}{3 \times 10^6} =$ ______________ ;

g. $5 \times 10^6 \times 7 \times 10^3 \div 10^5 =$ ________ .

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3.4. Roots

95. Most people have an understanding of the meaning of the terms square root and cube root. These two roots are probably the most often used by the average person. However, they are only two of thousands of roots.

a. 1.98×10^7 ;
b. 2.58×10^{14} ;
c. 2.58×10^4 ;
d. 1.5×10^{-3} ;
e. 1.5×10^{13} ;
f. 14 ;
g. 3.5×10^5 .

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96. The square root of a number such as 9 is the number 3, which when multiplied by itself gives the original number 9. Three is a root of 9 and is the square root because it is multiplied by itself only once.

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97. The number 8 is the square root of 64 because $8 \times$ ______ $= 64$.

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98. If the number 3 is multiplied by itself twice, $3 \times 3 \times 3$, the product is 27. The cube root of 27 is the number which, when multiplied by itself twice, is 27. Thus, the cube root of 27 is ______.

8

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99. The symbol which represents square root is $\sqrt{}$; higher roots such as 3rd, 4th, and 5th roots are indicated by placing the number 3, 4, or 5 as shown: $\sqrt[3]{}$, $\sqrt[4]{}$, $\sqrt[5]{}$.

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100. Indicate the following roots by placing the number under the correct symbol:

a. Square root of 64 ;
b. Cube root of 27 ;
c. Fourth root of 81 ;
d. Square root of 225 ;
e. Cube root of 1000 .

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101. Roots may be also represented as fractional exponents. Thus: $\sqrt{64}$ could be written $64^{1/2}$ or $\sqrt[3]{81}$ as $81^{1/3}$.

a. $\sqrt{64}$
b. $\sqrt[3]{27}$
c. $\sqrt[4]{81}$
d. $\sqrt{225}$
e. $\sqrt[3]{1000}$

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102. Rewrite the following using the common root sign:

a. $N^{\frac{1}{3}} =$ ______________ ;
b. $625^{\frac{1}{2}} =$ ______________ ;
c. $1728^{\frac{1}{3}} =$ ______________ ;
d. $A^{\frac{1}{3}}B^{\frac{1}{3}} =$ ______________ ;
e. $32^{\frac{1}{5}} =$ ______________ .

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103. Roots of smaller numbers may be found easily by trial and error methods. For example, $\sqrt[3]{8}$ may be quickly shown to be 2 and $\sqrt{25}$ is 5.

a. $\sqrt[3]{N}$;
b. $\sqrt{625}$;
c. $\sqrt[3]{1728}$;
d. $\sqrt[3]{AB}$;
e. $\sqrt[5]{32}$.

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104. Find the indicated roots of the following:

a. $\sqrt{64}$ = ______ ;
b. $\sqrt{81}$ = ______ ;
c. $\sqrt[4]{81}$ = ______ ;
d. $\sqrt{49}$ = ______ ;
e. $\sqrt[3]{1000}$ = ______ ;
f. $\sqrt[3]{64}$ = ______ .

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105. Note that in the last problem $\sqrt[3]{1000} = 10$. If we should write 1000 as 10^3, we could take the cube root of 10^3 by dividing the exponent by 3 to give 10. Also, $\sqrt{100}$ could be found by dividing the exponent of 10 by 2 to give 10.

a. 8 ;
b. 9 ;
c. 3 ;
d. 7 ;
e. 10 ;
f. 4 .

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106. In like manner, find the square root of:

a. 10,000 = ______ ;
b. 1,000,000 = ______ ;
c. 0.0001 = ______ .

Find the cube root of:

d. 0.001 = ______ ;
e. 1,000,000 = ______ ;
f. 1,000 = ______ .

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107. This affords an easy method of taking the square root of quite large numbers. For example, take the square root of 360,000; 360,000 may be written as 36×10^4. The square root of 36 is 6; the square root of 10^4 is 10^2, so we have 6×10^2 or 600.

a. 100 ;
b. 1000 ;
c. 0.01 ;
d. 0.1 ;
e. 100 ;
f. 10 .

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108. Please note that when extracting square roots, the exponent of 10 must be divisible by 2; and, when extracting cube roots, the exponent of 10 must be divisible by 3.

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109. Thus, in the above problem, you should not write 360,000 as 3.6×10^5 for purposes of taking either square roots or cube roots because you do not already know the square or cube root of 3.6, and the exponent 5 is not evenly divisible by either 2 or 3.

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110. The rule could be written:

> To take the square root of a large number, rewrite the number as a power of 10 with an exponent of 10 that is divisible by two.

The rule for extracting any other root would be similar.

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111. Take the cube root of 27,000,000. In order to have an exponent of 10 divisible by 3, we must write the number 27×10^6.

a. The cube root of 27 is ________________ ;

b. The cube root of 10^6 is ________________ ;

c. The cube root of 27,000,000 is __________ .

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112. By the same method, take the indicated root of:

a. $\sqrt{40,000}$ = ________________ ;
b. $\sqrt{8,100}$ = ________________ ;
c. $\sqrt[3]{64,000}$ = ________________ ;
d. $\sqrt{400}$ = ________________ ;
e. $\sqrt[3]{0.008}$ = ________________ ;
f. $\sqrt{0.09}$ = ________________ ;
g. $\sqrt{0.0036}$ = ________________ ;
h. $\sqrt[3]{125,000}$ = ________________ .

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a. 3 ;
b. 10^2 ;
c. 3×10^2
or 300 .

a. 200 ;
b. 90 ;
c. 40 ;
d. 20 ;
e. 0.2 ;
f. 0.3 ;
g. 0.06 ;
h. 50 .

SECTION I-4

LOGARITHMS

4.1. Introduction

1. Logarithms represent a method for simplifying the multiplication and division of very large numbers, those containing many digits or decimal places.

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2. In order to better understand logarithms, let us look at "powers-of-ten" once more. One hundred, in power-of-ten notation is 10^2; 1000 is 10^3; 0.001 is 10^{-3}. In these cases, 10 is called the base. Any number can be written as 10 with an exponent. This exponent is called the "logarithm" of the number.

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3. The logarithm, to use the base 10, of 1000 is the ____________ of ten when 1000 is written 10^3. Therefore, the logarithm of 1000, to the base 10, is 3.

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4. Logarithms which are to the base 10 are most often used in elementary mathematics and are called common logarithms. exponent

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5. The common logarithm of:

1 = 0	0.1 = -1
10 = 1	0.01 = -2
100 = 2	0.001 = -3
1000 = 3	0.0001 = -4
10,000 = 4	0.00001 = -5
100,000 = 5	0.000001 = -6

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6. There are two systems of logarithms in use today--common logarithms, which are to the base 10; and natural logarithms, which are to the base e; e, numerically, is 2.718.

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7. Common logarithms are to the base _____ and natural logarithms are to the base ____. Since it is often confusing to discuss them both at the same time, we shall limit our discussion to common logarithms. However, the rules for using them are the same, no matter what the base.

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8. The common logarithm of a number is the exponent of 10--the power to which 10 must be raised in order to be that number. For example, the logarithm of 100 is 2. Ten with the exponent 2 or, in other words, 10 raised to the second power, is the number ______.

10, e

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9. The number 10^3 is 1000, so the logarithm of 1000 is _____. The number 10^1 is 10, so the logarithm of 10 is ______.

100

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10. If you multiply $10^3 \times 10^1$, you add exponents. But if we use the term logarithms instead of exponents, you add the logarithms 3 and 1. The sum of these logarithms is 4. The number that has the logarithm 4 is ten with the exponent _____, so the number is _________. 3, 1

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11. Since the logarithm of 10 is 1 and the logarithm of 100 is 2, the logarithm of a number between 10 and 100 would be between 1 and _____. 4, 10,000

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12. The logarithm of 1 is 0.0000, since 10^0 is one. The logarithm of 10 is 1.0000, since 10^1 is ten. The logarithm of 5 is between the logarithms of 1 and of 10; so it is zero point something. 2

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13. Mathematicians have calculated the logarithms of a host of numbers and placed them in tables such as Table 4 so that we may use these tables to look up the logarithm for any number.

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14. Let us make a short table. ("Log 1" means "logarithm of 1".)

Log 1 = 0.0000
Log 5 = 0.6990
Log 10 = 1.0000
Log 50 = 1.6990
Log 100 = 2.0000
Log 500 = 2.6990
Log 1000 = 3.0000
Log 5000 = ______

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Table 4. Common Logarithms of Numbers

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

Table 4. (Continued)

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9898	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

NOTE: For numbers less than 10, look for 20, 30, 40, etc., in table.

15. We let you put in the last number because we were sure you could see that the part of the logarithm to the right of the decimal point was the same regardless of the power of 10 by which 5 was multiplied (50, 500, 5000). The same would be true for any other number multiplied by any power of 10. 3.6990

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16. If the logarithm of 20 is 1.3010, the logarithm of 200 is 2.________.

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17. The number to the left of the decimal point in the logarithm changes with the power of 10. Since 50 is between 10 (or 10^1) and 100 (or 10^2), and the exponent is the logarithm, we say $50 = 10^{1.6990}$. The one (1) to the left of the decimal point means that the number is between 10 and 100. For the number 500, since it is between 100 and 1000, the number to the left of the decimal point of the logarithm would be ______ and the number to the right of the decimal point would be ______; thus, the logarithm of 500 is ______. 3010

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18. If the logarithm of 30 is 1.4771, the logarithm of 3000 is ______. The logarithm begins with 3 because 3000 is greater than 1000 and less than 10,000. 2, 0.6990, 2.6990

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19. The number 2000 could be written $10^{3.3010}$. The logarithm of 2000 is __________. 3.4771

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<u>4.2. Characteristic and Mantissa</u>	3.3010
20. From Frames 15 to 17 we know that a logarithm has two parts. One part determines the position of the decimal point and is the whole number part of the logarithm. This is called the <u>characteristic</u>.	
- - - - - - - - -	
21. The characteristic of the logarithm of 500 is 2 because 500 is between 10^2, which is 100, and 10^3, which is 1000. By the same rule, the characteristic of the logarithm of 50 is ______ and of 5000 is _____.	
- - - - - - - - -	
22. The part of the logarithm that determines the place of the decimal point of the number is called the ________________.	1, 3
- - - - - - - - -	
23. Write the characteristics of the logarithms of the following numbers: 19, 56, 6, 145, 6394.	characteristic
- - - - - - - - -	
24. As you may have already guessed, the characteristics of numbers greater than one are positive whole numbers.	1, 1, 0, 2, 3
- - - - - - - - -	
25. The characteristics of numbers less than one are negative whole numbers. For example, 0.01 is 1/100 or $1/10^2$, which can be written 10^{-2}. Thus, the characteristic of the logarithm of the number 0.01 is __________.	
- - - - - - - - -	

26. You should recall that 0.01 can also be written 1×10^{-2}. When in doubt about the characteristic of a number, write the number as a power of 10 (a number between 1 and 10 times a power of 10). In this form, the characteristic of the logarithm of the number is the exponent of the base, 10. Thus, the characteristic of the logarithm of 1×10^{-2} is ______, as noted above. -2

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27. Write the characteristics for the logarithms of the following numbers: 0.5, 5, 50, 0.05, 0.005, 5000. -2

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28. A number such as 100 or 1000 is an even power of 10, so the characteristic is a whole number. Thus, the logarithm of 1000 is 3.0000. However, between 1000 and 10,000 is a host of numbers whose logarithms are greater than 3, but less than 4. -1, 0, 1, -2, -3, 3

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29. For the in-between numbers, the logarithm has two parts--the characteristic, which is the whole number, and the <u>mantissa</u>, the fraction. This last part of the logarithm, the mantissa, is found for any number by looking in a table of <u>common</u> logarithms and is the same regardless of the position of the decimal point in the number.

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30. The part of the logarithm of a number that determines the position of the decimal point is the __________.

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31. The part of the logarithm of a number that is not related to the position of the decimal point is the ______________. — characteristic

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32. When we call Table 4 a table of logarithms, we are not exactly right. It is a table of mantissas because, as we learned earlier, the mantissa is the same for numbers which have the same series of digits. — mantissa

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33. Thus, if 32.4 has the logarithm 1.5105, 324000 has the logarithm 5.5105. The part of the logarithm that is the same is the ____________.

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34. Thus, we can use a table for the mantissas of our logarithms and determine the __________ of each logarithm from the position of the decimal point. — mantissa

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35. Now let us find the mantissa of a number such as 264. In Table 4, look for the first two digits of 264--two, six--in the column labeled N. Then, move across the page to the column under 4 and read the number 4216. This is the mantissa of the logarithm of 264. The characteristic of the logarithm of 264 is ______. — characteristic

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36. When we add the mantissa 4216 as the fraction 0.4216 to the characteristic 2, we have the logarithm of 264 which is 2.4216. This is usually written log 264 = _______. — 2

- - - - - - - - - -

4.3. Logarithms and Antilogarithms

2.4216

37. To find the logarithm of any three-digit number, look up the mantissa, as noted above, and add it to the proper characteristic as determined by the size of the number.

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38. Note that if we change the decimal point of the number 264, used in the example above, the only part of the logarithm that changes is the characteristic.

Log 264 = 2.4216
Log 2640 = 3.4216
Log 2.64 = 0.4216
Log 0.264 = -1.4216 or 9.4216-10

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39. Now find the following:

a. Log 156 =	e. Log 0.056 =
b. Log 25 =	f. Log 843000 =
c. Log 327 =	g. Log 84.3 =
d. Log 7.3 =	h. Log 0.000843 =

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40. We mentioned earlier that fractions have characteristics that are negative numbers. Since this sign <u>applies only to the characteristics</u>, it is usually placed above the characteristic to remind the operator that it does not apply to the mantissa. Also, for reasons that will be more obvious later, a characteristic -2 is often written 8.0000-10 or $\bar{4}.9258$ is written 6.9258-10.

a. 2.1931
b. 1.3979
c. 2.5145
d. 0.8633
e. $\bar{2}.7482$ or 8.7482-10
f. 5.9258
g. 1.9258
h. $\bar{4}.9258$ or 6.9258-10

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41. For more practice, write the following:

a. Log 144 =	d. Log 2340 =
b. Log 1.44 =	e. Log 763 =
c. Log 0.0144 =	f. Log 67.8 =

- - - - - - - - -

42. Now let us reverse the process. Take a logarithm and find the number (called the "antilogarithm") to which it belongs.

a. 2.1584
b. 0.1584
c. $\bar{2}.1584$ or 8.1584-10
d. 3.3692
e. 2.8825
f. 1.8312

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43. To find the antilogarithm of the logarithm 1.3324, look for the mantissa 3324. It is for the number two one five. The characteristic 1 means that the number is between 10 and 100; so, the antilogarithm of 1.3324 is 21.5. The number whose logarithm is 1.3324 is ____________.

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44. Find the antilogarithms of the following logarithms:

21.5

a. Antilog 2.4232 = ______________ ;
b. Antilog 1.6794 = ______________ ;
c. Antilog 3.8176 = ______________ ;
d. Antilog 0.9741 = ______________ ;
e. Antilog 8.4378-10 = ____________ .

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45. For practice purposes, we have picked numbers which are found in our table. However, with real problems we are seldom so fortunate. For example, let us say we need the logarithm of the number 2357. Our table gives 2350 and 2360, so we must calculate the mantissa that is 7/10 of the difference between the one for 2350 and that for 2360. This is called "interpolating".

a. 265;
b. 47.8;
c. 6570;
d. 9.42;
e. 0.0274.

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46. The mantissa for 2360 is 0.3729.
The mantissa for 2350 is <u>0.3711</u>.
The difference is 0.0018.
7/10 of 0.0018 is 0.00126.
So, the mantissa for 2357 is 0.3711 plus 0.00126 or 0.37236.

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47. Now we can write the logarithm of 2357 as __________.

- - - - - - - - -

48. By interpolation, find the logarithms of the following numbers:

3.37236

a. Log 37.25 = __________ ;
b. Log 8.126 = __________ ;
c. Log 1728 = __________ .

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49. The reverse process, interpolating mantissas to find the antilogarithms, is done in the same manner.

a. 1.5711;
b. 0.9099;
c. 3.2375.

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50. For example, we wish to find the number whose logarithm is 1.6357. From the table:

The mantissa for 433 is	0.6365
The mantissa for 432 is	0.6355
The difference is	0.0010
Our mantissa is	0.6357
The mantissa for 432 is	0.6355
The difference is	0.0002

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51. Thus, our number is 2/10 of the difference between 432 and 433. Without the decimal point, then, we would write it 4322. The characteristic 1 means that our number is greater than ______ and less than _________; so, the number is actually _________.

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52. By interpolation of mantissas, find the antilogarithms for the following:

10,
100,
43.22

a. Antilog 2.639 = _______________ ;
b. Antilog 0.9481 = ______________ ;
c. Antilog 9.808-10 = ____________ ;
d. Antilog 3.9835 = ______________ .

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4.4. Multiplication and Division

a. 435.5;
b. 8.874;
c. 0.6427;
d. 9627.5.

53. The value of logarithms is most appreciated when a problem involves multiplying and dividing three or four times in succession. For practice, however, we would rather use small numbers in single acts of multiplication or division.

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54. To multiply by logarithms, add the logarithms of the numbers to be multiplied and find the antilogarithms of the sum.

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55. For example, multiply 25.3 by 42.2 as follows:

Log 25.3 = 1.4031
Log 42.2 = <u>1.6253</u>
The sum is ______ .

- - - - - - - - -

56. Now find the antilogarithm of 3.0284. | 3.0284

0.0294 is the mantissa of 107;
<u>0.0253</u> is the mantissa of 106;
______ is the difference.

0.0284 is our mantissa;
<u>0.0253</u> is the mantissa of 106;
______ is the difference.

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57. The antilogarithm of 0.0284 (without worrying yet about the decimal point) is 106 plus the fraction 31/41 by our interpolation. This fraction as a decimal is 0.756, which makes our number 106756. Now we can consider the characteristic 3 which means the number is between 1000 and 10,000. Thus, our answer is ______________. | 0.0041, 0.0031

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58. If you are so curious, and we hope you are, as to multiply the above numbers in the ordinary way, you will get the answer 1067.66 and will feel that the answer by logarithm multiplication is not very accurate. To allay such concern, let us discuss "significant figures". 1067.56

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59. Let us assume that you wish to paint a room. You measure its length at 14 ft 10 in., its width at 12 ft 2 in., and its height at 7 ft 11 in. One gallon of paint is advertised to cover 500 square feet. In this case you are interested only in whether it will take one gallon or two gallons of paint.

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60. Thus, instead of multiplying 14 ft 10 in. by 7 ft 11 in. to get a wall size, you probably multiply 15 ft by 8 ft because these numbers are close enough. For your purposes, 15 ft is as <u>significant</u> as 14 ft 10 in. and 8 ft is as ________________ as 7 ft 11 in.

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61. In your problem, then, you have two walls at 15 ft × 8 ft for a total of _____ ft^2. The other two walls, at 12 ft × 8 ft each, have a total of 192 ft^2, making the total area of the four walls ______ ft^2. This means that you buy only one gallon of paint. significant

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62. You have not used exact measurements, but you have used measurements which were ___________ for your problem. 240, 432

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63. In the same way, let us look at the problem of Frame 55. We were multiplying 25.3 by 42.2. Let us assume that 42.2 is an exact measure but the 25.3 is "rounded off to the nearest tenth". The number may be 25.29 or 25.31, and we call it 25.3. significant

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64. 25.29 × 42.2 = 1067.238
25.31 × 42.2 = 1068.082
Note that with this assumption we are unsure of the fourth digit. Is it 7 or 8? Is our answer 1067 or 1068?

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65. Since both of the original answers fall between these two numbers, we can accept either as correct for the problem.

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66. In the original problem, the answer by logarithms was 1067.56. The answer by multiplication was 1067.66. Any digits beyond 1067 were not only doubtful but, actually, had no real significance.

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67. A rule that is often used where the significance of certain digits in an answer <u>is in doubt</u> is:

> The number of significant digits in an answer is no greater than the number of significant digits of the smallest number used in the problem.

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68. Thus, in our original problem, 25.3 × 42.2, there were only three digits in the answer in which we should place full confidence--the one, zero, and six. We were not sure whether the next digit was 7 or 8.

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69. Seldom is this rule adhered to exactly. Most people who work with mathematics look at the reasonableness of the answer. They use those digits which make the answer reasonable, as we did in the paint problem.

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70. We would like for you to adopt this policy of "reasonableness" in the problems that follow.

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71. Using logarithms, multiply 63 × 147 × 64.8.

Log 63 is ____________ ;
Log 147 is ___________ ;
Log 64.8 is __________ ;
The sum is 5.7782.

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72. The antilogarithm of 0.7782 is ___________, neglecting the decimal point. The characteristic 5 means the number is between 100,000 and 1,000,000. Thus:

1.7993;
2.1673;
1.8116.

63 × 147 × 64.8 = ____________ .

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73. Using logarithms, multiply 534 × 186 × 14. The sum of the logarithms is __________. The characteristic of this logarithm means that the number is greater than ________ and less than __________.	six zero, 600,000

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74. The closest mantissa in our table to the mantissa of our number (1431) is 1430 which is for the digits 139. We do not expect our number to have significant digits beyond two or three; so, rather than interpolate for this small difference, we say the answer to 534 × 186 × 14 is ____________.	6.1431, 1,000,000, 10,000,000

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75. Multiply 0.51 × 12 × 0.043.	1,390,000

$$\begin{array}{lll}\text{Log } 0.51 = & \bar{1}.7076 \text{ or} & 9.7076\text{-}10 \\ \text{Log } 12 \quad = & & 1.0792 \\ \text{Log } 0.043 = & \bar{2}.6335 \text{ or} & \underline{8.6335\text{-}10} \\ & \text{The sum is} & \underline{\qquad\qquad} \end{array}$$

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76. Rewrite the sum of the logarithm, 19.4203-20, as a logarithm with a negative characteristic _________.	19.4203-20

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77. The antilogarithm of $\bar{1}.4203$ is about __________.	$\bar{1}.4203$

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78. The sums of the logarithms of the following numbers are:	0.263

Log 60 + Log 0.35 + Log 430 = ____________ .

Log 0.8 + Log 0.034 + Log 900 = ___________ .

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79. To divide by use of logarithms, subtract the logarithm of the divisor from the logarithm of the dividend. 3.9558, 1.3888

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80. For example, to divide 476 by 28:

Log 476 is	2.6776
Log 28 is	<u>1.4472</u>
The difference is	1.2304

The antilogarithm of 1.2304 is _______; thus, 467 ÷ 28 is __________.

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81. Divide 231 by 221. 17, 17

Log 231 is ____________ .

Log 221 is ____________ .

The difference is ____________ .

From the information available, we would say that the answer is a little closer to ________ than _______.

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82. Divide 1.43 by 0.621. 2.3636, 2.3444, 0.0192, 1.05, 1.04

Log 1.43 is 0.1553.

Log 0.0621 is $\bar{2}.7931$.

Since this is a difficult subtraction, we make it easier by rewriting both logarithms in the form used when the characteristic is negative.

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83. From the above problem,

$$\begin{array}{rr} \text{Log } 1.43 \text{ is} & 10.1553\text{-}10 \\ \text{Log } 0.0621 \text{ is} & \underline{8.7931\text{-}10} \\ \text{Subtract to give} & 1.3622 \end{array}$$

The antilogarithm of 1.3622 is ________.

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84. Divide 0.5 by 25. 23

Log 0.5 is __________.
Log 25 is __________.
The difference is __________.
The antilogarithm is __________.

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85. Now let us try this problem: 9.6990-10, 1.3979, $\bar{2}.3011$, 0.02

$$(257 \times 2.35 \times 10^{3} \times 9.81 \times 10^{-2}) \text{ divided by } (7.6 \times 10^{2} \times 3.5 \times 10^{-1}) = \underline{\quad ? \quad}.$$

To work this problem, the logarithms of the numbers in the first set of parentheses are __________ together. Subtract from this the sum of the logarithms in the ________ set of parentheses. Then find the __________ of this difference.

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86. The sum of the logarithms of the numbers in the first set of parentheses is ________. The sum of the logarithms of the numbers in the second set of parentheses is ________. The difference is __________. added, second, antilogarithm

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87. If we assume that the answer will have no more than three significant digits, it is ________. — 4.7727, 2.4249, 2.3478

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4.5. Powers and Roots — 223

88. You will recall that when a number is raised to a power, such as 6 raised to the fifth power (6^5), you multiply $6 \times 6 \times 6 \times 6 \times 6$. With one-digit numbers, this is not difficult. However, if you raise 253 to the fifth power, it is time consuming, at least, and the possibility of error is increased if you multiply in the usual manner.

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89. Logarithms allow you to raise a number to a power easily. Simply multiply the logarithm of the number by the power to which the number is to be raised. This is the same as adding as many logarithms as the power indicates.

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90. If you wish to raise 253 to the fifth power,
Log 253 is _________.
Multiply this logarithm by 5 to give ________.

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91. From the above frame, we have log 253^5 is 11.0155. The antilogarithm of the mantissa is about halfway between ______ and __________. By interpolation the next digit will be _______, making our answer, to four digits, _____________. — 2.4031, 11.0155

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92. The characteristic is 11. Thus, the number is 103,600,000,000. It is much less cumbersome to write such a number as a number between 1 and 10, times a power of 10, as discussed in Section 3.3, pages 84 to 91. — 103, 104, 6, 1036

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93. In power-of-ten notation, the number would be written 1.036×10^{11}. Note that the exponent of 10, the number _______, is exactly the same as the characteristic of the logarithm 11.0155 because 1×10^{11} is 100,000,000,000.

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94. Thus, it is easy to write an antilogarithm with power-of-ten notation. For example, given the logarithm 3.4669, the antilogarithm is written 2.93 (a number between 1 and 10) times 10 with an exponent 3 because the characteristic of the logarithm is 3. The antilogarithm of 3.4669, then, is ___________ in power-of-ten notation. — 11

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95. Using logarithms and giving answers to no more than three significant figures: — 2.93×10^3

a. $16^4 =$ ____________________ ;
b. $0.053^4 =$ _________________ ;
c. $25^5 =$ ____________________ ;
d. $0.635^3 =$ _________________ ;
e. $(2 \times 10^4)^3 =$ _____________ ;
f. $(2 \times 10^{-4})^3 =$ ____________ ;
g. $5^2 =$ _____________________ .

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96. Now let us find the square root of a number such as 64 by use of logarithms. What is $\sqrt{64}$? First find the logarithm of 64.

a. 6.55×10^{4} ;
b. 7.89×10^{-6} ;
c. 9.76×10^{6} ;
d. 2.56×10^{-1} or 0.256 ;
e. 8×10^{12} ;
f. 8×10^{-12} ;
g. 25 .

$$\text{Log } 64 = 1.8062$$

Divide 1.8062 by 2 (because we are taking the square root).

$$1.8062 \div 2 = 0.9031 .$$

The antilogarithm of 0.9031 is ____________, which you probably knew already;

$$\sqrt{64} = 8 .$$

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97. Now find the cube root of 343, $\sqrt[3]{343}$. Log 343 = 2.5353. Divide 2.5353 by 3 (because we are taking the cube root). 2.5353 ÷ 3 = 0.8451. The antilogarithm of 0.8451 is _________, which is the cube root of 343.

8

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98. In each of the above cases, we divided the logarithm of the number by the root--2 for square root, 3 for cube root--for which we were looking. Then we found the antilogarithm of the logarithm.

7

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99. Now we can make a general statement of the rule,

> To find a given root of a number by use of logarithms, divide the logarithm of the number by the root and then find the antilogarithm.

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100. For example, to find the square root of 144, Log 144 is ________. Divide this logarithm by ________ to give ________. The antilogarithm of ________ is ________.

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101. To find the cube root of a number, divide the ____________ of the number by _____. This will give the logarithm of the ________ ________ of the number. Finally, find the ______________ of this logarithm.

2.1584,
2,
1.0792,
1.0792,
12

- - - - - - - - -

102. The above problem was easy because you knew what the answer was. Now, what is the cube root of 46,700? Write in the next frame your step-by-step solution to the problem.

logarithm,
3,
cube root,
antilogarithm

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103.

- - - - - - - - -

104. The cube root of 46,700 is not exactly but is very nearly ________.

Log 46,700 =
4.6693;
4.6693 ÷ 3 =
1.5564;
antilogarithm
of 1.5564 =
very nearly 36.

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105. There are many symbols used in mathematics to replace words. We use + for plus, ÷ for divided by, etc. Another symbol you should become familiar with is ≃ or ≈, which means "approximately equal to". We could have used it in the last frame to write $\sqrt[3]{46,700}$ is __ 36. — 36

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106. The square root of 15,900 (as calculated with logarithms) is ≃___________. — ≃ or ≈

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107. Find the indicated roots of the following numbers to no more than three significant figures. The answers are in Frame 108. — 126

a.	$\sqrt{625}$	f.	$\sqrt{0.02}$
b.	$\sqrt[3]{1728}$	g.	$\sqrt{0.2}$
c.	$\sqrt{256}$	h.	$\sqrt{2}$
d.	$\sqrt{40}$	i.	$\sqrt{20}$
e.	$\sqrt[6]{64}$	j.	$\sqrt{200}$

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108.

a.	25	f.	0.141 or 0.142
b.	12	g.	0.447
c.	16	h.	1.41 or 1.42
d.	6.32	i.	4.47
e.	2	j.	14.1 or 14.2

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SECTION I-5

USE OF THE SLIDE RULE

5.1. Introduction

1. Basically, the slide rule is an instrument with which logarithms may be added or subtracted quickly. Thus, the same problems that we worked in Part 4 of this study may be solved without looking up the logs and antilogs in a table.

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2. To lay some foundations for our thinking, let us look at two ordinary pieces of wood with ten equally-spaced markings on each.

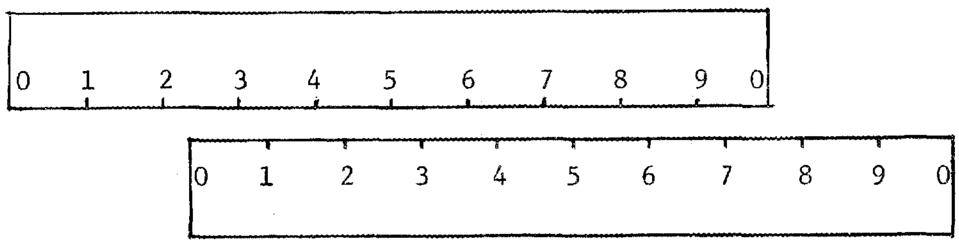

Fig. 5.1. Linear Spacing of Calibration Marks

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3. We can use these sticks to add small numbers (3 + 5) as shown below.
   (a) Place the left zero of the top stick in line with one of the numbers to be added, 3, on the bottom stick.
   (b) Find the other number, 5, on the top stick and read the sum of 3 + 5, which is 8, on the bottom stick.

(Continued)

3. (Continued)

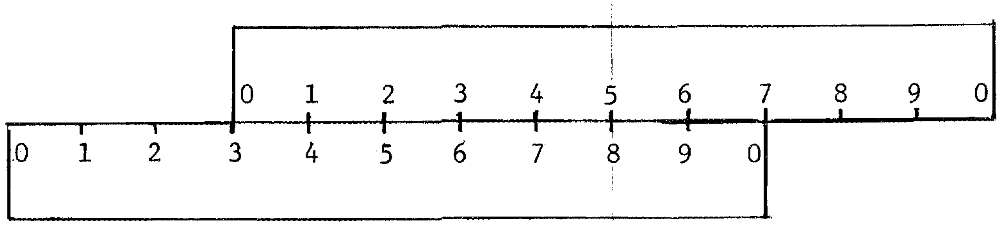

Fig. 5.2. Addition

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4. Now let us make our sticks more useful by making the large graduations equal 10 units, rather than one unit, as below.

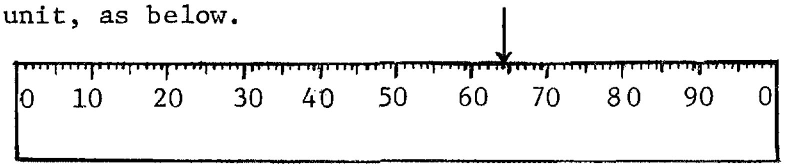

Fig. 5.3. Smaller Linear Graduations

Now, each large scale division has a value of 10, 20, 30, etc., and the arrow points to a value of _______.

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5. We have been referring to lines on sticks. Often it is more convenient to refer to the lines as a <u>scale</u>, meaning a series of steps. Each stick in Fig. 5.1 has a __________ or a series of equal steps from zero to ten. 64

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6. On the stick in Fig. 5.3, we have decided to increase the value of the steps of our ________ by a factor of ten, making the steps 0, 10, 20, 30, etc. The value of our scale is now from zero to _________. scale

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7. Using the same method as that used in Frame 3, let us add 27 and 64 using two sticks with scales graduated as in Fig. 5.3. scale, 100

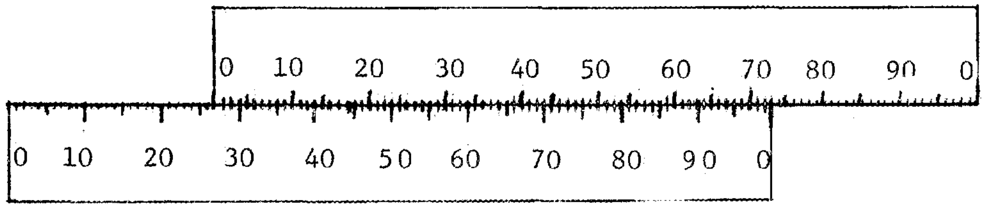

Fig. 5.4. Addition of Larger Numbers

Place the zero of the top scale opposite 27 on the bottom scale. Draw a straight line (hairline) so that it intersects both scales at 64 on the top scale. The answer, on the bottom scale, is _______.

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8. Each slide rule has a sliding member with a hairline etched on it. The purpose of this hairline is, as in the above frame, to allow numbers on the two scales to be matched more closely. 91

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9. Our scale (Frame 7) is approaching a limit for precise work because the lines are getting very close together. However, if high precision is not important, we could let each large scale division be worth 100 and each small one worth 10 to extend our ability to add larger numbers.

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10. Using the scale in Fig. 5.4, allow each large scale division to be worth 100. Now add 270 and 360. The answer on the lower scale is _____.

- - - - - - - - - -

11. Also, we could double the length of the rule and extend our ability to add without losing precision. 630

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12. We said that a slide rule is an instrument for adding logarithms. Thus, we cannot use rules with linear divisions but must use rules that are marked logarithmically, as below.

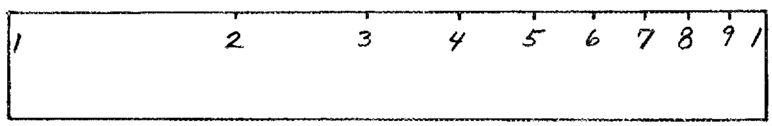

Fig. 5.5. A Logarithmic Calibration

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13. This spacing is produced as shown in Fig. 5.6. Note that the spacing is for the logarithm of the number. Thus, we can add logarithms (multiply numbers) in the same manner that we added linear numbers with a linear scale.

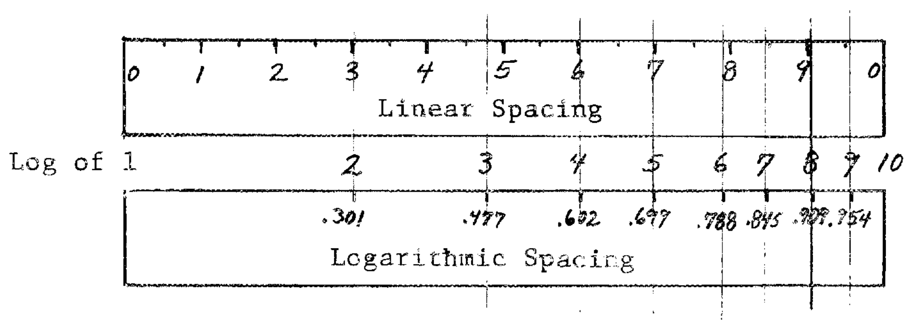

Fig. 5.6. Comparison of Logarithmic and Linear Calibrations

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14. Since one is the number whose logarithm is zero, we mark the left end of the stick, one. The log of 2 is 0.301, so the mark for 2 is 0.301 parts of the whole scale: 3 is 0.477 parts of the scale, etc. These numbers are the mantissas of the logarithms of numbers from one through ten.

\- - - - - - - - -

15. The mantissa for log 10 is ______, as it was for log 1. You should recall that, although characteristics change, the mantissas repeat in a "factor-of-ten" cycle. Each such cycle is a "decade". This scale, then, is a one ____________ (factor-of-ten) scale.

- - - - - - - - -

16. You will recall that the characteristic of a logarithm determines the decimal point of a number and the mantissa does not. Thus, a slide rule, which adds only the ___________ of the logarithms, cannot determine the size of a number; it can determine only the digits of the number. — zero, decade

- - - - - - - - -

17. The scale divisions on a slide rule are spaced to represent the ____________ of the logarithms of the numbers on the rule. — mantissas

- - - - - - - - -

18. The mantissa of the logarithm of 4 is 0.602. The number 4 on our logarithmetic scale is ________ parts of the total distance from the left end to the right end. — mantissas

- - - - - - - - -

19. Since our scale is logarithmetic, there is no zero. The left end is one, or ten, or 100, etc. The right end is always a factor of _________ larger than the left end. — 0.602

- - - - - - - - -

20. We refer to the left end as the left index and to the right end as the right _____________. — ten

- - - - - - - - -

21. There are a number of scales on each slide rule labeled A, B, C, D, etc., as shown in Fig. 5.7. For multiplication and division, we shall consider only the C and D scales. index

- - - - - - - - -

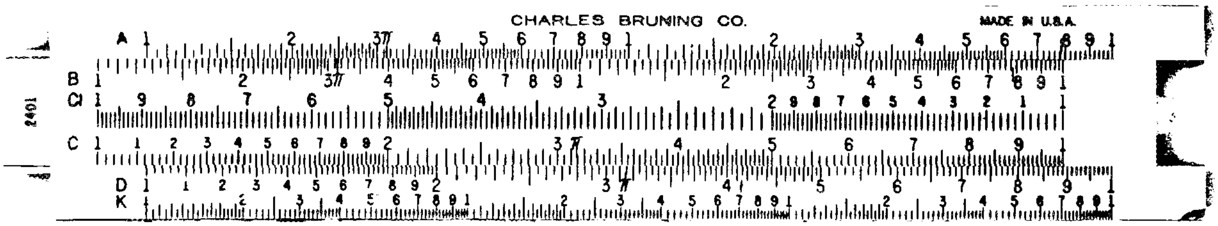

Fig. 5.7. A Slide Rule with Six Scales

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Perhaps we should mention that, in the following illustrations, we have used three different slide rules. We felt that we should use, in each illustration, the slide rule best suited to illustrate the text. The reader will probably find that his slide rule is very similar to one of the three that we have shown.

- - - - - - - - -

22. Since there are no decimal points, the left index number may represent any multiple of 10 less or greater than 1. The right index number, then, is a factor of 10 (greater, less) than the left index.

- - - - - - - - -

23. If the left index represents 10, between this index number and 2 we would have 11, 12, 13, 14, etc., each separated by 10 spaces (as shown on the C and D scales in Fig. 5.8 below). greater

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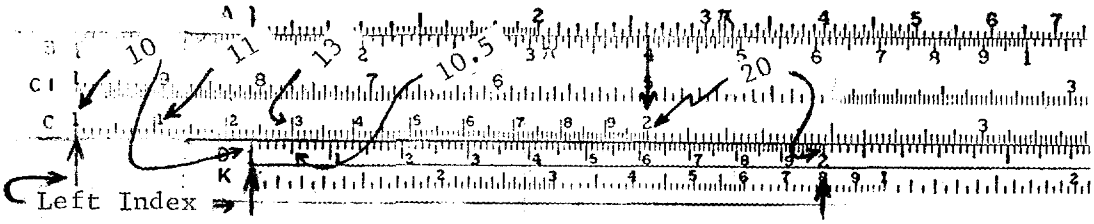

Fig. 5.8. Slide Rule Calibrations between the Left Index and Two

- - - - - - - - -

24. Note that although the divisions between the large numbers one and two are in units of ten, they are not linear but are spaced ____________________.

- - - - - - - - -

25. Note, in Fig. 5.8, that between one and two, it is possible to read a three-digit number. On the C scale, a small three is between the larger one and two. If the large one and two are designated 10 and 20, the small three is at a point we could call 13. This number 13 on the C scale is opposite a point on the D scale halfway between 10 and 11. The third digit in this case would be five; and the number on D, opposite 13 on C, would be 105 or 1.05 or 10.5 or 10,500, etc., depending on the problem we may be working. logarithmically

- - - - - - - - -

26. Thus, since decimal points (can, cannot) be read on a slide rule, we may write the digits 105; but it is better if we say one zero five rather than one hundred and five because one hundred and five is an exact number and indicates a decimal point; one zero five does not. We will discuss the determination of the decimal point in Frames 59, 60, and 61.

- - - - - - - - -

27. On the same figure, note that the number 146 on the C scale is opposite __________ on the D scale. Also:

cannot

a. 161 on C is opposite ________ on D.
b. 125 on C is opposite ________ on D.
c. 173 on C is opposite ________ on D.
d. 135 on C is opposite ________ on D.
e. 140 on C is opposite ________ on D.
f. 151 on C is opposite ________ on D.

\- - - - - - - - -

28. Now let us look at the portion of the scale that extends from 2 to 4, Fig. 5.9 below.

\- - - - - - - - -

118
a. 130 ;
b. 101 ;
c. 140 ;
d. 109 ;
e. 113 ;
f. 122 .

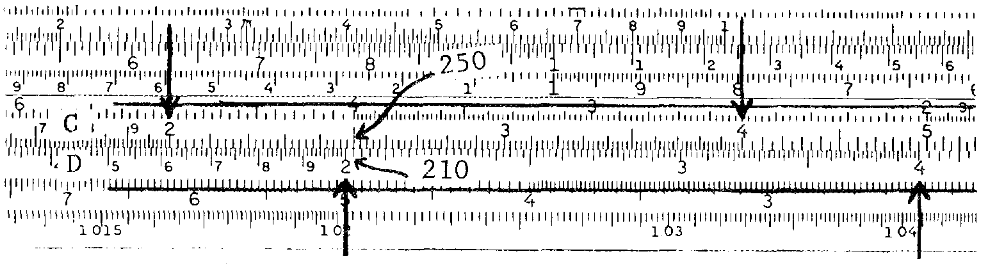

Fig. 5.9. C and D Scales between Two and Four

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29. The numbers in this part of the scale are closer together because from log 3, which is 0.4771, to log 4, 0.6021, is a linear distance proportional to their difference: 0.6021 - 0.4771 = 0.1250. The linear distance from log 1, which is zero, to log 2, 0.3010, is 0.3010--which is more than double the linear distance from log 3 to log 4.

\- - - - - - - - -

30. Since these numbers are closer together, the spacing is such that there are only five small divisions between the major divisions. Thus, each small division is two greater than the preceding one. Note that the number 250 on the C scale is opposite 202 on the D scale.

- - - - - - - - -

31. The first two digits of a number are marked exactly, but the third digits are marked exactly only to "two's" as in counting by "two's". Therefore, we must estimate the positions of odd digits like 1, 3, 5, etc.

- - - - - - - - -

32. Note that 266 on C is about halfway between 214 and 216 on the D scale. Not knowing exactly what the number is, we "interpolate". This is the same as "interpolation" with logarithms--we make a calculated guess or "estimate" that the number is ≈215.

- - - - - - - - -

33. Practice in reading numbers: (Use Fig. 5.9.)

a. 222 on C is opposite ________ on D.
b. 260 on C is opposite ________ on D.
c. 280 on C is opposite ________ on D.
d. 312 on C is opposite ________ on D.
e. 342 on C is opposite ________ on D.

- - - - - - - - -

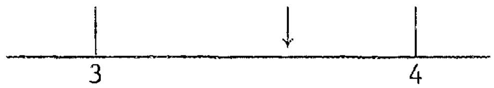
34. When the number we want is between two points on a slide rule and we try to mentally divide the distance between the two points such that we can make an estimate as to its position, we ________________.

a. 179 ;
b. 210 ;
c. 226 ;
d. 252 ;
e. 276 .

- - - - - - - - -

35. Given the two marks below, the arrow is pointing to about (3.5, 3.6, 3.7).

interpolate



This process of estimating is called ______________.

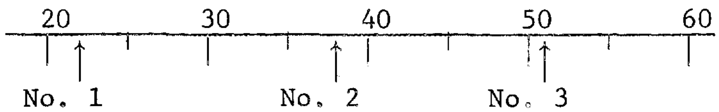
- - - - - - - - -

36. Recall that we did much the same thing arithmetically when we wanted a mantissa or an antilogarithm which was between numbers given in the table of logarithms.

3.6,
interpolation

- - - - - - - - -

37.



By interpolation, estimate the numbers to which the arrows point.

No. 1 is about ________ .

No. 2 is about ________ .

No. 3 is about ________ .

- - - - - - - - -

38. Between the numbers 4 and 10 on the slide rule, the spacing is such that it is difficult to divide the space between 40 and 41 or 64 and 65 into more than two parts (see Fig. 5.10).

22 ;
38 ;
51 .

- - - - - - - - -

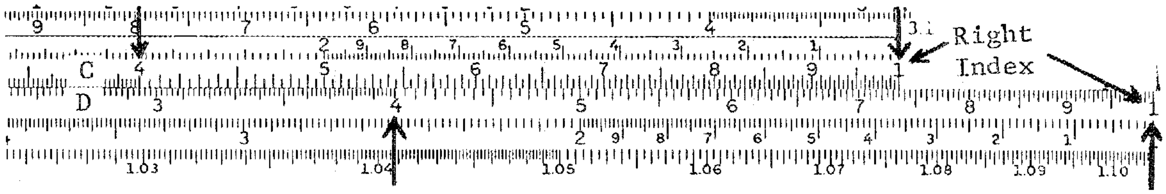


Fig. 5.10. Spacing of Numbers between Four and Right Index

- - - - - - - - -

39. In this part of the scale, we must interpolate for the third digit for all numbers except those which are exactly halfway between the longer spaces.

- - - - - - - - -

40. For practice in reading the following numbers, use Fig. 5.10 or your own slide rule with the right index placed in the same position as in Fig. 5.10.

a. 470 on C is opposite _______ on D.
b. 530 on C is opposite _______ on D.
c. 625 on C is opposite _______ on D.
d. 870 on C is opposite _______ on D.
e. 965 on C is opposite _______ on D.
f. 720 on C is opposite _______ on D.

- - - - - - - - -

41. You may have noted that it is difficult, especially where the spacing is close, to interpolate as precisely as you would like. Usually, if the third digit is not off more than one number, your answer is considered correct. For example, if you read 417 and the true answer is not more than 418 nor less than 416, you are considered correct.

a. 346 ;
b. 390 ;
c. 460 ;
d. 640 ;
e. 710 ;
f. 530 .

- - - - - - - - -

5.2. Multiplication

42. Now that we have had some practice at reading the C and D scales, let us do some actual addition of logarithms (multiplication). We shall continue to show figures of slide rules as needed; however, you should have one with which to work problems as we proceed.

- - - - - - - - -

43. On the slide rule in Fig. 5.11, the left index of the C scale is aligned with ________ on the D scale. The hairline has been moved to the right and is aligned with ________ on the C scale.

- - - - - - - - -

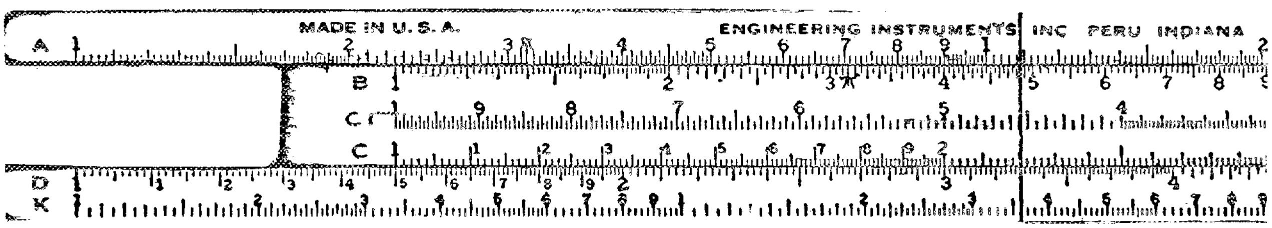


Fig. 5.11. Slide Rule for Answering Frame 43

- - - - - - - - -

44. If we add log 15 to log 22, their sum is the logarithm of the number on the D scale which is at the hairline. This number is __________.

one five
two two

- - - - - - - - -

45. We said we were adding logarithms. Try the same problem with your logarithm tables.

three three

Log 15 = _______

Log 22 = _______

Antilogarithm of 2.5185 is ______.

- - - - - - - - -

46. Note that when working with the complete logarithm, the characteristic allows us to determine the ____________. On a slide rule, we have only the mantissa, so the position of the decimal point is dependent on the numbers in the problem.	1.1761, 1.3424, 330

- - - - - - - - -

47. In Frame 44, the numbers on the slide rule were one five and two two. The actual problem might have been	decimal point

$$1.5 \times 2.2$$
$$1.5 \times 22$$
$$15 \times 220$$
$$1500 \times 0.22$$

or a host of others. In any case, the digits of the answer were three three.

- - - - - - - - -

48. When a problem is worked on a slide rule, the placement of the decimal point in the answer is (pick the correct answer):

   a. read on the slide rule;
   b. determined independently according to the numbers used in the problem;
   c. found by referring to a table of logarithms.

- - - - - - - - -

49. Practice problems:	b.

a. $12 \times 17 =$ __________ ; e. $25.8 \times 3.7 =$ __________ ;
b. $12 \times 21 =$ __________ ; f. $5.55 \times 13.2 =$ __________ ;
c. $1.76 \times 3.22 =$ ________ ; g. $2.24 \times 10^{-3} \times 3.68 \times 10^{5} =$ ______ ;
d. $176 \times 32.2 =$ _________ ; h. $1.25 \times 10^{3} \times 4.5 \times 10^{2} =$ ________ .

- - - - - - - - -

50. Now, suppose we wish to multiply 7.35 × 5.625. If we set the left index number of the C scale opposite 735 on the D scale, as in Fig. 5.12, the other number 5625 on the C scale does not contact the D scale.

a. 204 ;
b. 252 ;
c. 5.67 ;
d. 5670 ;
e. 95.5 ;
f. 75.3 ;
g. 8.25×10^2 or 825 ;
h. 5.63×10^5 .

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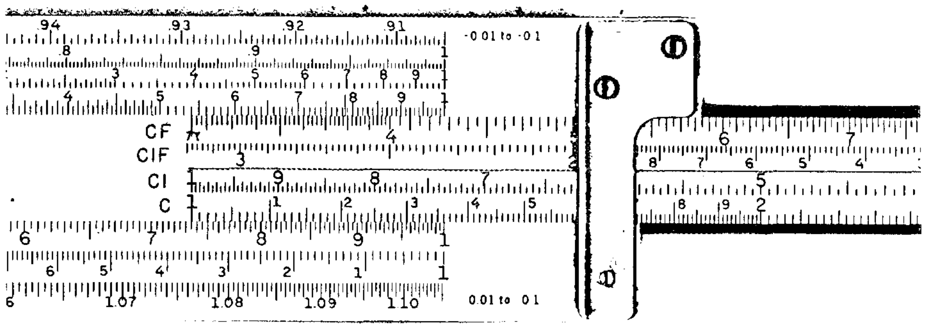

Fig. 5.12. Left Index of C Scale Set at 735 on the D Scale

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51. So, we "switch ends" of the slide and place the right index of the C scale at 735, as in Fig. 5.13. Now we can use 5625 on the C scale and read the answer on the D scale.

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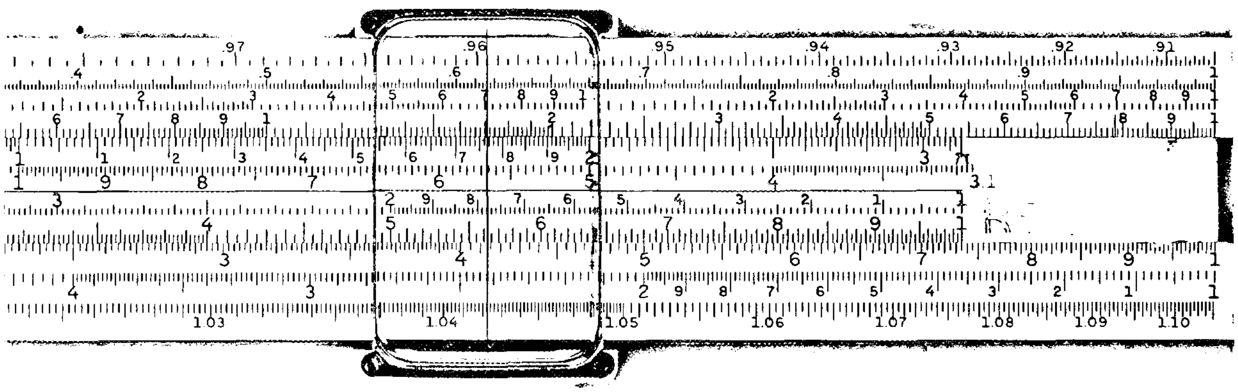

Fig. 5.13. Right Index of C Scale on 735 on D Scale

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52. When we use the right index of C, we are essentially shifting to a "new" D scale as illustrated in Fig. 5.14. But, since the scales read only mantissas, the "new" D scale is, of course, identical to the old scale. Thus, $7.35 \times 5.625 =$ __________.

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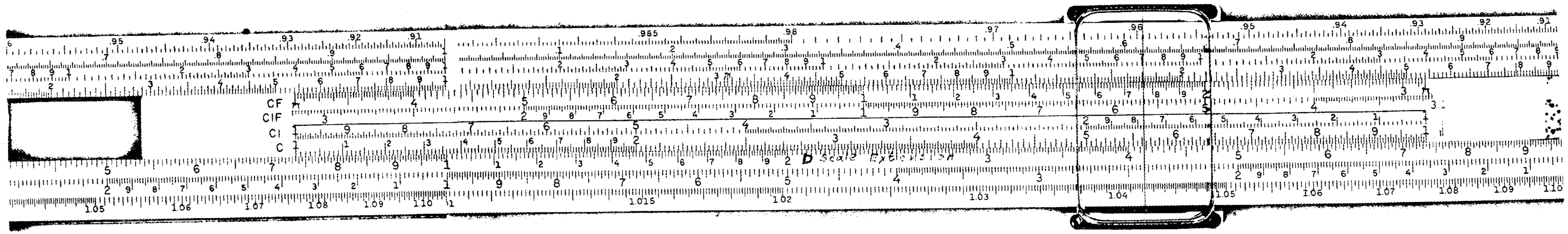

Fig. 5.14. Simulating an Extension of the D Scale

- - - - - - - - -

53. Practice problems:

41.3

a. $52.3 \times 6 =$ ________ ;
b. $9.8 \times 45 =$ ________ ;
c. $9.33 \times 10^{-2} \times 1.86 \times 10^{5} =$ ________ ;
d. $3.7 \times 10^{-1} \times 8.5 \times 10^{-2} =$ ________ ;
e. $0.667 \times 3 =$ ________ ;
f. $16 \times 0.375 =$ ________ ;
g. $8.75 \times 423 =$ ________ ;
h. $106 \times 16 =$ ________ .

- - - - - - - - -

54. Now let us discuss division. To divide numbers by using logarithms, subtract the logarithm of the divisor from the logarithm of the dividend. We perform the same manipulation with the slide rule.

a. 314 ;
b. 441 ;
c. 17,350
d. 3.14×10^{-2} or 0.0314 ;
e. 2 ;
f. 6 ;
g. 3700 ;
h. 1695 .

- - - - - - - - -

55. Let us look first at our linearly ruled sticks. In the figure below, we place one number on the lower scale and adjacent to it on the upper scale the number to be subtracted.

- - - - - - - - -

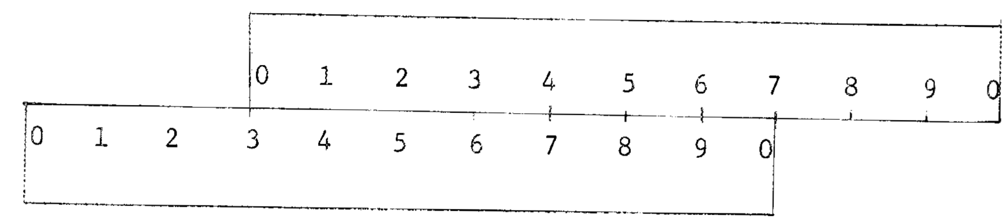

Fig. 5.15. Subtraction with Linear Scales

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56. Our answer is read on the lower scale opposite the left index of the upper scale. Let us, for example, subtract 6 from 9; the answer is 3, the number on the lower scale opposite the _____ _____ _______ on the upper scale.

- - - - - - - - -

57. Let us do the same with the logarithmic scale of the slide rule. Divide 6 by 2.5. As in Fig. 5.16 below, place the hairline of your rule on 6 on the D scale and move 2.5 on the C scale under the hairline. The answer _________ is read on the D scale opposite the left index of the C scale.

left index
number

- - - - - - - - -

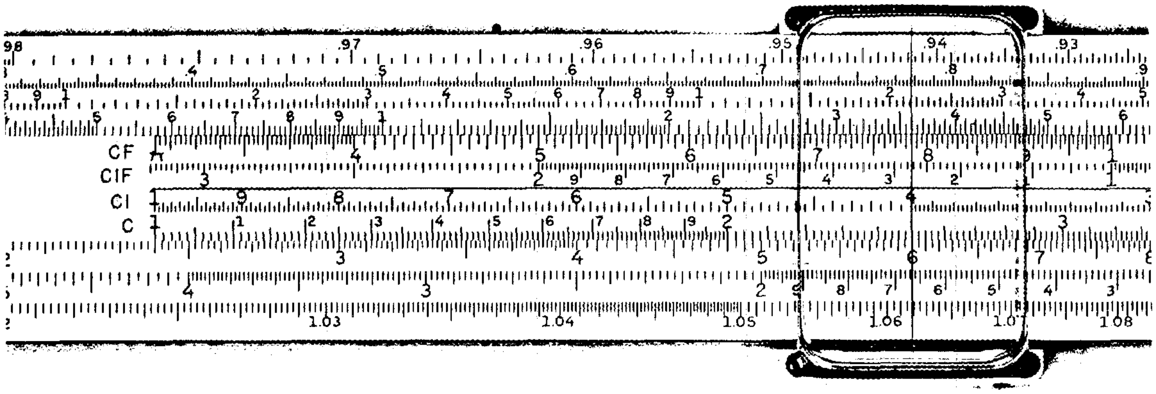

Fig. 5.16. Dividing 6 by 2.5

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58. If the numbers are such, as in Fig. 5.17, that the left index of C is off scale, read the answer on the D scale opposite the <u>right</u> index of the C scale. 2.4

- - - - - - - - - -

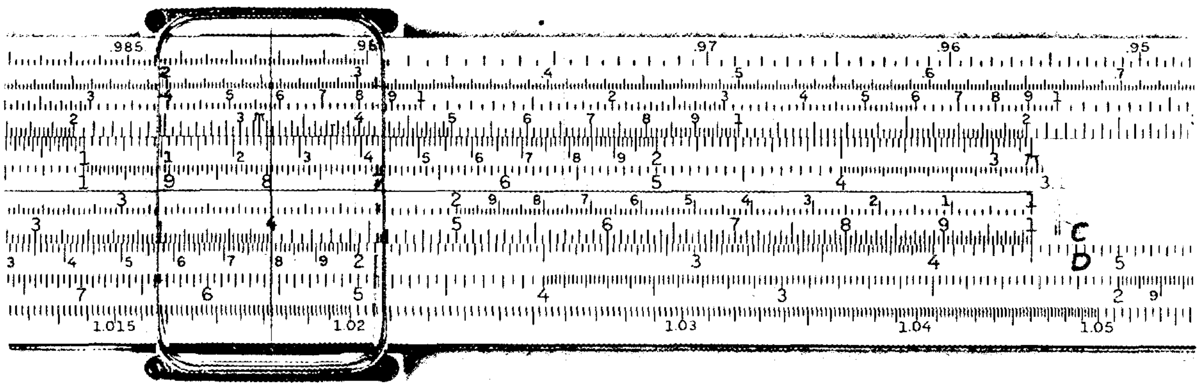

Fig. 5.17. Hairline at 4 on C Scale and 18 on D Scale

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59. In Fig. 5.17, we have divided one eight by four and the answer is ____________________.

- - - - - - - - - -

60. As in multiplication, the decimal point cannot be determined by the numbers on the slide rule. Usually the operator determines the location of the decimal point by quickly dividing numbers of roughly the same size. Using numbers in power-of-ten form is a help in this process.

four five

- - - - - - - - -

61. For example, in Fig. 5.17, we could be dividing 180,000 by 40. The digits in the answer are four five. If we write 180,000 as 18×10^4 and 40 as 4×10^1, it is obvious that the answer is ______.

- - - - - - - - -

62. A less simple problem would be to divide 14.6 by 0.0334. The digits in the answer, read opposite the right index of C, are four three seven. Write the problem

4.5×10^3 or 4500

$$\frac{14.6 \times 10^0}{3.34 \times 10^{-2}}$$

and it is easy to see that the answer is _____ $\times 10^2$ or ______________.

- - - - - - - - -

63. Practice problems:

4.37, 437

a. $673 \div 231 =$ ________ ;
b. $5.87 \times 10^3 \div 6.7 \times 10^5 =$ ________ ;
c. $1.873 \div 0.0562 =$ ________ ;
d. $12550 \div 172 =$ ________ ;
e. $2.62 \times 10^{-4} \div 3.59 \times 10^{-6} =$ ________ ;
f. $262 \div 327 =$ ________ ;
g. $3.89 \div 3.96 \times 10^2 =$ ________ ;
h. $3.89 \div 2.19 =$ ________ .

- - - - - - - - -

64. There is a saying that the more a slide rule is used the more accurate it becomes. The truth is that the more you use a slide rule, the more expert <u>you</u> become in its use and in the ability to interpolate with greater precision. So, if you wish to be really "good" with a slide rule--PRACTICE.

a. 2.91 ;
b. 8.76×10^{-3} ;
c. 33.4 ;
d. 73 ;
e. 73 ;
f. 0.802 ;
g. 9.83×10^{-3} ;
h. 1.775 .

- - - - - - - - -

65. There are times when problems involve successive acts of multiplication and division. With a little practice, it is easy to do such problems. Let us try one such as

$$500 \times \frac{273}{295} = \underline{\hspace{3em}} .$$

- - - - - - - - -

66. For this problem, we wish to combine the multiplication and division steps. It is immaterial whether you divide or multiply on the first step. Let us multiply 500×273, and the answer is on the D scale at the hairline. Now move 295 on the C scale over to the hairline and you have divided the product of 500×273 by 295. The answer is four six three. Placement of the decimal point makes it ________.

- - - - - - - - -

67. Practice problems:

463

a. $\frac{26 \times 13}{12} = \underline{\hspace{6em}}$;

b. $\frac{14 \times 22 \times 843}{16 \times 574} = \underline{\hspace{4em}}$;

c. $\frac{365 \times 31.5}{30} = \underline{\hspace{5em}}$;

d. $\frac{25 \times 92 \times 18}{41 \times 101} = \underline{\hspace{4em}}$.

- - - - - - - - -

68. We said that it was immaterial whether you divide first or multiply first. This is true, but you will find that there are times when one beginning makes the problem easier than another beginning.

a. 28.2 ;
b. 28.3 ;
c. 383 ;
d. 10 .

- - - - - - - - -

69. Let us use, for example, problem b of Frame 67. If you began the problem by dividing 14 by 16, you should have had no difficulties. However, if you multiplied 14 × 22 and then divided by 16, you now find that the most likely step, to multiply by 843, cannot be done directly because 843 is off the scale to the right.

- - - - - - - - -

70. The left index of C is on the number 1925. From our previous experience, we know that we can switch ends of the slide; that is, place the _________ _________ of the C scale on the number ___________ and proceed to multiply by 843 as usual.

- - - - - - - - -

71. There is another procedure which may be simpler in this case. Place the hairline at 1925, divide 1925 by 574, and then 843 is in the correct position to multiply. The answer _____________ will be the same regardless of the path taken.

right index,
1925

- - - - - - - - -

72. Work each of the problems left to right; then, work them right to left and check the answers. 28.3

a. $\dfrac{28 \times 3}{36 \times 5 \times 0.0206} =$ ____________ ;

b. $\dfrac{9.15 \times 37 \times 0.236}{2.25 \times 1.5} =$ ____________ ;

c. $\dfrac{0.214 \times 750 \times 0.1857}{0.995 \times 26 \times 1.235} =$ ________ .

- - - - - - - - -

73. It would be impractical in this study to mention all of the shortcuts for multiplication and division. However, before we leave the subject, we should mention the CI scale, often called the C-Inverted or reciprocal scale.

a. 22.6 ;
b. 23.6 or 23.7 ;
c. 0.942 .

- - - - - - - - -

74. Just above the C scale on most slide rules is the CI scale (shown in Fig. 5.18). It is also called the C-Inverted scale because it is exactly like the C and D scales except that it is reversed. Before we discuss its use, we should review the meaning of the term reciprocal.

- - - - - - - - -

75. When we were discussing common fractions, we mentioned that the reciprocal of a number is the fraction formed by writing 1 over the number. That is, the reciprocal of 2 is 1/2; of 68 is ______; of 1/2 is ______.

- - - - - - - - -

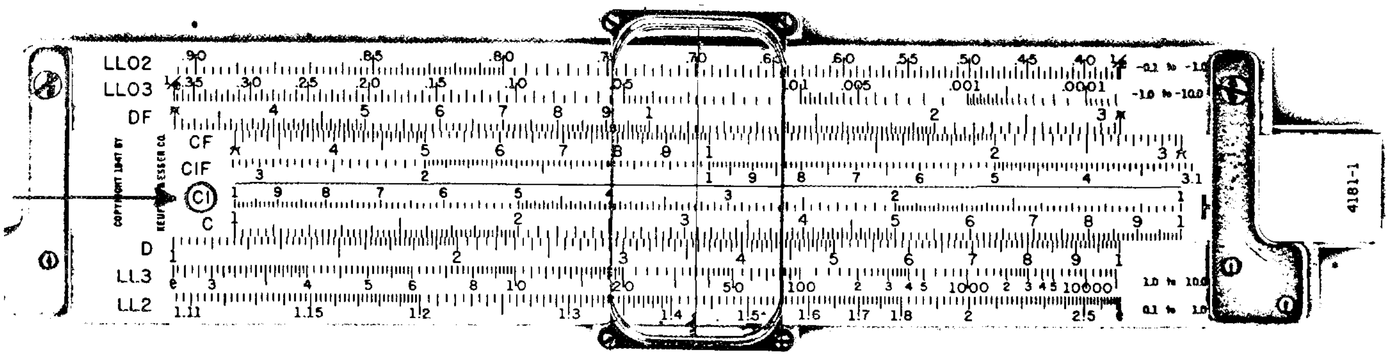

Fig. 5.18. Slide Rule with Arrow Pointing to CI Scale

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76. You will note from the last example that the reciprocal of 1/2 is 2. So, dividing a number N by the reciprocal of M, 1/M, is the same as multiplying N × M. That is, N ÷ 1/M = N × M/1 = ________.

1/68, 2

- - - - - - - - -

77. The number 134 divided by the reciprocal of 2, 1/2, is:

$$134 \div 1/2 = 134 \times 2/1 = \_\_\_\_\_\_\_\_ .$$

NM

- - - - - - - - -

78. Multiplying a number N by a reciprocal 1/M is the same as dividing N by M. N × 1/M = N/M. The number 134 multiplied by the reciprocal of 2 is:

$$134 \times 1/2 = 134/2 = \_\_\_\_\_\_\_\_ .$$

268

- - - - - - - - -

79.	Thus we can use the inverted C scale to multiply and divide by reciprocals. Set your slide rule to multiply 25 by 1/16. Set the right index on 25. Then multiply 25 by 16 on the CI scale. The answer on the D scale is ______________.	67
	- - - - - - - - -	
80.	In the above frame, instead of dividing 25 by 16 on the C and D scales, we multiplied 25 by the ________________ of 16 on the CI scale and read the answer as usual on the ___________ scale.	1.56
	- - - - - - - - -	
81.	Work the problem $45 \times 620 \times 72 = ?$ as follows: Right index of C on 45; Hairline to 62 on C and the answer, so far, is opposite 62 on the D scale. In order to multiply again, we would need to shift the right index of C to the hairline at ________ on the D scale.	reciprocal, D
	- - - - - - - - -	
82.	Instead of multiplying, let us <u>divide</u> by the _______________ of 72. Without moving the hairline, move 72 on the CI scale over to the hairline. Now read the answer on the D scale opposite the _______ index of the CI scale. The digits of the answer are _____ _____ _____. The answer is __________.	279
	- - - - - - - - -	

83. In the following problems, multiply the first two numbers as usual, divide by the reciprocal of the third, and multiply as usual on the fourth.

reciprocal, left, two zero one 2.01×10^6

a. $148 \times 24 \times 0.157 \times 0.032 =$ ________ ;

b. $1.2 \times 36 \times 0.58 \times 2 =$ ________ ;

c. $56 \times 0.21 \times 940 \times 0.375 =$ ________ ;

d. $842 \times 60 \times 0.125 \times 3 \times 10^{-3} =$ ________ .

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84. Work the following problems using the CI scale as much as possible.

a. 1.783 ; b. 50.2 ; c. 4140 or 4150 ; d. 18.94 .

a. $\dfrac{250}{90 \times 1.37} =$ ________ ;

b. $\dfrac{57.5}{2.8 \times 6.3} =$ ________ ;

c. $\dfrac{345}{146 \times 5.6} =$ ________ .

- - - - - - - - -

5.3. Scale of Common Logarithms

a. 2.03 ; b. 3.26 ; c. 0.422 .

85. One of the several scales on most slide rules is a scale of common logarithms. This L scale on the rule in Fig. 5.19 is located just above the CI scale and is used with the C scale for finding mantissas in the same way that a table of common logarithms is used.

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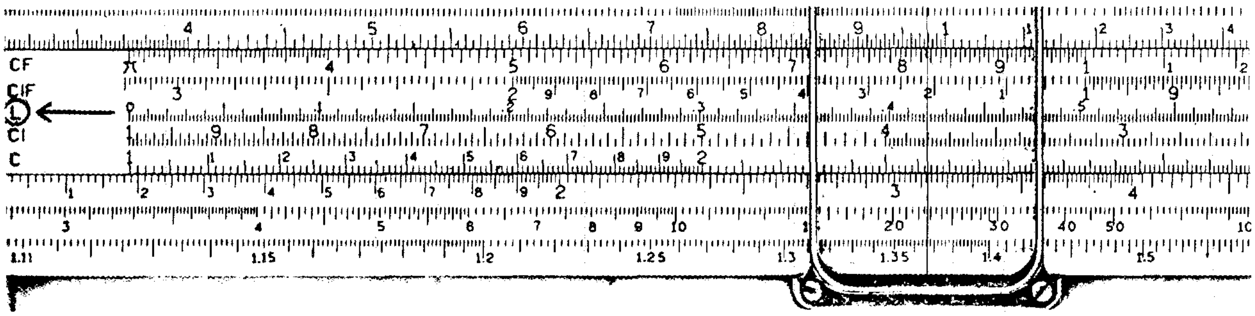

Fig. 5.19. The Scale of Common Logarithms

86. You will note that the hairline in Fig. 5.19 is at two six three on the C scale. The number at the hairline on the L scale is 0.42, which is the mantissa of log 263. So, to find the mantissa of the logarithm of a number, place the hairline on the number on the C scale and read the mantissa on the L scale.

87. Find the common logarithm scale on the rule you are using and write the logarithms of the following numbers:

a. Log 156 = ________;
b. Log 5.35×10^3 = ________;
c. Log 1.75 = ________;
d. Log 30.4 = ________;
e. Log 0.036 = ________;
f. Log 0.5225 = ________.

a. 2.193 ;
b. 3.728 ;
c. 0.243 ;
d. 1.483 ;
e. 8.556-10 ;
f. 9.718-10 .

5.4. Squares and Square Roots

88. The worth of the slide rule as an instrument for either extracting square and cube roots or for squaring and cubing numbers is determined by the need of the operator for precise or approximate answers.

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89. As you have probably already determined, the slide rule is not a precision instrument, especially if the precision desired is greater than two or three significant digits. However, it is an excellent instrument where few-digit significance is useful.

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90. While the above statement is certainly true with regard to multiplication and division, when you begin squaring two or three digit numbers, the precision with which you can read the answer is no more than three digits.

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91. For example, let us square the numbers 16 and 46 by regular arithmetic multiplication.

$$16^2 = \_\_\_\_\_\_ ;$$
$$46^2 = \_\_\_\_\_\_ .$$

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92. The square of 16 is 256, which can be read rather easily on the slide rule. However, the exact square of 46 is 2116. On the slide rule, you would probably need to interpolate between 2110 and 2120. You would not be able to read the 6 at all and would guess as to whether the <u>third</u> digit was one or two. 256, 2116

- - - - - - - - -

93. Let us look at the slide rule again. This time we shall use the A and D scales. Note in Fig. 5.20 that the slide has been removed to lessen the possible confusion of the intervening scales. On your slide rule you may find that the B and C scales are easier to use together. The A and B scales are identical, as are the C and D scales.

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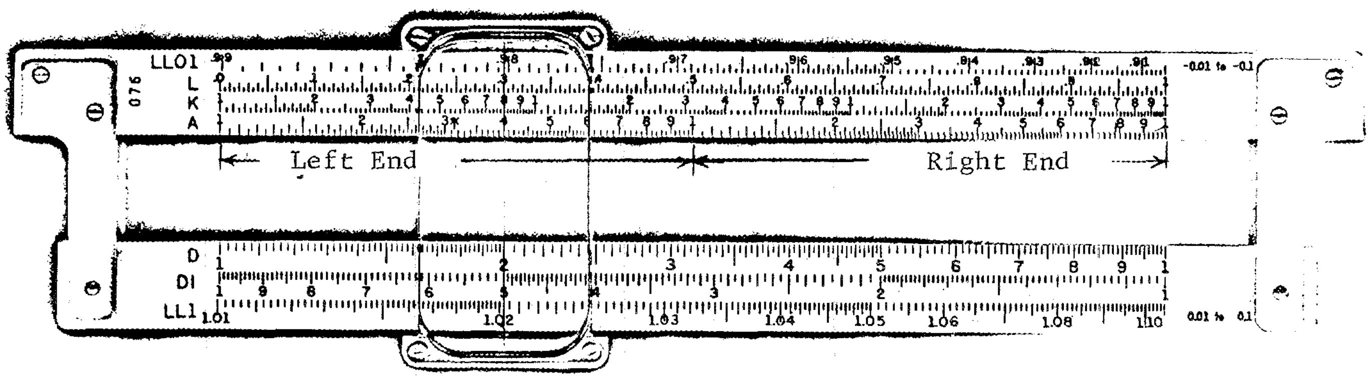

Fig. 5.20. The Two-Decade A Scale

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94. Note that the A scale is exactly as long as the D scale, but it is made up of two shorter one-through-ten scales. Since the right end of the D scale is a factor of ten greater than the left end, the right end of the A scale must be two factors of ten (10^2 or 100) greater than the left end.

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95. If the left end of A represents numbers from 1 to 10, the right end represents numbers from 10 to 100. If we were speaking of graph paper, we would call it two-decade paper. For the same reason, we can call the A scale a two-decade scale.

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96. Since the D scale is one decade long and the A scale is two decades long, the logarithm of the number under the hairline on the A scale is twice as large as the logarithm of the number under the hairline on the D scale.

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97. When we place the hairline over a number on the D scale and shift our attention to the number under the hairline on the A scale, we effectively multiply the logarithm of the D scale number by two. Thus, the number on the A scale is the square of the number on the D scale.

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98. In Fig. 5.20, we have placed the hairline at 2 on the D scale. On the A scale you read __________ which is 2^2.

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99. Refer now to Fig. 5.21 where we have placed the hairline on 16 on the D scale. At the hairline on the A scale you should read 256 because the hairline is slightly to the right of the line which represents the number 255. 4

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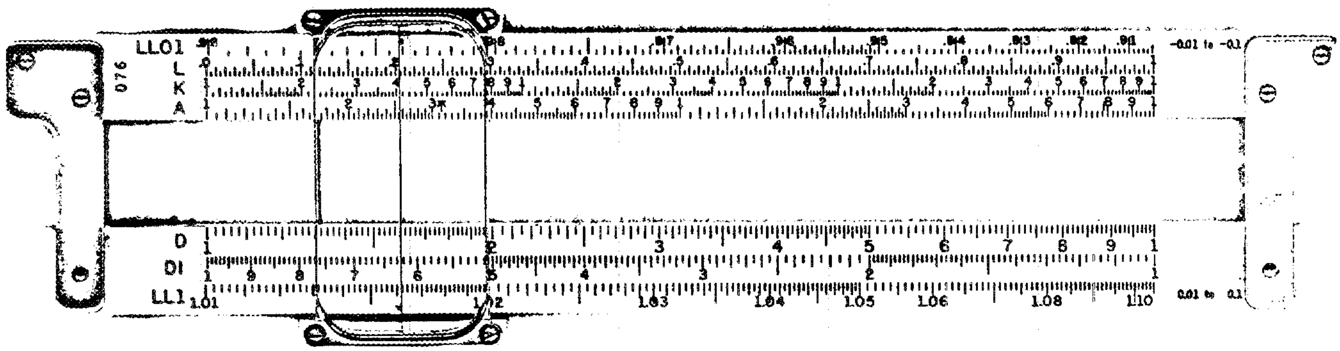

Fig. 5.21. Using the Left Half of A with D

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100. Since you have already determined in Frame 92 that 16^2 is 256, you will know that 255 is not quite right. We are trying to emphasize that interpolation on the shorter A scale is a little more difficult than on the D scale. However, we are still within the accuracy limit of plus or minus one for the third digit.

- - - - - - - - - -

101. Now let us move to the other end of the D scale and place the hairline at 46. In Fig. 5.22 we are using a 10-in. slide rule to make interpolation easier. The square of 46, as noted on the A scale, is ____________.

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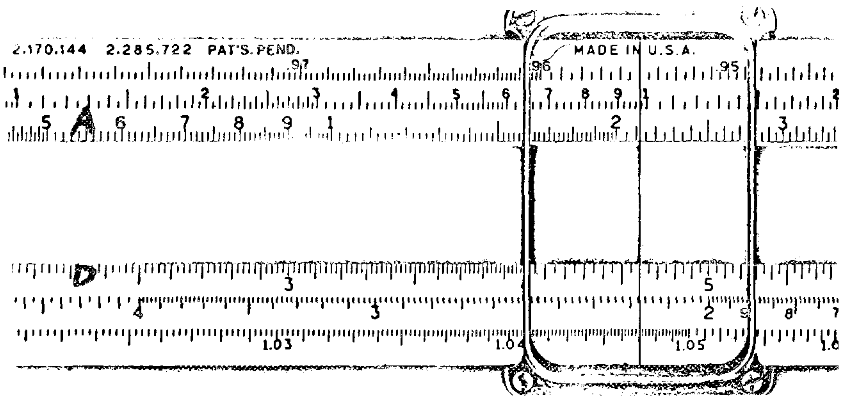

Fig. 5.22. Using the Right Half of A with D

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102. Again, the interpolation calls for a more critical decision; but, since we know from Frame 92 that 46^2 is 2116, we are well within the "plus-or-minus-one in the third place" accuracy that we should have. 2110 or 2120

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103. Actually, in this case and similar cases where the answer has four digits, you can be <u>sure</u> of the fourth digit. Since we are squaring 46, we can see that the last digit is a six by mentally multiplying the last digit of the multiplier and multiplicand:

$$6 \times 6 = 36 \ .$$

So, if we know that the answer is a four-digit number and we can read the first three from the slide rule, we can easily supply the fourth.

- - - - - - - - -

104. Thus, in Frame 101, we read on the slide rule the number either 2110 or 2120. We know that the answer is greater than _________ and less than _____________. As in Frame 103, we deduce that the fourth digit is six, so the correct answer to the problem 46^2 is _______.

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105. Practice problems: 2110, 2120, 2116

a. 12^2 = __________ ; j. 3.5^2 = __________________ ;

b. 13^3 = __________ ; k. 35^2 = __________________ ;

c. 1.3^2 = _________ ; l. 51^2 = __________________ ;

d. 130^2 = _________ ; m. 6.2^2 = _________________ ;

e. 1.8^2 = _________ ; n. 6.85^2 = ________________ ;

f. 18^2 = __________ ; o. $(7.2 \times 10^2)^2$ = __________ ;

g. 180^2 = _________ ; p. 0.083^2 = ________________ ;

h. 31^2 = __________ ; q. 0.23^2 = _________________ ;

i. 33^2 = __________ ; r. 0.114^2 = ________________ .

The answers are in the frame which follows.

- - - - - - - - -

106. a. 144 ; j. 12.2 or 12.3 ;

b. 169 ; k. 1225 ;

c. 1.69 ; l. 2600 ;

d. 1.69×10^4 ; m. 38.5 ;

e. 3.24 ; n. 47 ;

f. 324 ; o. 51.9×10^4 ;

g. 3.24×10^4 ; p. 68×10^{-4}

h. 960 or 961 ; q. 5.29×10^{-2} ;

i. 1089 ; r. 0.013 .

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107. Now let us reverse the process. When extracting square roots, there are two rules to remember. The first is: If your number has an odd number of digits to the left of the decimal point, use the left half of the A scale. If the number of digits is even, use the right half of the A scale.

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108. The other rule is: If you are extracting the square root of a decimal fraction, write the fraction as a power of ten where the exponent of 10 is divisible by 2. For example, 0.121 would be written 12.1×10^{-2}. The number 0.0121 would be written either 1.21×10^{-2} or 121×10^{-4}.

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109. In the first case, $\sqrt{12.1 \times 10^{-2}}$, place the hairline over 121 on the right hand A scale. The answer, on the D scale, is 3.47×10^{-1} $(-2 \div 2 = -1)$. The square root of 0.121 is

________.

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110. In the second case, $\sqrt{121 \times 10^{-4}}$, place the hairline over 121 on the left hand A scale. The answer, on the D scale, is 11×10^{-2} or, written as a decimal fraction, ______________.

3.47×10^{-1} or 0.347

- - - - - - - - -

111. To extract the square root of 40, place the hairline on 4 on the right half of the A scale (40 is two digits, even). Read six three two on the D scale. $\sqrt{40}$ = __________.

0.11

- - - - - - - - -

112. To extract the square root of 7.3, place the hairline at seven three on the <u>left</u> half of the A scale. While 7.3 is two digits, there is only one digit to the left of the decimal point (one digit, <u>odd</u>, use the <u>left</u> side). So, $\sqrt{7.3}$, read on the D scale, is ________. 6.32

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113. Practice problems: 2.7

a. $\sqrt{1.3}$ = ________ ; h. $\sqrt{550}$ = ________ ;
b. $\sqrt{13}$ = ________ ; i. $\sqrt{55}$ = ________ ;
c. $\sqrt{130}$ = ________ ; j. $\sqrt{5.5}$ = ________ ;
d. $\sqrt{0.13}$ = ________ ; k. $\sqrt{0.5}$ = ________ ;
e. $\sqrt{0.013}$ = ________ ; l. $\sqrt{0.05}$ = ________ ;
f. $\sqrt{17.4}$ = ________ ; m. $\sqrt{195}$ = ________ ;
g. $\sqrt{8.35}$ = ________ ; n. $\sqrt{348000}$ = ________ ;

The answers are in the frame below.

- - - - - - - - - -

114. a. 1.14 ; h. 23.45 ;
b. 3.6 ; i. 7.41 ;
c. 11.4 ; j. 2.345 ;
d. 0.36 ; k. 0.741 ;
e. 0.114 ; l. 0.2345 ;
f. 4.17 ; m. 13.95 ;
g. 2.89 ; n. 590 .

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115. There are several other scales that can be studied and many shortcut procedures that are useful in the working of particular problems. We leave these for the person who wishes to go further with the study of the slide rule. Our purpose was to give an introduction to the slide rule, and we feel that you now have a base for continued study. GOOD LUCK!

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