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A DEFENSE SYSTEM DESIGN MODEL

R. A. Uher

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ORNL-TM-2041

Contract No. W-7405-eng-26

Director's Division
CIVIL DEFENSE RESEARCH PROJECT

A DEFENSE SYSTEM DESIGN MODEL

R. A. Uher

Research sponsored under Advanced
Research Project Agency Order No. 907

FEBRUARY 1968

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ABSTRACT

A model is presented which optimizes strategic defense. Surviving population is considered the measure of the effectiveness of the defense system which can include active and passive elements. If the attack size and defense budget are specified, the model produces optimum active and passive defense mixes and allocates defense spending to regions of the United States according to population density.

A DEFENSE SYSTEM DESIGN MODEL

R. A. Uher*

I. INTRODUCTION

This paper presents a mathematical method for solving a problem of optimum strategic defense allocation. The model is limited since it does not include the interface with the strategic offense or national policies and objectives. Surviving population has been singled out as the measure of effectiveness of the defense system, which, in the light of the total strategic picture, is probably insufficient.

Other measures of defense effectiveness which we have not included are

1. The cost to the enemy to overcome the defense.

2. The time necessary for the enemy to develop the technology to overcome the defense.

3. The degree to which the system helps prevent the conflict from erupting in the first place.

The first of these requires that the offense build more weapons or other devices which have the capability of negating the defense. If he chooses to do this, the cost he incurs may be a measure of the defense quality.

The second measure of defense effectiveness recognizes that a more realistic picture of conflict is a dynamic one with changing threats and concepts of defense. A measure of effectiveness is the amount of time necessary for the offense to technologically negate the defense system. During this time, advanced technological defense concepts are developed to keep ahead of the offense.

Finally, the last measure of effectiveness is the ability of the defense to prevent an all-out nuclear war either by preventing damage caused by accidental launchings or by producing such a strong defense that an attack seems unwise in the eyes of the offense.

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*U. S. Army on loan to Oak Ridge National Laboratory.

As a measure of effectiveness some combination of these objectives is probably better than any one of them.

In the outline of the civilian defense allocation model to be presented, we assume a two-person, constant-sum game. The offense wants to destroy the most people by distributing his weapons over the United States in the "best" way while the defense desires to save the most people by allocating dollars to localities in the United States in some "best" way. The Lagrange Multiplier formalism is applied to find the solutions to the game. We make no attempt to solve the larger game where strategic offense and defense of N-countries are considered.

We are concerned with the problem of the design of defense systems at fixed budgets which save the most people. Although we restrict attention to surviving population, the analysis which follows can be applied to other kinds of surviving value.

A defense system can be categorized according to the behaviour of the offense and defense before and during engagement. There are three classes:

1. The Offense Favored System against which the offense has the last move. The offense chooses the strategy to minimize survivors against the optimum defense strategy. A fixed-shelter system is a good example of an offense favored system if the offense has good knowledge of the defense system.

2. The Defense Favored System in which the defense has the last move. The defense chooses the strategy to maximize survivors against the optimum offense strategy. An active defense is a defense favored system if the defense has good knowledge of the offense system.

3. The system in which neither the offense nor defense is favored. Both the offense and defense choose the strategy which optimizes survivors against all strategies of the opposing side. This system does not base its strategy on the strategy of the other side. It will be shown later that such a system can be treated by considering it to be simultaneously defense and offense favored.

The following discussion deals with the mathematical formulation of the problem, a method of solution, and application to specific defense systems. We include passive defense, active defense, and mixes of both systems. The limitations of the method are also discussed.

II. MATHEMATICAL FORMULATION OF PROBLEM

To solve the design problem, we divide the geographical area of the defended country into localities within which attacking weapon and defense effects are confined. Because of this confinement, the localities, or more properly, cells, are considered independent. The number of survivors after an attack size N and defense expenditure C is

$$S = \sum_i S_i(n_i, c_i) \quad , \tag{1}$$

where S_i is the number of survivors in the i'th cell, which has been attacked by n_i out of a total of N weapons and which has received c_i out of total defense expenditure C. The total attack is constrained to N weapons,

$$N = \sum_i n_i \quad , \tag{2}$$

and the set $\{n_i\}$ is called the offense strategy. Likewise, the defense expenditure is constrained to C dollars,

$$C = \sum_i c_i \quad , \tag{3}$$

and the set $\{c_i\}$ is called the defense strategy. There is an additional constraint; namely, if a defense cost c_i in the i'th cell can be spent among many elements of a defense system (such as active and passive defense, warning, rescue, etc.) then it must be spent in such a way as to maximize S_i. The actual results of this maximization procedure (that is the distribution of c_i among the component systems) at this cellular level will be called a tradeoff at the local level, while the results of the procedure summed over all cells will be called the tradeoff at the national level. Equations (2) and (3) are labeled the offense and defense constraints, respectively. When the terms offense $\{n_i\}$ or defense $\{c_i\}$

strategies are used, they will be used within the implicit boundary of offense and defense constraints unless otherwise indicated. The problem presented here fits the description of a two-person, constant-sum game. The game theoretic story appears in the appendix.

We formulate the offense favored problem. For each defense strategy $\{c_i\}$, an offense strategy $\{\hat{n}_i\}$ is chosen to minimize survivors

$$S\,[\{c_i\},\ \{\hat{n}_i\}] \ \leq\ S\,[\{c_i\},\ \{n_i\}] \quad . \tag{4}$$

The strategy $\{\hat{n}_i\}$ depends on the particular defense strategy chosen. The defense strategy $\{\hat{c}_i\}$ is chosen which maximizes the number of survivors against all offense strategies $\{\hat{n}_i\}$ which satisfy inequality (4)

$$S\,[\{\hat{c}_i\},\ \{\hat{n}_i\}] \ \geq\ S\,[\{c_i\},\ \{\hat{n}_i\}] \quad . \tag{5}$$

Inequalities (4) and (5) are represented in one relation with the introduction of new notation

$$S\,[\{\hat{c}_i\},\ \{\hat{n}_i\}] \ =\ \underset{\{c_i\}}{\mathrm{MAX}}\left(\underset{\{n_i\}}{\mathrm{MIN}}\ S\,[\{c_i\},\ \{n_i\}]\right). \tag{6}$$

Equation (6) is also referred to as the max-min solution.

The defense favored problem is formulated in two steps. For each offense strategy $\{n_i\}$, a defense strategy $\{c_i^*\}$ is chosen to maximize survivors

$$S\,[\{c_i^*\},\ \{n_i\}] \ \geq\ S\,[\{c_i\},\ \{n_i\}] \quad . \tag{7}$$

The offense strategy $\{n_i^*\}$ is picked to minimize survivors against all defense strategies $\{c_i^*\}$ satisfying inequality (7)

$$S\,[\{c_i^*\},\ \{n_i^*\}] \ \leq\ S\,[\{c_i^*\},\ \{n_i\}] \quad . \tag{8}$$

Inequalities (7) and (8) merge into one relation

$$S\,[\{c_i^*\},\ \{n_i^*\}] \ =\ \underset{\{n_i\}}{\mathrm{MIN}}\left(\underset{\{c_i\}}{\mathrm{MAX}}\ S\,[\{n_i\},\ \{c_i\}]\right). \tag{9}$$

Equation (9) is also referred to as the min-max solution. The defense favored solution is better for the defense than the offense favored since it may represent a higher survival level

$$S\,[\{\hat{c}_i\},\ \{\hat{n}_i\}] \ \le\ S\,[\{c_i^*\},\ \{n_i^*\}] \quad .$$

The defense system which is neither offense nor defense favored has been solved during the discussion of the first two categories of defense systems. The defense chooses the strategy $\{\hat{c}_i\}$ which maximizes survivors against the best offense strategy. The offense chooses the strategy $\{n_i^*\}$ which minimizes survivors against the best defense strategy.

An offense favored system means that the offense moves against a known defense strategy. A defense favored system requires that the defense fight a known offense strategy. Since a system which is neither offense nor defense favored means that offense and defense moves are hidden from each other, then the following relation holds

$$S\,[\{\hat{c}_i\},\ \{\hat{n}_i\}] \le S\,[\{\hat{c}_i\},\ \{n_i^*\}] \le S\,[\{c_i^*\},\ \{n_i^*\}] \quad .$$

When many cells and large forces are involved, the task of finding the optimum strategies is formidable. The introduction of Lagrange multipliers to soften the constraints simplifies things.

III. SOLUTIONS USING LAGRANGE FORMALISM

The local Lagrangian at cell i is defined by the equation

$$L_i(\lambda,\ \mu,\ c_i,\ n_i) \ = \ S_i(c_i,\ n_i) + \lambda n_i - \mu c_i \ , \tag{10}$$

where λ and μ are positive numbers independent of the index i. The total Lagrangian for the system of cells is

$$L(\lambda,\ \mu,\ C,\ N) = \sum_i L_i(\lambda,\ \mu,\ c_i,\ n_i) \quad . \tag{11}$$

Everett[1] has shown that for any defense strategy $\{c_i\}$, the minimization of the local Lagrangian with respect to n_i produces an optimum offense

strategy $\{\hat{n}_i\}$ against the defense strategy $\{c_i\}$

$$S\,[\{c_i\},\ \{\hat{n}_i\}] \ \leq\ S\,[\{c_i\},\ \{n_i\}] \quad . \tag{12}$$

For each offense strategy $\{n_i\}$, the maximization of the local Lagrangian with respect to c_i produces an optimum defense strategy $\{c_i^*\}$ against the offense stragegy $\{n_i\}$

$$S\,[\{c_i^*\},\ \{n_i\}] \ \geq\ S\,[\{c_i\},\ \{n_i\}] \quad . \tag{13}$$

Pugh[2] has pointed out that maximization of the local Lagrangian $L(\lambda,\ \mu,\ c_i,\ \hat{n}_i)$ with respect to c_i <u>may</u> produce the optimum defense strategy $\{\hat{c}_i\}$ to be used with an offense favored system. The choice of $\hat{c}_i$ subject to the inequality

$$L_i(\lambda,\ \mu,\ \hat{c}_i,\ \hat{n}_i) \ \geq\ L_i(\lambda,\ \mu,\ c_i,\ \hat{n}_i) \quad , \tag{14}$$

generally leads to the result*

$$S\,[\{\hat{c}_i\},\ \{\hat{n}_i\}] \ \geq\ S\,[\{c_i\},\ \{\hat{n}_i\}] \quad . \tag{15}$$

However, the answer must be verified employing a procedure described by Pugh.[2] Similar arguments can also be applied to the defense favored problem. The choice of n_i^* subject to the inequality

$$L_i(\lambda,\ \mu,\ c_i^*,\ n_i^*) \ \leq\ L_i(\lambda,\ \mu,\ c_i^*,\ n_i) \quad ,$$

generally leads to the result

$$S\,[\{c_i^*\},\ \{n_i^*\}] \ \leq\ S\,[\{c_i^*\},\ \{n_i\}] \quad , \tag{16}$$

which again must be verified.

These results are summarized in the following statement. For a *defense (offense) favored system a min-max (max-min)* solution of all local Lagrangians generally produces a min-max (max-min) solution of the survival problem. We now consider the existence of max-min or min-max solutions for the local Lagrangian.

---

*Hugh Everett, private communication.

The properties of the local survival function determine whether or not the max-min or min-max solutions of the local Lagrangian exist. The local survival function, $S(c,n)$, (we drop all i-subscripts when considering a single cell) is monotone increasing in the variable c with an upper bound of P (the cell population) and monotone decreasing with variable n, vanishing as n gets very large. This statement is sufficient to prove that the Lagrangian

$$L = S(n,c) + \lambda n - \mu c \quad , \tag{17}$$

always has a minimum with respect to the variable n for each value of c and a maximum with respect to the variable c for each value of n. This is illustrated in Fig. 1.

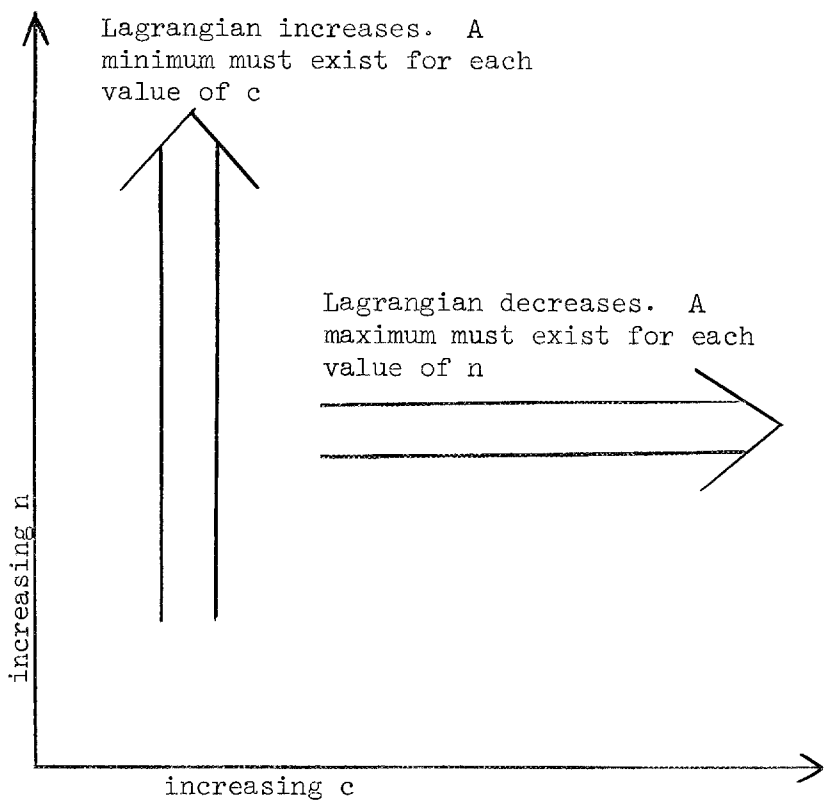

Fig. 1

If we call the value of c which produces a maximum of the Lagrangian for each value of n, $\bar{c}(n)$; and call the value of n which produces a minimum of the Lagrangian for each value of c, $\bar{n}(c)$; then, if the functions $\bar{n}(c)$ and $\bar{c}(n)$ are bounded, the respective max-min and min-max will exist. To prove this one uses the fact that S is bounded if one of the variables is fixed (the fixed variable corresponding to the bounded function $\bar{n}(c)$ or $\bar{c}(n)$ and the Lagrangian then either decreases as a function of c as the bound of $\bar{n}(c)$ is reached or increases as a function of n as the bound $\bar{c}(n)$ is reached.

If the functions $\bar{n}$ and $\bar{c}$ are not bounded, the min-max or max-min may or may not exist depending on the rate of change of the slope of the survival functions at large values of c or n. We must examine <u>each</u> of the particular survival functions to ascertain this condition.

The usual procedure for solving the optimum allocation problem is to pick values for λ and μ, maximize the minimum (minimize the maximum) of the local Lagrangian with respect to c_i and n_i <u>taking into account other offense</u> and <u>defense constraints</u> and sum the results to find the global constraints N and C.

IV. THE SURVIVAL FUNCTION

The cellular model for defense system optimization is valid if the local or cell survival function is independent of the offense and defense effects of other cells. Each element of a grid representation[3] of the U.S. population could be the basic cell. The geographical area, A, of each grid element is chosen so that the area of lethal weapons effects is within the area of the cell. This guarantees the cellular independence of the local survival function. The fatalities caused by the n'th weapon are calculated with the assumption that the survivors of the (n-1)'th weapon are distributed uniformly over the cell. It can be shown, however, that this condition is equivalent to saying that the weapons are randomly targeted against the area of the cell.

We assume that $I^{(i)}$ interceptors have been allocated to defend the i'th cell and each weapon has a penetration probability q if θ interceptors are expended. Although we restrict attention to all warhead payloads, the average effects of including decoys can be studied by varying θ. The attack size, n_e, necessary to exhaust the defense is defined by the equation

$$I^{(i)} = \theta \; n_e^{(i)} \quad . \tag{18}$$

Passive defense in a cell is characterized by a fixed blast shelter system having overpressure protection p_i. Although we assume full occupancy of shelters the method is easily extended to include other situations. The first weapon penetrating the active defenses of the i'th cell kills φ_i of the population

$$\varphi_i = \frac{A_w(p_i, Y)}{A} \quad , \tag{19}$$

where A_w is the lethal area of a weapon of yield Y against a population with overpressure protection p_i. If there are k-penetrating weapons, the expression for the number of survivors is

$$S_i(k) = P_i \; [1 - \varphi_i]^k \quad , \tag{20}$$

where P_i is the population of the i'th cell.

In the presence of an active defense the survival function can be written as a sum over terms having the form of $S_i(k)$ and weighted by factors representing probabilities of having various numbers of penetrating weapons. The survivors in the i'th cell with n_i attacking weapons is

$$S_i(n_i, c_i) = P_i \sum_{k=0}^{n_i} \bar{B} \; (q_i^{(1)} | q_i^{(2)} | k \; | n_i) \; [1 - \varphi_i]^k \quad , \tag{21}$$

where $\bar{B} \; (q_i^{(1)} | q_i^{(2)} | k | n_i)$ is the probability of k weapons penetrating out of n_i. The quantities $q_i^{(1)}$ and $q_i^{(2)}$ are single warhead penetration probabilities before and after exhaustion of the active defenses. Of

course, after exhaustion, the single warhead penetration probability is the single warhead reliability. The defense cost c_i depends upon the number of interceptors and the overpressure protection allotted to the cell.

We assume that overpressure is the only kill mechanism which means that fallout and thermal protection have been provided. The survival function will be described for three cases.

a. Passive Defense Only

b. Active and Passive Defense, Leakage Attacks

c. Active and Passive Defense, Exhaustion Attacks

a. Passive Defense Only

Since no interceptors defend the area, the penetration probability is just the single weapon realiability

$$q_i^{(2)} = q_o \quad , \tag{22}$$

and we may write the survival function

$$S_i(n_i, c_i) = P_i \sum_{k=0}^{n_i} B\,(q_o|k|n_i)\,[1 - \varphi_i]^k \quad , \tag{23}$$

where B is the probability of having k arrivals out of n_i launches

$$B\,(q_o|k|n_i) = \binom{n_i}{k} q_o^k\,(1 - q_o)^{n_i - k} \quad , \qquad \text{and} \tag{24}$$

where $\binom{n_i}{k}$ is the combinatorial symbol

$$\binom{n_i}{k} = \frac{n_i!}{k!(n_i-k)!} \quad .$$

If we substitute equation (24) into equation (23) and work out the algebra, the result is

$$S_i(c_i, n_i) = P_i\,[1 - q_o\varphi_i]^{n_i} \quad . \tag{25}$$

Equation (25) is the form of the survival function to be used with no active defense.

b. Passive and Active Defense, Leakage Attacks

If the weapons launched penetrate the active defense by leakage alone (i.e., the number of weapons is less than or equal to the number necessary to exhaust the defense)

$$n_i \le n_e^{(i)} \quad , \tag{26}$$

then the same argument applies as in the derivation of equation (25) except that the penetration probability ($q_i^{(i)} = q$) replaces the quantity ($q_i^{(2)} = q_o$) in equation (25)

$$S_i(c_i,n_i) = P_i\,[1 - q\varphi_i]^{n_i} \qquad n_i \le n_e^{(i)} \ . \tag{27}$$

c. Passive and Active Defense, Exhaustion Attacks

Another complication enters when the attacking weapons exhaust the defense. The probability of realizing k penetrators depends not only upon the total number of weapons but also upon the number necessary for exhaustion. If n_i weapons are launched, then $n_e^{(i)}$ of them penetrate with probability q while $(n_i - n_e^{(i)})$ penetrate with probability q_o. Under these circumstances, the probability of having k penetrators is

$$\bar{B}(q|q_o|k|n_i) = \sum_{k_1=0}^{k} B(q|k_1|n_i)\, B(q_o|k-k_1|n_e^{(i)} - n_i) \quad , \tag{28}$$

where

$$\begin{aligned} B(q|a|b) &= \binom{b}{a} q^a (1-q)^{b-a} && a \le b \\ &= 0 && a > b \ . \end{aligned} \tag{29}$$

Equation (28) is substituted into equation (21) and the result is

$$S_i(c_i,n_i) = P_i \sum_{k=0}^{n_i} (1-\varphi_i)^k \sum_{k_1=0}^{k} B(q|k_1|n_i)\, B(q_o|k-k_1|n_e^{(i)} - n_i) \qquad n_i > n_e^{(i)} \ . \tag{30}$$

Equation (30) simplifies if the indicated summations are done. The result for the survival function is

$$S_i(c_i,n_i) = P_i\,(1 - q\varphi_i)^{n_i} \qquad n_i \le n_e^{(i)}$$
$$= P_i\,(1 - q\varphi_i)^{n_e^{(i)}}\,(1 - q_o\varphi_i)^{n_i - n_e^{(i)}} \qquad n_i > n_e^{(i)}\,. \tag{31}$$

The functional dependence of φ_i on the defense cost c_i is discussed in the following sections.

V. PASSIVE DEFENSE

We assume that the blast shelter is the basic unit of a passive defense system. It is characterized by a mid-lethal overpressure (referred to as overpressure protection) at which 50% of the occupants become fatalities. All shelters in a cell have the same mid-lethal overpressure rating. Each cell or locality contains enough blast shelters so that all people in the cell can be sheltered given sufficient warning. The people in a cell not provided with blast protection have thermal and fallout shelters which have an incidental overpressure protection of about 10 psi. These thermal and fallout shelters are provided at zero cost.

If we assume that attacking weapons burst on the surface, the function φ_i has the approximate form[4]

$$\varphi_i = \frac{293\ Y^{2/3}}{A_i p_i}\ , \tag{32}$$

where p_i is the overpressure protection (psi) afforded the people in the i'th cell, Y is the yield of the weapon in megatons, and A_i is the geographical area of the cell in square kilometers. If all people in a locality do not reach shelter, the survival function in equation (25) must be modified to

$$S_i(c_i,n_i) = P_i[\{1 - f_i\}\{1 - q_o\varphi_i^{(o)}\}^{n_i} + f_i\{1 - q_o\varphi_i\}^{n_i}]\ , \tag{33}$$

where f_i is the fraction of the people in the i'th cell who are sheltered, $\varphi_i^{(o)}$ and φ_i is the fraction of those not sheltered and sheltered, respectively, who are fatalities.

The cost of blast shelters is not well established. For example, the Project Harbor Summary Report[5] lists the cost of a 1000 space, 100 psi shelter (10 sq. ft per space) at $175 per space, while Robbins and Narver[6] lists the cost of the Tunnel Grid (again in 10 sq. ft per space) at about $500 per space. One reason for this ambiguity is the lack of a precise definition of overpressure protection. A systems analyst must know the overpressure at which blast shelters fail while the shelter designer gives the overpressure at which the shelter will be safe. There have been several estimates of the variation of cost with overpressure. For overpressures in the range from 10 to 200 or 300 psi the cost varies with the square root of the overpressure. The LD-50 and LD-85 overpressures[7] for various design or rated overpressures are plotted in a graph in Fig. 2. Figure 3 shows the cost of a 7.5 sq. ft blast shelter space as a function of the mid-lethal overpressure. This graph results from costs used in several system studies.[7] In dollars, the cost per space is

$$C = 100 + 15\ p^{1/2} \quad . \tag{34}$$

This formula is valid to a design overpressure of about 200-300 psi. Beyond that, the cost-overpressure relationship may become linear.[8]

Equation (33) expresses the survival as a function of the fraction sheltered f_i and the overpressure protection p_i. The fraction sheltered depends upon the nature of the threat, the warning system and shelter accessibility. Therefore, it is conceivable that the defense can trade-off overpressure protection for increased shelter occupancy by diverting part of the defense funds to better the warning system and educational program. Unfortunately, the offense can also manipulate the occupancy factor f. There have been no studies of any significance addressing the shelter occupancy problem. Therefore, we assume that f_i is a parameter and we begin by taking $f_i = 1$ for all i.

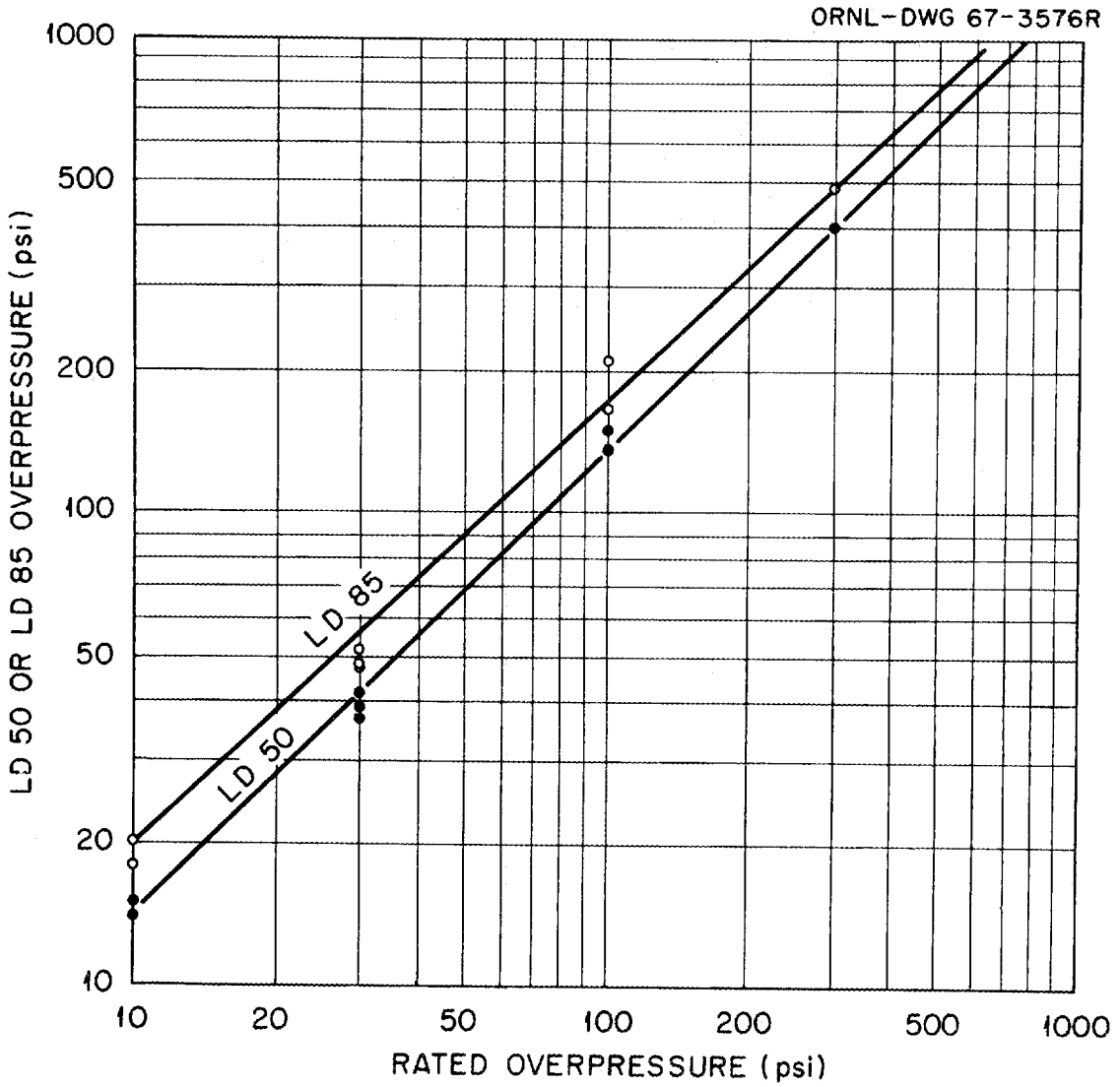

Fig. 2. LD-85 and LD-50 Overpressure as a Function of Design Overpressure.

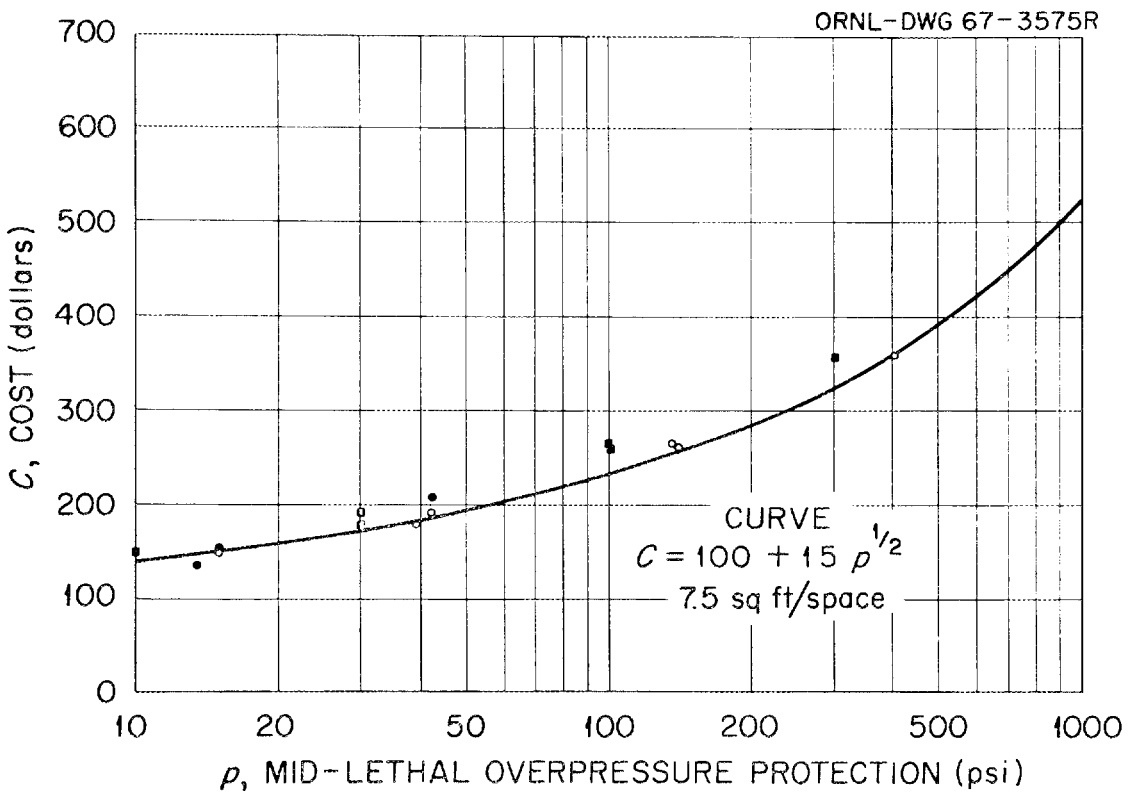

Fig. 3. The Cost of Blast Shelters as a Function of Mid-Lethal (LD-50) Overpressure Rating.

If the objective of the offense is the destruction of a certain number of people, he can overcome any passive defense system by increasing the number of weapons he intends to direct toward this objective. This increase in number of weapons by the offense can be used as a measure of the effectiveness of the defense. More specifically the defense strategy is chosen at some survival level to maximize the additional number of weapons needed by the offense to negate the defense. This kind of study has been done in relation to active defense.

VI. INCLUSION OF ACTIVE DEFENSE

The design of defense systems which contain active and passive elements is more complicated than the design of systems having only one element. Before the Lagrangian method outlined in section IV can be applied, the local survival function must contain the active and passive mix which maximizes survivors for any local defense cost. This procedure is called a tradeoff.

The geographical area covered by a terminal active defense is larger than the area of lethality of weapons which might consider penetrating this defense. If the basic cells are chosen to have areas slightly larger than lethality areas then defense effects extend over more than one cell, which are no longer independent. However, we may remedy this difficulty. The active defense system cost (radars, data processors, etc.) is expressed in the cost of an "average" interceptor. The cost of one "average" interceptor is the total active defense system cost divided by the total number of interceptors. We allot "average" interceptors to the cells in a manner consistent with the tradeoff and Lagrange solutions. Radar and associated equipment are added to complete the system. The costs of "real" interceptors, radars and all other equipment are tallied to obtain the real system cost. We require that this cost be identical to the cost of the "average" interceptor multiplied by the number of "average" interceptors.

To tradeoff interceptors for overpressure protection at the cellular level, we let s designate the "average" interceptor cost. The cost allotted to the i'th cell may then be expressed by the equation

$$c_i = sI_o^{(i)} \quad . \tag{35}$$

The passive defense cost depends upon the number of people protected (P_i) in the cell and the overpressure protection (p_i) to which they are protected. The tradeoff equation is

$$sI_o^{(i)} = sI^{(i)} + P_i(c_1 + c_2 p_i^{1/2}) \quad . \tag{36}$$

The first term on the right hand side is the active defense cost while the second term is the passive defense cost. If equation (36) is rearranged to express the overpressure protection in terms of the number of interceptors, we obtain

$$p_i = \left[\frac{s}{c_2 P_i} \left(I_o^{(i)} - I^{(i)}\right) - \frac{c_1}{c_2} \right]^2 . \tag{37}$$

The quantity, $I_o^{(i)}$, is the maximum number of interceptors bought for the i'th cell if nothing is spent for passive defense.

The survivors depend upon attack (n_i), total cost ($I_o^{(i)}$), and the mix of interceptors and overpressure protection [well defined by $I^{(i)}$ given $I_o^{(i)}$] and equation (37). This relation is expressed in functional form

$$S_i = S_i\left(n_i, I_o^{(i)}, I^{(i)}\right) . \tag{38}$$

The <u>best</u> active-passive mix is obtained by choosing from the set of $I^{(i)}$, for fixed values of n_i and $I_o^{(i)}$, the value of $I^{(i)}$ (call it $I_{max}^{(i)}$) such that

$$S_i(n_i, I_o^{(i)}, I_{max}^{(i)}) \geq S_i(n_i, I_o^{(i)}, I^{(i)}) \quad . \tag{39}$$

The value of $I_{max}^{(i)}$ satisfying equation (39) can be represented by the functional relationship

$$I_{max}^{(i)} = I_{max}^{(i)}\left(n_i, I_o^{(i)}\right) \quad . \tag{40}$$

Lagrange's method applied to this new survival function produces the "best" defense distribution, the local active and passive defenses, and the "best" weapon distribution. The national active defense cost is

$$C_A = s \sum_i I_{max}^{(i)} , \tag{41}$$

while the total defense cost is

$$C = s \sum_i I_o^{(i)} . \tag{42}$$

VII. THE NATIONAL DEFENSE CALCULATION

If the basic cells are elements of a grid superimposed over the United States, then they are characterized only by population in each element. Let $M(P)dP$ be the number of cells with population between P and P + dP. The graph in Fig. 4 shows the population of 5 km grid squares ranked according to population as a function of the rank. Figure 5 shows the total population for all 5 km grid squares up to a certain rank again, ranked according to population. The curve in Fig. 5 is the integral of the curve in Fig. 4. The function $M(P)dP$ can be obtained from these curves.

The solutions to the defense allocation problem are expressed as functions of P, first the weapon distribution

$$n = n(P) \qquad (\text{replacing } n_i) , \tag{43}$$

second, the defense cost distribution

$$I_o = I_o(P) \qquad (\text{replacing } I_o^{(i)}), \tag{44}$$

and, finally, the active/passive mix

$$I_{max} = I_{max}(P) \quad (\text{replacing } I_{max}^{(i)}). \tag{45}$$

The constraints are obtained by integrating equations (43), (44), or (45) over all possible values of P weighted by the function $M(P)dP$.

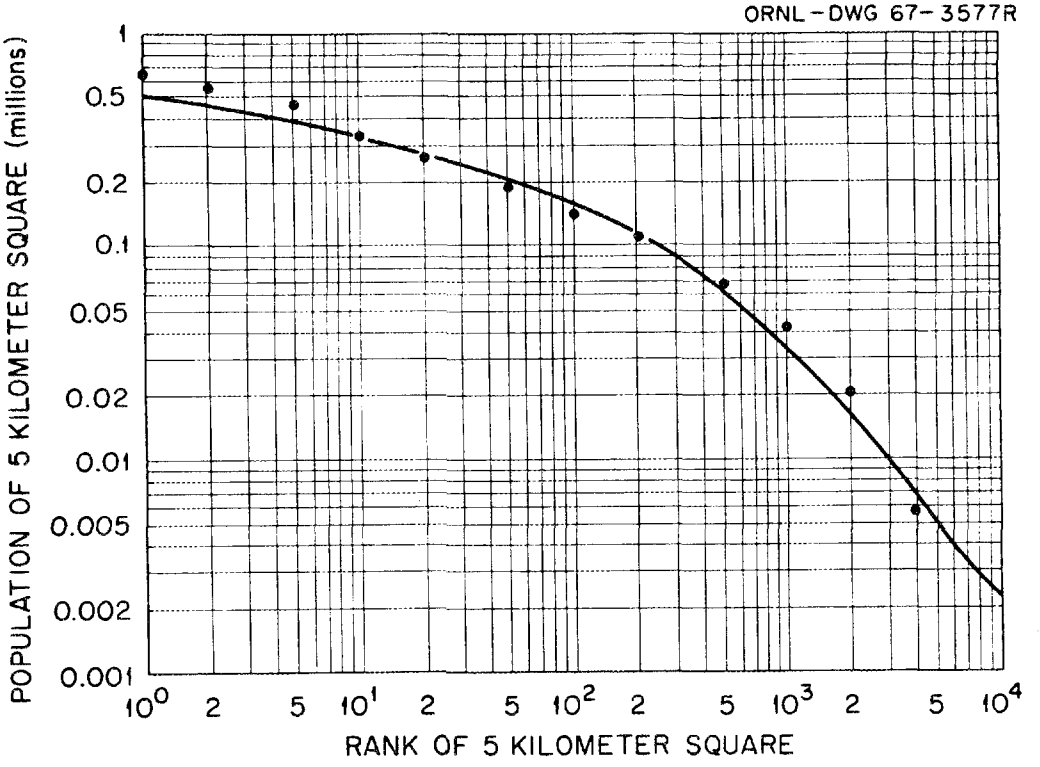

Fig. 4. Population of a 5-km Square as a Function of the Rank of the 5-km Square in a Rank-Ordering According to Population.

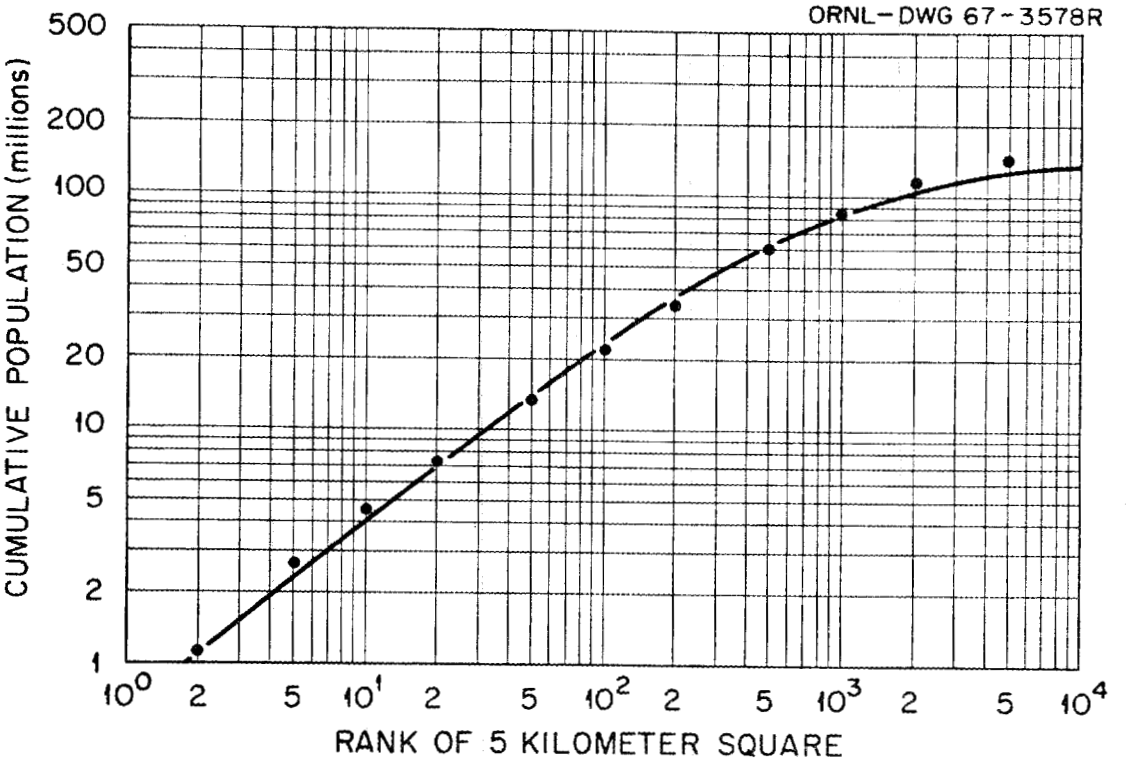

Fig. 5. Total Population of the 5-km Squares up to Rank N in a Rank-Ordering of the 5-km Squares According to Population.

The attack size is

$$N = \int_{o}^{P_{max}} M(P)\ n(P)dP \quad , \tag{46}$$

the total defense cost is

$$C = s \int_{o}^{P_{max}} M(P)\ I_{o}(P)dP \quad , \tag{47}$$

and the active defense cost is

$$C_{A} = s \int_{o}^{P_{max}} M(P)\ I_{max}(P)dP \quad , \tag{48}$$

where P_{max} is the grid element with the most people.

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APPENDIX A

The defense system design model presented in this paper is a two-person, constant-sum game. The players are the offense and the defense. The offense strategies $\{n_i\}$ are distributions of weapons over the geographical area of the United States subject to a total attack size constraint. Likewise, the defense strategies $\{c_i\}$ are allocations of defense dollars to these same areas constrained by total budget.

The game matrix has a very large dimension so that a complete specification of the payoffs is literally impossible except perhaps for the smallest attack sizes and defense expenditures. A second point well remembered is that the game can be played only once and the stakes are so high that the desired solution is not a mixed strategy where we must worry about a standard deviation in a probability, but rather a pure strategy. There are three games that must be considered.

1. The all-seeing offense game in which the offense sees the chosen defense strategy. If the offense chooses to play this game then the optimum solution for both sides is offense favored.

2. The all-seeing defense game in which the defense sees the offense chosen strategy. If the defense chooses to play this game then the optimum solution for both is defense favored.

3. The blind-offense and blind-defense game in which neither side sees the other's move. The optimum solution for both sides favors neither offense nor defense. In the language of game theory if the defense and offense have the rows and columns, respectively, of the payoff matrix which represents survival then the solutions to the three games described above are

a. The defense strategy for the offense favored solution.

b. The defense strategy for the defense favored solution.

c. The defense strategy for the "unfavored" solution.

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