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NUCLEAR DIVISION
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ORNL - TM - 2045

COPY NO. - 83

DATE - June 28, 1968

Calculation of Properties of Steam from the N. E. L. Steam Tables

E. J. Lee

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A set of FORTRAN function subroutines has been written for calculation of the specific volume and enthalpy of steam in the region, 350°C. - 800°C. temperature and 1-1000 bars pressure. Routines have also been written for the viscosity and thermal conductivity.

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INTRODUCTION

The basic reference for this report is the N. E. L. Steam Tables[1] where the following equation of state fitted to experimental values for the region, 350°C. - 800°C. and 1-1000 bars, is given:

$$W = \frac{P - RT\rho}{RT\rho^2} = \sum_{r=o}^{m} \sum_{s=o}^{n} a_{rs} \, T_r^*(X) \, T_s^*(y) \qquad (1)$$

where ρ = density in g/cm^3,

P = pressure in bars,

T = temperature in ^oK,

R = gas constant = 4.6152 bar cm^3/g^{o}K,

$X = \rho/\rho\ max$, $y = (\frac{1}{T} - \frac{1}{T max})/(\frac{1}{T\ min} - \frac{1}{T\ max})$,

$\rho\ max$ = maximum density = 0.70839 g/cm^3,

$T\ max$ = maximum temperature = 1178.15°K,

$T\ min$ = minimum temperature = 468.15°K,

the a_{rs} are the coefficients for the least squares fit to equation (1) and T_j^* (u) denotes the j^{th} degree shifted Chebyshev polynomial in u.

1Department of Scientific and Industrial Research, National Engineering Laboratory, Steam Tables 1964, prepared by R. W. Bain; Her Majesty's Stationery Office, Edinburgh (1964).

For the enthalpy, there is given in the Steam Tables[1] the derivation of the equation:

$$U = \frac{h - h_o}{RT} = \sum_{r=o}^{m+1} \sum_{s=o}^{m} b_{rs}\, T_r^*(x)\, T_s^*(y) \qquad (2)$$

where h = enthalpy in J/g,

R, T, $T^*(x)$ and $T^*(y)$ are as in (1) and

$h_o \equiv$ zero pressure enthalpy

$= 1809.25 + 1.48286\ T + 3.79025 \times 10^{-4}\ T^2 + 46.174\ \ell n\ T.$

The Steam Tables contain other equations for the calculation of the saturation pressure, viscosity and thermal conductivity which are suitable for direct evaluation as they stand. Routines have been provided for these but no details will be given herein other than a description of input and output. The routines provided for the user are all function type subroutines and common statements have been avoided. Input and output values are in appropriate engineering units.

METHODS OF CALCULATION

a. Specific volume from temperature and pressure

We use Newton's method to solve (1) for ρ.

Letting $f(\rho) = RT\ \rho^2\ W(\rho) + RT\rho - P,$ (3)

we take $\rho_o = P/RT$ and iterate

$\rho_{i+1} = \rho_i - f(\rho_i)/f'(\rho_i)$ until

$|(\rho_{i+1} - \rho_i)/\rho_i| < 10^{-4}$ with $f'(\rho_i)$

approximated by $(f(\rho_i + \delta) - f(\rho_i - \delta))/2\delta$ where $\delta = 10^{-4} \times \rho_i$. The specific volume, V_s, is then given by $V_s = 1/\rho$.

b. Enthalpy from temperature and pressure. From (2) the enthalpy is: $h = h_o + RTU(\rho)$ (4)

ρ is found as in a. above and substituted in (4) to give the enthalpy, h.

c. Pressure from temperature and enthalpy. Using Newton's method to solve (2) for ρ, let $f(\rho) = h - h_o - RTU(\rho)$ (5) with $\rho_o = 0.01$ and iterate as in a. above. Solving (1) for the pressure then gives $P = RT\rho^2 W(\rho) + RT\rho$.

d. Temperature from enthalpy and pressure

The method of interval-halving is used to solve (2) for T. We take $T_o = T_c + \delta_o$ where $\delta_o = 0.5 * (1073.15 - T_c)$ and $T_c \equiv$ saturation temperature evaluated by Newton's method applied to the saturation line equations given in the Steam Tables, p. 3, and iterate on $f(T_i) = h - h_o(T_i) - RTU(T_i, \rho(T_i))$ with $\delta_{i+1} = 0.5 * \delta_i$ and $T_{i+1} = T_{i+1} \pm \delta_{i+1}$ until $|f(T_i)| < 0.0001$. $\rho(T_i)$ is obtained as described in a. above.

FUNCTION SUBPROGRAMS

In the following T = temperature in ^{o}F. and P = pressure in lbs/in^2

a. SV(T, P)

SV = specific volume in ft^3/lb.

b. ENTH(T, P)

ENTH = enthalpy in Btu/lb.

c. ENTR(T, ENTH)

ENTR = pressure in lbs/in^2.

d. ENTM(ENTH, P)

ENTM = temperature in ^{o}F.

e. VISC(T, P)

VISC = viscosity in micropoises.

f. TCON(T, P)

TCON = thermal conductivity in milliwatts/m ^{o}K.

In a. through d. the return argument is set to 0.0 if the pressure exceeds the saturation pressure . In e. and f. the return argument is set to 0.0 if the temperature < 100^{o}C or pressure < 1 bar.

The shifted Chebyshev polynomials are defined by $T_o^*(u) = 1$, $T_1^*(u) = 2u-1$ and the recursion relation,

$$T_{j+1}^*(u) = 2T_1^*(u)\, T_j^*(u) - T_{j-1}^*(u),\quad j \geq 1 \qquad (7)$$

They were computed by numerical recursion from (7).

The Steam Tables give the values of the a_{rs}. The b_{rs} were computed from the a_{rs} using the relation derived in the Steam Tables, p. 19.

A check of the routines disclosed that they are generally accurate to about 1:1000 except near the saturation line where the accuracy is about 1:100. It was found that the results were of similar accuracy for temperatures $< 350^{o}$C., i.e., when extrapolating beyond the region over which the a_{rs} were fit. The a_{rs} and b_{rs} are listed in Table I.

TABLE I

a_{rs} 350-500 DEG C

-2.933857	-3.560472	-0.752570	-0.355238	-0.171618	-0.088253	-0.023708	-0.002024
2.178508	1.646874	0.257244	-0.286577	-0.191781	-0.118835	-0.020660	0.0
-0.410626	-0.712020	-0.121860	0.067088	0.080874	0.014949	0.0	0.0
0.027044	-0.046842	-0.028628	-0.032120	-0.006524	0.0	0.0	0.0
0.102056	0.134922	0.052882	0.007107	0.0	0.0	0.0	0.0
-0.026332	-0.049185	-0.008993	0.0	0.0	0.0	0.0	0.0
0.002725	0.001175	0.0	0.0	0.0	0.0	0.0	0.0

b_{rs} 350-500 DEG C

-.557699	-.694154	-.335840	-.191929	-.093314	-.035759	-.007474	-.000574
-.517544	-.701325	-.411388	-.260457	-.132506	-.048392	-.008937	-.000574
.028821	-.017969	-.066270	-.056070	-.033804	-.011927	-.001464	0
-.012767	-.014584	.006493	.010977	.005157	.000706	0	0
.002373	.000893	-.001177	-.001280	-.000231	0	0	0
.002571	.003030	.001343	.000201	0	0	0	0
-.000760	-.001221	-.000212	0	0	0	0	0
.000055	.000024	0	0	0	0	0	0

a_{rs} 500-800 DEG C

-2.684145	-3.110441	-0.406137	-0.143284	-0.063656	-0.048549	-0.012639	-0.000698
1.571884	0.553618	-0.584342	-0.801473	-0.454050	-0.215289	-0.047550	-0.003221
-0.410626	-0.712020	-0.121860	0.067088	0.080874	0.014949	0.0	0.0
0.027044	-0.046842	-0.028628	-0.032120	-0.006524	0.0	0.0	0.0
0.102056	0.134922	0.052882	0.007107	0.0	0.0	0.0	0.0
-0.026332	-0.049185	-0.008993	0.0	0.0	0.0	0.0	0.0
0.002725	0.001175	0.0	0.0	0.0	0.0	0.0	0.0

b_{rs} 500-800 DEG C

-.321009	-.261421	-.019557	-.002132	-.007125	-.006168	-.001326	-.000055
-.381747	-.452753	-.230511	-.152102	-.083838	-.031804	-.005620	-.000312
-.072072	-.202130	-.201676	-.137512	-.071325	-.024930	-.004294	-.000257
-.012767	-.014584	.006493	.010977	.005157	.000706	0	0
.002373	.000893	-.001177	-.001280	-.000231	0	0	0
.002571	.003030	.001343	.000201	0	0	0	0
-.000760	-.001221	-.000212	0	0	0	0	0
.000055	.000024	0	0	0	0	0	0

APPENDIX

SUBPROGRAM LIST (FORTRAN 63)

```
  FUNCTION SV(T,P)
  DATA(RMX=1.4116518),(TI=776.83228),(TIX=0.65936619),(G=4.6152)
  TP=0.55555556*(T-32.0)+273.15 $PP=P/14.5038
  IF(PP.GT.PCRIT(TP)) 7,1
1 RX=RA=PP/(G*TP) $DR=0.0001*RX $TT=TI/TP-TIX $W=CEBAA(TT)
  DO 5 I=1,20 $W=CEBAB(RMX*RX)    $RP=RX+DR
  WP=CEBAB(RMX*RP)    $RM=RX-DR $WM=CEBAB(RMX*RM)
  F=RX*RX*W+RX-RA $FP=RP*RP*WP+RP-RA $FM=RM*RM*WM+RM-RA
  DEL=2.0*DR*F/(FP-FM) $RX=RX-DEL $IF(ABSF(DEL/RX).LT.1.0E-4)6,5
5 CONTINUE
6 SV=0.0160185/RX
  RETURN
7 SV=0.0
  RETURN
  END SV

  FUNCTION ENTH(T,P)
  DATA(RMX=1.4116518),(TI=776.83228),(TIX=0.65936619),(G=4.6152)
  TP=0.55555556*(T-32.0)+273.15 $PP=P/14.5038
  IF(PP.GT.PCRIT(TP)) 7,1
1 TT=TI/TP-TIX $ $R=SGP(TP,PP) $W=CEBB(RMX*R,TT)
  ENTH=G*TP*W+(1809.25+1.48286*TP+TP*TP*3.79025E-04+46.174*LOGF(TP))
  ENTH=0.429923*ENTH
  RETURN
7 ENTH=0.0
  RETURN
  END ENTH

  FUNCTION ENTR(T,ENTH)
  DATA(RMX=1.4116518),(TI=776.83228),(TIX=0.65936619),(G=4.6152)
  TP=0.55555556*(T-32.0)+273.15 $TT=TI/TP-TIX $EN=2.326*ENTH
  H=EN-(1809.25+1.48286*TP+TP*TP*3.79025E-04+46.174*LOGF(TP))
  RT=G*TP $RX=0.01$DR=0.0001*RX $DO 5 I=1,20 $RP=RX+DR $RM=RX-DR
  FP=H-RT*CEBB(RMX*RP,TT) $FM=H-RT*CEBB(RMX*RM,TT)
```

```
      F=H-RT*CEBB(RMX*RX,TT) $DEL=2.0*DR*F/(FP-FM) $RX=RX-DEL
      DR=0.0001*RX
      IF(ABSF(DEL/RX).LT.1.0E-04) 6,5
    5 CONTINUE
    6 PP=RT*RX $W=CEBAA(TT)        $PP=PP*RX*CEBAB(RMX*RX)    +PP
      IF(PP.GT.1.010*PCRIT(TP)) 7,1
    7 ENTR=0.0
      RETURN
    1 ENTR=14.5038*PP
      RETURN
      END ENTR

      FUNCTION ENTM(ENTH,P)
      H=2.326*ENTH $PR=P/14.5038 $IF(PR.LT.221.2) 12,11
   11 T=700.0 $GO TO 3
   12 T=646.75 $DT=0.5 $DO 2 I=1,20
      TP=T+DT $TM=T-DT $FP=PCRIT(TP)-PR $FM=PCRIT(TM)-PR $F=PCRIT(T)-PR
      DEL=2.0*DT*F/(FP-FM) $T=T-DEL $DT=0.001*T
      IF(ABSF(DEL/T).LT.1.0E-04) 3,2
    2 CONTINUE
    3 TA=T $TB=1073.15 $DT=0.5*(1073.15-TA) $R=SGP(T,PR)
      U=FENT(T,R,H) $T=TA+DT $DO 5 I=1,25 $DT=0.5*DT
      R=SGP(T,PR) $V=FENT(T,R,H) $IF(U*V) 31,5,32
   31 TB=T $GO TO 33
   32 TA=T $U=V
   33 T=TA+DT
    5 CONTINUE
   61 ENTM=1.8*(T-273.15)+32.0
      RETURN
      END ENTM

      FUNCTION VISC(T,P)
      PB=P/14.5038 $IF(PB.LT.1.0) 1,2
    1 VISC=0.0
      RETURN
    2 TC=0.55555556*(T-32.0) $IF(TC.LT.100.0) 1,3
    3 V=0.407*TC+80.4 $R=SGP(TC+273.15,PB) $IF(TC.LT.340.0) 4,5
    4 VISC=V-R*(1858.0-5.9*TC)
```

```
      RETURN
    5 VISC=V+R*(R*(R*102.1+676.5)+353.0)
      RETURN
      END VISC

      FUNCTION TCON(T,P)
      PB=P/14.5038 $IF(PB.LT.1.0) 1,2
    1 TCON=0.0
      RETURN
    2 TC=0.55555556*(T-32.0) $IF(TC.LT.100.0) 1,3
    3 C=TC*(TC*(TC*(-4.51E-08)+1.04E-04)+0.0587)+17.6
      R=SGP(TC+273.15,PB) $TCON=C+R*(TC*(TC*(-2.771E-05)+0.4198)+103.51+
     1R*(2.1482E+14)/TC**4.20)
      RETURN
      END

      FUNCTION CEBAA(X)
      DIMENSION A(7,8),TX(7),TY(8),B(7,8)
      DATA(TX=1.0),(TY=1.0)
      DATA(A=-2.933857,2.178508,-0.410626,0.027044,0.102056,-0.026332,0.
     1002725,-3.560472,1.646874,-0.712020,-0.046842,0.134922,-0.049185,0
     2.001175,-0.752570,0.257244,-0.121860,-0.028628,0.052882,-0.008993,
     30.0,-0.355238,-0.286577,0.067088,-0.032120,0.007107,0.0,0.0,-0.171
     4618,-0.191781,0.080874,-0.006524,0.0,0.0,0.0,0.0,-0.088253,-0.118835,0
     5.014949,0.0,0.0,0.0,0.0,0.0,0.0,-0.023708,-0.020660,0.0,0.0,0.0,0.0 0.0,0.0,-0
     6.002024,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0)
      DATA(B=-2.684145,1.571884,-0.410626,0.027044,0.102056,-0.026332,0.
     1002725,-3.110441,0.553618,-0.712020,-0.046842,0.134922,-0.049185,0
     2.001175,-0.406137,-0.584342,-0.121860,-0.028628,0.052882,-0.008993
     3,0.0,-0.143284,-0.801473,0.067088,-0.032120,0.007107,0.0,0.0,-0.06
     43656,-0.454050,0.080874,-0.006524,0.0,0.0,0.0,0.0,-0.048549,-0.215289,
     50.014949,0.0,0.0,0.0,0.0,0.0,0.0,-0.012639,-0.047550,0.0,0.0,0.0,0.0,0.0,0.0,-
     60.000698,-0.003221,0.0,0.0,0.0,0.0,0.0,0.0,0.0)
      TY(2)=X+X-1.0 $DO 1 I=3,8 $TY(I)=2.0*TY(2)*TY(I-1)-TY(I-2)
    1 CONTINUE $IF(X.GT.0.3453965) 12,11
   11 M=0 $GO TO 13
   12 M=1
   13 CEBAA=0.0
```

```
      RETURN
      ENTRY CEBAB
      TX(2)=X+X-1.0 $DO 2 I=3,7 $TX(I)=2.0*TX(2)*TX(I-1)-TX(I-2)
    2 CONTINUE $CEBAA=0.0 $IF(M) 21,22
   21 DO 5 I=1,7 $DO 5 J=1,8 $Z=A(I,J)*TX(I)*TY(J) $CEBAA=CEBAA+Z
    5 CONTINUE
      RETURN
   22 DO 6 I=1,7 $DO 6 J=1,8 $Z=B(I,J)*TX(I)*TY(J) $CEBAA=CEBAA+Z
    6 CONTINUE
      RETURN
      END CEBAA

      FUNCTION CEBB(X,Y)
      DIMENSION B(8,8),TX(8),TY(8),C(8,8)
      DATA(TX=1.0),(TY=1.0)
      DATA(B=-0.5576989,-0.5175436,0.02882124,-0.01276703,0.002372795,0.
     1002571138,-0.0007595265,0.00005515198,-0.6941539,-0.7013254,-0.017
     29692,-0.01458357,0.000893273,0.003030141,-0.001220805,0.0000237816
     36,-0.33584,-0.4113879,-0.06626999,0.006493059,-0.001176619,0.00134
     42822,-0.0002123517,0.0,-0.1919288,-0.2604569,-0.05607008,0.0109770
     59,-0.001279544,0.0002013811,0.0,0.0,0,-0.09331409,-0.132506,-0.03380
     6373,0.005157087,-0.0002310768,0.0,0.0,0.0,-0.03575945,-0.04839214,
     7-0.01192671,0.0007059815,0.0,0.0,0.0.0.0,-0.0074735,-0.008937034,-
     80.001463534,0.0,0.0,0.0,0.0,0.0,-0.0005735125,-0.0005735125,0.0,0.
     90,0.0,0.0,0.0,0.0)
      DATA(C=-0.321009,-0.381747,-0.072072,-0.012767,0.002373,0.002571,-
     10.000760,0.000055,-0.261421,-0.452753,-0.202130,-0.014584,0.000893
     2,0.003030,-0.001221,0.000024,-0.019557,-0.230511,-0.201676,0.00649
     33,-0.001177,0.001343,-0.000212,0.0,-0.002132,-0.152102,-0.137512,0
     4.010977,-0.001280,0.000201,0.0,0.0,-0.007125,-0.083838,-0.071325,0
     5.005157,-0.000231,0.0,0.0,0.0,-0.006168,-0.031804,-0.024930,0.0007
     606,0.0,0.0,0.0,0.0,-0.001326,-0.005620,-0.004294,0.0,0.0,0.0,0.0,0
     7.0,-0.000055,-0.000312,-0.000257,0.0,0.0,0.0,0.0,0.0,0.0)
      TX(2)=X+X-1.0 $TY(2)=Y+Y-1.0 $DO 1 I=3,8
      TX(I)=2.0*TX(2)*TX(I-1)-TX(I-2) $TY(I)=2.0*TY(2)*TY(I-1)-TY(I-2)
    1 CONTINUE $CEBB=0.0 $IF(Y.GT.0.3453965) 11,21
   11 DO 2 I=1,8 $DO 2 J=1,8 $Z=B(I,J)*TX(I)*TY(J) $CEBB=CEBB+Z
    2 CONTINUE
```

```
      RETURN
   21 DO 3 I=1,8 $DO 3 J=1,8 $Z=C(I,J)*TX(I)*TY(J) $CEBB=CEBB+Z
    3 CONTINUE
      RETURN
      END CEBB

      FUNCTION SGP(TP,PP)
      DATA(RMX=1.4116518),(TI=776.83228),(TIX=0.65936619),(G=4.6152)
      RX=RA=PP/(G*TP) $DR=0.0001*RX $TT=TI/TP-TIX $W=CEBAA(TT)
      DO 5 I=1,20 $W=CEBAB(RMX*RX)     $RP=RX+DR
      WP=CEBAB(RMX*RP)      $RM=RX-DR $WM=CEBAB(RMX*RM)
      F=RX*RX*W+RX-RA $FP=RP*RP*WP+RP-RA $FM=RM*RM*WM+RM-RA
      DEL=2.0*DR*F/(FP-FM) $RX=RX-DEL $IF(ABSF(DEL/RX).LT.1.0E-4)6,5
    5 CONTINUE
    6 SGP=RX
      RETURN
      END SGP

      FUNCTION FENT(T,R,H)
      DATA(RMX=1.4116518),(TI=776.83228),(TIX=0.65936619),(G=4.6152)
      FENT=H-G*T*CEBB(RMX*R,TI/T-TIX)-(1809.25+1.48286*T+T*T*3.79025E-04
     *+46.174*LOGF(T))
      RETURN
      END

      FUNCTION PCRIT(T)
      Y=647.26-T $IF(Y) 5,5,6
    5 PCRIT=2000.0
      RETURN
    6 Z=T+0.01 $IF(T.GT.373.15) 2,1
    1 U=28.59051-3.561215*LOGF(Z)+0.0024804*Z-3142.31/Z $GO TO 3
    2 X=Z*Z-293700.0 $Y=Y**1.25$U=1.3869E-04
      D=2.7550431E-11 $U=U*X*(EXPF(D*X*X)-1.0)
      U=5.432368+(U-2005.1)/Z-0.0044*EXPF(-0.0131588*Y)
    3 PCRIT=EXPF(2.3025851*U)
      RETURN
      END PCRIT
```

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