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ORNL- TM- 1312
Volume I

57

STRUCTURAL ANALYSIS OF SHIPPING CASKS

VOL. I. ANALYSIS OF A SHIPPING CASK SUBJECTED TO INTERNAL PRESSURE

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ORNL-TM-1312

Contract No. W-7405-eng-26

General Engineering and Construction Division

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VOL. 1. ANALYSIS OF A SHIPPING CASK SUBJECTED TO INTERNAL PRESSURE

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MARCH 1966

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Abstract

Formulas to be used by the design engineer to calculate the maximum stress produced in a cask used to ship radioactive materials when it is subjected to internal pressure are presented in this report. These formulas were developed for use as a guide to indicate compliance with the proposed Atomic Energy Commission regulation pertaining to the design requirement for resistance to internal pressure; however, use of these formulas does not guarantee compliance with this regulation. Formulas, which take into account the elastic nature of the lead shielding, are derived to determine the thickness of the cavity wall for casks of both prismatic and cylindrical configuration, the formulas are tabulated, and example problems are given. Creep of the lead shield is evaluated, and it is concluded from this evaluation that the creep problem tends to be self-alleviating.

1. INTRODUCTION

This report provides the design engineer with formulas for use in calculating the maximum stresses produced in a cask when it is subjected to internal pressure. These formulas are oriented primarily toward the requirements of the proposed Atomic Energy Commission regulation 10 CFR 71 related to internal pressure.[1]

The proposed regulations pertaining to the shipment of radioactive materials and the shipping casks used to contain radioactive material during transport govern licensees of the Atomic Energy Commission. One of these regulations pertains to the design requirements for resistance to internal pressure, and it states that:

> Packaging (the cask) shall be capable of withstanding an internal gauge pressure within the containment vessel of 20 pounds per square inch or twice the operating gauge pressure, whichever is greater, without generating stress in any material of the packaging in excess of its yield strength.

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[1]Code of Federal Regulations, Title 10 Part 71, "Protection Against Radiation in the Shipment of Irradiation Fuel Elements," Federal Register Document 61-9151, September 1961.

At the present time, essentially all large casks designed and used in the United States are made of lead encased in steel and have a cylindrical or prismatic configuration. The materials most generally used in cask construction are chemical lead and the 300-series stainless steel, but tungsten, depleted uranium, carbon steel, iron, and concrete are sometimes used for shielding purposes. Some typical properties of chemical lead are given in Tables 1 through 3, and some properties of type-347 stainless steel are given in Table 4.

Table 1. Properties of Chemical Lead[a]

Melting point, °F	618
Tensile strength (cast), psi	3000
Modulus of elasticity in tension, psi	2.0×10^6
Poisson's ratio	0.4 to 0.45
Elastic limit, psi	200
Creep in % per hour at temperatures from room to 150°F for	
200 psi stress	0.000004 to 0.0006
300 psi stress	0.000015 to 0.0050
400 psi stress	0.000030 to 0.0230
Coefficient of expansion	
Linear (48 to 212°F, mean)	0.0000163 per °F
Cubical (48 to 212°F)	0.0000488 per °F
Cubical (liquid at melting to 675°F)	0.0000717 per °F
Increase in volume on melting	4%

[a]All properties given for room temperature unless noted otherwise.

Table 2. Effect of Temperature on Tensile Properties of Chemical Lead Annealed at 212°F

Temperature (°F)	Tensile Strength (psi)	Elongation (%)	Reduction in Area (%)
68	1920	31	100
180	1140	24	100
302	710	33	100
383	570	20	100
509	280	20	100

Table 3. Maximum Allowable Fiber Stress in Extruded Chemical Lead Pipe

Temperature (°F)	Maximum Allowable Fiber Stress (psi)
68	200
104	180
140	162
176	144
212	127
230	118
266	100
302	80

Table 4. Properties of Type 347 Stainless Steel[a]

Modulus of elasticity, psi	28,000,000
Mean coefficient of thermal expansion (32 to 600°F)	0.0000095 per °F
Tensile strength of annealed plate, psi	90,000
Yield strength of annealed plate (0.2% offset), psi	35,000
Poisson's ratio	0.3

[a] All properties given for room temperature unless noted otherwise.

Cavities in casks for spent fuel elements are quite large, and the trend seems to be toward larger casks. Some sizes characteristic of typical irradiated fuel containers are given in Table 5. It should be noted that the inner shell of all casks listed in the table is stainless steel. Therefore, it would be economically wise to make this inner shell as thin as possible to safely resist the internal pressure. One way to reduce the thickness of the inner shell is to consider the shield as a load bearing member. This may be of particular advantage when the actual operating pressure of the cask is less than the minimum design pressure of 20 psig prescribed by CFR Title 10 Part 71 and when the cask is of prismatic configuration.

Table 5. Typical Characteristics of Irradiated Fuel Containers

Cask Designation	Supplier	Owner	Empty Weight (tons)	Inside Dimensions (in.)	Thickness of Inner Shell[a] (in.)	Nominal Thickness of Lead (in.)	Thickness of Outer Shell[b] (in.)	Design Pressure (psig)
Chalk River	O. G. Kelley	USAEC	45	30 by 132 35 high	1/2 cladding	6 3/4	1/2 cladding	20
Yankee	Edlow Lead	Westinghouse	70	38 diameter 119 high	3/8	10	1 3/8	100
Dresden	Stearns Roger	Stanray	71	41 diameter 142 high	3/4	9 1/2	3/4 cladding	120
GE - Universal	Knapp Mills	GE	21	13 diameter 129 high	1/2	8 3/8	3/4 cladding	100
Piqua - Elk River	Knapp Mills	USAEC	28	30 diameter 83 high	3/4	8	1 c	100
M-130	Knapp Mills	U. S. Navy	110	55 diameter 132 high	1	10 1/2	1 c	300
MTR Fuels	National Lead	National Lead	12	23 diameter 30 high	1/2	8 1/2	1/2 c	20

[a]Stainless steel.

[b]Stainless steel unless noted otherwise.

[c]Carbon steel.

The shielding material itself is usually very weak. However, because of its large thickness (6 to 12 in.), it may have a reasonable load carrying capacity. If lead is the shielding material, its creep under load at elevated temperatures should be evaluated.

2. SUMMARY

To define the state of stress in a structure, the magnitudes and directions of the three principal stresses must be determined. When at least two of these three stresses are different from zero, a strength theory must be used to determine the proximity to yielding. The maximum shear stress theory discussed in Chapter 3 is used in this report.

In the calculations for casks with a prismatic configuration, the walls of the cask are analyzed as a frame consisting of the liner and shield loaded internally by the pressure. For this configuration, there are two cases that must be investigated: one is for the liner and shield bonded together and the other is for the liner and shield not bonded. The procedures for developing the formulas for calculating the stresses in the first case are:

1. determination of the maximum bending moment in the frame,
2. determination and distribution of membrane forces in the frame,
3. determination of the principal stresses in the shield, and
4. determination of the principal stresses in the liner.

The procedures for developing the formulas for calculating the stresses for the second case are:

1. determination of bending moments and membrane forces in shield and liner,
2. determination of principal stresses in the shield, and
3. determination of principal stresses in the liner.

The formulas for these two cases are derived in Chapter 4 of this report, and for convenience, they are tabulated in Table 6.

In the calculations for casks with a cylindrical configuration, the liner and shield are divided into four regions. These four regions are: region 1, the liner and shield remote from discontinuities; region 2, the junction of the cylindrical portion of the liner to the offset for the lid; region 3, the junction of the bottom head to the cylindrical portion of the liner; and region 4, the bottom head remote from discontinuities. The stresses that exist in these four regions are calculated in Chapter 5, and for convenience, they are tabulated in Table 7.

Table 6. Formulas for Determination of Stresses in Cask with Prismatic Configuration

	Three Principal Stresses			Stress Intensity
	σ_1	σ_2	σ_3	$S = [\sigma_{max} - \sigma_{min}]^a$
Shield and liner bonded				
Stress in shield at Liner-shield interface	$\sigma_1 = \sigma_{b_s} + \sigma_{F_{1_s}}$ from Eqs. 27 and 29[b] $\sigma_1 = \frac{p}{2}\left[\left(\frac{w^3 + d^3}{w + d}\right)\left(\frac{\overline{Y} - t_L}{6I}\right) + \left(\frac{\alpha}{1 + \alpha}\right)\frac{w}{t_s}\right]$	$\sigma_2 = \sigma_{F_{2_s}}$ from Eq. 30[a] $\sigma_2 = \left(\frac{\alpha}{1 + \alpha}\right)\left(\frac{d}{w + d}\right)\frac{pw}{2t_s}$	$\sigma_3 = \sigma_{r_s}$ from Eq. 31[c] $\sigma_3 = p$	$S = \sigma_1 - \sigma_3$ $S = \frac{p}{2}\left[\left(\frac{w^3 + d^3}{w + d}\right)\left(\frac{\overline{Y} - t_L}{6I}\right) + \left(\frac{\alpha}{1 + \alpha}\right)\frac{w}{t_s} + 2\right]$
Outer surface	$\sigma_1 = \sigma_{F_{1_s}} - \sigma_{b_s}$ from Eqs. 26 and 29[d] $\sigma_1 = \frac{p}{2}\left[\left(\frac{\alpha}{1 + \alpha}\right)\frac{w}{t_s} - \left(\frac{w^3 + d^3}{w + d}\right)\left(\frac{t_L + t_s - \overline{Y}}{6I}\right)\right]$	$\sigma_2 = \sigma_{F_{2_s}}$ from Eq. 30[b] $\sigma_2 = \left(\frac{\alpha}{1 + \alpha}\right)\left(\frac{d}{w + d}\right)\frac{pw}{2t_s}$	0	See footnote e.
Stress in liner	$\sigma_1 = \sigma_{b_L} + \sigma_{F_{1_L}}$ from Eqs. 32 and 33[b] $\sigma_1 = \frac{p}{2}\left[\frac{n}{6}\left(\frac{w^3 + d^3}{w + d}\right)\frac{\overline{Y}}{I} + \left(\frac{1}{1 + \alpha}\right)\frac{w}{t_L}\right]$	$\sigma_2 = \sigma_{F_{2_L}}$ from Eq. 34[b] $\sigma_2 = \left(\frac{1}{1 + \alpha}\right)\left(\frac{d}{w + d}\right)\left(\frac{pw}{2t_L}\right)$	$\sigma_3 = \sigma_r$ from Eq. 35[c] $\sigma_3 = p$	$S = \sigma_1 - \sigma_3$ $S = \frac{p}{2}\left[\frac{n}{6}\left(\frac{w^3 + d^3}{w + d}\right)\frac{\overline{Y}}{I} + \left(\frac{1}{1 + \alpha}\right)\frac{w}{t_L} + 2\right]$
Shield and liner not bonded				
Stress in shield at Liner-shield interface	$\sigma_1 = \sigma_{b_s} + \sigma_{F_{1_s}}$ from Eqs. 45 and 46[b] $\sigma_1 = \frac{p}{2}\left[\frac{(w - g)}{t_s} + \left(\frac{w^3 + d^3}{w + d}\right)\left(\frac{1}{t_s^2}\right)\right]$	$\sigma_2 = \sigma_{F_{2_s}}$ from Eq. 47[b] $\sigma_2 = \frac{pwd}{2t_s(w + d)}$	$\sigma_3 = \sigma_{r_s}$ from Eq. 48[c] $\sigma_3 = p$	$S = \sigma_1 - \sigma_3$ $S = \frac{p}{2}\left[\frac{(w - g)}{t_s} + \left(\frac{w^3 + d^3}{w + d}\right)\left(\frac{1}{t_s^2}\right) + 2\right]$
Outer surface	$\sigma_1 = \sigma_{F_{1_s}} - \sigma_{b_s}$ from Eqs. 45 and 46[d] $\sigma_1 = \frac{p}{2}\left[\frac{(w - g)}{t_s} - \left(\frac{w^3 + d^3}{w + d}\right)\left(\frac{1}{t_s^2}\right)\right]$	$\sigma_2 = \sigma_{F_{2_s}}$ from Eq. 47[b] $\sigma_2 = \frac{pwd}{2t_s(w + d)}$	0	See footnote e.
Stress in liner	$\sigma_1 = \sigma_{b_L} = \sigma_{F_{1_L}}$ from Eqs. 49 and 50[b] $\sigma = \frac{p}{2}\left[\frac{1.5g^2}{t_L^2} + \frac{g}{t_L}\right]$	$\sigma_2 = \sigma_{F_{2_L}}$ from Eq. 51 $\sigma_2 = 0$	$\sigma_3 = \sigma_r$ from Eq. 52[c] $\sigma_3 = p$	$S = \sigma_1 - \sigma_3$ $S = \frac{p}{2}\left[\frac{1.5g^2}{t_L^2} + \frac{g}{t_L} + 2\right]$

[a]Algebraic difference (i.e., if $\sigma_1 > \sigma_2 > \sigma_3$, then $\sigma_{max} - \sigma_{min} = \sigma_1 - \sigma_3$). Tensile stresses are considered positive, compressive stresses negative.

[b]A tensile stress.

[c]A compressive stress.

[d]Positive sign for tensile stress, negative sign for compressive stress.

[e]These differences cannot be determined until the actual of σ_1, σ_2, and σ_3 are calculated. For these cases it is not necessarily true that $\sigma_1 > \sigma_2 > \sigma_3$.

Table 7. Formulas for Determination of Stresses in Cask of Cylindrical Configuration

	Three Principal Stresses: σ_1	Three Principal Stresses: σ_2	Three Principal Stresses: σ_3	Stress Intensity[a] $S = (\sigma_{max} - \sigma_{min})$
Stress in region 1				
Liner	$\sigma_1 = \sigma_\theta$ from Eq. 64[b] $\sigma_1 = \frac{p}{1+\beta}\left(\frac{a}{t_L}\right)$	$\sigma_2 = \sigma_x$ from Eq. 65[b] $\sigma_2 = \frac{pa}{2t_L}$	$\sigma_3 = \Sigma\sigma_r$ from Eq. 66[c] $\sigma_3 = p$	$p\left[\frac{1}{A^*}\left(\frac{a}{t_L}\right) + 1\right]$ *where A = 1 + β or 2 whichever is less
Shield	$\sigma_1 = \sigma_\theta$ from Eq. 61[b] $\sigma_1 = \frac{\beta p}{\beta+1}\left(\frac{b^2+a^2}{b^2-a^2}\right)$	$\sigma_2 = \sigma_x$ from Eq. 63 $\sigma_2 = 0$	$\sigma_3 = \Sigma\sigma_r$ from Eq. 62[c] $\sigma_3 = \frac{\beta p}{\beta+1}$	$\frac{\beta p}{\beta+1}\left(\frac{b^2+a^2}{b^2-a^2} + 1\right)$
Stress at region 2 in liner				
Outside surface	$\sigma_1 = \Sigma\sigma_\theta$ from Eqs. 64, 75, 77, and 78[a] $\sigma_1 = \frac{p}{1+\beta}\left(\frac{a}{t_L}\right) - \frac{2E_{reg.\ 2}\lambda_L a}{t_L} + \frac{2M_{reg.\ 2}\lambda_L^2 a}{t_L} - \frac{6\upsilon_L M_{reg.\ 2}}{t_L^2}$	$\sigma_2 = \Sigma\sigma_x$ from Eqs. 65 and 76[a] $\sigma_2 = \frac{pa}{2t_L} - \frac{6M_{reg.\ 2}}{t_L^2}$	$\sigma_3 = \Sigma\sigma_r$ from Eq. 62[c] $\sigma_3 = \frac{\beta p}{\beta+1}$	See footnote d.
Inside surface	$\sigma_1 = \Sigma\sigma_\theta$ from Eqs. 64, 75, 77, and 78[a] $\sigma_1 = \frac{p}{1+\beta}\left(\frac{a}{t_L}\right) - \frac{2F_{reg.\ 2}\lambda_L a}{t_L} + \frac{2M_{reg.\ 2}\lambda_L^2 a}{t_L} - \frac{6\upsilon_L M_{reg.\ 2}}{t_L}$	$\sigma_2 = \Sigma\sigma_x$ from Eqs. 65 and 76[b] $\sigma_2 = \frac{pa}{2t_L} + \frac{6M_{reg.\ 2}}{t_L^2}$	$\sigma_3 = \Sigma\sigma_r$ from Eq. 66[c] $\sigma_3 = p$	See footnote d.
Stress at region 3 in liner				
Cylindrical wall outside surface	$\sigma_1 = \Sigma\sigma_\theta$ from Eqs. 64, 90, 91, and 93[a] $\sigma_1 = \frac{p}{1+\beta}\left(\frac{a}{t_L}\right) - \frac{2F_{reg.\ 3}}{t_L}\lambda_L a + \frac{2M_{reg.\ 3}}{t_L}\lambda_L^2 a - \frac{6\upsilon_L M_{reg.\ 3}}{t_L^2}$	$\sigma_2 = \Sigma\sigma_x$ from Eqs. 65 and 92[a] $\sigma_2 = \frac{pa}{2t_L} - \frac{6M_{reg.\ 3}}{t_L^2}$	$\sigma_3 = \Sigma\sigma_r$ from Eq. 62[c] $\sigma_r = \frac{\beta p}{\beta+1}$	See footnote d.
Cylindrical wall inside surface	$\sigma_1 = \Sigma\sigma_\theta$ from Eqs. 64, 90, 91, and 93[a] $\sigma_1 = \frac{p}{1+\beta}\left(\frac{a}{t_L}\right) - \frac{2F_{reg.\ 3}}{t_L}\lambda_L a + \frac{2M_{reg.\ 3}}{t_L}\lambda_L^2 a + \frac{6\upsilon_L M_{reg.\ 3}}{t_L^2}$	$\sigma_2 = \Sigma\sigma_x$ from Eqs. 65 and 92[b] $\sigma_2 = \frac{pa}{2t_L} + \frac{6M_{reg.\ 3}}{t_L^2}$	$\sigma_3 = \Sigma\sigma_r$ from Eq. 66[c] $\sigma_r = p$	See footnote d.
Head inside surface	$\sigma_1 = \Sigma\sigma_r$ from Eqs. 94 and 95[b] $\sigma_1 = \frac{F_{reg.\ 3}}{t_h} + \frac{6M_{reg.\ 3}}{t_h^2}$	$\sigma_2 = \Sigma\sigma_\theta$ from Eqs. 96 and 97[b] $\sigma_2 = \frac{\upsilon_L F_{reg.\ 3}}{t_h} + \frac{6M_{reg.3}}{t_h^2}$	$\sigma_3 = \Sigma\sigma_x$ from Eq. 98[c] $\sigma_3 = p$	$\frac{F_{reg.\ 3}}{t_h} + \frac{6M_{reg.\ 3}}{t_h^2} + p$
Head outside surface	$\sigma_1 = \Sigma\sigma_r$ from Eqs. 94 and 95[a] $\sigma_1 = \frac{F_{reg.\ 3}}{t_h} - \frac{6M_{reg.\ 3}}{t_h^2}$	$\sigma_2 = \Sigma\sigma_\theta$ from Eqs. 96 and 97[a] $\sigma_2 = \frac{\upsilon_L F_{reg.\ 3}}{t_h} - \frac{6M_{reg.3}}{t_h^2}$	$\sigma_3 = \Sigma\sigma_x$ $\sigma_3 = 0$	See footnote d.
Stress at region 4 head				
Inside surface	$\sigma_1 = \Sigma\sigma_r$ from Eqs. 99, 101, and 103[a] $\sigma_1 = \frac{6M_{reg.\ 3}}{t_h^2} + \frac{F_{reg.\ 3}}{t_h} - \frac{3pa^2(3+\upsilon_L)}{8t_h^2}$	$\sigma_2 = \Sigma\sigma_\theta$ from Eqs. 100, 102, and 104[a] $\sigma_2 = \frac{6M_{reg.\ 3}}{t_h^2} + \frac{F_{reg.\ 3}}{t_h} - \frac{3pa^2}{2t_h^2}$	$\sigma_3 = \Sigma\sigma_x$ from Eq. 105[c] $\sigma_3 = p$	See footnote d.
Outside surface	$\sigma_1 = \Sigma\sigma_r$ from Eqs. 99, 101, and 103[a] $\sigma_1 = -\frac{6M_{reg.\ 3}}{t_h^2} + \frac{F_{reg.\ 3}}{t_h} + \frac{3pa^2(3+\upsilon_L)}{8t_h^2}$	$\sigma_2 = \Sigma\sigma_\theta$ from Eqs. 100, 102, and 104[a] $\sigma_2 = -\frac{6M_{reg.3}}{t_h^2} + \frac{F_{reg.\ 3}}{t_h} + \frac{3pa^2}{2t_h^2}$	$\sigma_3 = \Sigma\sigma_x$ $\sigma_3 = 0$	See footnote d.

[a]Algebraic difference (i.e., if $\sigma_1 > \sigma_2 > \sigma_3$, then $\sigma_{max} - \sigma_{min} = \sigma_1 - \sigma_3$). Tensile stresses are considered positive, compressive stresses negative.

[b]A tensile stress.

[c]A compressive stress.

[d]These differences cannot be determined until the actual values of σ_1, σ_2, and σ_3 are calculated. For these cases it is not necessarily true that $\sigma_1 > \sigma_2 > \sigma_3$.

If the shielding material is lead and the pressure load is to be applied for extended periods of time (weeks), the creep of the lead must be evaluated. An evaluation of creep for a lead shield is given in Chapter 6, and it is concluded from the results of this evaluation that creep is not really a problem because it tends to be self-alleviating.

3. STRENGTH THEORY

The state of stress in a structure is completely defined when the magnitudes and directions of the three principal stresses have been determined. If at least two of these stresses are different from zero, a strength theory must be employed to determine the proximity to yielding. For ductile materials, the distortion energy theory more accurately predicts yielding than the maximum shear stress theory, but both are better predictors than the maximum normal stress theory commonly used until fairly recent times. Section III of the ASME Code[1] is based on the maximum shear stress theory because it is slightly more conservative and easier to apply than the distortion energy theory. For this reason, the maximum shear stress theory is used in this report.

The maximum shear stress theory states that yielding occurs when the maximum shear stress in a structural member becomes equal to the maximum shear stress in a tensile specimen subject to its yield point stress. The maximum shear stress in a structural member equals half the algebraic difference between the maximum and the minimum principal stresses. Therefore, if it is understood that $\sigma_1 > \sigma_2 > \sigma_3$ where σ_1, σ_2, and σ_3 are the three principal stresses, the maximum shear stress, τ_{max}, in a structural member equals $(\sigma_1 - \sigma_3)/2$. In a simple tensile test, $\sigma_2 = \sigma_3 = 0$; therefore, at yielding, the maximum shear stress $= (S_{yp} - 0)/2 = S_{yp}/2$, where S_{yp} is the tensile stress in the specimen at yielding. Hence, $(\sigma_1 - \sigma_3)/2 = S_{yp}/2$ or $\sigma_1 - \sigma_3 = S_{yp}$ is the failure criterion.

A new term, "stress intensity," is used in Section III of the ASME Code and will be used in this report. Stress intensity, S, is defined as twice the maximum shear stress and is therefore equal to the absolute value of $\sigma_1 - \sigma_3$. It is directly comparable to S_{yp}. Thus, if failure is not to occur, the absolute value of the algebraic difference between the maximum and the minimum principal stresses must not exceed the yield point of the structural material at the operating temperature; that is, $S = \sigma_1 - \sigma_3 \leq S_{yp}$.

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[1]ASME Boiler and Pressure Vessel Code, Section III, Rules for Construction of Nuclear Vessels, American Society of Mechanical Engineers, New York, 1963.

4. CALCULATIONS FOR CASKS WITH PRISMATIC CONFIGURATION

The general configuration of a prismatic cask is illustrated in Fig. 1. The height of the cavity, h, is considered to be much larger than the width, w, and the width is much larger than the depth, d. The cask is composed of three components: the liner, the shield, and the jacket. The cask is considered to have an internal pressure of p.

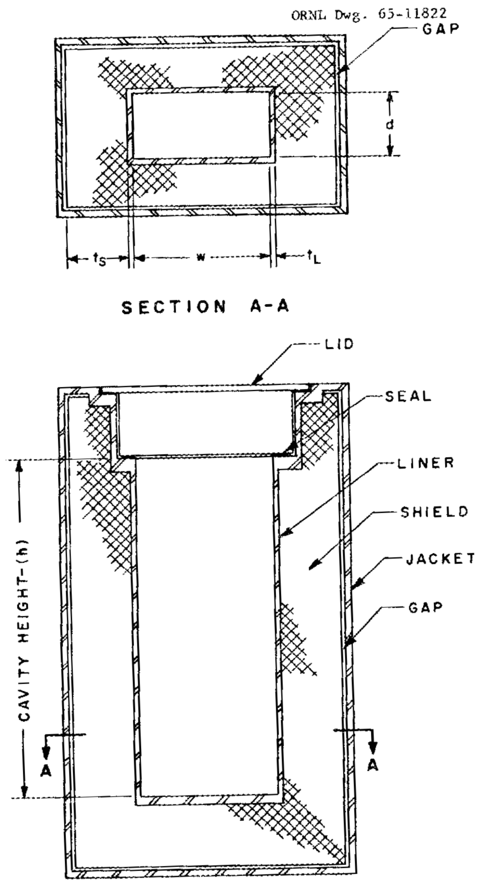

Fig. 1. General Configuration of a Prismatic Cask.

In order to simplify the calculations and results and at the same time obtain a conservative result, it was assumed that:

1. the jacket is not in contact with the shielding material,
2. the thickness of the liner is small when compared with that of the shielding,
3. the pressure load between the liner and shielding is uniform over the common surface of contact,

4. the height, h, is enough larger than the width, w, $(h > 2w)$ so that end effects on the side walls can be neglected.[1] This is conservative because the ends actually help support the load.

Method of Calculations

Since the end effects are assumed negligible, the cask walls can be analyzed as a frame consisting of the liner and shield loaded internally by the pressure p. There are two cases that must be investigated: one is the case where the liner and shield are bonded together and the other is the case where no bond exists. The procedures that are followed in developing the formulas for calculating the stresses for the first case are:

1. determination of the maximum bending moment in the frame,
2. determination and distribution of the membrane forces in the frame,
3. determination of the principal stresses in the shield, and
4. determination of the principal stresses in the liner.

The procedures for developing the formulas for calculating the stresses for the second case are:

1. determination of bending moments and membrane forces in shield and liner,
2. determination of principal stresses in the shield, and
3. determination of principal stresses in the liner.

The formulas for these two cases are derived in the following calculations and summarized in Table 6 of Chapter 2. Their application is indicated in the example calculations for a prismatic cask in this chapter.

Calculations for Bonded Liner and Shield

Determination of Maximum Frame Moment

The determination of the maximum bending moment in the frame is based on ref. 2. The load diagram for the frame is illustrated in Fig. 2.

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[1]S. P. Timoshenko, p. 116 in Strength of Materials, Part II, D. Van Nostrand Co., Inc., 3rd ed., 1956.

[2]S. P. Timoshenko, p. 190 in Strength of Materials, Part I, D. Van Nostrand Co., Inc., 3rd ed., 1955.

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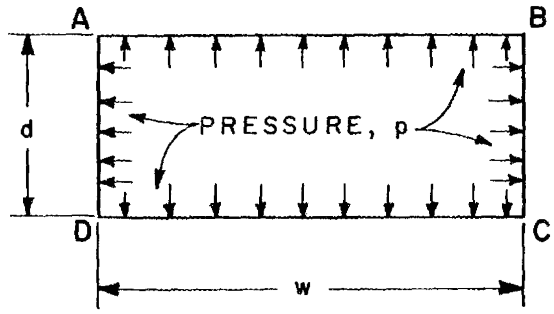

Fig. 2. Load Diagram for Frame of Prismatic Cask with Bonded Liner and Shield.

Two conditions must exist at the corners (A, B, C, or D) in order to maintain continuity. The bending moments in two adjacent spans, such as spans AB and AD, at their junction must be equal, and the changes in slope of these two adjacent spans at the junction must be numerically equal but opposite in sign if the sign convention of Fig. 3 is used. Figure 3 shows the free-body diagrams of spans AB and AD. From this,

$$M_{AD} = M_{AB} \text{ , and} \tag{1}$$

$$\theta_{AD} + \theta_{AB} = 0 \ . \tag{2}$$

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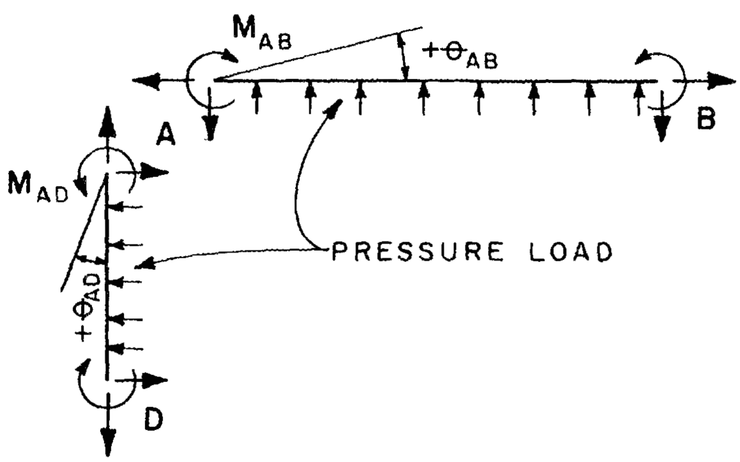

Fig. 3. Free-Body Diagrams of Spans AB and AD.

The slope of span AB at A,

$$\theta_{AB} = \frac{pw^3}{24EI} - \frac{M_{AB}w}{2EI} . \tag{3}$$

The slope of span AD at A,

$$\theta_{AD} = \frac{pd^3}{24EI} - \frac{M_{AD}d}{2EI} . \tag{4}$$

By utilizing Eqs. 1, 2, 3, and 4,

$$M_{AB} = M_{AD} = \frac{p}{12}\left[\frac{w^3 + d^3}{w + d}\right] . \tag{5}$$

The bending moment diagram may be drawn by parts and is shown in Fig. 4.

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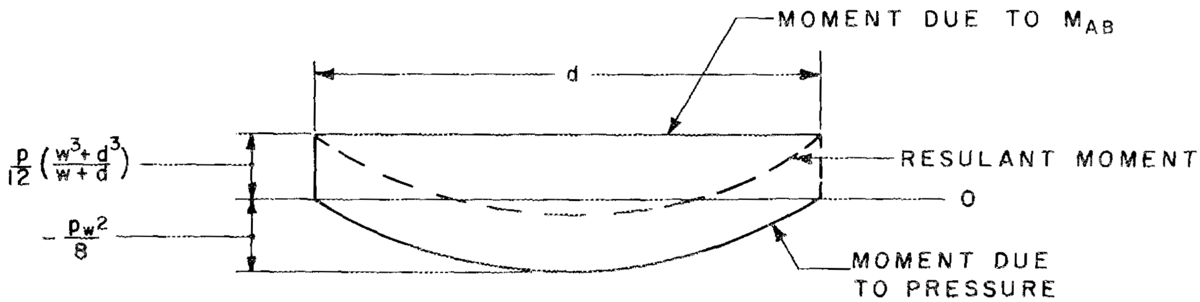

Fig. 4. Bending Moment Diagram for Frame of Prismatic Cask with Bonded Liner and Shield.

The resultant moment at any point is the algebraic sum of the ordinates of the two parts under consideration. The maximum moment at the center of the span will occur when the end moment is a minimum. This will be when the derivative of the moment with respect to span length equals zero; that is, when the moment at the center of span AB is a maximum,

$$\frac{dM_{AB}}{d(d)} = 0 .$$

Dividing the numerator by the denominator in the right-hand member of Eq. 5,

$$M_{AB} = \frac{p}{12}\,(w^2 - wd + d^2) . \tag{6}$$

Taking the derivative of Eq. 6 with respect to d,

$$\frac{dM_{AB}}{d(d)} = \frac{p}{12}\,(-w + 2d)\ . \tag{7}$$

Setting Eq. 7 equal to zero and solving for d,

$$d = w/2\ . \tag{8}$$

When $d = w/2$,

$$M_{AB} = \frac{p}{12}\,(w^2 - \frac{w^2}{2} + \frac{w^2}{4}) = \frac{pw^2}{16}\ .$$

The moment at the center is then $-pw^2/16$; that is, when $d = w/2$, the magnitudes of the moments at the ends and at the center of the beam AB are equal. As span length d approaches span length w (see Fig. 2), the end moment approaches its maximum value of pw/12 and the magnitude of the moment at midspan approaches $pw^2/24$. Thus, the maximum bending moment in spans AB, BC, CD, or AD is at the end and is equal to

$$\frac{p}{12}\,(w^2 - wd + d^2) \text{ or } \frac{p}{12}\,(\frac{w^3 + d^3}{w + d})\ .$$

Determination and Distribution of Membrane Forces

Each span is under direct tensile or membrane forces as a result of the pressure forces on the adjacent spans and ends. A differential element cut from the inside corner of the frame at joint A in span AD is illustrated in Fig. 5.

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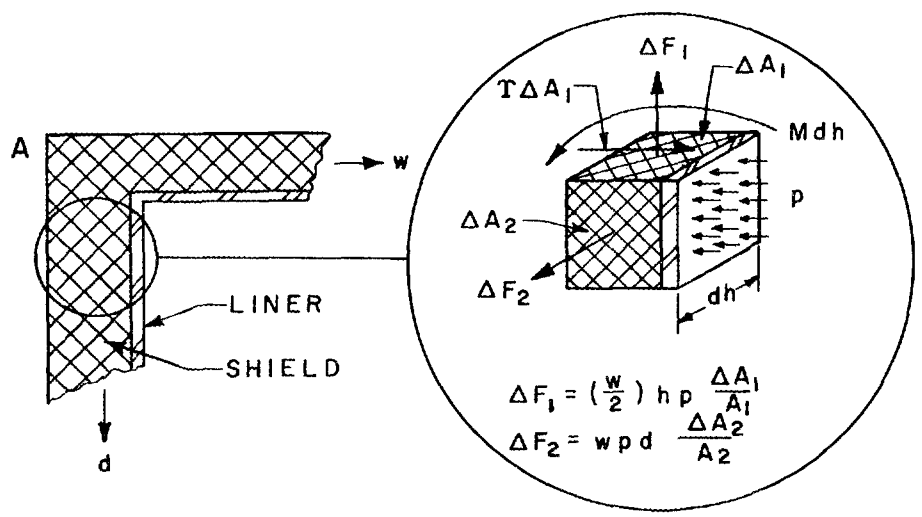

Fig. 5. Differential Element Cut From Inside Corner of Frame At Joint A in Span AD of Prismatic Cask with Bonded Liner and Shield.

The membrane force F_1 in span AD is a result of the pressure forces on spans AB and CD. In addition to the membrane force F_1, there exists another membrane force, F_2, which acts perpendicularly to F_1. This is caused by the pressure load on the ends or heads of the cask. This force F_2 is similar to the longitudinal force in a cylinder, and the force F_1 is similar to the circumferential force in a cylinder. The differential forces ΔF_1 $(\Delta F_1 = F_1 \frac{\Delta A_1}{A_1})$ and $(\Delta F_2 = F_2 \frac{\Delta A_2}{A_2})$ and moments are noted in Fig. 5. The values of F_1 and F_2 are:

$$F_1 = \frac{whp}{2} , \tag{9}$$

and

$$F_2 = wpd . \tag{10}$$

The forces F_1 and F_2 are carried by the liner and shield. When the liner and shield are bonded, the part of the force carried by each is determined by utilizing the fact that the increase in the length of the liner equals the increase in the length of the shield. The increase in length of a member due to an axial load,

$$\delta = \frac{Px}{AE} \tag{11}$$

where

P = load, lb,

x = length of member, in.,

A = the transverse cross sectional area of the member, in.^{2}, and

E = the modulus of elasticity, psi.

Since the liner and shield are assumed bonded,

$$\delta_s = \delta_L , \tag{12}$$

or

$$\frac{F_{1_s} d}{A_s E_s} = \frac{F_{1_L} d}{A_L E_L} . \tag{12a}$$

Since $d_s = d_L$ and $\frac{A_s}{A_L} = \frac{t_s}{t_L}$, Eq. 12a reduces to

$$\frac{F_{1_s}}{t_s E_s} = \frac{F_{1_L}}{t_L E_L} . \tag{13}$$

Solving Eq. 13 for F_{1_s},

$$F_{1_s} = \frac{E_s}{E_L}\left(\frac{t_s}{t_L}\right) F_{1_L} \ . \tag{13a}$$

Also,

$$F_{1_s} + F_{1_L} = F_1 \ . \tag{14}$$

Solving Eqs. 13a and 14 for F_{1_s},

$$F_{1_s} = \frac{\left(\frac{E_s t_s}{E_L t_L}\right) F_1}{1 + \left(\frac{E_s t_s}{E_L t_L}\right)} \ . \tag{15}$$

If α is defined as

$$\alpha = \frac{E_s t_s}{E_L t_L} \ , \tag{16}$$

Eq. 15 reduces to

$$F_{1_s} = \frac{\alpha}{1 + \alpha} F_1 . \tag{17}$$

From Eqs. 14 and 17 it follows that

$$F_{1_L} = \frac{1}{1 + \alpha} F_1 \ . \tag{18}$$

In a similar manner it is found that

$$F_{2_s} = \frac{\alpha}{1 + \alpha} F_2 \ , \tag{19}$$

and

$$F_{2_L} = \frac{1}{1 + \alpha} F_2 \ . \tag{20}$$

Determination of Stresses in Shield

Stresses in Shield due to Moment M_{AD}. The stress due to a bending moment is defined by

$$\sigma_b = \frac{Mc}{I} \ , \tag{21}$$

where

σ_b = bending stress, psi,

M = bending moment, in.-lb,

C = distance from neutral axis to extreme fiber, in.,

I = moment of inertia, in.^{4}

For the case where the liner and shield are bonded, the liner and shield constitute a beam of two materials that carries the bending load. In order to determine the distribution of stress, the composite beam must be converted to an equivalent section one-material beam.[3] A cross section of the composite beam and an equivalent one-material section are illustrated in Fig. 6.

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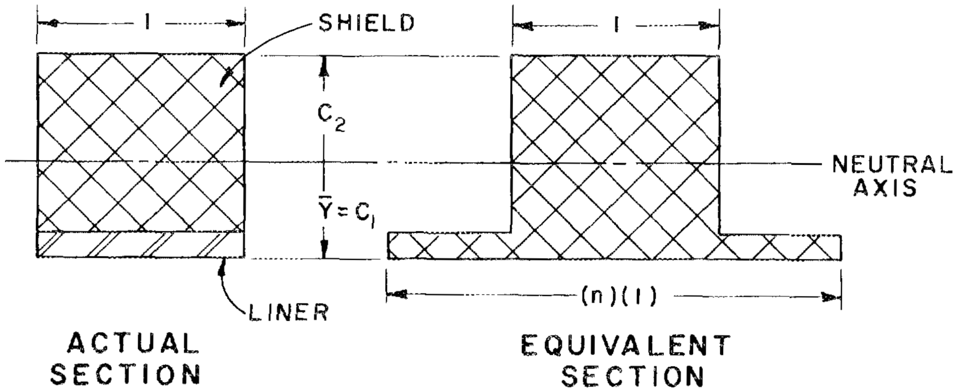

Fig. 6. Cross Section of Composite Beam and Equivalent One-Material Section of the Same Beam.

The actual section illustrated in Fig. 6 is converted into an equivalent section of shield material where

$$n = \frac{E_L}{E_s} . \quad (22)$$

The distance $\bar{Y}$ from the base to the neutral axis,

$$\bar{Y} = \frac{\Sigma A(Y)}{\Sigma A} , \quad (23)$$

where

A = cross-sectional area of the component,

Y = distance from base to centroid of A.

Therefore, for an actual section 1 in. wide converted to an equivalent section,

$$\bar{Y} = \frac{n(t_L)(\frac{t_L}{2}) + t_s(t_L + \frac{t_s}{2})}{nt_L + t_s} = \frac{\frac{nt_L^2}{2} + t_L t_s + \frac{t_s^2}{2}}{nt_L + t_s} . \quad (23a)$$

[3]P. G. Laurson and W. J. Cox, pp. 322-323 in Mechanics of Materials, Wiley and Sons, Inc., 3rd ed., New York, 1947.

Once $\overline{Y}$ is known, the moment of inertia of the equivalent section with respect to the neutral axis may be calculated.

$$I = 1/3[n\overline{Y}^3 - (n - 1)(\overline{Y} - t_L)^3 + (t_s + t_L - \overline{Y}^3]. \tag{24}$$

By definition from Fig. 6,

$$C_1 = \overline{Y}, \tag{25a}$$

and

$$C_2 = t_L + t_s - \overline{Y} . \tag{25b}$$

Since C_2 is larger than $C_1 - t_L$, the largest bending stress in the shield is a distance C_2 from the neutral axis and this stress (from Eqs. 5, 21, and 25),

$$\sigma_{C_2} = \frac{\frac{p}{12}\left(\frac{w^3 + d^3}{w + d}\right)(t_L + t_s - \overline{Y})}{I} \text{ (compressive)}, \tag{26}$$

where $\overline{Y}$ and I are given by Eqs. 23a and 24, respectively.

The bending stress in the shield at the liner to shield interface is

$$\sigma_b = \frac{\frac{p}{12}\left(\frac{w^3 + d^3}{w + d}\right)(\overline{Y} - t_L)}{I} \text{ (tensile)}, \tag{27}$$

where $\overline{Y}$ and I are given by Eqs. 23a and 24, respectively.

Stresses in Shield due to F_{1_s} and F_{2_s}. The stress due to a tensile load,

$$\sigma = \frac{P}{A} , \tag{28}$$

where

σ = stress, psi,

P = load, lb,

A = area over which P acts, in.^{2}

Therefore, the stress in the shield due to F_{1_s} (see Eqs. 9, 17, and 28),

$$\sigma_{F_{1_s}} = \left(\frac{\alpha}{1 + \alpha}\right)\frac{pw}{2t_s} \text{ (tensile)}, \tag{29}$$

and the stress due to F_{2_s} (see Eqs. 10, 19, and 28),

$$\sigma_{F_{2_s}} = \left(\frac{\alpha}{1 + \alpha}\right)\left(\frac{d}{w + d}\right)\left(\frac{pw}{2t_s}\right) \text{ (tensile)} . \tag{30}$$

Stress in Shield Due to Pressure. The pressure load transmitted to the shield is assumed to be equal to the pressure. Therefore, the radial stress on the shield,

$$\sigma_{r_p} = p \text{ (compressive)} . \tag{31}$$

Determination of Stresses in Liner

Stresses in Liner due to Moment M_{AD}. From Eqs. 5, 21, 22, and 28a, the maximum bending stress in the liner when the liner and shield are bonded together,

$$\sigma_{b_L} = \frac{\frac{pn}{12}\left(\frac{w^3 + d^3}{w + d}\right)\bar{Y}}{I} \text{ (tensile)}, \tag{32}$$

where $\bar{Y}$ and I are given by Eqs. 23a and 24, respectively.

Stresses in Liner due to F_{1_L} and F_{2_L}. From Eqs. 9, 18, and 28, the stress in the liner due to F_{1_L},

$$\sigma_{F_{1_L}} = \left(\frac{1}{1 + \alpha}\right) \frac{pw}{2t_L} \text{ (tensile)} . \tag{33}$$

From Eqs. 10, 20, and 28, the stress in the liner due to F_{2_L},

$$\sigma_{F_{2_L}} = \left(\frac{1}{1 + \alpha}\right)\left(\frac{d}{w + d}\right)\left(\frac{pw}{2t_L}\right) \text{ (tensile)} . \tag{34}$$

Stress in Liner due to Pressure. The liner is subjected to the full pressure, p. Therefore, the radial stress due to pressure,

$$\sigma_{r_p} = p \text{ (compressive)} . \tag{35}$$

Calculations for Liner and Shielding Not Bonded

Determination of Bending Moments and Membrane Forces in Shield and Liner

A view of a corner of the frame ABCD under pressure loading with the condition greatly exaggerated is illustrated in Fig. 7, and free-body diagrams of a portion of the liner and shield at the corner are shown in Fig. 8.

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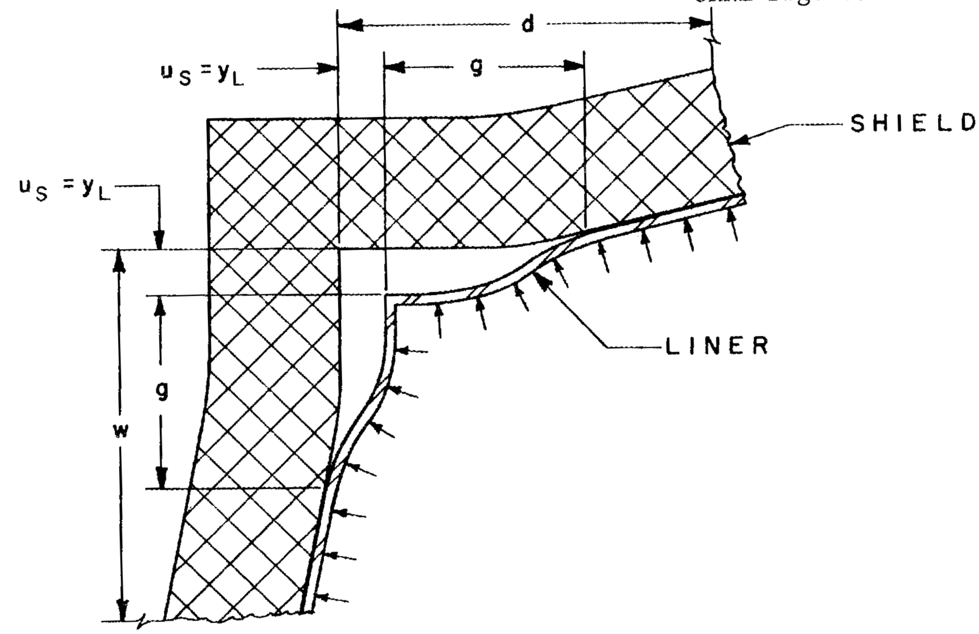

Fig. 7. Corner of Frame ABCD Under Pressure Loading When Liner and Shield are not Bonded Together.

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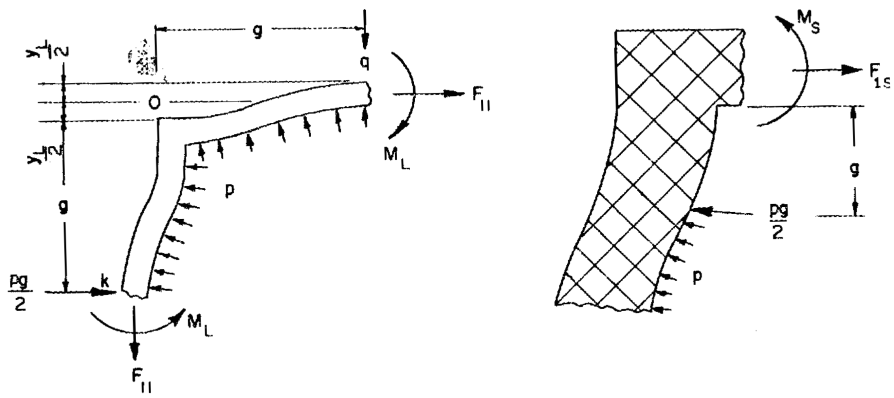

Fig. 8. Free-Body Diagrams of Portion of Liner and Shield at the Corner for the Case Where Liner and Shield are not Bonded Together.

To arrive at formulas for F_{1_L}, F_{1_S}, F_{2_S}, M_L, and M_S, it was assumed that:

1. the total bending moment given by Eq. 5 must be resisted by the shield,

$$M_S = \frac{p}{12}\left(\frac{w^3 + d^3}{w + d}\right) ;$$

2. the increase in the length of the span, u_s, of the shield is produced by the total pressure, and

$$u_s = \frac{pwd}{4t_s E_s} ; \tag{36}$$

3. the elongation of the liner due to F_{1_L} is negligible;
4. the change in slope of the liner from point 0 to points k or q is zero;
5. the membrane force F_{2_L} is zero.

The deflection, y_L, of the segment 0q or 0k in Fig. 8 may be calculated by considering it to consist of two cantilevers of length g/2, one supported at 0 and the other at q, with both carrying a uniformly distributed load of p lb/in.[2] Assuming this and applying ref. 4,

$$y_L = \frac{p\ell^4}{8E_L I_L} , \tag{37}$$

where ℓ is the length of the cantilever in in.
Therefore, the deflection,

$$y_L = \frac{pg^4}{64E_L I_L} . \tag{38}$$

From Fig. 7 it is apparent that

$$u_s = y_L . \tag{39}$$

For a beam 1 in. wide,

$$I_L = \frac{t_L^3}{12} . \tag{40}$$

Simultaneous solution of Eqs. 36, 38, 39, and 40 along with Eq. 16 yields

$$g = \sqrt[4]{\frac{4}{3} \frac{t_L^2}{\alpha} wd} , \tag{41}$$

where g is the unsupported length of the liner, as shown in Fig. 7. When g is known, the forces F_{1_s} and F_{1_L} and the moment M_L can be calculated.

$$F_{1_s} = \left(\frac{w - g}{2}\right) hp . \tag{42}$$

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[4]P. G. Laurson and W. J. Cox, p. 164 in Mechanics of Materials, Wiley and Sons, Inc., 3rd ed., New York, 1947.

$$F_{1_L} = \frac{g}{2} hp \ . \tag{43}$$

$$M_L = \frac{pg^2}{8} \ . \tag{44}$$

Determination of Stresses in Shield

Stresses in Shield due to Moment M_s. The bending stress in the shield for the case where there is no bond between the liner and the shield,

$$\sigma_{b_s} = \frac{M_s c_s}{I_s} = \frac{p}{2}\left(\frac{w^3 + d^3}{w + d}\right) \text{ (tensile at inner surface) } . \tag{45}$$

Stresses in Shield due to Membrane Forces F_{1_s} and F_{2_s}. The stress in the shield due to F_{1_s},

$$\sigma_{F_{1_s}} = \frac{F_{1_s}}{A_1} = \frac{\left(\frac{w - g}{2}\right) hp}{ht_s} = \left(\frac{w - g}{2}\right) \frac{p}{t_s} \text{ (tensile) } . \tag{46}$$

The stress in the shield due to F_{2_s},

$$\sigma_{F_{2_s}} = \frac{F_{2_s}}{A_2} = \frac{pwd}{2t_s(w + d)} \text{ (tensile) } . \tag{47}$$

Stress in Shield due to Pressure. Assume that all of the pressure is transmitted through the liner to the shield. Therefore, the radial stress due to pressure,

$$\sigma_{r_p} = p \text{ (compressive)}. \tag{48}$$

Determination of Stresses in Liner

Stress in Liner due to Moment M_L. The bending stress in the liner due to M_L,

$$\sigma_{b_L} = \frac{M_L c_L}{I_L} = 3/4 \ \frac{pg^2}{t_L^2} \text{ (tensile) } . \tag{49}$$

Stresses in Liner due to Membrane Forces F_{1_L} and F_{2_L}. The stress in the liner due to F_{1_L},

$$\sigma_{F_{1_L}} = \frac{F_{1_L}}{A_1} = \frac{gp}{2t_L} \text{ (tensile) } . \tag{50}$$

The stress in the liner due to F_{2_L},

$$\sigma_{F_{2_L}} = 0 \; . \tag{51}$$

Stress in Liner due to Pressure. The radial stress in the liner due to pressure,

$$\sigma_{r_p} = p \text{ (compressive)} \; . \tag{52}$$

Application of Results of Calculations

The approach to be followed in using the results of the preceding calculations to determine the minimum thickness of the liner is to:

1. select the required shield thickness based on shielding requirements,
2. assume a liner thickness of t_L,
3. calculate the maximum principal stresses (σ_1, σ_2, and σ_3) in the shield and liner,
4. compute S ($S = \sigma_1 - \sigma_3$) and compare with the yield point of the material in question,
5. repeat above four steps until the calculated stresses are acceptable ($S \leq S_{yp}$).

Examples for Casks with Prismatic Configuration

The material properties are obtained from Tables 1 through 4, and reference is made to Table 6 for the formulas for stress. Assume a cask of prismatic configuration with the characteristics listed below.

Dimensions: h = 72 in., w = 36 in., d = 24 in., and t_s = 10 in.

Design pressure = 20 psi

Design temperature = 212°F

Shielding material: chemical lead, $E_s = 2 \times 10^6$ psi, $\upsilon_s = 0.4$

Liner material: type 347 stainless steel, $E_L = 28 \times 10^6$ psi, $\upsilon_L = 0.3$

Assume that t_L = 0.5 in.

Determination of Constants α, g, n, $\overline{Y}$, and I

$$\alpha = \frac{E_s t_s}{E_L t_L} = \frac{2 \times 10^6 \times 10}{28 \times 10^6 \times 0.5} = 1.429.$$

$$g = \sqrt[4]{\frac{4}{3} \frac{t_L^2}{\alpha} wd} = \sqrt[4]{\frac{4}{3} \frac{(0.25)}{1.43} (36)(24)} = 3.767 \text{ in.}$$

$$n = \frac{E_L}{E_s} = \frac{28 \times 10^6}{2 \times 10^6} = 14.$$

$$\overline{Y} = \frac{\frac{nt_L}{2} + t_L t_s + \frac{t_s^2}{2}}{nt_L + t_s} = \frac{\frac{14(0.25)}{2} + (0.5)(10) + \frac{100}{2}}{14(0.5) + 10} = 3.338 \text{ in.}$$

$$I = \frac{1}{3} [n\overline{Y}^3 - (n - 1)(\overline{Y} - t_L)^3 + (t_s + t_L - \overline{Y})^3]$$
$$= \frac{1}{3} [14(3.338)^3 - 13(2.838)^3 + (7.162)^3] = 196.972 \text{ in.}^4$$

Determination of Stress Intensities Where Liner and Shield Are Not Bonded

Stresses in Shield at Liner-Shield Interface.

$$\sigma_1 = \frac{p}{2} \left[\frac{w - g}{t_s} + \frac{w^3 + d^3}{w + d} \left(\frac{1}{t_s^2}\right)\right]$$

$$= \frac{20}{2} \left[\frac{36 - 3.767}{10} + \frac{36^3 + 24^3}{36 + 24} \frac{1}{100}\right] = 133 \text{ psi.}$$

$$\sigma_2 = \frac{pwd}{2t_s(w + d)} = \frac{(20)(36)(24)}{2(10)(36 + 24)} = 14.4 \text{ psi.}$$

$\sigma_3 = -20$ psi.

The stress intensity $= \sigma_1 - \sigma_3 = 153$ psi.

Stresses in Shield at Outer Surface.

$$\sigma_1 = \frac{p}{2} \left[\frac{w - g}{t_s} - \frac{w^3 + d^3}{w + d} \left(\frac{1}{t_s^2}\right)\right] = -68.6 \text{ psi.}$$

$\sigma_2 = 14.4$ psi.

$\sigma_3 = 0.$

The stress intensity = 83 psi.

Stresses in Liner.

$$\sigma_1 = \frac{p}{2}\left[\frac{1.5g^2}{t_s^2} + \frac{g}{t_L}\right] = \frac{20}{2}\left[\frac{1.5(3.767)^2}{(0.5)^2} + \frac{3.767}{0.5}\right] = 927 \text{ psi.}$$

$\sigma_2 = 0.$

$\sigma_3 = -20$ psi.

The stress intensity is 947 psi.

Determination of Stress Intensities where Liner and Shield Are Bonded

Stresses in Shield at Liner-Shield Interface.

$$\sigma_1 = \frac{p}{2}\left[\frac{w^3 + d^3}{w + d}\left(\frac{\bar{Y} - t_L}{6I}\right) + \frac{\alpha}{1 + \alpha}\left(\frac{w}{t_s}\right)\right]$$

$$= \frac{20}{2}\left[\frac{36^3 + 24^3}{36 + 24} \cdot \frac{3.338 - 0.5}{6(196.972)} + \frac{1.429}{2.429}\,\frac{36}{10}\right]$$

$= 43.52$ psi.

$$\sigma_2 = \left(\frac{\alpha}{1 + \alpha}\right)\left(\frac{d}{w + d}\right)\left(\frac{pw}{2t_s}\right)$$

$$= \frac{1.429}{2.429}\;\frac{24}{36 + 24}\;\frac{20(36)}{2(10)} = 8.5 \text{ psi.}$$

$\sigma_3 = -20$ psi.

The stress intensity is 63.52 psi.

Stresses at Shield Outer Surface

$$\sigma_1 = \frac{p}{2}\left[\frac{\alpha}{1 + \alpha}\left(\frac{w}{t_s}\right) - \frac{w^3 + d^3}{w + d}\left(\frac{t_s + t_L - \bar{Y}}{6I}\right)\right]$$

$$= \frac{20}{2}\;\frac{1.429}{2.429}\left(\frac{36}{10}\right) - \frac{36^3 + 24^3}{36 + 24} \cdot \frac{10 + 0.5 - 3.338}{6(196.972)}\Bigg]$$

$= -39.9$ psi.

$$\sigma_2 = \left(\frac{\alpha}{1 + \alpha}\right)\left(\frac{d}{w + d}\right)\left(\frac{pw}{2t_s}\right) = 8.5 \text{ psi (as above).}$$

$\sigma_3 = 0.$

The stress intensity is 48.5 psi.

Stresses in Liner.

$$\sigma_1 = \frac{p}{2}\left[\frac{n}{6}\ \frac{w^3 + d^3}{w + d}\ \frac{\bar{Y}}{I} + \frac{1}{1 + \alpha}\ \frac{w}{t_L}\right]$$

$$= \frac{20}{2}\left[\frac{14}{6}\ \frac{36^3 + 24^3}{36 + 24}\ \frac{3.338}{196.972} + \frac{1}{1 + 1.429} \cdot \frac{36}{0.5}\right] = 695 \text{ psi.}$$

$$\sigma_2 = \left(\frac{1}{1 + \alpha}\right)\left(\frac{d}{w + d}\right)\left(\frac{pw}{2t_L}\right)$$

$$= \frac{1}{2.429}\ \frac{24}{36 + 24}\ \frac{20(36)}{2(0.5)} = 118 \text{ psi.}$$

σ_3 = -20 psi.

The stress intensity is 715 psi.

Summary of Example Calculations

The stresses and the stress intensities for the prismatic cask example are tabulated in Table 8 for easy comparison. It should be noted that the bonded cask meets the stress requirements in all respects while the 153 psi stress intensity in the shield of the unbonded cask is not acceptable.

Table 8. Tabulation of Stresses for Example Prismatic Cask where t_s = 10 in.

	Stresses (psi)			Intensity (psi)	Allowable Stress (psi)
	σ_1	σ_2	σ_3		
Shield and liner not bonded					
Shield at liner-shield interface	133	14.4	-20	153	127
Shield outer surface	-68.6	14	0	83	127
Liner inner surface	927	0	-20	947	35,000
Shield and liner bonded					
Shield at liner-shield interface	43.5	8.5	-20	63.5	127
Shield outer surface	-40	8.5	0	48.5	127
Liner inner surface	695	118	-20	714	35,000

Portions of the preceding example for a prismatic cask were repeated for a shielding thickness of 11.5 in. For this shielding thickness, the largest stress intensity in the shield of the unbonded cask is 126 psi. The time required to close the gap between the jacket and shield is 2,085 years or 18,265 hours.

The preceding analysis for a prismatic cask is conservative for a cask with a rectangular cavity in a cylindrical jacket provided the minimum thickness of the shield is used for t_s in the analysis; that is, that t_s is the smaller of t_a and t_b illustrated in Fig. 9.

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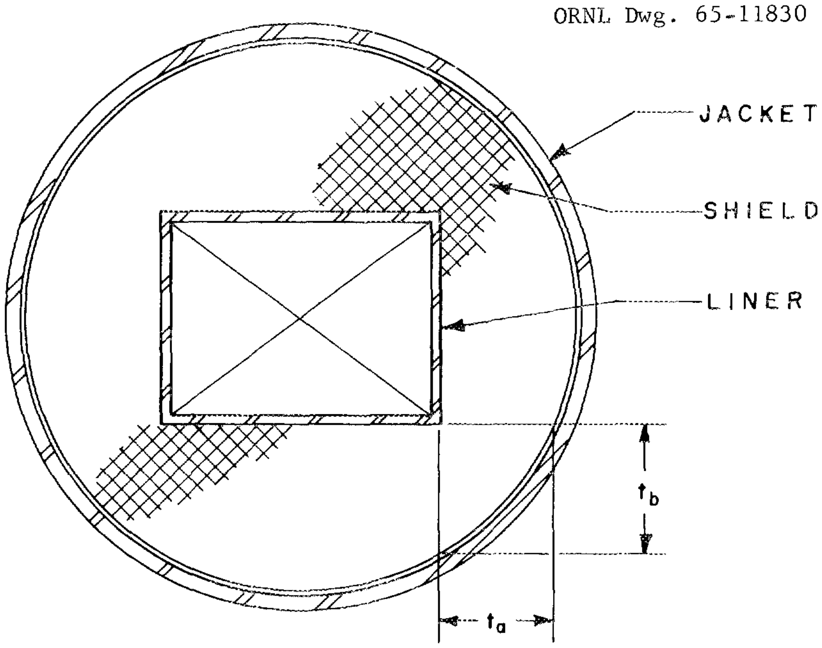

Fig. 9. Cross Section of Rectangular Cask Cavity in Cylindrical Jacket.

5. CALCULATIONS FOR CASKS WITH CYLINDRICAL CONFIGURATION

The general configuration of a cylindrical cask is illustrated in Fig. 10.

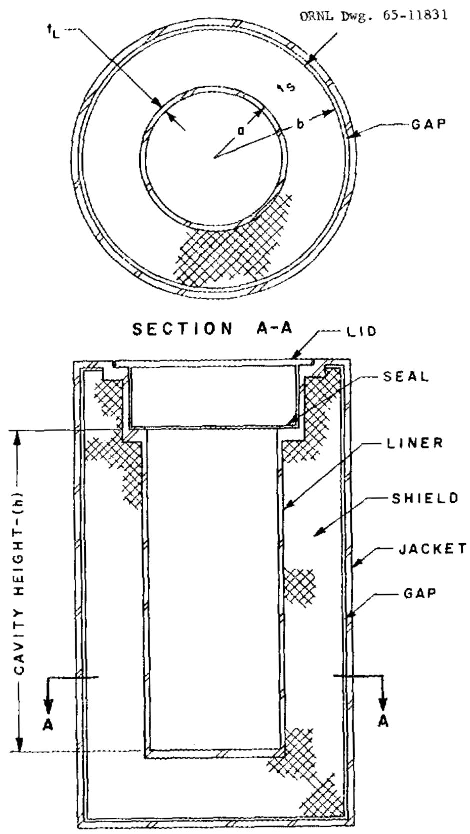

Fig. 10. General Configuration of a Cylindrical Cask.

In order to simplify the calculations and at the same time obtain conservative results, it was assumed that:

1. the jacket is not in contact with the shield,
2. the cylindrical portions of the liner and shield are in contact,
3. the thickness of the liner is such that the thin-wall formulas apply ($a/t_L > 10$),
4. the thickness of the shield is such that the thick-wall formulas apply ($b/a > 1.1$),
5. the height of the cavity, h, is greater than 2a,
6. the pressure load between the liner and shield is uniform over the common surface of contact.

Method of Calculations

Formulas are developed for calculating the principle stresses in the four regions of the liner and shield illustrated in Fig. 11. The four regions are: region 1, the liner and shield remote from discontinuities; region 2, the junction of the cylindrical portion of the liner to the lid offset; region 3, the junction of the bottom head to the cylindrical portion of the liner; and region 4, the bottom head remote from discontinuities.

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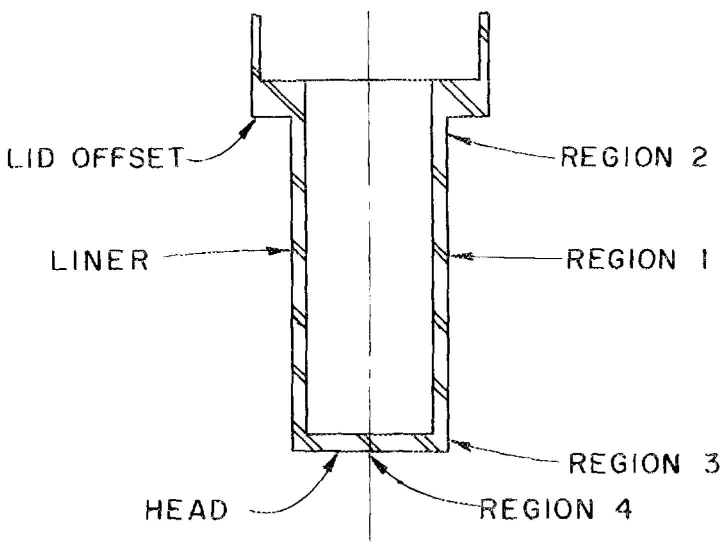

Fig. 11. Four Regions of Liner and Shield in Cylindrical Cask.

The stresses that exist in the liner at each region are:

region 1, membrane stresses due to pressure;

region 2, membrane stresses due to pressure and
discontinuity stresses due to the change in section;

region 3, membrane stresses due to pressure and
discontinuity stresses due to the change in section;

region 4, bending stresses due to pressure and
discontinuity stresses due to changes in section of region 3.

The stresses that exist in the shield were investigated in region 1 only because (1) discontinuity stresses do not exist in the shield in regions 2 and 3 and (2) the shield and liner are assumed not to be in contact in region 4. The formulas for each region of the cylindrical cask are derived in the following calculations and tabulated in Table 7 of Chapter 2. Their application is indicated in the example calculations for a cylindrical cask in this chapter.

Stresses in Region 1

The liner and shield are in contact in this region, and the radial displacement of the outer surface of the liner is equal to the radial displacement of the inner surface of the shield. The total pressure load, p, is resisted jointly by the liner and the shield. The internal pressure on the shield is denoted by p_s, and the effective internal pressure on the liner is denoted by p_L. Therefore,

$$p_s + p_L = p. \tag{53}$$

The radial displacement for a thin-walled cylinder subjected to an internal pressure and no longitudinal forces,

$$\delta_r = \frac{pr^2}{Et} , \tag{54}$$

where

p = internal pressure, psi,

r = radius of cylinder, in.,

E = modulus of elasticity, psi,

t = wall thickness, in.

For a thick-walled cylinder subjected to radial pressure and no longitudinal forces, the radial displacement at the inner surface,[1]

$$\delta_r = \frac{pa}{E}\left(\frac{b^2 + a^2}{b^2 - a^2} + \upsilon\right) , \tag{55}$$

where

p = pressure, psi,

a = inside radius, in.,

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[1]S. P. Timoshenko, p. 210 in Strength of Materials, Part II, D. Van Nostrand Company, Inc., 3rd ed., 1956.

b = outside radius, in.,

E = modulus of elasticity, psi,

υ = Poission's ratio.

The radial displacement for the liner equals the radial displacement of the inner surface of the shield. Therefore,

$$\frac{p_L a^2}{E_L t_L} = \frac{p_s a}{E_s}\left[\frac{b^2 + a^2}{b^2 - a^2} + \upsilon_s\right] \quad . \tag{56}$$

Solving Eq. 56 for p_s/p_L,

$$\frac{p_s}{p_L} = \frac{E_s a}{E_L t_L}\left(\frac{1}{\frac{b^2 + a^2}{b^2 - a^2} + \upsilon_s}\right) , \tag{57}$$

and defining,

$$\frac{p_s}{p_L} \equiv \beta \equiv \frac{\frac{a}{t_L}\,\frac{E_s}{E_L}}{\left(\frac{b^2 + a^2}{b^2 - a^2} + \upsilon_s\right)} \quad . \tag{58}$$

From Eqs. 53 and 58, the effective internal pressure on the liner,

$$p_L = \frac{p}{1 + \beta} , \tag{59}$$

and the effective internal pressure on the shield,

$$p_s = \frac{\beta p}{1 + \beta} \quad . \tag{60}$$

Once the effective internal pressures have been determined, the stresses can be calculated.

Stresses in Shield. The stresses for a thick-walled cylinder subjected to internal pressure are:[2]

1. maximum circumferential stress,

$$\sigma_\theta = p_s\left(\frac{b^2 + a^2}{b^2 - a^2}\right) = \frac{\beta p}{\beta + 1}\left(\frac{b^2 + a^2}{b^2 - a^2}\right) ; \tag{61}$$

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[2]R. J. Roark, p. 276 case 27 in Formulas for Stress and Strain, McGraw-Hill Book Co., 3rd ed., 1954.

2. maximum radial stress,

$$\sigma_r = p_s = \frac{\beta p}{\beta + 1} ; \tag{62}$$

3. maximum longitudinal stress,

$$\sigma_x = 0. \tag{63}$$

Stresses in Liner. The stresses for a thin-walled vessel subjected to internal pressures are:[3]

1. maximum circumferential stress,

$$\sigma_\theta = \frac{p_L r}{t_L} = \frac{p}{1 + \beta} \left(\frac{a}{t_L}\right) ; \tag{64}$$

2. maximum longitudinal stress assuming no help from shield,

$$\sigma_x = \frac{p_L r}{2t_L} = \frac{pa}{2t_L} ; \tag{65}$$

3. maximum radial stress,

$$\sigma_r = p . \tag{66}$$

Stresses in Region 2

In addition to membrane stresses, discontinuity stresses exist at the junction of the cylindrical portion of the liner and the offset step for the lid, as illustrated in Fig. 12.

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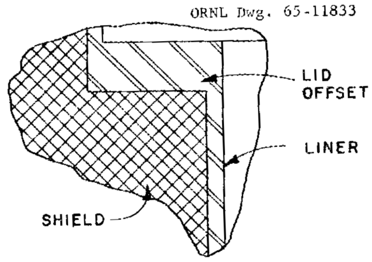

Fig. 12. Junction of Cylindrical Portion of Liner and Offset for Lid.

Because of the many designs that could possibly be used at the lid offset, assume that the attachment between the liner and the offset is completely rigid; that is, that no deflection or rotation of the joint is

[3]R. J. Roark, p. 268 case 1 in Formulas for Stress and Strain, McGraw-Hill Book Co., 3rd ed., 1954.

permitted. This is a conservative assumption because the joint is never completely rigid.

A radial force F and a bending moment M of magnitudes such that the deflection and the slope at the junction are zero must be applied at the junction, as illustrated in Fig. 13.

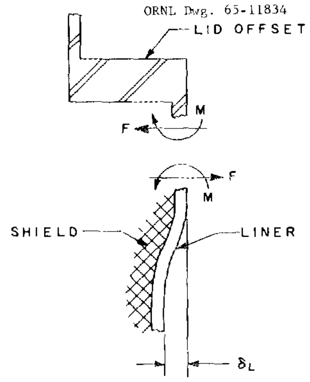

Fig. 13. Radial Force and Bending Moment at Junction of Cylindrical Portion of liner and Offset for Lid.

The deflection of the liner remote from discontinuities,

$$\delta_L = \frac{p_L r^2}{E_L t_L} = \frac{pa^2}{E_L t_L} \quad \frac{1}{1+\beta} \quad . \tag{67}$$

The moment M and the force F must prevent this deflection and also maintain a zero slope at the junction.

Deflections due to F and M, respectively,[4]

$$\delta_F = \frac{F}{2D_L \lambda_L^3} \ , \tag{68}$$

$$\delta_M = - \frac{M}{2D_L \lambda_L^2} \ , \tag{69}$$

where

M = moment, in.-lb per inch of circumference,

F = force, lb per inch of circumference,

$$D_L = \frac{E_L t_L^3}{12(1 - \upsilon_L^2)} \ ,$$

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[4]S. P. Timoshenko, pp. 12, 126 and 127 in Strength of Materials, Part II, D. Van Nostrand Co., 3rd ed., 1956.

$$\lambda_L = \sqrt[4]{\frac{3(1 - \nu_L^2)}{r^2 t_L^2}} \ .$$

Setting Eq. 67 equal to Eq. 68 plus 69,

$$\frac{pa^2}{E_L t_L} \left(\frac{1}{1 + \beta}\right) = \frac{(F - \lambda_L M)}{2\lambda_L^3 D_L} \ . \tag{70}$$

The slope produced by F must be counteracted by M;[4] that is,

$$F - 2\lambda M = 0. \tag{71}$$

Substituting for D_L in Eq. 70 and rearranging,

$$F - \lambda_L M = \frac{p}{2\lambda_L (1 + \beta)} \ . \tag{72}$$

Simultaneous solution of Eqs. 71 and 72 yields,

$$M = \frac{p}{2\lambda_L^2 (1 + \beta)} \ , \tag{73}$$

and

$$F = \frac{p}{\lambda_L (1 + \beta)} \ . \tag{74}$$

Once the discontinuity force and moment have been determined, the discontinuity stresses in the liner can be determined. The maximum circumferential stress at the junction of the liner and offset for the lid due to F,[5]

$$\sigma_\theta = \frac{2F\lambda_L a}{t_L} \text{ (compressive)} \ . \tag{75}$$

At the junction, F produces no longitudinal stress.

At the junction, the moment M produces stresses (1) in the longitudinal direction, (2) in the circumferential direction due to the increase in radius affected by it, and (3) in the circumferential direction due to the deformation of a transverse section of an elemental longitudinal strip. These stresses are a maximum at the junction, and they are:[5]

1.

$$\sigma_x = \frac{6M}{t_L^2} \ , \tag{76}$$

[5]R. J. Roark, p. 271 in Formulas for Stress and Strain, McGraw-Hill Book Co., 3rd ed., 1954.

2. $$\sigma_\theta = \frac{2M\lambda_L^2 a}{t_L} \text{ (tensile)} ,\qquad (77)$$

3. $$\sigma_\theta = \nu\sigma_x = \frac{6\nu M}{t_L^2} .\qquad (78)$$

The stresses given by Eqs. 76 and 78 are tensile on the inner surface of the liner and are compressive on the outer surface.

The total principal stresses in the liner at region 2 are obtained by adding the membrane stresses given by Eqs. 64, 65, and 66 to the discontinuity stresses given by Eqs. 75, 76, 77, and 78.

Stresses in Region 3

Discontinuity stresses in addition to membrane stresses exist at the junction of the cylindrical portion of the liner and the bottom head. In order to satisfy the conditions of continuity at the junction of the cylindrical wall and head, the change of slope of an elemental longitudinal strip in the cylindrical wall must equal the change of slope of the head, and the radial deflection of the flat head must be equal to the radial deflection of the cylindrical wall. In the analysis, it was assumed that:

1. a gap exists between the head and shielding so that the head must resist the total pressure,
2. the radial deflection of the head is zero.

A sectional view at region 3 is illustrated in Fig. 14, and the forces and moments acting at the junction are illustrated in Fig. 15.

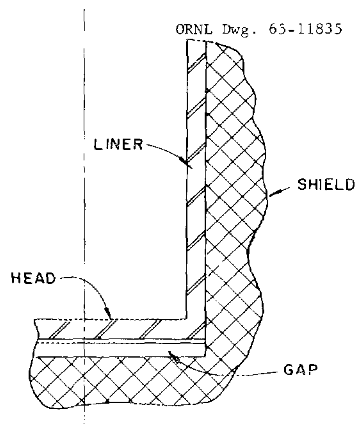

Fig. 14. Sectional View at Region 3.

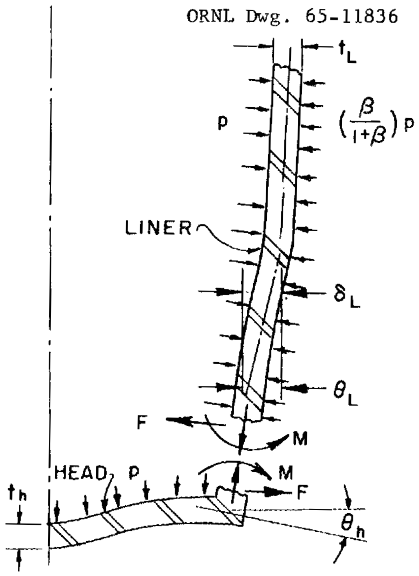

Fig. 15. Forces and Moments Acting at Junction in Region 3.

The change in slope at the edge of the head,[6]

$$\Delta\theta_h = -\frac{3p\pi a^2(m_L - 1)a}{2\pi E_L m_L t_h^3} + \frac{12(m_L - 1)Ma}{E_L m_L t_h^3}$$

$$= \frac{(m_L - 1)a}{E_L m_L t_h^3}[-1.5\ pa^2 + 12M]\ . \tag{79}$$

The change in slope at the edge of the liner,[5]

$$\Delta\theta_L = \frac{F}{2D_L\lambda_L^2} - \frac{M}{\lambda_L D_L}\ . \tag{80}$$

Since both slopes were originally zero,

$$\Delta\theta_L = \Delta\theta_h\ . \tag{81}$$

Substituting Eqs. 79 and 80 into Eq. 81,

$$\frac{(m_L - 1)a}{E_L m_L t_h^3}[1.5\ pa^2 - 12M] + \frac{F}{2D_L\lambda_L^2} - \frac{M}{\lambda_L D_L} = 0\ . \tag{82}$$

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[6]R. J. Roark, pp. 194 and 197 in Formulas for Stress and Strain, McGraw-Hill Book Co., 3rd ed., 1954.

By definition, $m_L = \frac{1}{\upsilon_L}$ and $D_L = \frac{E_L t_L^3}{12(1 - \upsilon_L^2)}$. Using these definitions and solving Eq. 82 for M,

$$M = \frac{4(1 + \upsilon_L)Ft_L^3 + pa^3\lambda_L^3 t_L^3}{8(1 + \upsilon_L)\lambda_L t_h^3 + 8a\lambda_L^2 t_L^3} . \tag{83}$$

The other continuity condition that the radial deflection equals zero is employed to obtain a second equation involving M and F. The radial deflection of the liner without the restraint of the head,[3]

$$\delta = \frac{p_L r^2}{E_L t_L} = \frac{p}{(1 + \beta)} \frac{a^2}{E_L t_L} . \tag{84}$$

The force F and the moment M must produce a resultant deflection equal and opposite to that produced by the pressure. The deflection, δ, due to a radial load, F,[5]

$$\delta_F = \frac{F}{2D_L \lambda_L^3} . \tag{85}$$

The deflection, δ, due to a radial moment, M,[5]

$$\delta_M = - \frac{M}{2D_L \lambda_L^2} . \tag{86}$$

Setting Eq. 84 equal to Eqs. 85 plus 86,

$$\frac{p}{(1 + \beta)} \frac{a^2}{E_L t_L} = \frac{F}{2D_L \lambda_L^3} - \frac{M}{2D_L \lambda_L^2} . \tag{87}$$

Since $D_L = \frac{E_L t_L^3}{12(1 - \upsilon_L^2)}$, Eq. 87 reduces to

$$M = \frac{F}{\lambda_L} - \frac{pa^2\lambda_L^2 t_L^2}{6(1 + \beta)(1 - \upsilon_L^2)} . \tag{88}$$

Simultaneous solution of Eqs. 83 and 88 yields

$$F = \frac{p[(1 + \beta)(a\lambda_L t_L)^3 + 4a\lambda_L t_L^3 + 4(1 + \upsilon_L)t_h^3]}{4(1 + \beta)\lambda_L[2a\lambda_L t_L^3 + t_h^3]} . \tag{89}$$

Once F has been determined, M may be found from Eq. 88.

After M and F have been calculated, the discontinuity stresses at region 3 in the cylindrical portion of the liner are determined from the circumferential stress due to F, the circumferential stress due to M, the longitudinal stress due to M, and the secondary circumferential stress due to M.[5] The circumferential stress due to F,

$$\sigma_{\theta_F} = \frac{2F}{t_L} \lambda_L a \text{ (compressive)} . \tag{90}$$

The circumferential stress due to M,

$$\sigma_{\theta_M} = \frac{2M}{t_L} \lambda_L^2 a \text{ (tensile)} . \tag{91}$$

The longitudinal stress due to M,

$$\sigma_{x_m} = \frac{6M}{t_L^2} . \tag{92}$$

The secondary circumferential stress due to M,

$$\sigma_{\theta_M} = \upsilon_L \sigma_{x_M} = \frac{6\upsilon_L M}{t_L^2} . \tag{93}$$

The stresses given by Eqs. 92 and 93 are tensile at the inner surface and compressive at the outer surface.

The total principal stresses in the liner at region 3 are obtained by adding the membrane stresses given by Eqs. 64, 65, and 66 to the discontinuity stresses given by Eqs. 90, 91, 92, and 93.

The discontinuity stresses at region 3 in the head of the liner are the radial stress due to F, the radial stress due to M, the circumferential stress due to F, the circumferential stress due to M, and the traverse stress. The radial stress due to F,

$$\sigma_{r_F} = \frac{F}{t_h} \text{ (tensile)} . \tag{94}$$

The radial stress due to M,

$$\sigma_{r_M} = \frac{6M}{t_h^2} . \tag{95}$$

The circumferential stress due to F,

$$\sigma_{\theta_F} = \frac{\upsilon_L F}{t_h} \text{ (tensile)} . \tag{96}$$

The circumferential stress due to M,

$$\sigma_{\theta_M} = \frac{6M}{t_h^2} \cdot \qquad (97)$$

The traverse stress,

$$\sigma_x = p \text{ (compressive)} . \qquad (98)$$

The stresses given by Eqs. 95 and 97 are tensile on the inside surface and compressive on the outside surface. The total principal stresses in the head at region 3 are found by adding like stresses given by Eqs. 94 through 98.

Stresses in Region 4

A free-body diagram of the head is illustrated in Fig. 16. The stresses in region 4 are affected by the moment M, the force F, and the pressure p.

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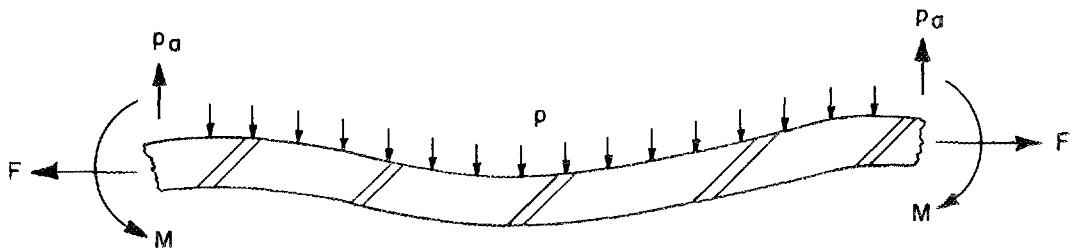


Fig. 16. Free-Body Diagram of Head in Region 4.

Stresses due to M are the radial stress,[6]

$$\sigma_{r_M} = \frac{6M}{t_h^2} ; \qquad (99)$$

and the circumferential stress,

$$\sigma_{\theta_M} = \frac{6M}{t_h^2} . \qquad (100)$$

Stresses due to F are the radial stress,

$$\sigma_{r_F} = \frac{F}{t_h} \text{ (tensile)}; \qquad (101)$$

and the circumferential stress,

$$\sigma_{\theta_F} = \frac{F}{t_h} \text{ (tensile)} \tag{102}$$

Stresses due to p are the radial stress,[6]

$$\sigma_{r_p} = \frac{3pa^2}{8t_h^2}(3 + \upsilon_L) \ ; \tag{103}$$

the circumferential stress,

$$\sigma_{\theta_p} = \frac{3pa^2}{8t_h^2}(3 + \upsilon_L) \ ; \tag{104}$$

and the traverse stress,

$$\sigma_p = p \text{ (compressive)} \ . \tag{105}$$

The stresses given by Eqs. 99 and 100 are tensile on the inside surface and compressive on the outside surface whereas the stresses given by Eqs. 103 and 104 are compressive on the inside and tensile on the outer surface. The total principal stresses at region 4 are found by adding like stresses given by Eqs. 99 through 105.

Application of Results of Calculations

The approach that could be followed in using the results of the preceding calculations consists of:

1. selecting the required shield thickness based on shielding requirements,
2. assuming a liner thickness, t_L, and a head thickness, t_h,
3. calculating the maximum principal stresses (σ_1, σ_2, and σ_3) in the shield and liner,
4. computing S ($S = \sigma_{max} - \sigma_{min}$) and comparing with the yield point of the material in question,
5. repeating the above four steps until the calculated stresses are acceptable ($S \leq S_{yp}$).

Example for Cask with Cylindrical Configuration

The material properties are obtained from Tables 1 through 4, and reference is made to Table 7 for the formulas for the stresses. Assume a cask of cylindrical configuration with the characteristics listed below.

Dimensions: $a = 27.5$ in., $t_s = 10.5$ in., $b = 38$ in., and $h = 132$ in.

Design pressure = 100 psi

Design temperature = 212°F (wall temperature of liner)

Shielding material: chemical lead, $E_s = 2 \times 10^6$ psi, $\upsilon_s = 0.4$

Liner material: type 347 stainless steel, $E_L = 28 \times 10^6$ psi, $\upsilon_L = 0.3$

Assume that $t_L = 1.25$ in. and $t_h = 1.5$ in.

Therefore,

$$\beta = \frac{E_s a}{E_L t_L} \frac{1}{\left(\frac{b^2 + a^2}{b^2 - a^2}\right) + \upsilon_s} = \frac{2 \times 10^6 \times 27.5}{28 \times 10^6 \times 1.25} \frac{1}{\left(\frac{38^2 + 27.5^2}{38^2 - 27.5^2}\right) + 0.4}$$

$$= 0.437,$$

and

$$\lambda_L = \sqrt[4]{\frac{3(1 - \upsilon_L^2)}{a^2 t_L^2}} = \sqrt[4]{\frac{3(1 - 0.3^2)}{27.5^2 \times 1.25^2}} = 0.2194.$$

Determination of Stresses

Stresses in Shield in Region 1.

$$S = \frac{\beta p}{\beta + 1}\left(\frac{b^2 + a^2}{b^2 - a^2} + 1\right) = \frac{0.437 \times 100}{0.437 + 1} (3.199 + 1) = 127 \text{ psi.}$$

Stresses in Liner at Liner Lid Offset in Region 2.

$$F_{\text{reg. 2}} = \frac{\beta p}{\lambda_L(1 + \beta)} = \frac{p}{0.2194(1 + 0.437)} = 3.172p \ .$$

$$M_{\text{reg. 2}} = \frac{p}{2\lambda_L^2(1 + \beta)} = \frac{p}{2 \times 0.2194^2(1 + 0.437)} = 7.23p \ .$$

The stress in the liner at the outside surface in region 2,

$$\sigma_1 = \frac{p}{1+\beta}\left(\frac{a}{t_L}\right) - \frac{2F_{reg.\ 2}\lambda_L a}{t_L} + \frac{2M_{reg.\ 2}\lambda_L^2 a}{t_L} - \frac{6\upsilon_L M_{reg.\ 2}}{t_L^2}$$

$$= \left(\frac{100}{1.437}\right)\left(\frac{27.5}{1.25}\right) - \frac{2(3.172)(100)(0.2194)(27.5)}{1.25}$$

$$+ \frac{2(7.23)(100)(0.2194^2)(27.5)}{1.25} - \frac{(0.6)(0.3)(7.23)(100)}{1.25^2}$$

$$= 1530 - 3062 + 1531 - 833 = -834 \text{ psi},$$

$$\sigma_2 = \frac{pa}{2t_L} - \frac{6M_{reg.\ 2}}{t_L^2} = \frac{100(27.5)}{2(1.25)} - \frac{6(7.23)(100)}{1.25^2} = 1833 - 2776 = -943 \text{ psi},$$

$$\sigma_3 = \frac{\beta p}{1+\beta} = \frac{0.437}{1.437}(100) = -30 \text{ psi}.$$

Therefore, the stress intensity on the outside surface of the liner in region 2 is $\sigma_{max} - \sigma_{min} = 943$ psi.

The stress in the liner at the inside surface of region 2,

$$\sigma_1 = \frac{p}{1+\beta}\left(\frac{a}{t_L}\right) - \frac{2F_{reg.\ 2}\lambda_L a}{t_L} + \frac{2M_{reg.\ 2}\lambda_L^2 a}{t_L} + \frac{6\upsilon_L M_{reg.\ 2}}{t_L^2}$$

$$= 1530 - 3062 + 1531 + 833 = +832 \text{ psi},$$

$$\sigma_2 = \frac{pa}{2t_L} + \frac{6M_{reg.\ 2}}{t_L^2} = 1833 + 2776 = +4609 \text{ psi},$$

$$\sigma_3 = p = -100 \text{ psi}.$$

Therefore, the stress intensity on the inside surface of the liner in region 2 is 4709 psi.

Stress in Liner in Region 3.

$$F_{reg.\ 3} = \frac{p[(1+\beta)(a\lambda_L t_L)^3 + 4a\lambda_L t_L^3 + 4(1+\upsilon_L)t_h^3]}{4(1+\beta)\lambda_L[2a\lambda_L t_L^3 + t_h^3]},$$

$$= \frac{100[1.437(27.5 \times 0.2194 \times 1.25)^3 + 4(27.5)(0.2194)(1.25^3) + 4(1.3)(1.5^3)}{4(1.437)(0.2194)[2(27.5)(0.2194)(1.25^3) + 1.5^3]}$$

$= 1453.3.$

$$M_{reg.\ 3} = \frac{F_{reg.\ 3}}{\lambda_L} - \frac{pa^2\lambda_L^2 t_L^2}{6(1+\beta)(1-\upsilon_L^2)}$$

$$= \frac{1453.3}{0.2194} - \frac{100(27.5^2)(0.2194^2)(1.25^2)}{6(1.437)(0.91)} = 5899 \ .$$

The stress in the liner at the outside surface of the cylindrical wall,

$$\sigma_1 = \frac{p}{1+\beta}\,\frac{a}{t_L} - \frac{2F_{reg.\ 3}}{t_L}\lambda_L a + \frac{2M_{reg.\ 3}\lambda_L^2 a}{t_L} - \frac{6\upsilon_L M_{reg.\ 3}}{t_L^2}$$

$$= \left(\frac{100}{1.437}\right)\left(\frac{27.5}{1.25}\right) - \frac{2(1453.3)}{1.25}(0.2194)(27.5) + \frac{2(5899)(0.2194^2)(27.5)}{1.25} - \frac{6(0.3)(5899)}{1.25^2}$$

$$= 1530 - 14030 + 12493 - 6796 = -6803,$$

$$\sigma_2 = \frac{pa}{2t_L} - \frac{6M_{reg.\ 3}}{t_L} = 1833 - 22652 = -20{,}819 \text{ psi},$$

$$\sigma_3 = \frac{p}{\beta+1} = -30.$$

Therefore, the stress intensity on the outside surface of the cylinder portion of the liner at region 3 is 20,789 psi.

The stress in the liner at the inside surface of the cylindrical wall,

$$\sigma_1 = \frac{p}{1+\beta}\left(\frac{a}{t_L}\right) - \frac{2F_{reg.\ 3}}{t_L}\lambda_L a + \frac{2M_{reg.\ 3}}{t_L}\lambda_L^2 a + \frac{6\upsilon_L M_{reg.\ 3}}{t_L^2}$$

$$= 1530 - 14030 + 12493 + 6796 = +6789,$$

$$\sigma_2 = \frac{pa}{2t_L} + \frac{6M_{reg.\ 3}}{t_L^2} = 1833 + 22652 = +24{,}485,$$

$$\sigma_3 = p = -100.$$

Therefore, the stress intensity on the inside surface of the cylindrical portion of the liner at region 3 is 24,585 psi.

The stress in the liner at the inside surface of the head,

$$\sigma_1 = \frac{F_{reg.\ 3}}{t_h} + \frac{6M_{reg.\ 3}}{t_h} = \frac{1453.3}{1.5} + \frac{6(5899)}{1.5^2} = 962 + 15730 = +16{,}692,$$

$$\sigma_2 = \frac{\upsilon_L F_{reg.\ 3}}{t_h} + \frac{6M_{reg.\ 3}}{t_h^2} = 27.9 + 15730 = 16,009,$$

$$\sigma_3 = p = -100.$$

Therefore, the stress intensity on the inside surface of the head in region 3 is 16,792 psi.

The stress in the liner at the outside surface of the head,

$$\sigma_1 = \frac{F_{reg.\ 3}}{t_h} - \frac{6M_{reg.\ 3}}{t_h^2} = 962 - 15730 = -14,768,$$

$$\sigma_2 = \frac{\upsilon_L F_{reg.\ 3}}{t_h} - \frac{6M_{reg.\ 3}}{t_h} = 279 - 15730 = -15,451,$$

$$\sigma_3 = 0.$$

Therefore, the stress intensity on the outside surface of the head in region 3 is 15,451 psi.

Stress in Region 4. The stress in the liner at the inside surface of the head,

$$\sigma_1 = \frac{6M_{reg.\ 3}}{t_h^2} + \frac{F_{reg.\ 3}}{t_h} - \frac{3pa^2(3 + \upsilon_L)}{8t_h^2}$$

$$= 15730 + 962 - \frac{3(100)(27.5^2)(3.3)}{8 \times 1.5^2} = -24,902 \text{ psi},$$

$$\sigma_2 = \frac{6M_{reg.\ 3}}{t_h^2} + \frac{\upsilon_L F_{reg.\ 3}}{t_h} - \frac{3pa^2}{8t_h^2}(3 + \upsilon_L) = 15730 + 279 - 41594 = -25,585,$$

$$\sigma_3 = p = -100.$$

Therefore, the stress intensity on the inside surface of the head in region 4 is 24,585 psi.

The stress in the liner at the outside surface of the head,

$$\sigma_1 = -\frac{6M_{reg.\ 3}}{t_h^2} + \frac{F_{reg.\ 3}}{t_h} + \frac{3pa^2(3 + \upsilon_L)}{8t_h^2}$$

$$= -15730 + 962 + 41594 = +26,826 \text{ psi},$$

$$\sigma_2 = -\frac{6M_{reg.\ 3}}{t_h^2} + \frac{\upsilon_L F_{reg.\ 3}}{t_h} + \frac{3pa^2}{8t_h^2}(3 + \upsilon_L) = -15730 + 279 + 41594$$

$$= 26,143,$$

$\sigma_3 = 0.$

Therefore, the stress intensity on the outside surface of the head in region 4 is 26,826 psi.

Summary of Example Calculations

As the above calculations show, the head thickness can be reduced slightly with a resulting stress intensity closer to the limit of 35,000 psi. The thickness of the cylindrical portion of the liner cannot be reduced because the allowable stress in the shield would be exceeded. The stress intensities calculated for the cylindrical cask example are given in Table 9.

Table 9. Tabulation of Stresses for Example Cylindrical Cask

	Calculated Stress Intensity (psi)	Allowable Stress (psi)
Stress in Shield in region 1	127	127[a]
Stress in liner in region 2		
Outside surface	934	35,000
Inside surface	4,709	35,000
Stress in liner in region 3		
Outside surface of cylindrical wall	20,789	35,000
Inside surface of cylindrical wall	24,585	35,000
Inside surface of head	16,792	35,000
Outside surface of head	15,451	35,000
Stress in liner in region 4		
Inside surface of head	24,585	35,000
Outside surface of head	26,826	35,000

[a]Elastic limit stress.

6. CREEP EVALUATION FOR LEAD SHIELD

If the shielding material is lead and the pressure load is to be applied for extended periods of time (weeks), the creep of the lead must be evaluated. The design limit for creep in pressure vessels set by Section VIII of the ASME Boiler and Pressure Vessel Code[1] is 1% in 10,000 hours. If this 1% creep limit is used, the allowable operation time can be determined as given below.

The secondary creep rate increases with increasing stress, and for a constant temperature, the relationship between them is most commonly expressed by the empirical equation:[2]

$$\dot{\epsilon}_c = B\sigma^{\eta} (\eta > 1), \tag{106}$$

where

$\dot{\epsilon}_c$ = creep strain rate, $\frac{\text{in./in.}}{\text{hr}}$,

σ = stress in psi,

B = temperature dependent constant with units of $\frac{(\text{in.}^2/\text{lb})^{\eta}}{\text{hr}}$,

η = temperature dependent constant.

In logarithmic form Eq. 106 becomes

$$\log \dot{\epsilon}_c = \log B + \eta \log \sigma .$$

Given two sets of data for $\dot{\epsilon}_c$ and σ at a specified temperature, one can write

$$\log (\dot{\epsilon}_c)_1 = \log B + \eta \log \sigma_1 ,$$

and

$$\log (\dot{\epsilon}_c)_2 = \log B + \eta \log \sigma_2 .$$

Subtracting the first from the second gives

$$\log (\dot{\epsilon}_c)_2 - \log (\dot{\epsilon}_c)_1 = \eta \log \sigma_2 - \eta \log \sigma_1 ,$$

---

[1]ASME Boiler and Pressure Vessel Code, Section VIII, Rules for Construction of Unfired Pressure Vessels, American Society of Mechanical Engineers, New York.

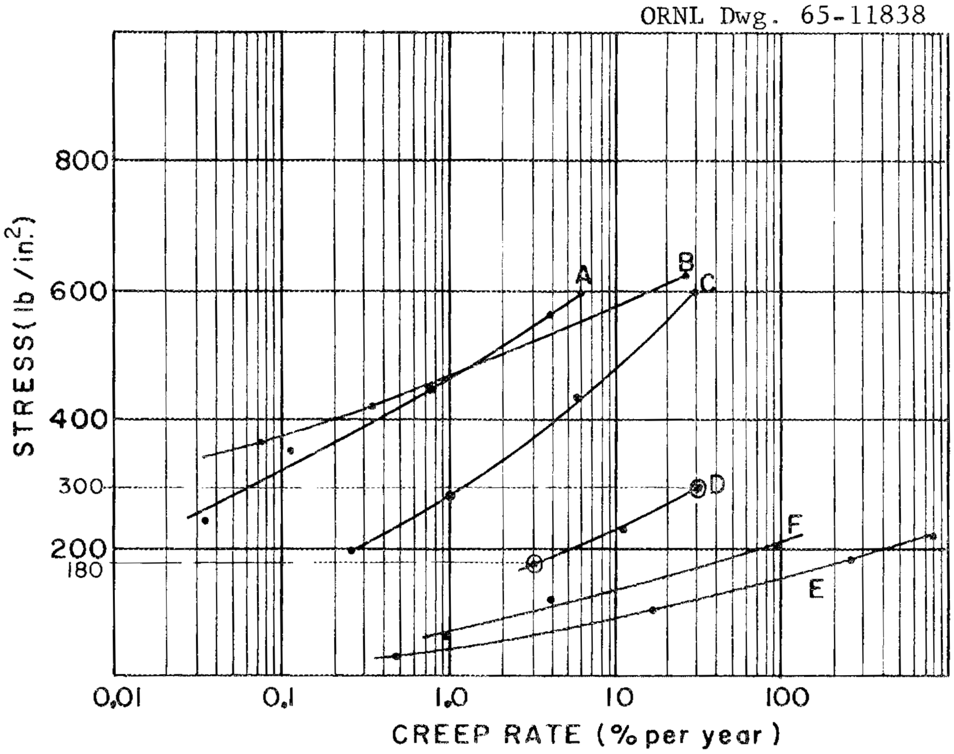
[2]C. W. Richards, pp. 28, 291, and 292 in Engineering Materials Science, Wadsworth Publishing Co., San Francisco, 1961.

or

$$\eta = \frac{\log \frac{(\dot{\epsilon}_c)_2}{(\dot{\epsilon}_c)_1}}{\log \frac{\sigma_2}{\sigma_1}} .$$

Once η is determined, substitution of consistent data for $\dot{\epsilon}_c$ and σ into Eq. 106 will yield B.

The data shown in Fig. 17 may be used to establish an equation like Eq. 106 but with $\dot{\epsilon}_c$ having units of $\frac{\text{in./in.}}{\text{yr}}$ and B having units of $\frac{(\text{in.}^2/\text{lb})^\eta}{\text{yr}}$.



A = 6% antimonial lead, 30°C
B = chemical lead, 30°C
C = common lead, 30°C
D = chemical lead, 100°C
E = 6% antimonial lead, 100°C
F = common lead, 100°C

Fig. 17. Creep Data for Rolled Lead (from American Smelting and Refining Company).

If the shielding material is assumed to be chemical lead, use of the encircled data on Fig. 17 yields $B = 10^{-11.5}$ and $\eta = 4.5$. Therefore, the creep rate equation for chemical lead at a temperature of 212°F is assumed to be

$$\dot{\epsilon}_c = 10^{-11.5}\,\sigma^{4.5} \tag{107}$$

where

$\dot{\epsilon}_c$ = creep rate, $\frac{\text{in./in.}}{\text{yr}}$,

σ = stress, psi.

From Table 3, the allowable stress for chemical lead corresponding to an operating temperature of 212°F is 127 psi. Therefore,

$$\begin{aligned}\dot{\epsilon}_c &= 10^{-11.5}(127)^{4.5}\\ &= 2.93 \times 10^{-2.4}\\ &= 0.00888\ \frac{\text{in./in.}}{\text{yr}}\ ,\\ &= 0.00888\ \frac{\text{in./in.}}{8760\ \text{hr}}\ ,\\ &= 1.014 \times 10^{-6}\ \frac{\text{in./in.}}{\text{hr}}\ ,\\ &= 1.014\%\ \text{in}\ 10{,}000\ \text{hr}.\end{aligned}$$

Upon observing this result of 1.014% in 10,000 hr, one suspects that the allowable stress of 127 psi for chemical lead at a temperature of 212°F was established by using the aforementioned ASME specification.[1]

In the development of the formulas that are presented for determining the stresses in the shield, it was assumed that a gap exists between the jacket and the shield. If the shield creeps for a sufficient length of time, this gap is closed so that contact between the shield and jacket is established. Creep is affected by this contact in two ways. First, the contact results in a redistribution of the load among the liner, shield, and jacket. This causes a reduction in the maximum stress in the shield, which in turn reduces the creep rate. Second, the contact improves the heat transfer characteristics of the cask. This reduces the temperature of the shield and this also reduces the creep rate. Therefore, one may conclude that creep in the shield tends to be self alleviating.

A free-body diagram of a span of length w is illustrated in Fig. 18. From Eq. 8, the minimum end moment was found to be $pw^2/16$. The time

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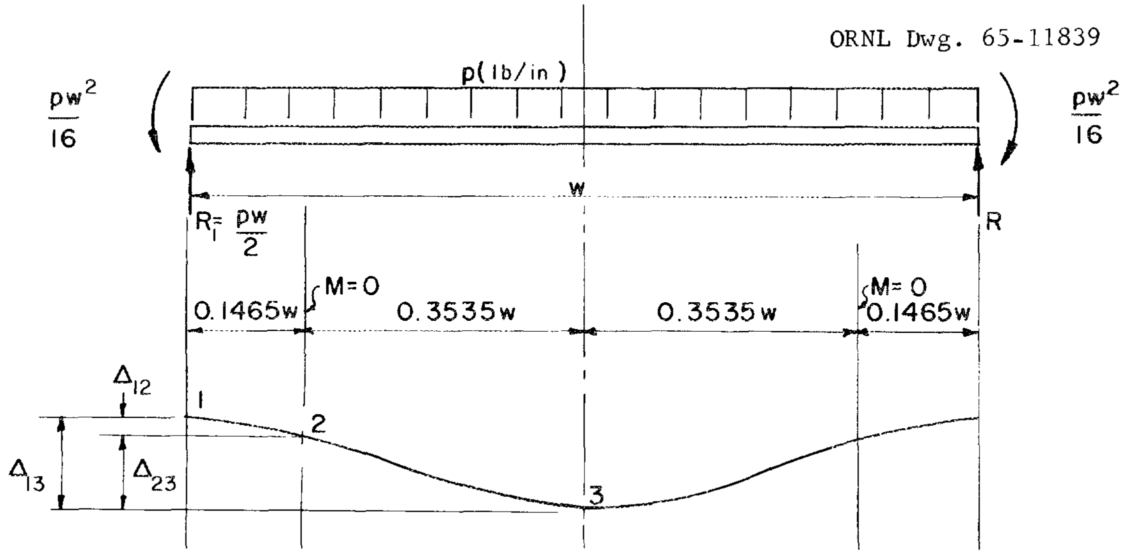

Fig. 18. Free-Body Diagram of Span of Length w.

required to close the gap between the jacket and the shield will be calculated, and the elastic deflection will be calculated by using the minimum moment $pw^2/16$ at the end of the span. The deflection due to creep will also be calculated by assuming the span to be simply supported. Use of the end moment permits the following equation to be written to establish the points of zero moment in the span.

$$M_x = -\frac{pw^2}{16} + \frac{pw}{2}x - \frac{px^2}{2} = 0;\ x^2 - wx + \frac{w^2}{4} = -\frac{w^2}{8} + \frac{w^2}{4}$$

$$(x - 0.5w)^2 = \pm 0.3535w;\ x = 0.1465\ w.$$

Segments 1-2 and 2-3 shown in Fig. 18 may be treated as cantilever beams supported at point 1 and fixed at point 3. The deflection of point 2 with respect to point 1 consists of two parts: (1) the deflection of point 2 due to the rotation of the joint at point 1, and (2) the deflection due to the deformation of segment 1-2. Use of the end moment $pw^2/16$ and of Eq. 8 in Eq. 3 gives an angle of rotation of the joint at point 1 of $pw^3/96EI$. This rotation contributes $\frac{pw^3}{96EI}$ (0.1465w) in. to the deflection Δ_{12}. The magnitude of the end deflection of a cantilever beam of length ℓ fixed (no rotation) at one end and carrying a uniformly distributed load of p lb/in. is $p\ell^4/8EI$ (ref. 3). Thus, the deformation of segment 1-2

[3]P. G. Laurson and W. J. Cox, p. 164 in Mechanics of Materials, Wiley and Sons, Inc., 3rd ed., New York, 1947.

contributes $\frac{p(0.1465w)^4}{8EI}$ to the deflection Δ_{12}. Hence,

$$\Delta_{12} = \frac{pw^3}{96EI}(0.1465w) + \frac{p}{8EI}(0.1465w)^4$$

$$= 1.5836 \times 10^{-3} \frac{pw^4}{EI} .$$

Similarly, $\Delta_{23} = \frac{p}{8EI}(0.3535w)^4 = 1.9519 \times 10^{-3} \frac{pw^4}{EI}$, and therefore the elastic deflection,

$$\Delta_{13} = (1.5836 + 1.9519)\ 10^{-3} \frac{pw^4}{EI} = 3.5355 \times 10^{-3} \frac{pw^4}{EI} . \qquad (108)$$

In calculating the deflection due to creep, it was assumed that plane sections before bending remain plane after bending in the plastic as well as in the elastic range. This usual assumption permits the derivation related to the deflection of the free end of a cantilever beam subject to creep that follows. The deflection of a cantilever beam is illustrated in Fig. 19.

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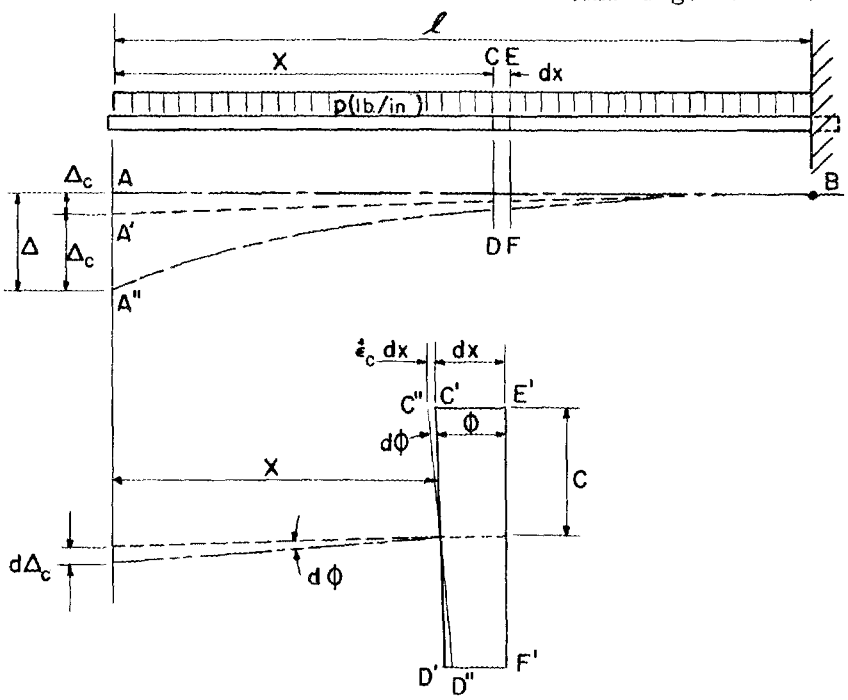

Fig. 19. The Deflection of a Cantilever Beam.

In Fig. 19, the lines AB, A'B, and A''B represent the neutral surfaces of the beam subjected to no deflection, elastic deflection, and elastic plus creep deflection, respectively. Likewise, CD and EF are sections of the nondeflected beam dx units apart and x units from the free end. C'D' and E'F' are for the beam subjected only to elastic deflection and C''D'' represents the orientation of section CD with respect to C'D' after the occurrence of creep. From Fig. 19, it is evident that $cd\phi = \epsilon_c dx$ and that $x d\phi = d\triangle_c$. Hence, the differential creep deflection,

$$d\triangle_c = \frac{\epsilon_c}{c} x dx. \tag{109}$$

Eq. 106 may be written

$$\dot{\epsilon}_c = \frac{\epsilon_c}{T} = B\sigma^{\eta},$$

where

$\dot{\epsilon}_c$ = creep, in./in.,

T = time, yr.

Substituting $\dot{\epsilon}_c = B\sigma^{\eta}T$ and $\sigma = \frac{Mc}{I} = \frac{px^2c}{2I}$ into Eq. 109,

$$d\triangle_c = \frac{BTp^{\eta}c^{\eta-1}}{2^{\eta}I^{\eta}} x^{2\eta+1} dx,$$

whence,

$$\triangle_c = \frac{BTp^{\eta}c^{\eta-1}}{2^{\eta}I^{\eta}} \int_0^L x^{2\eta+1} dx,$$

and the end deflection due to creep,

$$\triangle_c = \frac{Bp^{\eta}c^{\eta-1}L^{2\eta+2}}{2^{\eta}I^{\eta}(2\eta + 2)} T. \tag{110}$$

For chemical lead at a temperature of 212°F, Eq. 110 becomes

$$\triangle_c = \frac{(10^{-11.5})p^{4.5}(c^{3.5})L^{11}}{2^{4.5}(I^{4.5}) \times 11} T. \tag{111}$$

Since $I = \frac{bt^3}{12}$ where b is assumed to be 1 in., Eq. 111 reduces to

$$\triangle_c = 8.066 \times 10^{-11} \frac{p^{4.5}L^{11}}{t^{10}} T. \tag{112}$$

For purposes of illustrating the use of Eqs. 108 and 112 in evaluating the severity of creep, suppose a prismatic cask has an internal pressure of 20 psi, a lead shield 10 in. thick, a span length $w = 36$ in., and an operating temperature of 212°F.

By referring to Table 1, it may be seen that shrinkage on casting is taken in practice to be 7/64 to 5/16 in. per foot. Certainly the lead shield with its large thickness relative to the liner thickness will subject the liner walls to compressive forces sufficient to shorten if not buckle them. In the light of these two considerations, let it be assumed that a gap of $t_s/60$ in. will develop between the shield and the jacket during fabrication of the cask. Note that t_s is the thickness of the shield in inches. If this gap is to be closed, the sum of the elastic deflection plus the deflection due to creep must be no less than $t_s/60$.

Since $I = t^3/12$ and E for lead $= 2 \times 10^6$, from Eq. 108,

$$\Delta_{13} - 3.5355 \times 10^{-3} \frac{pw^4(12)}{(2 \times 10\ )t^3} = \frac{2.1213 \times 10^{-8} pw^4}{t^3}.$$

Since half a simply supported beam may be treated as a cantilever, from Eq. 112,

$$\Delta_c = 8.066 \times 10^{-11} \frac{p^{4.5}}{t_s^{10}} (w/2)^{11}\ T$$

$$= 3.9385 \times 10^{-14} \frac{p^{4.5} w^{11} T}{t_s^{10}}\ .$$

Therefore,

$$\frac{t_s}{60} = 2.1213 \times 10^{-8} \frac{pw^4}{t_s^3} + 3.9385 \times 10^{-14} \frac{p^{4.5} w^{11}}{t_s^{10}}\ T. \qquad (113)$$

Substituting $p = 20$, $t_s = 10$, and $w = 36$ into Eq. 113,

$$\frac{10}{60} - 2.1213 \times 10^{-8} \frac{(20)(36)^4}{10^3} = 3.9385 \times 10^{-14} \frac{(20)^{4.5}(36)^{11}}{10^{10}}\ T,$$

for which it is found that $T = 0.5055$ yr or 4428 hr. It will therefore require almost 4500 hr of operation of the cask at design pressure to close the gap between the shield and the jacket.

NOMENCLATURE

A = cross sectional area, in.^{2}

ΔA = incremental area, in.^{2}

a = outside radius of liner and inside radius of shield, in.

B = temperature dependent constant with units of $\frac{(\text{in.}^2/\text{lb})^{\eta}}{\text{hr}}$ or $\frac{(\text{in.}^2/\text{lb})^{\eta}}{\text{yr}}$

b = outside radius of shield, in.

c = distance from neutral axis of cross section to outermost fiber

$D = \frac{Et^3}{12(1 - \upsilon^2)}$, lb/in.^{2}

d = depth of prismatic cavity, in.

E = modulus of elasticity, psi

E_L = modulus of elasticity of liner, psi

E_S = modulus of elasticity of shield, psi

F = force, lb per in. of length

ΔF = incremental membrane force, lb

g = unsupported length of liner, in.

h = height of prismatic cask cavity, in.

I = moment of inertia, in.^{4}

L = subscript referring to liner

ℓ = length of cantilever

M = bending moment, in.-lb per inch of length

M_{AB} = bending moment in beam AB at joint A

M_{AD} = bending moment in beam AD at joint A

m_L = the reciprocal of υ

$n = E_L/E_S$

P = load on a tensile member, lb

p = internal design pressure, psi

p_s = internal design pressure on shield, psi

p_L = effective internal design pressure $(p - p_s)$ on the liner, psi

r = radius of liner, in.

S = stress intensity equal to $\sigma_1 - \sigma_3$ where $\sigma_1 > \sigma_2 > \sigma_3$

S_{yp} = tensile stress of a material at yielding

s = subscript referring to shield

T = time, yr

t = material thickness, in.

u_s = increase in length of span of shield, in.

w = width of prismatic cask cavity, in.

x = longitudinal coordinate or length of a tensile member, in.

Y = distance from base to centroid of A in Fig. 6, in.

$\overline{Y}$ = distance from inner surface of liner to the neutral axis of section of composite beam consisting of liner and shield, in.

y_L = deflection of liner as shown in Fig. 9, in.

$\alpha = \frac{E_s t_s}{E_L t_L}$

$\beta = \frac{p_s}{p_L} = \frac{E_s a}{E_L t_L} \frac{1}{\left(\frac{b^2 + a^2}{b^2 - a^2}\right) + \upsilon_s}$

δ = elongation of member due to tensile load, in.

δ_L = radial displacement (deflection) of cylindrical liner, in.

δ_s = radial displacement (deflection) of cylindrical shield, in.

$\dot{\epsilon}_c$ = creep strain rate, $\frac{\text{in./in.}}{\text{hr}}$ or $\frac{\text{in./in.}}{\text{yr}}$

η = temperature dependent constant

θ = angle of rotation or slope

θ_{AB} = the slope or angle of rotation of beam AB at joint A

θ_{AD} = the slope or angle of rotation of beam AD at joint A

$\lambda = \sqrt[4]{\frac{3(1 - \upsilon^2)}{r^2 t^2}}$

υ_L = Poisson's ratio for the liner material

υ_s = Poisson's ratio for the shielding material

σ = stress, psi

$\sigma_1, \sigma_2, \sigma_3$ = three principal stresses in structural member, psi

σ_b = bending stress in psi

σ_r = radial stress in psi

σ_x = longitudinal stress in psi

σ_θ = hoop or circumferential stress in psi

$\sigma_{F_{1s}}$ = stress in shield due to membrane force F_1 on shield, psi

$\sigma_{F_{2s}}$ = stress in shield due to membrane force F_2 on shield, psi

τ_{max} = maximum shear stress

ϕ = angle of rotation

Σ = summation

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