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A FAILURE ANALYSIS FOR THE LOW-TEMPERATURE PERFORMANCE OF DISPERSION FUEL ELEMENTS

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ABSTRACT

An analytical approach is proposed which allows the burnup (by fission) of uranium required to cause failure in a uranium dioxide-stainless steel dispersion fuel element to be calculated.

The analysis is developed by assuming the matrix of the fuel element to be made up of a uniform, close-packed array of spherical UO_2 particles, each surrounded by and associated with a hollow stainless steel sphere. Equations are then written for the amount of fission gas released into the stainless steel cavity in terms of the UO_2 particle size and density and the burnup. The release mechanism is by recoil only, since diffusion is unimportant for the particle sizes and temperatures (> 1000°F) of interest. The gas atoms recoiled from the UO_2 particle are assumed to diffuse from the stainless steel shell into the cavity. The pressure thus exerted inside the stainless steel sphere is then computed by the application of a real gas law. A suitable failure criterion for an internally pressurized, heavy-walled metal sphere appears to be when the sphere becomes entirely plastic. An equation for the pressure at failure and displacements of the sphere are written in terms of the UO_2 loading and the yield strength of the steel. By combination with the previous expressions, the burnup required to cause failure is calculated. Thus, the effects of UO_2 density and particle size, temperature, strength of the matrix material, and UO_2 loading on the burnup at failure are predicted by the theory.

Some of the assumptions necessary in the analysis are only indirectly supported by experiment, and approximations of the behavior of particulate UO_2 under irradiation are necessary to develop the theory. The analysis is somewhat limited by lack of knowledge of the strength properties of materials while under neutron and fission-fragment bombardment. However, predictions made on the basis of strengths of steels after neutron irradiation have been correlated successfully with the few experimental results available at temperatures up to 1000°F.

The theory predicts a decrease in the burnup lifetime of the fuel with increasing volume fraction of UO_2, decreasing UO_2 particle size, and increasing temperature. Decreasing the UO_2 particle density is predicted to be the strongest parameter in increasing the performance of these fuels because of the "built in" void space provided for the fission gases by the lower density UO_2 particles.

The relative effects of the various fuel parameters on the performance of the fuel elements, as predicted by the theory, can be useful in the design of dispersion fuels.

INTRODUCTION

Enriched uranium oxide-stainless steel dispersion fuel elements have been developed for the U. S. Army Pressurized-Water Reactors,[1] the Vallecitos Boiling Water Reactor, and the Organic Moderated Reactor Experiment. These fuels are required to operate to high burnups at temperatures between 500 and 1000°F.

In the past, irradiation experiments on UO_2 dispersion-type fuel elements have not yielded sufficient quantitative information on the performance of these fuels to determine the relative influence of important fabrication variables and fuel design parameters, such as the UO_2 particle size, shape and density, the fuel loading, and the properties of the matrix material. It has therefore been impossible to fabricate fuels capable of known performance, and, further, it has been difficult to optimize fuel element parameters for the maximum performance in terms of allowable burnup before failure.

It is the purpose of this paper to propose an analytical model which will permit quantitative predictions of the achievable burnup for these fuels and compare the effect of the following parameters on the burnup lifetime.

A. UO_2 density.

B. UO_2 particle size.

C. Volume fraction of UO_2 in the stainless steel matrix.

D. Temperature of the fuel-bearing matrix.

E. Strength of the matrix.

EXPERIMENTAL OBSERVATIONS

Examination of irradiated fuels, both before and after failure, lead to important conclusions regarding the behavior of the fissile material during the irradiation and the mechanism of failure. It has been found[2,3] that the UO_2 particles tend to sinter early in the irradiation and that internal porosity disappears. At high burnup the pores appear again,[3,4,5] probably as a result of precipitation of fission gases to form bubbles or voids. Failure results from gross expansion of the matrix and propagation of cracks between the UO_2 particles. Release of fission products to the coolant usually accompanies such failures. It appears that the basic cause for the failure is fission-gas pressure between the UO_2 particle and the surrounding matrix. At some burnup, stresses exceeding the fracture stress exist in the "web" of steel between the UO_2 particles, thus causing crack initiation and propagation from particle to particle.

In the following, an attempt will be made to quantitatively account for the effect of the sintering of the UO_2 and to predict the burnup of uranium atoms (by fissioning) at which the fission-gas pressure should cause fracture in a stainless steel matrix surrounding UO_2 particles. The observed appearance of the gas-filled voids in the UO_2 at high burnup is difficult to account for analytically, since the burnup-temperature-pressure relationship for its occurrence is unknown. The effect of neglecting this factor in the analysis should tend to cause the predictions to be somewhat optimistic (higher than experiment).

ANALYTICAL APPROACH

A. Formation of Fission Gases

There are usually two fission-product atoms formed per fission event. Since approx 12 at. % of these are gaseous (xenon and krypton) at the temperatures under consideration (> 1000°F), there are 4×10^{-27} moles of gas per atom of uranium in uranium irradiated to 1% burnup. There are approx 2.46×10^{22} atoms of uranium per cubic centimeter of 100% dense UO_2; therefore, the number of moles of gas formed per cubic centimeter of UO_2 per percent burnup of uranium atoms, N, is 9.7×10^{-5} of which 86.5% are isotopes of xenon and 13.5% are isotopes of krypton.[6] If the UO_2 particle is approximately a sphere and it sinters to 100% of theoretical density early in the irradiation, its volume is $4/3\ \pi\ a_o^3\ d$, where a_o is the pre-irradiation radius of the UO_2 particle and d is the fraction of theoretical density of the UO_2 particle before irradiation. The amount of xenon and krypton formed in a UO_2 particle at a particular percent burnup, B_u, is then

$$n_{Xe}^f = (0.865)\frac{4}{3}\pi a_o^3 d N B_u,$$
$$n_{Kr}^f = (0.135)\frac{4}{3}\pi a_o^3 d N B_u. \qquad (1)$$

B. Release of the Fission Gases

Two mechanisms, recoil and diffusion, may contribute to the quantity of gas within the region between the UO_2 sphere and the stainless steel.

The range of fission products in a material[7] is

$$\overline{R} = 0.56\ A^{1/3}\ R_{air}. \qquad (2)$$

The effective atomic mass of UO_2 is calculated from

$$A_{UO_2} = A_U \frac{\rho_{UO_2} d}{\rho_U}, \qquad (3)$$

where ρ = density and on appropriate substitution yields

$$A_{UO_2} = 146\ d. \qquad (4)$$

The ranges of xenon and krypton in air are 1.8 and 2.3 cm, respectively.[7] In order to get $\bar{R}$ in units of length, the range as given by Eq. 2 is divided by the density of the UO_2 particle, ρ_{UO_2} d. Combination of Eqs. 2 and 4 and division by ρ_{UO_2} d yields

$$\bar{R}_{Xe} = 4.86\ d^{-2/3},$$
$$\bar{R}_{Kr} = 6.21\ d^{-2/3}. \qquad (5)$$

The fraction of the gases formed in the UO_2 that recoil out has been shown to be approximated by[8]

$$f = 0.25 \frac{V_a}{V_p}, \qquad (6)$$

where V_a = volume of the material from which nuclei may recoil, V_p = volume of the particle.

The volume of the particle after a short neutron exposure is

$$V_p = \frac{4}{3} \pi a_o^3 d, \qquad (7)$$

and the volume of the annulus from which nuclei may recoil is

$$V = \frac{4}{3} \pi \left[d\ a_o^3 - \left(d^{1/3} a_o - \bar{R} \right)^3 \right]. \qquad (8)$$

The fraction of the gases released by recoil is then

$$f = 0.25 \left[\frac{d\ a_o^3 - \left(d^{1/3} a_o - \bar{R} \right)^3}{d\ a_o^3} \right], \qquad (9)$$

which, for xenon and krypton, may be reduced to

$$f_{Xe} = 0.25 \left[1 - \left(1 - \frac{4.86}{a_o d} \right)^3 \right],$$
$$f_{Kr} = 0.25 \left[1 - \left(1 - \frac{6.21}{a_o d} \right)^3 \right]. \qquad (10)$$

Since the recoil atoms are injected into a "damage zone" in the stainless steel surrounding the UO_2 particle, an estimate must be made of how much diffuses from the stainless steel into the space between the UO_2

particle and the stainless steel. There are no known data on diffusion of xenon and krypton in stainless steel during fission-product bombardment. There is, however, indirect evidence that these gases diffuse out of stainless steel even at low temperatures.[9,10] For the purposes of this computation, it will be assumed that all of the xenon and krypton atoms that recoil out of the UO_2 into the stainless steel diffuse into the space between the UO_2 particle and the stainless steel.

Information on the diffusion of the fission gases xenon and krypton from UO_2 is contradictory; however, the available data allow reasonable approximations of the maximum diffusion coefficient to be made. A recent summary[11] indicates at 1000°F the diffusion constant for Kr^{85} in UO_2 should be approx 10^{-21} cm^2/sec. Booth[12] indicates that the diffusion of fission products from UO_2 should be represented by

$$f = \frac{4}{a_s}\sqrt{\frac{Dt}{\pi}}, \qquad (11)$$

where

f = fraction released,

D = diffusion constant,

t = time, and

a_s = radius of equivalent sphere.

Thus, for a two-year irradiation ($t = 6.3 \times 10^7$ sec) the fraction released at 1000°F should be

$$f = \frac{5.6 \times 10^{-7}}{a_s}, \qquad (12)$$

so that for radii (a_s) larger than approx 10^{-4} cm, the contribution of the diffusion process to the gas in the annulus between the UO_2 and the stainless should be insignificant compared to the recoil contribution. Measurements at this laboratory of the BET surface area of spherical UO_2 particles prior to irradiation indicate surface areas such that $a_s = 4.1 \times 10^{-4}$ cm for particles with an average geometric radius of 6.4×10^{-3} cm. However, in view of the evidence for the sintering of UO_2 early in the irradiation, the radius of the equivalent sphere will be assumed to be approximately the geometric radius, and the contribution due to diffusion from UO_2 particles of radius greater than 10^{-4} cm neglected.

The total number of moles of gas released into the region is by combination of Eqs. 1 and 10

$$n^r_{Xe} = 0.865 \frac{\pi}{3} a_o^3 \; d \; N \; B_u \left[1 - \left(1 - \frac{4.86}{a_o d}\right)^3\right],$$
$$n^r_{Kr} = 0.135 \frac{\pi}{3} a_o^3 \; d \; N \; B_u \left[1 - \left(1 - \frac{6.21}{a_o d}\right)^3\right]. \qquad (13)$$

C. Pressure in the Cavities

To compute the pressure exerted in the cavity by this amount of gas, a form of the Van der Waals' equation of state may be used[13]

$$p = 14.7 \; RT \left[\frac{n_{Xe}}{V-n_{Xe}b_{Xe}} + \frac{n_{Kr}}{V-n_{Kr}b_{Kr}}\right], \qquad (14)$$

where

p = pressure (psi),

R = gas constant

T = absolute temperature,

b = Van der Waals' constants, and

V = free volume in the cavity.

D. Volume in the Cavities

The free volume in the cavity is the total volume of the cavity at pressure, p, less the volume occupied by the UO_2 particle, that is,

$$V = \frac{4}{3} \pi \; (a^3 - d \; a_o^3), \qquad (15)$$

where a is the radius of the cavity at pressure p.

E. Stresses Produced by Fission-Gas Pressure

The UO_2-stainless steel dispersion may be considered to be made up of a close-packed array of UO_2 spheres of initial radius a_o, each surrounded and associated with a hollow stainless steel sphere of initial radius b_o, as shown in Fig. 1. The problem then reduces to properly describing the failure of such an array of hollow spheres due to the internal fission-gas pressure. According to the development in Appendix A, the fuel element

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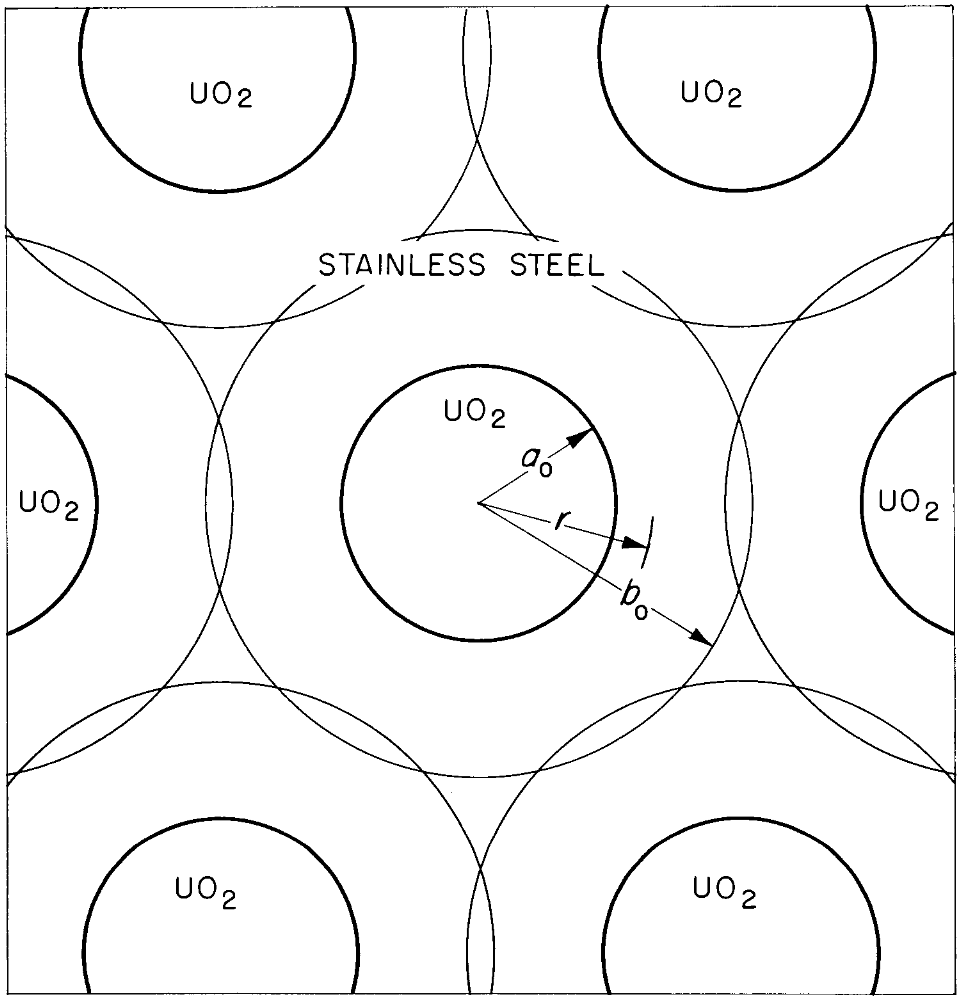

Fig. 1. Model Representing an Ideal Dispersion.

should fail when the outer fibers of the hollow steel spheres first become plastic. The pressure at which this occurs is given by Eq. A29, and the inside radius of the spheres is given by Eq. A28. From Appendix A, these equations are

$$p = \frac{2}{3}\, Y \ln \frac{1}{F},$$

and

$$a = a_\circ \left\{ 1 + \frac{Y}{E} \left[(1 - \nu)\frac{1}{F} - \frac{2}{3}(1 - 2\nu) \ln \frac{1}{F} \right] \right\}.$$

Thus, the burnup at which failure of the fuel element should occur on the basis of this analysis may be obtained by combining Eqs. 13, 14, 15, and Eqs. A28 and A29 from Appendix A. The result with R = 0.08207 liter atm/mole °K, b_{Xe} = 0.0511 liter/mole, b_{Kr} = 0.0398 liter/mole, N = 0.097 mole/liter % B_u, and ν = 0.3 is

$$(K\, k_2\, k_4 + k_1\, k_4 + k_2\, k_3)\, B_u^2$$

$$-(K\, k\, k_4 + K\, k\, k_2 + k\, k_1 + k_2\, k_3)\, B + K\, k^2 = 0 \;, \qquad (16)$$

where

$$K = 22.8\, \frac{Y}{Td} \ln \frac{1}{F} \;,$$

$$k \quad \left[1 + \frac{Y}{E} \left(\frac{0.7}{F} - 0.267 \ln \frac{1}{F} \right) \right]^3 - d \;,$$

$$k_1 = 0.865 \left[1 - \left(1 - \frac{4.86}{a_\circ d} \right)^3 \right] ,$$

$$k_2 = 1.07 \times 10^{-3}\, d \left[1 - \left(1 - \frac{4.86}{a_\circ d} \right)^3 \right],$$

$$k_3 = 0.135 \left[1 - \left(1 - \frac{6.21}{a_\circ d} \right)^3 \right] ,$$

$$k_4 = 1.3 \times 10^{-4}\, d \left[1 - \left(1 - \frac{6.21}{a_\circ d} \right)^3 \right] ,$$

and the units used are

T = °K,

Y = psi,

$a_\circ$ = microns,

d = fraction, and

F = volume fraction.

At this point, all that remains is to describe the "in pile" yield strength, Y, and the modulus of elasticity, E, with temperature for the particular stainless steel of interest, and the burnup required to cause failure may be computed from Eq. 16. Since the in-pile strength properties are not known, data on irradiated materials must be used. The available results[14,15] (see Appendix A, ref 2, also) indicate that a reasonable approximation for the yield strength at temperatures below 1000°F is approximately twice the out-of-pile values. Since there is no reason to expect the modulus to be significantly changed by irradiation, the out-of-pile values will be assumed valid.

It is interesting to note that the assumption of no work hardening appears better in the case of irradiated material than unirradiated material, since the yield stress approaches the ultimate stress after irradiation indicating that the work-hardening coefficient is small. Another important observation from the theory is that the "swelling" of the fuel just prior to failure should be small.

RESULTS

The effects of the properties of four stainless steels at temperatures up to 1000°F, the UO_2 particle size and density, and the volume fraction of UO_2 in the matrix have been machine calculated using Eq. 16 and some of the results are presented in Figs. 2–13.

The effect of temperature on the burnup required to cause failure of fuel elements utilizing four common stainless steels is shown in Fig. 2. The values of yield stress and modulus for the steels used in computing the results are given in Table I. Note that typical stress values have been doubled to represent the effect of the neutron irradiation. The temperature dependence of the burnup is seen to be relatively small, and, as might be expected, materials with higher yield stresses and lower moduli are predicted to perform better than the other steels. The values for burnup at failure for the "ideal" dispersions described by Eq. 16 and presented in Fig. 2 are quite high and, of course, should not be taken to represent burnup limits for real fuels. The effect of departures from the ideal dispersion is discussed later in this report.

Figures 3–7 present the predicted effect of the UO_2 loading on the burnup at failure of a type 304 stainless steel fuel element at 800°F. The radius of the UO_2 particles is taken as a parameter. Each of the

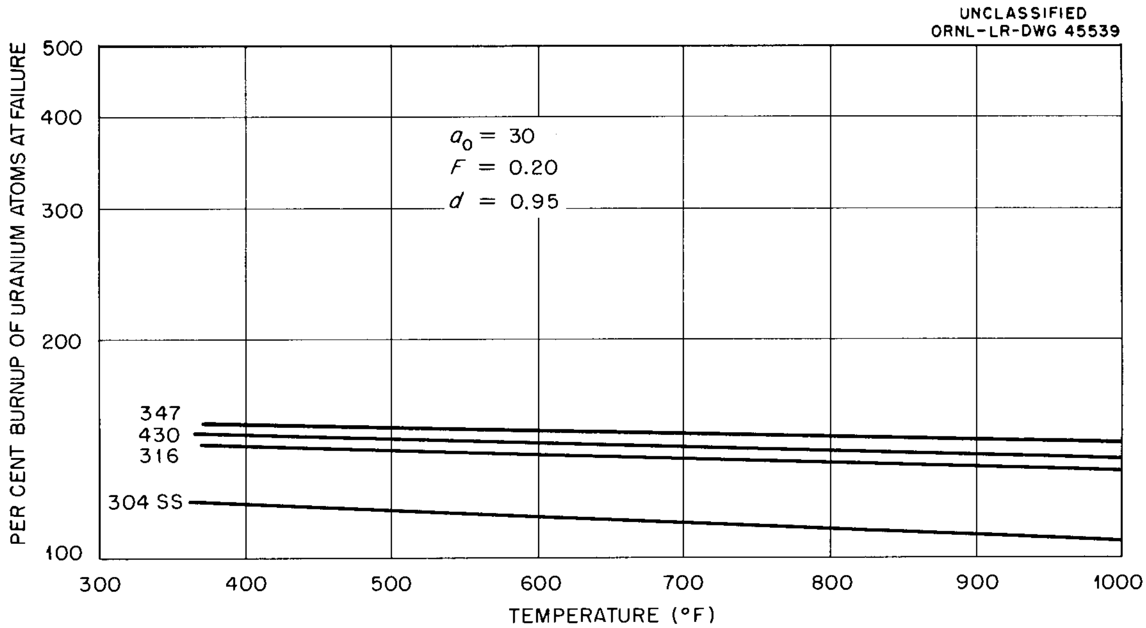

Fig. 2. Comparison of Stainless Steels.

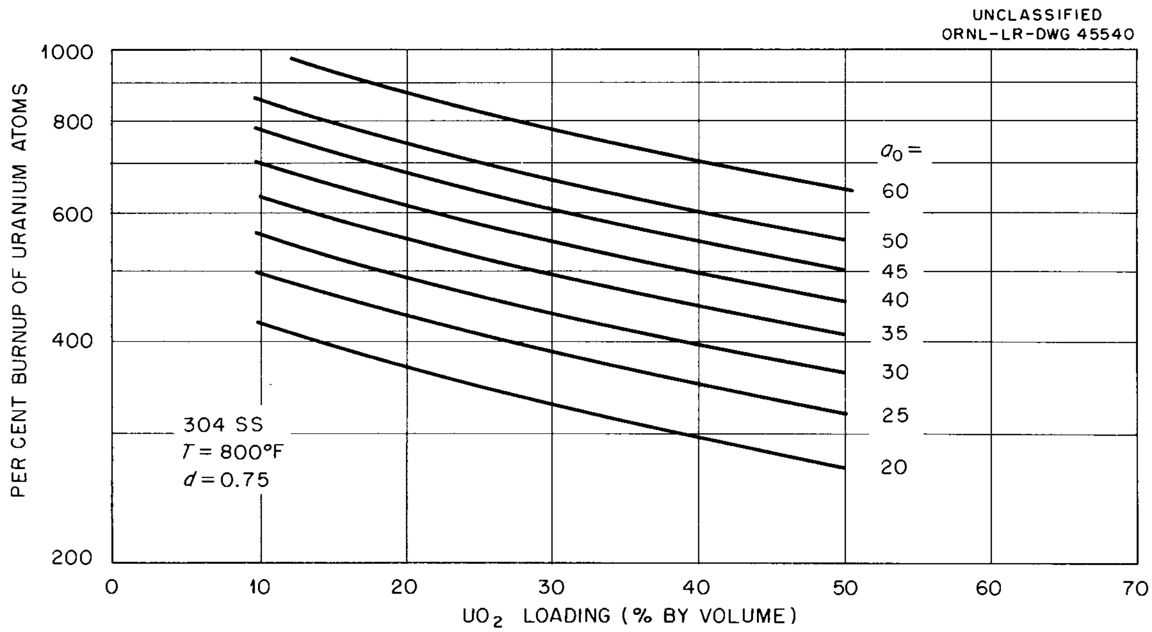

Fig. 3. Effect of UO_2 Loading.

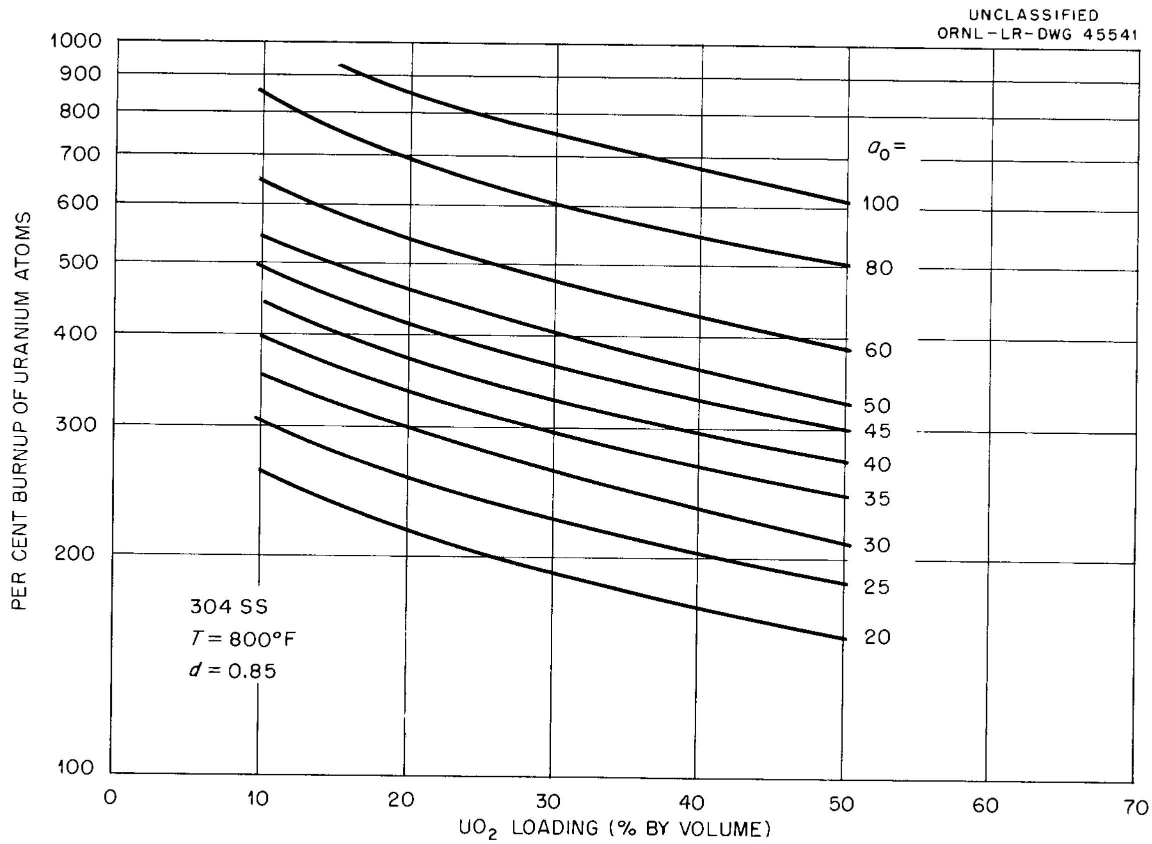

Fig. 4. Effect of UO_2 Looding.

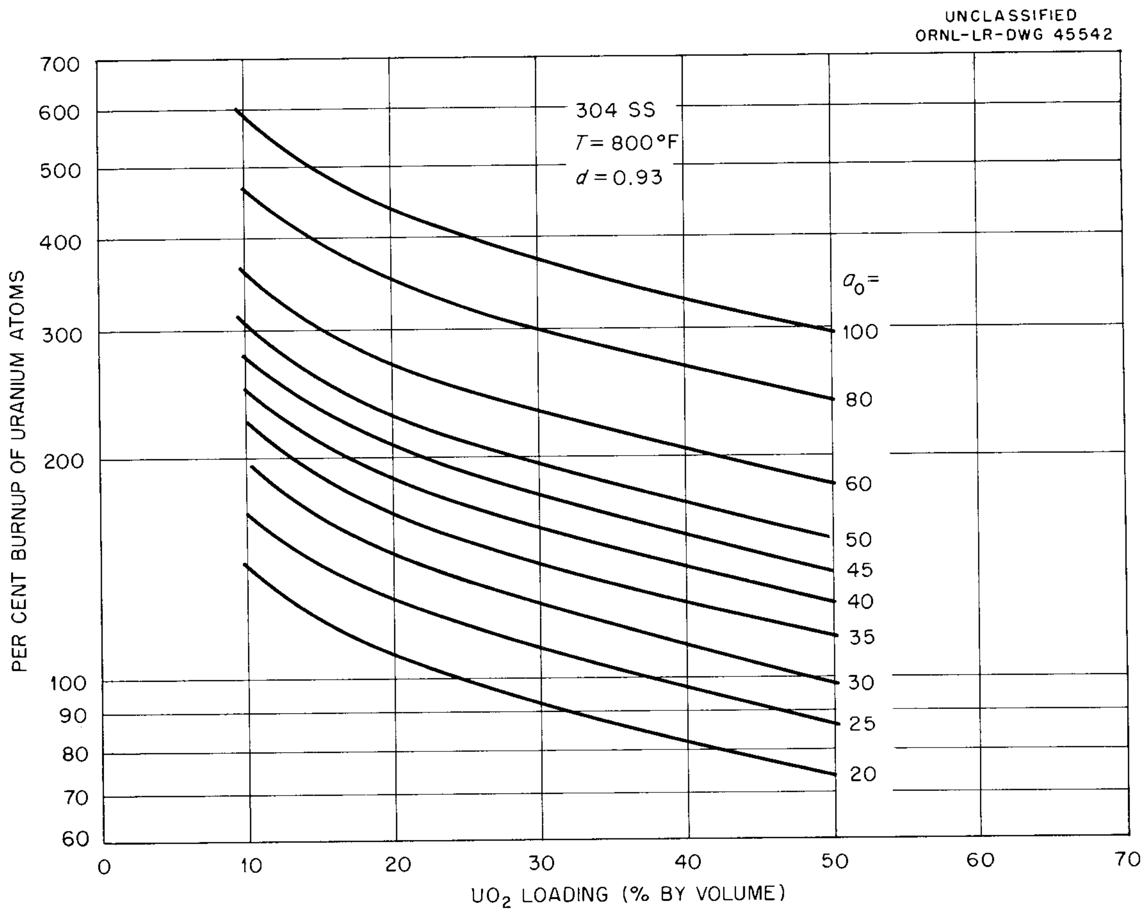

Fig. 5. Effect of UO_2 Loading.

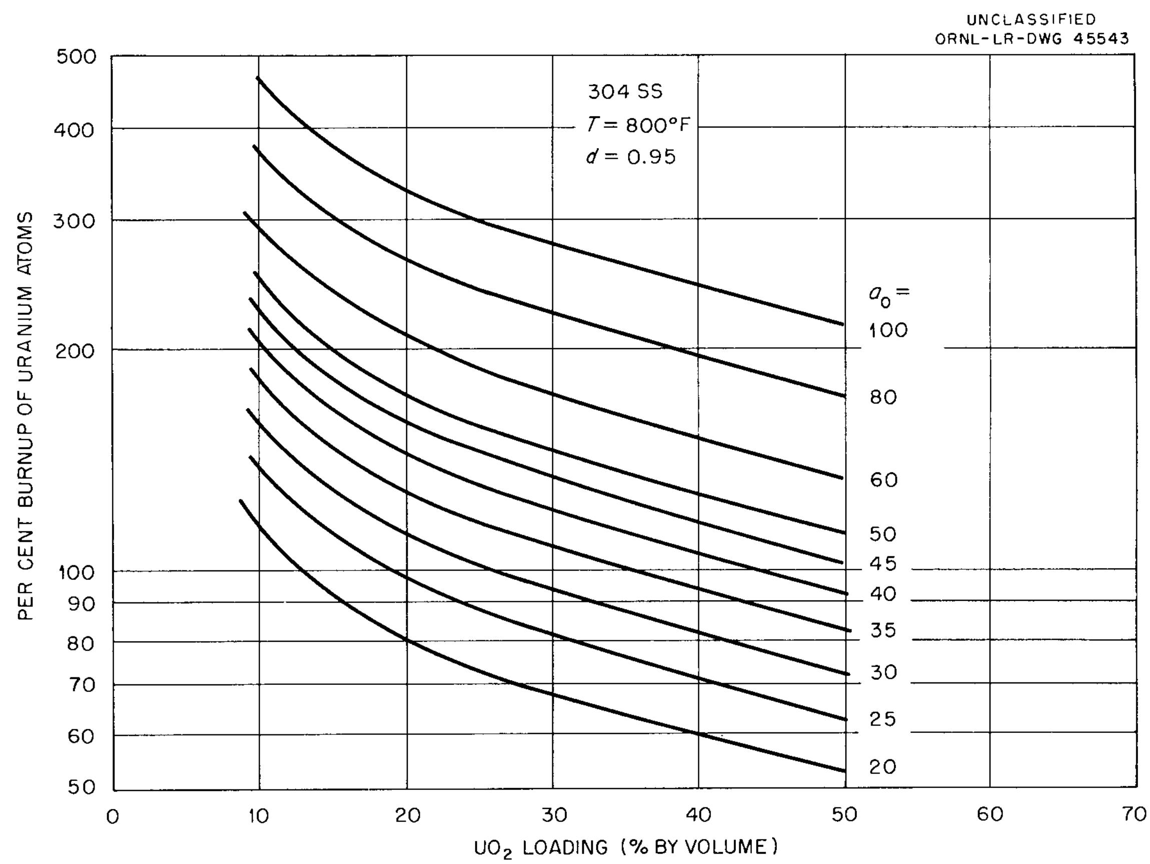

Fig. 6. Effect of UO_2 Loading.

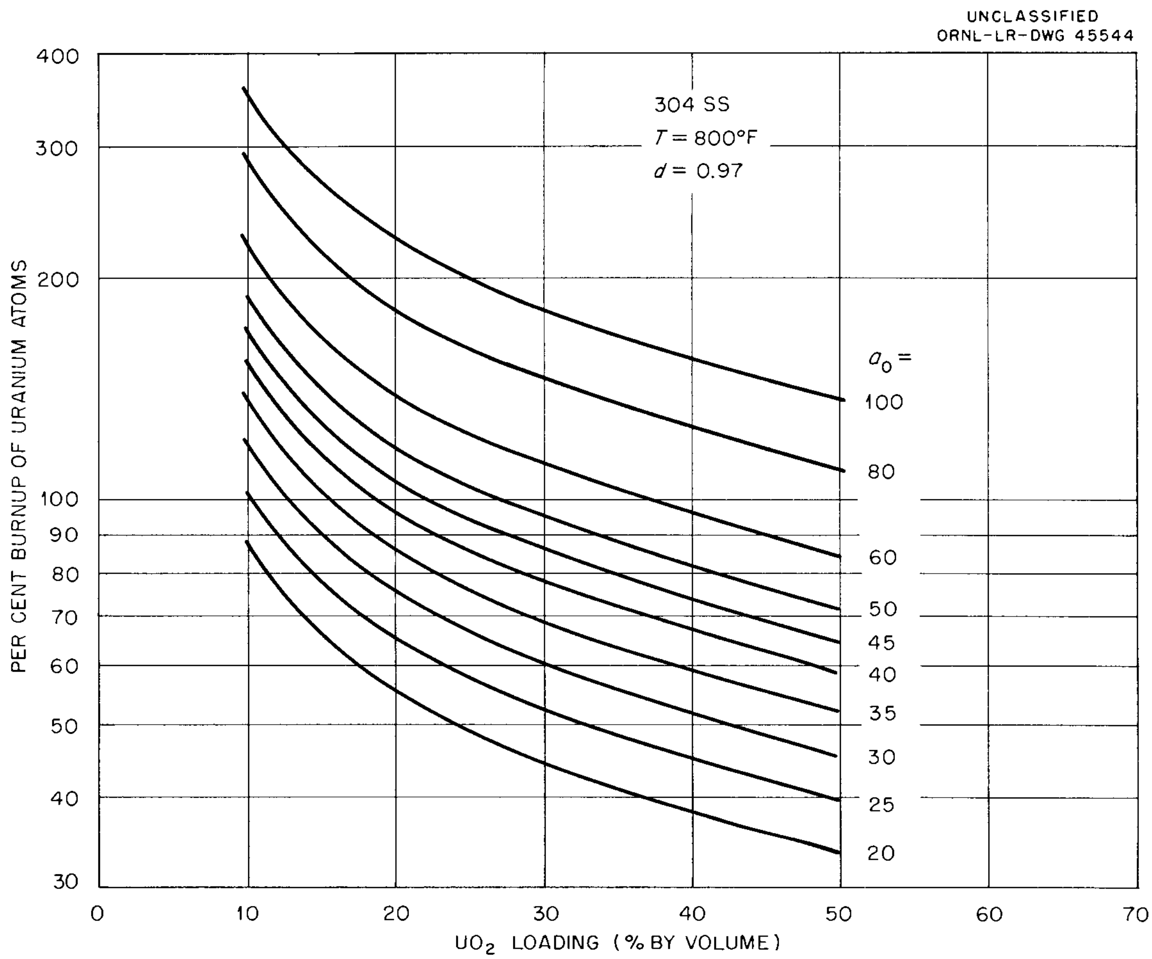

Fig. 7. Effect of UO_2 Loading.

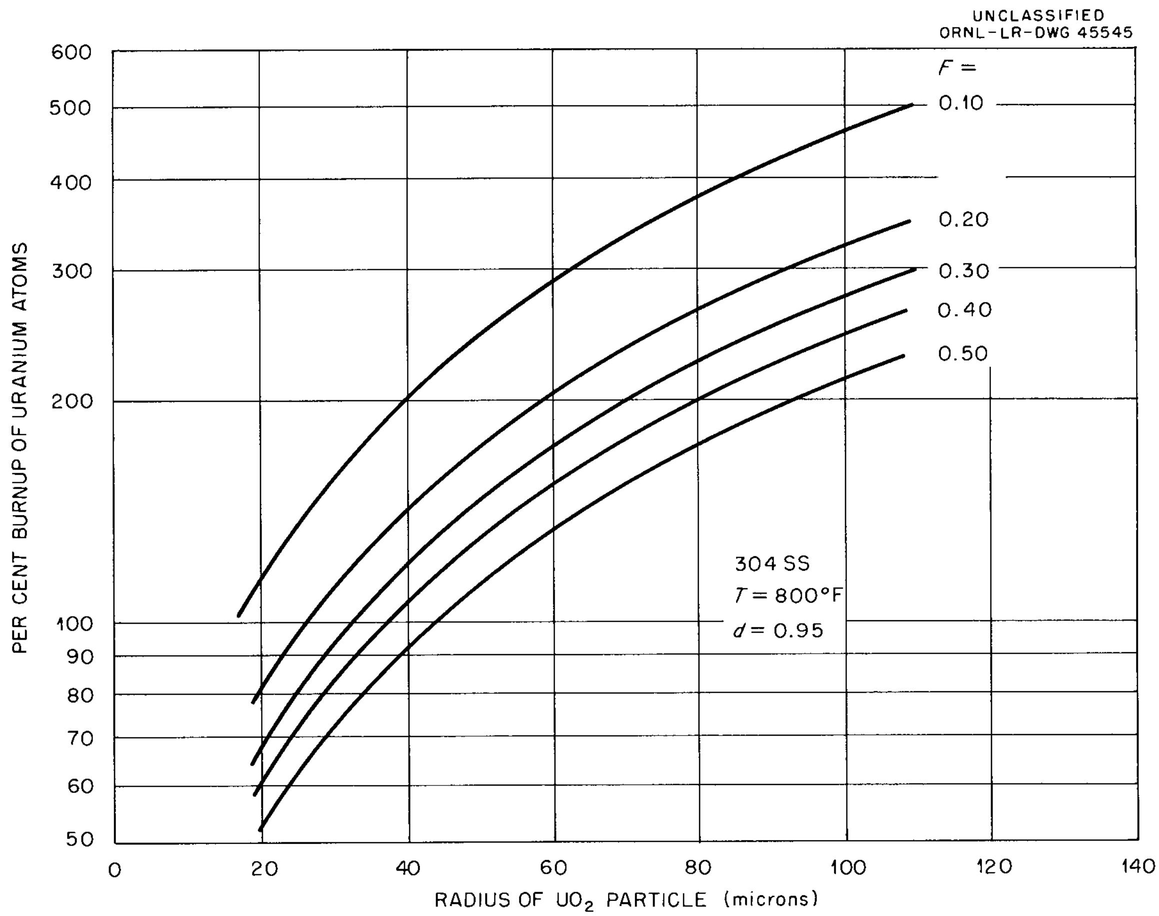

Fig. 8. Effect of UO_2 Particle Size.

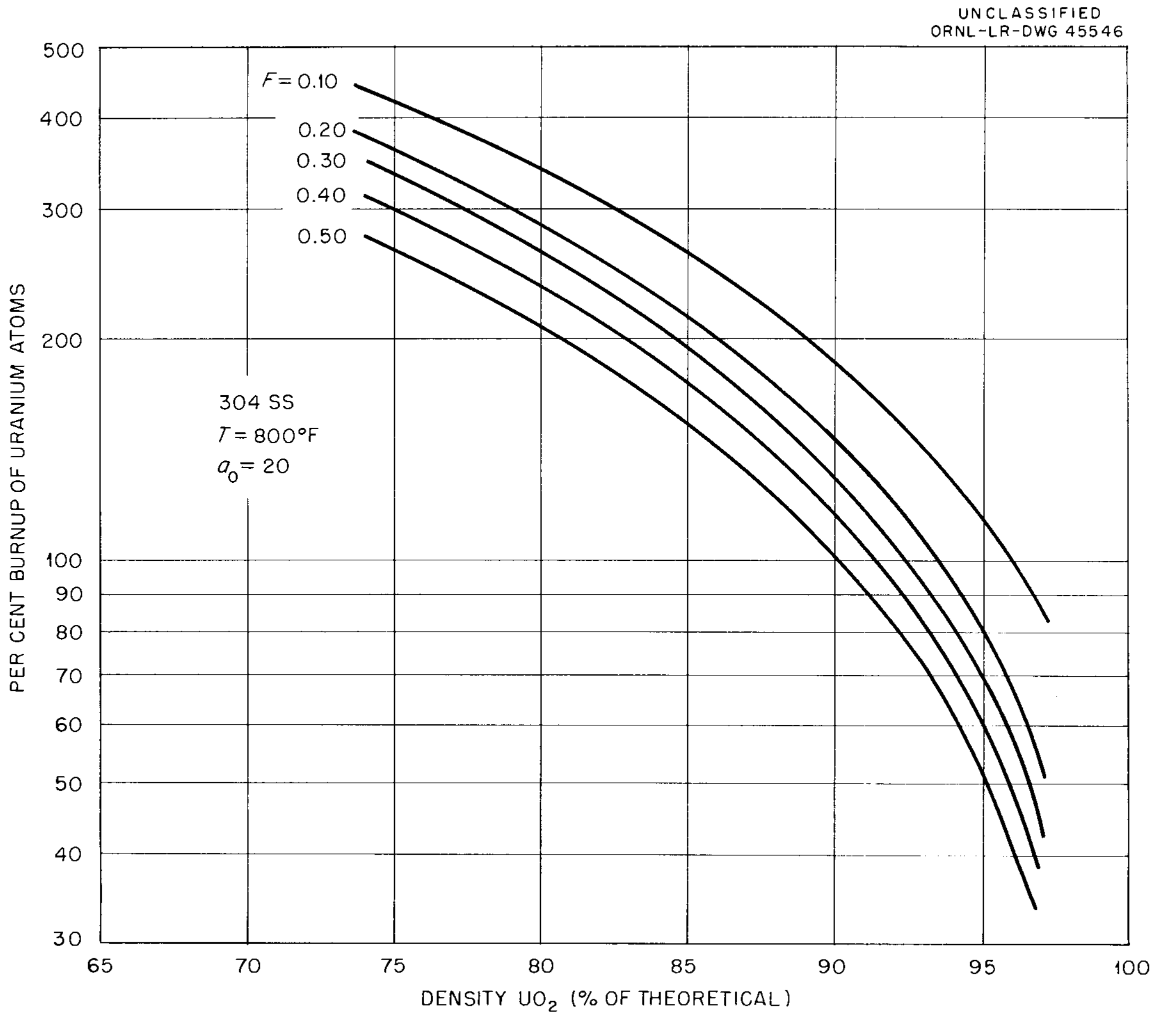

Fig. 9. Effect of UO_2 Density and UO_2 Loading.

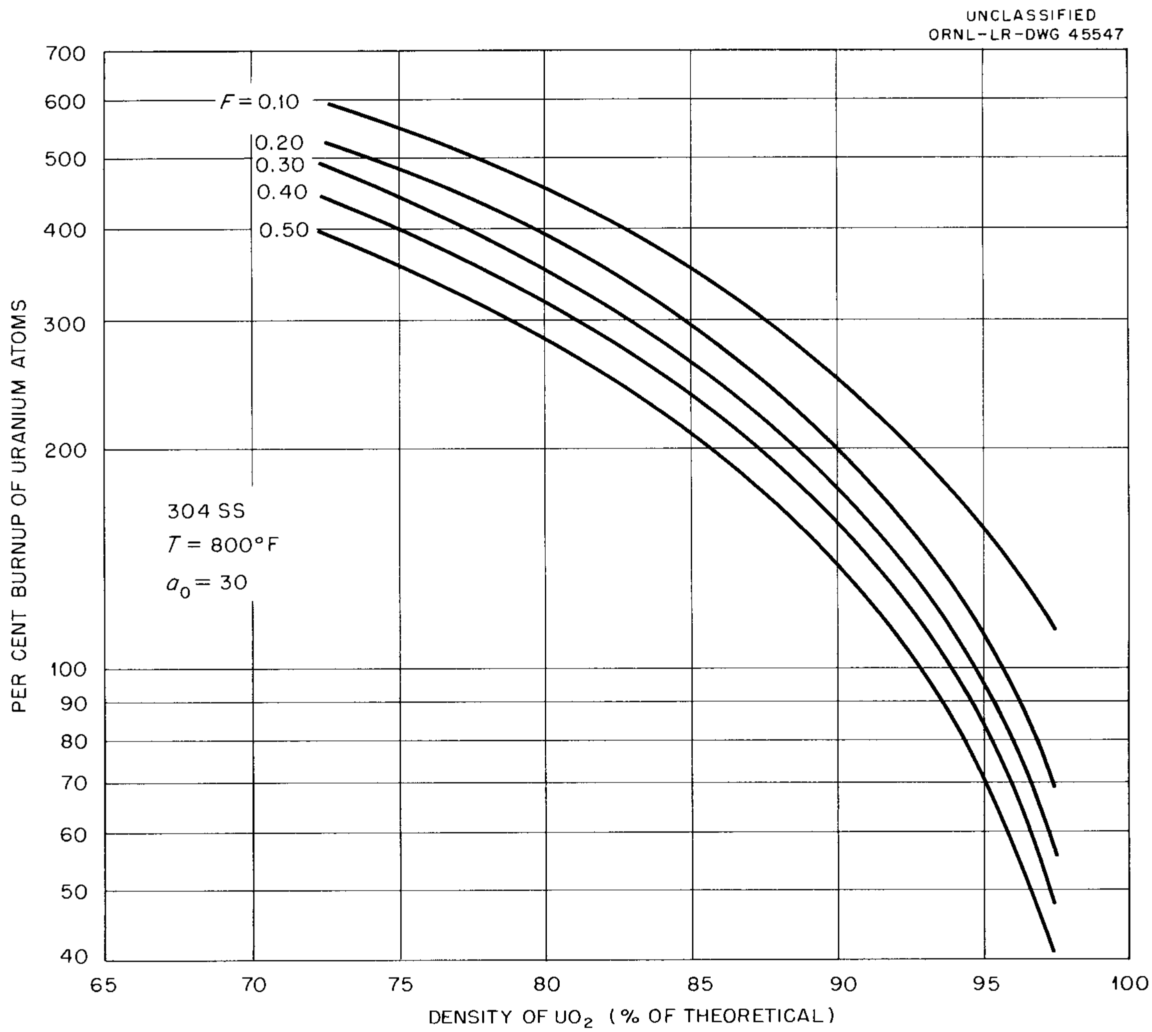

Fig. 10. Effect of UO_2 Density and UO_2 Loading.

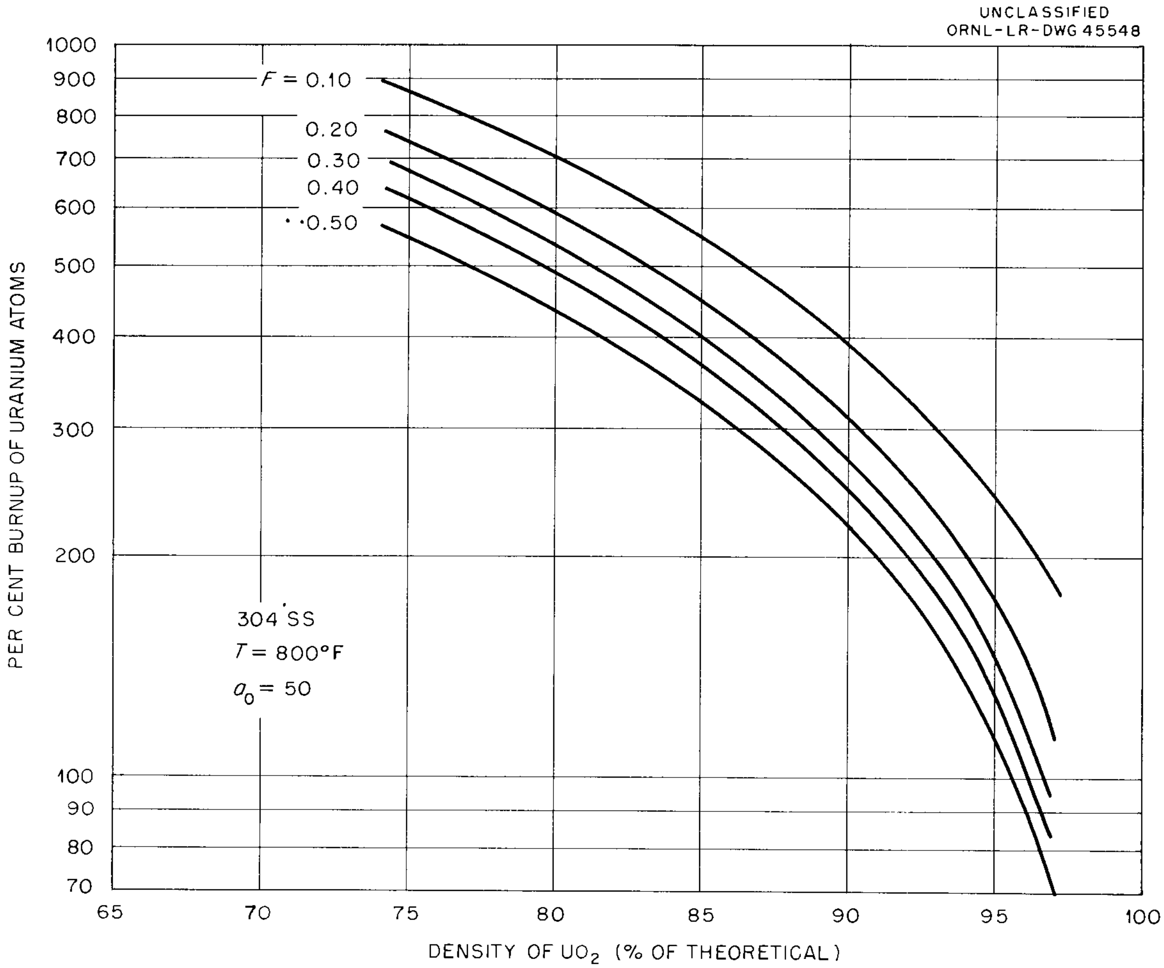

Fig. 11. Effect of UO_2 Density and UO_2 Loading.

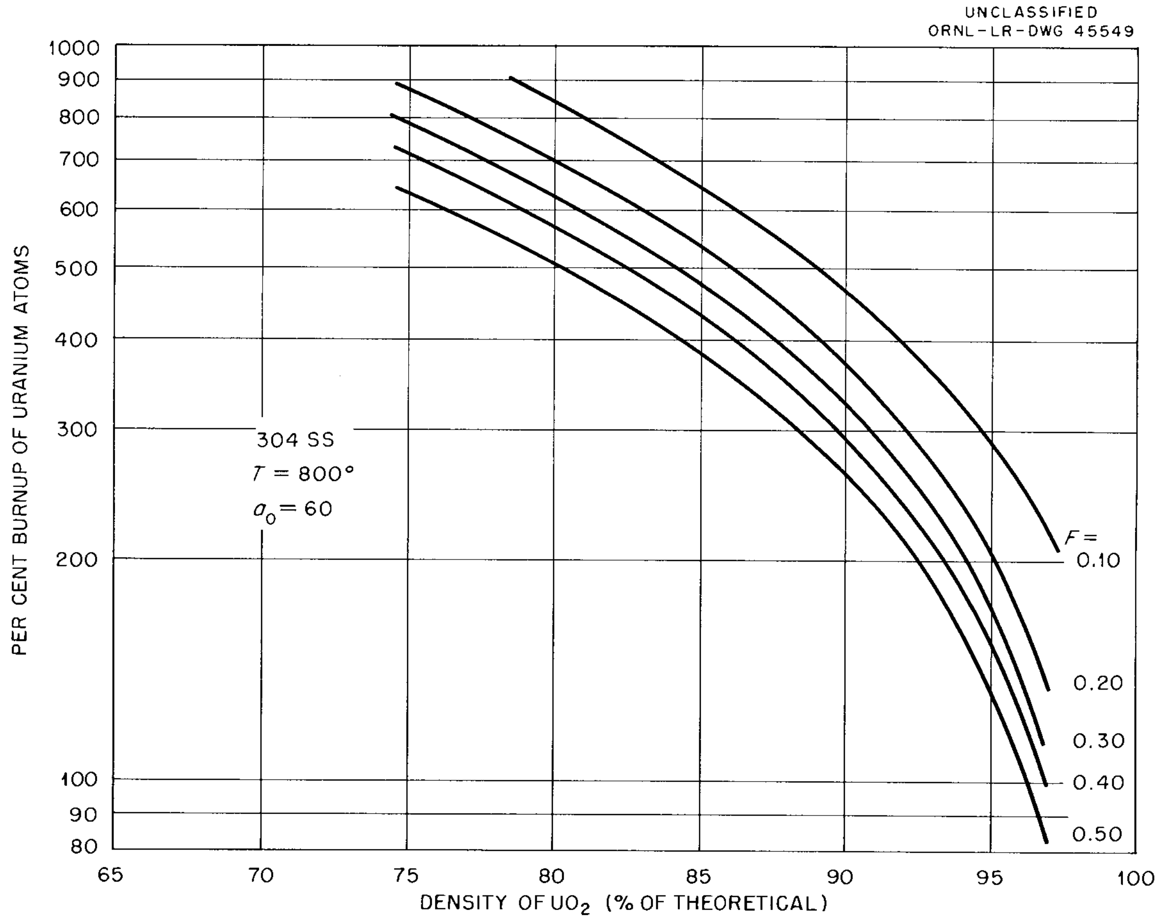

Fig. 12. Effect of UO_2 Density and UO_2 Looding.

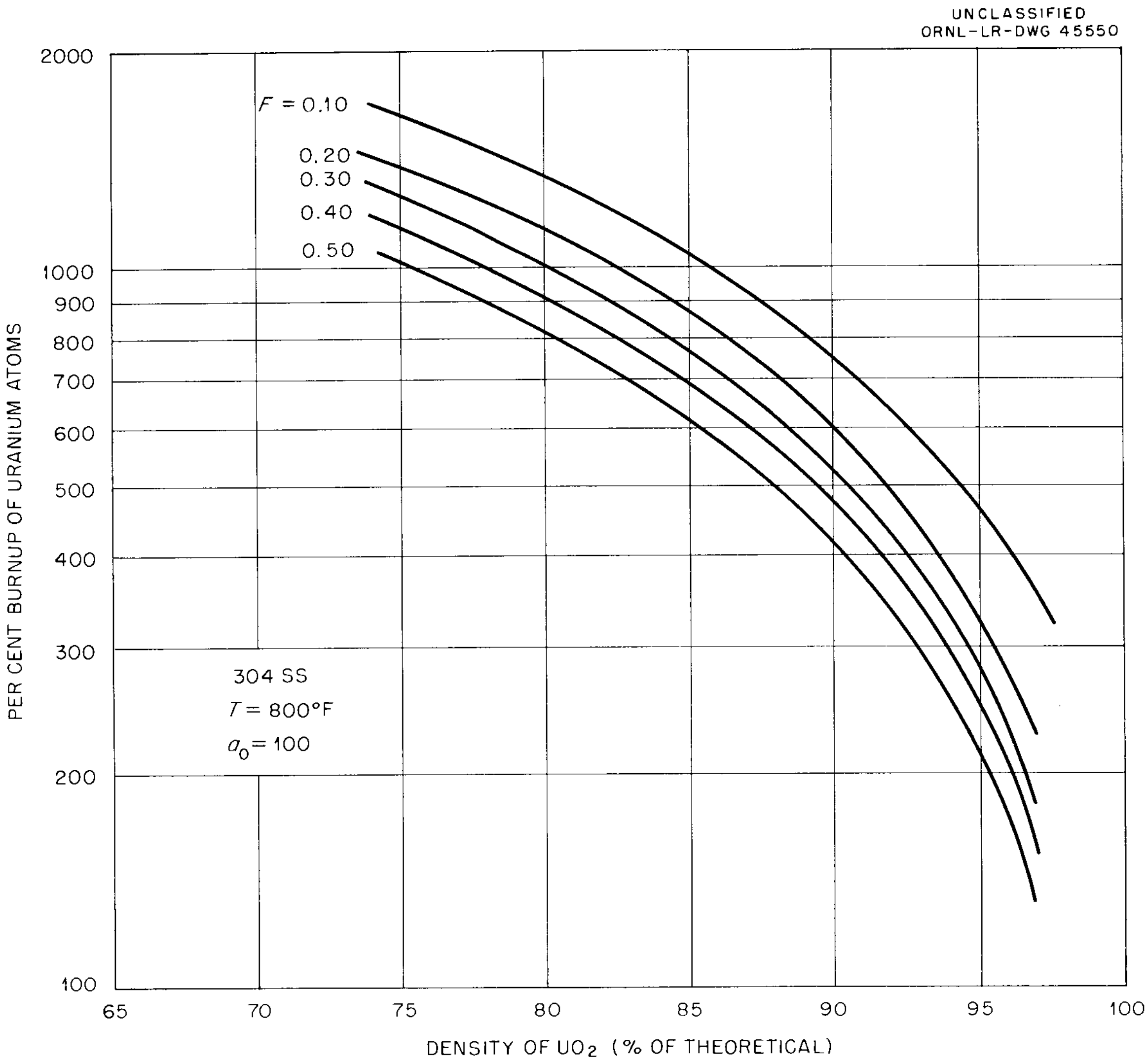

Fig. 13. Effect of UO_2 Density and UO_2 Loading.

TABLE I. Properties of Irradiated Stainless Steels

T		Type 304 Stainless Steel		Type 347 Stainless Steel		Type 316 Stainless Steel		Type 430 Stainless Steel	
(°F)	(°K)	Y (psi)	Y/E (psi)	Y (psi)	Y/E (psi)	Y (psi)	Y/E (psi)	Y (psi)	Y/E (psi)
400	477	44 000	1.70×10^{-3}	72 000	2.76×10^{-3}	62 000	2.38×10^{-3}	72 000	2.48×10^{-3}
600	589	34 000	1.36×10^{-3}	68 000	2.72×10^{-3}	56 000	2.24×10^{-3}	68 000	2.42×10^{-3}
800	700	29 000	1.21×10^{-3}	62 000	2.58×10^{-3}	46 000	1.92×10^{-3}	62 000	2.30×10^{-3}
1000	811	28 000	1.27×10^{-3}	60 000	2.72×10^{-3}	44 000	2.00×10^{-3}	48 000	2.18×10^{-3}

figures represents identical conditions for different UO_2 particle densities. As might be expected, increasing the UO_2 loading decreases the burnup at failure. Curves have been plotted on these charts for several values of UO_2 particle radius so that they may be used as fuel element design curves if desired. For a particular UO_2 density, an "equiburnup" line (horizontal line at some burnup) may be drawn. The values of UO_2 loading at which the curves for various values of a_o intersect the "equiburnup" line are combinations of F and a_o, which should produce the same performance.

A cross plot from the previous figures is shown in Fig. 8. Burnup is plotted vs UO_2 particle radius for various UO_2 loadings. Again the steel considered is type 304 stainless steel at 800°F, and the pre-irradiation UO_2 density is 0.95 of theoretical. Increasing the radius of the UO_2 particles is seen to have a stronger effect on increasing the burnup performance than does reducing the UO_2 loading.

The most significant parameter in influencing the burnup performance of dispersion fuels is the pre-irradiation density of the UO_2 particles. The effect of this parameter is illustrated in Figs. 9-13. Increasing the UO_2 density decreases the burnup required to cause failure of the fuel element. This effect is most pronounced at high values of density. This effect is a result of the assumption that the UO_2 "sinters" or shrinks early in the irradiation. This shrinkage produces a void around the UO_2 particle which allows the fission gas to exist at a lower pressure, thereby increasing the burnup performance. The increased gas release by recoil from the lower density UO_2 is accounted for in the analysis and causes the effect of decreasing density on the burnup to be less pronounced at the lower UO_2 densities.

COMPARISON WITH EXPERIMENT

Experimental data on the failure of UO_2-stainless steel fuel elements at temperatures up to 1000°F are meager. Data from a summary by Haynes[15] are shown in Table II with the pertinent known fuel parameters for the irradiations indicated. The results in the column marked "Predicted Burnup at Failure" were estimated from the analytical results for ideal dispersions in the following manner.

TABLE II. Comparison of Calculations with Experimental Results

No.	UO_2 Loading % by Volume	Matrix Material	Burnup % U^{235}	Estimated Surface Temp. (°F)	Assumed Center Temp. (°F)	Experimental Results	Predicted Burnup at Failure % U^{235}
1	14	Type 304 stainless steel	51	190	400	No failure	90
2	14	Type 304 stainless steel	57	190	400	No failure	90
3	21	Type 304 stainless steel	35 ± 15	186	400	No failure	78
4	21	Type 304 stainless steel	20 ± 15	186	400	No failure	78
5	24	18% Cr–12% Ni–bal Fe	35	797	1000	Swelled, released fission gas	63
6	18	18% Cr–12% Ni–bal Fe	21	824	1000	Swelled, released fission gas	69
7	24	18% Cr–12% Ni–bal Fe	40	850	1000	No failure	63
8	18	18% Cr–12% Ni–bal Fe	50	554	800	Swelled slightly	73
9	7.7	18% Cr–12% Ni–bal Fe	20	662	800	No failure	110

The important departures from the ideal dispersion result from inhomogeneity in the UO_2 distribution and "stringering" or elongation of the UO_2 particles as a result of rolling the fuel specimens in fabrication. The effect of inhomogeneity is to increase the apparent UO_2 loading in certain areas in the fuel, thus promoting earlier failure than for the uniform ideal dispersion. This effect, in terms of the reduction in burnup at failure, can be read directly from curves such as Fig. 6. The stringering produces two effects which must be considered. Because the surface-to-volume ratio of an elliptical particle is greater than for a spherical one, the fraction of gas released by recoil will be greater. In addition, the stress (σ_θ) in Eq. A4, Appendix A, will be increased along the long axis of the elliptical steel shell by an amount roughly equivalent to the ratio $m/2a_\circ$, where m is the major semiaxis of the ellipse and $a_\circ$ is the radius of the originally spherical UO_2 particle that was elongated by the rolling. To calculate the decrement in performance, the ideal gas law serves as a reasonable substitution for Eq. 14. The burnup is then proportional to Y in Eq. 19, and, since $|\sigma_\theta| \approx |\sigma_r|$ at $r = a_\circ$, the deviation from the results given by Eq. 16 due to increased stresses may be obtained.

In computing the results in Table II, an inhomogeneity factor of 1.6 times the average volume fraction UO_2 and a value for $m/2a_\circ$ of 1.6 were used. These values were estimated from photomicrographs of specimens of material from the irradiations numbered 1 through 4 in Table II, and the same values were assumed for the other irradiations for which there were no photomicrographs available. In all cases, an average UO_2 particle radius of 30 μ was used in the computations. The effect of stringering on the amount of gas released by recoil for these conditions should be to further reduce the predicted burnup by 6%. The calculated results are somewhat high in comparison to the experimental results for those samples that failed and are also above the burnup that did not cause failure for those samples that did not fail which is, of course, not in disagreement with these data. The disagreement between the predicted and the experimental results may be partially the result of the lack of knowledge of the exact experimental conditions existing for the irradiations numbered 5 through 7 in Table II. However, it is probable that the higher predicted values are, in part, a result of neglecting the effect of swelling of the UO_2 at high burnup. Although the experimental data do not prove the theory incorrect, there is insufficient evidence to prove that the assumptions necessary in developing the analysis lead to the exact analytical description of the behavior of these fuels under irradiation. Further experimentation in both the areas involving the assumptions used in the analysis and in proof testing of fabricated fuel elements to failure will be required to firmly establish the validity of the proposed model.

CONCLUSIONS

The results of this analytical treatment indicate the following:

1. The most significant parameter affecting the performance of dispersion fuels is the pre-irradiation UO_2 density. The lower the density the higher the allowable burnup.

2. The UO_2 particle size is also an important parameter. Increasing the particle size increases the allowable burnup.

3. Decreasing the volume fraction of UO_2 is less effective in increasing the performance than is increasing the particle size. A desirable method of increasing the U^{235} content of these fuels should be to increase the volume fraction of UO_2 and decrease the UO_2 density slightly to maintain the performance level.

4. Failure of the fuel should be characterized by a rather sudden expansion of the matrix. Swelling of the fuel prior to this expansion should be small.

The implications and results of this analysis suggest several areas of experimentation and research.

The analytical approach itself could be improved if data of a more conclusive nature in the following areas were available on which to base more accurate assumptions:

1. mechanical properties of steels under neutron irradiation at elevated temperatures,

2. effect of fission-fragment bombardment on the ductility of materials,

3. diffusion of fission gases in steels while under fission-fragment bombardment, and

4. effect of fissioning on the properties and dimensional behavior of small UO_2 particles.

This analytical treatment strongly indicates that the effect of UO_2 particle density on the burnup required to cause failure should be studied experimentally. Development of methods of fabricating more nearly "ideal" dispersions using large UO_2 particles should prove quite profitable in terms of increasing the burnup lifetime of dispersion fuels.

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APPENDIX A

THE FAILURE CRITERION FOR AN ARRAY OF HOLLOW STEEL SPHERES AT LOW TEMPERATURE

According to Hill,[1] the stresses at distance r from the center of a hollow metal sphere of inside radius a_o and outside radius b_o in spherical polar co-ordinates are

$$\sigma_r = -p\left(\frac{b_o^3}{r^3} - 1\right) \Big/ \left(\frac{b_o^3}{a_o^3} - 1\right), \tag{A1}$$

$$\sigma_\theta = \sigma_\phi = p\left(\frac{b_o^3}{2r^3} + 1\right) \Big/ \left(\frac{b_o^3}{a_o^3} - 1\right). \tag{A2}$$

The von Mises yield criterion

$$Y = \frac{1}{\sqrt{2}} \left[(\sigma_\theta - \sigma_r)^2 + (\sigma_\theta - \sigma_\phi)^2 + (\sigma_r - \sigma_\phi)^2\right]^{1/2}, \tag{A3}$$

reduces to

$$Y = \sigma_\theta - \sigma_r, \tag{A4}$$

since $\sigma_\theta = \sigma_\phi$.

The radial displacement outwards is

$$\mu = \frac{p}{E}\left[(1 - 2\nu)\, r + \frac{(1 + \nu)b_o^3}{2r^2}\right] \Big/ \left(\frac{b_o^3}{a_o^3} - 1\right), \tag{A5}$$

where ν = Poisson's ratio.

Then for no work hardening, from Eqs. A1, A2, and A4,

$$Y = \sigma_\theta - \sigma_r = \frac{3pa_o^3}{2r^3} \bigg/ \left(1 - \frac{a_o^3}{b_o^3}\right) . \tag{A6}$$

As the pressure increases, yielding begins at the inner surface, where $r = a_o$, at which time the pressure is from Eq. A6,

$$p = \frac{2}{3} Y \left(1 - \frac{a_o^3}{b_o^3}\right) . \tag{A7}$$

The displacements of the internal and external surfaces are at this pressure

$$\mu_o(a_o) = \frac{Y}{E} a_o \left[\frac{2(1 - 2\nu)a_o^3}{3b_o^3} + \frac{1 + \nu}{3}\right] , \tag{A8}$$

$$\mu_o(b_o) = \frac{Y}{E} a_o \left[\frac{(1 - \nu)a_o^2}{b_o^2}\right] . \tag{A9}$$

Failure of the stainless steel is not probable at this pressure, since it has been shown that although the ductility of the matrix is impaired by the neutron and fission-product bombardment plastic deformation will occur before failure.[2]

As the fission-gas pressure continues to increase, a plastic region of radius c spreads outwards into the hollow sphere. In the elastic region

$$\sigma_r = -A\left(\frac{b_o^3}{r^3} - 1\right) , \quad \sigma_\theta = \sigma_\phi = A\left(\frac{b_o^3}{2r^3} + 1\right) . \tag{A10}$$

At the plastic-elastic boundary, the material is just yielding so that A can be evaluated from $Y = \sigma_\theta - \sigma_r$ and is

$$A = \frac{2}{3} Y \frac{c^3}{b_o^3} . \tag{A11}$$

By substitution of Eq. A11 into Eq. A10, the elastic stress is then

$$\sigma_r = -\frac{2Yc^3}{3b_\circ^3}\left(\frac{b_\circ^3}{r^3} - 1\right), \qquad (A12)$$

$$\sigma_\theta = \sigma_\phi = \frac{2Yc^3}{3b_\circ^3}\left(\frac{b_\circ^3}{2r^3} + 1\right), \qquad (A13)$$

for $c \lesseqgtr r \lesseqgtr b_\circ$.

The displacement is

$$\mu = \frac{A}{E}\left[(1 - 2\nu)r + \frac{(1 + \nu)b_\circ^3}{2r^2}\right], \qquad (A14)$$

for $c \lesseqgtr r \lesseqgtr b_\circ$.

In the plastic region, equilibrium exists,

$$\frac{\partial\sigma_r}{\partial r} = \frac{2}{r}(\sigma_\theta - \sigma_r) . \qquad (A15)$$

Integration with $Y = \sigma_\theta - \sigma_r$ yields

$$\sigma_r = 2Y \ln r + B, \qquad (A16)$$

and since σ_r is continuous across the plastic boundary,

$$B = 2Y \ln c - \frac{2Y}{3}\left(1 - \frac{c^3}{b_\circ^3}\right). \qquad (A17)$$

Hence,

$$\sigma_r = -2Y \ln\frac{c}{r} - \frac{2}{3}Y\left(1 - \frac{c^3}{b_\circ^3}\right), \qquad (A18)$$

$$\sigma_\theta = \sigma_\phi = Y - 2Y \ln\frac{c}{r} - \frac{2}{3}Y\left(1 - \frac{c^3}{b_\circ^3}\right), \qquad (A19)$$

for $a \lesseqgtr r \lesseqgtr c$.

The internal fission-gas pressure needed to produce plastic flow to a radius c is obtained by substituting $r = a$ into Eqs. A18 and A19.

$$p = 2Y \ln\frac{c}{a} + \frac{2}{3}Y\left(1 - \frac{c^3}{b_\circ^3}\right). \qquad (A20)$$

For the values of b_o/a_o of concern here, the strains at the inner surface of the shell are small as long as the plastic zone has not spread throughout the entire metal sphere. When the plastic zone has just spread throughout the sphere, $c = b_o$.

Then from Eq. A20

$$p = 2\,Y \ln \frac{b_o}{a} , \tag{A21}$$

and the internal displacement is

$$\mu(a_o) = \frac{Y}{E}\, a_o \left[(1 - \nu)\, \frac{b_o^3}{a_o^3} - 2(1 - 2\nu) \ln \frac{b_o}{a_o} \right] . \tag{A22}$$

From Eq. A5, the external displacement is

$$\mu_o(b_o) = \frac{Y}{E}\, b_o \,(1 - \nu). \tag{A23}$$

From Eq. A20,

$$\frac{\partial p}{\partial c} = 2\,\frac{Y}{c} \left(1 - \frac{c^3}{b_o^3} \right) . \tag{A24}$$

Thus, when $c = b_o$, the pressure reaches a maximum, and during subsequent expansion the pressure is

$$p = 2\,Y \ln \frac{b}{a} \tag{A25}$$

which decreases monotonically.

In the absence of work hardening, the fuel element has failed, since at the pressure indicated in Eq. A25, the fuel element instantly expands until the fracture strain is reached. Until this pressure is reached, the strains at the inner and outer surfaces of the sphere are small so that

$$\frac{b}{a} \cong \frac{b_o}{a_o} . \tag{A26}$$

The volume fraction of UO_2 in the fuel is

$$F = \frac{a_o^3}{b_o^3} , \left(\frac{1}{F} \right)^{1/3} = \frac{b_o}{a_o} . \tag{A27}$$

and on substitution into Eqs. A22 and A25,

$$a = a_\circ + \mu(a_\circ), \qquad \text{(A28)}$$

$$a = a_\circ \left\{ 1 + \frac{Y}{E} \left[(1 - \nu)\frac{1}{F} - \frac{2}{3}(1 - 2\nu) \ln \frac{1}{F} \right] \right\}, \qquad \text{(A29)}$$

$$p = \frac{2}{3} Y \ln \frac{1}{F} .$$

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