ORNL-2915
UC-34 – Physics and Mathematics
TID-4500 (15th ed.)

MATHEMATICS PANEL

PROGRESS REPORT

FOR PERIOD SEPTEMBER 1, 1958 TO DECEMBER 31, 1959

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A. S. Householder, Chief

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MATHEMATICS PANEL PROGRESS REPORT

MATHEMATICS AND NUMERICAL ANALYSIS

F. L. Bauer[1] A. C. Downing C. T. Fike
W. Gautschi A. S. Householder R. P. Kanwal[2] A. Schopf[3]

The investigations described in this section fall in three areas: magnetohydrodynamical shock waves, the numerical solution of ordinary and partial differential equations, and the numerical treatment of matrices.

In the first area, a paper has been prepared and submitted for publication,[4] with the following purpose:

1. to derive the shock conditions in magnetohydrodynamics in their full generality, that is, for the case when the flow is unsteady and three-dimensional;
2. to decompose the flow and field quantities along the normal to the shock surface and along its Gaussian coordinates and thereby express separately these quantities downstream of the shock wave in terms of their known values upstream of the shock; in this rearranged form of the equations, various effects of the shock wave can be easily read off;
3. to discuss the differential effects of the shock conditions on the same lines as was done by the author for nonconducting gas dynamics.

While at the National Bureau of Standards W. Gautschi began (jointly with H. A. Antosiewicz of the Bureau) a survey of the most important numerical methods that are available to handle any differential equation, and the survey has been completed at ORNL. The methods are conveniently grouped into those of Runge-Kutta type and difference methods. Particularly for the latter, an exposition is given that differs from those usually found in the literature, as emphasis is given to mathematical foundations rather than algorithms.[5] The preparation of the survey grew out of a series of lectures presented by the authors to university teachers during two successive training programs at the National Bureau of Standards.

A study has been initiated of new difference methods designed for particular classes of differential equations, in particular for differential equations having periodic solutions. The corresponding methods turn out to be variants of the ordinary (classical) difference methods depending on a (small) parameter $\mu = h/T$, where h is the integration step and T the period of the solution in question. It was shown, in fact, that as $\mu \to 0$ the new methods reduce to the old ones. This fact is used to define certain optimal difference methods, the development of which is now under way.

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[1]Temporary employee, now at Gutenberg Universität, Mainz, Germany.

[2]Now at Pennsylvania State University.

[3]Deceased.

[4]R. P. Kanwal, *On Sound and Shock Waves in Magnetogasdynamics* (to be published).

[5]W. Gautschi and H. A. Antosiewicz, *Numerical Methods in Ordinary Differential Equations* (to be published).

The basic technique for obtaining approximate solutions of multiregion reactor problems is to cover the regions with a rectangular grid and then solve a finite difference problem for the flux at the nodes of the grid. Implicit in this method is the assumption that if the grid size were allowed to shrink to zero the finite difference solutions would converge to a solution of the differential problem. An investigation has been made into the justification of this assumption. Previous work has demonstrated convergence only for the one-region Dirichlet problem.[6,7] Recently a related question has been answered for the multigroup multiregion reactor calculation. It has been shown[8] that if the equations for the separate groups are solved cyclicly – the so-called "outer" iteration – then the process converges to a solution of the multiregion problem. But this assumes an exact solution for a partial differential problem for each group, not just a solution of the corresponding finite difference problem. The solution of the latter problem is obtained usually by a relaxation procedure which is referred to as the "inner" iteration.

The study in progress has been concerned initially with the convergence of this inner iteration. Clearly, convergence cannot be demonstrated for every formulation of the finite difference problem. So the first effort was to find a formulation for which convergence might be demonstrated, and attention has been fixed upon a method for treating interfaces which admits an additive analog of Green's second identity. With this tool it has been possible to show that the finite difference solutions will converge to a function satisfying the diffusion equation in the interior of each region and, moreover, with the usual interface conditions, to show that the solution of the differential problem is unique. The behavior at the interfaces has also been studied, and it appears that suitable convergence properties can be established there, although one step in the proof has not yet been justified.

The conjectured advantages of triangularizing a matrix by orthogonal multipliers[9] have not been substantiated by experiment on a number of test matrices, some selected more or less at random and some especially constructed, of order up to about 15 or so. Simple Gaussian elimination turned out to be faster and in nearly all cases at least slightly more accurate. For general use the method seems to stand clearly superior to all others.

A method of successive approximation, which appeared recently in the literature,[10] seemed to have certain advantages over known methods and led to a renewed appraisal of

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[6] R. Courant, K. Friedrichs, and H. Lewy, *Math. Ann.* 100, 32–74 (1928).

[7] I. G. Petrovskiĭ, *Uspekhi Mat. Nauk* 8, 161–70 (1941).

[8] G. J. Habetler and M. A. Martino, *The Multigroup Diffusion Equations of Reactor Physics*, KAPL-1886 (1955).

[9] A. S. Householder, *Math. Prog. Rep. Mar. 1, 1957, to Aug. 31, 1958*, ORNL-2652, p 5.

[10] N. Gastinel, *Comp. rend.* 246, 2571–74 (1958).

related methods. It turned out[11] that for the class of "methods of projection," including the method of steepest descent and the method of relaxation, a simple application of the theory of norms gave rigorous bounds for convergence rates and that the new method converged no faster than the method of relaxation but, step for step, required more computation. An incidental by-product of this investigation was a new proof and certain useful generalizations of an inequality due to Kantorovich that limits the rates. In particular, the inequality itself was shown[11] to be a consequence of a fundamental one due to Wielandt which concerns the localization of the characteristic roots of a matrix. For publication of other related results, notes left by A. Schopf are being assembled.

This problem of localization is basic to that of bounding the errors in computed values of the roots, and hence of the vectors. Localizotion theorems are essentially of two sorts: exclusion theorems, which delimit regions in the complex plane in which no roots can lie; and inclusion theorems, which delimit sets each known to contain at least one root. Historically, exclusion theorems were first stated explicitly by Gershgorin, and have been refined by Perron, Brauer, and others, and it is known that they can be extended when stated in terms of norms.[12] In some cases an exclusion theorem becomes an inclusion theorem, which is of more practical interest in bounding the errors. The literature in both areas is quite extensive, but the most general inclusion theorems previously known were those due to Wielandt, some of whose results have been announced without proof (see ref 13 for a brief survey of the literature).

It tums out that all known inclusion theorems, including those of Wielandt, can be obtained and generalized, and the proofs greatly simplified, by a simple application of the theory of norms. In the simplest situation, if a vector x is known to approximate a characteristic vector of A, then by forming the iterates $x_i = A^i x$, and the moments $\mu_{ij} = x_i^H x_j$, it is possible to determine a circle in the complex plane within which a root must lie. More generally, however, with any x, inclusion circles and regions of other types can be found. At present the results are formuloted for the most part in rather abstract terms, and techniques for making the most effective application in practice remain to be worked out. One restriction is that the theorems so far formulated presuppose that the matrix is similar to a diagonal matrix. Other cases are of much less practical importance, and the formulations will be much more complicated, but the same principles will apply.

Closely related to this problem is the sensitivity of the roots to small changes in the elements of the matrix and the extent to which a similarity transformation can yield a matrix for which the sensitivity is less or greater. This latter question is of importance

[11] A. S. Householder and F. L. Bauer, *Numerische Mathematik* 2, 55–59 (1960).

[12] A. S. Householder, *J. Assoc. Computing Mach.* 3, 314–24 (1956).

[13] F. L. Bauer and A. S. Householder, *Numerische Mathematik* 2, 42–53 (1960).

because a first step in the computation of such roots is usually to make one or a series of such similarity transformations, in order to replace the original matrix by one whose form is as simple as possible, perhaps even obtaining the characteristic polynomial explicitly. It is found that in a certain sense the sensitivity cannot be increased by a factor greater than the condition number of the transforming matrix P, which is to say by the factor $||P||\ ||P^{-1}||$. In particular, if P is unitary the sensitivity cannot be increased at all, since in this case $||P|| = ||P^{-1}|| = 1$ (ref 14). With regard to the sensitivity in general, if E is a matrix of (presumably small) errors and if the perturbed matrix $A + E$ can be diagonalized by the matrix X, then every root of A lies in a circle about a root of $A + E$ whose radius cannot exceed $||E||\ ||X||\ ||X^{-1}||$ (ref 15). Again the same method would apply to the case of a perturbed matrix $A + E$ that cannot be diagonalized, but the formulation is much more complicated.

Rutishauser has developed a method of accelerating his LR transformation for finding characteristic roots and vectors of a Hermitian matrix, and it seemed plausible that a somewhat similar device might also be effective for Bauer's staircase iteration which is applicable to an arbitrary matrix. This method iterates on a matrix rather than on a vector, and in general will provide as many of the roots as there are linearly independent columns in the matrix operand. Some numerical experiments were carried out to test this conjecture, and it was found that acceleration did indeed occur but only to one of the roots, usually the one of smallest modulus. Further analysis revealed the reason for this and made it clear that acceleration was possible only to one root, and this only when the matrix operand is square.[16]

[14]C. T. Fike, *J. Assoc. Computing Mach.* 6, 360–62 (1959).

[15]F. L. Bauer and C. T. Fike, *Norms and Exclusion Theorems* (to be published).

[16]F. L. Bauer, C. T. Fike, and A. S. Householder, *Acceleration of Bernoulli Type Iterations* (to be published).

PROGRAMMING

SUMMARY

The programming activities of the Mathematics Panel, other than those performed as direct services for other divisions, fall roughly into two main categories: one may be called programming research, and the other consists in the testing of numerical algorithms and the preparation of general-purpose programs.

The first class includes the study of programming processes, and the design, construction, and revision of programming systems, which expedite these processes as normally used in the preparation of numerical codes. Attention has been focused during the period on two main projects: the completion and revision of the Oracle Binary Internal Translator (ORBIT)[1] and the design and construction of a translator for ALGOL (ALGOrithmic Language) for use on the Oracle. ALGOL[2] is a universal algebraic coding language expected to be used extensively both in this country and abroad.

Contributions to general-purpose programming are discussed in the section entitled "General Programming." Some of these are in the form of complete self-contained programs that perform frequently needed mathematical operations. These are canned programs that are available for use by any interested persons. Others are in the form of subroutines which are stored in compiler libraries and are automatically incorporated into programs which call for them.

ORACLE BINARY INTERNAL TRANSLATOR (ORBIT)

G. C. Caldwell[3] E. L. Cooper A. H. Culkowski A. C. Downing
M. Feliciano A. A. Grau J. Harrison

Completion and Revision. – The original ORBIT translator was completed and put into service early in 1959. A revision which was undertaken at the same time has also been completed and is now in service. Lectures and courses have been given during the period and are listed in "Training and Seminars."

Revision of ORBIT was required because memory requirements in the first version imposed severe restrictions on the depth of nesting of iterations, on any correction or change in the main processing segment, and on the addition of desirable new features to the system. The main segment consisted of a large list of subroutines which were needed in random fashion during processing, and so it was not possible to break it up into two or more segments which could be used successively.

The following changes and additions, some of which are discussed in greater detail below, were made at the time of revision or since:

1. preprocessing of iteration statements to release space for other use in the main segment;
2. simplification of the end-of-page and end-of-program sentinels and of the parameter input format (this was made possible by a new one-character paper-tape input order built into the Oracle);
3. inclusion in the language of the basic elementary functions of analysis: sine, cosine, tangent, arcsine, arccosine, arctangent, exponential, natural logarithm, and hyperbolic sine and cosine, and addition of a random-number generator;
4. introduction of new input and output orders for ORBIT programs;

[1] J. J. Andrews, J. Harrison, and A. A. Grau, "Oracle Binary Internal Translator (ORBIT)," *Math. Prog. Rep. Mar. 1, 1957, to Aug. 31, 1958*, ORNL-2652, p 10.

[2] "Preliminary Report – International Algebraic Language," ed. by A. J. Perlis and K. Samelson, *Communs. Assoc. Computing Mach.* 1(12), 8–22 (1958).

[3] Research participant from North Carolina State College, Raleigh.

5. an error-monitor segment that reports programming errors and in many cases permits their correction at the console;
6. an edit segment that permits the systematic presentation of the original and compiled program side by side for ready reference;
7. new statements in the ORBIT language that permit the use of automatic segment switching in multisegment ORBIT programs;
8. provision for the use of magnetic-tape files as auxiliary storage with ORBIT programs.

The present ORBIT program consists of five segments: the first segment loads a program and processes iteration statements while doing so, the second does the main processing, the third prepares all information for the final Oracle compiler step, the fourth is the error-monitor segment, and the fifth is the program-edit segment.

Error-Monitor Segment. – Twenty-two types of programming errors are detected and reported by the ORBIT error monitor. Of these all but three can be corrected at the console by punching a paper tape with the correction of the statement in error or by furnishing appropriate information on the console keyboard.

Among the types of errors handled are typing errors that result in invalid pairs of successive characters, unmatched parentheses, numbers with too many digits, and the misspelling of unary functions.

The error is pinpointed by a report of the number of the line in which the error occurs. More than one statement may be used to correct an error in a statement.

Program-Edit Segment. – The edit is a record furnished at the programmer's option of the original ORBIT program that has been processed and of the final machine program into which it has been translated; these are recorded side by side on the page. The main part of the program is followed by the list of constants and their addresses and the statement label table.

POGO[1] output instructions are used in the edit segment, so that either typewriter or curve-plotter output may be obtained. The edit is found to be a valuable aid in debugging.

Multisegment ORBIT Programs. – If a program is too long to fit into the full 2048-word memory of the Oracle, it is sometimes possible to divide it into two or more pieces, called "segments," which are then called successively into high-speed memory. The combined program is conveniently stored on magnetic tape, and by suitable coding any segment may automatically call into memory any other.

A basic principle of ORBIT programming is that if two ORBIT programs have identical initialization declarations, variables with the same identifiers will be assigned identical memory locations. This permits the programming of multisegment ORBIT programs with almost full cross reference.

To make automatic the switching of segments in ORBIT multisegment programs, two new statements have been introduced into the language; these have the function of locating another segment of the program on the magnetic tape, placing it into the high-speed memory, and transferring control to its beginning.

A recent development permits the almost automatic preparation on magnetic tape of multisegment programs during the compiling process. It is now possible, therefore, for a programmer without a detailed knowledge of the technical inner behavior of the Oracle to construct ORBIT programs with almost an arbitrary number of segments.

Magnetic-Tape Files. – Ideally, the programmer who uses an algebraic coding language such as ORBIT should not be required to know very much about the hardware characteristics of the machine. He should be able to use as auxiliary storage the Oracle magnetic tapes without a detailed knowledge of the machine coding involved in their use. How to use efficiently magnetic tape for auxiliary storage with an algebroic language such as FORTRAN, ORBIT, or ALGOL is a problem that has not yet been solved completely.

A partial solution for this problem has, however, been obtained for the Orocle and is presently in use with the ORBIT system. A single ORBIT statement defines the number of words in each block or record on tape. The addresses of locations on the tape are numbered in sequence from one record to the next. After this is done, variables may be recorded on or reod from magnetic tape by referring only to the addresses thus assigned. The auxiliary storoge thus in effect defines a new class of voriables whose values are stored at those addresses.

Tape manipulation is minimized by retoining an entire block or record always in high-speed storage, and, consequently, tape is moved only when two consecutive references refer to distinct records.

This solution to the auxiliary storage problem takes advantage of a property of the Oracle magnetic-tape system which is almost unique – the ability to write a block of information at any point on the tape without having to copy every succeeding block.

Manual. – The *Programmers' Manual*[4] for the revised ORBIT system was planned for use both as a reference volume on ORBIT and as a programming primer using the ORBIT language as the vehicle. It is divided into five chapters, which deal with the language, programming, compiling and use of programs, a description of the complete system, and an appendix with tables that summarize information needed in using the system.

New material, which will be incorporated in any revision of the Manual, is issued in the form of ORBIT memoranda. Three such memoranda have been issued, dealing with extensions, multisegment programming, and magnetic-tape files.

Status. – No extensive changes are planned in the ORBIT system as it now exists and as it is now operated. Some minor modifications will probably be made from time to time at the suggestion of users. Most of these will be in the form of additions. Some such work is now being done by users of the system, not members of the Mathematics

[4]A. A. Grau *et al., Programmers Manual — Oracle Binary Internal Translator, Algebraic Programming System for the ORACLE,* ORNL CF-59-9-20.

Panel, giving facilities that provide for the use of alphameric material in ORBIT output, a section of code that will make possible the automatic preparation of multisegment programs, and the use of the curve plotter as an ORBIT output medium. Information concerning each of these topics will be issued in the form of ORBIT memoranda.

Attention is now being directed toward the construction and use of an ALGOL translator. In its design and construction, the experience gained in preparing ORBIT plays a substantiol part, although it has not been possible to take over directly any great part of the ORBIT system itself.

AN ORACLE TRANSLATOR FOR ALGOL

G. Atta E. L. Cooper M. Feliciano A. A. Grau

Introduction. – ALGOL (ALGOrithmic Language) is a result of collaboration on an international level. Preliminary specifications for this universal algebraic programming language were drawn up at a meeting in Zürich, Switzerland, in May–June 1958 by a joint committee of the Association for Computing Machinery and the Gesellschaft der Angewandte Mathematik und Mechanik. A second meeting, in January 1960, is expected to result in a final report on the language.

Like existing algebraic coding languages, such as FORTRAN, IT, and ORBIT, the language is independent of a particular machine and may be used for programming modern high-speed computers of all types. Since one consideration that led to ALGOL was a desire for standardization, it may be expected in time to replace other algebraic languages now in use. It has all the important features possessed by most of these and in addition has some powerful new features of its own.

ALGOL is also a useful device for the exchange and publication of numerical algorithms and computing information. A *Handbook for Automatic Computation*, consisting of at least five volumes, is in preparation by Springer-Verlag; members of the Mathematics Panel are among the contributors and editors. The entire first volume will be devoted to an exposition of ALGOL and ALGOL translation, and ALGOL will be used as the vehicle for presenting algorithms throughout. Without such a common programming language, this set of books, which fills a definite need in this area of technology, is not possible.

A program written in ALGOL may be used on a particular machine if a translator program for that machine is available. The latter must be specifically designed to transform the former into a program in machine language. An ALGOL translator is an asset to a machine, and since the Mathematics Panel has always been interested in many phases of numerical analysis, translators for the machines in the Oak Ridge area which could be used for testing algorithms to be used in the Handbook, for running ALGOL programs originating elsewhere, and for general programming are needed. Hence it was decided to attempt the construction of an ALGOL translator for the Oracle. A translator for the IBM-7090 is in preparation elsewhere.

The character set of any given machine generally is not the character set of the ALGOL reference language. Consequently, it is necessary to use a hardware adaptation of the ALGOL language isomorphic to it. The rules for making the adaptation for the Oracle have been worked out.

One difficulty faced in the construction of a translator for the Oracle is its limited memory capacity (2048 words). FORTRAN set aside 8000 words for tables needed in the course of processing. Oracle magnetic tapes are not well adapted to the storage of information that is needed in random fashion.

The system finally devised is particularly useful on a smaller machine. Attempts to imitate extensively systems used elsewhere have been given up. The translation process has been simplified as much as possible. All functions of any complexity are handled in target programs by subroutines. The work involved in translation has been allotted to various stages in such a way that the memory requirements of any program and the necessary tables are minimal. Final testing will indicate whether some additional modification will be necessary.

No particular difficulty is anticipated in the eventual addition to the system of features of ORBIT such as multisegment programs and magnetic-tape files.

Structure of the Translator. – The translator consists of four main parts and the Oracle compiler. The first segment takes a program punched on paper tape in the Oracle hardware adaptation of ALGOL described below and stores it in a systematic manner on magnetic tape. For this it uses an internal bi-hexadecimal representation of reference ALGOL.

The second part of the translator processes do and for statements. A do statement has the effect of copying part of a program with substitutions. A for statement corresponds to an iteration statement in ORBIT or a DO statement in FORTRAN and is processed by replacing it by suitable ALGOL statements of other types. The result is a program still in ALGOL language, which does not contain do or for statements.

Tables of identifiers, arrays, switches, and functions are next formed. At this point variables are replaced by relative addresses or pseudo-addresses.

The main part of the translation follows. Details of the sequential handling of information using the stacking principle are of more than passing interest and are discussed more fully below.

Oracle Hardware Language. – For some of the functions required in ALGOL, no convenient symbols are available; therefore English words or phrases are used. Such ALGOL "symbols," however, are then either printed in a different style of type or, in the case of handwriting, underlined.

In the Oracle adaptation of ALGOL, this principle is extended to all symbols which are not part of the input-output repertoire. In order to indicate that a given English word, phrase, or abbreviation represents a single ALGOL symbol, it is enclosed in a pair of apostrophes. Thus **go to**, $<$, $\neq$, and $\geq$ become in Oracle ALGOL, 'go to', 'ls', 'nteq', and 'greq'.

Processing of Do and For Statements. – The processing of a do statement is a copy-with-substitution operation which is carried out in the original ALGOL program.

The for statement, although the analog in ALGOL of the iteration statement in ORBIT and the DO statement in FORTRAN, involves some generalization. The pilot model of the Oracle translator will initially handle only the most frequently occurring case of the for statement, and the discussion below is limited to this case. Other cases depend on this one.

The for statement may be considered to be an abbreviation for two sets of statements of other types which are placed one before and one after the statements subject to the for loop. At some point, anywhere from a point where the program is still in ALGOL to the point where the final target program takes shape, a substitution of the equivalent of these two sets must be made in the program. In existing systems the loop is always executed at least once, and the value of the indexing variable is immaterial on exit. Following the lead of European computer groups, the following meaning is assigned to the for statement:

Let a for statement apply to the statement Σ (which may itself be a compound statement made up of any number of simple statements):

$$\textbf{for } v := a\ (b)\ c : \Sigma \ . \tag{1}$$

The statement defines a sequence of values of v. If b is positive, the sequence consists of all members of the sequence

$$a\ ,\quad a+b\ ,\quad a+2b\ ,\quad \ldots\ ,\quad a+kb\ ,\quad \ldots$$

which are not greater than c, and if b is negative, of those that are not less than c. The statement Σ is executed in turn for each member of the sequence thus defined, and the value of v on exit is the last member of the sequence. If there are no elements in the sequence, v is considered undefined on exit from the loop. In this case, the statement Σ is never executed.

In the case where b is positive, the statements (1) above must be translated into:

```
          v: = a;
L1:       if (v > c); go to L2;
          Σ;
          v: = v + b;
          go to L1;
L2:       v: = v - b
```

The labels $L1$ and $L2$ are taken from a list reserved for this purpose. In general, a, b, and c may be more general expressions. If they are, additional variables and statements are introduced into the program.

Sequential Translation. – Main processing in the ALGOL translator for the Oracle is done by methods based on the principles laid down by Bauer and Samelson.[5] In the application of these principles, unpublished specifications for the ALGOL translator used at Mainz, Germany, drawn up by M. Paul, were of assistance.

Basic to these methods is the *stacking principle*, whereby information that is needed in translation is stored, when necessary, in a last-in first-out fashion. Two stacks in particular play a leading role, the *control stack* and the *auxiliary stack*. Incoming information which cannot be processed immediately is factored into two parts which are stored in these.

The control stack determines a state of the translator which determines the manner in which information is handled. The total behavior of the translator may be described by listing all possible states and all possible types of incoming information. In this way a double-entry table is obtained, with about 40 states and 30 types of incoming information. Only a small fraction of the possibilities suggested by this can occur in a valid program. A given state and a given type of information determine a set of subroutines for execution. The number of such subroutines does not exceed a few dozen.

The methods here outlined handle satisfactorily all phases of the translation process other than linkages (go to statements). To process these it is necessary to make two lists of identifiers and addresses during the course of translation and to make a final pass in which addresses are assigned to transfer orders.

Subroutines. – Floating-point operations as in ORBIT must be handled by subroutines. With the stacking principle in mind, and in an effort to obtain as short a translator program as possible because of memory limitations, it was necessary to redesign the subroutines used in ORBIT for ALGOL use. The same is true of the unary functions which will be incorporated into an ALGOL system from the outset.

Status. – A pilot model of an ALGOL translator for the Oracle is in an advanced state of preparation. The loading segment is completed, the segment processing for statements debugged, and the main processing segment completely designed; most of the coding for the latter has been done.

Most of the translator has been written in a language consisting of ALGOL augmented by additional primitive elements. To a considerable extent this dispenses with the need for flow charts. ALGOL itself is not sufficient for the writing of compilers and translators, since the notion of address, for example, must be among the primitives of

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[5] F. L. Bauer and K. Samelson, *Sequentielle Formelübersetzung* (to be published in *Elektronische Rechenanlagen*; will also be published in English under possibly another title).

such a language, and at some point machine characteristics must play a part. However, it would be possible to use most of the specifications for the Oracle ALGOL translator to construct a translator for another machine, if the latter had a high-speed memory of about the same capacity or greater and had either magnetic tape or drum auxiliary memory.

GENERAL PROGRAMMING

Complete self-contained programs, generally referred to as "canned" programs, prepared or extensively revised during the period of the report, are described below. In addition, contributions in the form of subroutines and service programs added to the Oracle library are summarized; the complete list of these is given in Table 1.

Unless otherwise noted, a separate report is available for each canned program, each subroutine of the Compiler Library, and each Oracle service program. These may be consulted for more detailed information than is found below.

Linear Regression and Correlation Code II (CP OOD) (A. H. Culkowski and N. M. Dismuke). – This program was completed in the beginning of 1959, with options added shortly thereafter.

By the method of weighted least squares, k sets of coefficients, $b_{l,k}$, will be calculated, along with a number of basic statistical estimates. There is a separate set of coefficients for each y_k, but the set of x's is assumed to be the same for each y_k.

The observation matrix (X, Y, W) consists of functions of observations. A specific function is associated with each column, and the function is determined by means of a formula that indicates which variables of an explicit observation are to be combined. A formula may contain any of the following operations singly or in combination: absolute value, exponential, natural logarithm, power, sine, cosine, tangent, hyperbolic cosine, sine and tangent, division, multiplication, negation, and addition.

The total number of columns may not exceed 41; however, the number of observations is limited only by time. In the last column are the weights, and these may be specified as part of the empirical data, or, if given by an explicit functional representation, they may be calculated by the program.

The user may request any or all of the following results: raw regression coefficients, transpose or normal matrix, standard regression coefficients, correlation matrix, means, standard deviations, standard deviation ratios, inverse matrix, y computed or predicted, residuals, multiple correlation coefficient squared, mean sum of squared residuals or the standard error squared, sums of squares and sums of products of deviations from the means matrix, weighted sum of the squared residuals, weighted sum of the residuals, variances of parameter estimates, variances of a mean prediction, and variances of a single prediction.

Various options have been devised to save time or possibly help to debug a program that does not run. There is an option to use input numbers previously computed by some other program, which are in the correct form and order on the proper magnetic-tape drive.

If a subsequent case is to be run using some or all of the original variables per observation, as the previous case, another option will permit the use of the data already computed.

If the matrix to be normalized or transposed already exists in the proper form and on the correct magnetic-tape drive, there is an option to begin the program from that point.

An option to put out the observation matrix is a convenience which serves as a check in determining whether all formulas have been correctly stated and all constants correctly given. Any errors in input are readily apparent here.

If it is required to find variances for single or mean predictions, or both, for observations other than the original but using the original means, inverse, and standard error squared, an option is available to do this.

It is also possible to use computed raw regression coefficients to obtain new dependent variables for other sets of independent variables requiring the same regression coefficients. In this way also, derivatives of a function may be found, since new column functions may be used.

To date, approximately 374 curve fittings have been run.

Bairstow Iteration Polynomial Equation Solver (CP OOF) (A. A. Grau). – The Bairstow Iteration Equation Solver is designed to solve polynomial equations with real coefficients of any degree for both real and complex roots. It employs the iterative method developed by Bairstow[6] and by Hitchcock.[7] Basically, the method finds quadratic factors of the polynomial; these are then solved for roots by the quadratic formula.

The program permits alteration of a number of parameters in the program to change accuracy requirements. Since the method is not guaranteed to converge in all cases, provision is also made for a complete monitor of the iteration steps. Practically, the method almost always gives good results. It has been used for solving polynomial equations that have arisen in several areas of the Laboratory.

Characteristic-Value, Vector Code for Real Symmetric Matrices (J. Wehe). – This package code, which calculates the characteristic values and vectors, and certain associated residuals, of a fixed-point real symmetric matrix, was prepared for the IBM-704 computer. Plane rotations reduce the matrix to triple-diagonal form using the method of Givens,[8] and a Sturm sequence is used to calculate the characteristic values. In these respects the program is similar to a corresponding program for use on the Oracle. The vectors are determined by use of back substitution as described by Wilkinson.[9]

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[6] L. Bairstow, *Investigations Relating to the Stability of the Aeroplane*, Reports and Memoranda No. 154 of the Advisory Committee for Aeronautics (1914).

[7] F. L. Hitchcock, "Finding Complex Roots of Algebraic Equations," *J. Math. and Phys.* 17, 55–58 (1938).

[8] W. Givens, *Numerical Computation of the Characteristic Values of a Real Symmetric Matrix*, ORNL-1574 (1954).

[9] J. H. Wilkinson, "The Calculation of Eigenvalues of Codiagonal Matrices," *Computer J.* 1, 90 (1958).

The program deals with a slightly simpler problem than the Oracle code but could, if desired, be expanded to handle the same type of problem. Subroutines were available for the IBM-704 for both the Givens and the Wilkinson method for finding vectors. The experience obtained agreed with that of Wilkinson[9] and that of Rotenberg[10] in confirming the superiority of the latter method. The method has, however, the disadvantage of producing only one vector for each distinct characteristic root.

Linear Equation Solver (C. T. Fike). – An Oracle code for solving simultaneous linear equations was completed which incorporates recent ideas on the subject. Its general characteristics are speed and accuracy. The basic method used is that of Gaussian elimination with interchanges, which is supplemented by an iterative procedure. The improvement technique makes use of the fact that if x_1 is an approximate solution to the problem $Ax = b$, then $x_1 + y_1$ is generally a better approximation, where y_1 is an approximate solution to $Ay = b - Ax_1$. A report on this program has not yet been issued.

Power Method Matrix Code (C. T. Fike). – A program using double-precision fixed-point arithmetic for finding the maximum characteristic value of an arbitrary matrix and the corresponding right and left vectors by the power method was prepared for the Oracle. Tests show that although the method works it is very slow and would not be feasible for much production work. A report has not been written for this program.

Subroutines and Service Programs. – Additions to the Oracle Compiler Library (given in Table 1) include a set of eight single and double integration routines and new sine and cosine routines; routines to compute the exponential integral, elliptic functions, confluent hypergeometric function, and complementary error function; codes employing (8,32) arithmetic to solve systems of linear equations and invert matrices; new input and output codes, some using POGO output orders; conversion routines, including conversion from one type of floating point to another, from Oracle floating point to IBM-704 floating point; and curve-plotter routines. Needed for the ORBIT system are arctangent, integer division, and tape handling routines.

New service programs include POGO output changer, POGO memory dump with zero suppression, transfer monitor, an item conversion routine required by a change in item format, and a dictionary translator.

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[10]P. Fox, A. Rotenberg, and E. Witherell, *Eigenvalue, Eigenvector Code*, SHARE Report No. 396, NUMXEW (1958).

Table 1. Additions to Libraries

Item Number	Compiler Item	Contributor
	A. Oracle Compiler Library	
	Elementary Functions, Fixed-Point	
Trigonometric		
00 15F	Fixed-point sin x function	C. T. Fike
	Elementary Functions, Floating-Point	
Exponential and Logarithmic		
00 1E1	Exponential integral (8,32) $E_1(x)$	N. B. Alexander
00 1E2	Exponential integral (8,32) $E_2(x)$	N. B. Alexander
00 1E3	Exponential integral (8,32) $E_3(x)$	N. B. Alexander
	Polynomials and Special Functions	
00 154	Polynomial division (40,40)	A. A. Grau
00 123	Confluent hypergeometric function	M. T. Robinson*
00 124	Complementary error function	M. T. Robinson
00 12C	Complete elliptic integral $e(k, \pi/2)$	N. B. Alexander
00 12D	Complete elliptic integral $f(k, \pi/2)$	N. B. Alexander
00 155	Quadratic formula (40,40)	A. A. Grau
	Operations on Functions and Solutions of Differential Equations	
00 11D	(40,40) exponential integral	A. C. Downing
00 13E	Gauss-Laguerre (40,40) integration	C. T. Fike
00 151	Double integration over unit circle fixed-point	C. T. Fike
00 152	Double integration over unit circle fixed-point (greater accuracy)	C. T. Fike
00 153	Double integration (x), $f(x,y)$ fixed-point (Gaussian)	C. T. Fike
00 156	Double integration, fixed-point (Gaussian)	C. T. Fike
00 157	Double integration (square root $1 - x^2$), fixed-point (Gaussian)	C. T. Fike
00 158	Single fixed-point integration	C. T. Fike
00 159	Single fixed-point integration (greater accuracy)	C. T. Fike
	Operations on Matrices and Vectors	
00 047	(8,32) Equation solver	E. C. Long
00 048	(8,32) Equation inverse	E. C. Long
00 04A	(8,32) Equation solver and inverse	E. C. Long
00 04B	(8,32) Equation solver and inverse	E. C. Long
00 15C	Memory-to-memory matrix transpose	C. T. Fike
00 203	Matrix triangular factorization	C. T. Fike
	Input	
Fixed-Point		
00 202	Fixed-point fraction input code	C. T. Fike
Alphameric		
00 12E	Load one alphameric character to sentinel	E. Cooper
00 133	One alphameric decimal word to (40,40)	A. Culkowski
Others		
00 201	Integer input code	C. T. Fike

Table 1 (continued)

Item Number	Compiler Item	Contributor
	Output	
Fixed-Point		
00 13B	Fixed-point output of an (8,32) number	W. R. Busing*
Floating-Point		
00 20F	(40,40) floating-point	C. T. Fike
Others		
00 11C	Lead zero delete and punch word in Q	E. C. Long
00 127	POGO conversion with intermediate storage	E. Cooper
00 128	POGO	E. Cooper
00 129	POGO	E. Cooper
00 12A	POGO	E. Cooper
00 12B	POGO	E. Cooper
00 161	Roman numeral output	M. T. Robinson
	Conversions	
Floating-Point to Floating-Point		
00 073	(7,3) to (28,12)	E. C. Long
00 11E	(8,32) to (28,12)	E. C. Long
00 11F	(28,12) to (8,32)	E. C. Long
00 120	(40,40) to (28,12)	E. C. Long
00 121	(28,12) to (40,40)	E. C. Long
00 141	Oracle (8,32) floating-point to IBM-704 floating-point	M. T. Robinson
00 142	Oracle (40,40) floating-point to IBM-704 floating-point	M. T. Robinson
Fixed-Point to Floating-Point		
00 12F	4-2-2-1 BCD to binary fixed-point	M. T. Robinson
00 13F	Oracle fixed-point to IBM-704 floating-point	M. T. Robinson
Others		
00 150	Double precision decimal to binary	C. T. Fike
	Curve Plotter	
00 130	Coordinate, label, and point plotter	J. Reynolds**
00 131	Word plotter	J. Reynolds
00 132	Horizontal or vertical line plotter	J. Reynolds
	ORBIT Subroutines	
00 137	Arc tan x (8,32)	M. Feliciano
00 13A	Integer division	M. Feliciano
00 16A–00 16F	ORBIT tape handling subroutines	A. C. Downing
00 173–00 179	ORBIT tape handling subroutines	A. C. Downing

Table 1 (continued)

Item Number	Compiler Item	Contributor
	B. Service Program Library	
SP 012	POGO output changer	E. Cooper
SP 013	POGO memory dump with zero suppression	A. A. Grau
SP 014	Transfer monitor	A. A. Grau
SP 109	Item conversion code	E. C. Long
SP 10A	Directory translator	G. L. Broyles
	C. Canned Program Library	
CP 00F	Bairstow iteration equation solver	A. A. Grau

*Solid State Division.
**Instrumentation and Controls Division.

BIOLOGY AND MEDICINE

M. A. Kastenbaum A. W. Kimball

Cytology and Genetics

A model has been proposed for representing the induction of exchanges in chromosomes by ionizing particles. The model is based on the postulate that broken ends can rejoin only at a finite number of sites within the nucleus. The mathematical implications of this model have been studied, and distribution functions have been obtained for the number of exchanges per cell under various assumptions about the form of the distribution of the number of sites per cell. Data from experiments with *Tradescantia* and *Vicia faba* have been used to test the adequacy of the model.

Recombination experiments with *Neurospora crassa* often yield plate counts that must be combined in the form of a ratio in order to estimate recombination frequencies. Exact confidence limits may be computed with the aid of tables of the binomial distribution. Under certain conditions, however, these limits may be computed from simple formulas, the use of which requires only a table of the normal distribution. These formulas may also be used in estimating sample sizes for limits of prescribed length. A full description of the procedures will appear elsewhere.[1]

For use in connection with longevity studies on populations of irradiated animals, a study was made of estimation procedures for mortality intensity. This parameter is defined as the negative derivative with respect to time of the logarithm of the survival

[1]A. W. Kimball, "Confidence Intervals for Recombination Frequencies in *Neurospora* Experiments," submitted for publication in *Biometrics*.

curve and is important in the construction of the Gompertz plot, a widely used representation of mortality data. The results of the study show that two estimators of the actuarial type are biased and may lead to distortions of the Gompertz plot. An unbiased estimate is available,[2] but it depends on an assumption not likely to be valid in animal experiments of moderate size. This estimate has been modified to agree more closely with the conditions that usually prevail in animal experiments, and the complete results of the study have been submitted for publication.[3]

Methods for fitting both simple and generalized multihit survival curves were applied to radiation experiments with several different organisms. Included were studies on the effects of neutron and gamma radiation on germination rates in timothy seeds, ultraviolet radiation on hatchability of *Habrobracon* eggs, x radiation on the killing of *Tetrahymena pyriformis*, and other studies. Fitting of the generalized curves was facilitated by an approximate linearization of the incomplete beta function reported earlier.[4]

In connection with further studies on the postirradiation period in x-ray mutagenesis in *Paramecium aurelia*, a disproportionate subclass number analysis of variance was carried out in order to combine the results of several experiments and to strengthen conclusions related to time dependence of modifying factors.

A study of the effects of ultraviolet radiation of the nucleolus in grasshopper neuroblasts had as its end point the length of time from irradiation to prometaphase. For each dose the ratio of time for unirradiated to time for irradiated cells was fitted to log dose. Confidence intervals were found for the dose which yields a 50% increase in the time interval. Tests of significance were then applied to the results for irradiation in several mitotic stages at two different wavelengths. Several statistical tests were utilized in connection with a color-mutant investigation in maize. They included the computation of "exact" probabilities for several 2×2 tables and the estimation of parameters for models proposed by Luria and Delbruck.

Experiments with *Vicia faba* using fractionated doses of x radiation have been used to study the effect of dose on the rejoining time of chromosomes broken by ionizing particles. An experiment was designed, based on the "up-and-down" method, for studying the effect of varying the amount of total dose allotted to each fraction with maximum efficiency. Several new procedures were developed for use in connection with *Drosophila* experiments. These included methods for special partitions of heterogeneity chi-squares, models for estimating relative radiation sensitivities of sperm types from multiple classifications of phenotypes, and maximum-likelihood solutions for several estimation problems.

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[2] H. L. Seal, "The Estimation of Mortality and Other Decremental Probabilities," *Skand. Actuarietidskr.* 37, 137–62 (1954).

[3] A. W. Kimball, "Estimation of Mortality Intensities in Animal Experiments," submitted for publication in *Biometrics*.

[4] *Math. Prog. Rep. Mar. 1, 1957, to Aug. 31, 1958*, ORNL-2652, p 21–23.

Weighted regression analyses were performed on *Neurospora* data to determine the relationship between the number of mutants per 10^6 survivors and time at two different temperatures. The weights applied were computed on the assumption that the observations were Poisson-distributed. The slopes of the linear fits which resulted were compared by applying the Student's t test.

The relative loss of a gene locus following x irradiation of maize pollen may be expressed as a function of the length of the chromosome arm on which it is located and its proximity to the centromere.[5] Since the two chromosome arms are of different length, it was of interest to know the distance from the centromere for which the loss on the long arm is equal to the maximum loss on the short arm. The problem was handled quite adequately by applying elementary calculus techniques.

A reversion of prophase chromatin to the interphase state has been observed in living neuroblasts of grasshopper embryos after x irradiation.

Experiments with *Vicia faba* root tips were initiated to determine whether the same effect occurs in this plant. Groups of seedlings were irradiated with various doses of x rays, and the distributions of cells by phases were then compared with those of cells from normal unirradiated root tips by employing the techniques of exact partitioning of chi-square in contingency tables.[6]

Estimates of LD_{50} for chick embryos 20 hr after exposure to various doses of 250-kvp x rays were computed at incubation ages of two to sixteen days. Two series of experiments, one each in the summer and winter, were carried out with eggs from the same flock, as a test of the seasonal effect. The proportions surviving each dose were transformed to angles by the arc sine transformation, and the resulting angles were fitted linearly against dose. The LD_{50} estimates were then computed from these fitted lines and appropriate variances, and confidence intervals were attached to each estimate.

Further studies on embryonic development and its possible impairment as a result of irradiation were carried out. These studies produced series of data representing quantitative measurements, the analysis of which involved the use of standard estimation and analysis-of-variance techniques.

Pathology and Physiology

Computations on the Operation Greenhouse experiment were finished, and the final report is nearing completion. Studies on radiation-induced lymphatic and hematopoietic tumors have been extended to include the evaluation of dose rate and fractionation effects, and, in these experiments, disease incidence and other mortality parameters were

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[5]A. C. Faberge, "The Possibility of Forecasting the Relative Rate of Induced Loss for Endosperm Markers in Maize," *Genetics* 42, 454–72 (1957).

[6]M. A. Kastenbaum, "A Note on the Exact Partition of Chi-Square in Contingency Tables," to be published in *Biometrics* in September 1960.

estimated. Experiments on protein metabolism in the rat-mouse chimera yielded a large number of exponential decay curves which were subjected to analysis by the method of least squares. Because glomerular nephrosclerosis in mice is greatly enhanced by irradiation, experiments have been initiated to study the development of the disease histologically. To assist in the interpretation of the data and to help design future experiments, several models were fitted to preliminary results and compared. Several analyses of variance, some from disproportionate data, were carried out on the results of an experiment on the effects of antigen-inactivated spleen cells on the lymphoid tissues of irradiated mice.

Methods for correcting estimates of disease incidences for intercurrent mortality have already been reported.[7] An approximate variance formula given in that report was not entirely satisfactory, and no conclusions could be drawn about the small-sample properties of the estimates. To provide better information on these points a large-scale random sampling experiment was designed and was carried out on the Oracle. In addition, a true asymptotic variance formula was derived. The results of the experiment permitted the comparison of the two variance formulas with the actual variances for a wide range of parameters and permitted the derivation of a correction factor for each of the two variance formulas that, in practice, should provide sample variances that have very little bias. A complete report of this work is in preparation.

Radiation Immunology

Studies on primary and secondary antibody responses in mice to a variety of stimuli and related experiments on concentration of serum protein fractions yielded a large amount of data for statistical analysis. Methods used included the analysis of variance and the application of multiple-comparisons procedures for separating mean responses. Standard analysis-of-variance techniques were applied to data resulting from electrophoretic studies of the sera from mice after two types of treatment, one with S,β-aminoethylisothiuronium·Br·HBr prior to irradiation, and one with bone marrow following irradiation. The purpose of this study was to determine whether the delayed mortality in irradiated mice receiving foreign bone marrow results from an in vivo antigen-antibody reaction reflecting incompatibility between the host and donor tissues.

Microbial Protection and Recovery

Survival and mutation experiments with *Aspergillus terreus* spores were performed to provide answers to specific questions about threshold effects for both killing and

[7] A. W. Kimball, "Estimation of Disease Incidences in Populations Subject to Multiple Causes of Death," *Bull. Inst. Internat. Statist.* **36**, 193–204 (1958).

mutagenesis and about the interdependence, if any, between the two responses, particularly in the presence of a protective compound. By constructing successive orthogonal contrasts of the dose-survival fractions and computing the corresponding chi-square test statistics, the threshold doses for killing were estimated. The mutation data were fitted by the method of weighted least squares.

Mammalian Recovery

A series of dose-survival experiments with human cells in tissue culture have been performed under varied conditions chosen to provide information about a model which has been proposed to explain the observed response. Multihit curves have been fitted to the data for each experiment, and new models have been suggested. Experiments are being planned to test these new models. Epithelial cells of mice given 2500 r of x rays undergo abnormal enlargement in all dimensions; crypt cells appear to show the greatest enlargement. Cell populations of intestinal mucosa and stroma of mice 9–12 weeks old were examined for the relationship between abnormal cell enlargement and doses ranging from 500 to 5000 r. The data were analyzed by analysis-of-variance methods. Preliminary analyses have been carried out on data collected for the purpose of comparing, in several strains of mice, erythrocyte life span of normal unirradiated mice with that of bone-marrow-treated, irradiated mice of the same strain. The life of an erythrocyte appears to drop off exponentially with time. Therefore a logarithmic transformation was performed on the observations, and the transformed data were fitted linearly against time. The problem has been coded for the Oracle, and the program yields estimates of the regression coefficients and the 50% point, as well as variances and confidence intervals on the latter.

Mammalian Genetics and Development

Cytological studies of irradiated testes in mice have been extended to very low dose ranges. Counts of type A and type B spermatogonia, resting primary spermatocytes, and Sertoli cells were made in testis sections from mice exposed to Co^{60} gamma rays and 14.1- and 3.5-Mev neutrons. The statistical procedures involved several analyses of variance, fitting of various regression functions, and computation of RBE's and confidence intervals for the two energy levels of neutrons. For the specific-locus mutation study in mice, numerous tests of significance and interval estimates were calculated in connection with the comparison of mutation rates for acute and chronic irradiation.

Biophysics

Counts made on three different polonium-beryllium sources in different locations by use of two detectors yielded seven decay curves which were fitted by least squares. Tests were made for departures from linearity, and the seven fits, although not balanced with respect to the three factors, were combined to provide single half-life estimates for comparison with values already published. Several experiments on the photoprotection effect in *E. coli* B were analyzed by regression methods to estimate the dependence of the effect on temperature and intensity. By use of Oracle subroutines for elliptic integrals, specifically written for the problem, several sets of single- and double-integration formulas were evaluated as part of a study to determine the correction to the inverse-square law required by the finite size of ionization chambers when these instruments are used to measure radiation intensity. Further mathematical and physical details of this work have been published elsewhere.[8,9]

Cell Physiology

Peptides isolated from rat liver microsomes may be separated from contaminating amino acids, sugars, and salts by countercurrent dialysis. Inasmuch as a single such dialysis can achieve only fractional purification, more complex systems of multiple dialysis, in which a linear series of cells is used, have been devised. The system of immediate interest may be described as follows. Let each dialysis cell be a stage, and each dialysis period be a cycle. After each cycle, the "dialout" (solution of escaped solute, outside the dialysis sac) at each stage is concentrated and used as the "dialin" (solution inside the sac) in the succeeding stage, whereas the dialin at each stage is concentrated and placed in the sac of the previous stage. The dialin at the first stage is retained at the first stage, and the dialout at the final stage is taken out of the system. In the nomenclature of stochastic processes, the first stage is a reflecting barrier and the last stage an absorbing barrier.

To determine the probability with which a particle of the isolate will be at a specified stage in the system after a given number of cycles, the system was described in terms of a Markov process with a stochastic matrix resulting from the product of a diffusion matrix and a transfer matrix. Algèbraically explicit solutions have been worked out[10] for any number of stages and cycles in a system of this type. Special cases involving a fixed number of stages were coded for the IBM-610, and numerical solutions were subsequently obtained at various cycles for different initial probabilities.

UT-AEC Agricultural Research Program

Studies on large mammals at the UT-AEC farm involved an evaluation of two methods for collecting samples of bull semen and the estimation of an LD_{50} (30-day) for cattle subjected to various doses of whole-body irradiation. Among the variables measured in

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[8] *Biol. Semiann. Prog. Rep. Aug. 15, 1958*, ORNL-2593, p 95–96.

[9] *Biol. Semiann. Prog. Rep. Feb. 15, 1959*, ORNL-2702, p 172.

[10] M. A. Kastenbaum, "The Separation of Molecular Compounds by Countercurrent Dialysis: A Stochastic Process," accepted for publication in *Biometrika* in June 1960.

the first study were volume of semen, total sperm, and sperm concentration for each sample collected by both methods. For each variable a Student's *t* test of the weighted differences between the two methods was carried out. The results of these tests indicated that no significant difference between the methods had been demonstrated by the data. The second study involved the irradiation of 30 head of cattle both in the spring and in the fall of 1959. Estimates of the LD_{50} were made after first transforming the percentage killed at each dose to angles by the arc sine transformation. The transformed data were then fitted linearly against dose and the appropriate LD_{50} values estimated from the line. Variances and confidence intervals were then computed for each LD_{50} estimate.

HEALTH PHYSICS

D. A. Gardiner M. A. Kastenbaum

Ecology

The data obtained from soil samples of the upper and lower lake-bed regions of White Oak Lake were analyzed for the purpose of comparing the chemical constituents of the two regions, and also for the purpose of studying the trend for the years 1956, 1957, and 1958 of the concentration of Sr^{90} in the soil.[1] The techniques used were those of the analysis of variance.

Also by the analysis of variance, samples of the native vegetation collected over a two-year period were analyzed for Sr^{90}, Cs^{137}, Co^{60}, and the more abundant cations. These data were examined in conjunction with the soil data, mentioned above, for the purpose of exploring the soil-to-plant movement of radionuclides on the lake bed. For each species the two annual samples were compared[1] for the concentration of Sr^{90} and Cs^{137}.

In an agricultural experiment, three species of forage crops were planted in a randomized block experiment in the White Oak Lake bed agricultural plot.[1] Prior to planting, soil samples were collected within each plot and analyzed for radionuclides and other cations. Each species was harvested at maturity and the total plot yield recorded. The crops were compared within and among blocks by analyses of variance, and the relationship between total accumulation of radionuclides per unit area and yield of vegetation was approximated.

Internal Dosimetry

In another study, a new and different method for estimating the "relative damage factor" has been proposed. The relative damage factor, *N*, is used by health physicists

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[1] S. I. Auerbach *et al.*, *H-P Ann. Prog. Rep. July 31, 1959*, ORNL-2806, p 18–27.

to express the expected relative degree of malfunction of an organ injured by a radioactive isotope. At present the relative amounts of radionuclides deposited in an organ may be measured autoradiographically, but this method is not entirely satisfactory because it provides no information as to the nonuniform distribution of the radioactive material within the organ. The study was designed to determine in detail the relative amounts of uranium deposited in the histological structure of the kidney of the rat. The amounts of uranium in the kidney are measured by two methods: (1) separate ashing and radiochemical analysis of cortex and medulla from half of each kidney; (2) counting the alpha tracks in representative sections of cortex and medulla from the other half. The proposed estimate of the relative damage factor is the ratio of counts per unit volume in the cortex or in the medulla to the counts per volume in the whole kidney.[2]

Urinalysis

The results of an extensive study to compare the methods of enriched-uranium urinalysis in use at the plants in the Oak Ridge area have been reported.[3] During the progress of this study statistical consulting service was given on topics such as (1) the formulation of models for measurement, (2) the propagation of random errors, (3) the analysis of variance, and (4) tests of hypotheses. During the study, interest in the wide variation in the determination of recovery factors for the Y-12 procedure prompted another study.[4] Since the total amount of uranium in urine is not detected in the urinalysis, recovery factors, estimated from experimental data, are applied to correct the amount of uranium detected. An experiment to test the hypothesis that the recovery factor is constant over a specific range of uranium concentration in urine was conducted and the hypothesis was rejected. In the course of this investigation, extra-Poisson variation in amounts increasing with increasing uranium concentration was discovered. The reasons for this extra variation are now being investigated.

Waste Disposal

The initial phases[5] of a laboratory program to determine the points of optimum operation of the lime-soda softening process for removing Cs^{137} and Sr^{90} and subsequent

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[2]E. S. Jones *et al.*, *H-P Ann. Prog. Rep. July 31, 1959*, ORNL-2806, p 194.

[3]Process Analysis Department, Technical Division, Y-12 Plant, "Comparison of Enriched-Uranium Urinalysis Procedures," Y-B92-125 (Dec. 11, 1959).

[4]W. C. Johnson *et al.*, "Investigation of the Y-12 Electrodeposition Procedure of Enriched-Uranium Urinalysis," EPS-Y-423, MIT Engineering Practice School, Oak Ridge, Tennessee (Oct. 19, 1959).

[5]D. A. Gardiner, *Math. Prog. Rep. Mar. 1, 1957, to Aug. 31, 1958*, ORNL-2652, p 25–26.

discoveries[6] have been reported. A paper is being prepared[7] which will contain a detailed account of the now completed program. By the "method of steepest ascent" the optimum treatment for Cs^{137} removal was determined to be one of mixing the waste with an amount of lime equal to 135% of the stoichiometric requirement, 474 ppm excess soda ash, and 605 ppm of −200 mesh Grundite clay. This treatment removed 95% of the Cs^{137}. The optimum treatment for the removal of Sr^{90} was one of mixing the waste with 250% of the stoichiometric lime requirement, 520 ppm excess soda ash, and 357 ppm of −200 mesh Grundite clay. This removed 96% of the Sr^{90} and also 92% of the Cs^{137}. It is intended to use this latter treatment as a point of departure in a program to discover the optimum point of operation in the ORNL waste-treatment plant.

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[6]K. E. Cowser *et al.*, *H-P Ann. Prog. Rep. July 31, 1959*, ORNL-2806, p 3–6.

[7]D. A. Gardiner and K. E. Cowser, "Optimization of Radionuclide Removal from Low-Level Process Wastes by the Use of Response Surface Methods," (to be submitted to *Health Physics* for publication).

METALLURGY

N. B. Alexander S. E. Atta A. C. Downing D. A. Gardiner

Fuel Element Plates. – The diffusion of uranium into the aluminum cladding of fuel element plates is one of two problems concerning fuel element plates on which assistance was given by members of the Mathematics Panel. Assuming a constant coefficient of diffusivity, an analytical expression was obtained for estimating the concentration of uranium diffusing into the cladding. Curves showing the concentration as a function of depth (in terms of dimensionless quantities) for various diffusion constants were obtained from the literature of heat conduction, and a FORTRAN program was prepared to permit a nonlinear least-squares estimation of the diffusion coefficient when experimental data are available.

The second problem was one of determining an optimum particle size of uranium oxide for the fabrication of fuel element plates. It was assumed that the particles were spherical in shape and were distributed throughout the plate according to the uniform probability distribution. Then the probability density function of the shortest distance between any two particles was analytically obtained.

Zircaloy-2. – Round tensile specimens of Zircaloy-2 and other close-packed hexagonal metals have been observed to become elliptical when strained. Computing machine programs were written to (1) demonstrate the ellipticity of cross sections of the strained tensile specimen, (2) compute the natural axial and contractile strains for characterizing the material, and (3) compute the reduction in area for each specimen.

Segregation in Uranium Metal. – The tendancy for impurities in a piece of metal to distribute themselves more or less regularly according to some pattern is known as segregation. A study of this phenomenon was conducted on metal castings of a geometry common to castings produced at the Y-12 plant. The fabrication procedure suggested that the cross-section pattern of segregation could be described by a quadratic expression in two dimensions of the casting. An experiment was designed, and eight samples of metal from each of five castings were obtained under the supervision of the Y-12 Process Analysis Department. The samples were analyzed for content of 27 elements, and a quadratic model was fitted to the more important of the elements by the method of least squares. The segregation patterns were then described by plotting contours of equal purity or content, as determined by the estimated quadratic functions, on cross-section outlines of the castings. D. A. Gardiner and A. M. Christman are preparing a report of this program for publication in a statistical journal.

Absorption Spectra of Fused Salts. – A spectrophotometer equipped with an automatic paper-tape punch measures the absorption spectra and records the observations digitally on paper tape. An Oracle program was written to process the tape, store the information, and produce a specified output. The program is divided into three phases. In phase I, the input data are cross-referenced, converted to Oracle language, and stored on magnetic tape. Phase II allows for one, two, or three sets of data to be called in from magnetic tape and combined into a single set of data according to the wishes of the experimenter. Phase III is the output phase, which allows from one to six sets of information calculated from the sets produced in phase II to be read out in any or all of four different media, Oracle paper tape, printed table, tape for off-line curve plotter, and Oracle curve plotter.

REACTORS

N. B. Alexander M. M. Callahan H. P. Carter D. J. Wehe

Several calculations relative to reactor technology were made by the Mathematics Panel. For the most part, the problems were completely formulated by the originators and presented to the Mathematics Panel for numerical computation. Almost all of the work reported in this section was done on the IBM-704.

Two computer programs were designed for studying the proposed fuel element cladding for the Experimental Gas Cooled Reactor. The first computes the stresses and deflections of cylindrical shells subjected to nonsymmetrical temperatures or pressures. The temperature or pressure functions are expressed by Fourier series, and the stresses and displacements are then determined by the evaluation of a corresponding set of Fourier series. The second program gives an estimation of the mechanical life of metal-clad fuel elements. To accomplish this, the rate at which the gas pressure builds up is computed, and the rate at which the clad creeps when the pressure inside the element exceeds the ambient pressure is estimated; then, using the elastic and creep properties of the clad, the clad expansion may be estimated.

Calculations to determine the equilibrium concentrations of fuel isotopes and fission products in a gas-cooled, graphite-moderated, thorium-breeder reactor have been made. These concentrations are a function of the processing rates, power, burnup, and other engineering variables; so a set of equations describing the fuel rate, discharge rate, recycling time, and amount of each fuel isotope and fission products was iterated until the concentration of U^{233} was constant. The initial concentration of each fuel element for each stream in the reactor was computed from the fraction of neutrons absorbed by all elements in the region in a lethargy group and the microscopic cross sections of absorption, fission, and capture of the elements in the reactor.

A program to determine the neutron spectrum shifting in going from the middle of the moderator volume to the surface of the fuel element in a one-dimensional heterogeneous reactor is being developed. Essentially, the desired results can be obtained from the solution of a large system of coupled integral equations.

In order to compare the concentration of slurry particles near an efflux channel with the concentration at large distances, the system of second-order differential equations governing the trajectories of slurry particles in an ideal incompressible fluid was set up for numerical integration. The fluid velocity is the driving force for the particles, and for simple geometries this is known analytically. Since the coefficient of the second derivative of particle position is very small, being proportional to its mass, this makes the numerical integration process unstable except for very small integration intervals. Ultimately, the work was abandoned because of the large amount of computing time estimated to be necessary to obtain a satisfactory solution for even a two-dimensional model.

PHYSICS

N. B. Alexander	H. P. Carter
D. E. Arnurius	A. H. Culkowski
S. E. Atta	N. M. Dismuke
N. A. Betz	A. C. Downing
M. M. Callahan	C. T. Fike

The high-intensity, fast-chopper, time-of-flight neutron spectrometer has been used to measure neutron cross sections. A sequence of steps must be taken to process the raw data and yield the quantities of physical interest. Although the precise course followed in processing the data depends on the isotope, the type of cross section, and the energy region, as well as the details of the experiment, some standardization is possible. The spectrometer and some representative measurements are described elsewhere.[1]

Experimental data are recorded on teletype tape as seven-character Oracle words representing a three-hex-digit channel number and a four-hex-digit count. A typical tape is composed of an identification word followed by up to 2048 data words. As many as four such tapes may be required to carry out the data processing of one experiment. The first link is an Oracle program which makes some preliminary corrections and adjustments to the raw data and converts them to energy vs cross section or vs transmission, the variables of the later links in the data processing.

Raw data are read directly into the Oracle, where they may be (1) corrected for counter dead time, (2) dead-time corrected and averaged, or (3) simply arranged in a convenient form and put on magnetic tape. In the next pass, the program converts the data to energy vs cross section or vs transmission and records statistical weight. For total and scattering measurements, both cross section and transmission are computed, and for capture measurements, simply energy vs cross section is computed. Output of the data for visual inspection is via narrow magnetic tape, which is printed in tabular form. Provision has been made for the addition of routines to permit direct plotting of transmission and cross-section graphs on the Oracle CRT curve plotter. The actual supplying of these data to later links is a detail which is not yet settled.

The resonances in neutron cross sections characterize the isotope under study, and hence the determination of resonance parameters is an important function of the processing of neutron cross-section data. Several fittings and function evaluations are useful from time to time to study the sensitivity of the cross sections to certain resonance parameters. A general multilevel, many-exit-channel fitting of experimental cross-section data using known resonance theory[2] involves many more parameters than can be handled computationally. Under specific conditions, approximations can be made which enable parameters to be estimated.

The simplest case is a single-level Breit-Wigner fit, which computationally is a four-parameter nonlinear least-square fitting. Somewhat more complex is a fitting which includes interference with potential scattering; computationally, this increases the number of parameters, and the functional dependence of cross section on its parameters is through a longer sequence of numerical steps. Still another approximation applicable to fissionable nuclei, which involves a few levels and an arbitrary number of channels, has been worked out by Vogt.[3] Computationally, it is a function evaluation requiring a

[1]G. G. Slaughter, R. C. Block, and J. A. Harvey, *Phys. Semiann. Prog. Rep. Mar. 10, 1958*, ORNL-2501, p 26; *Phys. Semiann. Prog. Rep. Sept. 20, 1958*, ORNL-2610, p 22; *Phys. Ann. Prog. Rep. Mar. 10, 1959*, ORNL-2718, p 28.

[2]E. P. Wigner, *Phys. Rev.* 70, 15 (1946); *Phys. Rev.* 70, 606 (1946); E. P. Wigner and L. Eisenbud, *Phys. Rev.* 72, 29 (1947).

[3]E. Vogt, *Phys. Rev.* 112, 203 (1958); *Low Energy Neutron Cross Sections of Fissionable Nuclei*, TPI-98 (1959).

complex matrix inversion for every value of energy; but the fitting is, in fact, only intelligent guessing and re-evaluation. It may be possible to devise a true fitting procedure. Vogt's approximation has been programmed for the IBM-704. Output is on magnetic tape or curve plotter. At present, comparison with experimental data is not made by machine.

The fitting referred to above is complicated further by Doppler broadening and by resolution broadening. Because of these two effects, the experimentally observed resonances have been broadened; the effects must therefore be included in the fitting or evaluation. At present the fitting and the broadening are being studied separately. An estimate of machine time based on a least-square fitting including 20 levels and both Doppler and resolution broadening gave a time of the order of a day per iteration.

An approach which has been used is breakdown of the consideration into three problems according to the comparison of the resonance width, Γ, and the combined Doppler and resolution widths, Δ. For $\Delta < 0.1\Gamma$ the fitting is carried out and compared with experimental data corrected for resolution and Doppler broadening. For $\Delta > \Gamma$, an area analysis is applied to the transmission dip. A simplification occurs because the area is independent of resolution; however, the difficulties of Doppler broadening, sample thickness, and level interference are still present. The most difficult case is that for $0.1\Gamma < \Delta < \Gamma$, when no obvious simplifications occur. This case is under study using a directed trial-and-error approach: for a given set of parameters an evaluation of the cross section is carried out and compared with experimental data.

Phenomenological nuclear potentials from neutron scattering on O^{16} (ref 4) have been constructed and various configurations examined using the IBM-704 in some parameter studies.[5] The Woods-Saxon form of the potential was used along with a Thomas-type spin-orbit energy term to effect a phase-shift analysis. In an attempt to postulate a potential configuration which would be consistent with *f*-state energy levels, a velocity-dependent type of potential formulated by Ross, Lawson, and Mark[6] was used. Integration of the radial Shrödinger equation was programmed by the Numerov method. The total computation was arranged in sequential sections to permit parameter adjustments at intermediate stages.

Ion trajectories in electrostatic fields were traced to determine the best shape of equipotential grid placed in the ion beam cross section – the shape that would focus the ions with the least spherical aberration. Calculations indicate that a sine-shaped grid produces the best focusing. The work is to be reported in detail.[7]

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[4] J. L. Fowler and H. O. Cohn, *Phys. Rev.* 109, 89 (1958).

[5] E. G. Corman and J. L. Fowler, *Bull. Am. Phys. Soc.* 5, 34 (1960).

[6] A. A. Ross, R. D. Lawson, and H. Mark, *Phys. Rev.* 104, 401 (1956).

[7] N. B. Alexander, R. M. Chester, and D. G. Maeder, "Spherical Aberration of Grid-Type Electrostatic Lenses" (to be published).

Rotational frequencies of the vinylacetylene molecule (CH_2CHCCH) cannot be fitted to equations for the rotational energy states which approximate the effects of centrifugal distortion upon the moments of inertia. A rigorous description of the energy levels, including five terms for description of centrifugal distortion, has been adapted to this molecule. Attempts to fit the eight coefficients in the formula to the 15 data have to date yielded physically unacceptable solutions. An IBM-704 nonlinear least-square fitting program was used with FORTRAN-coded function and derivatives. Initial estimates were obtained on the Oracle using the linear regression code.

In the *calibration of a carbon resistance thermometer* for use at liquid-helium temperatures, some Oracle computations were carried out. Several sets of simultaneous equations were solved to obtain coefficients in a least-square fit to calibration data. Tables of resistance vs temperature and of the slope of resistance vs temperature were produced.

The spherical-harmonics method and the Gauss quadrature method have been applied to obtain by a limit process the exact solution of the Boltzmann equation for the Milne problem with linearly anisotropic scattering and absorption in plane geometry.[8]

The results are being used to calculate, as functions of the multiplication constant and the anisotropy constant, tables for (1) the distance of the extrapolated end point; (2) the linear extrapolation distance; (3) some factors of normalization, the density, and the current density at the boundary of the medium; (4) the angular distributions of neutrons at the boundary of the medium; and (5) the transient and asymptotic parts of the current density and of the density of neutrons in the interior of the medium.

In all cases integrals are involved which contain one special weight function. It has a singularity at the upper limit of integrotion which can be avoided by two methods of calculation. Carrying out the two methods of computation serves also as a check on the calculations and provides an estimote of the rounding errors introduced.

One of the central problems in *radiation damage* is the detailed mechanics of the collisions between like atoms in which the moving atom has kinetic energy in the 100- to 100,000-ev range. Physically this is a purely classical problem once the interatomic potential is known; all the information needed is contained in the integral for the deflection angle, Θ. Since the interatomic potential is unknown, a first approach to this problem has been undertaken by choosing a reasonable form for the potential in terms of an adjustable parameter.

The integration for Θ was carried out for various values of the potential parameter. The integral for Θ had a singularity in the integrand which could be removed by a suitable transformation. For a discussion of the results, see ref 9.

[8]W. Kofink, *Studies of the Spherical Harmonics Method in Neutron Transport Theory*, ORNL-2334 (May 17, 1957); *Studies of the Spherical Harmonics Method in Neutron Transport Theory, Part II*, ORNL-2358 (July 11, 1957); *Studies of the Spherical Harmonics Method in Neutron Transport Theory, Part III*, ORNL-2850 (Oct. 21, 1959).

[9]D. K. Holmes, G. Leibfried, and O. S. Oen, *Solid State Ann. Prog. Rep. Aug. 31, 1959*, ORNL-2829, p 1.

Detailed *tables* for the *components* of the *magnetic field produced* by a *semi-infinite current sheet* were prepared. The tables include values of the magnetic vector potential as well as the axial and radial components of the magnetic field, for a cylindrical current sheet of unit radius. Particular attention was given to the field values in the vicinity of the end of the sheet, where table values are given at intervals of 0.01 in the r and z directions. In this region, eight significant figures were obtained. To secure this accuracy, the usual mathematical formulation in terms of complete and incomplete elliptic integrals is not satisfactory, since the field values then depend upon differences of similar quantities. The expressions used for the calculation and their derivations were published, together with the tables, as an ORNL report.[10]

A smaller table, giving the radial and axial components for a single circular loop, was also prepared. The magnetic vector potential for a single loop is equal to the radial component of H of the semi-infinite current sheet whose end coincides with the loop, provided the total current in the loop is numerically equal to the current density in the circular sheet. Similarly, the scalar potential for a single loop may be obtained from the tables for a current sheet because the scalar potential is equal to the axial component of the field due to a current sheet in the half space not containing the current sheet.

[10]N. B. Alexander and A. C. Downing, *Tables for a Semi-Infinite Circular Current Sheet*, ORNL-2828 (Sept. 29, 1959).

CHEMISTRY

N. B. Alexander   D. A. Gardiner   H. P. Carter

Concentrations of daughter elements in the decay chains of several heavy nuclei were computed as functions of flux density and of time. Such calculations are distinguished by a characteristic numerical difficulty in obtaining meaningful results for daughters whose relative concentrations are small.

The basic technique for the numerical resolution of spectral fine structure is to use a nonlinear least-square residual procedure to fit the observed spectrum. For the uranyl ion the spectrum between 3500 and 5000 Å arises from approximately 12 electronic transitions. Each band is overlapped on the high- and the low-frequency sides by two or three other bands, with the result that the parameters for any individual band cannot be estimated directly from the summed spectrum. Two mathematical models suggest themselves for approximating each band – one model based upon the Gaussian function and the other upon a resonance function. For a single band the resonance function will fit the tails better but give too narrow a peak. The effectiveness of these two models in resolving the observed spectrum was tested by preparing two programs for the IBM-704, making use of an existing general-purpose least-square routine for estimating nonlinear parameters. The numerical procedure did not converge for the resonance model. Possibly a combination of the two models would be best, but this was not tested because the size of the high-speed storage on the IBM-704 (8196 words) is too small. Using the Gaussian model, the calculation is presently performed on the Oracle using a program prepared by members of the Chemistry Division.[1] This program is more efficient – by a factor of one-third in cost – than the program available for the IBM-704.

The Process Analysis Department of the Y-12 Plant received general statistical assistance on a problem in accountability.[2] Advice and counseling were given on sampling procedures, probability distribution questions, and methods of estimating the propagation of random errors.

[1]W. R. Busing and H. A. Levy, *A Crystallographic Least Squares Refinement Program for the IBM 704,* ORNL CF-59-4-37 (Apr. 19, 1959).

[2]Y-B92-121 (Secret).

ORACLE OPERATIONS

Machine operations have continued on a three-shift five-day-week schedule. The good operating time and the operating ratio (good computing time/scheduled computing time) are as follows:

	Monthly Computing Time (hr)	Operating Ratio (%)
High	523	95.9
Average	422	94.2
Low	403	89.0

Ninety programmers used the machine; 28.9% were Mathematics Panel personnel.

Figures 1 and 2 indicate the hours of machine time used by various divisions during the last half of calendar year 1959 on the Oracle and IBM-704 respectively. The length of the bar represents the total time and is the sum of Mathematics Panel programmed time (crosshatched) and time programmed by other personnel.

The ORBIT translator code has been placed on the Oracle compiler tape, and the ORBIT subroutine library has been combined with the compiler library.

Modifications to the compiler include a more efficient item loader routine, a magnetic-tape edit routine for compiled programs, a complete rearrangement of the subroutine library to reduce compiling time, and the inclusion of 72 new subroutines to the library.

Programs were written to rearrange the subroutines in any desired order, to list slaves of subroutines, to list items which slave a given item, to obtain an edited directory of the library, and to delete or insert subroutines into the library.

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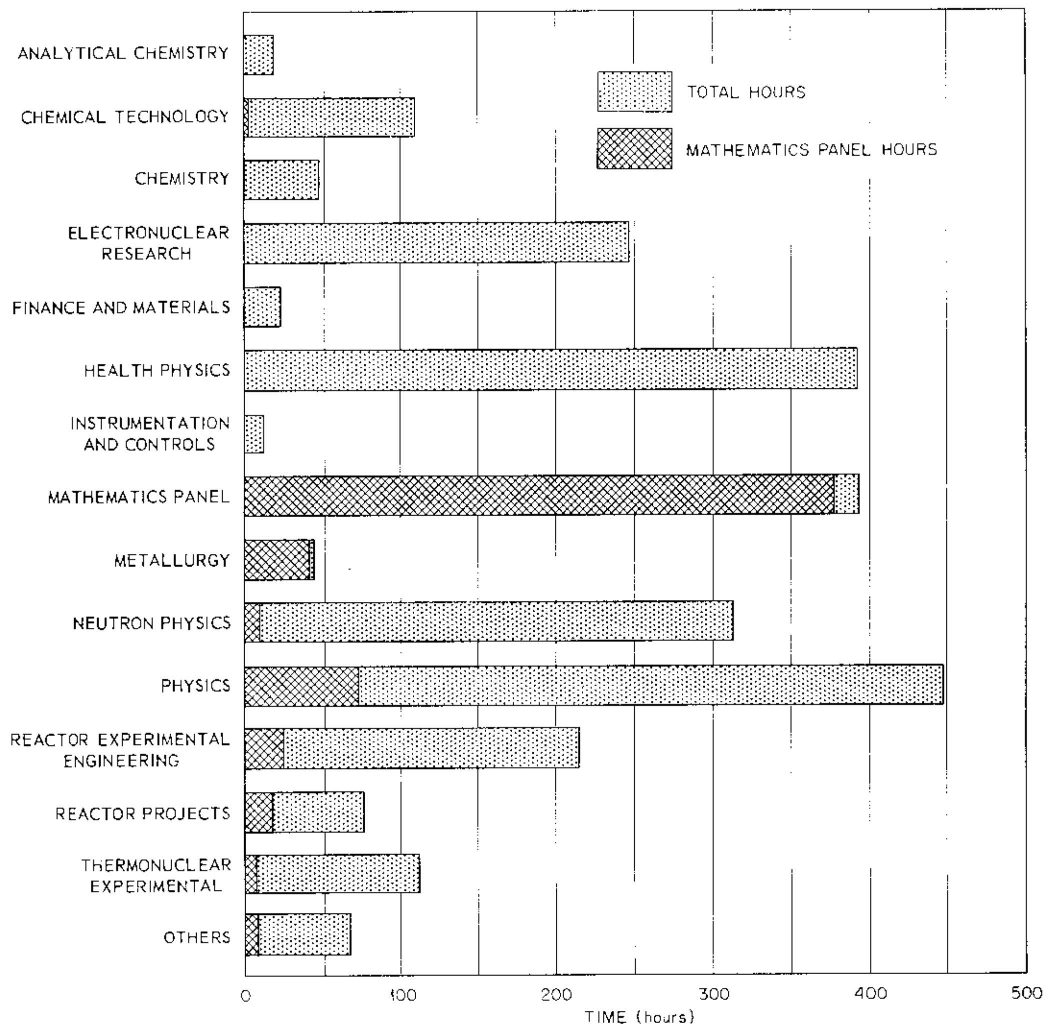

Fig. 1. Oracle Time Used by Divisions During Last Six Months of 1959.

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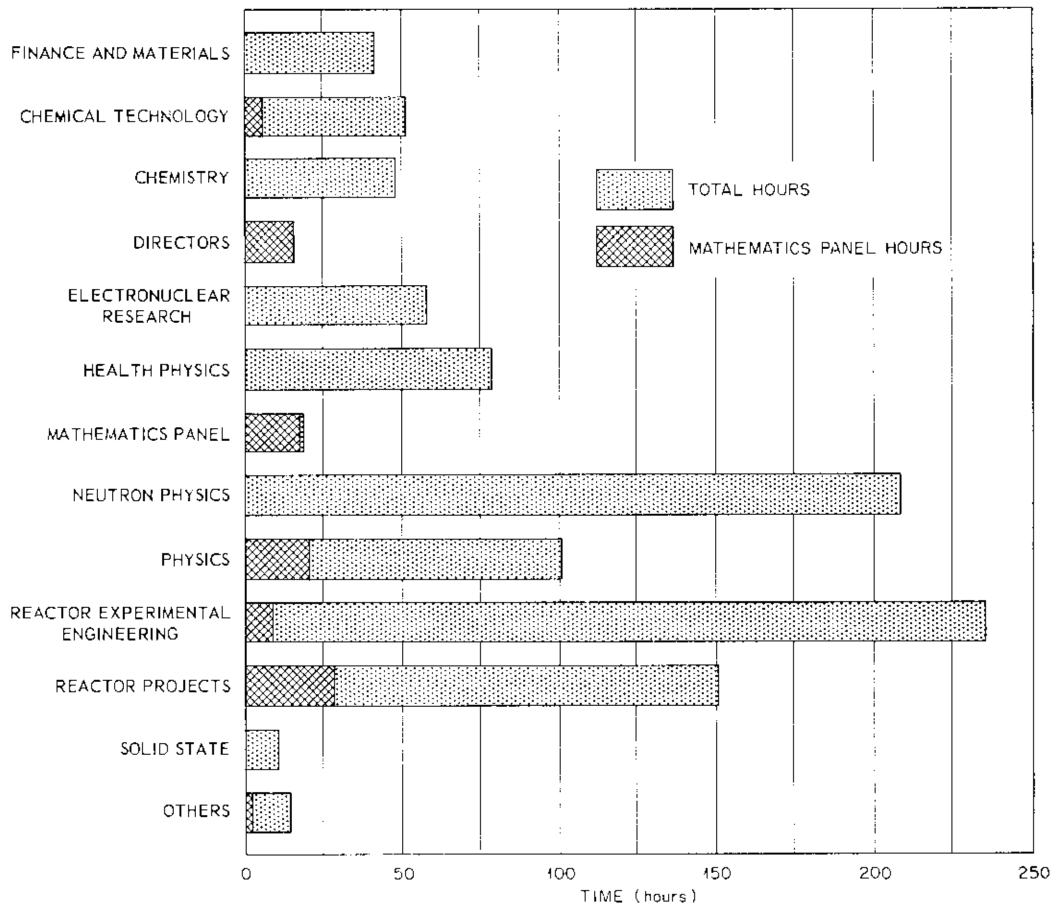

Fig. 2. IBM-704 Computer Time Used by Divisions During Last Six Months of 1959.

TRAINING AND SEMINARS

ORBIT Programming Course

J. Harrison

A series of six lectures were attended by 23 persons representing nine divisions. Eleven of the participants had no previous programming experience.

The lectures, which did not presuppose previous acquaintance with the Oracle, were of an elementary nature and contained many exercises and solutions. Emphasis was placed on the ease of programming directly from flow charts, and actual ORBIT programs were translated on the Oracle.

An operator ORBIT course was presented to all Oracle operators.

Numerical Analysis

C. T. Fike

A series of lectures, 2 on numerical integration and 12 on finite differences and interpolation, were attended by about 50 members of the Laboratory staff.

Statistical Problems of Radiation Measurement

D. A. Gardiner

Seven 2-hr lectures were given to students of the ORINS Radioisotopes Techniques School. These lectures began with the concept of the probability distribution. Then the evolution from the binomial distribution to the Poisson was made, and the use of the normal distribution as an approximation to the Poisson distribution was explained. By introducing the concepts of expectation and variance of linear forms, the students were shown how to describe the distributions of many of the functions useful in radiation measurement. Through the use of these distributions the students were able to construct confidence intervals for their functions and make simple tests of hypotheses. Formulas for the expectations and variances of products and quotients of Poisson variables were also briefly explained.

Oracle Coding Course

E. C. Long

Two beginning courses in coding, each stressing basic coding for the Oracle, were presented in ten 4-hr lectures to a total of 60 persons.

Desk Calculator Course

D. A. Gardiner A. W. Kimball E. Leach M. A. Kastenbaum

During July 1959 members of the Statistics Section conducted two courses in the use of desk calculators at the request of the Biology Division. Thirty-four members of the division participated.

Seminars

The following topics were presented at Statistics Section seminars:

D. A. Gardiner

"Some Remarks on Interlaboratory Testing," November 1959.

"An Iterative Least-Squares Procedure for Fitting the Sum of a Gaussian and an Exponential to Absorption Spectra," March 1959.

M. A. Kastenbaum

"The Analysis of Quantal Experiments Involving Two Treatments," December 1959.

"Causal Path Analysis," October 1959.

"An Application of Stochastic Processes to the Separation of Molecular Compounds by Countercurrent Dialysis," January 1959.

"Simultaneous Regression Equations in Experimentation," October 1958.

A. W. Kimball

"Measures of Association: Fact or Fiction?" November 1959.

"The Estimation of Mortality Intensities from Controlled Experiments," Part I, September 1959; Part II, October 1959.

"Defining and Measuring the Association Between Two Diseases," February 1959.

N. Dismuke

"Oracle and IBM-704 Codes of Interest to Statisticians," February 1959.

G. E. Albert, University of Tennessee

"Orthogonal Polynomials," January 1960.

"Information Theory in Statistics," March 1959.

"Extension of the Neyman-Pearson Theory of Tests to Discontinuous Variates," November 1958.

F. A. Gifford, U.S. Weather Bureau Office, Oak Ridge, Tennessee

"Some Statistical Problems in the Theory of Turbulent Diffusion," March 1959.

R. McMillan and E. Johnson, Statistical Services Department, Y-12

"Use of Statistics in Certification of Material," December 1959.

J. S. Olson, Health Physics Division

"Random and 'Contagious' Spatial Distributions of Organisms, with an Example of Monte Carlo Study of Colony Growth," April 1959.

G. S. Watson, Australian National University

"Cytoplasmic Inheritance," November 1958.

The following topics were presented at Mathematics Panel seminars:

A. C. Downing

"On the Convergence of Multiregion Reactor Calculations," December 1959.

A. A. Grau

"ORBIT Language and Compiler" (two lectures), February 1959.

The following topic was presented at a Biology Division seminar:

A. A. Grau

"Group Theory," December 1959.

LECTURES AND PAPERS

Carter, H. P., "Distributive Lattices Induced by Finite Groups," Southeastern Section, Mathematical Association of America, Johnson City, Tennessee, March 1959.

Gardiner, D. A., "Response Surface Methodology," Pensacola-Mobile Section of the American Society for Quality Control, Mobile, September 1958.

Gardiner, D. A., "Optimization of Removal of Radionuclides from Process Wastes," Mathematical Analyses of Atomic Energy Operations, USAEC, Washington, December 1959.

Gardiner, D. A., and S. E. Atta, "An Iterative Least Squares Procedure for Fitting the Sum of a Gaussian and an Exponential to Absorption Spectra," Institute of Mathematical Statistics and the Association for Computing Machinery, Cleveland, April 1959.

Grau, A. A., "Numerical Methods for Solving Polynomial Equations," University of Tennessee, Knoxville, December 1958; North Carolina State College, Raleigh, November 1958; University of South Carolina, Columbia, January 1959; Alabama Polytechnic Institute, Auburn, February 1959.

Grau, A. A., "Ternary Operations in Boolean Algebras and Lattices," University of Kentucky, Lexington, March 1959.

Householder, A. S., "On Certain Methods of the Factorization of Matrices," IBM Research Center, Ossining, New York, October 1958.

Householder, A. S., "A Class of Inclusion Theorems for Proper Values of Matrices," American Mathematical Society Annual Meeting, Philadelphia, January 1959.

Householder, A. S., "Inclusion and Exclusion Theorems for Characteristic Roots of Matrices," University of Tennessee, Knoxville, February 1959.

Householder, A. S., "Some Methods for Expanding the Characteristic Polynomial," Argonne National Laboratory, Lemont, Illinois, February 1959.

Householder, A. S., "International Cooperation in Computing," Mid-Southeastern Section Meeting, Association for Computing Machinery, Atlanta, May 1959.

Householder, A. S., "Some Remarks on the Inversion of Matrices," Institut für Angewandte Mathematik, Gutenberg University, Mainz, Germany, June 1959; British Computer Society, Cambridge University, Cambridge, England, June 1959.

Householder, A. S., "Moments and Characteristic Roots," International Conference on Information Processing, Paris, France, June 1959.

Householder, A. S., "Advanced Numerical Analysis," Summer Lectures, University of Michigan, Ann Arbor, June–July 1959.

Householder, A. S., "Numerical Analysis," Summer NSF Institute on Mathematics, Washington University, Topeka, July 1959.

Householder, A. S., "The Numerical Solution of Linear Equations" and "Life, Living, and Livelihood" (two lectures), Peabody National Science Foundation Institute, Nashville, July 1959.

Householder, A. S., "On Certain Iterative Methods for Solving Linear Systems," Office of Ordnance Research, Duke University, Durham, September 1959.

Householder, A. S., "Life, Living, and Livelihood," Central Association of Science and Mathematics Teachers, Chicago, November 1959.

Kastenbaum, M. A., "An Application of Stochastic Processes to the Separation of Moleculor Compounds by Countercurrent Dialysis," Johns Hopkins University, Baltimore, April 1959; Virginia Polytechnic Institute, Blacksburg, April 1959; Medical College of Virginia, Richmond, May 1959.

Kimball, A. W., "Estimation from Observations Having Specified but Unassigned Expectations in a Radiation Experiment with *Paramecium aurelia*," American Statistical Association and Biometric Society, Chicago, December 1958.

Kimball, A. W., "Methods for Fitting Multihit Survival Curves," University of Richmond, February 1959.

Kimball, A. W., "Approximote Linearization of the Incomplete Beta Function," Biometric Society and Institute of Mathematical Statistics, Pittsburgh, March 1959.

Kimball, A. W., "Errors of the Third Kind in Statistical Consulting," Fourth Conference on Design of Experiments, Army Quartermaster Research and Engineering Center, Notick, Massachusetts, October 1958.

PUBLICATIONS

A. S. Householder, "Generated Error in Rotational Tridiagonalization," *J. Assoc. Comp. Mach.* **5**, 335–38 (1958).

A. S. Householder, "Unitary Triangulorization of a Nonsymmetric Matrix," *J. Assoc. Comp. Mach.* **5**, 339–42 (1958).

A. S. Householder (with F. L. Bauer), "On Certain Methods for Expanding the Characteristic Polynomial," *Numerische Mathematik* **1**, 29–37 (1959).

A. S. Householder (with K. Fan), "A Note Concerning Positive Matrices and M-matrices," *Monatsh. Math.* **63**, 3 (1959).

A. S. Householder, "Dandelin, Lobacevskii, or Graeffe?" *Am. Math. Soc. Monthly* **66**, 464–66 (1959).

A. S. Householder, "Minimal Matrix Norms," *Monatsh. Math.* **63**, 344–50 (1959).

A. S. Householder (with F. L. Bauer), "Moments and Characteristic Roots," *Numerische Mathematik* **2**, 42–53 (1960).

A. S. Householder, *A Glossary for Numerical Analysis*, ORNL-2704 (Mar. 19, 1959).

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M. A. Kastenbaum and D. E. Lamphiear, "Calculation of Chi-Square to Test the No-Three-Factor Interaction Hypothesis," *Biometrics* **15**, 107–15 (1959).

M. A. Kastenbaum, "A Confidence Interval of the Abscissa of the Point of Interaction of Two Fitted Linear Regressions," *Biometrics* **15**, 323–24 (1959).

A. C. Upton, F. F. Wolff, J. Furth, and A. W. Kimball, "A Comparison of the Induction of Myeloid and Lymphoid Leukemia in X-Irradiated RF Mice," *Cancer Research* **18**, 842–48 (1958).

E. Novitski and A. W. Kimball, "Birth Order, Parental Ages, and Sex of Offspring," *Am. J. Human Genet.* **10**, 268–75 (1958).

A. W. Kimball, "Disease Incidence Estimation in Populations Subject to Multiple Causes of Death," *Bull. Intern. Statist. Inst.* **36**, 193–204 (1958).

J. Furth, A. C. Upton, and A. W. Kimball, "Late Pathologic Effects of Atomic Detonation and Their Pathogenesis," *Radiation Research*, Suppl. 1, 243–64 (1959).

A. W. Kimball and E. Leach, "Approximate Linearization of the Incomplete Beta Function," *Biometrika* **46**, 214–18 (1959).

A. A. Grau, M. Feliciano, G. C. Caldwell, E. L. Cooper, A. Culkowski, and J. Harrison, *Programmers Manual, ORACLE Binary Internal Translator* (*ORBIT*), *Algebraic Programming System for the ORACLE*, ORNL CF-59-9-20 (Sept. 22, 1959).

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ORNL-1435	Period Ending October 31, 1952
ORNL-1498	Period Ending January 31, 1953
ORNL-1588	Period Ending June 30, 1953
ORNL-1662	Period Ending December 31, 1953
ORNL-1751	Period Ending June 30, 1954
ORNL-1842	Period Ending December 31, 1954
ORNL-1928	Period Ending June 30, 1955
ORNL-2037	Period Ending December 31, 1955
ORNL-2134	Period Ending June 30, 1956
ORNL-2283	Period Ending February 28, 1957
ORNL-2652	Period Ending August 31, 1958