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THE EFFECTIVE RESONANCE INTEGRAL OF U^{238} METAL

L. Dresner

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The Effective Resonance Integral of U^{238} Metal

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ABSTRACT

The homogeneous effective resonance integral of U^{238} metal has been calculated to be 10.1 barns at $T = 300^{\circ}K$. This value agrees well with experiment.

The Effective Resonance Integral of U^{238} Metal

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INTRODUCTION

In a previous paper (1) the author calculated the effective resonance integral of homogeneous mixtures of moderator and U^{238} using the theory of Wigner et al. (2). In these calculations the effect of Doppler broadening of resonance lines was included exactly. A condition of validity of Wigner's theory is that the width of the resonance line be narrow compared to the average logarithmic energy loss per collision. In mixtures with considerable moderator scattering this condition is fairly well fulfilled. In the pure metal, however, the case may be otherwise, especially in the lowest resonances.* It is the purpose of this paper to investigate in detail the departures from Wigner's theory caused by the failure of the thin resonance assumption.

THEORY

Let us consider the slowing down of neutrons in an infinite, homogeneous medium in which the ratio of the scattering to the total cross section, η, is constant. The asymptotic solution to this problem has been given by Weinberg and Noderer (3). They give for the resonance escape probability to lethargy u, the expression

$$p = \exp(\lambda u) \qquad (1)$$

*I am indebted to R. Hellens for first bringing this point to my attention.

where λ satisfies the transcendental equation

$$1 + \lambda = \eta \frac{1 - \alpha^{1+\lambda}}{1 - \alpha} \tag{2}$$

and $\alpha = (A - 1/A + 1)^2$. A is the mass of the moderating nuclei. When $\eta = 1$, $\lambda = 0$; λ can be expanded as a power series in $1 - \eta$ around $\eta = 1$ as follows:

$$-\lambda = \frac{1 - \eta}{\xi} + \frac{\xi - \gamma}{\xi^2}(1 - \eta)^2 + \ldots \tag{3}$$

where $\xi = 1 + \frac{\alpha \ln \alpha}{1 - \alpha}$, and $\gamma = 1 - \frac{1}{2}\frac{\alpha(\ln \alpha)^2}{(1 - \alpha)\xi}$. Written in terms of the cross sections, λ is, to first order in Σ_a/Σ_t,

$$\lambda = -\frac{\Sigma_a}{\xi \Sigma_t} \tag{4}$$

and to second order

$$\lambda = -\frac{\Sigma_a}{\xi \Sigma_s + \gamma \Sigma_a} \tag{5}$$

These are, respectively, the Wigner and Goertzel-Greuling approximations to λ (4).

If η varies slowly with lethargy, i.e., if its fractional change in a lethargy interval $\ln\left(\frac{1}{\alpha}\right)$ is $<< 1$, so may λ. Then one can write as an approximate value of the resonance escape probability to lethargy u

$$p = \exp\left(\int_0^u \lambda(u')du'\right) \qquad (6)$$

where $\lambda(u')$ is related to $\eta(u')$ by Eq. (2). Equation (6) is valid irrespective of the value of η, so long as η and λ are slowly varying.*

Plotted in Fig. 1 is $\xi\lambda$ vs. η with λ calculated exactly, as well as with Eqs. (4) and (5) for A = 238. We see that for $\eta \gtrsim 0.5$ the Goertzel-Greuling approximation is entirely adequate, the error at $\eta = 0.5$ being 5%. For $\eta \gtrsim 0.8$ Wigner's approximation is quite good, the error being 7.5% for $\eta = 0.8$.

In the event that $\eta \geq 0.5$ for a particular resonance, so that the Goertzel-Greuling approximation at least is valid, we can immediately write down the value of the contribution to the effective resonance from that resonance. Following exactly the method and notation of Reference (1) we have

$$\left(\int \sigma_a \frac{dE}{E}\right)_{eff} = \frac{\sigma_p \Gamma_\gamma}{E}\left(\frac{\Gamma}{\Gamma_n + \epsilon\Gamma_\gamma}\right) J(\xi,\beta) \qquad (7)$$

except that now $\beta = \frac{\sigma_p}{\sigma_o}\left(\frac{\Gamma}{\Gamma_n + \epsilon\Gamma_\gamma}\right)$. ϵ is 1 in the Wigner approximation and γ/ξ ($\sim 2/3$ for A >> 1) in the Goertzel-Greuling approximation.

Finally, one more expression is useful. That is the contribution to the effective resonance integral in the Wigner or Goertzel-Greuling approximations from the portion of resonance line with the natural line shape for which $|E - E_o| \gtrsim C\left(\frac{1}{2}\Gamma\right)$. If $\sigma_o >> \sigma_p$, as is nearly always the case, this result is

*A similar expression has been given by Hurwitz. See AECD-3645, p. 370-1.

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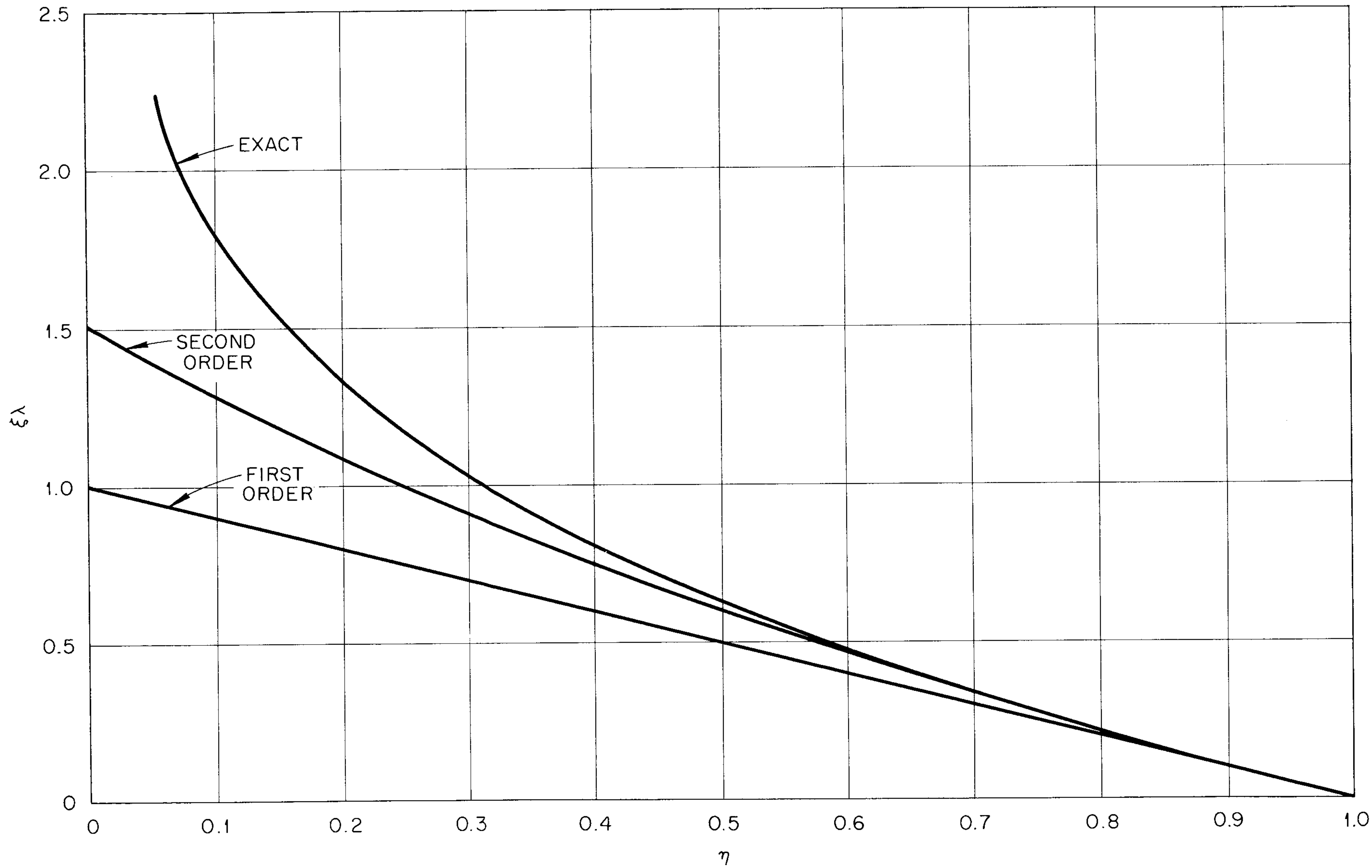

Fig. 1. $\xi\lambda$ vs. η in Various Approximations for $A = 238$.

$$\left(\int \sigma_a \frac{dE}{E}\right)'_{\text{eff}} = \sigma_p \cdot \left(\frac{\sigma_o}{\sigma_p}\right)^{1/2} \frac{\Gamma_\gamma}{E} \left(\frac{\Gamma}{\Gamma_n + \epsilon \Gamma_\gamma}\right)^{1/2} \times$$

$$\left\{\frac{\pi}{2} - \tan^{-1}\left[\left(\frac{\sigma_p}{\sigma_o}\right)^{1/2} C \left(\frac{\Gamma}{\Gamma_n + \epsilon \Gamma_\gamma}\right)^{1/2}\right]\right\} \qquad (8)$$

RESULTS

It will be necessary to treat each of the uranium resonances separately because they are not all handled in the same way. The resonance data used in these calculations is that reported in BNL-325 (5) except that Γ_γ is taken as .025 ev for all resonances. We begin with the resonance at 6.70 ev. Let us consider $\xi\lambda$ and η as functions of the departure in lethargy from exact resonance measured in units of $\ln\left(\frac{1}{\alpha}\right)$ at T = 300°K. The fractional change in both λ and η in one unit of $\ln\left(\frac{1}{\alpha}\right)$ in lethargy is small, never exceeding about 25%. At $\Delta u = \pm 12 \ln\left(\frac{1}{\alpha}\right)$ the Goertzel-Greuling approximation to is within 1% of the exact value; moreover, the Doppler broadened line shape, $\psi(x,t)$, obtained from the tables of Rose, et al., (6), is within 0.5% of the natural line shape, $(1 + x^2)^{-1}$. Here x is the departure from resonance in units of the half-width. Hence, the part of the integral in Eq. (6) within $\pm 12 \ln\left(\frac{1}{\alpha}\right)$ of exact resonance is calculated numerically, using the Doppler broadened line shape. The rest is obtained from Eq. (8). Using this method a contribution to the resonance integral of 3.9 barns is found at T = 300°K, compared to only 2.7 barns from Wigner's theory.

The fractional changes in λ and η for the 21.0 ev resonance are also small never exceeding about 35% in one unit of $\ln\left(\frac{1}{\alpha}\right)$ in lethargy. For $\Delta u = \pm 5 \ln\left(\frac{1}{\alpha}\right)$ the Goertzel-Greuling approximation to λ is accurate to less than 1%; also the Doppler broadened and natural line shapes agree within less than 1%. Hence, we again calculate the central part of the integral numerically and the tail with Eq. (8) at T = 300°K. The method of this paper gives 1.3 barns, compared to 1.0 barns from Wigner's theory.

Of the remaining resolved resonances, those at 81.6, 90, 118, 140, 166, 212, 242, 258, 278, 297, 368, and 418 ev were considered narrow enough to use Wigner's theory to calculate the contributions to the effective resonance integral. An index of narrowness is the ratio $\frac{\epsilon E}{\frac{1}{2}\Gamma}$ which is the value of x corresponding to one unit of $\ln\left(\frac{1}{\alpha}\right)$ in lethargy. In the resonances mentioned above this ratio was always of the order of 100 or more.

On the other hand, the resonances at 37.0, 66.5, 104, and 192 ev have smaller ratios, $\frac{\epsilon E}{\frac{1}{2}\Gamma}$. In these resonances the ratio Γ_n/Γ is never less than 0.5, so that $\eta \gtrsim 0.5$ always. Hence, the Goertzel-Greuling approximation was employed throughout. The contributions from these resonances are augmented by 7%, 10%, 4.5%, and 3%, respectively, over their values calculated by Wigner's theory. On this basis the contribution of all but the first two resolved resonances to the effective resonance integral is increased about 0.1 barns from 1.7 barns to 1.8 barns, about a 5% increase Since these resonances only contribute about 20% of the total effective resonance this small increase will affect the total by only about 1%.

The unresolved resonances at high energy also contribute about 20% of the total effective resonance integral. For them $\eta \geq \Gamma_n/\Gamma \gtrsim 0.65$, while $\frac{\epsilon E}{\frac{1}{2}\Gamma} \gtrsim 150$. Since the Wigner theory is valid for both η close to 1 and for large $\frac{\epsilon E}{\frac{1}{2}\Gamma}$ we shall make no correction to the contribution of the unresolved resonances. For $\eta \gtrsim .65$ the correction cannot exceed 15%; this will effect the total by about 3% at most and probably very much less because $\frac{\epsilon E}{\frac{1}{2}\Gamma} >> 1$. The unresolved resonances are accounted for as described in Reference (1).

CONCLUSIONS

The results of these calculations are summarized in Table 1. It can be seen that only the first two resonances are substantially affected; between them they account for most of the 1.6 barns difference between Wigner's theory and this work. This work recommends a value of 10.1 barns as the homogeneous effective resonance integral of U^{238} metal. While it is difficult to estimate the uncertainties in the various entries we have attempted to indicate their order of magnitude. The uncertainties attached to the first three entries as well as the fifth are based on reported uncertainties in experimental resonance and thermal cross section data; the uncertainty in the fourth entry is based on reasonable estimates of the average resonance parameters of the unresolved resonances. The uncertainty in the total is about 10%.

According to Macklin and Pomerance (7) the best experimental value of the effective resonance integral is 10.2 barns with an uncertainty of about 15%. The agreement of the computed value with experiment is excellent but probably fortuitous.

Table 1. Contributions to the Effective Resonance Integral of U^{238}

	Wigner's Theory	This Work	Estimated Error
6.7 ev resonance	2.7 barns	3.9 barns	(10%)
21.0 ev resonance	1.0 barns	1.3 barns	(10%)
Remaining resolved resonances	1.7 barns	1.8 barns	(10%)
Unresolved resonances	1.7 barns	1.7 barns	(20%)
"1/v" contribution	1.4 barns	1.4 barns	(1%)
Total	8.5 barns	10.1 barns	(10%)

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4. See, for example, AECD-3645, Vol. 1, p. 365 ff.

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