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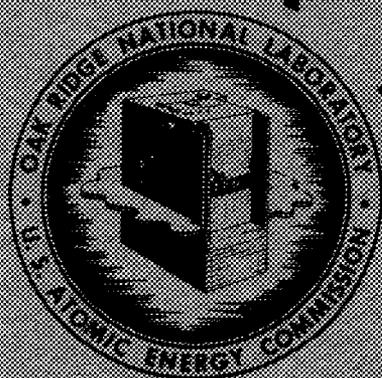
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A COMPARISON OF ONE-DIMENSIONAL
CRITICAL MASS COMPUTATIONS WITH
EXPERIMENTS FOR COMPLETELY
REFLECTED REACTORS

F. G. Prohammer



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A COMPARISON OF ONE-DIMENSIONAL CRITICAL MASS COMPUTATIONS
WITH EXPERIMENTS FOR COMPLETELY REFLECTED REACTORS

F. G. Prohammer

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Table of Contents

	Pg. No.
I. Introduction	1
II. Multigroup Machine Calculations	1
III. Conversion of Experimental Cylindrical Reactors to Equivalent Spheres	3
IV. Geometrical Comparison of Cylindrical and Spherical Reactors	4
V. Unreflected Spherical Reactor	8
App. A. Critical Assemblies Used in Comparison of Age-Diffusion and Goertzel-Selengut Programs	9
App. B. Critical Conditions for Water-Reflected Cylindrical Reactors -- Experiments	10
App. C. Nuclear Constituents and Critical Sizes of Spherical Reactors -- Calculations	13

A COMPARISON OF ONE-DIMENSIONAL CRITICAL MASS COMPUTATIONS
WITH EXPERIMENTS FOR COMPLETELY REFLECTED REACTORS

Abstract

Critical mass calculations for completely reflected cylindrical reactors would be greatly facilitated if a one-dimensional computational program used for spherical reactors could be applied. A comparison of cylindrical and spherical reactors has shown that the cylindrical reactors can be converted to equivalent spheres by a relationship which assumes that the buckling of a sphere is equal to that of the cylinder making allowance for reflector savings δ . The relationship is expressed as

$$\frac{\pi}{R + \delta} = \left[\left(\frac{2.4048}{r - \delta} \right)^2 + \left(\frac{\pi}{h + 2\delta} \right)^2 \right]^{1/2}$$

where R and r are the radii of the sphere and cylinder, respectively, and h is the cylinder height.

A COMPARISON OF ONE-DIMENSIONAL CRITICAL MASS COMPUTATIONS WITH EXPERIMENTS FOR COMPLETELY REFLECTED REACTORS

I. Introduction

The completely reflected cylindrical reactor which occurs frequently in reactor designs poses a difficult problem in critical mass calculations because the differential equation for the neutron density in such a system is not spatially separable. The problem is further complicated by the fact that most of the machine reactor computational programs presently available are one-dimensional. As a result, a short cylinder is usually converted into a sphere, and the critical mass of this equivalent sphere is then computed by machine calculations. It is the purpose of this analysis to find a suitable means for obtaining an equivalent sphere for machine computation of short, water-moderated, water-reflected cylinders.

II. Multigroup Machine Calculations

The multigroup machine calculational program available was a 30-group Coertzel-Selengut program known as Eyewash and coded for the Univac. Although this code has been discussed in detail by Alexander and Given,¹ it is briefly described here as applied to this program. The Eyewash program divides the energy region between 10 Mev and thermal into 30 groups. In each group the differential equation for the diffusion of neutrons is replaced by a difference equation. These difference equations are coupled by two slowing-down densities, one which treats hydrogen as a light element and another which treats all the remaining elements as heavy elements. The difference equations have only a radial spatial dependence, making this program suitable only for spherically symmetric reactors.

1. J. H. Alexander and N. D. Given, "A Machine Multigroup Calculation. The Eyewash Program for Univac," ORNL-1925 (Sept. 6, 1955).

A calculation is performed by supplying the number N of nuclei per cubic centimeter of each element present in each region and also the size of the lattice spacing for the difference equations, which is the same throughout all regions. The machine then delivers an answer which is the theoretical number of neutrons per fission ν_c necessary to make the given reactor critical. The multiplication constant k is taken as $k = 2.48/\nu_c$. Since the experimental reactor is not critical when k is different from unity, one of two things must be done. If the critical size of a reactor with fixed constituents is required, the size of the lattice spacing may be adjusted until k is as near unity as desired. On the other hand, if the critical concentration of a reactor of fixed size is required, the number of nuclei per cubic centimeter may be altered until k is as near unity as desired.

The machine code that Eyewash was designed to replace was another 30-group machine program known as Medusa and also coded for the Univac. Medusa differs from Eyewash primarily in that it uses only one slowing-down density, treating all elements as heavy elements. In this paper the Medusa program is referred to as the age-diffusion program and the Eyewash program as the Coertzel-Selengut program.

Because the age-diffusion program had been used previously for hydrogenous reactors, a few test calculations were made with both programs. The experiments of Thomas, Fox, and Callihan² provided excellent test reactors for the comparison of these two programs, since the critical assemblies were aqueous spheres with water reflectors. With both the size and concentrations fixed, a calculation of k provided a direct test of the validity of both programs. The values of k

2. J. T. Thomas, J. K. Fox, and Dixon Callihan, "A Direct Comparison of Some Nuclear Properties of U-233 and U-235," ORNL-1992 (Nov. 28, 1955).

are shown in Table 1, and the details of the reactors are given in Appendix A. The values of k for the Goertzel-Selengut code are very satisfying, and they, together with the k's for the age-diffusion code, provide a measure of the effect of treating hydrogen as a heavy element. In the remainder of this report only the Goertzel-Selengut code is used.

Table 1. Comparison of k Values Determined with Age-Diffusion and Goertzel-Selengut Programs

Test Reactor	k	
	Age-Diffusion	Goertzel-Selengut
1	1.130	0.992
2	1.133	0.992
3	1.148	1.007
4	1.152	1.001
5	- - -	1.010
6	- - -	1.015

III. Conversion of Experimental Cylindrical Reactors to Equivalent Spheres

The critical experiments of Beck, Callihan, Morfitt, and Murray³ provide ample information for the determination of a suitable criterion for obtaining an equivalent sphere for a short cylindrical reactor. The pertinent data for 76 of the experimental reactors are given in Appendix B. All of these experiments were performed with just critical cylindrical cores of water solutions of uranyl fluoride (UO_2F_2) contained in thin aluminum or stainless steel cans, which in turn were immersed in much larger cylinders of water. In this

3. C. K. Beck, A. D. Callihan, J. W. Morfitt, and R. L. Murray, "Critical Mass Studies, Part III," K-343 (April 19, 1949).

analysis the material of the cans could not be taken into account, thus it was ignored except for a grouping of results into aluminum-contained and stainless steel--contained reactors.

Among the 76 cylindrical reactors there were only 24 different critical concentrations. The critical sizes of corresponding spherical reactors with most of the 24 concentrations were computed with the Goertzel-Selengut program. The results in terms of the sphere radius R vs. concentration are plotted in Fig. 1, together with an experimental curve for spherical reactors.⁴ (The nuclear constituents and critical sizes of the converted spherical reactors for the 24 concentrations are tabulated in Appendix C.) From Fig. 1 a spherical reactor can be associated with each of the 76 cylindrical reactors in Appendix B through the common concentrations. The relationship between the geometric properties of cylinders and associated spheres is depicted by the curves marked "aluminum container" and "stainless steel container" in Fig. 2, in which D/d is plotted as a function of h/d (D = sphere diameter, d = cylinder diameter, h = cylinder height). The next step is to find a suitable criterion for duplicating as nearly as possible the two curves.

IV. Geometrical Comparison of Cylindrical and Spherical Reactors

In seeking the criterion, four methods for obtaining equivalent spheres for the cylindrical reactors were considered. In the first method a sphere with its volume equal to that of the original cylinder was chosen:

$$\frac{4}{3} \pi R^3 = \pi r^2 h$$

This relationship (labeled "equal volume" in Fig. 2) is a reasonably good fit up to an h/d of unity.

4. A. D. Callihan, J. W. Morfitt, and J. T. Thomas, "Small Thermal Homogeneous Critical Assemblies," P/834, Proceedings of International Conference at Geneva, Volume 5, Session 20A (United Nations, 1955).

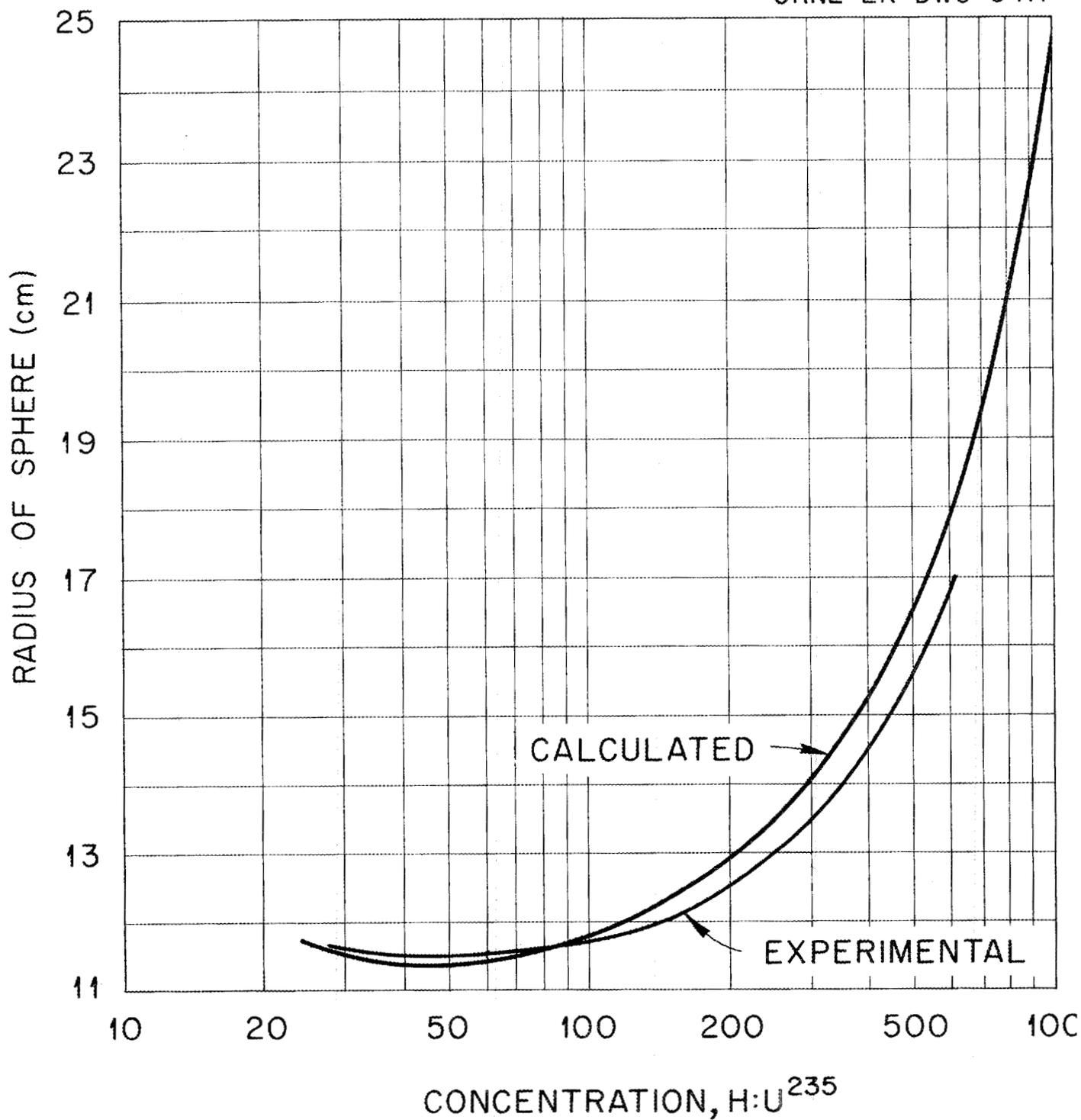


Fig. 1. Effect of Concentration on Critical Radius of Water-Reflected Spheres.

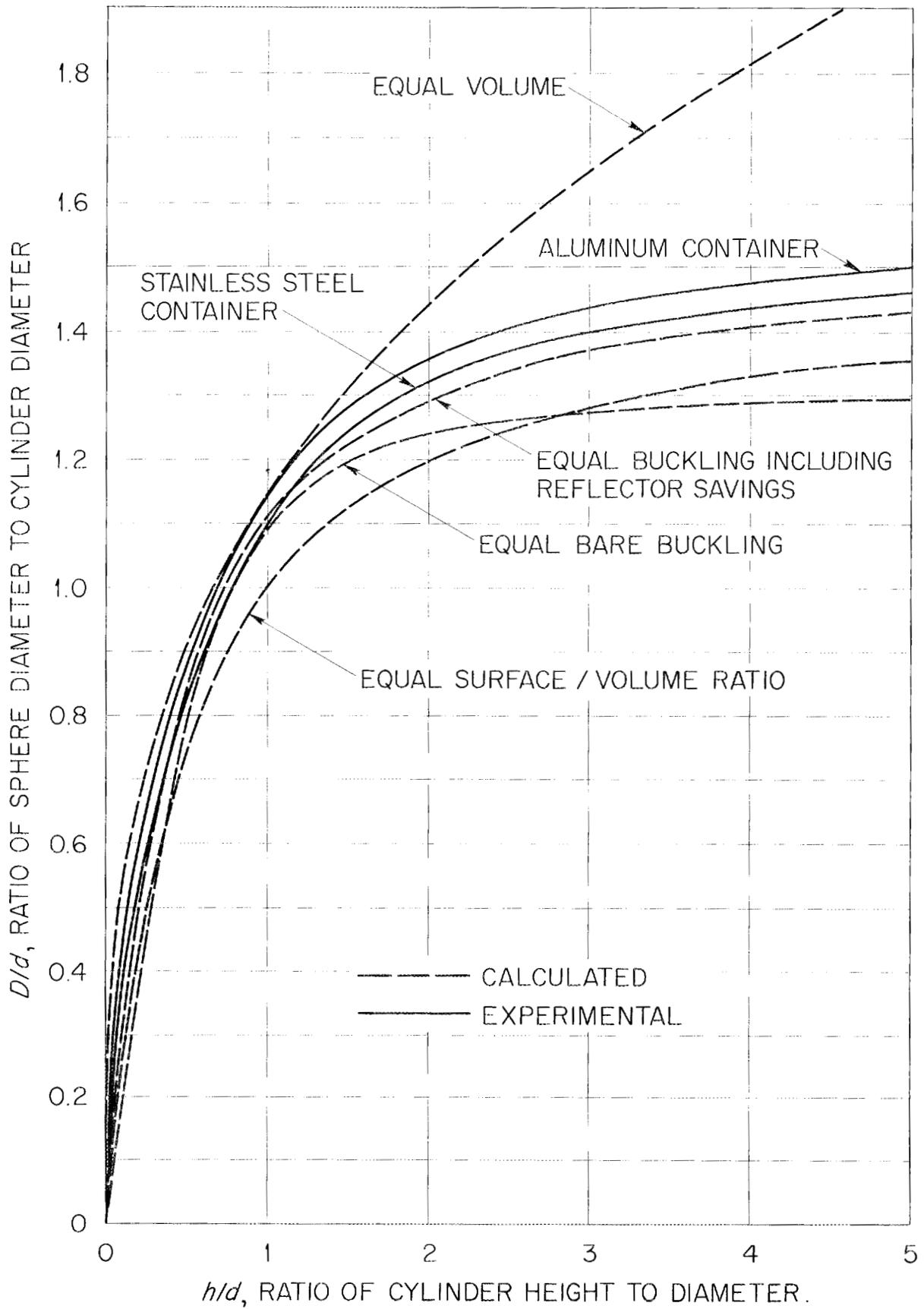


Fig. 2. Geometrical Comparison of Cylindrical and Spherical Reactors.

In the second method a sphere with its surface-to-volume ratio equal to that of the original cylinder was selected:

$$\frac{4\pi R^2}{\frac{4}{3}\pi R^3} = \frac{2\pi r^2 - 2\pi rh}{\pi r^2 h}$$

This relationship (labeled "equal surface-volume ratio" in Fig. 2) gives the poorest fit of the four methods. Better results might have been expected since the leakage of neutrons is proportional to the surface area while the production of neutrons is proportional to the volume.

A sphere with its buckling equal to that of the original cylinder was chosen in the third method:

$$\frac{\pi}{R} = \left[\left(\frac{2.4048}{r} \right)^2 + \left(\frac{\pi}{h} \right)^2 \right]^{1/2}$$

This relationship (labeled "equal bare buckling" in Fig. 2) gives fair values up to an h/d of unity.

Finally a fourth method was chosen in which the buckling of a sphere was again equal to that of the original cylinder but the reflector savings δ was also included:

$$\frac{\pi}{R + \delta} = \left[\left(\frac{2.4048}{r - \delta} \right)^2 + \left(\frac{\pi}{h + 2\delta} \right)^2 \right]^{1/2}$$

The reflector savings was chosen equal to the thermal-diffusion length in water, $L = 2.85$ cm. This relationship is labeled "equal buckling including reflector savings" in Fig. 2.

This fourth curve is by far the closest fit to the aluminum and stainless steel curves and consequently represents the criterion sought. In fact, if the reflector savings were increased, the last curve could be made to duplicate

very nearly either the aluminum or the stainless steel curve, yielding a different reflector savings for each.

V. Unreflected Spherical Reactor

In this analysis some mention should be made of unreflected reactors. The Goertzel-Selengut program is best suited for reflected reactors, especially those with infinite reflectors, for with an unreflected reactor a different extrapolation distance for each of the 30 groups really should be chosen in requiring the flux to be zero at the extrapolated boundary. Among the experiments with spherical critical assemblies, Thomas² performed an experiment with an unreflected sphere, which again provided the necessary experimental information. Since the size and concentration of this critical sphere were known, one over-all extrapolation distance could be chosen for all 30 groups and its value adjusted until the multiplication constant was very nearly unity. The Goertzel-Selengut computation showed this extrapolation distance to be approximately 2.8 cm, or very nearly equal to the thermal-diffusion length used for neutrons in water.

Appendix A

Critical Assemblies Used in Comparison of Age-Diffusion
and Goertzel-Selengut Programs*

Reactor	Sphere Diameter (cm)	Critical Temperature (°C)	Critical Volume (liters)	Fuel Concentration ($\frac{\text{g of U}^{235}}{\text{g of Solution}}$)	Density (g/cc)
1	26.4	27.5	9.660	0.08557	1.1118
2	26.4	39.5	9.675	0.08714	1.1143
3	26.4	74	9.713	0.09266	1.1234
4	26.4	85.5	9.726	0.0974	1.1268
5	32.0	54	17.042	0.04900	1.0639
6	32.0	64.5	17.049	0.04983	1.0651

*J. T. Thomas et al., ORNL-1992, op. cit., p. 8.

Appendix B

Critical Conditions of Water-Reflected Cylindrical Reactors -- Experimental*

Reactor No.	Cylinder Diameter (in.)	Concentration		Density (g/cc)	Cylinder Height (cm)
		H/U ²³⁵	$\frac{\text{g of U}^{235}}{\text{g of Solution}}$		
Aluminum-Contained					
1	6	52.9	0.293	1.566	70.9
2		58.8	0.2745	1.51	71.8
3	6-1/2	26.2	0.4179	1.98	44.5
4		52.9	0.293	1.566	39.2
5		56.7	0.281	1.51	40.4
6		119.0	0.1685	1.26	52.6
7	8	26.2	0.4179	1.98	21.5
8		29.9	0.394	1.926	20.7
9		52.9	0.293	1.566	19.5
10		58.8	0.2745	1.51	20.5
11		99.5	0.1924	1.32	22.4
12		192.0	0.1145	1.17	28.1
13		290.0	0.0801	1.10	40.1
14	10	52.9	0.293	1.566	13.4
15		328.7	0.715	1.101	22.4
16		499	0.0488	1.070	35.2
17	15	52.9	0.293	1.566	7.90
18		56.7	0.281	1.51	8.50
19		221.0	0.1014	1.14	11.30
20		499.0	0.0488	1.070	16.90
21		755.0	0.033	1.04	27.10
22		999.0	0.0252	1.03	44.30
Stainless Steel--Contained					
23	6-1/2	31.6	0.385	1.88	49.0
24		43.9	0.326	1.65	47.1
25		62.7	0.264	1.50	47.9
26		86.4	0.213	1.35	53.8
27		123.2	0.164	1.25	80

Appendix B (cont.)

Reactor No.	Cylinder Diameter (in.)	Concentration		Density (g/cc)	Cylinder Height (cm)
		H/U ²³⁵	$\frac{\text{g of U}^{235}}{\text{g of Solution}}$		
28	7	24.4	0.430	2.02	35.4
29		31.6	0.385	1.88	34.0
30		43.9	0.326	1.65	32.7
31		61.1	0.268	1.47	32.6
32		86.4	0.213	1.35	36.0
33		123.2	0.164	1.25	42.3
34		174	0.124	1.19	57.3
35	8	24.4	0.430	2.02	23.4
36		26.2	0.418	1.98	23.5
37		31.6	0.385	1.88	22.6
38		43.9	0.326	1.65	21.9
39		56.7	0.281	1.51	22.2
40		58.8	0.275	1.51	20.8
41		62.7	0.264	1.50	22.9
42		86.4	0.213	1.35	23.8
43		99.5	0.192	1.32	24.2
44		123.2	0.164	1.25	26.0
45		174	0.124	1.19	30.1
46		192	0.115	1.17	29.4
47		226	0.0995	1.15	36.3
48	320	0.0732	1.10	60.1	
49	9	24.4	0.430	2.02	18.4
50		31.6	0.385	1.88	18.1
51		43.9	0.326	1.65	17.8
52		62.7	0.264	1.50	18.0
53		86.4	0.213	1.35	18.7
54		123.2	0.164	1.25	19.9
55		174	0.124	1.19	22.2
56		226	0.0995	1.15	25.3
57		320	0.0732	1.10	33.0
58	10	24.4	0.430	2.02	15.3
59		31.6	0.385	1.88	15.3
60		43.9	0.326	1.65	14.9
61		62.7	0.264	1.50	15.2
62		86.4	0.213	1.35	15.4
63		123.2	0.164	1.25	16.8
64		174	0.124	1.19	18.1
65		226	0.0995	1.15	20.0
66		320	0.0732	1.10	25.0

Appendix B (cont.)

Reactor No.	Cylinder Diameter (in.)	Concentration		Density (g/cc)	Cylinder Height (cm)
		H/U ²³⁵	$\frac{\text{g of U}^{235}}{\text{g of Solution}}$		
67	12	62.6	0.264	1.49	12.3
68		174	0.124	1.19	14.9
69		226	0.0995	1.15	16.5
70		320	0.0732	1.10	18.5
71		499	0.0488	1.07	26.3
72		755	0.0330	1.04	48.7
73	15	56.7	0.281	1.51	10.1
74		221	0.1014	1.14	13.0
75		499	0.0488	1.07	20.0
76		755	0.0330	1.04	28.8

*This information is copied from Report K-343 (Ref. 3), p. 72.

Appendix C

Nuclear Constituents and Critical Sizes of Spherical Reactors -- Calculated

Table C-1. Nuclear Constituents of Spherical Reactors with Various Concentrations

Concentration, H/U ²³⁵	N* x 10 ²⁴			
	Uranium	Fluorine	Hydrogen	Oxygen
24.4	0.002384	0.004767	0.05441	0.03197
26.2	.004471	.004541	.05556	.03232
29.9	.002082	.004165	.05832	.03333
31.6	.001986	.003972	.05850	.03322
43.9	.001476	.002952	.06040	.03315
52.9	.001260	.002518	.06213	.03358
56.7	.001164	.002329	.06159	.03313
58.8	.00137	.002275	.06251	.03353
61.1	.001081	.002162	.06174	.03303
62.7	.001087	.002173	.06356	.03395
86.4	.0007891	.001578	.06360	.03338
99.5	.0006969	.001394	.06472	.03375
119.0	.0005826	.001165	.06458	.03345
123.2	.0005625	.001125	.06459	.03342
174	.0004049	.0008098	.06591	.03377
192.0	.0003676	.0007352	.06584	.03366
221.0	.0003172	.0006344	.06554	.03341
226	.0003140	.0006280	.06632	.03379
290.0	.0002418	.0004836	.06542	.03319
320	.0002210	.0004419	.06613	.03351
328.7	.0002160	.0004320	.06636	.03361
499	.0001433	.0002866	.06674	.03366
755.0	.00009418	.0001884	.06641	.03339
999.0	.00007123	.0001425	.06652	.03340

*N = number of nuclei per cubic centimeter of each element present in each region.

Table C-2. Critical Sizes of Spherical Reactors with Various Concentrations -- Aluminum Containers

Concentration, H/U ²³⁵	Reactor No.	Lattice Point Number at Boundary		Lattice Spacing,** Δr^* (cm)	ν_c^{**} (neutrons/fission)
		Of Core, B ₁ *	Of Reflector, B ₂ *		
26.2	3	15	30	0.7700	2.50
				0.7900	2.45
				0.7800	2.48
	7	15	30	0.7500	2.55
				0.7800	2.48
	29.9	8	15	30	0.7600
52.9	1	15	30	0.6881	2.69
				0.7500	2.50
				0.7700	2.45
58.8	2	15	30	0.7500	2.52
				0.7400	2.54
				0.7600	2.49
99.5	11	15	30	0.7800	2.49
119.0	6	15	30	0.7800	2.53
				0.8000	2.48
192.0	12	15	30	0.8300	2.53
				0.8600	2.46
221.0	19	15	30	0.8500	2.54
				0.8700	2.49
290.0	13	16	30	0.8400	2.57
				0.8700	2.50
328.7	15	16	30	0.8700	2.54
				0.9100	2.46
499	16	17	30	0.9500	2.52
				0.9700	2.48
755.0	21	18	30	1.1000	2.52
				1.1190	
999	22	19	30	1.2817	2.50
				1.2917	2.49

*B₁ x Δr = core radius; B₂ x Δr = reflector radius.

**The values of r plotted in Fig. 1 for $\nu_c = 2.48$ are interpolated values.

Table C-3. Critical Sizes of Spherical Reactors with Various Concentrations -- Stainless Steel Containers

Concentration, H/U ²³⁵	Reactor No.	Lattice Point Number at Boundary		Lattice Spacing,** Δr^* (cm)	ν_c^{**} (neutrons/fission)
		Of Core, B ₁ *	Of Reflector, B ₂ *		
24.4	28	15	30	0.7700	2.52
				0.7850	2.48
31.6	49	15	30	0.7200	2.66
				0.7600	2.54
				0.8000	2.45
31.6	23	15	30	0.78648	2.42
				0.7600	2.48
				0.7900	2.41
31.6	50	15	30	0.7700	2.46
				0.7880	2.42
				0.7500	2.51
31.6	59	15	30	0.7600	2.48
				0.7600	2.49
				0.7700	2.46
43.9	24	15	30	0.7500	2.51
				0.7700	2.46
43.9	51	15	30	0.7900	2.55
				0.8200	2.48
56.7	73	14	30	0.7900	2.55
				0.8200	2.48
61.1	31	15	30	0.7850	2.45
				0.7700	2.49
62.7	25	15	30	0.7600	2.47
				0.7500	2.50
62.7	52	15	30	0.7600	2.47
				0.7550	2.49
86.4	26	15	30	0.7600	2.53
				0.7850	2.47
86.4	53	15	30	0.7700	2.50
				0.7800	2.48

Table C-3 (cont)

Concentration, H/U^{235}	Reactor No.	Lattice Point Number at Boundary		Lattice Spacing,** $r^*(cm)$	ν_c^{**} (neutrons/fission)
		Of Core, B_1^*	Of Reflector, B_2^*		
123.2	27	15	30	0.8000	2.49
				0.8050	2.78
	54	15	30	0.7900	2.52
				0.8000	2.49
174	55	15	30	0.8300	2.49
221	74	15	30	0.8900	2.45
				0.8720	2.49
226	56	16	30	0.8100	2.51
				0.8250	2.47
320	57	16	30	0.8800	2.51
				0.8900	2.49
499	71	17	30	0.9000	2.48
				0.9700	
755	72	18	30	0.9600	1.90
				0.88	2.66
				1.1500	2.47
				1.1450	2.46
	76	18	30	1.1400	2.47

* $B_1 \times \Delta r$ = core radius; $B_2 \times \Delta r$ = reflector radius.

**The values of r plotted in Fig. 1 for $\nu_c = 2.48$ are interpolated values.