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# RADIATION FLUX TRANSFORMATION AS A FUNCTION OF DENSITY OF AN INFINITE MEDIUM WITH ANISOTROPIC POINT SOURCES

C. D. Zerby

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# RADIATION FLUX TRANSFORMATION AS A FUNCTION OF DENSITY OF AN INFINITE MEDIUM WITH ANISOTROPIC POINT SOURCES


## Abstract

The derivation is presented of a general equation used to transform the radiation current, flux, or dose rate from an anisotropic point source in an infinite homogeneous medium at a given density to the values at any density of the medium. These equations are then used to describe the method of transforming the Tower Shielding Facility data to apply at any altitude.


## Introduction

The purpose of this report is to present a method of transforming the Tower Shielding Facility dose rate data so as to apply at higher altitudes. In Section I of this report a rigorous method of transformation is given for the case of the dose or current obtained by a point detector where the radiation comes from a point source. In Section II the transformation of the TSF data is described using the transformations obtained in Section I. Section III describes the additional experiments needed at the TSF to make full use of the transformations.

## I. Transformation of Data from a Point Source

The steady state transport equation for an anisotropic point source located at the origin of the coordinate system in an infinite homogeneous medium takes the form

$$\vec{\Omega}\cdot\vec{\nabla}_r F(\vec{r},E,\vec{\Omega}) + N\sigma_t(E)F(\vec{r},E,\vec{\Omega}) = \int_{\Omega'}\int_{E'} N\sigma_s(E)f(E,E',\vec{\Omega},\vec{\Omega}')F(\vec{r},E',\vec{\Omega}')d\vec{\Omega}'dE'$$

$$+ S(E,\vec{\Omega})\,\frac{\delta(\vec{r})}{r^2}\,\frac{\delta(1-\vec{\Omega}\cdot\hat{r})}{2} \qquad (1)$$

where

$\vec{\Omega}$ = a unit vector,

$F(\vec{r},E,\vec{\Omega})$ = the number of neutrons per $cm^2$ per sec per unit energy at energy E per unit solid angle in the direction $\vec{\Omega}$ at position $\vec{r}$,

N = atomic density of the medium, $cm^{-3}$,

$\sigma_t(E)$ = total microscopic cross section, $cm^2$, per atom at neutron energy E,

$\sigma_s(E)$ = microscopic scattering cross section per atom at neutron energy E, $cm^2$,

$f(E,E',\vec{\Omega},\vec{\Omega}')dEd\Omega dE'd\Omega'$ = the probability of scattering of the neutrons from direction $\vec{\Omega}'$ in $d\vec{\Omega}'$ with energy E' in dE' to the direction $\vec{\Omega}$ in $d\vec{\Omega}$ with energy E in dE,

$S(E,\vec{\Omega})$ = source strength: neutrons per unit energy at energy E per unit solid angle in direction $\vec{\Omega}$, per sec,

$\hat{r}$ = unit vector in the direction $\vec{r}$.

All the terms in Eq. 1 have the units $cm^{-3}$ $sec^{-1}$ per unit energy.

Transformation of Eq. 1 to length measured in mean free paths (mfp) is easily accomplished by dividing through by $\left[N\sigma_t(E_o)\right]^3$, where $E_o$ is some arbitrary fixed energy $\left[N\sigma_t(E_o)\right.$ has units of $\left.cm^{-1}\right]$. Let $\vec{\lambda} = N\sigma_t(E_o)\vec{r}$ as the new position vector with length measured in mfp. Then Eq. 1 becomes

$$\vec{\Omega}\cdot\vec{\nabla}_\lambda G(\vec{\lambda},E,\vec{\Omega}) + \frac{\sigma_t(E)}{\sigma_t(E_o)} G(\vec{\lambda},E,\vec{\Omega}) = \int_{\Omega'}\int_{E'} \frac{\sigma_s(E)}{\sigma_t(E_o)} f(E,E',\vec{\Omega},\vec{\Omega}')G(\vec{\lambda},E',\vec{\Omega}')dE'd\Omega'$$

$$+ S(E,\vec{\Omega}) \frac{\delta(\vec{\lambda})}{\lambda^2} \frac{\delta(1-\vec{\Omega}\cdot\hat{\lambda})}{2\pi} \qquad (2)$$

where

$$G(\vec{\lambda},E\vec{\Omega}) = \frac{F\left[\vec{\lambda}(r),E,\vec{\Omega}\right]}{\left[N\sigma_t(E_o)\right]^2} = \text{neutrons per (mfp)}^2 \text{ per sec per unit energy}$$

at energy E per unit solid angle in direction $\vec{\Omega}$ at position $\vec{\lambda}$,

$$\vec{\nabla}_\lambda = \frac{\vec{\nabla}_r}{N\sigma_t(E_o)} = \hat{i}\,\frac{\partial}{\partial\lambda_x} = \hat{j}\,\frac{\partial}{\partial\lambda_y} + \hat{k}\,\frac{\partial}{\partial\lambda_z}\,.$$

The transformation of variables in the source term is easily recognized when it is remembered that the Dirac delta function has reciprocal units of the argument and $\hat{r} = \hat{\lambda}$. All terms in Eq. 2 now have the units of $(\text{mfp})^{-3}\ \text{sec}^{-1}$ per unit energy.

It is most significant that Eq. 2 is independent of the atomic density. Physically this means that the neutron current at position $\vec{\lambda}$ in units given to the function $G(\vec{\lambda},E,\vec{\Omega})$ is independent of atomic density of the transporting media (other than through the dependence of $\vec{\lambda}$).

In the way of an example: let two experiments be done with the same point source in the same infinite medium with the exception that in the first experiment the atomic density of the media is $N_1$ and in the second experiment the density is $N_2$. Now if the neutron current is measured in each experiment at a position such that

$$\vec{\lambda}_1 = \vec{\lambda}_2 \tag{3}$$

then

$$G_1(\vec{\lambda}_1,E,\vec{\Omega}) = G_2(\vec{\lambda}_2,E,\vec{\Omega}) \tag{4}$$

Equations 3 and 4 are reduced to conventional units by substituting in the

definitions for $\vec{\lambda}$ and G. Thus Eq. 3 becomes

$$N_1\sigma_t(E_o)\vec{r}_1 = N_2\sigma_t(E_o)\vec{r}_2 \tag{5}$$

or

$$r_2 = \frac{N_1}{N_2}\ r_1 = \frac{\rho_1}{\rho_2}\ \vec{r}_1 \tag{6}$$

where $\rho_1$ and $\rho_2$ are the densities of the media in the first and second experiment, respectively. From Eq. 4

$$\frac{F_1(\vec{r}_1,E,\vec{\Omega})}{\left[N_1\sigma_t(E_o)\right]^2} = \frac{F_2(\vec{r}_2,E,\vec{\Omega})}{\left[N_2\sigma_t(E_o)\right]^2} \tag{7}$$

or

$$F_2(\vec{r}_2,E,\vec{\Omega}) = \frac{N_2^{\ 2}}{N_1^{\ 2}}F_1(\vec{r}_1,E,\vec{\Omega}) = \frac{\rho_2^{\ 2}}{\rho_1^{\ 2}}F_1(\vec{r}_1,E,\vec{\Omega}) \tag{8}$$

Equations 6 and 8 are best combined for clarity in the form

$$F_{\rho_2}(\vec{r}_2 = \frac{\rho_1}{\rho_2}\vec{r}_1,E,\vec{\Omega}) = \frac{\rho_2^{\ 2}}{\rho_1^{\ 2}}F_{\rho_1}(\vec{r}_1,E,\vec{\Omega}) = \frac{r_1^{\ 2}}{r_2^{\ 2}}F_{\rho_1}(\vec{r}_1,E,\vec{\Omega}) \tag{9}$$

The practical use of Eq. 9 is made in the case where the neutron flux (or tissue dose rate) is desired along a particular direction from the source but for different densities of medium. The flux (or dose rate) is measured at various distances $r_1$ along the axis in the medium with density $\rho_1$, then the flux (or dose rate) at any other density and source-detector separation distance is obtained by the proper integration of Eq. 9. The flux is obtained by

$$\phi_{\rho_2}(\vec{r}_2 = \frac{\rho_1}{\rho_2}\vec{r}_1, E) = \frac{r_1^2}{r_2^2}\phi_{\rho_1}(\vec{r}_1, E) \tag{10}$$

where $\phi(\vec{r},E)$ is neutron flux (neutrons per $cm^2$ per sec per unit energy at position $\vec{r}$). In the case of dose rate, integration of Eq. 9 gives

$$D_{\rho_2}(\vec{r}_2 = \frac{\rho_1}{\rho_2}\vec{r}_1) = \frac{r_1^2}{r_2^2}D_{\rho_1}(\vec{r}_1) \tag{11}$$

where $D(\vec{r})$ is the tissue dose rate at position $\vec{r}$.

In the above, specific reference was made to neutrons; however, the term photons could equally well have been used. Therefore Eqs. 9, 10, and 11 also apply to gamma radiation.

It should be pointed out that the three equations (Eqs. 9, 10, and 11) hold rigorously for mixtures as long as the weight percentages of the various elements remain the same when the density changes.

The equations (Eqs. 9, 10, and 11) hold only in the case of steady state. There appears to be no simple transformation relations for those cases where the source is time dependent.

## II. Transformation of the Data from the Tower Shielding Facility

The above development was idealized because of the simplicity in geometry, making possible a rigorous development. Nevertheless, the results are of considerable importance because of the many calculations which use the identical geometry.

The TSF geometry is more complicated than that described above. It consists of a cylindrical reactor tank 12 ft in diameter and 6 ft high with

a hemispherical bottom and a detector tank which is a 5-ft cube located at some distance from the reactor tank. Both tanks are filled with water and are suspended in the air. The radiation field is established by placing the reactor near the inside surface of the reactor tank so a beam of radiation is concentrated in a particular direction with respect to the source-detector axis. Measurements are usually made with a detector located inside the detector tank and on the normal at the midpoint of one surface.

For purposes of discussion here consider both tanks to be suspended in a body of air of infinite extent. Actually, this is not quite true because of the influence of the ground on the radiation field; however, it is presumed that the measurements can be corrected to remove ground effects.

For the case of the TSF, the density-independent transport equation takes the form

$$\vec{\Omega}\cdot\vec{\nabla}_{\lambda}G(\vec{\lambda},E,\vec{\Omega}) + \frac{K(\vec{\lambda})\sigma_t(\vec{\lambda},E)}{\sigma_t(E_o)}\,G(\vec{\lambda},E,\vec{\Omega})$$

$$= \int_{\Omega'}\int_{E'} \frac{K(\vec{\lambda})\sigma_s(\vec{\lambda},E)}{\sigma_t(E_o)}\, f(E,E',\vec{\Omega},\vec{\Omega}')G(\vec{\lambda},E',\vec{\Omega}')dE'd\Omega' + S(\vec{\lambda},E,\vec{\Omega}) \qquad (12)$$

where

$S(\vec{\lambda},E,\vec{\Omega})$ = source strength per $(\text{mfp})^3$ per sec per unit energy at energy E per unit solid angle in direction $\vec{\Omega}$

= 0 for $\vec{\lambda}$ outside of the reactor,

$\sigma(\vec{\lambda},E)$ = microscopic cross section per atom at neutron energy E and at position $\vec{\lambda}$, $cm^2$,

$K(\vec{\lambda})$ = variable that enters because of the change of atomic type and density from inside to outside the tanks

= 1 for $\vec{\lambda}$ outside the tanks.

Equation 12 is independent of atomic density when all dimensions are measured in units of mfp including the dimensions of the tanks. Physically this means that two experiments give the same value of $G(\vec{\lambda},E,\hat{\Omega})$, provided all the dimensions involved are identical when measured in units of mfp.

If the TSF data are transformed by Eqs. 9, 10, and 11, the dimensions of the tanks are also transformed by an equation similar to Eq. 6, that is

$$x_2 = \frac{\rho_1}{\rho_2} x_1 \tag{13}$$

where $x_1$ is any tank dimension. Of course this is not the desired transformation property because the tanks would not change dimensions with change in air density; however, all is not completely lost when the situation is analyzed qualitatively. The question that needs answering is under what condition is the use of Eqs. 9, 10, and 11 significantly affected when constant tank dimensions are to be maintained (as opposed to those of Eq. 13) with a change in air density? An insight to the answer can be had by noting the dimensions of the respective tanks in units of mfp. Table 1 gives the dimensions at several energies at S. T. P. conditions for both gamma rays and neutrons. In the case of the neutrons an average cross section was used.

In all cases except at low energy the dimensions of the tanks are so small they appear as points in the media; therefore, one would expect the

distribution of 1st, 2nd, 3rd, etc. collisions to be very nearly the same as for a point source in the infinite media except in the case where the radiation beam is subtended by the detector tank. In all cases the major contributions to the dose measured in the detector tank comes from particles which have made one, two, or three scatterings in the air.[1] An exception to this is the case in which the radiation beam is directed close to 180 deg with respect to the source-detector axis. In that case the contribution of at least the first-scattered radiation will be reduced depending on the size of the reactor tank. Thus in most cases the size of the tanks will not affect the intensity of the radiation coming into the detector tank.

One additional comment should be made about the influence of the size of the detector tank: from measurements of the relaxation length of the radiation in the detector tank it appears that detector measurements made near the surface and near the center of one face of the reactor tank are not influenced by radiation entering the tank from the other surfaces. From this it is inferred that changes in the detector tank size would not alter the detector measurements provided the change did not influence the air-scattered flux to any great extent.

The above qualitative arguments can be summed up as follows: when the energy distribution of the source radiation is predominantly above 0.1 Mev, the TSF data which is obtained with the radiation beam pointed in a direction that misses the detector tank and yet does not approach angles of 180 deg can be transformed to apply at various altitudes by using Eqs. 9, 10, and 11.

1. This is inferred from preliminary flux calculations using the Monte Carlo method of calculation.

The transformation of a measurement obtained with the beam at 180 deg will give an underestimate of the measurements at higher altitude and an overestimate at lower altitude. This presents an experimental limitation which is difficult to remove and makes shield shaping somewhat doubtful in the small region about the 180 deg direction.

## III. Experiments at the Tower Required to Make Use of the Transformations

To use the transformation given in Eq. 11, the data already obtained at the TSF for one separation distance should also be obtained at several shorter separation distances so the data could be interpolated to obtain dose at higher altitudes.

Suppose the dose at 30,000 ft is required for a separation distance $r_2 = 64$ ft. The separation distance $r_1$ required for measurement at the altitude of the TSF ($\sim$1,000 ft) is

$$r_1 = \frac{\rho_2}{\rho_1} r_2 = \frac{0.00456 \times 64}{0.01175} = 24.8 \text{ ft}$$

At the separation distance of 24.8 ft the minimum angle at which the beam can point with respect to the source-detector axis and still miss the detector tank (and have the transformations hold) is approximately 6 deg for a monodirectional beam. The angle will be somewhat larger for the distributed beam obtained at the TSF. The situation is worse for transformations to still higher altitudes since the separation distance becomes smaller and the minimum angle becomes larger. One solution to this problem would be a smaller detector tank.

When using the transformations, great care should be taken to select the correct separation distance. At present it is measured from the center of the reactor tank to the back face of the detector tank. The correct separation distance to be used in the transformations is indicated on Fig. 1 and is selected so the distance B to C will be equal to the radius of the reactor tank after the transformation. The relation between the two separation distances is established if

$r_1'$ = separation distance as it is measured now,

$r_1$ = separation distance to be used in the transformations,

$\theta$ = angle at which the radiation beam points with respect to the source-detector axis,

$\ell$ = one half the side dimension of the detector tank,

$x$ = radius of the reactor tank.

Then the correct value of $r_1$ for measurements at the side faces of the detector tank is

$$r_1 = r_1' - x\left(1 - \frac{\rho_2}{\rho_1}\right)\cos\theta + \ell \qquad (14)$$

In conclusion it should be pointed out that the transformation (Eq. 10) applies at the surface D to E shown in Fig. 1 and, since D to E is a short distance in mfp for the higher energy particles, the flux will not vary in magnitude to any extent over the surface. Now, at a given energy E and distance y (shown in Fig. 1), the reading made by the detector is directly proportional to the flux on the surface D to E. Therefore, if the flux is transformed by Eq. 10 so is the reading made by the detector but the distance y remains <u>fixed</u>.

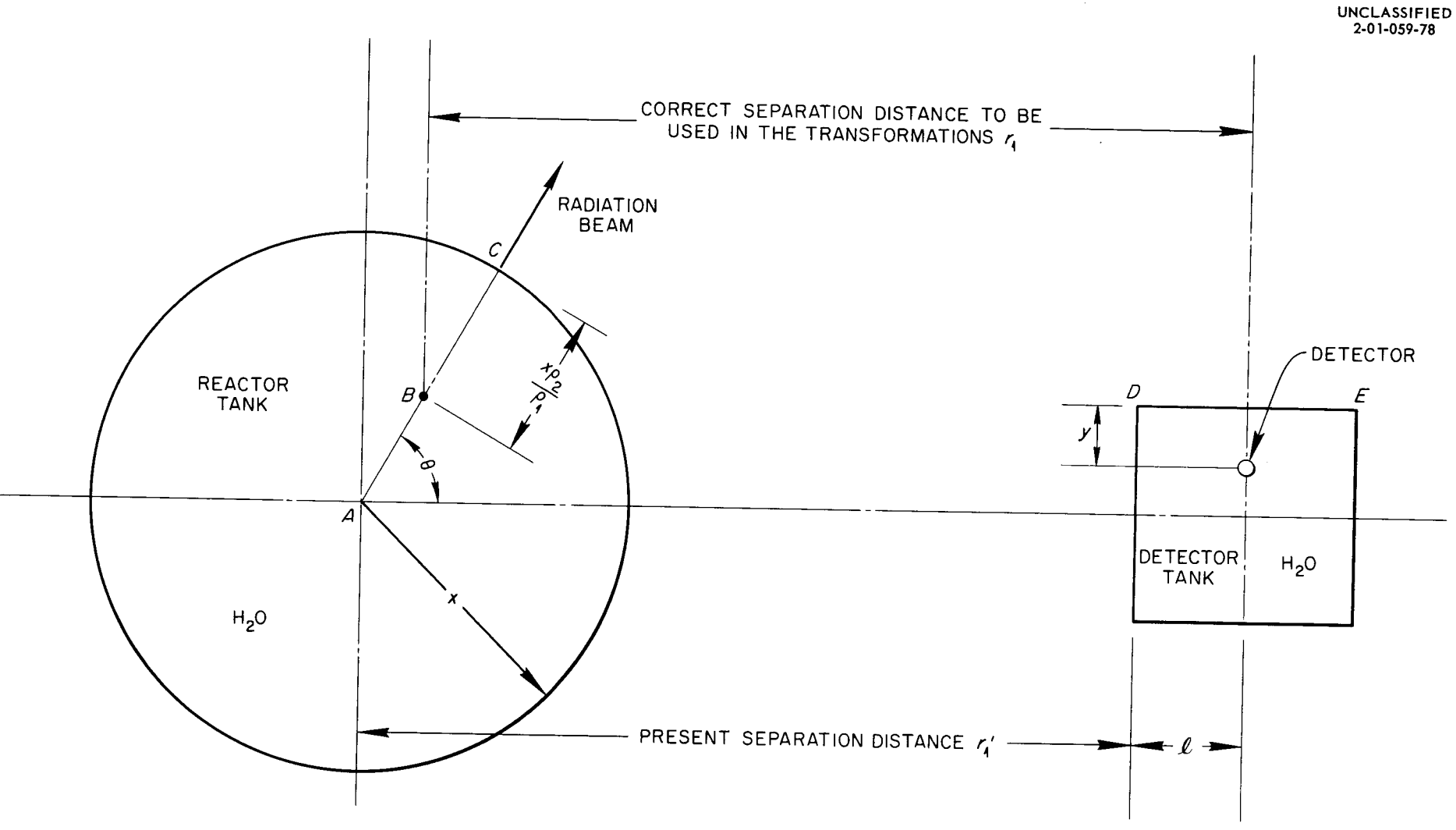


Fig. 1. Top View of Reactor Tank and Detector Tank at the Tower Shielding Facility.

Table 1. Dimensions of Reactor and Detector Tanks

| Energy Mev | Diameter of Reactor Tank (mfp) | | Side Dimensions of Detector Tank (mfp) | |
|---|---|---|---|---|
| | Gamma Rays | Neutrons | Gamma Rays | Neutrons |
| 10 | 0.010 | 0.025 | 0.004 | 0.011 |
| 5 | 0.013 | 0.028 | 0.005 | 0.012 |
| 1 | 0.030 | 0.046 | 0.013 | 0.019 |
| 0.5 | 0.041 | 0.068 | 0.017 | 0.028 |
| 0.1 | 0.073 | 0.085 | 0.030 | 0.035 |
| 0.01 | 1.804 | 0.1049 | 0.752 | 0.044 |