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Physics

ANGULAR CORRELATIONS IN ORBITAL CAPTURE

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ANGULAR CORRELATIONS IN ORBITAL CAPTURE

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Introduction

Observation of the lifetime and spectrum of a beta-transition does not usually suffice to determine the difference in angular momentum between the parent and daughter nuclei. The angular momentum change is even harder to ascertain in orbital capture, for which the transition energy is much less readily available. When beta-decay is followed by a gamma-transition, the beta-gamma angular correlation can help determine angular momentum values. An experimentally much more difficult angular correlation that can be observed in beta-decay is that between the electron and the recoiling nucleus. In principle, angular correlations exist that involve the inner bremsstrahlung connected with the electron emission, but they are experimentally unobservable in the presence of the background of external bremsstrahlung. In the case of orbital capture, there are no electrons emitted, but other radiations are present - X-rays associated with the transition of an outer atomic electron into the hole in an inner orbit left by the captured electron, and inner bremsstrahlung. It is the purpose of this report to investigate all the angular correlations that exist in connection with the orbital capture process, and to inquire as to the possibility of obtaining nuclear information from them. The following angular correlations between electromagnetic radiations are present*:

1) X-ray and following gamma-ray;

2) inner bremsstrahlung photon and following gamma-ray;

3) X-ray and inner bremsstrahlung photon.

This investigation was prompted by an experimental attempt in this laboratory to measure the second process.[1]

1) X-ray - Gamma-ray

This process differs in one respect from any previously investigated: While all cases dealt with involved either only nuclear radiations or only atomic radiations, here we link an atomic radiation with a nuclear radiation. Nevertheless, a correlation does exist in principle because the angular momentum of the captured electron (final state of the X-ray) is related to the angular momentum of the nucleus after capture (initial state of the gamma-ray) by the angular momentum of the beta-decay field.

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*Also to be found is a correlation between the recoiling nucleus and a gamma-ray following the capture process. Inasmuch as the only particle directly emitted in orbital capture is the neutrino, the nucleus acquires a recoil momentum equal in magnitude and opposite in direction to that of the neutrino. This recoil can be detected, though the experiment is extremely difficult. The correlation is the analogue of the beta-gamma correlation in electron emission. Anisotropy does not occur unless we go to a neutrino angular momentum greater than one-half; this means taking a higher term in the retardation expansion, with a large resultant loss in intensity. The experiment does not now appear feasible.

[1] E. D. Klema, private communication.

Let

j_1, μ_1 = total angular momentum and z-component thereof for atomic state from which X-ray originates.

L_X, M_X = total angular momentum and z-component thereof for X-ray proper.

j_2, μ_2 = total angular momentum and z-component thereof for captured electron (state to which X-ray goes)

L_β, M_β = total angular momentum and z-component thereof for beta-decay field.

j', μ' = total angular momentum and z-component thereof for neutrino.

J_1, m_1 = total angular momentum and z-component thereof for nucleus before capture.

J_2, m_2 = total angular momentum and z-component thereof for nucleus after capture(from which γ-ray originates).

L_γ, M_γ = total angular momentum and z-component thereof for gamma-ray.

J_3, m_3 = total angular momentum and z-component thereof for final nuclear state.

The Hamiltonian for the orbital capture process is written as the contraction of two tensors of equal rank L_β, one of which, A, contains the dependence on the leptons, the other, B, the dependence on the nuclear quantum numbers. Making use of the Wigner-Eckart theorem to separate out the geometrical factors (magnetic quantum number dependence), the matrix element of the Hamiltonian can be expressed as

$$(j'\mu' J_2 m_2 \mid H_\beta \mid j_2 \mu_2 J_1 m_1) = (j' \| A_{L_\beta} \| j_2)(j_2 \| B_{L_\beta} \| j_1) \sum_{M_\beta} (-)^{M_\beta} C^{j' L_\beta j_2}_{\mu' M_\beta \mu_2} C^{J_2 L_\beta J_1}_{m_2 -M_\beta m_1}$$

The Clebsch-Gordan coefficients vanish unless

$$\mu' + M_\beta = \mu_2 \qquad \text{and} \qquad m_2 - M_\beta = m_1$$

so that for given μ_2, m_2, and M_β the quantum numbers μ' and m_1 are determined, and the product of Clebsch-Gordan coefficients

$$\sum_{M_\beta} (-)^{M_\beta} C^{j' L_\beta j_2}_{\mu' M_\beta \mu_2} C^{J_2 L_\beta J_1}_{m_2 - M_\beta m_1} = f(j', j_2, J_1, J_2, L_\beta; m_2, \mu_2)$$

connects m_2 and μ_2.

Writing down the matrix elements for the X-ray and γ-ray and summing in the usual manner[2], we obtain a correlation function

$$W \sim \sum_{\nu \text{ even}} C^{L_X L_X \nu}_{1\ -1} C^{L_\gamma L_\gamma \nu}_{1\ -1} P_\nu (\cos\theta)$$

$$W(j_2 j_2 L_X L_X; \nu j_1)\, W(j_2 j_2 L_\beta L_\beta; \nu j')\, W(J_2 J_2 L_\gamma L_\gamma; \nu J_3)\, W(J_2 J_2 L_\beta L_\beta; \nu J_1)$$

The crucial selection rule in this expression is the condition $\Delta(j_2 j_2 \nu)$ for the non-vanishing of the first two Racah coefficients. It states that the angular correlation will be isotropic unless the captured electron has $j_2 > \frac{1}{2}$. Unfortunately, orbital capture ordinarily occurs predominantly with $j_2 = \frac{1}{2}$ electrons, $j_2 > \frac{1}{2}$ electrons contributing appreciably only for very low energy transitions with $L_\beta \geqslant 2$.[3] Thus, while the correlation does exist, its detection is prohibitively difficult in practice.

2) Inner Bremsstrahlung Photon - Gamma-ray

The inner bremsstrahlung process can be schematized as radiation

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[2]L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. 25, 729 (1953).

[3]H. Brysk and M. E. Rose, ORNL 1830 (1955).

of a photon by an atomic electron which proceeds to a virtual state, followed by capture from the virtual state.*

Case (2) looks like case (1), with the photon replacing the X-ray, but there is one salient difference: the intermediate state in the correlation chain is now a virtual state. Since the virtual state is not observable, we must sum over all such states, and sum coherently. Instead of

$$\sum_{\mu_2} \left| \left(j_2 \mu_2 \middle| H_{L_X}^{M_X} \middle| j_1 \mu_1 \right) \left(j' \mu' \middle| A_{L_\beta}^{M_\beta} \middle| j_2 \mu_2 \right) \right|^2$$

we now have

$$\left| \sum_{E_2, j_2, \mu_2} \frac{\left(j_2 \mu_2 \middle| H_{L_P}^{M_P} \middle| j_1 \mu_1 \right) \left(j' \mu' \middle| A_{L_\beta}^{M_\beta} \middle| j_2 \mu_2 \right)}{E_1 - E_2} \right|^2$$

so that there will be interference terms between virtual states with different quantum numbers (unprimed and primed), and the correlation function will contain a sum over virtual states which makes it depend on relative magnitudes of matrix elements:

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*The alternative path - emission of a positron into a virtual state, followed by annihilation of the positron with an atomic electron giving rise to a photon - can be taken into account simply by including the negative energy states in the sum over virtual states. The reversal of the time scale in the alternative path is inconsequential.

$$W \sim \sum_{E_2, E_2', j_2, j_2'} (-)^{j_2+j_2'} \sqrt{(2j_2+1)(2j_2'+1)} \; \frac{(j_2 \| H_{L_P} \| j_1)(j' \| A_{L_\beta} \| j_2)}{E_1 - E_2}$$

$$\frac{(j_2' \| H_{L_P} \| j_1)^* \, (j' \| A_{L_\beta} \| j_2')^*}{E_1 - E_2'}$$

$$\sum_{\nu \text{ even}} C_{1\ -1}^{L_P\, L_P\, \nu} \; C_{1\ -1}^{L_\gamma\, L_\gamma\, \nu} \; P_\nu(\cos\theta)$$

$$W(j_2 j_2' L_P L_P; \nu\, j_1)\; W(j_2 j_2' L_\beta L_\beta; \nu\, j')\; W(J_2 J_2 L_\gamma L_\gamma; \nu\, J_3)\; W(J_2 J_2 L_\beta L_\beta; \nu\, J_1)$$

The first two Racah coefficients give the selection rule $\Delta\,(j_2 j_2' \nu)$, which requires for anisotropy that at least one of j_2 and j_2' be $> \frac{1}{2}$. This is a less rigid requirement than in case (1) - linear in the ratio of the matrix elements of $j_2 = 3/2$ to those of $j_2 = \frac{1}{2}$ instead of quadratic - but is still ample to discourage experimental detection.

3) X-ray - Inner Bremsstrahlung Photon

This correlation can give no nuclear information since it depends on atomic parameters exclusively. It can provide a check on some theoretical conclusions about the inner bremsstrahlung process[4], but this is more conveniently done by an examination of the bremsstrahlung spectrum. Nevertheless, it is presented here for its formal interest.

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[4]R. J. Glauber and P. C. Martin, Phys. Rev. 95, 572 (1954).

The process is of the standard form of an angular correlation between two successive electromagnetic radiations (state 1: origin of X-ray; state 2: terminal of X-ray, origin of photon; state 3: terminal of photon - the actual time sequence is irrelevant), except that state 3 is a virtual state. This last fact makes it necessary to include in the description the capture of the electron from the virtual state, and to carry out a coherent sum over all possible 3 states. The capture process makes itself evident through reduced matrix elements acting as coefficients in the sum; its geometrical features have no effect. It is found that only states with $j_3' = j_3 + 2n$, n = integer, interfere with each other. Since a $j_3 + 2$ state will make a very much smaller contribution than a j_3 state (vide the orbital capture probabilities), we can neglect interference between states of different j_3 (though there may remain interference between states with the same j_3 but different energy E_3). Then

$$W \sim \sum_{E_3 E_3' j_3} (-)^{j_2 - j_3} \frac{(j_3 \| H_{L_P} \| j_2)(j' \| A_{L_\beta} \| j_3)}{E_2 - E_3} \frac{(j_3 \| H_{L_P} \| j_2)^*(j' \| A_{L_\beta} \| j_3)^*}{E_2 - E_3'}$$

$$\sum_{\nu \text{ even}} C_{1\,-1}^{L_P L_P \nu} \; C_{1\,-1}^{L_X L_X \nu} \; P_\nu(\cos\theta)$$

$$W(L_X L_X j_2 j_2; \nu j_1)\, W(L_P L_P j_2 j_2; \nu j_3)$$

We again encounter the condition $\Delta(j_2 j_2 \nu)$. This time, however, it is not necessarily disabling. It is satisfied if a $p_{3/2}$ electron emits an electric dipole photon to go into a virtual $s_{1/2}$ state from which it is captured. In fact, the sequence p electron ⟶ s electron (via electric dipole photon) makes the dominant contribution to the inner bremsstrahlung process at low energies.[4]

Conclusion

Angular correlations between the radiations connected with the orbital capture process exist in general. They are found not to represent at present a useful tool for the determination of nuclear parameters, however, because the selection rules enforce isotropy in most cases of interest. The conditions necessary for the observation of anisotropy (where nuclear information is available) are currently experimentally unattainable.