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SHIELD OPTIMIZATION

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E. P. Blizard

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SHIELD OPTIMIZATION

A shielding problem can be said to be solved when the radiations from a given array of sources has been reduced to some tolerable limit at a specified location. In general if a problem can be solved at all, there are many different ways in which it can be solved, so that it is of interest to inquire which of these solutions is the best. While for stationary applications "best" might imply cheapest, for mobile power applications weight is the primary consideration and "best" therefore means lightest. For this reason mobile reactor shields are usually optimized with respect to total weight.

The weight of the shield alone, however, is not the sole consideration, since other features of the shield can affect the overall installation weight. Thus the location of the center of gravity of a submarine shield is quite important to the stability of the craft. For this situation it might be possible to evaluate the effect of a high centroid by including in the shield weight the extra keel ballast required.

In an airplane it is common to consider the shield as divided, partly around the reactor, and partly around the crew. Strictly from considerations of shield weight, it might appear desirable to have the two parts of the shield of about the same weight, but this highly divided situation means a large moment of inertia about the transverse axes, with concomitant increase in control surface requirements.

A large reactor shield [redacted], the alternative to a a highly divided neutron [redacted] increase the [redacted] area of the fuselage as to increase the power requirements for a given performance. This also has an equivalent weight penalt[redacted]

The distance from reactor to crew position is another variable in airplane design. The greater is this distance the greater is the inverse square attenuation, but the fuselage becomes longer, hence heavier and harder to propel through the air. There is a proper balance in this parameter also.

The influence of all these variables can be taken into account in optimizing a shield with respect to weight. For the purposes of the present discussion, it will be understood that this is the case, but the weight will nevertheless be treated as a simple integral over all parts of the shield. Thus

$$W = \int_{\text{shield}} \rho(x,y,z)\, dV \tag{1}$$

where in the simple case $\rho(x,y,z)$ would be a density and dV a volume element. In general W could also include other parts which are functions of variables other than (x,y,z). This will not change the fundamental method of optimization, hence will not be treated explicitly now.

Although there must exist another expression, analogous to Eq. (1), for the dose rate at the position occupied by the nearest personnel (e.g., crew position in an airplane), it would be excessively complicated and would involve unknown cross sections. It is, however, not difficult to express the variation of the dose rate as the integral of variations which are measurable throughout the shield. For the purposes of the present discussion it will be assumed that there are just two functions which specify the shield configuration, one for the interior and the other for the periphery. The variation in the dose rate at the crew position is then expressed by the following equation:

$$\delta D = \int_{\substack{\text{shield}\\ \text{volume}}} D'_\alpha(x,y,z)\, \delta\alpha(x,y,z)\, dV + \int_{\substack{\text{shield}\\ \text{surface}}} D'_t(x,y,z)\, \delta t(x,y,z)\, dS \tag{2}$$

where $\alpha(x,y,z)$ is a point function describing the composition of the shield,

and $t(x,y,z)$ is a function describing the surface (S) of the shield

$\delta t(x,y,z)$, the variation in $t(x,y,z)$, is taken to be normal to the shield surface at (x,y,z) and positive for increase of shield thickness.

$D'_\alpha(x,y,z)$ and $D'_t(x,y,z)$ are functional derivatives of D with respect to α and t, and are to be described in more detail in the next paragraph.

dV, dS are volume and surface elements, the latter being taken at the variable (usually outer) surface.

The functional derivatives[3] D'_α and D'_t are defined in the following manner. Consider a small volume ϵ about the point (x,y,z) in the shield interior, and change the value of $\alpha(x,y,z)$ within this volume by an amount $\delta\alpha(x,y,z)$. This will produce a change in D by an amount $\delta_\alpha D$. The functional derivative[3] is then defined as

$$D'_\alpha(x,y,z) = \lim_{\epsilon\delta\alpha \to 0} \frac{\delta_\alpha D}{\delta\alpha(x,y,z)} \qquad (3)$$

It is easily seen that $D'_\alpha(x,y,z)$ is one true observable of a shield measurement. Suppose $\alpha(r)$ represents the volume fraction of lead in the lead-water region of a spherically symmetric shield. Then the measurement consists of inserting a spherical shell of thickness $\gamma = \frac{\epsilon}{4\pi r^2}$ at some radius r of the shield and observing the change in the dose rate, D*. Then, since $\delta\alpha = 1$,

---

* Theoretically, of course, any small piece of lead at or near the proper radius could be used. The spherical shell is chosen so that the maximum effect will be observed for a given spread, or uncertainty, in the radius at which the lead is located.

$$D'_\alpha(r) \cong \frac{\delta_\alpha D}{4\pi r^2 \gamma} \qquad (4)$$

It is convenient to introduce in this connection a "replacement length, ℓ," much like a relaxation length,* which describes the effect on D of replacing water with lead at r.

$$\ell(r) = -\lim_{\gamma \to 0} \frac{\gamma D}{\delta_\alpha D} \qquad (5)$$

$$\approx -\frac{\gamma D}{\delta_\alpha D} \qquad (5a)$$

whence

$$D'_\alpha(r) = -\frac{D}{4\pi r^2 \ell(r)} \qquad (7)$$

It is of interest at this point to inquire why "ℓ" might be a function of r. It is, after all, merely an indication of the effect of replacing a

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* The similarity between ℓ and a relaxation length λ is seen from the following:

The law for plane exponential attenuation of a beam of intensity I is

$$I = I_o e^{-x/\lambda}$$

$$\frac{dI}{dx} = -\frac{1}{\lambda} I_o e^{-x/\lambda}$$

$$\lambda = -\frac{I dx}{dI} \qquad (6)$$

This equation is then to be compared to Eq. (5a) in which γ corresponds to dx, $\delta_\alpha D$ to dI, and D to I.

small thickness of water with lead. At first glance this might be thought to be independent of the radius at which the lead is introduced, and if this were the case it would be clearly an advantage to locate the heavy material at the smallest radius so that the weight per unit thickness would be a minimum. There are, however, very good reasons why the lead cannot advantageously be located too close to the source in a reactor shield, to wit:

1) Capture and inelastic scattering gamma rays will be produced in the lead, so that locating this material too near the source, i.e., in too high a neutron flux, will cause this secondary gamma-ray source to be excessive.

2) Intermediate energy neutrons will proceed through the lead relatively unimpeded, producing a strong secondary source beyond the lead.

3) In general, secondary gamma rays will be produced throughout the shield. The lead must be so located that it intercepts these gamma rays, i.e., it cannot be concentrated too near the source.

Next consider a small element of shield surface σ near the point (x,y,z) at which the thickness is increased by an amount $\delta t(x,y,z)$ measured in the direction normal to the surface. This will produce a change in D by an amount $\delta_t D$, giving the following definition

$$D_t'(x,y,z) = \lim_{\sigma\delta t \to 0} \frac{\delta_t D}{\sigma \delta t(x,y,z)} \tag{8}$$

The simplest exposition of the nature of $D_t'(x,y,z)$ is obtained by the use again of the spherical shield. For this case, σ becomes $4\pi r_o^2$, or the area of the outermost shield surface, and $D_t'(x,y,z)$ becomes simply $D_r'(r_o)$

$$D_t'(x,y,z) = D_r'(r_o) = \frac{1}{4\pi r_o^2}\left(\frac{\partial D}{\partial r}\right)_{r=r_o} \tag{9}$$

$$= \frac{-D}{4\pi r_o^2 \lambda(r_o)} \tag{10}$$

where $\lambda(r_o)$ is simply the relaxation length for total dose rate of the material at the outer shield edge.

The shield weight is of course assumed to be affected by the variations in α and t, and it is therefore of interest to write down the effect as exactly as possible:

$$\delta W = \int_{\substack{\text{shield}\\ \text{volume}}} \frac{\partial\rho(x,y,z)}{\partial\alpha}\,\delta\alpha\, dV + \int_{\substack{\text{shield}\\ \text{surface}}} \rho(x,y,z)\ \delta t(x,y,z)\ dS\,. \tag{11}$$

The shield weight is optimized subject to the constraint of constant dose rate, so that,

$$\delta W + \frac{\Lambda}{D_o}\,\delta D = 0 \tag{12}$$

where Λ is an arbitrary constant to be determined by the conditions of the problem and D_o is the allowed dose at the occupied space, that is, the value of D when the shield is adequate or,

$$\int_{\substack{\text{shield}\\ \text{volume}}} \left[\frac{\partial\rho(x,y,z)}{\partial\alpha(x,y,z)} + \frac{\Lambda}{D_o}\, D'_\alpha(x,y,z)\right] \delta\alpha dV +$$

$$+ \int_{\substack{\text{shield}\\ \text{surface}}} \left[\rho(x,y,z) + \frac{\Lambda}{D_o}\, D'_t(x,y,z)\right] \delta t(x,y,z)\ dS = 0 \tag{13}$$

Equation (13) indicates a requirement for shield optimization in terms of variations in $\alpha(x,y,z)$ and in $t(x,y,z)$. Thus if

$$\Phi(\alpha,t) = W + \frac{\Lambda}{D_o}\, D \tag{14}$$

is the integral to be minimized, the left hand side of Eq. (13) expresses the variation in this integral due to variations in $\alpha(x,y,z)$ and $t(x,y,z)$. Thus if the functions $\alpha(x,y,z)$ and $t(x,y,z)$ are changed to $\alpha(x,y,z) + \delta\alpha(x,y,z)$ and $t(x,y,z) + \delta t(x,y,z)$, the corresponding variation in Φ is expressed by the LHS of Eq. (13). But for Φ to be a minimum (a maximum would be easily recognized and discarded), this variation must be zero, as is expressed by Eq. (13), for any arbitrary variation functions $\delta\alpha(x,y,z)$ and $\delta t(x,y,z)$. The only way to insure this, since $\delta\alpha$ and δt are completely unspecified, is to require that the square-bracketed quantities in Eq. (13) are identically zero. That is, for all points within the volume of the shield

$$\frac{\partial\rho}{\partial\alpha} + \frac{\Lambda}{D_o} D_\alpha' = 0 \tag{15}$$

and for every point on its surface,

$$\rho + \frac{\Lambda}{D_o} D_t' = 0. \tag{16}$$

For a spherical shield, these requirements become, for the volume of the shield,

$$\frac{\partial\rho}{\partial\alpha} = \frac{\Lambda}{4\pi r^2 \ell(r)} \tag{17}$$

and since $\frac{\partial\rho}{\partial\alpha}$ is simply the difference in densities of the two components, the requirement becomes simply that

$$r^2 \ell(r) = \text{constant} = C \tag{18}$$

and for the shield surface,

$$\rho(r_o) = \frac{\Lambda}{4\pi r_o^2 \lambda(r_o)} \tag{19}$$

Λ can of course be eliminated between Eqs. (17) and (19), so that the optimum configuration is completely determined. It should be emphasized that $\ell(r)$ and $\lambda(r_o)$ are to be determined in the optimum configuration itself, and that Eqs. (17) and (19) serve only to identify the optimum shield when it has been achieved. It is nevertheless obvious that it is possible in cases of physical interest, at least, by comparing actual $\ell(r)$'s with those specified by Eq. (17), to determine in which direction to change α to approach the optimum.

The Neutron-to-gamma Ratio

Eqs. (17) and (19) specify the optimum values of "$\ell(r)$" and $\lambda(r_o)$, which are measures of the shield effectiveness in terms of the dose rate D. The instruments which are used measure neutron and gamma-ray dose rates separately, however, so it is convenient to derive from the basic equations the desired effectivenesses for these two dose components. For this purpose, let the dose rate be represented as the sum of a neutron and a gamma ray component,

$$D = N + \Gamma \tag{20}$$

and define two new replacement lengths in terms of these individual components:

$$\ell_n(r) = \lim_{\gamma \to 0} \frac{\gamma N}{\delta_\alpha N} \tag{21}$$

$$\ell_\gamma(r) = \lim_{\gamma \to 0} \frac{\gamma \Gamma}{\delta_\alpha \Gamma} . \tag{22}$$

Similarly, for the relaxation lengths at the shield exterior,

$$\lambda_n(r_o) = - N \left.\frac{dr}{dN}\right)_{r=r_o} \tag{23}$$

$$\lambda_\gamma(r_o) = -\Gamma \left.\frac{dr}{d\Gamma}\right)_{r=r_o} \tag{24}$$

and, for completeness, add

$$\lambda(r_o) = -D \left.\frac{dr}{dD}\right)_{r=r_o} . \tag{25}$$

From these definitions it is not difficult to show that

$$\frac{D}{\lambda(r_o)} = \frac{N}{\lambda_n(r_o)} + \frac{\Gamma}{\lambda_\gamma(r_o)} \tag{26}$$

and

$$\frac{D}{\ell(r)} = \frac{N}{\ell_n(r)} + \frac{\Gamma}{\ell_\sigma(r)} \tag{27}$$

In a lead-water shield, the replacement of water by lead makes little difference to the neutron dose. As a consequence it is a reasonable approximation to let

$$\ell_n(r) >> \ell_\gamma(r)$$

If furthermore $N \sim \Gamma$, then it is easy to show that Eq. (18) would be replaced by

$$r^2 \ell_\gamma(r) = c \frac{\Gamma}{D} \tag{28}$$

It follows at once from the above for an optimized spherically symmetric shield that the ratio of neutron to gamma dose rates is given by

$$\frac{N}{\Gamma} = \frac{\dfrac{\rho_o r_o^2}{\rho' r^2 \ell_\gamma(r)} - \dfrac{1}{\lambda_\gamma(r_o)}}{\dfrac{1}{\lambda_n(r_o)} - \dfrac{\rho_o r_o^2}{\rho' r^2 \ell_n(r)}} \qquad (29)$$

where $\rho_o = \rho(r_o)$

$$\rho' = \frac{\partial \rho}{\partial \alpha}$$

Parameter Optimization

In many cases of shield design it is sufficient to specify the shield in terms of a finite number of parameters. In the previous case, the functions α and t could be considered as infinite sets of parameters, having values to be individually determined at every point in space. Considerable simplification results if only a few parameters need be considered.

In a typical aircraft divided shield where the shield is very asymmetric, the parameters might include:

1,2) The two reactor shield thickness parameters T_o and T' if the thickness as a function of a polar angle ψ is expressed by $T = T_o - T'\psi$.

3) The effective angle of a lead shadow shield.

4-9) The six thicknesses of crew shield front, sides, and rear for neutrons and for gamma rays.

10) Some function of the reactor shield diameter, to take account of the the extra drag occasioned by excessive frontal area, etc.

The general method for shield optimization in which a finite number of parameters is adequate is now outlined.

The weight is expressed as a function of the parameters, thus:

$$W = W(x,y,z,\ldots) \tag{30}$$

The dose rate at the occupied space is likewise expressed in general as a function of the same parameters.

$$D = D(x,y,z\ldots) \tag{31}$$

Furthermore it is required that the dose rate in the chosen design be just equal to some tolerable level D_o. That is,

$$D(x,y,z,\ldots) - D_o = 0 \tag{32}$$

In addition there may be other conditions, such as that the crew compartment volume is fixed (while its shape may not be). These are expressed also in general form, thus

$$F_1(x,y,z,\ldots) = 0 \tag{33}$$

$$F_2(x,y,z,\ldots) = 0 \tag{33a}$$

Although it will often be more convenient to combine the special conditions expressed by Eqs. (33,a,..) with either Eq. (30) or Eq. (32), the general method for optimizing does not require this.

If a new function

$$\Phi = W + \omega D + \omega_1 F_1 + \omega_2 F_2 + \ldots \tag{34}$$

is defined, where ω, ω_1, ω_2, etc. are constants to be determined from the characteristics of the function W, D, F_1,F_2, etc., then it is postulated that W is then optimized subject to the conditions of Eqs. (32) and (33,a..)

if

$$\frac{\partial \Phi}{\partial x} = \frac{\partial \Phi}{\partial y} = \frac{\partial \Phi}{\partial z} = \ldots = 0 \qquad (35)$$

In order to make this postulate apparent, Eq. (34) is first simplified by combining the functions F_1, F_2, etc. with either W or D. Each one of these will be such that it can be lumped with either the dose rate or the weight. Thus if F_1 represents the weight penalty for increasing the reactor-to-crew separation, then $W + \omega_1 F_1$ would represent the shield weight proper plus some amount to allow for the extra fuselage. A similar penalty to be added to the dose, say F_2, could refer to some physical disadvantage, associated, for example, with an uncomfortable crew space. Taking these into account, let

$$W' = W + \omega_1 F_1 + \omega_3 F_3 + \ldots \qquad (36)$$

$$D' = D + \frac{\omega_2}{\omega} F_2 + \frac{\omega_4}{\omega} F_4 + \ldots \qquad (37)$$

also,

$$D' - D_o = 0 \qquad (32a)$$

$$\Phi = W' + \omega D' \qquad (38)$$

For variation in any two of the parameters x and y,

$$\frac{\partial \Phi}{\partial x} \delta x = \frac{\partial W'}{\partial x} \delta x + \omega \frac{\partial D'}{\partial x} \delta x \qquad (39)$$

$$\frac{\partial \Phi}{\partial y} \delta y = \frac{\partial W'}{\partial y} \delta y + \omega \frac{\partial D'}{\partial y} \delta y \qquad (40)$$

Now Eq. (32a) constrains δx and δy so that for a given δx, a concomitant δy must be such that

$$\frac{\partial D'}{\partial x}\,\delta x + \frac{\partial D'}{\partial y}\,\delta y = 0 \tag{41}$$

The variation in weight which this causes must be zero if the weight has been optimized, that is,

$$\delta W = \frac{\partial W}{\partial x}\,\delta x + \frac{\partial W}{\partial y}\,\delta y = 0 \tag{42}$$

On multiplication of (41) by an arbitrary constant ω it is seen by comparison of this and (42) with expressions (39) and (40) that the sum of the latter is zero. Furthermore, it is evident that (39) is equal to the negative of (40). Accordingly, both (39) and (40) must be zero, and by analogy so are all other partial derivatives of Φ, that is, if W' is optimized and D' is fixed, then

$$\frac{\partial \Phi}{\partial x} = \frac{\partial \Phi}{\partial y} = \frac{\partial \Phi}{\partial z} = \ldots = 0 \tag{43}$$

or

$$\frac{\dfrac{\partial W'}{\partial x}}{\dfrac{\partial D'}{\partial x}} = \frac{\dfrac{\partial W'}{\partial y}}{\dfrac{\partial D'}{\partial y}} = \ldots = \omega \tag{44}$$

Optimization of a Box-shaped Shield

As an example of optimization of a shield in which it is possible to specify the configuration by a finite set of parameters, consider the following:

It is required to shield a given volume of radioactive material (pure gamma emitter) with a minimum weight of shielding material. Both the source and the shield are to be in the shape of rectangular parallelepipeds. The source is to be located in a vehicle at a position well behind an operating crew. As a consequence radiation leaving the front, sides, and rear of the source contribute differently to the dose rate at the crew position, for which the shield is to be designed.

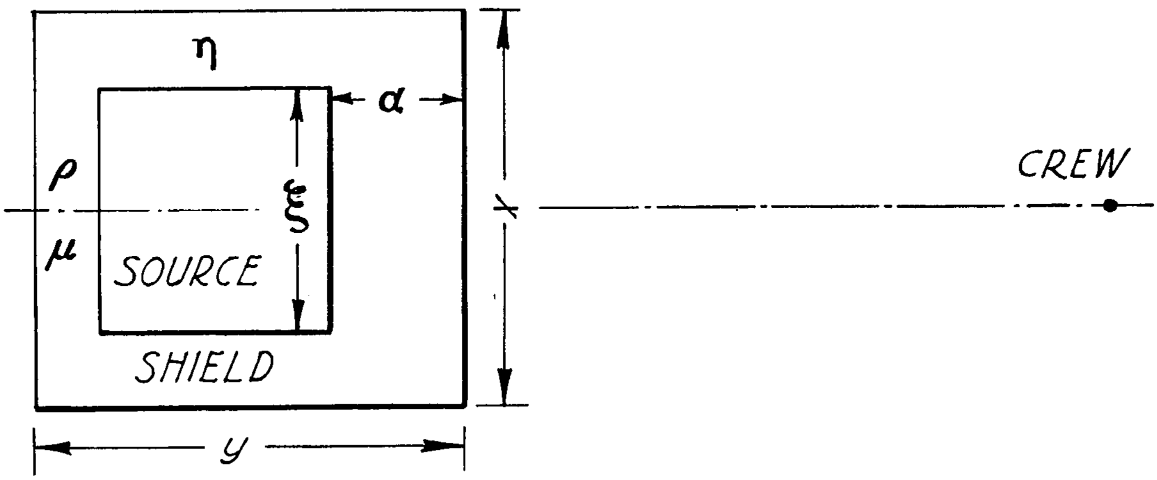

Fig. 1 - Box-shaped Source in Shield

Figure 1 shows the configuration of the source in its shield. Both are assumed to have square cross sections.

The conditions of the problem are stated mathematically as follows:

$$W = \rho(x^2y - V) \tag{45}$$

$$V = \xi^2\eta = \text{constant} \tag{46}$$

$$D = D_F + D_S + D_R \tag{47}$$

$$D_F = F\xi^2e^{-\mu\alpha} \tag{48}$$

$$D_S = S\xi\eta e^{-\mu\left((x-\xi)/2\right)} \tag{49}$$

$$D_R = R\xi^2 e^{-\mu(y-\eta-\alpha)} \tag{50}$$

$$D(\xi,\alpha,x,y,\eta) - D_o = 0 \tag{51}$$

where

ρ is the density of shield material

W is the shield weight

x,y,ξ,η are the dimensions of shield and source, as shown in Fig. 1.

V is the source volume

D is the total dose rate at crew position

D_F, D_S, D_R are the dose rates due to radiation leaving the front, the sides, and the rear of the source

F,S,R are constants which describe the attenuation of these radiation components

μ is the attenuation coefficient of the shield material

D_o is the tolerable dose rate at the crew position

Optimization is first carried out for those variables of which W is not explicitly a function.

$$\frac{\partial D}{\partial \alpha} = -\mu F\xi^2 e^{-\mu\alpha} + \mu R\xi^2 e^{-\mu(y-\eta-\alpha)} = 0 \tag{52}$$

$$\alpha = \frac{y-\eta}{2} + \frac{1}{2\mu} \ln\frac{F}{R} \tag{53}$$

$$= \frac{y-\eta}{2} + C$$

where

$$C = \frac{1}{2\mu} \ln\frac{F}{R}$$

Next ξ is eliminated from the expression for the dose by means of Eq. (46), and optimization is carried out with respect to η.

$$D = \frac{2v\sqrt{FR}}{\eta} e^{-\mu/2\,(y-\eta)} + S\sqrt{V\eta}\, e^{-\mu\left(\frac{x}{2} - \frac{1}{2}\sqrt{\frac{V}{\eta}}\right)^{*}} \tag{54}$$

* Note that $Fe^{-\mu C} + Re^{\mu C} = 2\sqrt{FR}$

$$\frac{1}{V}\frac{\partial D}{\partial \eta} = 0 = 2\sqrt{FR}\left(\frac{\mu}{2} - \frac{1}{\eta^2}\right) e^{-\mu/2\ (y-\eta)} + \frac{S}{2}\left(\frac{1}{\sqrt{V\eta}} - \frac{\mu}{2\,\eta}\right. e^{-\frac{\mu}{2}\left(x - \sqrt{\frac{V}{\eta}}\right)} \tag{55}$$

For a source box large compared to a relaxation length in the shield material,

$$\mu >> 1/\eta$$

and if $V \sim \eta^3$,

then the following simplification is permissible:

$$\frac{4\sqrt{FR}}{S} = e^{\frac{\mu}{2}\left(y-\eta-x+\sqrt{\frac{V}{\eta}}\right)}$$

$$y - \eta - x + \sqrt{\frac{V}{\eta}} = A \tag{56}$$

where

$$A = 1/\mu \,\ln\,(16FR/S^2)$$

Two more equations follow from Eq. (43)

$$\frac{\partial \Phi}{\partial x} = 2\rho xy - \omega\,\frac{\mu S\sqrt{V\eta}}{2}\; e^{-\mu\left(\frac{x}{2} - \frac{1}{2}\sqrt{\frac{V}{\eta}}\right)} = 0 \tag{57}$$

$$\frac{\partial \Phi}{\partial y} = \rho x^2 - \frac{\omega\mu V\sqrt{FR}}{\eta}\, e^{-\mu/2\ (y-\eta)} = 0 \tag{58}$$

The four remaining unknowns x, y, η, and ω are fixed by Eqs. (51 and 54), (56), (57), and (58).

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