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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

FOR PERIOD ENDING APRIL 30, 1952

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OAK RIDGE NATIONAL LABORATORY
OPERATED BY
CARBIDE AND CARBON CHEMICALS COMPANY
A DIVISION OF UNION CARBIDE AND CARBON CORPORATION

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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

for Period Ending April 30, 1952

A. S. Householder, Chief

C. L. Perry, Editor

DATE ISSUED

JUL 9 1952

OAK RIDGE NATIONAL LABORATORY
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Mathematics Panel quarterly progress reports previously issued in this series are as follows:

ORNL-345	December, January, February, 1948-1949
ORNL-408	Period Ending July 31, 1949
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MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

SUMMARY

C. P. Hubbard, formerly of the U. S. Navy Electronics Laboratory, San Diego, California, is a new addition to the permanent staff. W. A. Rutledge, of Alabama Polytechnic Institute, has joined the Mathematics Panel for six months to assist on algebraic problems arising in the development of subroutines for the digital computer. J. Z. Hearon, of the University of Chicago, is here as Research Participant for three months to work on biometric problems.

W. J. Youden, of the Statistical Engineering Laboratory, National Bureau of Standards, spent April 21 to 24 in Oak Ridge consulting with chemists and others at the Laboratory, at K-25, and at Y-12 on problems relating to statistical design. The time was much too short, and a second visit is being planned.

Sangren and Hubbard have spent some time at the Argonne National Laboratory using the REAC for solving several systems of differential equations relating to reactor kinetics and to chemical systems. Coding for the computation of the field factors in beta decay to be done on the Whirlwind is nearing completion.

Householder represented Oak Ridge at a meeting held at the Washington AEC offices on April 14 to consider needs of the AEC installations for computing facilities. Los Alamos appears to have the greatest need, and their MANIAC, now beginning to operate, will by no means satisfy these needs. KAPL and Westinghouse also were thought to need some of the time of a high-speed machine. Both Oak Ridge and Argonne felt that when their own machines get under way the bottleneck will be in personnel rather than in machines.

Construction of subroutines for the Oak Ridge machine continues, and fabrication of the machine proceeds at a satisfactory rate.

The following lectures were given under the ORINS Traveling Lecture Plan:

A. S. Householder

"The Solution of Large Linear Systems," University of Tennessee, February 6.

"Mathematics and Biology," University of Tennessee, February 19.

"Nervous Systems of Computing Machines," University of Chicago, February 15.

"Iterative Methods for Solving Equations," University of Maryland, April 3.

W. C. Sangren

"Sturm-Liouville Systems with Discontinuous Coefficients," University of Georgia, February 21; Alabama Polytechnic Institute, March 5; University of Alabama, March 6; University of Oklahoma, April 2.

"Factorization Methods for Differential Equations," University of Georgia, February 22; Alabama Polytechnic Institute, March 5.

"Mathematics at Oak Ridge," University of Georgia, February 20.

A. W. Kimball

"Elements of the Theory of Sequential Analysis," Georgia Institute of Technology, February 19; Alabama Polytechnic Institute, February 21.

In addition, the following papers and reports were published during the quarter:

J. Moshman, "Testing a Straggler Mean in a Two-Way Classification Using the Range," *Annals of Mathematical Statistics* **23**, 126.

W. C. Sangren, "Generalized Fourier Integrals," *Proceedings of the International Congress of Mathematicians*, Vol. II, 1950.

W. C. Sangren, *Calculations for Homogeneous Reactors*, ORNL-1205, April 1, 1952.

UNCLASSIFIED PROJECTS

METHODS OF COMPUTATION FOR USE WITH A HIGH-SPEED AUTOMATIC-SEQUENCED COMPUTER

Participating Members of Panel. W. Givens, W. A. Rutledge.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1232.

Problem 2. Numerical Inverting of Matrices of High Order.

Status. The series of lectures on the first von Neumann-Goldstine paper was concluded during the quarter. The second paper, giving statistical estimates, will be made obsolete by forthcoming work of another author. At present the problem is inactive.

Problem 3. Numerical Computation of Characteristic Values and Characteristic Vectors.

Results. The method explained in the last quarterly report has been reduced to a sequence of algorithms, which will serve as a suitable base for coding the calculation of characteristic values of real symmetric matrices of large order. Error bounds have been found both for the sum of squares and for the individual characteristic values. These are favorable and guarantee the stability of the method. Numerical results are too complex to be summarized readily but the accuracy obtainable can be roughly indicated: the characteristic values of a matrix of order 100, scaled so that the sum of squares of its elements is slightly less than one, will be obtainable by this method with an accuracy of better than six decimal places with the ORAC. For large matrices the dominant term in the error bound is a constant multiple of the five-halves power of the order.

Status. A manuscript giving the detailed results is in preparation. It is planned to continue with error bounds for the characteristic vectors.

Problem 4. Basic Matrix Operations.

Background and Status. Efficiency demands that a complete set of subroutines be prepared to be available when the automatic computer is placed in operation. Therefore a study of proper and efficient coding methods is being carried out on basic matrix operations; in particular, multiplication and inversion, and coded sequences are being prepared for these operations.

Several coded sequences for multiplication of general matrices have been prepared. These are designed to use different methods of scaling according to the situation encountered. Preliminary work on matrix inversion is under way.

CALCULATION OF PERCENTAGE POINTS OF THE LOGARITHMIC NORMAL DISTRIBUTION

Participating Members of Panel. J. Moshman, E. A. Forbes, E. B. Carter (K-25).

Background and Status. The logarithmic normal probability density function has the form

$$f(x) = \frac{1}{c(x - a)\sqrt{2\pi}} \exp\left\{-\frac{1}{2c^2}\left(\log\frac{x - a}{b}\right)^2\right\}$$

and is a function of the three pameters a, b, and c.

If μ is the mean and μ_i the ith moment about the mean, then

$$\mu = b\omega^{1/2} + a \ ,$$

$$\mu_2 = b^2\omega(\omega - 1) \ ,$$

$$\mu_3 = b^3\omega^{3/2}(\omega - 1)^2 \ (\omega + 2) \ ,$$

where $\omega = e^{c^2}$.

It can easily be shown that ω is the only real root of the equation

$$\omega^3 + 3\omega^2 - (4 + \alpha_3^2) = 0 \ ,$$

where $\alpha_3 = \mu_3/\mu_2^{3/2}$ and is the skewness of the distribution.

If y is a normally distributed variable with zero mean and unit variance, that is,

$$y = \frac{1}{c}\log\frac{x - a}{b} \ ,$$

then the unit log normal deviate t in terms of y may be written.

Specifically, for some $0 \le \gamma \le 1$ there corresponds a y_γ such that

$$\gamma = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y_\gamma} e^{-y^2/2} \, dy \ .$$

Then

$$t_\gamma = \frac{x - \mu}{\sqrt{\mu_2}} = \frac{e^{y_\gamma c - (c^2/2)} - 1}{\sqrt{\omega - 1}} \ .$$

Values of t_γ have been computed for $\alpha_3 = 0(0.01)3.00$ and $\gamma = 0.005$, 0.01, 0.025, 0.05, 0.10, 0.90, 0.95, 0.975, 0.99, and 0.995, and thus upper and lower critical values for the indicated values of γ have been obtained.

The tables were computed on IBM equipment and differenced. Where second differences were out of line, the values were checked by hand using greater decimal accuracy throughout and all differences were brought into line.

The log normal distribution is potentially of considerable value in dealing with skewed distributions. A paper illustrating some practical applications is being prepared for issuance with the table.

TEST OF A PROBABILITY MODEL FOR CELL FORMATION IN *NEUROSPORA CONIDIA*

Origin. K. C. Atwood, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. It is postulated that there exists a nuclear pool containing two kinds of nuclei, A and B. It is assumed further that when a cell forms it chooses nuclei at random from the pool. Let p be the proportion of A nuclei in the pool, and let the total number of nuclei available be very large compared with the number of nuclei selected for use by the cells. This latter assumption merely assures that the probability of choosing an A nucleus is the same for every cell formed.

Let the results of one experiment be expressed as follows:

M = number of cells formed,

x_n = number of cells having n nuclei $\left(\sum_{n=1}^{m} x_n = M\right)$,

m = maximum number of nuclei per cell,

a_{nj} = number of A nuclei in the jth cell having a total of n nuclei ($j = 1, \ldots, x_n$; $a_{nj} \le n$).

Then the probability of the sample is given by

$$P(S) = \prod_{n=1}^{m} \prod_{j=1}^{x_n} C^n_{a_{nj}} \, p^{a_{nj}} (1 - p)^{n - a_{nj}} .$$

This is a bona fide probability function, since clearly

$$\sum_{a_{nj}=0}^{n} P(S) = 1$$

$$(n = 1, \ldots, m;\ j = 1, \ldots, x_n) .$$

It can be shown that the maximum likelihood estimate of p is given by

$$\hat{p} = \frac{\sum_{n=1}^{m} \sum_{j=1}^{x_n} a_{nj}}{\sum_{n=1}^{m} n x_n} ,$$

that is, $\hat{p}$ is the total number of A nuclei divided by the total number of nuclei.

In actual experiments the x_n are known but the individual a_{nj} are not known. Instead, estimates of the proportions of A and B homokaryons and the proportion of heterokaryons are available. Let these estimates be k_A/M, k_B/M and k_{AB}/M, respectively. Then, the expected values of the k's are given by

$$E(k_A) = x_1 p + x_2 p^2 + \ldots + x_m p^m ,$$

$$E(k_B) = x_1(1 - p) + x_2(1 - p)^2 + \ldots + x_m(1 - p)^m , \tag{1}$$

$$E(k_{AB}) = M - E(k_A) - E(k_B) .$$

Data of this nature must be regarded as a sample from a trinomial distribution with probability

$$\frac{M!}{k_A!\ k_B!\ k_{AB}!} P_A^{k_A} P_B^{k_B} (1 - P_A - P_B)^{M-k_A-k_B} \quad , \tag{2}$$

where $MP_A = E(k_A)$ and $MP_B = E(k_B)$.

To test the hypothesis that cells select nuclei at random, the maximum likelihood estimate of p from Eq. 2 would be used to obtain expected frequencies from Eqs. 1 that in turn would be combined with the experimental frequencies to provide a χ^2 with 2 degrees of freedom. Unfortunately, the maximum likelihood equation obtained from Eq. 2 presents formidable computational difficulties. Accordingly, a graphical minimum χ^2 solution was performed for each of seven experiments. In five of these, substantial departures from the null hypothesis were found and led to the conclusion that it is very unlikely that cells select nuclei at random from the nuclear pool.

There is some theoretical basis for explaining this departure, which always results in an excess of homokaryons over and above what would be expected on the basis of random selection. An attempt will be made to construct a new probability model based on this information. If the attempt is successful, the new model will also be tested in a manner similar to the one just described.

OXYGEN AND TEMPERATURE EFFECTS ON RADIATION-INDUCED LETHAL MUTATIONS IN *DROSOPHILA MELANOGASTER*

Origin. W. K. Baker, Biology Division.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. Populations of *drosophila melanogaster* were exposed to 3000 r of x radiation at temperatures varying from 5 to 35°C in atmospheres of air, oxygen, and nitrogen. The number of flies per population varied from 150 to 740. After treatment the lethal mutants in each population were counted. An analysis of the effects of oxygen concentration and temperature on the percentage of lethals was desired.

A χ^2 analysis of the type described previously[1] revealed the presence of about 19% extraneous variation. Accordingly, binomial weights were used in the analysis of variance and the results are shown in Table 2. The arcsin transformation was employed, and the means in Table 1 have been retransformed to original units. Both oxygen concentration and temperature were found to affect the mutation rate, and there was no apparent interaction.

(1)"Analysis of Recombination Percentages for Several Experiments with Bacteriophage T2H," *Mathematics Panel Quarterly Progress Report for Period Ending January 31, 1952*, ORNL-1232, p. 11.

TABLE 1

Experimental Results

TEMPERATURE (°C)	PERCENTAGE OF MUTANTS		
	NITROGEN	AIR	OXYGEN
5	7.1	10.1	16.8
15	6.0	7.2	10.4
25	4.8	6.8	13.2
30	5.4	5.9	10.5
35		8.4	11.0

TABLE 2

Analysis of Variance

SOURCE OF VARIATION	DEGREES OF FREEDOM	MEAN SQUARE
Among oxygen concentrations (A)	2	5079.6*
Among temperatures (B)	4	544.1**
Among experiments within temperatures (C)	8	110.7
AB interaction	7	179.2
AC interaction	15	86.5
Total	36	

*Significant at 5% level.
**Significant at 1% level.

An attempt was made to separate the temperature means for each oxygen concentration, and it was found in each case that the 5°C mean differed significantly from the other four temperature means but that the latter did not differ significantly among themselves.

These results will be published in an appropriate journal.

DERIVATION OF A LIKELIHOOD RATIO TEST FOR A NULL HYPOTHESIS IN AN IRRADIATION EXPERIMENT WITH CORN POLLEN

Origin. D. Schwartz, Biology Division.

Participating Member of Panel. A. W. Kimball.

Background. Corn pollen are exposed to x radiation in atmospheres of air and nitrogen. At maturity the seeds are examined and are classified as normal or as one of two types of mutants called whole losses and mosaics. The results of such an experiment may be tabulated as follows:

EXPOSURE	NORMALS	WHOLE LOSSES	MOSAICS	TOTAL
Air	a_1	a_2	a_3	n
Nitrogen	b_1	b_2	b_3	m

where

$$n = a_1 + a_2 + a_3$$

and

$$m = b_1 + b_2 + b_3$$

are the total number of seeds examined in the two exposure groups. Let the probabilities associated with the experiment be:

EXPOSURE	NORMALS	WHOLE LOSSES	MOSAICS
Air	p_1	p_2	p_3
Nitrogen	q_1	q_2	q_3

where

$$p_1 + p_2 + p_3 = 1$$

and

$$q_1 + q_2 + q_3 = 1 .$$

On the basis of the sample it is desired to test the null hypothesis,

$$\frac{p_2}{q_2} = \frac{p_3}{q_3} ,$$

against an alternative hypothesis that places no restriction on the parameters.

Status. The likelihood of the sample is given by the product of two trinomial distributions:

$$L = kp_1^{a_1} \; p_2^{a_2} \; (1 - p_1 - p_2)^{n-a_1a_2} \; q_1^{b_1} \; q_2^{b_2} \; (1 - q_1 - q_2)^{m-b_1-b_2} \; , \tag{1}$$

where

$$k = \frac{n! \; m!}{a_1! \; a_2! \; a_3! \; b_1! \; b_2! \; b_3!} \; .$$

The null hypothesis reduces to

$$p_2 = \frac{q_2(1 - p_1)}{(1 - q_1)}$$

Substituting this in Eq. 1 one obtains the likelihood under the null hypothesis

$$L_0 = kp_1^{a_1} \; (1 - p \;)^{n-a_1} \; q_1^{b_1} \; q_2^{a_2+b_2} \; (1 - q_1)^{-(n-a_1)} \; (1 - q_1 - q_2)^{n+m-a_1-a_2-b_1-b_2} \tag{2}$$

Solving the simultaneous equations

$$\frac{\partial \log L_0}{\partial p_1} = 0 \; ,$$

$$\frac{\partial \log L_0}{\partial q_1} = 0 \; ,$$

$$\frac{\partial \log L_0}{\partial q_2} = 0$$

for p_1, q_1, and q_2, the maximum likelihood estimates under the null hypothesis are

$$\hat{p}_1 = \frac{a_1}{n} \; , \qquad \hat{q}_1 = \frac{b_1}{m} \; ,$$

$$\hat{q}_2 = \frac{(m - b_1) \; (a_2 + b_2)}{m(m + n - a_1 - b_1)} \; .$$

Under the alternative hypothesis the estimates are the familiar ones:

$$\hat{\hat{p}}_1 = \frac{a_1}{n}, \qquad \hat{\hat{p}}_2 = \frac{a_1}{n},$$

$$\hat{\hat{q}}_1 = \frac{b_1}{m}, \qquad \hat{\hat{q}}_2 = \frac{b_2}{m}.$$

Substituting these estimates in their respective likelihoods, one finds

$$L\ (\max) = k\left(\frac{a_1}{n}\right)^{a_1} \left(\frac{a_2}{n}\right)^{a_2} \left(\frac{n - a_1 - a_2}{n}\right)^{n-a_1-a_2} \left(\frac{b_1}{m}\right)^{b_1} \left(\frac{b_2}{m}\right)^{b_2} \left(\frac{m - b_1 - b_2}{m}\right)^{m-b_1-b_2},$$

$$L_0(\max) = k\left(\frac{a_1}{n}\right)^{a_1} \left(\frac{n - a_1}{n}\right)^{n-a_1} \left(\frac{b_1}{m}\right)^{b_1} \left(\frac{m - b_1}{m}\right)^{m-b_1}$$

$$\cdot \frac{(a_2 + b_2)^{a_2+b_2}\ (n + m - a_1 - a_2 - b_1 - b_2)^{n+m-a_1-a_2-b_1-b_2}}{(n + m - a_1 - b_1)^{n+m-a_1-b_1}},$$

whereupon the likelihood ratio is given by

$$\lambda = \frac{L_0\ (\max)}{L\,(\max)}$$

$$= \frac{(n - a_1)^{n-a_1}\ (m - b_1)^{m-b_1}\ (a_2 + b_2)^{a_2+b_2}\ (n + m - a_1 - a_2 - b_1 - b_2)^{n+m-a_1-a_2-b_1-b_2}}{a_2^{a_2}\ (n - a_1 - a_2)^{n-a_1-a_2}\ b_2^{b_2}\ (m - b_1 - b_2)^{m-b_1-b_2}\ (n+m - a_1 - b_1)^{n+m-a_1-b_1}}.$$

The exact distribution of λ would be impossible to evaluate in most practical problems. However, a theorem of Wilks[2] indicates that $-2 \ln \lambda$ has an asymptotic χ^2 distribution with one degree of freedom. For purposes of computation it is convenient to make the following substitutions:

$$x_1 = a_2 + a_3 ,$$

$$x_2 = b_2 + b_3 ,$$

$$x_3 = a_2 + b_2 ,$$

$$x_4 = a_3 + b_3 ,$$

$$x_5 = -(a_2 + a_3 + b_2 + b_3) ,$$

$$x_6 = -a_2 ,$$

$$x_7 = -a_3 ,$$

$$x_8 = -b_2 ,$$

$$x_9 = -b_3 .$$

Then

$$\chi^2 = -2 \sum_{i=1}^{9} x_i \ln |x_i|$$

with one degree of freedom may be referred to readily available χ^2 tables to determine the level of significance.

In the particular experiment under consideration,

$$a_1 = 5959 , \qquad b_1 = 4459 ,$$

$$a_2 = 159 , \qquad b_2 = 43 ,$$

$$a_3 = 40 , \qquad b_3 = 3 ,$$

which yields $\chi^2 = 5.719$. For one degree of freedom this corresponds to a significance level of 0.0169, from which one would conclude that the null hypothesis is likely erroneous. Because of the large sample sizes involved in this experiment the χ^2 approximation is probably very accurate.

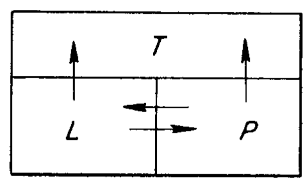
ESTIMATION OF VOLUME OF LYMPH SPACE

Origin. J. Furth, R. H. Storey, Biology Division.

Participating Members of Panel. J. Moshman, J. Z. Hearon, G. J. Atta, H. B. Goertzel.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1091.

Background and Status. Assume a three compartment system



where P represents the plasma or blood compartment, L the generalized lymph compartment, and T the tissues of the body. A substance is introduced in P, interchanged back and forth with L, and from both in only one direction into T.

The following notation will be used, with the subscripts p and l denoting compartments P and L, respectively:

A = activity per unit volume,

V = volume,

t = time,

k = rate of metabolism (time^{-1}).

[2] S. S. Wilks, *Mathematical Statistics*, Princeton University Press, 1943, p. 151.

The process may be described by the differential equations

$$\frac{d(A_p V_p)}{dt} = f_{pl}(\xi) - k_p V_p A_p \ , \qquad (1)$$

$$\frac{d(A_l V_l)}{dt} = f_{lp}(\xi) - k_l V_l A_l \ , \qquad (2)$$

where $f_{pl}(\xi)$ represents the "transport" rate from P to L as a function of some vector variable ξ, and $f_{lp}(\xi)$ is similarly defined for the rate from L to P. Obviously,

$$f_{pl}(\xi) = - f_{lp}(\xi) = f(\xi) \ . \qquad (3)$$

If Eqs. 1 and 2 are integrated, the following equations are obtained:

$$V_p A_p \ e^{k_p t} = \int_0^t f(\xi) \ e^{k_p \xi} \ d\xi + V_p^{(0)} A_p^{(0)} \ , \qquad (4)$$

$$V_l A_l \ e^{k_l t} = - \int_0^t f(\xi) \ e^{k_l \xi} \ d\xi + V_l^{(0)} A_l^{(0)} \ , \qquad (5)$$

where the superscript indicates the values when $t = 0$.

Several cases of immediate practical significance can be distinguished.

Case I: $0 \neq k_l = k_p = k$.

By adding Eqs. 4 and 5,

$$A_p V_p + A_l V_l = R_0 \ e^{-kt} \ , \qquad (6)$$

where R_0 is the total activity injected at $t = 0$. It should be noted that the volumes need not be assumed constant.

Case II: $k_l = k \neq 0$, $k_p = 0$.

By adding Eqs. 1 and 2,

$$\frac{d(A_p V_p)}{dt} + \frac{d(A_l V_l)}{dt} = - k A_l V_l \ , \qquad (7)$$

and by direct integration and transposition of Eq. 7

$$A_p V_p + A_l V_l + k V_l \int_0^t A_l \ dt = R_0 \ . \qquad (8)$$

Alternatively, Eq. 1 may be written as

$$\int_0^t f(\xi) \ e^{k\xi} \ d\xi = \int_0^t d(A_p V_p) \ e^{k\xi} \ d\xi \qquad (9)$$

and integrated by parts, to give

$$\int_0^t f(\xi) \ e^{k\xi} \ d\xi = e^{kt} \ V_p A_p - V_p^{(0)} A_p^{(0)} - k \int_0^t V_p A_p \ e^{k\xi} \ d\xi \ . \qquad (10)$$

Then, by substituting Eq. 10 in Eq. 5 and simplifying,

$$V_p A_p + V_l A_l = R_0\, e^{-kt} + kV_p \int_0^t A_p\, e^{-k(t-\xi)}\, d\xi \quad . \tag{11}$$

Case III: $0 \neq k_p \neq k_l \neq 0$.

From Eq. 1

$$\int_0^t d(V_p A_p)\, e^{k_l \xi}\, d\xi = \int_0^t f(\xi)\, e^{k_l \xi}\, d\xi - k_p V_p \int_0^t A_p\, e^{k_l \xi}\, d\xi \ , \tag{12}$$

and from Eq. 5

$$\int_0^t d(V_p A_p) . e^{k_l \xi}\, d\xi = V_l(0) A_l(0) - V_l A_l\, e^{k_l t} - k_p V_p \int_0^t A_p\, e^{k_l \xi}\, d\xi \ . \tag{13}$$

But from Eqs. 9 and 10

$$\int_0^t d(V_p A_p)\, e^{k_l \xi}\, d\xi = e^{k_l t}\, V_p A_p - V_p(0) A_p(0) - k_l V_p \int_0^t A_p\, e^{k_l \xi}\, d\xi \ . \tag{14}$$

Hence Eqs. 13 and 14 give

$$\begin{aligned} V_l(0) A_l(0) - V_l A_l\, e^{k_l t} - k_p V_p \int_0^t A_p\, e^{k_l \xi}\, d\xi \\ = e^{k_l t}\, V_p A_p - V_p(0) A_p(0) - k_l V_p \int_0^t A_p\, e^{k_l \xi}\, d\xi \quad , \end{aligned} \tag{15}$$

or

$$e^{k_l t}\, (V_p A_p + V_l A_l) = R_0 + (k_l - k_p) V_p \int_0^t A_p\, e^{k_l \xi}\, d\xi \quad .$$

The mathematical models are being considered in the light of experimental data from a series of experiments on dogs, as described in the reference.

COMPARISON OF X AND BETA IRRADIATION EFFECTS ON CHROMOSOME FRAGMENTATION

Origin. M. E. Gaulden, Biology Division.

Participating Members of Panel. J. Moshman, G. J. Atta.

Background and Status. Two series of cells were irradiated with 64 r of x rays and 64 rep of beta rays. It was desired to examine the differential effects of the two types of radiation on cell fragmentation with emphasis on single and double breaks. Since fragmentation is a function of the time elapsed after irradiation, it was decided for biological reasons to compare the two sets of cells 44, 220, 264, and 352 min after irradiation.

Each series was partitioned into groups of adjacent time periods having homogeneous responses, and then corresponding groups with x and beta irradiation were compared. No significant differences were found for the criteria (1) cells with fragments and (2) number of single breaks. There were markedly significant differences between 352 min after beta irradiation and a corresponding time after x irradiation for (1) the incidence of double breaks, (2) the ratio of single to double breaks, and (3) the total number of breaks. These are obviously not independent criteria. This project was completed.

HEAT EFFECT ON CELL EXTINCTION

Origin. M. E. Gaulden, Biology Division.

Participating Members of Panel. J. Moshman, G. J. Atta.

Reference. *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, -1091, -1151, -1232.

Background and Status. Data obtained previously have been accumulated on the effect of staining and irradiation on cell extinctions of grasshopper embryos. The present experiment, which is related to similar research at Brookhaven National Laboratory, was designed to determine the significance of treatment with heat on cell extinctions. Twenty-five embryos received the heat treatment and an equal number served as controls. Standard statistical procedures indicated, with probability of error of less than 1%, that the heat treated cells had a greater mean extinction than the controls. This project has been completed.

KINETICS OF THE HBr-$HBrO_3$ REACTION

Origin. O. E. Myers, Chemistry Division.

Participating Members of Panel. J. H. Fishel, C. P. Hubbard, W. C. Sangren.

Reference. *Mathematics Panel Quarterly Progress Reports*, ORNL-1151 and 1232.

Background and Status. Considerable difficulty has been encountered during the integration of the differential equations both in integrating numerically and with analog computation. The interval of integration was too small to continue with the numerical integration. With analog equipment (specifically the REAC) the difficulty was with scaling. This project will be continued.

PROFESSIONAL SELECTION PROGRAM

Origin. H. B. Hurt, Health Division.

Participating Member of Panel. K. P. Graw.

Background and Status. The Health Division recently gave a set of psychological tests to forty scientists working at ORNL. Numerous statistics have been computed for the Health Division from the data gathered during the tests. This project will be continued.

NUMERICAL EVALUATION OF INTEGRALS

Origin. R. B. Birkhoff, Health Physics Division.

Participating Members of Panel. N. Edmonson, S. E. Ezzell.

Background and Status. The purpose of this work is the numerical evaluation of two integrals arising in the study of the energy loss of fast electrons in thin films. The first of these integrals,

$$\phi(\lambda) = \int_0^\infty C(v)\ B_I(v)\ \Phi\,\{[\lambda + 9.33]\ C(v) - 9.33\}\ dv\ , \tag{1}$$

has been evaluated for values of λ ranging from −3 to 9 by steps of one unit. In this integral,

$$C(v) = \frac{1}{1 + 0.0287v}\ ,$$

the values of $B_I(v)$ are taken from curve I of a graph given by Telegdi,[3] and the function ϕ is determined from Fig. 1 of Landau.[4]

The results obtained from the computation of the integral Eq. 1 justified the computation of the integral

$$\phi(\lambda) = \int_0^\infty C(v)\ B_I(v) \sum_{\nu=1}^{4} \frac{C_\nu\ \gamma_\nu}{\sqrt{\gamma_\nu^2 + 18.8}} \exp\left\{-\frac{[\{\lambda + 9.33\}\ C(v) - 9.33 - \lambda_\nu]^2}{\gamma_\nu^2 + 18.8}\right\}. \tag{2}$$

This integral is a more exact description of the physical situation being considered. Here the numbers C_ν, γ_ν, and λ_ν are given in Table 1 of the paper by Blunck and Leisegang.[5] The functions $C(v)$ and $B_I(v)$ are the same as in Eq. 1. The computation of this integral is in progress.

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(3) V. L. Telegdi, "Alpha-Alpha Correlations in the Photodisintegration of C^{12} and the Resonant Absorption of Electromagnetic Radiation of Non-E. D. Character," *Phys. Rev.* **84**, 600 (1951).

(4) L. Landau, "On the Energy Loss of Fast Particles by Ionization," *J. Phys.* (*U.S.S.R.*) **8**, 204 (1944).

(5) O. Blunck and S. Leisegang, "Zum Energieverlust schneller Elektronen in dunnen Schichten," *Z. Physik* **128**, 500 (1950).

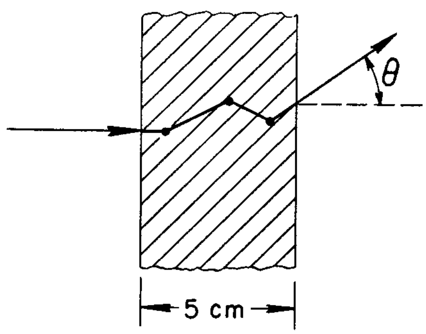
NEUTRON PENETRATION IN A 5-cm SLAB OF PARAFFIN

Origin. T. Burnett, W. S. Snyder, Health Physics Division.

Participating Member of Panel. K. P. Graw.

Reference. "Monte Carlo Estimate of Collision Distribution in Tissue," *Mathematics Panel Quarterly Progress Report*, ORNL-1232.

Background and Status. The neutron histories that were calculated for the monte carlo estimate of collision distribution in tissue were used to find the emerging energy spectrum of neutrons from a 5-cm slab of paraffin.



Every neutron entered the slab normal to the surface (as shown in the figure) with an energy 0.0033 Mev $< E <$ 0.0068 Mev (mean E = 0.005 Mev). The velocity distribution and angular distribution of the emerging neutrons were tabulated for a set of about 1000 histories. Similar statistics were tabulated for a set of about 1000 histories of neutrons entering the slab with higher energies (mean E = 0.5 Mev, 0.33 Mev $< E <$ 0.67 Mev).

THERMAL NEUTRON DAMAGE

Origin. W. S. Snyder, Health Physics Division.

Participating Members of Panel. K. P. Graw, C. P. Hubbard.

References. *Mathematics Panel Quarterly Progress Report*, ORNL-1151; W. S. Snyder, "Calculations for Maximum Permissible Exposure to Thermal Neutrons," *Nucleonics*, Vol. 6, No. 2, p. 46.

Background and Status. The distribution of energy absorbtion due to thermal neutron collisions in a 30-cm slab of tissue is $F(x)$ where

$$F(x) = \alpha \int_0^{30} f(y)\, E_1(\gamma\, |x - y|)\, dy + \beta f(x)$$

and

$$f(w) = 13.11\, e^{-0.4502w} - 5.140\, e^{-5.166w} .$$

The first term of $F(x)$ represents the energy absorption owing to gamma rays produced while the term $\beta f(x)$ is the energy absorbed owing to the production of protons. The Panel has computed $F(x)$ over the range $0 \le x \le 30$ cm for two sets of values of α and γ representing upper and lower bounds for maximum permissible exposure.

EVALUATION OF THE LATERAL DISTRIBUTION OF ENERGY DISSIPATED BY A MOVING ION

Origin. J. Neufeld, W. S. Snyder, Health Physics Division.

Participating Members of Panel. V. C. Carlock, K. P. Graw.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1232.

Background and Status. The computations described in ORNL-1232 have been extended to include a tabulation of the total energy loss from $x = X$ to

$x = \infty$ for all X. The extended computations were for the same homogeneous media as were used in the previous calculations. The results of the computation will appear in an ORNL report by Neufeld and Snyder to be published during the next quarter. This project will be continued.

SOME PARAMETER ESTIMATES IN A NEAR WATER SOLUTION

Origin. L. C. Noderer, Long-Range Planning Group.

Participating Members of Panel. E. B. Carter and F. P. Barr, Numerical Analysis Laboratory, K-25, N. M. Dismuke, Mathematics Panel.

Background and Status. In connection with a study of the effect of holes in a reactor on the passage of neutrons, there was interest in estimates for certain parameters characterizing the slowing down of 2.5-Mev neutrons in a near water solution. Since neutron histories in tissue[6] were available, the following calculations for 2.5-Mev neutrons in tissue were made:

$$\left[\frac{\sum l^2}{N} \Big/ \left(\frac{\sum l}{N} \right)^2 \right]_{\text{avg}} = 2.39 \;,$$

$$\left[\frac{6\tau}{\sum l^2} \right]_{\text{avg}} = 1.99 \;,$$

$$\left[\frac{\sum l^2}{N} \right]_{\text{avg}} = 10.1 \;,$$

where

l = distance traveled by a neutron between collisions,

N = number of collisions a given neutron has from 2.5 Mev until it is absorbed or slowed to less than 1 ev,

$\sum$ means summation over the N collisions for a given neutron,

6τ = square of the crow-flight distance traveled by a neutron from 2.5 Mev until it is absorbed or slowed to less than 1 ev,

$[\;]_{\text{avg}}$ means an average over the total population of 1013 neutron histories.

ESTIMATION OF TOTAL PROTON TRACK LENGTH IN PHOTOGRAPHIC EMULSION

Origin. E. P. Blizard, J. L. Meem, Physics Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. In certain physics experiments, photographic emulsions are exposed to neutron bombardment. By microscopic examination, the experimenters would like to estimate the total proton track length in the emulsion.

Some recent results of Cornfield and Chalkley[7] are useful in dealing with this problem. They considered any three-dimensional figure of arbitrary shape with total surface

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(6) "Monte Carlo Estimate of Collision Distributions in Tissue," *Mathematics Panel Quarterly Progress Report for Period Ending January 31, 1952*, ORNL-1232, p. 16.

(7) J. Cornfield and H. W. Chalkey, "A Problem in Geometric Probability," *J. Wash. Acad. Sci.* **41**, 226-229 (1951).

area S, and let the figure be completely contained in a three-dimensional space of volume X. They then considered the experiment of throwing a needle of fixed length, r, at random into the space of volume X. Each time the needle intersects the surface of the arbitrary figure one "cut" is said to have occurred. These authors have shown that if this experiment is performed n times, the expected number of cuts is given by

$$E(c) = rnS/2X \; . \qquad (1)$$

This result combined with one other provides a method for estimating the volume-to-surface ratio in cases where irregular figures are involved. With a slight modification, however, it can be used to estimate the total proton track length in a photographic emulsion.

Equation 1 has been shown to hold also for the case in which S is the total surface area of a number of irregular figures contained in the volume X. For this discussion it is assumed that the emulsion is a rectangular parallelepiped and that each of the three dimensions is divided into H, W, and L segments each of length h, w, and l, respectively. Then the emulsion is divided into HWL identical rectangular parallelepipeds each of volume hwl, and thus the total volume of the emulsion is $X = HWL\ hwl$. The total surface area of the HWL parallelpipeds is

$$S = 2HWL\ (hw + hl + wl) \quad .$$

If one needle (proton track) of length r_i is thrown at random into the emulsion, it can be seen from Eq. 1 that

$$E(c) = \frac{r_i S}{2X} \; .$$

Likewise, if n needles of different lengths are thrown at random into the emulsion, it follows that

$$E(c) = \frac{S}{2X} \sum_{i=1}^{n} r_i = \frac{SR}{2X} \; ,$$

where

$$R = \sum_{i=1}^{n} r_i \; .$$

Clearly then an estimate of the total length of the proton tracks is given by

$$\hat{R} = \frac{2X}{S} K \; ,$$

where K is the number of intersections of the tracks with the surfaces of the small rectangular parallelepipeds. Substituting for X and S,

$$\hat{R} = \frac{hwl}{hw + hl + wl} K \; .$$

Since h, w, and l are known, the standard error of $\hat{R}$ is given by

$$\sigma_{\hat{R}} = \frac{hwl}{hw + hl + wl} \sigma_K \; .$$

As h, w, or l approaches zero, $\sigma_{\hat{R}}$ approaches zero. This must be true since in this case one actually measures the length of each proton track by dividing it into infinitesimally small segments. In practice this means that for σ_K independent of $E(K)$, greater accuracy can be achieved by making the dimensions of the small parallelepipeds smaller. If σ_K is a function of $E(K)$, not much can be said concerning the relationship between $\sigma_{\hat{R}}$ and h, w, and l until some experimental data are available. If K follows the Poisson law, the percentage error in $\hat{R}$ would be estimated by

$$\frac{\sigma_{\hat{R}}}{\hat{R}} = \frac{1}{\sqrt{K}} .$$

Preliminary experiments are under way that will provide a basis for determining $\sigma_{\hat{R}}$ and for deciding on optimum choices of the small parallelepiped dimensions.

BETA DECAY (Field Factors)

Origin. M. E. Rose, P. R. Bell, Physics Division.

Participating Members of Panel. C. L. Perry, H. B. Goertzel, N. M. Dismuke, C. P. Hubbard.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1232.

Status. During the quarter a flow chart was developed for calculating the field factors. Perry and Dismuke spent a week in Cambridge with members of the applications group of the Digital Computer Laboratory of Massachusetts Institute of Technology investigating the feasibility of calculating the field factors on the Whirlwind and learning something about coding and available subroutines for this machine. The investigation indicated that the required accuracy, 0.1%, could be achieved by using an interpretive subroutine called the (24, 6, 0), which calculates with seven-decimal-digit floating-point numbers and requires approximately 50 ordinary Whirlwind operations per floating-point operation in place of the four-decimal-digit fixed-point numbers used in normal Whirlwind arithmetic. Further, it appeared that the problem would fit into the memory in two parts, at most. A detailed examination of parts of the calculation was begun.

Several methods for calculating the gamma function of complex argument were compared by using for measure of goodness such considerations as calculating time, roundoff error, and routine length. The Sterling approximation seemed best, but an error analysis showed that this method calculated with the (24, 6, 0) subroutine would not yield the desired accuracy. To resolve this difficulty, it was decided that the IBM calculations for $(p/w)(F_\nu/F_0)$, which include the complex gamma function, would be used.

The functions Y_+ and Y_- are given by[8]

$$\left\{\begin{matrix} Y_+ \\ Y_- \end{matrix}\right\} = (s_\nu + iy)\, e^{-ipr_0}\, {}_1F_1(s_\nu + 1 + iy,\, 2s_\nu + 1;\, 2ipr_0) \left\{\begin{matrix} e^{i\eta_+} \\ e^{i\eta_-} \end{matrix}\right\} = \left\{\begin{matrix} R_+ + iI_+ \\ R_- + iI_- \end{matrix}\right\}$$

[8] M. E. Rose, "Relativistic Wave Functions in the Continuous Spectrum for the Coulomb Field," *Phys. Rev.* **51**, 484 (1937).

where ${}_1F_1(a, b; x)$ denotes the confluent hypergeometric series.[9] The code was written for

1. $(W - 1)\ I_-^2 + (W + 1)\ R_+^2$

2. $-(W - 1)\ I_-^2 + (W + 1)\ R_+^2$

3. $(W - 1)\ I_+^2 + (W + 1)\ R_-^2$

4. $-(W - 1)\ I_+^2 + (W + 1)\ R_-^2$

5. $-\sqrt{W^2 - 1}\ I_+R_+ + \sqrt{W^2 - 1}\ I_-R_-$

6. $-\sqrt{W^2 - 1}\ I_+R_+ - \sqrt{W^2 - 1}\ I_-R_-$

e^{-ipr_0} and ${}_1F_1(s_\nu + 1 + iy, 2s_\nu + 1; 2ipr_0)$ were calculated by Taylor series by using as many terms as required to obtain the desired accuracy. A few calculations were made by using desk calculators and following the proposed scheme for Whirlwind calculation. These computations showed that for some parameter values enough significant digits were lost in the addition or subtraction indicated in Eqs. 1 to 6 that 0.1% precision could not be expected.

Since the time of the conference at MIT, the Whirlwind group has developed a subroutine called the (39, 6, 0), which calculates with 12-decimal-digit floating-point numbers and requires approximately 75 ordinary Whirlwind operations per floating-point operation. Using this subroutine would improve the situation some, but might not remove the difficulty. However, the following plan has been agreed upon. Equations 1 to 6 will be calculated using the new subroutine. The code will instruct the Whirlwind to print out answers along with the number of significant digits. If there are regions for which the table is not sufficiently accurate, these calculations will be repeated on the ORAC when it is available for computations.

NEUTRON DECAY THEORY

Origin. L. C. Biedenharn, H. Reynolds, A. H. Snell, Physics Division.

Participating Members of Panel. J. H. Fishel, C. P. Hubbard, C. Perhacs.

Background and Status. In order to evaluate some theories of neutron decay, it is necessary to perform the following integrations.

$$I_1 = 2\int_a^b \sqrt{E(E+2)}\ (E+1)\ (1.530 - E)^2 \left\{ \frac{1 + \dfrac{E(E+2)\cos 2\theta}{(1.530 - E)^2}}{\sqrt{1 - \dfrac{E(E+2)\sin^2\theta}{(1.530 - E)^2}}} \right\} dE$$

[9] The Greuling approximations for the field factors represent $e^{-x}[{}_1F_1(a, b; x)]^2$ by expanding $e^{-x/2}$ and ${}_1F_1(a, b; x)$ in Taylor series, squaring, multiplying these together, and keeping one term for $(\pi p/W)\ L_\nu$, three terms for $(\pi p/W)\ M_\nu$, and two terms for $(\pi p/W)\ N_\nu$.

$$I_2 = \int_a^b \sqrt{E(E+2)}\,(E+1)\,(1.530-E)^2\,\frac{E(E+2)}{(E+1)(1.530-E)}\cdot$$

$$\left\{\frac{1+\dfrac{E(E+2)\cos 2\theta}{(1.530-E)^2}}{\sqrt{1-\dfrac{E(E+2)\sin^2\theta}{(1.530-E)^2}}}\sin^2\theta - 2\cos^2\theta\sqrt{1-\frac{E(E+2)\sin^2\theta}{(1.530-E)^2}}\right\} dE$$

where

$$a = 0.4626$$

$$b = \tan^2\theta\left\{1+\frac{1.530}{\sin^2\theta}-\sqrt{\frac{(1.530)^2}{\sin^2\theta}+\frac{3.060}{\sin^2\theta}+1}\right\}$$

Because of the complexity of the integrands, and since the integrals are desired for several values of the parameter θ, the integration is being performed on IBM equipment.

It is expected that the work will be completed during the next quarter.

ENERGY LEVELS OF NUCLEI

Origin. E. D. Klema, Physics Division.

Participating Member of Panel. C. P. Hubbard.

Background and Status. One theory gives the energy level of an excited nucleus as $E_{n,j}$, where

$$E_{n,j} = \alpha\,[(n-1)(n+2)]^{1/2} + \beta(AZ^4)^{-1/3} j(j+1) \quad ,$$

n = vibration quantum number,

j = rotation quantum number,

A = mass number of nucleus,

Z = charge of nucleus,

and α and β are parameters for which the best estimates are 0.387 and 1.44, respectively.

The Panel, using more recent observed energy levels, is assisting in calculating better estimates for α and β to evaluate the theory.

CALCULATION OF INTERNAL CONVERSION COEFFICIENTS WITH SCREENING

Origin. M. E. Rose, Physics Division.

Participating Member of Panel. M. R. Arnette.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1232.

Status. The coding of the problem for the SEAC and checking of the code on the SEAC has continued. Owing to scaling difficulties in the computations, the coding time and the machine computation time will be longer than was originally estimated. The present estimate is that the coding will be completed in June 1952 and that the calculations will take about 200 hr of SEAC time.

DETERMINATION OF THE FAST NEUTRON FLUX OF THE X-10 GRAPHITE PILE

Origin. D. K. Holmes, Physics Division.

Participating Members of Panel. J. Moshman, H. B. Goertzel, E. B. Carter (K-25).

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1232.

Status. A series of 294 complete neutron histories was run on IBM machines. About 20 min of machine time was required per history. The preliminary results are now being analyzed in an attempt to find means for simplifying and speeding up the machine procedure. It is tentatively planned to continue the random walks in the pile as if it were uniformly composed of the graphite moderator. The resulting estimate of the flux would then be corrected for the actual presence of the fuel rods.

CALCULATION OF RACAH COEFFICIENTS FOR THE ANGULAR DISTRIBUTION IN NUCLEAR REACTIONS

Origin. M. E. Rose and L. C. Biedenharn, Physics Division.

Participating Members of Panel. S. L. Hull, R. C. Weaver, M. Tsagaris, P. J. Brown, E. A. Forbes.

References. L. C. Biedenharn, *Tables of the Racah Coefficients*, ORNL-1098, April 8, 1952; *Physics Division Quarterly Progress Report for Period Ending March 20, 1951*, ORNL-1005; *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, -1091, -1151, and -1232.

Background and Status. The computation described in ORNL-1232 has been completed, and the *Tables of Racah Coefficients* was published during the quarter. Biedenharn has developed some new recurrsion relations which are being used to check the values in the tables. An errata sheet containing all errors found in the tables is being prepared for publication as a supplement to ORNL-1098.

T + D AND He^3 + D CROSS SECTIONS

Origin. A. Simons, Physics Division.

Participating Member of Panel. H. B. Goertzel.

Reference. *Mathematics Panel Quarterly Progress Report for the Period Ending July 31, 1951*, ORNL-1091.

Background and Status. The calculation of the cross section for the T + D and He^3 + D reaction from a formula involving exact coulomb wave functions fitted empirically at the nuclear radius, completed in 1951, was extended to the region of low energy – specifically for the energy range 400 to 240 kev. This project was completed.

MIXED INTERNAL CONVERSION CORRELATION COEFFICIENTS

Origin. M. E. Rose, Physics Division.

Participating Member of Panel. C. Perhacs.

Background and Status. This problem is an extension of the previously reported angular correlation coefficients. The values obtained in the intermediate stages of the unmixed angular correlation computations were used as starting values for the mixed correlation computations. The mixed correlation coefficients were obtained for the same parameter range of l, k, z as for the unmixed coefficients. The computations have been completed and checked.

REACTOR RESPONSE CURVES

Origin. J. Trimmer, Reactor Technology Division.

Participating Members of Panel. R. C. Weaver, C. L. Perry, W. C. Sangren.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1151.

Background and Status. The Mathematics Panel was asked to find the steady-state solution of the differential equation

$$\frac{du}{dt} + \frac{u}{1 + m \sin \beta t} = 1$$

for several values of β and m ($|m| < 1$). It was found that any solution to this differential equation is an almost periodic function asymptotic to the unique periodic solution determined by m and β alone. Thus the steady-state solution was found by evaluating the periodic solution over one period (period = $2\pi/\beta$). The solutions were evaluated by the REAC at Argonne and checked in part by numerical integration. This project was completed.

SECRET PROJECTS

MACHINE COMPUTATION OF SOLUTIONS FOR THE NEUTRON TRANSPORT EQUATION

Origin. W. K. Ergen (ANP Project), A. S. Householder (Mathematics Panel).

Participating Member of Panel. N. Edmonson.

Background and Status. The purpose of this investigation is to determine the feasibility of computing solutions of the transport equation on the Oak Ridge Automatic Computer. The treatment of the transport equation given by Safonov[10] is being studied at present.

MISCELLANEOUS COMPUTATIONS FOR ANP PROJECT

Origin. Members of ANP research staff.

Participating Members of Panel. E. A. Forbes, P. J. Brown, R. C. Weaver, M. Tsagaris.

Background and Status. This is a computing project set up to handle the computations necessary for the ANP project. Computations have recently

[10] G. Safonov, *Certain Multi-group Equations for Critical Reactors*, RM-462, Sept. 26, 1950.

been made of heat transfer, bare reactor, and reflected reactor problems. Computations have also been made for the Critical Reactor Group. This computing group is set up to function for as long as any need for it exists.

DEPIGMENTATION AS A BIOLOGICAL DOSIMETER – OPERATIONS GREENHOUSE

Origin. A. C. Upton, Biology Division.

Participating Members of Panel. J. Moshman, K. P. Graw.

Background and Status. A marked tendency to lose pigmentation has been observed among the surviving Greenhouse mice. The pattern rather consistently starts at the head and shoulders and proceeds to the rump – colors range from the natural dark brown to pure white. A sample of mice from all stations having survivors was selected and individually observed. The coat was divided into six sections and each section was assigned a number between 0 and 4, inclusive, which indicated the degree of depigmentation (zero reflected the original brown and 4 a pure white).

A linear discriminant function was set up in the form

$$y = \sum_{i=1}^{6} \lambda_i p_i$$

where p_i is the value of the pth digit, $0 \le p_i \le 4$, y is the dose received, and λ_i is the corresponding coefficient. It is hoped greying at some or all anatomical locations may prove significant in discriminating between dose groups. Calculations are now under way to compute the λ's and assess their significance.

TWO-GROUP THREE-REGION COMPUTATIONS FOR A PRODUCTION REACTOR

Origin. J. R. Parks, Long-Range Planning Group.

Participating Members of Panel. H. B. Goertzel, E. N. Lawson.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1232.

Background and Status. The calculation of the core multiplication constant, the radial buckling constant, and the fast- and slow-neutron fluxes and currents that was started last quarter has been completed. In addition computations have been completed for 16 new production reactor designs.

BULK SHIELDING FACILITY

Origin. L. Meem, Physics Division.

Participating Member of Panel. V. C. Carlock.

Background and Status. The determination of the saturated activities of a large number of foils irradiated in the bulk shielding facility was completed.

KINETICS OF HRE

Origin. T. A. Welton (Physics Division of HRP), R. B. Briggs (Research Director's Division).

Participating Members of Panel. N. D. Given, C. P. Hubbard, W. C. Sangren.

References. W. C. Sangren, *Kinetic Calculations for Homogeneous Reactors*, ORNL-1205, April 1, 1952; *Mathematics Panel Quarterly Progress Reports*, ORNL-1151 and -1232; *Homogeneous Reactor Project Quarterly Progress Reports*, ORNL-1221 and -1280.

Background and Status. The results of the investigation through February have been published in ORNL-1205. With the aid of the REAC at Argonne National Laboratory numerous power and temperature curves resulting from step and linear changes in reactivity have been obtained. The REAC was also used for a preliminary investigation of the kinetic consequences of changing the parameters associated with a one-group assumption for the delayed neutrons.

CHARACTERISTICS OF EXPLOSION AND DETONATION OF COMBUSTIBLE GASES GENERATED BY THE ISHR

Origin. T. H. Pigford, Reactor Experimental Engineering Division.

Participating Member of Panel. N. D. Given.

Background and Status. The object of this calculation is to determine as accurately as possible the characteristics of explosions and detonations of combustible gases generated by the ISHR. By an iterative method, solutions of a number of algebraic equations are obtained that give explosion pressures, impact pressures, and detonation velocities. The calculations of pressures resulting from constant-volume explosion, in which the effect of dissociation of the reaction products is considered, have been completed. The rest of the calculation is in progress.

BOILING REACTOR STUDIES

Origin. P. R. Kasten, Reactor Experimental Engineering Division.

Participating Member of Panel. E. N. Lawson.

Background and Status. Static calculations have been performed to determine the critical mass of the Teapot reactor under various operating conditions. Also, the effectiveness of a central control rod has been calculated for various rods and thimble diameters for these operating conditions. Calculations are now being made to determine parameter relations for kinetic studies of boiling reactors.

KINETICS OF BOILING REACTORS

Origin. P. R. Kasten, Reactor Experimental Engineering Division.

Participating Members of Panel. N. D. Given, C. P. Hubbard, W. C. Sangren.

Background and Status. Numerous systems of nonlinear differential equations describing the kinetics of boiling reactors were solved with the REAC and the results were turned over to Kasten. Preliminary calculations were carried out with desk computers to determine the parameter values to be used in the computations of the differential equations.

CIRCULATION IN BOILING REACTORS

Origin. P. C. Zmola, Reactor Experimental Engineering Division.

Participating Member of Panel. E. N. Lawson.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1232.

Background and Status. Calculations and graphing for Boiling Reactor designs based on formulas derived by Zmola have been completed.