ORNL-1091
Progress

9A

[redacted]

MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

FOR THE PERIOD ENDING JULY 31, 1951

OAK RIDGE NATIONAL LABORATORY
U.S. ATOMIC ENERGY COMMISSION

OAK RIDGE NATIONAL LABORATORY
OPERATED BY
CARBIDE AND CARBON CHEMICALS COMPANY
A DIVISION OF UNION CARBIDE AND CARBON CORPORATION
UCC
POST OFFICE BOX P
OAK RIDGE, TENNESSEE

[redacted]

[redacted]

ORNL-1091

This document consists of 31 pages.

Copy 7 of 159. Series A

Contract No. W-7405, eng 26

MATHEMATICS PANEL
QUARTERLY PROGRESS REPORT
for the Period Ending July 31, 1951

A. S. Householder, Chief

DATE ISSUED: NOV 12 1951

OAK RIDGE NATIONAL LABORATORY
operated by
CARBIDE AND CARBON CHEMICALS COMPANY
A Division of Union Carbide and Carbon Corporation
Post Office Box P
Oak Ridge, Tennessee

[illegible]

[illegible]

[redacted]

[redacted] MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

INTERNAL DISTRIBUTION

ORNL-1091
Progress

1. G. T. Felbeck (C&CCC)
2-3. Chemistry Library
4. Physics Library
5. Biology Library
6. Health Physics Library
7. ~~Metallurgy Library~~
8-9. Training School Library
10-13. Central Files
14. C. E. Center
15. C. E. Larson
16. W. B. Humes (K-25)
17. W. D. Lavers (Y-12)
18. A. M. Weinberg
19. J. A. Swartout
20. [redacted]
21. E. J. Murphy
22. F. C. VonderLage
23. A. S. Householder
24. A. H. Snell
25. E. H. Taylor
26. A. Hollaender
27. [redacted] Briant
28. [redacted]
29. M. T. Kelley
30. K. Z. Morgan
31. W. H. Pennington
32. R. Crook
33. M. L. Nelson
34. E. O. Wollan
35. E. P. Blizard
36. R. S. Livingston
37. [redacted]
38. G. H. Clewett
39. C. D. Susano
40. J. Moshman
41. C. L. Perry
42. R. A. Charpie
43. W. C. Sangren
44. N. M. Dismuke
45. N. Edmonson
46. A. W. Kimball
47. M. R. Arnette
48. D. D. Cowen
49. [redacted]

EXTERNAL DISTRIBUTION

50-52. Aircraft Nuclear Propulsion Project, Oak Ridge
53-64. Argonne National Laboratory
65. Armed Forces Special Weapons Project
66-71. [redacted] Energy Commission, Washington
72. [redacted] Memorial Institute
73-76. Brookhaven National Laboratory
77. [redacted]
78-79. Carbide & Carbon Chemicals Company (C-31 Plant)
80-81. [redacted] (K-25 Plant)
82-85. Carbide & Carbon Chemicals Company (Y-12 Area)
86. Chicago Patent Group
87. Chief of Naval Research
88. [redacted] University (Havens)

[redacted]

[redacted]

[redacted]

89-91. du[redacted]t Company
92. H. K. Ferguson Company
93-96. General Electric Company, Richland
97. Hanford Operations Office
98-101. Idaho [redacted] Office
102. Iowa State College
103. Kellex Corporation
104-107. Knolls Atomic Power Laboratory
108-110. Los Alamos
111. Mallinckrodt Chemical Wo[redacted]
112-114. Mound Laboratory
115. National Advisory Committee for Aeronautics
1[redacted]al Bureau of Standards
117. Naval Medical Research Institute
118. Naval Research Laboratory
11[redacted] New Brunswick Laboratory
[redacted] Operations Office
12[redacted]. North American Aviation, Inc.
[redacted]5. Pa[redacted], Washington
12[redacted] [redacted]ion
12[redacted] Sa[redacted] Operations O[redacted]
128. [redacted]ratory
12[redacted] [redacted] - Headquarters
130[redacted] S. Naval Radiological Defense Laborat[redacted]
131-13[redacted] [redacted]ty of California Radiation Laboratory
136-137. [redacted]iversity of Rochester
138-141. We[redacted]inghouse Electric Corpo[redacted]
142-144. W[redacted]ight-Patterson Air Force Base
145-159. T[redacted] Information Service, Oak Ridge

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

Mathematics Panel quarterly progress reports previously issued in this series are as follows:

ORNL-345	December, January, February, 1948-1949
ORNL-408	Period Ending July 31, 1949
ORNL-516	Period Ending October 31, 1949
ORNL-634	Period Ending January 31, 1950
ORNL-726	Period Ending April 30, 1950
ORNL-818	Period Ending July 31, 1950
ORNL-888	Period Ending October 31, 1950
ORNL-979	Period Ending January 31, 1951
ORNL-1029	Period Ending April 30, 1951

[redacted]

[redacted]

TABLE OF CONTENTS

[redacted]

[redacted]

TABLE OF CONTENTS (continued)

[redacted]

[redacted]

1. SUMMARY

Mrs. Gwen Wicker, formerly of the Biology Division, has joined the Mathematics Panel as a permanent employee. Dr. Ernest Ikenberry, of Alabama Polytechnic Institute, is with the Mathematics Panel for the summer as a Research Participant.

Dr. Householder has collaborated with the Argonne computer group in completing the logical design of the control circuity for the Oak Ridge digital computer. A summary of the design problem, which was completed this quarter, is included in this report (p. 2).

The largest computing task undertaken by the Panel this quarter was the calculation of the angular correlation coefficients (p. 10). The computations are being performed on the IBM card programmed calculators at K-25, and completion of the task is expected during the following quarter. The calculation of the threshold values for the angular correlation coefficients is pending and will probably follow the angular correlation coefficient calculations.

Dr. N. Edmonson, Dr. E. Ikenberry, and R. R. Coveyou are investigating and redesigning the computations being performed for ANP (Aircraft Nuclear Propulsion Project). The results of these investigations will be reported in an ANP report. An outline of one of the new methods proposed for the calculations is included in this report (p. 3). Ann Forbes, Rooney Weaver, Mildred Guernsey, Marina Tsagaris, and Phyllis Brown are on temporary loan to ANP to assist in computations.

Drs. H. L. Lucas and J. Z. Hearon visited the Laboratory May 14-15 and May 28-29, respectively, for consultation on biometric problems. Dr. J. W. Tukey (p. 11) consulted with the Panel June 11-12 on several statistics problems. Herman Kahn visited the Panel July 16-17 and while here discussed Monte Carlo methods with A. S. Householder, G. E. Albert, Lewis Nelson, and J. Moshman.

Professors W. M. Whyburn and V. A. Hoyle of the University of North Carolina visited the Panel in May to discuss B. M. Drucker's work. Mr. Drucker is an ORINS Fellow from the University of North Carolina who is writing his thesis at ORNL on some problems relative to the numerical solution of differential equations.

Drs. W. C. Sangren and C. L. Perry presented mathematics colloquia entitled *Generalized Sturm-Liouville Expansions* and *Programming for Automatic Computing Machines* at the University of Tennessee May 16, 1951 and May 2, 1951, respectively.

Dr. A. W. Kimball presented a paper entitled *Microbiological Assays with Nonparallel Response Curves* at the Gordon Research Conferences, New Hampton, New Hampshire, July 26, 1951.

. . . . .

[redacted]

2. SPECIAL PROJECTS

THE OAK RIDGE COMPUTER

Work on the design of the control system for the Oak Ridge digital computer has progressed considerably during the past quarter with A. S. Householder spending most of June and July at Argonne working on the project with Dr. D. A. Flanders and Mr. James Alexander. Mr. Howard Parsons of Los Alamos Scientific Laboratory and Dr. Flanders met with Dr. Householder at ORNL on May 28 and 29, and Mr. Parsons gave a description of the design of the control system of the Los Alamos computer. This part of the Los Alamos machine already has been built and tested.

A tentative list of the operations to be built into the Oak Ridge machine is as follows: (1), eight memory to accumulator transfers, with or without clearing, and by addition or subtraction of the number or its magnitude; (2) a single memory to quotient register transfer; (3) multiplication with or without round-off; (4) division, in which initial contents of the quotient register may or may not be retained as an added 39 digits of the dividend; (5) 12 distinct types of shift to be described below; (6) three types of transfer of control, each to the left or right, one type being unconditional, one type being conditional based upon sign, and one type being conditional based upon overflow in the accumulator; and (7) transfers from either accumulator or quotient register of the left or right address, left or right order, both addresses, or the entire word. The possible transfer of half words would be useful in half-precision operations.

If the original contents of the digital positions of the accumulator register are α_0, α_1 ... α_{39} and of the quotient register are β_0, β_1 ... β_{39}, the shifts are characterized by specifying the new contents after a single shift. The following table lists these:

LEFT SHIFTS

(1)	α_1	$\alpha_2\ldots\alpha_{39}$	0	; 0	0 ... 0	α_0
(2)	α_1	$\alpha_2\ldots\alpha_{39}$	0	; β_1	$\beta_2\ldots\beta_{39}$	α_0
(3)	α_1	$\alpha_2\ldots\alpha_{39}$	β_0	; β_2	$\beta_2\ldots\beta_{39}$	α_0
(4)	α_1	$\alpha_2\ldots\alpha_{39}$	0	; β_0	$\beta_1\ldots\beta_{38}$	β_{39}

(5) same as (4) but stops when $\alpha_1 \neq \alpha_x$.

RIGHT SHIFTS

(1)	0	$\alpha_0\ldots\alpha_{37}$	α_{38} ;	β_0	$\beta_1\ldots\beta_{38}$	β_{39}
(2)	0	$\alpha_0\ldots\alpha_{37}$	α_{38} ;	α_{39}	$\beta_0\ldots\beta_{37}$	β_{38}
(3)	β_{39}	$\alpha_0\ldots\alpha_{37}$	α_{38} ;	α_{39}	$\beta_0\ldots\beta_{37}$	β_{38}
(4)	α_x	$\alpha_0\ldots\alpha_{37}$	α_{38} ;	β_0	$\beta_1\ldots\beta_{38}$	β_{39}
(5)	α_x	$\alpha_0\ldots\alpha_{37}$	α_{38} ;	α_{39}	$\beta_0\ldots\beta_{37}$	β_{38}
(6)	α_x	$\alpha_0\ldots\alpha_{37}$	1 ;	β_0	$\beta_1\ldots\beta_{38}$	β_{39}
(7)	α_x	$\alpha_0\ldots\alpha_{37}$	1 ;	α_{39}	$\beta_0\ldots\beta_{37}$	β_{38}

The digit α_x resides in an auxiliary toggle which at the outset is set equal to α_0 and remains fixed throughout the operation. This is therefore the sign digit of the initial contents of the accumulator. Hence the fifth left shift proceeds until a further shift would produce a number outside the range of the machine, i.e., ≥ 1 or <-1. The number of shifts will be registered in the shift counter and transferred into the quotient register when the shift is complete. Right shifts 4, 5, 6, and 7 are power shifts (yielding a product by a negative power of 2). Shift 6 is a round-off right shift; shift 7 is accidental and possibly useless.

Mr. William J. Gerhard has recently gone to Argonne to assist in the construction of the Oak Ridge machine.

NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS; MULTIPOINT BOUNDARY PROBLEMS

In the course of an examination of the method [R. R. Coveyou, *The Calculation of Eigenvalues of Differential Systems by Numerical Integration*, Y-F10-72 (Aug. 30, 1951)] used currently by the ANP Physics Group for the numerical integration of the age-diffusion equation of reactor theory, an observation was made that seems to be of more general interest.

The major difficulty encountered in the numerical integration of multipoint boundary value problems is that the starting values for the numerical integration are not uniquely determined by the boundary conditions at the origin. Hence, some method must be found for adjusting the obtained solution in order that all boundary conditions be met. Several techniques exist which will do this; this discussion gives the details of one method which seems interesting and and convenient.

Consider the system

SYSTEM I

$$\phi_0''(x) + A_0(x)\phi_0'(x) + B_0(x)\phi_0(x) + S_0(x) = 0$$

$$\phi_1''(x) + A_1(x)\phi_1'(x) + B_1(x)\phi_1(x) + S_1(x) = 0$$

$$a_0\phi_0(0) + b_0\phi_0'(0) = 0$$

$$c_0\phi_0(I) + d_0\phi_0'(I) = c_1\phi_1(I) + d_1\phi_1'(I)$$

$$e_0\phi_0(I) + f_0\phi_0'(I) = e_1\phi_1(I) + f_1\phi_1'(I)$$

$$0 = g_1\phi_1(B) + h_1\phi_1'(B) ,$$

with

$$(c_0f_0 - d_0e_0)(c_1f_1 - d_1e_1) \neq 0 .$$

Systems of this type appear, for instance, in the solution of reactor equations.

Along with System I, we consider the associated homogeneous system.

SYSTEM II

$$\psi_0''(x) + A_0(x)\psi_0'(x) + B_0(x)\psi_0(x) = 0$$

$$\psi_1''(x) + A_1(x)\psi_1'(x) + B_1(x)\psi_1(x) = 0$$

$$\psi_0(0) = b_0$$

$$\psi_0'(0) = -a_0$$

$$c_0\psi_0(I) + d_0\psi_0'(I) = c_1\psi_1(I) + d_1\psi_1'(I)$$

$$e_0\psi_0(I) + f_0\psi_0'(I) = e_1\psi_1(I) + f_1\psi_1'(I)$$

Define

$$W_0(x) = \psi_0'(x)\phi_0(x) - \psi_0(x)\phi_0'(x) \quad (1)$$

$$W_1(x) = \psi_1'(x)\phi_1(x) - \psi_1(x)\phi_1'(x) \quad (2)$$

The following consequences are readily derived:

SYSTEM III

$$W_0'(x) + A_0(x)W_0(x) = S_0(x)\psi_0(x)$$

$$W_1(x) + A_1(x)W_1(x) = S_1(x)\psi_1(x)$$

$$W_0(0) = 0$$

$$W_1(I) = \frac{c_0 f_0 - d_0 e_0}{c_1 f_1 - d_1 e_1} W_0(I) \; .$$

Now, consider the system

SYSTEM IV

$$\psi_1(x)\phi_1'(x) - \psi_1'(x)\phi_1(x) + W_1(x) = 0$$

$$\psi_0(x)\phi_0'(x) - \psi_0'(x)\phi_1(x) + W_0(x) = 0$$

$$g_1\phi_1(B) + h_1\phi_1'(B) = 0$$

$$c_0\phi_0(I) + d_0\phi_0'(I) = c_1\phi_1(I) + d_1\phi_1'(I) \; .$$

If the ψ's and W's satisfy II and III respectively, then the ϕ's calculated from IV are easily seen to be the unique solution of I.

The application of the method is as follows: Systems II and III are integrated from the origin outwards (numerically, in most cases). Then, at the completion of this integration, one integrates System IV from B inward towards its origin (also numerically).

The method has the striking advantage over most commonly used methods of being nontentative in character. A special case of this method is implicitly contained in the work of the KAPL reactor physics group or the numerical integration of the age-diffusion equations. There, however, the basic idea is applied to the approximating difference equations and is well masked in algebraic complication.

• • • • •

3. SPECIAL PROBLEMS

COUNTER STATISTICS

Participating Members of Panel. G. E. Albert and Lewis Nelson.

References. (*a*) G. E. Albert and M. L. Nelson, Mathematics Panel quarterly report ORNL-979, p. 10; (*b*) M. G. Kendall, *Advanced Theory of Statistics*, vol. 1, Lippincott, Philadelphia, 1944.

Background. It is assumed that a sequence f of events is distributed in time in such a way that the number $N_p(t)$ occurring in any time interval of length t is a chance variable having the Poisson distribution with mean at where a is some positive constant; i.e.,

$$Pr[N_p(t) = x] = e^{-at} \frac{(at)^x}{x!} , \quad (1)$$

$$x = 0, 1, 2, \ldots .$$

The sequence f may be a sequence of radioactive particles striking a counter or a sequence of vehicle wheels passing over a traffic counter pickup, etc.

A counting device generates a second sequence g of events, since, due to locking effect, the counter does not register all events of f. Two types of locking effect have received consideration in the literature [reference (*a*)].

The participants appear to have been the first persons to consider the statistical problem of inferring the value of the mean rate a of events in f from an actual count $N_r(t)$ of the sequence g by modern statistical methods. Confidence intervals for the rate a for a type I counter were reported in reference (*a*).

Status. 1. *A New Dead Time Model.* It has been recognized for some time that actual counters do not behave in accordance with either of the idealized locking effects called types I and II but that these idealizations are likely opposite extremes for the true situation. With this in mind the following model for the locking effect is proposed:

Let two constants π and u be given subject to the restrictions $u > 0$ and $0 \leqq \pi \leqq 1$.

(i) An event E in f can be registered as an event in g only if none has been registered during a time u preceding E.

(ii) An event E in f will be registered in g if it follows its predecessor in f by more than time u.

(iii) If an event E in f follows its predecessor in f by time less than or equal to u, the registration of E as an event in g is subject to chance, the probability of registration being π and of nonregistration being $1 - \pi$.

The choice $\pi = 0$ reduces this model to the usual type I model, while the choice $\pi = 1$ reduces it to the usual type II model.

2. *Distribution Theory.* The first problem is to determine the distribution

$$Pr[N_r(t) = x] \ , \quad x = 0, 1, 2, \ldots \ , \qquad (2)$$

or some operational transform of it, of the number $N_r(t)$ of events (registrations) in the sequence g in a time interval of length t. This is achieved by reducing the model to a type II model (see the lemma below) and applying the theory of the type II counter as given by Feller.

Lemma: Under the restrictions imposed by the new model, if further, $0 < \pi$, then the counter behaves in all respects like a type II counter with the true rate a in (1) replaced by an effective rate $a\pi$.

Feller's theory of the type II counter is now applicable for the determination of the Laplace transform of the distribution (2) of registrations $N_r(t)$ in a time interval of fixed length t, and to the determination of the transform of the distribution of random times T to a fixed number n of registrations, i.e., the distribution

$$Pr[T \leqq t | N_r(T) = n] \ . \qquad (3)$$

The case $\pi = 0$ is included by continuity of the transforms at $\pi = 0$.

3. *Confidence Intervals for a.* The distributions (2) and (3) have been normalized by the Cornish-Fisher method [see reference (*b*), pp. 156-159], and confidence intervals for the product $b = au$ have been obtained for counting experiments in which either the observation interval t is fixed in advance and the number of registrations $N_r(t)$ are observed, or a number n of registrations to be allowed is fixed in advance and the random time T to the condition $N_r(T) = n$ is observed. In both cases the count N_r should be fairly large, say greater than 50, since the Cornish-Fisher normalization method is asymptotic on N_r.

The complete results are much too complicated for practical use. It may be possible to prepare easily used charts from these results at some future date. A relatively simple result is available for the case in which $N_r \geqq 100$ and $b \leqq 0.1$.

Let x_p be that number such that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x_p} e^{-\frac{1}{2}x^2} \, dx = p \ .$$

Useful values are tabulated below:

p	x_p
.975	1.960
.025	-1.960
.950	1.645
.050	-1.645
.750	0.674
.250	-0.674

Let

$$A_p = u \left\{ N_r + x_p\sqrt{N_r} + \frac{1}{3}(x_p^2 - 1) + \frac{x_p^3 - 7x_p}{36\sqrt{N_r}} \right\} \qquad (4)$$

and

$$L_p = \frac{A_p}{t - (N_r - 1)u}\left\{1 + \frac{\pi}{2}\left[\frac{N_r u}{t - (N_r - 1)u}\right]^2\right\} . \quad (5)$$

If $N_r \geqq 100$, $b \leqq 0.1$, and $p_1 < p_2$, the interval

$$L_{p_1} < b \leqq L_{p_2} \quad (6)$$

will be an approximate $100(p_2 - p_1)$ percent confidence interval for b when the observation time t is fixed in the experiment. The same result holds in a fixed count experiment with t changed to the random observed time T.

For very large N_r, insignificant terms in (4) and (5) may be dropped. It is noteworthy that the counter type parameter π enters (4) and (5) only in a rather insignificant way. The entire term

$$\frac{\pi}{2}\left[\frac{N_r u}{t - (N_r - 1)u}\right]^2$$

is only a small correction factor. Thus, in most problems the counter type is probably immaterial in the estimation of b.

BASIC STUDIES IN THE MONTE CARLO METHOD

Participating Member of Panel. G. E. Albert.

References. (*a*) Mathematics Panel quarterly report ORNL-1029, p. 17; (*b*) G. E. Albert, memorandum to A. S. Householder, *A General Approach to the Monte Carlo Estimation of the Solutions of Certain Fredholm Integral Equations*, Part I; (*c*) S. S. Wilks, *Mathematical Statistics*, Princeton University Press, Princeton, 1943.

Background. Reference (*b*) is a mathematical rather than physical exposition of various well-known ways of estimating solutions of certain types of integral equations and of estimating functionals of such solutions. The medium for estimation is a fairly general stochastic process depending upon an integral-valued parameter. This memorandum has been given a fairly wide circulation in order to invite comments, suggestions for further study, and information on estimation methods that have been overlooked.

Status. Part II of the memorandum [reference (*b*)] is in preparation. It will include some modifications of material given in Part I, a comprehensive study of the use of representative (stratified) sampling [reference (*c*)] to reduce estimation errors, and, at the suggestion of Herman Kahn, it will include a study of the use of correlated processes.

Research on the use of representative sampling is tentatively completed. The fundamental idea is discussed below.

The basic idea of estimating the solution $\phi(x)$ of

$$\phi(x) = g(x) + \lambda \int_a^b K(x,y)\phi(y)dy$$

is to expand it in its Neumann series,

$$\phi(x) = g(x) + \sum_{n=1}^{\infty} \lambda^n \int_a^b \cdots \int_a^b K(x,x_1)\ldots K(x_{n-1},x_n)g(x_n)dx_1\ldots dx_n \ ; \qquad (1)$$

introduce a stochastic process which furnishes a distribution

$$Pr(k = n), \qquad n = 0, 1, 2, \ldots \qquad (2)$$

on the integers and a distribution of chain points $x_1, x_2, \ldots, x_k$,

$$f(x_1, x_2, \ldots, x_n | k = n), \qquad (3)$$

conditioned by the value of k (both are usually conditioned by the value of x); draw an integer k at random from (2); draw a chain at random from the appropriate density (3); use the chain and (3) to estimate the kth term of (1); and divide that estimate by $Pr(k)$ to obtain an estimate of $\phi(x)$. Repeating the drawing M times and averaging the estimates of ϕ so obtained gives the final estimate.

In representative sampling the integers $n = 0, 1, 2, \ldots$ are partitioned into a finite number of groups, say S, and $M_1, M_2, \ldots, M_S$ chains are drawn from each group. If $M = \Sigma M_i$ and the M_i's are proportional to the probabilities of the groups as given by summing (2) over the groups, the variance of the average will be smaller than if straight random sampling for all M chain lengths is used. Further reduction is achieved in the variance by stratifying the sampling of the chain points.

There is more to this procedure of representative sampling than meets the eye. To illustrate, the totally impractical problem of estimating a definite integral of the form

$$I = \int_0^1 g(x)dx \qquad (4)$$

will be discussed briefly. It will be assumed that $g(x)$ is bounded and non-negative on (0, 1).

Let $f(x)$ be any frequency function defined and positive over (0, 1). Partition (0, 1) by

$$0 = x_0 < x_1 < \ldots < x_S = 1$$

and define.

$$P_j = \int_{x_{j-1}}^{x_j} f(x)dx \ ,$$

$$f_j(x) = \begin{cases} f(x)/P_j & \text{for } x_{j-1} \leqq x \leqq x_j \\ 0 & \text{elsewhere} \end{cases} \qquad (5)$$

for each $j = 1, 2, \ldots, S$. Suppose that N and $N_j = NP_j$ and $j = 1, 2, \ldots, S$ are integers. For each j, draw N_j values of x, say

$$x_{1,j}, \ldots, x_{N_j,j} \text{ from } x_{j-1} \leqq x \leqq x_j,$$

using the conditional distribution (5). The quantity

$$\bar{I}_j = \frac{1}{N_j} \sum_{n=1}^{N_j} \frac{g(x_{n,j})}{f(x_{n,j})}$$

will be an unbiased estimate of

$$I_j = \frac{1}{P_j} \int_{x_{j-1}}^{x_j} g(x)dx$$

and

$$\bar{I}_R \equiv \sum_{j=1}^{S} \frac{N_j}{N} I_j$$

will be an unbiased estimate of I.

Let $\bar{I}$ be the average of N estimates $g(x)/f(x)$ of I with values of x drawn purely at random from (0, 1), using $f(x)$. The variances of $\bar{I}$ and $\bar{I}_R$ are are related by

$$\text{var}\ (\bar{I}) = \text{var}\ (\bar{I}_R) + \frac{1}{N} \sum_{j=1}^{S} P_j (I_j - I)^2 \ . \qquad (6)$$

It is clear from (6) that var $(\bar{I})$ > var $(\bar{I}_R)$ unless $I_j = I$, for $j = 1, 2, \ldots, S$. Moreover, there will exist a partitioning of (0, 1) for which var $(\bar{I})$ > var $(\bar{I}_R)$ unless $I_j = I$ holds for every j and every possible partitioning; that is, unless the frequency function $f(x)$ is chosen as

$$f_0(x) \equiv \frac{g(x)}{I} \ .$$

This is the well-known "importance function" for the integral I. Using it, var $(\bar{I}) = 0$. This cannot be achieved unless I is known at the beginning. It can be approximated, though, by a step function or other simple function $h(x)$ that approximates $g(x)$ and the choice

$$f_1(x) = \frac{h(x)}{\int_0^1 h(x)dx} \ .$$

Representative sampling over the partition used to define $h(x)$ should then lead to an estimate $\bar{I}_R$ of I of very small variance with a small N.

The above device will be explored at some length in Part II of reference (6).

PROGRAMMING FOR AUTOMATIC COMPUTING MACHINE

Participating Member of Panel. W. Givens.

Reference. Mathematics Panel quarterly report ORNL-1029, p. 7.

Status. Studies on the design and operation of a high-speed digital computer have been continued with particular reference to the inversion of matrices of large order.

ANGULAR CORRELATION BETWEEN CONVERSION ELECTRONS AND GAMMA RAYS

Origin. Drs. M. E. Rose and G. B. Arfken, Physics Division.

Participating Members of Panel. N. M. Dismuke, C. Perhacs, J. Fishel, N. Given, and C. Perry.

References. (*a*) Mathematics Panel quarterly report ORNL-1029, pp. 18, 21; (*b*) Mathematics Panel quarterly report ORNL-979, p. 38; (*c*) M. E. Rose and G. B. Arfken, "Angular Correlation with Conversion Electrons," *Physics Division Quarterly Progress Report for Period Ending June 20, 1951,* ORNL-1092 (in press).

Background and Status. A physical and mathematical description of the angular correlation coefficients is given in the current Physics Division quarterly progress report. These angular correlation coefficients are being calculated for the same atomic numbers, energies, and multipoles as were used in the Harvard Mark I computation of the K-shell internal conversion coefficients. The matrix elements found as an intermediate calculation, by Mark I, are being used in the computation of the angular correlation coefficients. The calculations are being performed on the IBM machines at K-25. The angular correlation coefficient calculations for the electric and magnetic dipoles and quadrupoles have been completed. The Panel expects to finish the computations for the 3rd, 4th, and 5th order multipoles in August.

CALCULATION OF INTERNAL CONVERSION COEFFICIENTS WITH SCREENING

Origin. Dr. M. E. Rose, Physics Division, and Dr. Gerald Goertzel, Physics Department, New York University.

Participating Members. Dr. Gerald Goertzel, Physics Department, New York University; M. S. Montalbano, Div. 11.2, National Bureau of Standards; M. R. Arnette, Mathematics Panel.

References. *Calculation of Internal Conversion Coefficients for the L-Shell,* a paper by M. E. Rose and G. H. Goertzel, December 18, 1949; and all preceding Mathematics Panel quarterly reports (listed on p. iv).

Background and Status. The calculations of the internal conversion coefficients for the K, L_I, L_{II}, L_{III} shells, for electric and magnetic multipoles of all orders, and for all combinations of energy (Mev) k = .05, .10, .15, .2, .4, .6, .8, 1.0, 1.5, 2.0 and atomic number z = 5, 15, 25, 35, 45, 55, 65, 75, 85, 95 are to be completed on the National Bureau of Standards eastern automatic computer (SEAC). The programming for these calculations is almost complete. The program is being coded for the SEAC and checked for coding errors.

CALCULATION OF RACAH COEFFICIENTS FOR THE ANGULAR DISTRIBUTION IN NUCLEAR REACTIONS

Origin. Drs. M. E. Rose and L. C. Biedenharn, Physics Division.

Participating Member of Panel. R. Crook.

References. (*a*) L. C. Biedenharn, C. Perry, and M. Rankin, "Tabulation of the Racah Coefficients," *Physics Division Quarterly Progress Report for Period Ending June 20, 1951,* ORNL-1092 (in press); (*b*) Mathematics Panel quarterly report ORNL-1029, p. 22.

Status. The calculations of the tables of Racah coefficients designated $S = 2$ and $S = 3$ have been completed and checked. The next and last table ($S = 5/2$) calculations are planned for the next quarter.

HARMONIC ANALYSIS

Origin. B. S. Borie, Jr., Metallurgy Division.

Participating Members of Panel. J. H. Fishel and C. Perry.

References. Mathematics Panel quarterly reports: ORNL-979, p. 42; ORNL-1029, p. 22.

Background and Status. The Wayne University calculations (on a differential analyzer) were checked by a numerical evaluation of the Fourier integrals by IBM calculation. The present method used to evaluate the Fourier integrals is to replace the experimentally determined function by a polygonal function. The exact evaluation of Fourier integrals is then made for the polygonal function. The IBM time per integral is approximately six minutes.

T + D AND He^3 + D CROSS SECTIONS

Origin. W. Kunz and Dr. W. M. Good, Physics Division.

Participating Member of Panel. N. Given.

Background and Status. The cross sections for the T + D and He^3 + D reactions were calculated from a formula involving exact coulomb wave functions fitted empirically at the nuclear radius.

Completed.

ANALYSIS OF SCINTILLATION SPECTROMETER DATA

Origin. P. R. Bell, Physics Division.

Participating Members of Panel. N. Given and C. Perry; J. W. Tukey, Consultant.

References. Mathematics Panel quarterly reports: ORNL-408, p. 16; ORNL-818, p. 11; ORNL-979, p. 40; and ORNL-1029, p. 39.

Background and Status. The scintillation spectrometer measures a function $f(x)$ of the energy spectrum $F(y)$ of a gamma- or beta-ray beam. The mathematical assumption for the relation between these functions is

$$f(x) = \int_0^\infty K(x,y)F(y)dy \ , \qquad (1)$$

where $K(x,y)$ describes the smearing by the spectrometer. In ORNL-408 a method is described for finding an approximate solution to equation (1) when

$$K(x,y) = e^{-(x-y)^2} \ .$$

A subsequent attempt to satisfactorily interpret the spectrometer data was described in ORNL-818. The results found in this attempt were unsatisfactory. Dr. Tukey has suggested the following scheme for analyzing the spectrometer data.

Let f_i, $i = 1, 2, \ldots, n$ designate the measurements made at $x = ih$ (h constant), using the spectrometer. Further assume that $K(x,y) = k(ih - y)$. Making these replacements in equation (1) produces

$$f_i = \int_{-\infty}^{\infty} k(ih - y)F(y)dy \ . \qquad (2)$$

Dr. Tukey suggested that the constants c_{i-j} be found such that

$$\sum_{j=i-3}^{i+3} c_{i-j} k(jh - y) = k^*(ih - y) \qquad (3)$$

is as near a "delta" function as is possible. Then for

$$b_i = \sum_j c_{i-j} f_j$$

we will have

$$b_i = \int_{-\infty}^{\infty} k^*(ih - y)F(y)dy \ ,$$

and b_i will be an approximate value for $F(ih)$.

The Panel is at present finding constants c_{i-j}, that make the function k^* as near a "delta" function as is possible [7 or less c_{i-j} are used], for the smearing function

$$k(ih - y) = \int_{(i-\frac{1}{2})h}^{(i+\frac{1}{2})h} e^{-(x-y)^2} dx \ .$$

DETERMINATION OF FAST NEUTRON FLUX IN ORNL PILE

Origin. Dr. D. K. Holmes, Physics Division.

Participating Members. J. Moshman, Mathematics Panel, and E. B. Carter, K-25, Central Statistical Laboratory.

References. Mathematics Panel quarterly reports: ORNL-726, p. 12; ORNL-818, p. 17; ORNL-888, p. 17; ORNL-979, p. 32; and ORNL-1029, p. 33.

Status. In machine state of calculation.

MONTE CARLO ESTIMATE OF COLLISION DISTRIBUTIONS IN TISSUE; MONTE CARLO ESTIMATE OF AGES IN H_2O

Origin. Drs. W. S. Snyder and J. Neufeld, Health Physics Division; E. P. Blizard, Physics Division.

Participating Members of Panel. K. A. Pflueger and N. M. Dismuke.

References. All the previous Mathematics Panel quarterly reports (listed on p. iv).

Background and Status. The slowing down histories of 2000 neutrons emitted from a point source in an infinite medium of tissue were obtained by the Monte Carlo method. The neutrons leave the source at an energy of 10 Mev in a collimated beam. The history of a neutron was followed from the source until the neutron was absorbed or had an energy after collision of less than 1 ev.

During the past quarter these histories were used to find the collision density in a 30-cm slab of tissue; to find the collision damage in a 30-cm slab; to find the distribution of the angle of deflection from the collimated beam after the neutrons had slowed down to an energy (in ev) after collision of $10^7/e \leq E < 10^7/\sqrt{e}$; and to find the age in H_2O and the age in tissue.

STUDY OF RADIOACTIVITY ABSORPTION IN GAMBUSIA

Origin. Dr. L. A. Krumholz, Health Physics Division.

Participating Member of Panel. A. W. Kimball.

Background. This experiment was planned to study the effects of several different factors in the uptake of radioactivity in gambusia. Little is known about the relevant factors associated with such absorption so that the present experiment is of an exploratory nature. It was desired to include as many factors as possible with the limited amount of material and time available. Some of the factors which are believed to be relevant are temperature, food level, activity level in food, activity level in water, length of exposure, and kind of isotope used.

In a preliminary conference it was agreed that three levels of each factor would be required since nonlinear effects, if present, should be detected. The inclusion of all six of the factors previously mentioned, each at three levels, would require $3^6 = 729$ fish for one replication of the experiment. The possibility that this number might be reduced by fractional replication was investigated. It is physically possible to count only about 27 fish in one working period, so that a 1/27 replicate would have been desirable. However, C. R. Rao [*Sankhyā* **10**, p. 81 (1950)] has shown that a 1/9 replicate is needed in order for all main effects and first order interactions to be estimable when second and higher order interactions are absent. It was also discovered that it would not be feasible to randomize the temperature and isotope factors in a manner suitable for the use of fractional replication.

The design finally adopted provides for three separate experiments, one for each isotope. The 3^4 combinations of the four factors other than temperature were placed in three blocks (corresponding to tanks containing fish in separate jars) of 27, each in such a manner that two degrees of freedom of one-third order interaction were confounded with blocks. By performing three runs with tanks at three different temperatures in each run, one complete replication for a particular isotope was achieved. This design meets the requirement that no more than 27 fish have to be counted in any one working period.

Status. The experiment is now in progress.

ESTIMATION OF PROBIT DIFFERENCES IN MOUSE PROTECTION EXPERIMENT

Origin. Dr. J. B. Kahn, Biology Division.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. In this experiment 480 mice separated into two groups of 240 per group were exposed to X radiation in a range of doses. One group served as controls, and each mouse in the other group was administered morphine sulfate 30 minutes before exposure to radiation. At the conclusion of the experiment it was discovered that some of the mice had contracted an infection, so that in some cases cause of death was in doubt. None of the infected mice are included in the results which are shown in Table 1.

TABLE 1

Mortality After Irradiation

DOSE (r)	NO. KILLED/NO. EXPOSED	
	CONTROLS	TREATED
590	11/40	
630	31/40	0/20
670	34/40	2/20
710	20/20	10/40
760	40/40	6/36
820		19/40
880	20/20	18/36
940		34/40
1000		20/20

In order to reduce the data statistically, probit curves were fitted to log dose for each series. The L. D. (lethal dose) 50 estimates and 95% confidence intervals which were obtained are:

	CONTROLS	TREATED
L. D. 50 point estimate	609	830
95% confidence interval	595-620	819-842

Because of the wide gap between the two confidence intervals it is clear without a formal statistical test that the L. D. 50's in the two groups differ significantly.

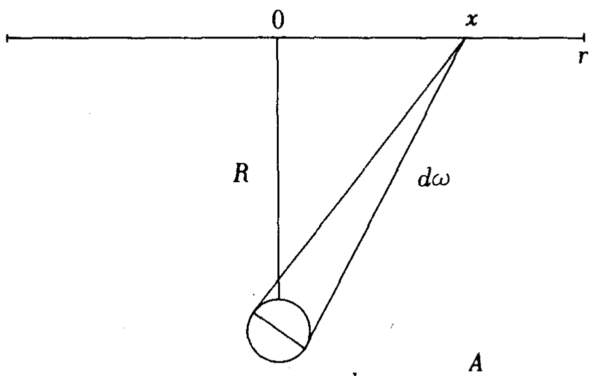
ESTIMATION OF SOURCE ACTIVITY

Origin. R. W. Rogers, Biology Division.

Participating Member of Panel. Jack Moshman.

Background and Status. A disc source of radius r is situated at a distance R from a spherical target of diameter A. Knowing the activity impinging on the counter, what is the activity of the source?

Let the source be emitting n counts/cm^2 sec. Consider the solid angle $d\omega$ subtended by the target at a distance x from the center, 0, of the source disc.



$$d\omega = \frac{A}{R^2 + x^2}$$

The proportion of emitted particles impinging on the target is

$$\frac{d\omega}{4\pi} = \frac{1}{4\pi} \frac{A}{R^2 + x^2} .$$

But in the differential annular ring on the disc of area $2\pi\ x\ dx$, $2\pi\ x\ n\ dx$ particles are being emitted. Hence the activity dN on the counter from the differential ring is

$$dN = \frac{1}{4\pi} \frac{A}{R^2 + x^2} 2\pi\ n\ x\ dx = \frac{n}{2} \frac{A\ x\ dx}{R^2 + x^2} ,$$

and the total counts impinging on the target is

$$N = \frac{nA}{2} \int_0^r \frac{x}{R^2 + x^2}\, dx$$

$$= \frac{nA}{4} [\ln (R^2 + r^2) - \ln R^2] ,$$

whence the activity n, knowing N, is

$$n = \frac{4N}{A[\ln (R^2 + r^2) - nR^2]} .$$

ANALYSIS OF VARIANCE

Origin. Dr. M. E. Gaulden, Biology Division.

Participating Members of Panel. Jack Moshman and Carl Perhacs.

Background. Entries in a table of experimental results were called n_{ijk} and d_{ik}. The quantity v_{ijk} was defined by

$$v_{ijk} = \sqrt{\frac{n_{ijk}}{d_{ik}}} ,$$

except where $n_{ijk} = 0$. In that case,

$$v_{ijk} = \sqrt{\frac{1}{4d_{ik}}} .$$

The range of v_{ijk} is $0 < v_{ijk} < 1$, and v_{ijk} was calculated exact to four decimal places. To obtain the transformation $y_{ijk} = 2 \arcsin v_{ijk}$, the values of v_{ijk} were compared directly to a punched-card table of $\sin \theta$, $0 \leq \theta \leq \pi/2$, in which θ is given to four decimal places. Then θ was read off directly as $y_{ijk}/2$. Even when the comparison of $\sin \theta$ and v_{ijk} resulted in two values of $\sin \theta$, matching the same value of v_{ijk}, either value of θ gave y_{ijk} to the desired accuracy of three significant digits.

Once the transformation was made, the y_{ijk}'s were summed and squared, and an analysis of variance table was made.

Status. This problem is completed.

HEAT EFFECT ON CELL EXTINCTION

Origin. Dr. M. E. Gaulden, Biology Division.

Participating Members of Panel. J. Moshman and J. Fishel.

Reference. Mathematics Panel quarterly report ORNL-1029, p. 33.

Background. A previous experiment (see reference) gave negative results, partially due to the use of only seven different embryos, although 81 cells were examined. A new design was prepared which called for 24 embryos and 120 cells. In the actual experiment 119 cells from 23 embryos were examined. It is hoped that the increased number of degrees of freedom between embryos (22), compared with the previous number (7), will establish the statistical significance of the heat effect as is theoretically hypothesized.

Status. The data have been coded and cell extinctions are being computed on IBM machines.

ISODOSE CURVES FOR BETA EMITTERS

Origin. Dr. Marshall Brucer, ORINS Medical Division.

Participating Member of Panel. J. Moshman.

Reference. Mathematics Panel quarterly report ORNL-1029, p. 34.

Status. A Monte Carlo procedure appears to be impractical because of a lack of the necessary physical data. An experimental approach is being considered.

APPROXIMATE DETERMINATION OF LYMPH SPACE

Origin. R. H. Storey and Dr. J. Furth, Biology Division.

Participating Members of Panel. J. Moshman and J. Z. Hearon.

Background. An important factor in radiation physiology is the change in permeability between the blood vessels and the interstitial tissues and lymph space. Experiments have shown that substances injected into the blood disappear into the lymph at a rate depending on the permeability. As a first step in measuring the magnitude of permeability changes, it is necessary to measure the lymph space.

If homologous albumin tagged with I^{131} is injected into the circulation some of it leaves the blood stream and enters the lymph. If we let R^* be the total activity injected, then we may estimate V_p, the plasma volume, by

$$V_p = \frac{R^*}{R_p^{(0)}} , \tag{1}$$

where $R_p^{(0)}$ is the radioactivity per unit volume in the plasma at 5 to 10 minutes after injection. The 5- to 10-minute time period insures reasonable mixing of the I^{131} in the plasma, but little I^{131} leaves the blood stream.

The transfer equations between lymph and plasma may be written as

$$\frac{dR_p^{(t)}}{dt} = \frac{hA}{V_p} \left[\frac{R_l^{(t)}}{C_l} - \frac{R_p^{(t)}}{C_p} \right] - k_p R_p^{(t)} \tag{2}$$

$$\frac{dR_l^{(t)}}{dt} = \frac{hA}{V_l}\left[\frac{R_p^{(t)}}{C_p} - \frac{R_l^{(t)}}{C_l}\right] - k_l R_l^{(t)} \quad (3)$$

where

$R_p^{(t)}$, $(R_l^{(t)})$ = radioactivity per milliliter of plasma (lymph) at time t.

C_p, (C_l) = concentration of albumin in plasma (lymph).

hA = transfer coefficient.

V_p, (V_l) = volume of plasma (lymph).

k_p, (k_l) = exponential rate of plasma (lymph) albumin metabolism.

If it is assumed $k_p = k_l = k$, and the volumes remain constant, then (2) and (3) may be combined into

$$R^* e^{-kt} = V_p R_p^{(t)} + V_l R_l^{(t)} , \quad (4)$$

providing there is uniform mixing of the tagged albumin in and between lymph and blood plasma. Uniform mixing to a reasonable extent does exist after about 10 hours after which time an equilibrium sets in between concentration of I^{131} in the blood plasma and in the lymph.

Equation (4) has two unknowns, k and V_l. R^* is known, $R_p^{(t)}$ and $R_l^{(t)}$ are measured directly in a counter, and V_p is estimated from equation (1). Radioactive decay is implicitly removed from consideration since all samples of plasma and lymph are measured in the counter at the same time.

A typical illustration of the behavior of plasma and lymph radioactivity per milliliter per second is shown in Fig. 1. The straight portions of the two curves after equilibration at 12 hours represents metabolic decay. We may then estimate k from the graph by letting

$$-k = \frac{R_p^{(\tau_2)} - R_p^{(\tau_1)}}{\tau_2 - \tau_1} = \frac{R_l^{(\tau_2)} - R_l^{(\tau_1)}}{\tau_2 - \tau_1} \quad (5)$$

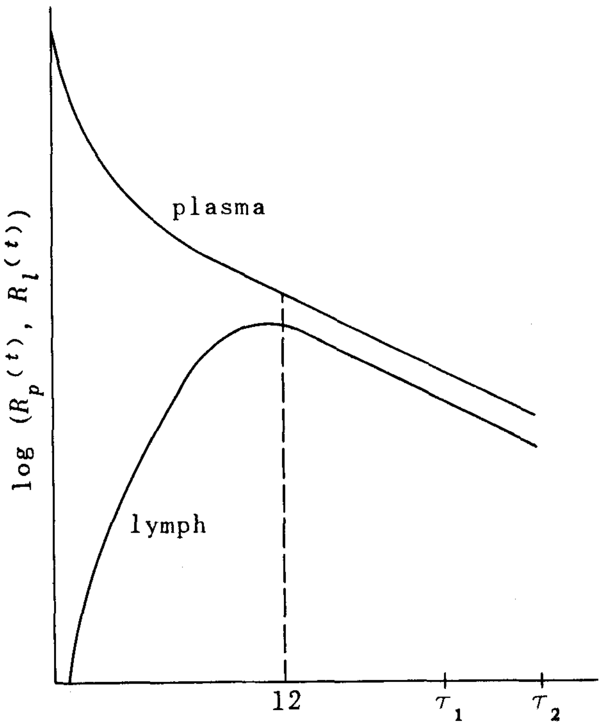


Fig. 1

and then solving equation (4) for V_l, obtaining

$$V_l = \frac{R^* e^{-kt} - V_p R_p^{(t)}}{R_l^{(t)}}, \tag{6}$$

$$t \geq 12.$$

It has further been found that

$$R_p^{(t)} = 0.6\, R_l^{(t)} , \tag{7}$$

$$t \geq 12,$$

in data from about 10 normal dogs with a range of the factor of about 0.5 to 0.7. This approximation enables an estimate of lymph space without tapping the lymphatics.

Status. More refined procedures are being examined to determine bounds on the accuracy of estimates from equations (6) and (7). A preliminary report on the procedure has been submitted to *Science* and a more detailed report is in preparation.

CIRCULATION CALCULATIONS

Origin. Dr. C. W. Sheppard, Biology Division.

Participating Members of Panel. W. C. Sangren and E. Ikenberry.

Background and Status. The problems involved are part of a collaborative study of the circulatory mixing and disappearance from the circulation of injected K^{42}. W. S. Wilde and Richard Overman are doing the experimental work, and C. W. Sheppard is doing the theoretical work.

Two mathematical problems were investigated. The first problem consists of finding $a_i(x,t)$ and $a_e(x,t)$ which satisfy

$$\frac{\partial a_e}{\partial t} = \rho'(a_i - a_e) , \tag{1}$$

$$a_e(x,0) = 0 , \tag{2}$$

$$\frac{\partial a_i}{\partial t} = \rho(a_e - a_i) - Q\,\frac{\partial a_i}{\partial x} , \tag{3}$$

$$a_i(x,0) = a_0(x) \tag{4}$$

and the condition of boundedness for the solutions. The constants ρ, ρ', and Q, and the initial distribution $a_0(x)$ are assumed to be given. Using standard Laplace transform procedure it is possible to show that

$$\int_0^\infty e^{-st} a_i(x,t)\,dt = \bar{a}_i(x,s)$$

$$= e^{-(c/Q)x} \int_{-\infty}^{x} \frac{a_0(z)}{Q}\, e^{(c/Q)z} dz ,$$

where

$$c = s + \rho - \frac{\rho\rho'}{s + \rho'}$$

In order to obtain $a_i(x,t)$, i.e., invert the transform, it appears desirable to put in the given function $a_0(z)$ and perform the integration on the right-hand side of this last equation. The integration and the following inversion has been carried out when $a_0(z)$ is a constant, a step function, and a square pulse. When $a_i(x,t)$ is obtained in this fashion, $a_e(x,t)$ may be obtained from equation (3). The results appear to agree well with known physical results.

It might be noted that the peculiar fashion in which the parameter s enters the transform through c also occurs when the method of separation of variables is used. This makes the separation of variables method inconclusive.

The second problem consists of finding $a(x,t)$ which satisfies

$$\frac{\partial a}{\partial t} = -\rho(x)a - Q\frac{\partial a}{\partial x}$$

and

$$a(x,0) = a_0(x) \quad ,$$

where $\rho(x)$ is a given periodic function of x. The general solution has been obtained and examined.

The analysis appears to be completed for these two problems.

DETERMINATION OF STOPPING POWERS FOR IONS

Origin. Drs. W. S. Snyder and J. Neufeld, Health Physics Division.

Participating Members of Panel. K. A. Pflueger, N. Given, and C. Perry.

References. (*a*) Mathematics Panel quarterly report ORNL-979, p. 42; (*b*) Mathematics Panel quarterly report ORNL-1029, p. 23; (*c*) J. Neufeld, *Ionization and Excitation Losses of Charged Particles of Intermediate Energies*, ORNL-884 (Dec. 14, 1950).

Status. The computations and graphing were completed during the first part of the quarter.

THORIUM BREEDER STUDIES

Origin. Dr. J. Lane, N. Lansing, and Dr. S. Visner, Long Range Planning Group, Homogeneous Reactor Project.

Participating Member of Panel. B. S. McGill.

Status. Barbara McGill is on temporary loan to the above-mentioned group to perform 2-group calculations. These calculations are expected to be completed in August.

PENDING PROBLEMS

Problem. Threshold Values of the Angular Correlation Coefficients.

Origin. Drs. M. E. Rose and G. B. Arfken, Physics Division.

Problem. Reactor Response.

Origin. Dr. J. Trimmer, Reactor School.

Problem. Analysis of Fluid Flow in the Homogeneous Reactor.

Origin. Drs. G. Wislicenus and D. Fax, Homogeneous Reactor Project.

Problem. Numerical Evaluation of Trigonometric Series.

Origin. Dr. B. S. Borie, Jr., Metallurgy Division.

Problem. Crystal Analysis by Neutron Diffraction.

Origin. Dr. H. Levy, Chemistry Division.

INACTIVE PROBLEMS

Problem. Quantum Mechanics Integrations.

Origin. F. C. Prohammer, Physics Division.

NEUTRON AGE IN WATER

Origin. E. P. Blizard, Physics Division.

Participating Members of Panel. Kathryn Pflueger and N. Dismuke.

References. Mathematics Panel quarterly reports: ORNL-345, p. 11; ORNL-888, p. 25; ORNL-1029, p. 21.

Status. The age of 5-Mev neutrons (slowing down from 5 Mev to 1 ev) in H_2O was obtained from histories of of 10-Mev neutrons (slowing down from 10 Mev to 1 ev) in tissue by picking those which had a collision in the interval 3.6- to 6.1 Mev range. The average energy of collisions in this interval was found to be 5.0 Mev. A minor tabulation error was found in the results listed in ORNL-1028 for 10-Mev neutrons, so the corrected results are listed in Table 1 along with results for 5-Mev neutrons. The deviation indicated is standard deviation estimated from a sample of 1995 neutrons for the 10-Mev case and from a sample of 838 neutrons for the 5-Mev case.

TABLE 1

Neutron Age

ENERGY LOSS	AGE IN WATER (cm^2)	AGE IN WATER (from MonP-219) (cm^2)	AGE IN TISSUE (cm^2)
10 Mev to 1 ev	178.3 ± 2.3	183	180.5 ± 2.4
5.6 Mev to 1 ev		78	
Avg. (5.0) Mev to 1 ev	79.7 ± 12.3	68	79.7 ± 13.4
4.0 Mev to 1 ev		57	

[redacted]

KINETICS OF THE HOMOGENEOUS REACTOR PROJECT

Origin. Dr. T. A. Welton, Physics Division.

Participating Member of Panel. W. C. Sangren.

Background and Status. It is apparent that the most general equations of motion for the Homogeneous Reactor Experiment cannot be solved or effectively investigated without making a number of assumptions and simplifications. The assumptions which are most generally made by the present Physics Group for the Homogeneous Reactor Project and by J. Stein have reduced the equations of motion to various systems of nonlinear differential equations.

There are many known "practical" ways to investigate a system of two first order nonlinear differential equations. Many of these ways have been studied with consideration of their possible application to the equations of the HRE. Unfortunately few of these "practical" methods are easily extended to more than two first order nonlinear differential equations.

One obvious way to solve nonlinear differential equations is to use numerical integration by hand or machine. This is being carried out in some simple cases.

Most of the work so far on the nonlinear equations has centered about the question of stability of the resulting motion. This work has generally fallen into two parts: (1) the stability of the linearized equations which result by assuming small displacements from the equilibrium position, and (2) the stability of reduced sets of nonlinear equations.

More details concerning this work will be found in the next quarterly report of the HRP.

STABILITY OF THE REACTOR SIMULATOR

Origin. Dr. J. Palmer, Instrument Department, Engineering and Maintenance Division.

Participating Member of Panel. W. C. Sangren.

Reference. Mathematics Panel quarterly report ORNL-1029, p. 15.

Background and Status. The mathematical problem which involved system (2), as given in the above reference, apparently is no longer of physical interest. On the other hand the problem which involved system (1) is being investigated by the Physics Group of the Homogeneous Reactor Project. The results of some of this work will appear in future HRP quarterly reports.

Relative to the approximating linear system (1)′, it is possible to conclude that the linear system is stable. An elementary argument for this can be based upon the fact that with one and two delayed neutron groups the system can be showed rigorously stable by using the Nyguist criteria.

COMPUTATION OF NEUTRON DISTRIBUTION AND k_{eff} FOR A CYLINDRICAL REACTOR WITH BARE ENDS AND REFLECTED CONVEX SURFACE

Origin. Dr. N. M. Smith, ANP Physics Division.

[redacted]

Participating Member of Panel. N. Edmonson.

Background and Status. It is desired to calculate the neutron distribution and the effective multiplication constant k_{eff} for a reflected cylindrical reactor with bare ends by transforming the partial differential equation

$$-D\nabla^2\phi(\mathbf{r},u) + \sigma_a\phi(\mathbf{r},u) = \frac{\partial q}{\partial u} + S(\mathbf{r},u) \qquad (1)$$

into difference equations suitable for multigroup computations on the Y-12 IBM equipment. In this equation

$\mathbf{r}$ = the position vector of a point in the core-reflector combination. The core-reflector configuration is a right circular cylinder of length $2H$. The coordinate system is cylindrical, with the central axis of of the core the z axis and the origin of coordinates at the midpoint of the central axis. The distance from the central axis is r.

u = lethargy of a neutron = $\ln \dfrac{10^7}{E}$, E being the energy of a neutron.

$\phi(\mathbf{r},u)$ = neutron flux per cm^3 per unit lethargy.

$q(\mathbf{r},u)$ = neutron slowing down density per cm^3 per unit lethargy.

$S(\mathbf{r},u)$ = neutron source density per cm^3 per unit lethargy.

σ_a = total microscopic absorption cross-section.

D = diffusion coefficient.

The neutron flux $\phi(\mathbf{r},u)$ and the slowing down density $q(\mathbf{r},u)$ are assumed symmetrical relative to the axis of the cylinder. This assumption reduces the space variables in (1) to r and z. It is further assumed that in equation (1) r and z may be separated. This leads to the expression of the z dependence of (1) by a factor

$$\cos \frac{\pi z}{2(H + d)},$$

where H is the half-cylinder length and d is the linear-extrapolation length. It is assumed that d has the same value for all lethargies and for both the core and the reflector.

It is assumed that the neutron flux $\phi(\mathbf{r},u)$ is finite for $r = 0$ and is equal to zero at the extrapolated boundary of the reflector. The core-reflector boundary conditions are: (1) $\phi(\mathbf{r},u)$ is continuous; and (2) $-D(\partial\phi/\partial r)$ is continuous.

This problem is being carried on as rapidly as possible.

DOSE ESTIMATION FROM FIELD EXPERIMENTS

Origin. Dr. A. D. Conger, Biology Division.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. As part of the recent Operation Greenhouse experiments at Eniwetok, several specimens of *Tradescantia* were exposed to bomb effects at distances which were believed to correspond roughly with doses employed previously in the laboratory. One of the primary purposes of the experiment was to correlate laboratory dose effect with bomb dose effect, and an essential part of this comparison was the estimation of equivalent laboratory doses from specimens exposed to the bomb. The measurements consist of chromosome and chromatid aberrations observed in plant cells prepared from *Tradescantia* buds exposed to different types of irradiation under different conditions. The details of the experiment will be reported elsewhere. The statistical methods of dose estimation are described here.

In all cases the results from the control experiments in the laboratory were such that aberration frequency (or some simple function of aberration frequency) could be expressed linearly against dose. Thus to a set of n points, a curve of the form

$$y = \alpha + \beta x \ ,$$

where y is aberrations per cell and x is dose, was fitted by the method of least squares. If y is normally and independently distributed with mean $(\alpha + \beta x)$ and variance σ^2, and if the error in laboratory dose is considered negligible with respect to the error in y, the maximum likelihood estimates a and b of α and β, respectively, are likewise normally and independently distributed.

From the field is given a response y_0, and the determination of a confidence interval for the corresponding x_0 is required. If we let

$$v = y_0 - a - bx_0 \ ,$$

it is seen that v is normally distributed with zero mean and variance

$$\sigma_v^{\,2} = \sigma^2 \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma\,(x - \bar{x})^2} \right].$$

Thus

$$t = \frac{v/\sigma_v}{\sqrt{ns^2/\sigma^2}} \ , \qquad (1)$$

where s^2 is an estimate of σ^2 based on $(n - 2)$ degrees of freedom, and t is distributed as Student's ratio with $(n - 2)$ degrees of freedom. If the 100λ percent point of Student's distribution is chosen for t, equation (1) becomes a quadratic equation in x_0, the roots of which provide the limits for the $100(1 - \lambda)$ percent confidence interval for x_0.

In some cases two types of aberrations were counted on the same sample, and, since it has been shown that these are independent, presumably a better confidence interval for the dose, which is necessarily identical for both counts, could be obtained by combining the two estimates. In this situation there are two control experiments with curves

$$y = \alpha + \beta x$$

and

$$y' = \alpha' + \beta' x$$

fitted to, say, n and m points respectively. Then

$$v = y_0 - a - bx_0$$

and

[redacted]

$$v' = y_0' - a' - b'x_0 ,$$

where y_0 and y_0' are the field responses and are both normally distributed with zero means, and

$$V = v + v'$$

will likewise be normally distributed with zero mean and variance

$$\sigma_V^2 = \sigma_v^2 + \sigma_{v'}^2 ,$$

since v and v' are independent. In a manner identical with that described for the single experiment, one confidence interval for x_0 based on $(m + n - 4)$ degrees of freedom can be found which replaces the separate confidence intervals based on $(m - 2)$ and $(n - 2)$ degrees of freedom which would be obtained if the data were not combined.

OPERATION GREENHOUSE

Origin. Drs. J. Furth and A. C. Upton, Biology Division.

Participating Members of Panel. J. Moshman and G. J. Atta.

References. Mathematics Panel quarterly reports: ORNL-979, p. 29; ORNL-1029, p. 37.

Status. Mice which were exposed to radioactivity and neutron bombardment during a test (Operation Greenhouse) of atomic explosives in the Pacific have been flown to Oak Ridge for study. The mice were individually weighed, and a complete randomization process was performed in the assignment of the mice in individual cages to trays in units of four cages. The trays were placed at random in the air-conditioned room set aside for these animals.

Punch cards were prepared with full background data for all mice in the longevity and lethality studies. Preliminary tabulations of 28-day mortality indicated there was no significant sex, weight, or age differences. There was, however, a decided difference in mortality due to the trays in which the animals were placed. Mice were placed in six trays, one on top of the other, at each station. The mortality in the bottom trays was significantly less than in the others. There was also a decided difference due to the animal's position in the tray. Arranged in three rows of 15 animals each, the row furthest from the blast had a significantly lower mortality than the other two rows.

Cataract studies are progressing. At about 1½ to 2 months following the the exposure, virtually all the exposed animals, both those given a predominantly gamma dose and those given a predominantly neutron dose, had low-grade radiation-induced cataracts. The appearance was so sudden that it was impossible to obtain any reliable regression of time-of-onset on dose. More careful steps are being taken to obtain such a regression for the appearance of higher grade opacities.

It is too early to obtain any data relative to neoplasis and leukemia incidence.

At the request of members of the Naval Medical Research Institute,

[redacted]

[redacted]

further analyses are being made of the interactions between sex, age, weight, and position on the mortality data.

DETERMINATION OF THE DIFFUSION LENGTH FOR THERMAL NEUTRONS IN THE STANDARD GRAPHITE PILE

Origin. Dr. H. R. Ritchie, Health Physics Division; E. D. Klema, Physics Division.

Participating Members of Panel. N. Given and C. L. Perry.

References. (*a*) D. Hall *et al.*, *Preliminary Material for Manual on Graphite Testing*, Chicago report CL-573 (April, 1943); (*b*) J. Chernick and I. Kaplan, *A Review of Graphite Testing Procedures and Their Applications to the BNL Reactor*, BNL-77 (Nov. 15, 1950); (*c*) Mathematics Panel quarterly report ORNL-1029, p. 23.

Status. Addition diffusion length calculations were made to determine the dependence of the diffusion length on the energy of the neutrons.

. . . . .

[redacted]

[redacted]

[redacted] MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

[redacted]

Thi [redacted] as de [redacted]

[redacted]

Th [redacted] ecting the [redacted] tes. Its t [redacted] its c [redacted] rized pe [redacted] were c [redacted] under applicable federal la [redacted]

[redacted]