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BOILING REACTORS: A PRELIMINARY INVESTIGATION

J. M. Stein

P. R. Kasten

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TABLE OF CONTENTS

Abstract ---------- 1

Introduction ---------- 2

Power Output ---------- 5

The Control Problem ---------- 7

Problem ---------- 9

- Case I ---------- 26
  - Case IA ---------- 27
  - Case IB ---------- 31
  - Case IC ---------- 38
- Case II ---------- 42
  - Case IIA ---------- 42
  - Case IIB ---------- 43
  - Case IIC ---------- 46
- Case III ---------- 46
- Case IV ---------- 47
  - Case IVA ---------- 49
  - Case IVB ---------- 49
  - Case IVC ---------- 52
  - Case IVD ---------- 53

Discussion ---------- 53

Necessary Research ---------- 61

Alternate Systems ---------- 62

Summary ---------- 63

Nomenclature ---------- 64

References ---------- 66

[redacted]

[redacted]

[redacted]

LIST OF TABLES

Table I - Thermodynamic Relations for the System Considered ---------- 14

Table II - Values of Decay Constant Associated with the Delayed Neutron Emission of U^{235} Fission ---------- 26

Table III - Corresponding Values of u, σ, t, and ρ for Case IA for One-half Cycle ---------- 28

Table IV - Effect of U^{235} Concentration on Reactor Size and Safety ------- 60

LIST OF FIGURES

Figure 1 - Schematic Diagram of a Boiling-Type Reactor ---------- 3

Figure 2 - Enthalpy of Vapor in Reactor Fluid vs. Fluid Density ---------- 17

Figure 3 - Fluid Vapor Fraction vs. Fluid Density for Constant Vapor Mass in the Steam Chest ---------- 18

Figure 4 - Plot of Vapor Fraction vs. Equilibrium Pressure ---------- 19

Figure 5 - Solution Reactivity vs. Fluid Density, for $k_{co} = 1.06$ --------- 21

Figure 6a - Graph of Equation (26), Power Ratio, σ, vs. Excess k in System for Constant Rate of k Input; $\beta = \tau = \Delta k = 0$ ----- 29

Figure 6b - Variation of Fluid Density with the Power Ratio, for Constant Rate of k Input; Based on Figure 6a ---------- 29

Figure 6c - Variation of Fluid Density vs. Time During which Solution Reactivity, k, is Increased; Based on Figure 6a ---------- 30

Figure 6d - Variation of Power Ratio with Time. k is Linearly Increased, Based on Figure 6a ---------- 30

Figure 7 - Graph of Equations (27a) and (27b), Power Ratio, σ, vs. Solution Excess Reactivity, u, with an Instantaneous Δk of 0.01, and with and without a Linear Increase in k; $\beta = \tau = 0$ ---------- 32

Figure 8a - Plot of Power Ratio, σ, vs. Fluid Density, ρ, with C = 0, Based on Figure 7 ---------- 33

Figure 8b - Plot of Power Ratio vs. Fluid Density for C = 0.002215, Based on Figure 7 ---------- 34

[redacted]

[redacted]

Figure 9a - Plot of Fluid Density vs. Time for C = 0, Based on Figures 7 and 8a ---------- 35

Figure 9b - Plot of Fluid Density vs. Time for C = 0.002215, Based on Figures 7 and 8b ---------- 35

Figure 10a - Plot of Power Ratio vs. Time for C = 0, Based on Figures 8a and 9a ---------- 36

Figure 10b - Plot of Power Ratio vs. Time for C = 0.002215, Based on Figures 8b and 9b ---------- 37

Figure 11a - Graph of Equation (28), Power Ratio, σ, vs. Excess Reactivity, u, for C = 0, $k = k_{co} + 0.02$, $\beta = \tau = 0$ ---------- 39

Figure 11b - Plot of Power Ratio vs. Time, Based on Figure 11a ---------- 40

Figure 11c - Plot of Fluid Density vs. Time, Based on Figure 11a ---------- 41

Figure 12a - Plot of Power Ratio, σ, vs. Excess Reactivity, u, for Continuously Increasing k and Various Δk, Considering Delayed Neutron Effects, $\tau = 0$, C = 0.002215 ---------- 44

Figure 12b - Power Ratio vs. Time for Continuous Increase in k, Various Δk, $\beta = 0.005$, $\tau = 0$, Based on Figure 12a ---------- 45

Figure 12c - Plot of Fluid Density vs. Time During which k Increases Linearly, Based on Figure 12a ---------- 45

Figure 13a - Plot of Power Ratio, σ, vs. Excess Reactivity, u, for Continuous k Increase with Time, C = 0.002215, τ = 0.004 and 0.01 Seconds, $\beta = 0$ ---------- 48

Figure 13b - Plot of Power Ratio, σ, vs. Time for $\tau = 0.004$, Based on Figure 13a ---------- 48

Figure 14a - Plot of Power Ratio, σ, vs. Prompt Excess Reactivity, u, for $\beta = 0.00406$, $\tau = 0.01$, C = 0.002215 and 0, and a Delayed Neutron Group with $\lambda = 0.154$ ---------- 50

Figure 14b - Plot of Power Ratio vs. Time Based on Figure 14a ---------- 50

Figure 15 - Plot of Power Ratio vs. Prompt Excess Reactivity for $\Delta k = 0.0043036$, $\beta = \tau = 0$ and $\beta = 0.00406$, $\tau = 0.01$ ---------- 51

Figure 16 - Power Ratio vs. Prompt Excess Reactivity for $\Delta k = 0.0088$, C = 0, $\beta = 0.00406$, $\tau = 0.01$ ---------- 54

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ABSTRACT

Kinetic equations have been developed for a boiling-type reactor and inherent reactor stability studies have been made on the basis of these equations. Solution reactivity changes cause nuclear power oscillations which are negatively damped by bubble delay times but critically damped by the delayed neutron effect. The over-all result is a highly damped oscillation for instantaneous reactivity changes of about 0.009 and delay times of the order of 0.01 second. Features of the proposed reactor are presented and compared with circulating types, the "boiler" appearing quite attractive if investigations of bubble-rise velocities, effects of transient conditions of vapor mass in steam chest, and bubble time delay functions yield favorable results.

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INTRODUCTION

Homogeneous-type reactors have aroused considerable interest concerning their use for the production of usable power and fissionable material as evidenced by feasibility studies (1) (2) and experiments leading toward a power producing homogeneous reactor (3). Although not restricted to a certain type, homogeneous reactors discussed to any degree have dealt with circulating types. While boiling-type reactors have been treated conversationally, no explicit evidence concerning their stability has been noted. Because a reactor of this type may promise a minimum of associated equipment, compactness in shield design, and lowered maintenance hazards, studies should be undertaken to evaluate its worth. This report presents and attempts to answer some of the questions concerning stability and feasibility of such a reactor. Recently, a report on a boiling slurry-type reactor has been issued (4) which presents a number of details not covered here, and also another viewpoint as to the operating cycle.

In the boiling-type homogeneous reactor envisioned (see Figure 1), the heat of the fission reaction vaporizes the liquid within the reactor liquid, the generated steam rising through the liquid and vapor spaces and then condensed by heat exchangers. Since heat is transferred by a condensing vapor to a liquid, the effective driving force for transfer is higher than for liquid to liquid heat exchange under appropriately similar conditions. Although the transfer coefficient in the latter case increases with liquid velocity, practical attainable velocities limit the coefficient so the net result is a smaller size heat exchanger when the condensation cycle is employed. This advantage can be increased if dropwise condensation will occur using organic

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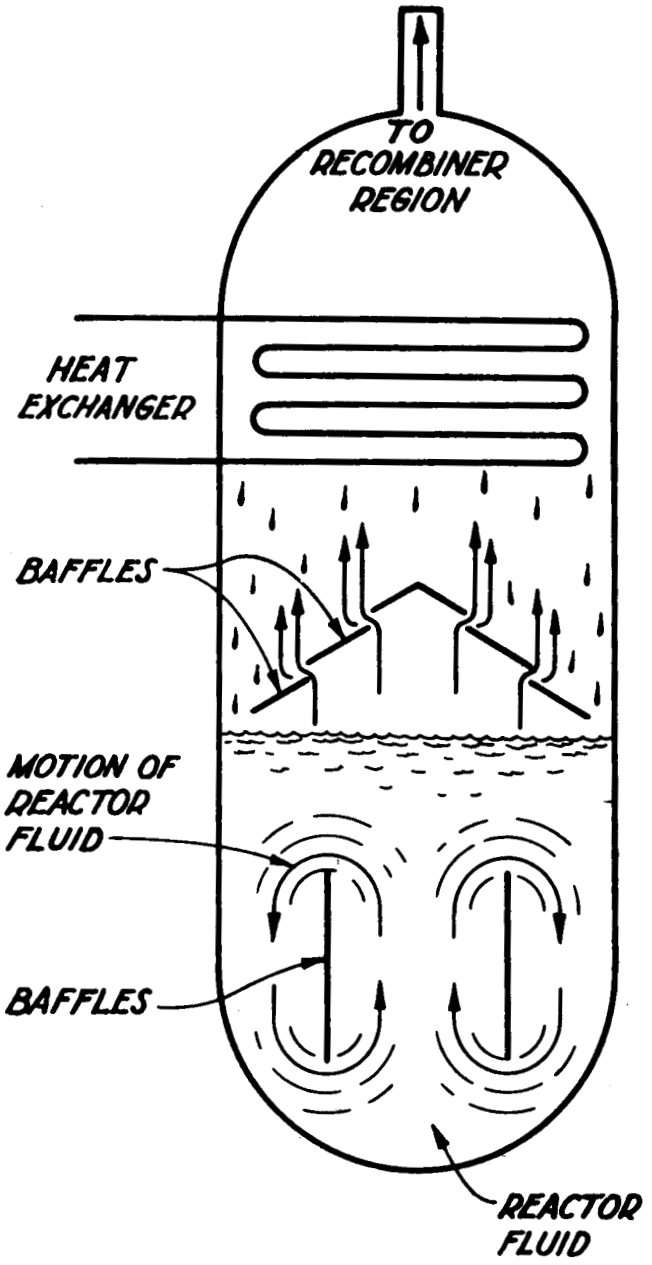

Figure 1

Schematic Diagram of Boiling-Type Reactor

additives as promotors. Heat transfer by condensation of a vapor also reduces erosion problems as compared with heat transfer by a rapidly moving hot liquid.

Another physico-mechanical advantage involves the possibility of using natural convection within the reactor fluid to remove the generated steam bubbles from the core. In this way the pumping power is substantially less than that of circulating-type reactors, and is attained without troublesome pump and piping equipment. Maintenance problems would thus be reduced, as would be the outlay of equipment necessary inside the radiation shield, thus promoting a compact shield with minimized radiation hazards to maintenance personnel. A boiling system is also desirable because of the pressurizer effect of the steam chest, tending toward thermodynamic equilibrium between temperature and pressure inside the reactor. This relationship would give pressure relief by vapor condensation, and hence aid in smooth operation of the reactor.

Since some of the reactivity poisons resulting from the fission reaction are gases having low aqueous solubilities, a boiling-type reactor affords an inherent method for their removal from the reaction zone. Also, in any homogeneous reactor employing water or heavy water as the reactor moderator and medium, decomposition gases will be formed by the action of the fission products upon the moderator. These gases create no explosion or reactivity problems in the "boiler" other than those associated with their constant removal from the steam chest, since the relative gas generation rate with respect to that of the vapor would be quite small, the proportion of gas to vapor being well within the explosion limit of electrolytic gas-steam mixtures. However, the heat

exchanger tubes would act as a steam filter, condensing the vapors passing upward, leaving a gas-vapor mixture relatively rich in electrolytic-gas above the exchanger. It seems reasonable that this gas mixture could be burned or catalytically recombined in the volume above the exchanger, with little danger of flashback because of the quenching action of the steam as the burning progressed downward. Alternatively, the heat exchanger within the reactor could be a partial condenser, the resulting steam-gas mixture led to a total condenser after which flame recombination of the gases is accomplished. Thus, if small order explosions occur in the recombiner, only a fraction of their effect is transmitted to the reactor.

POWER OUTPUT

To achieve large blocks of power from boiling-type reactors, reasonable specific power is desired. To obtain such desired power per unit fluid volume it is necessary that vapor be generated within the main body of liquid and removed from it at the required rate. Since natural rise rates of gases in water are of the order of one foot per second relative to the liquid, large power reactors would have to be built as shelves of liquid, or superimposed upon the natural bubble rise rate must be some high natural or forced liquid circulation rate. As illustrated in Figure 1, some type of baffle arrangement above the liquid surface could be employed so condensate would enter the reactor liquid so as to aid the circulation. These baffles could also act as entrainment separators. Large sized reactors would also be advantageous in obtaining the desired natural circulation rates, and natural or low enrichment uranium usage designates large reactor sizes. Under such conditions the neutron flux would be so distributed that the major proportion of bubbles

would be generated in the central regions of the core, establishing considerable density gradients which would aid circulation. The reactor volume required would depend upon the physical geometry desired, temperature of operation, moderator employed, the enrichment of the uranium, vapor holdup, and physical and mechanical limitations. It may be that the size of a reactor employing natural uranium as the fissile material is such that thermal stresses involved dictate the construction of a shell of uneconomical design, involving fabrication difficulties. If compactness were desired, a highly enriched core material would be required.

Vapor holdup in a boiling reactor may increase the size appreciably over that required in a comparable circulating-type reactor, the fraction of vapor holdup in the reactor depending upon the operating power, the specific power and the absolute velocity of the vapor bubbles. To illustrate the interdependence of the vapor fraction, the specific power and the absolute bubble-rise velocity, consider a reactor operating at P kw. Letting

h_{fg} = heat of vaporization of liquid, Btu/lb.

ρ_g = density of vapor, lb/cu. ft.

the cubic feet per second of vapor formed would be

$$\frac{3413\ P}{3600\ h_{fg}\ \rho_g} \qquad (1)$$

The above expression equals

$$\frac{v_g\ V_r}{t_{ave}} \qquad (2)$$

where

v_g = volume fraction of vapor in reactor proper

V_r = volume of reactor fluid, cu. ft.

t_{ave} = average time for a steam bubble to enter steam chest, measured from its formation.

Equating the above expressions and defining $\bar{R}$ as the average distance a bubble has to travel in escaping from the reactor fluid zone, the average bubble velocity required would be

$$\bar{v} = \frac{\bar{R}\ 3413\ P/V_r}{3600\ h_{fg}\ \rho_g\ v_g} \tag{3}$$

If the bubble originally starts at zero velocity and accelerates uniformly, the average terminal velocity required would be $2\bar{v}$.

Since $\bar{R}$, h_{fg}, and ρ_g are substantially constant, Equation (3) illustrates the interdependence of specific power, bubble velocity, and vapor holdup, with P/V varying directly as the product $v_g\bar{v}$. Thus, at any given specific power, increasing the vapor holdup lowers the required bubble-rise velocity. Corresponding values of the above variables under operating conditions can be found only by experimentation.

THE CONTROL PROBLEM

In any homogeneous reactor, it is desirable to have the control mechanism as an inherent property of the system, requiring no extraneous control rods with their associated instrumentation. It is also conceivable that this be necessary, since a large change in the reactivity may occur so rapidly

as to render mechanical controls useless. Fortunately, it appears that in aqueous homogeneous-type reactors the moderator density varies sufficiently with reactivity change so as to be an inherent control. In circulating types this is to be accomplished by operation under conditions such that the density change with temperature is of the proper magnitude. In the boiling-type, the change of density would be accomplished mainly by varying the volume of the vapor phase of the reactor medium. Here any reactivity change would vary the bubble production rate and thus the density of the solution, leading to new equilibrium conditions. Hence the variation of the bubble holdup would act as an automatic control toward solution reactivity changes.

In the above it was tacitly assumed that no time lag exists between nuclear power generation and vapor bubble formation and growth. Such a delay would result in an increase in generated nuclear power with reactivity increase, without any instantaneous corresponding bubble density change, which would be detrimental to automatic control. Thus for long bubble delay times, the nuclear power may increase to such a level that the eventual density changes would cause excessive pressure inside the vessel, or a low order atomic explosion may result. As the pressure is increased within the reactor, relief by vapor condensation would occur, but the extent of this relief at any time depends upon the rate of condensation versus pressure increase rate. Such information is very meager, and experimental data on such phenomena are required, particularly for large pressure increases in small time intervals.

Aiding reactor stability are the delayed neutron emitters which serve two functions, namely: (1) they keep sudden reactivity changes from being

[redacted]

instantaneously fully effective, and (2) they act as powerful damping factors on nuclear oscillations set up by solution reactivity changes.

PROBLEM

In order to be more direct, and obtain an insight into what the operating stability of a boiling-type reactor might be, a specific problem will be considered throughout the remainder of this report. The question of what fuel solution to use in homogeneous reactors is of some concern, since not all solutions of various uranium salts are stable in the desired range of reactor operation, and consideration must be given to neutron economy and corrosion problems involved. One of the likely homogeneous reactor systems consists of a mixture of D_2O-UO_2SO_4 operating at 250° C and 1000 psia [1] [5]. This system was considered in studies of large scale reactors, producing 10^6 kw of heat energy, with low enrichment uranium as the fissile material. Hence, consider the problem:

A bare, homogeneous boiling-type thermal reactor operating at 250° C is to produce 10^6 kw of heat energy, the reactor fluid volume being 10 per cent vapor under steady-state conditions. D_2O will be the moderator-solvent, and natural uranium the fissile material. What will occur within the system if an effect is introduced so as to increase the effective material multiplication constant of the reactor fluid?

Generalized considerations of all factors involved in the above problem lead to unknown quantities and mathematical complexities difficult to analyze; hence, in this study it is assumed that the rate of product heat removal from

[redacted]

the reactor is constant, that the required bubble escape rate is attainable, and that the mass of vapor in the steam chest remains constant. Also, D_2O is considered as having the thermodynamic properties of ordinary water.

According to Noderer and Lansing [6], the critical radius of a bare spherical reactor using natural uranium ($\eta = 1.350$) as UO_2SO_4 in a 250° C D_2O solution is 14.4 feet with the critical multiplication constant being 1.06. To find the critical radius when 0.1 of the reactor volume is vapor, consider

$$k_c = 1 + B^2M^2 \tag{4}$$

where B^2 is the so-called buckling factor, and M^2 the migration area. As a fair approximation, $M^2 = M_o^2\left(\frac{\rho_o}{\rho}\right)^2$, where ρ is the reactor fluid density, and for a sphere, $B^2 = (\pi/R)^2$ where R is the sphere radius. Thus, for k_c to remain constant, $R\rho$ must stay the same. Hence, for $v_g = 0.1$, the critical sphere radius is about 16 feet. This gives an insight into the size of reactor required; i.e., under steady-state operation, the reactor volume, V_r, is 17,120 cu. ft., which includes the vapor volume, $V_g = 1712$ cu. ft.

A cylindrical shaped reactor appears to be a practical starting design for a boiling-type reactor. Under this condition

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{L}\right)^2 \tag{5}$$

where L is the height of the cylinder and R is its radius. Letting m be the mass of the reactor fluid, $\rho = m/V_r$, and with $V_r(0) = \pi R^2 L_o$ where L_o is the fluid height under steady-state conditions, differentiation of Equation (4)

[redacted]

with respect to ρ yields:

$$\frac{dk_c}{d\rho} = B^2 \frac{dM^2}{d\rho} + M^2 \frac{dB^2}{d\rho} = -\frac{2B^2M^2}{\rho} + M^2 \frac{dB^2}{dL}\frac{dL}{d\rho}$$

$$= -\frac{2M^2}{\rho}\left[B^2 - \left(\frac{\pi}{L}\right)^2\right] = -\frac{2M^2}{\rho}\left[\frac{B^2\left(\frac{2.405}{R}\right)^2}{\left(\frac{\pi}{L}\right)^2 + \left(\frac{2.405}{R}\right)^2}\right]$$

$$= \frac{-2(k_c - 1)}{\rho\left[1 + \left(\frac{\pi^2 R^3 \rho}{2.405\, m}\right)^2\right]} = \frac{-2(k_c - 1)}{\rho\left[1 + \left(\frac{\rho \pi R}{\rho_o\, 2.405\, L_o}\right)^2\right]} \qquad (6)$$

Letting $L_o = 2R$, Equation (6) becomes

$$\frac{d\,k_c}{d\,\rho} = \frac{-2(k_c - 1)}{\rho\left[1 + 0.426\left(\frac{\rho}{\rho_o}\right)^2\right]} \qquad (6a)$$

Integration of Equation (6a) under the condition that $k_c = k_{co}$, when $\rho = \rho_o$ results in

$$k_c - 1 = \frac{(k_{co} - 1)}{1.426}\left[\left(\frac{\rho_o}{\rho}\right)^2 + 0.426\right] \qquad (7)$$

It is still necessary to find the correspondence between multiplication constant, time, power and vapor fraction of fluid. From the two group diffusion theory for one region thermal reactors, the time dependent equation for the power as a function of k is

$$\frac{dP}{dt} = \lambda_o\left[(1 - \beta)\,k - k_c\right] P + \sum_i \lambda_i \mu_i \qquad (8)$$

$$\frac{d\mu_i}{dt} + \lambda_i \mu_i = \beta_i\, k\, \lambda_o\, P \qquad (8a)$$

[redacted]

where

β = fraction of neutrons which result from decay of fission products

β_i = fraction of neutrons having decay characteristic λ_i ($\Sigma_i \beta_i = \beta$)

λ_o = reciprocal lifetime of prompt neutrons = $\Sigma_s v_s/(1 + B^2 \tau_f)$

λ_i = decay constant of delayed neutron emitter

μ_i = concentration of delayed neutron emitter

Σ_s = thermal absorption cross-section, per cm.

τ_f = Fermi Age, sq. cm.

B^2 = buckling factor

k = material multiplication constant if finite pile had no leakage

k_c = multiplication constant required for criticality, if finite pile had no leakage

P = power dissipation, kw.

t = time, sec.

To find a relation between fluid density and vapor fraction of reactor fluid, return to the definition of v_g, or

$$v_g = V_f/V_r \tag{9}$$

where

V_f = volume of vapor in reactor fluid, cu. ft.

V_r = volume of reactor fluid, cu. ft.

The density of the reactor fluid can then be expressed as:

$$\rho = \rho_l (1 - v_g) + \rho_g v_g$$

[redacted]

where

ρ_l = liquid density, lb/cu. ft.

ρ_g = vapor density, lb/cu. ft.

Defining $v_g(o)$ as the fraction vapor volume under steady-state operation, and ρ_o as the corresponding reactor fluid density,

$$\frac{\rho_o}{\rho} = \frac{a - v_g(o)}{a - v_g} \tag{11}$$

where

$$a = \rho_l(\rho_l - \rho_g) \tag{12}$$

It remains to find the time dependent correspondence between ρ and P. In the steady-state, the nuclear power production is directly proportional to the rate of heat removal from the reactor solution. Neglecting any change in the heat content of the liquid, if the heat extraction rate remains constant at P_o, the heat content of the vapor bubbles is time dependent upon the nuclear power as given below:

$$\frac{dH}{dt} = \alpha \int_0^\infty \lambda(\epsilon) \left[P(t - \epsilon) - P_o\right] d\epsilon \tag{13}$$

where

α = Btu/kw-sec. = 3413/3600

P_o = power removal from reactor unit, kw

$\lambda(\epsilon)$ = normalized distributed bubble time delay function

[redacted]

ϵ = time delay of bubble formation, sec.

$H = V_f h_{fg} \rho_g$, enthalpy of vapor in fluid, Btu.

h_{fg} = enthalpy of vaporization, Btu/lb.

Assuming that the total volume of the reactor unit is twice the original liquid volume, and using the thermodynamic properties of steam and water as presented by Keenan and Keyes [7], Equations (9) and (11), and constancy of vapor mass in the steam chest, the relations between H, ρ, v_g and p given in Table I were obtained.

TABLE I

Thermodynamic Relations for the System Considered

p #/in^2	T °F	h_{fg} Btu/#	ρ_l #/ft^3	ρ_g #/ft^3	$\rho_l - \rho_g$ #/ft^3	$a = \rho_l/\rho_l - \rho_g$	V_l ft^3	$V_s + V_f$ ft^3	V_f ft^3	v_g	ρ #/ft^3	$H = V_f \rho_g h_{fg}$ Btu
580	482.6	736.1	49.8	1.25	48.55	1.025	15,408	15,408	1712	0.1	44.91	1,575,000
600	486.2	731.6	49.8	1.30	48.50	1.027	15,392	15,424	2255	0.128	43.60	2,145,000
650	494.9	720.5	49.3	1.41	47.89	1.030	15,517	15,299	3157	0.169	41.23	3,207,000
700	503.1	709.9	48.8	1.53	47.27	1.0315	15,643	15,173	3984	0.203	39.16	4,326,000
800	518.2	688.8	47.8	1.76	46.04	1.039	15,907	14,909	5182	0.246	36.52	6,283,000
900	530.2	668.8	47.18	2.00	45.18	1.044	16,046	14,770	6210	0.279	34.56	8,307,000
1000	544.6	649.4	46.3	2.34	43.96	1.052	16,253	14,563	7247	0.308	32.69	11,013,000
1200	567.2	611.7	44.85	2.76	42.09	1.063	16,667	14,149	7946	0.323	31.16	13,415,000

The manner of obtaining the data presented in the above table is illustrated below.

Let V_l = total liquid volume, ft^3

V_f = vapor volume in reactor fluid

V_s = vapor volume in steam chest

$V_{st} = V_s + V_f$

Since the mass of vapor in the steam chest is assumed constant,

$$V_s \rho_g = V_s(o) \rho_g(o) = 13{,}694\ (1.24) = 17{,}120 \text{ lbs. vapor} \tag{14a}$$

Also, the total mass in the reactor unit is a constant, or

$$\rho_l\ V_l + \rho_g\ V_{st} = \rho_l(o)\ V_l(o) + \rho_g(o)\ V_{st}(o)$$

$$= 49.8\ (15{,}408) + 1.25\ (15{,}408) = 786{,}578 \text{ lbs. fluid} \tag{14b}$$

Also,

$$V_f + V_s + V_l = 30{,}816 \text{ ft}^3 \tag{14c}$$

Combining Equations (14a, b, c) yields

$$786{,}578 = (\rho_l - \rho_g)V_l + \rho_g\ (30{,}816) \tag{14d}$$

With the aid of the thermodynamic properties of steam [7], V_l can be calculated at various equilibrium pressures. Thus, at 600 psi.,

$$V_l = \frac{786{,}578 - \rho_g\ (30{,}816)}{\rho_l - \rho_g} = 15{,}392 \text{ ft}^3$$

Knowing V_l, V_{st} can be calculated from Equation (14c), or

$$V_{st} = 30{,}816 - V_l = 15{,}424 \text{ ft}^3 \text{ at 600 psia.}$$

Since $(V_s + V_f)\rho_g = V_{st}\rho_g$, with the aid of Equation (14a) there results

$$17{,}120 + V_f\ \rho_g = 15{,}424\ \rho_g$$

$$V_f = 2255 \text{ ft}^3 \text{ at } 600 \text{ psia.}$$

It follows that

$$v_g = \frac{V_f}{V_f + V_l} = \frac{2,255}{17,647} = 0.128$$

$$\rho = \rho_l (1 - v_g) + \rho_g v_g = (\rho_l - \rho_g)(a - v_g)$$

$$= 48.5 (1.027 - 0.128) = 43.6 \text{ lb/ft}^3$$

The pertinent information presented in Table I is plotted in Figures 2, 3, and 4.

The resulting curve in Figure 2 is closely approximated by the following equation:

$$H + c = \frac{a}{\rho - b} \tag{15}$$

where

$$c = 7.0598 \times 10^6$$
$$a = 205.326 \times 10^6$$
$$b = 21.1317 \times 10^6$$

Equations (7), (8), (8a), (13), and (15) mathematically describe the problem at hand, but they must be simplified for solution. Concerning Equations (8) and (8a), it has been shown by Stein in ORNL-925 [3] that for times up to 0.1 of a second they are accurately represented by:

$$\frac{d\rho}{dt} = \lambda_o \left[(1 - \beta) k - k_c\right] P + \lambda_o \beta k_{co} P_o \tag{16}$$

[redacted]
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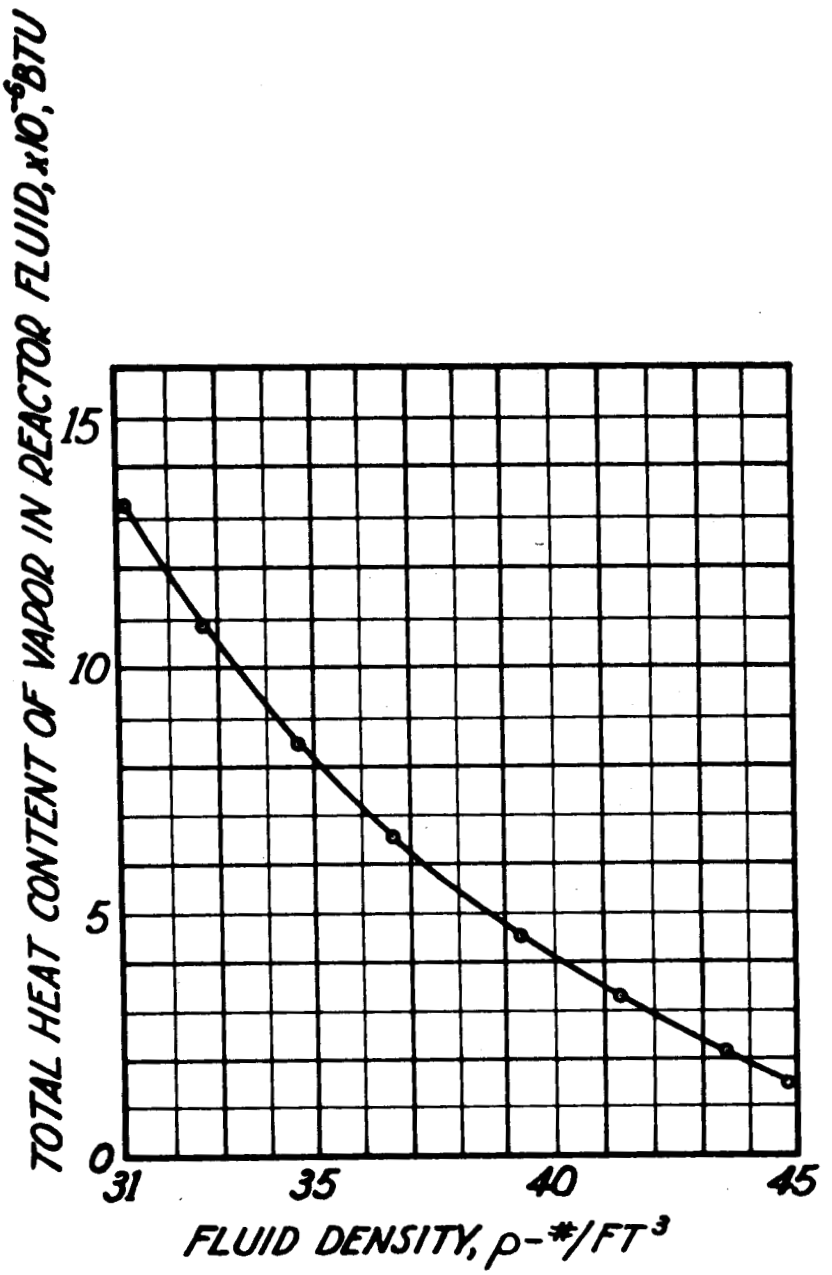

Figure 2

Enthalpy of Vapor in Reactor Fluid vs. Fluid Density

[redacted]
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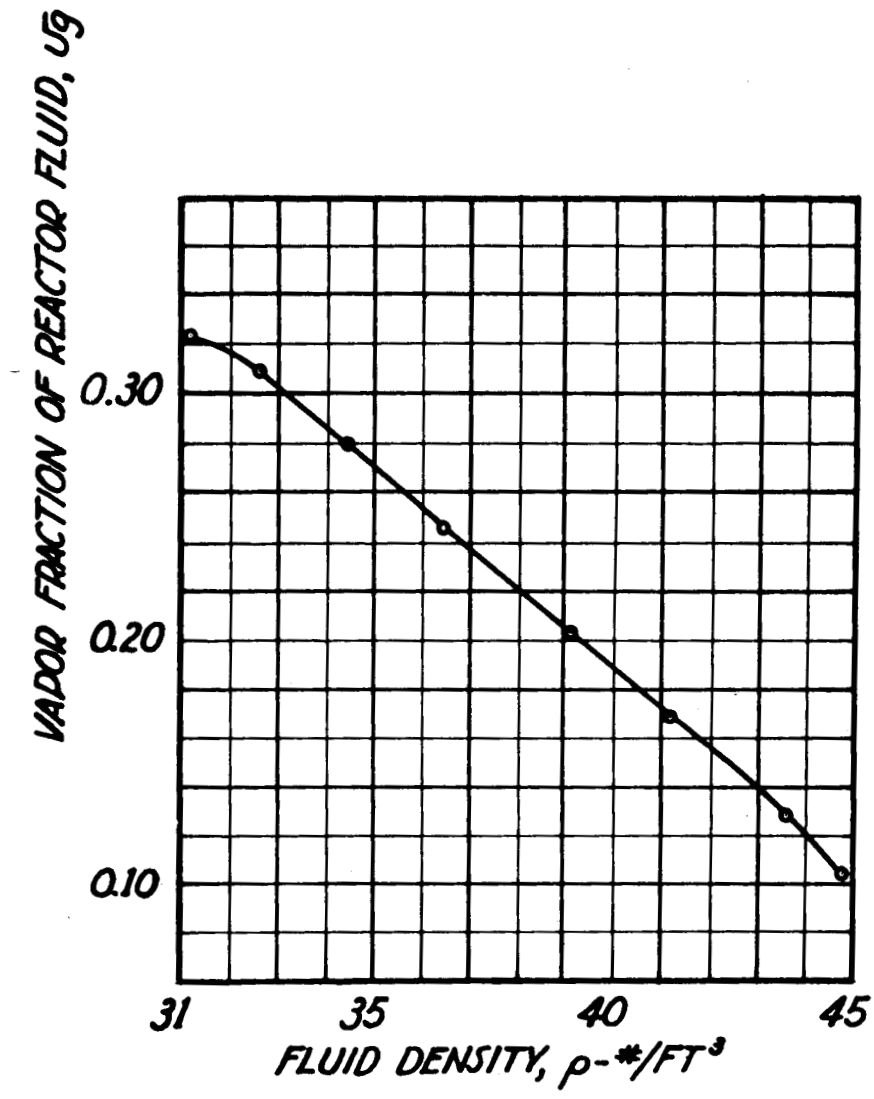

Figure 3

Fluid Vapor Fraction vs. Fluid Density for Constant Vapor Mass in the Steam Chest

[redacted]
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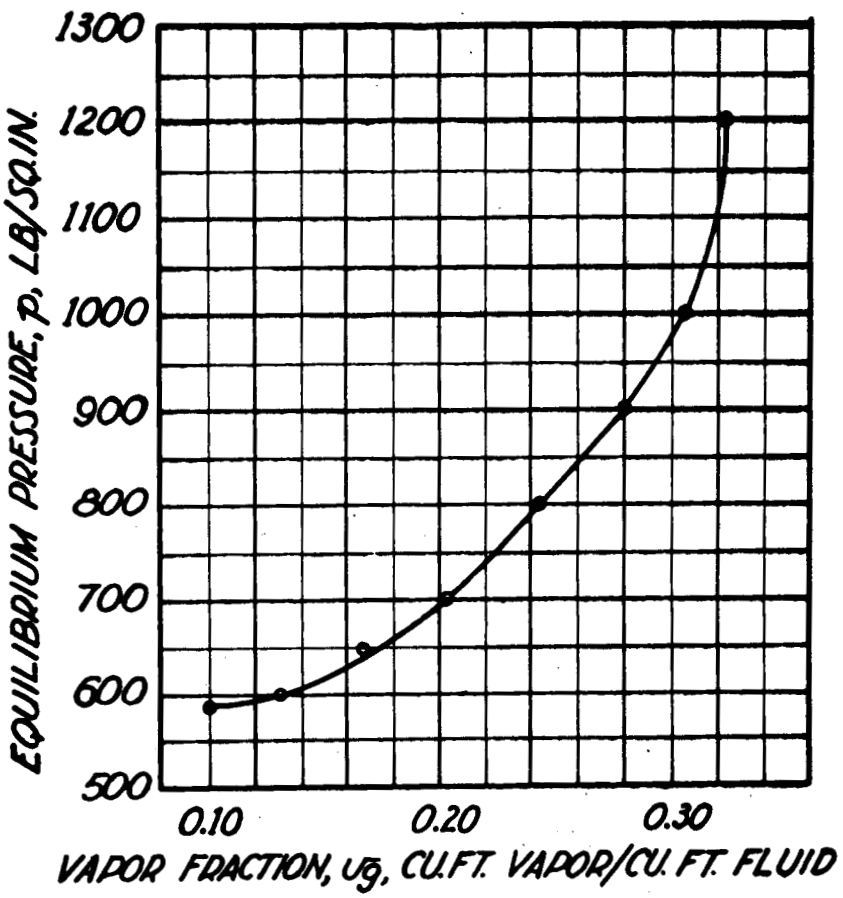

Figure 4

Plot of Vapor Fraction Versus Equilibrium Pressure

[redacted]

Also, in Equation (13) the form of the normalized distributed time delay function is unknown, and complicated functions make the resulting equations too difficult to handle conveniently. Hence, as a first approximation, let the vapor generated at time $t + \tau$ result from nuclear power at time t, where τ is some discreet value. This corresponds to saying the function $\lambda(\epsilon)$ is a delta function of argument $t - \tau$. Under this assumption, Equations (13) and (15) combine to yield:

$$a \frac{d\left(\frac{1}{\rho - b}\right)}{dt} = \alpha \left[P(t - \tau) - P_o\right] \tag{17}$$

For small τ, the above is approximated by:

$$a\left(\frac{d \frac{1}{\rho - b}}{dt}\right) = \alpha \left[P - \tau \frac{dP}{dt} - P_o\right] \tag{17a}$$

Because of the form of Equation (15), the solution of the kinetic equation is greatly facilitated if Equation (6) can be represented by

$$k_c = A + \frac{B}{\rho - b} \tag{18}$$

With A = 1.020022 and B = 0.95062, Equation (18) closely approximates Equation (6) for $\rho \geq 36$, as illustrated in Figure 5. The lower limit on ρ corresponds to a v_g of 0.255 (see Figure 3), so the range covered by Equation (18) appears sufficient to allow its use. Letting $\sigma = P/P_o$, the resulting simplified kinetic equations for the reactor are:

$$\frac{d\sigma}{dt} = \lambda_o \left[(1 - \beta) k - k_c\right]\sigma + \lambda_o \beta k_{co}$$

$$\frac{d\left(\frac{1}{\rho - b}\right)}{dt} = \alpha \frac{P_o}{a} \left[\sigma - \tau \frac{d\sigma}{dt} - 1\right]$$

[redacted]

[redacted]
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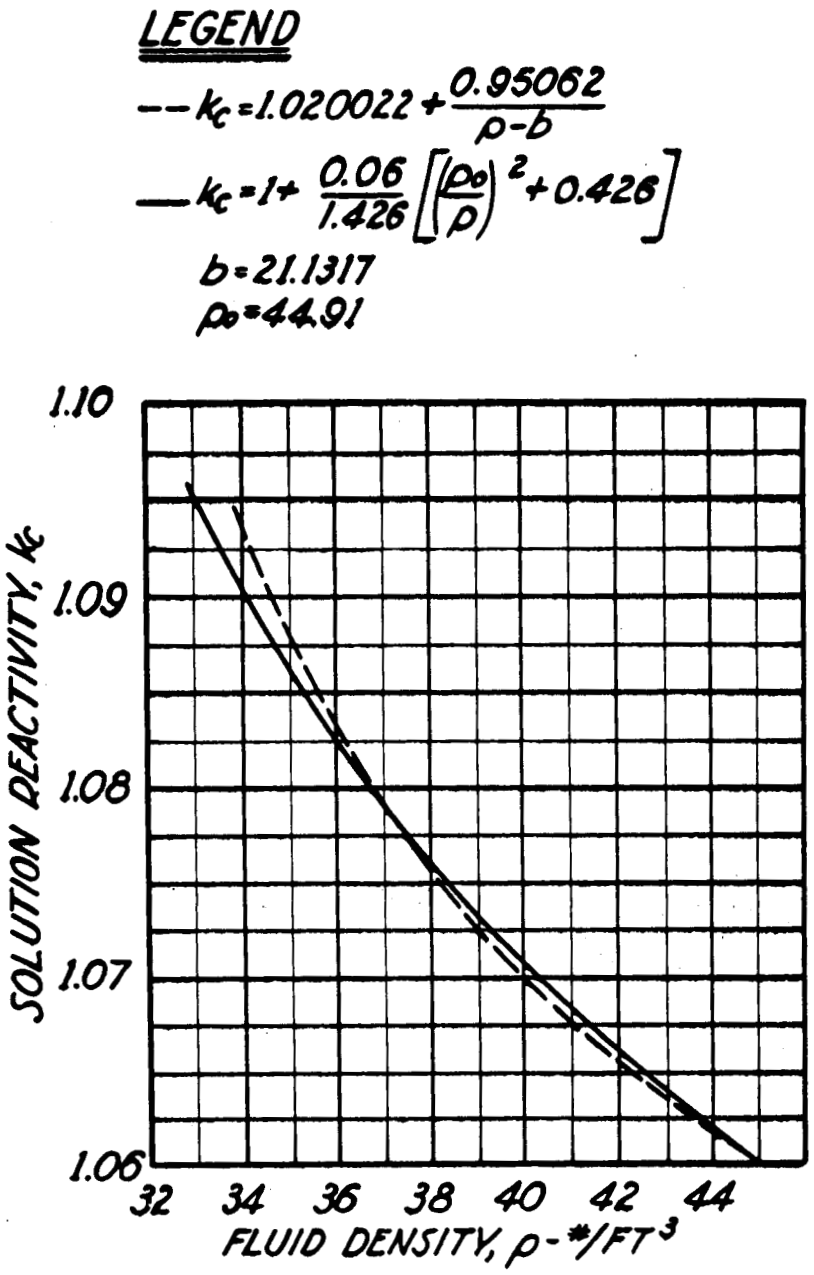

Figure 5

Solution Reactivity versus Fluid Density, for $k_{co} = 1.06$

[redacted]

$$k_c = A + B/(\rho - b) \tag{19}$$

Sudden reactivity changes will appear as changes in the material multiplication constant, k. The most dangerous fashion in which k could change is in the form of a step function, or instantaneously. However, k would change with time for normal alterations in solution reactivity. These two types of changes can be incorporated in k by letting

$$k = k_o + Ct \tag{20}$$

where any instantaneous increase in reactivity is represented in k_o, while k increase with time is represented in the linear function Ct.

Combination of Equations (19) and (20) yields non-linear equations difficult to solve directly on a time basis. Resort is therefore made to the phase-plane method, which eliminates time directly. Thus, letting

$$w = 1/(\rho - b)$$

$$u = k(1 - \beta) - k_c = (k_o + Ct)(1 - \beta) - A - Bw \tag{21}$$

$$K = \frac{P_o \alpha}{a}$$

Equation (19) can be replaced by

$$\frac{d\sigma}{dt} = \lambda_o (u\sigma + \beta k_{co})$$

$$\frac{du}{dt} = (1 - \beta) C - BK\left[\sigma - 1 - T\frac{d\sigma}{dt}\right] \tag{21a}$$

Dividing one equation by the other, their phase-plane representation becomes

[redacted]

$$\frac{du}{d\sigma} = BK\tau + \frac{(1 - \beta)\ C - BK\ (\sigma - 1)}{\lambda_o\ [u\sigma + \beta k_{co}]} \tag{22}$$

Plotting the above equation, the time required to travel any portion of the resulting curve can be found by graphically integrating Equation (19a), or:

$$\int_{t_1}^{t_2} dt = \int_{u_1}^{u_2} \frac{du}{C(1 - \beta) - BK\ [\sigma - 1 - \tau\ \lambda_o\ (u\sigma + \beta k_{co})]} \tag{23}$$

$$\int_{t_1}^{t_2} dt = \int_{\sigma_1}^{\sigma_2} \frac{d\sigma}{\lambda_o\ [u\sigma + \beta k_{co}]} \tag{24}$$

For the problem at hand,

$$\begin{aligned}
\alpha &= 3413/3600 \\
\lambda_o &= 10^4 \\
\rho_o &= 44.91 \\
a &= 205.326 \times 10^6 \\
A &= 1.020022 \\
b &= 21.1317 \\
B &= 0.95062 \\
k_{co} &= 1.06 \\
K &= 0.00461731 \\
P_o &= 10^6
\end{aligned} \tag{25}$$

It remains to choose values for C, β , and τ . If one assumes that feed material can be introduced into the boiler at about 7000 gallons per minute, and if the greatest concentration of natural uranium in make-up solution is 500 grams per liter, the maximum value C can have is 0.002215 per second. 7000 gpm was arbitrarily chosen, and is large enough to be beyond any probable input flow rate. Similarly, 500 gm. U/l. was arbitrarily chosen as a uranium concentration sufficiently greater than the 100 gm. U/l. in the reactor, and yet reasonably removed from the saturated solution concentration so as to eliminate the possibility of precipitation. Using the above values, the C calculated represents an extreme condition. The value 0.002215 was obtained as follows:

From reactor theory,

$k = \eta \epsilon p f$

η = prompt fission neutrons/neutron absorbed in U^{235} = 2.09

ϵ = fast effect = 1

p = resonance escape probability = 0.865 in this problem (6)

f = thermal utilization factor, or neutrons absorbed in U^{235}/neutron absorbed

In the problem at hand the reactor solution contains about 100 grams of uranium per liter, and k = 1.06. One can then write

$$1.06 = 2.09\ (1)\ 0.865\ \frac{0.71}{0.71 + A}$$

or $A = 0.50092$

The value for C in the expression $k = k_o + Ct$ can be obtained by differentiation, or

$$\frac{dk}{dt} = C$$

To obtain a maximum value, consider that in adding more uranium only the U^{235} is of importance. Then

$$k = 1.8078\ \frac{N_{25}}{N_{25} + 0.50092} \qquad (25a)$$

where N_{25} is the concentration of the U^{235}.

From the above

$$\left(\frac{dk}{dt}\right)_{max} = 1.8078 \frac{(0.50092)}{(1.211)^2} \frac{dN_{25}}{dt}$$

If concentrated feed solution contains 500 grams uranium per liter, and can be introduced at a maximum rate of about 7000 gallons per minute, then

$$\left(\frac{dN_{25}}{dt}\right)_{max} = \frac{7000\ (5)\ (0.71)}{15,408\ (7.48)} = 0.216 \frac{\text{gms } U^{235}}{\text{l - min.}}$$

and

$$\left(\frac{dk}{dt}\right)_{max} = C = 0.1329/\text{min.}$$

or

$$C = 0.002215/\text{sec.}$$

A feed rate of 7000 gallons per minute would correspond to about 16.5 minutes required to fill the tank. Also, for diluting purposes, if pure moderator were fed into the reactor at this rate, the initial drop in k would be about 0.027/minute.

The term "delayed neutrons" has come into usage because certain of the fission products formed from fission of U^{235} emit neutrons some time following fission [8]. These delayed neutrons are emitted with a half-life associated with that of the beta decay preceding the neutron emission, the fraction of delayed neutrons and its decay constants given in Table II.

TABLE II

Values of the Decay Constant Associated with the Delayed Neutron Emission of U^{235} Fission

β_i	λ_i (sec^{-1})
0.00025	13.86
0.00085	1.61
0.00241	0.456
0.00213	0.154
0.00166	0.0315
0.00025	0.0215
$\sum_i \beta_i = \beta =$ 0.00755	

Hence, the maximum value for β is 0.00755.

Concerning a value for τ, Bowman and Parish [9] have stated that the rate of bubble formation in internally heated aqueous solutions reaches a maximum about 0.003 seconds after a power pulse. Lane [10] reported information that steam bubbles form with explosive violence in the order of 0.003 seconds. On the basis of the above, τ will be assumed to have a value of about 0.003 seconds.

The effect of reactivity changes upon the nuclear system of the reactor depends on the values of the above parameters. To evaluate their importance, several cases will be presented.

Case I

$\beta = 0$, $\tau = 0$. This corresponds to disregarding delayed neutrons and also time delay in bubble formation with respect to nuclear power. The solution

reactivity is considered to change (1) instantaneously, (2) linearly with time, and (3) by a combination of (1) and (2).

Case IA. $k = k_{co} + 0.002215t$ ($u = 0$ when $\sigma = 1$). This represents an increase of solution reactivity with time. Here, using the parameter values as given in Equation (25), Equation (22) can be integrated, yielding

$$u^2 = \left[1.3208 \ln \sigma - 0.87786 (\sigma - 1)\right] \times 10^{-6} \tag{26}$$

which is graphically represented in Figure 6a. This shows the nuclear power oscillating about a new equilibrium value, 1.505 P_o. By reading corresponding values of u and σ from Figure 6a, Equations (23) and (24) can be graphically integrated to obtain the time required to travel any portion of the curve. Both equations are useful since it is easier to evaluate time by Equation (23) when $u\sigma$ is close to the value of βk_{co}. Otherwise Equation (24) is simplest to use. Once having corresponding values of u and t, the variation of ρ with time can be found from the definition of u, or

$$u = (k_o + Ct)(1 - \beta) - A - \frac{B}{\rho - b}$$

In this manner the relations between u, σ, t, and ρ were obtained, the corresponding values being given in Table III for one-half of the first cycle.

TABLE III

Corresponding Values of u, σ, t, and ρ for Case IA, for One-half Cycle

σ	$u \times 10^4$	$\frac{1}{C - BK(\sigma - 1)}$	$\frac{1}{\lambda_o u \sigma}$	t (sec)	ρ lb/ft^3
1	0	451.4	--	0	44.91
1.01	0.600	460.5	1.500	0.0300	44.91
1.04	1.292	490.2	0.7442	0.0599	44.91
1.10	1.952	562.9	0.4657	0.0943	44.90
1.30	2.884	1112.8	0.2667	0.162	44.87
1.5047	3.109	--	0.2138	0.206	44.82
1.70	2.939	-1166.4	0.20015	0.246	44.76
1.90	2.402	- 576.3	0.2191	0.288	44.68
2.10	1.196	- 382.7	0.3981	0.341	44.54
2.15	0.387	- 353.0	1.2018	0.370	44.45
2.155	0	- 350.3	--	0.384	44.42

The above results are plotted and extended to several cycles in Figures 6b, 6c, and 6d. From the above figures it is seen that under the conditions presented, the nuclear power oscillates with a period of about 0.77 seconds, having a maximum σ of 2.155. The change of fluid density with time is fairly regular, no abrupt changes taking place so as to get excessive pressures (see Figures 3 and 4). From Figure 6c, it is noted that k can be increased at the rate cited here for about nine seconds before the fluid density decreases to 37 lb/cu. ft.

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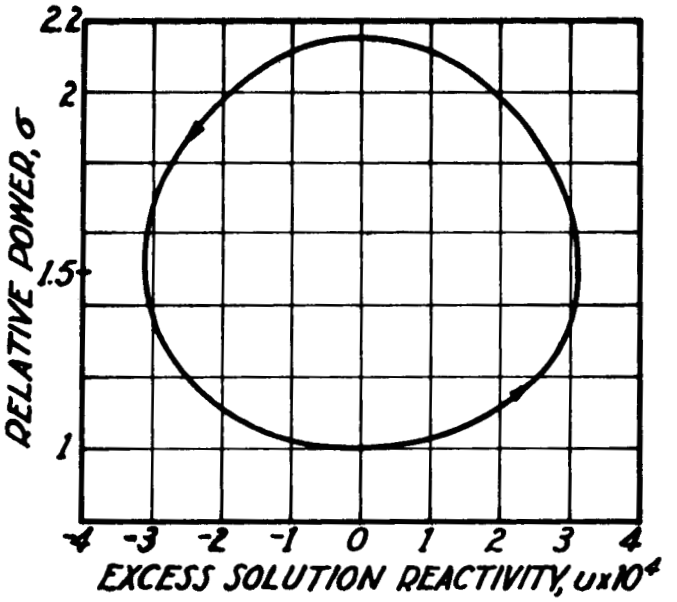

FIG. 6a

Graph of Equation (26), Power Ratio, σ, vs. Excess k in System for Constant Rate of k Input; $\beta = \gamma = \Delta k = 0$

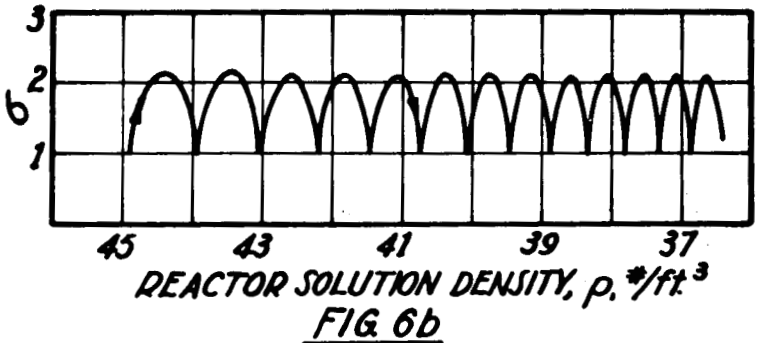

FIG. 6b

Variation of Fluid Density with the Power Ratio, for Constant Rate of k Input; Based on Figure 6a

[redacted]

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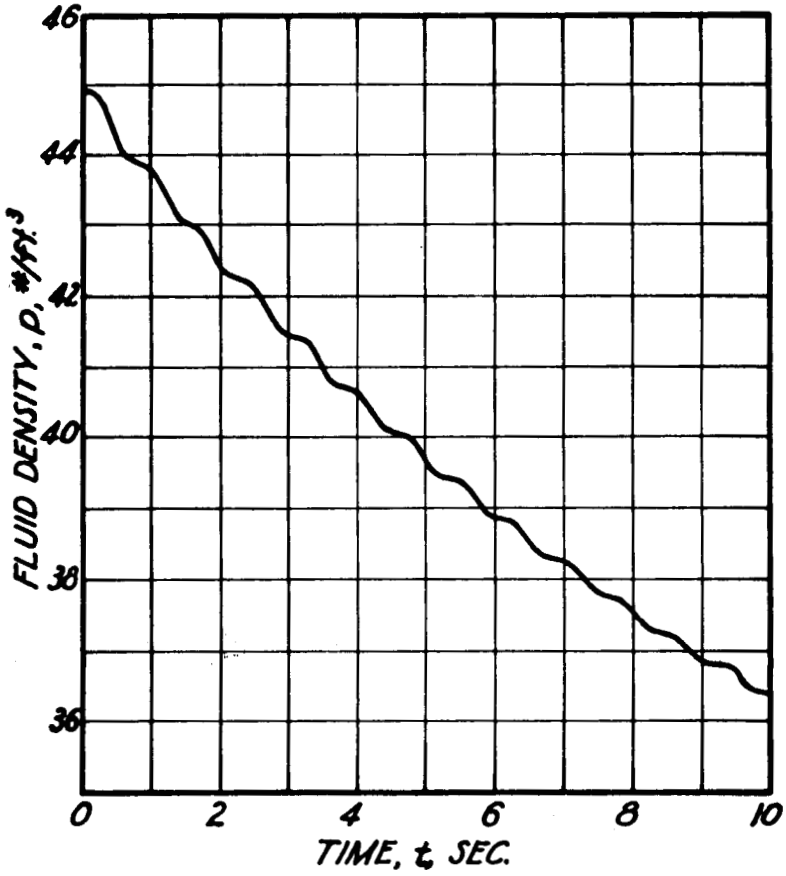

FIG. 6C

Variation of Fluid Density Versus Time During Which Solution Reactivity, k, is Increased; Based on Fig. 6a

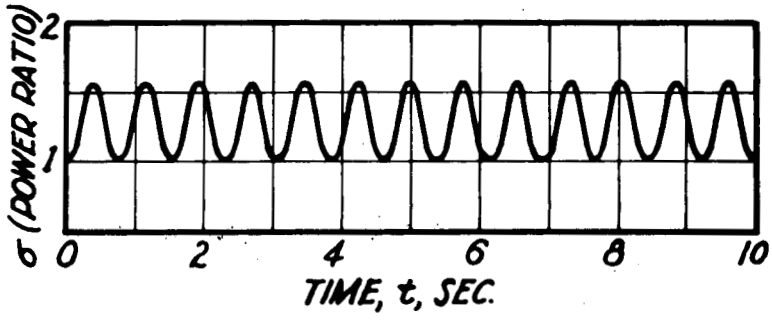

FIG. 6d

Variation of Power Ratio With Time. k is Linearly Increased; Based on Figure 6a

Case IB. $k = k_{co} + 0.01 + Ct$ ($u = 0.01$ when $\sigma = 1$); $C = 0$ and $C = 0.002215$. This concerns the influence of an instantaneous solution reactivity change on the nuclear characteristics and also the effect of C under such conditions. Insertion of the above values in Equation (22) yields the following upon integration:

$$u^2 - 10^{-4} = 0.87786 \times 10^{-6} \left[\ln \sigma - \sigma + 1\right] \tag{27a}$$

with $C = 0$, and

$$u^2 - 10^4 = \left[1.3208 \ln \sigma - 0.87786 (\sigma - 1)\right. \times 10^{-6} \tag{27b}$$

when $C = 0.002215$. These two equations are graphed in Figure 7, which illustrates that for an instantaneous Δk of 0.01 the effect of C is quite small on an excess reactivity-power level plot. The influence of C appears in the power level, fluid density, and time relationships. These were obtained in the same manner indicated in Case IA, and then plotted. By comparing Figures 8a and 9a with 8b and 9b, it is seen that the presence of C results in a change to lower fluid densities as time increases, whereas pure oscillatory motion occurs when $C = 0$. The period of each cycle is illustrated in Figures 10a and 10b, being about 3.3 seconds with $C = 0.002215$ and 4.8 seconds when $C = 0$. (This difference in period is brought about because the power pulse time intervals and heights are about the same in both instances, while the equilibrium power is P_o when $C = 0$ and $1.505\ P_o$ when $C = 0.002215$.)

Comparing Figures 6d and 10b, the influence of the instantaneous k change on the time cycle is very noticeable. Thus, for $k = k_{co} + Ct$, the power varies with time in a nearly sinusoidal manner, whereas each cycle is definitely

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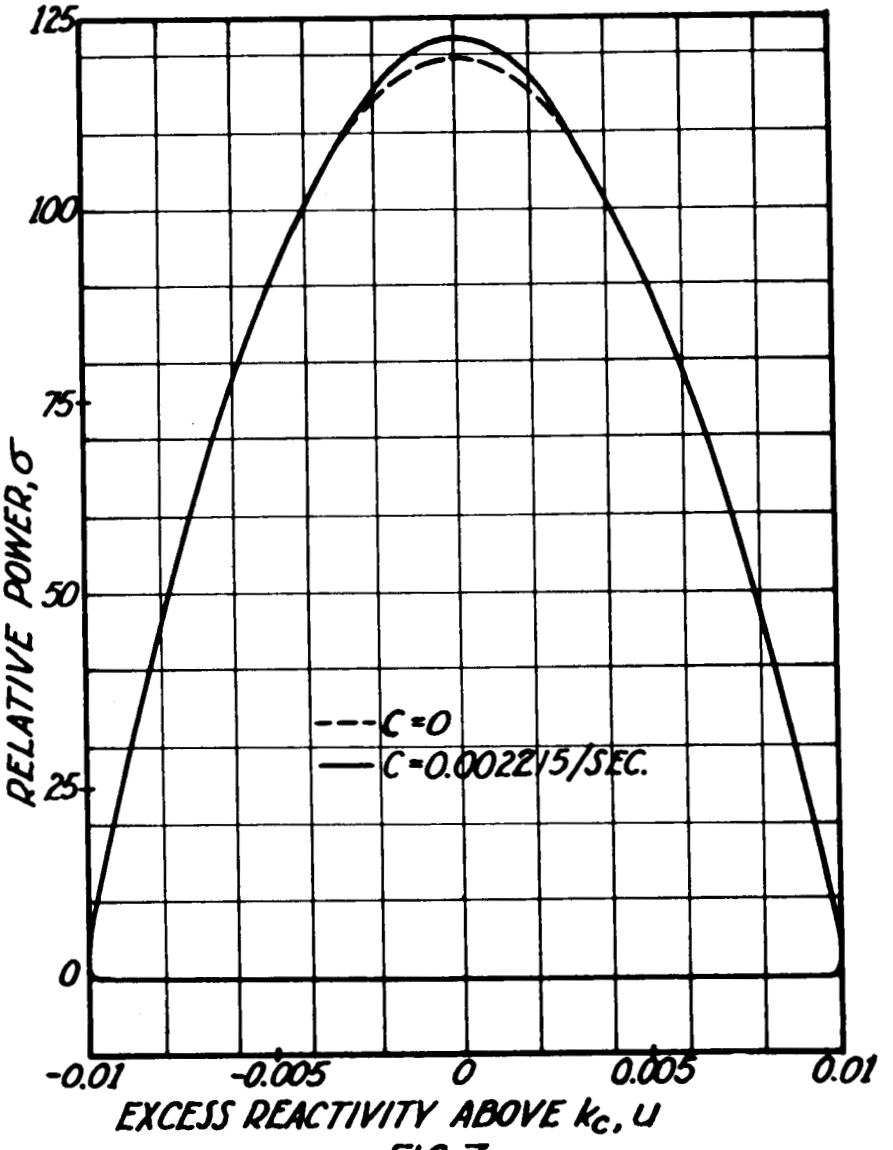

FIG. 7

Graph of Equations (27a) and (27b), Power Ratio, σ, Versus Solution Excess Reactivity, u, with an Instantaneous Δk of 0.01, and with and without a Linear Increase in k; $\beta = \tau = 0$

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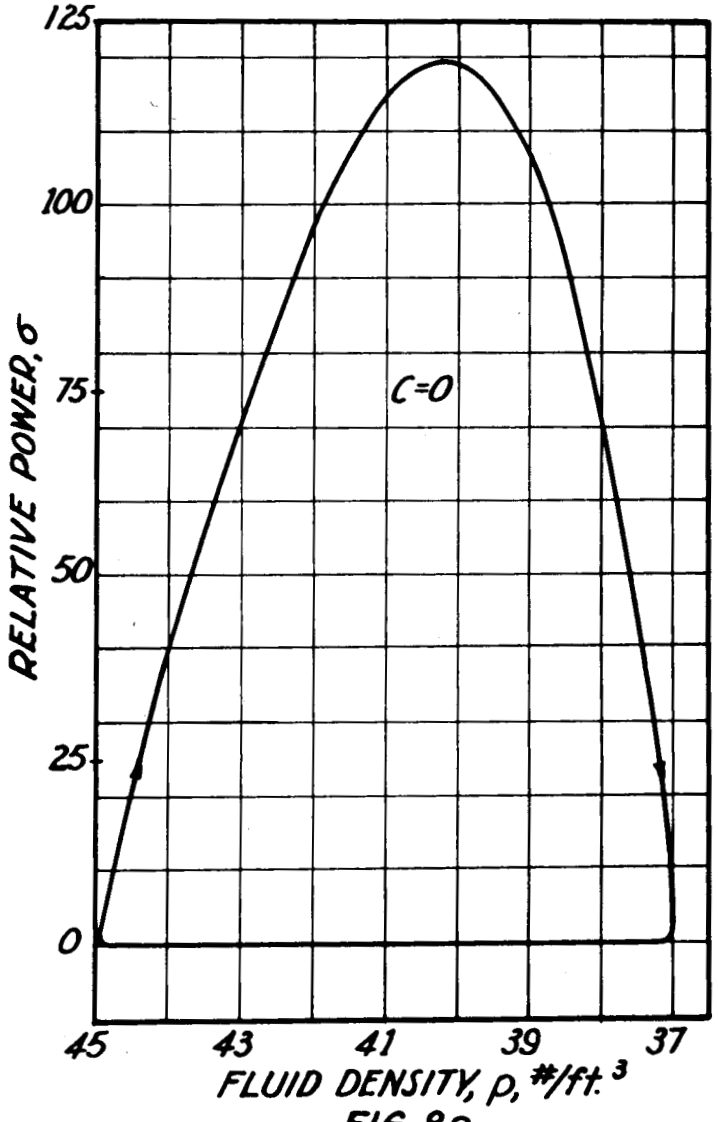

FIG. 8a

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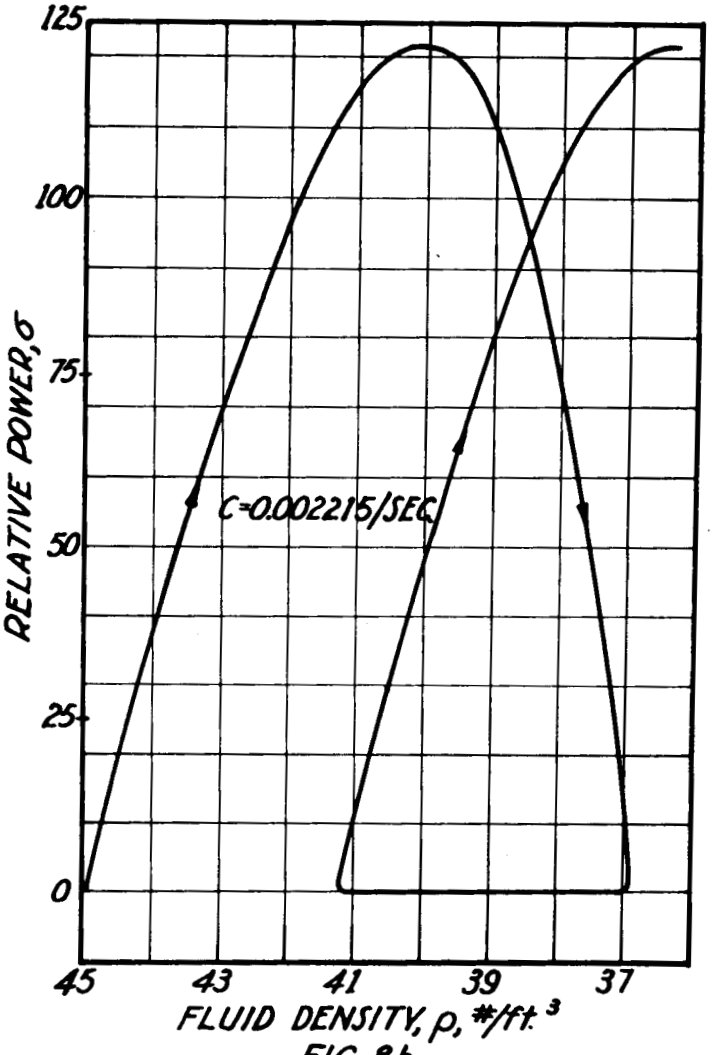

FIG. 8b

Plot of Power Ratio Versus Fluid Density for
C = 0.002215; Based on Figure 7

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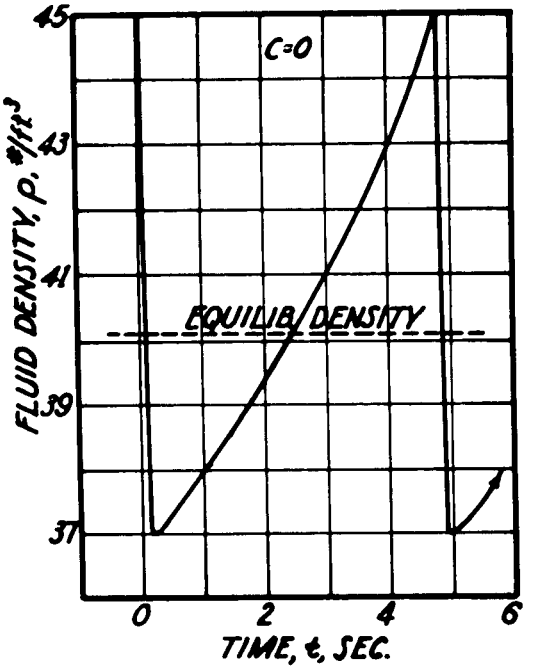
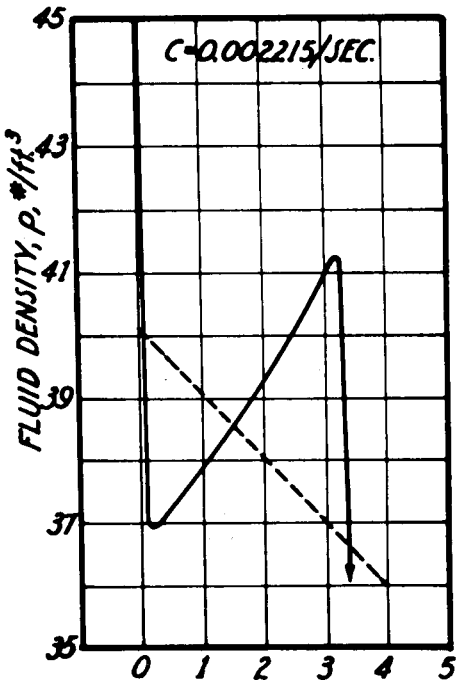


FIG. 9a

Plot of Fluid Density Versus Time for C = 0; Based on Figures 7 and 8a



Plot of Fluid Density vs. Time, for C = 0.002215; Based on Figs. 7 and 8b

FIG. 9b

[redacted]
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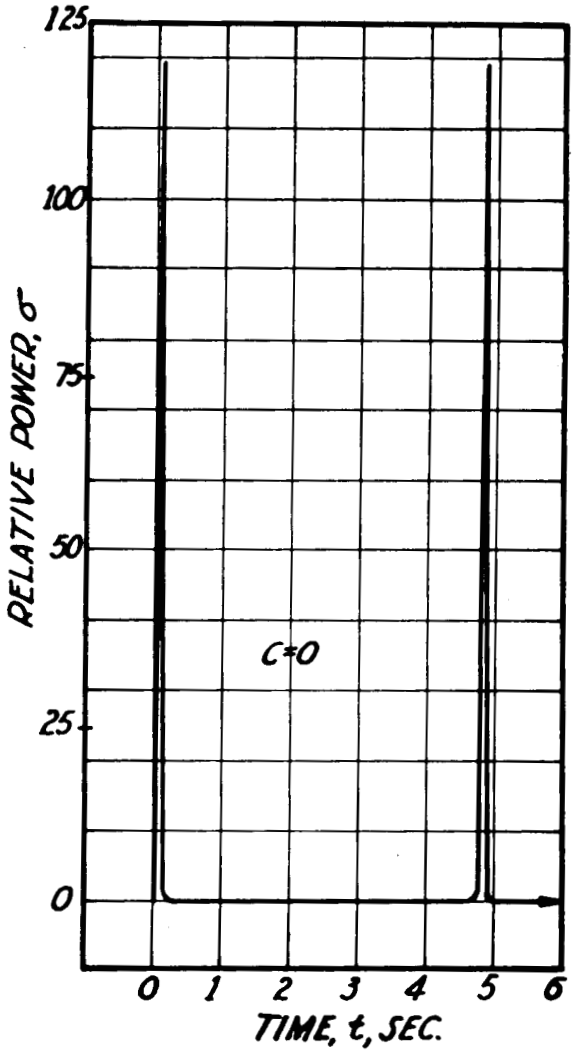

FIG. 10a

Plot of Power Ratio Versus Time for C = 0; Based on Figs. 8a and 9a

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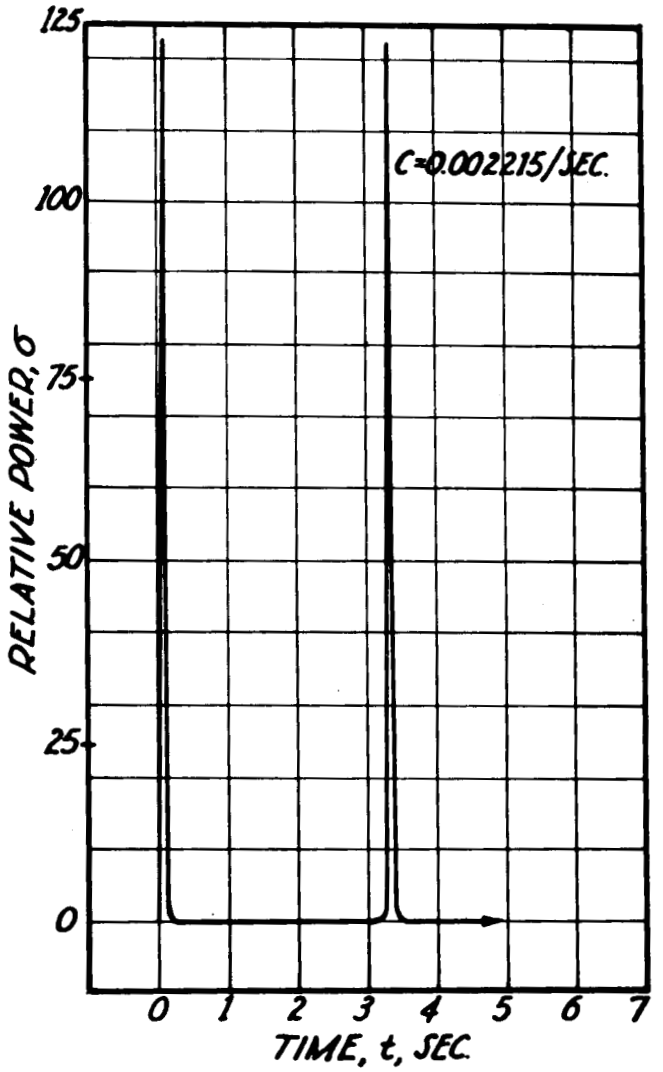

FIG. 10b

Plot of Power Ratio Versus Time for C = 0.002215;
Based on Figs. 8b and 9b

[redacted]

unsymmetrical for $k = k_{co} + 0.01 + Ct$. In the latter case the time to reach maximum nuclear power is 0.0612 seconds, the total period being 3.25 seconds, whereas for $k = k_{co} + Ct$ the corresponding times are 0.384 and 0.768 seconds. The density changes in the two instances are given in Figures 6c and 9b, illustrating the comparatively very rapid and significant density changes occurring when an instantaneous Δk is introduced.

<u>Case IC</u>. $k = k_{co} + 0.02$ ($u = 0.02$ when $\sigma = 1$). This is the same as Case IB with $C = 0$, except the Δk here is 0.02 instead of 0.01. For this case Equation (22) yields

$$u^2 - 4 \times 10^4 = 0.87786 \times 10^{-6} \left[\ln\sigma - \sigma + 1\right] \tag{28}$$

which is shown in Figure 11a below. The maximum nuclear power is about 462 P_o as compared to 119 when $\Delta k = 0.01$, or a four-fold increase in the height of the power pulse. This leads to more asymmetry in the power-time cycle, for, as shown in Figure 11b, it now takes 0.038 seconds to reach maximum power and about 0.2 seconds to complete the cycle. The accompanying density change is plotted in Figure 11c, which shows that the density goes down to about 33 lb/cu.ft., and therefore outside the region of applicability of Equation (18). However, the equilibrium fluid density is about 37 lb/cu. ft., so if there were a nuclear damping affect keeping the density from undershooting this equilibrium value, an instantaneous Δk of 0.02 would be permissible.

In all of the above situations, increasing reactivity starts sustained nuclear power oscillations. These oscillations are highly undesirable since very small increases in k at various times could increase the amplitude of existing oscillations. It is therefore necessary to have within the system

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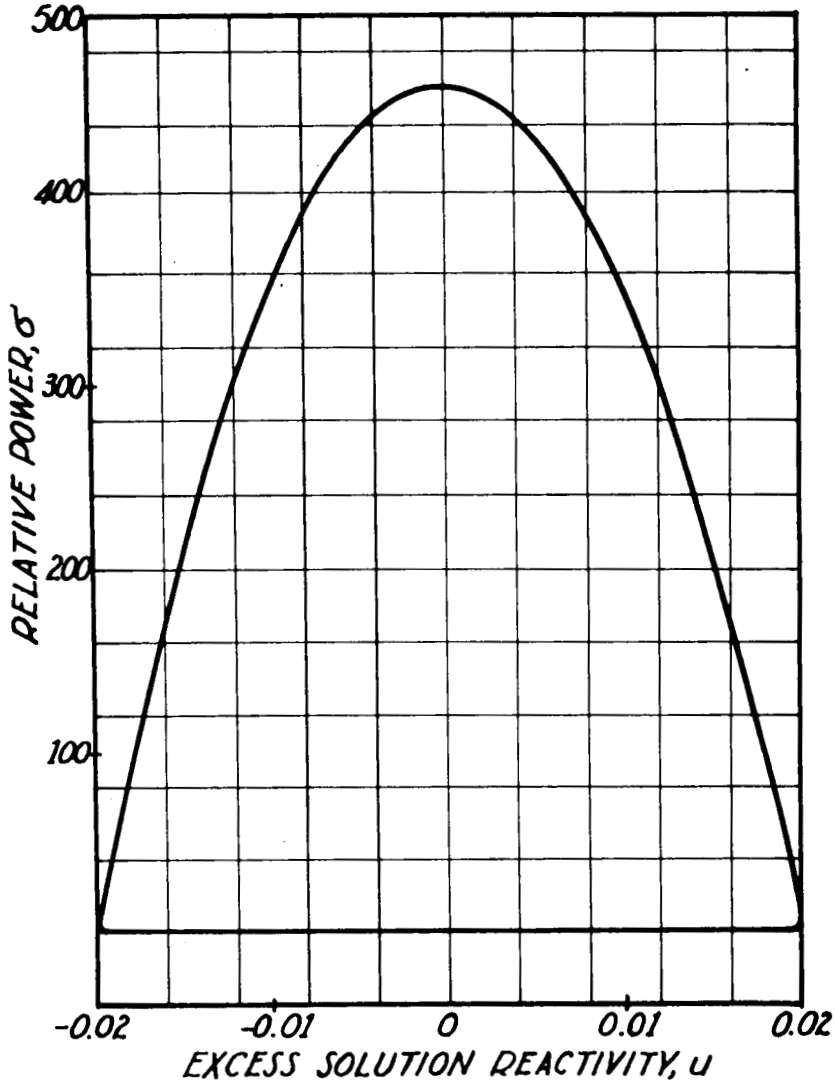

FIG. 11a

Graph of Equation (28), Power Ratio, σ, Versus Excess Reactivity, u, for C = 0, $k = k_{co} + 0.02$, $\beta = \tau = 0$

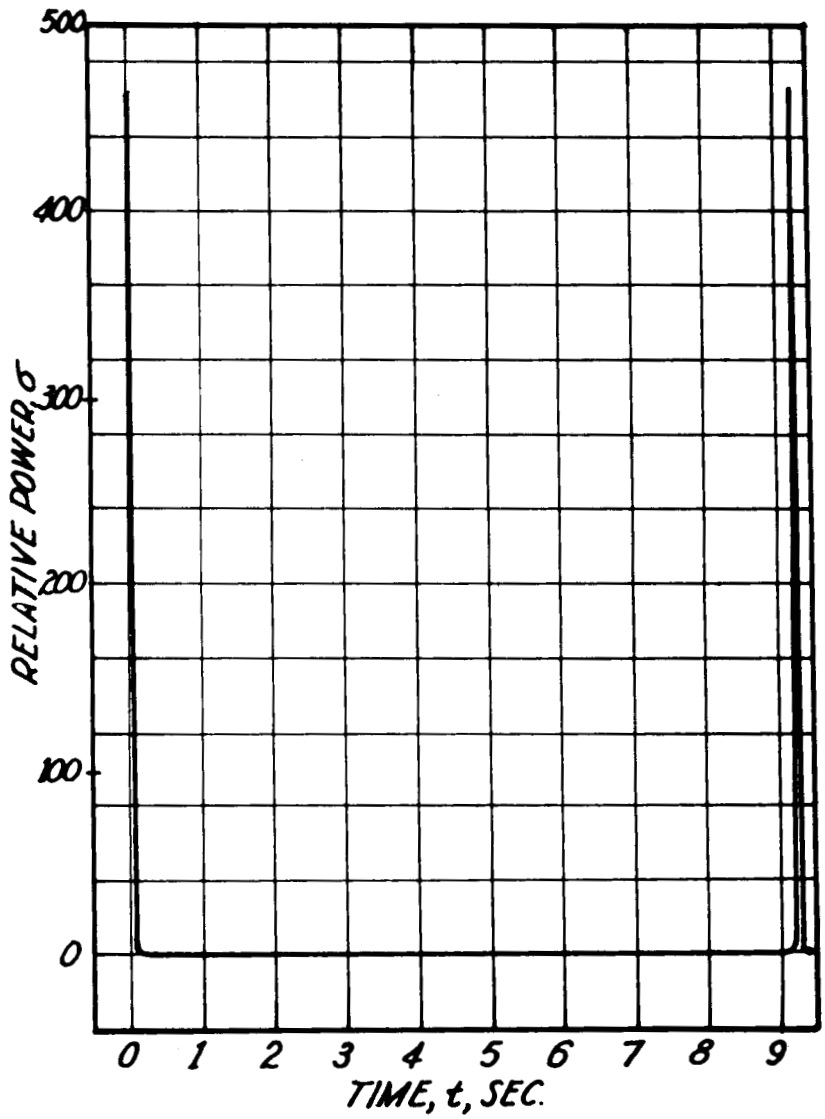

FIG. 11b

Plot of Power Ratio Versus Time; Based on Fig. 11a

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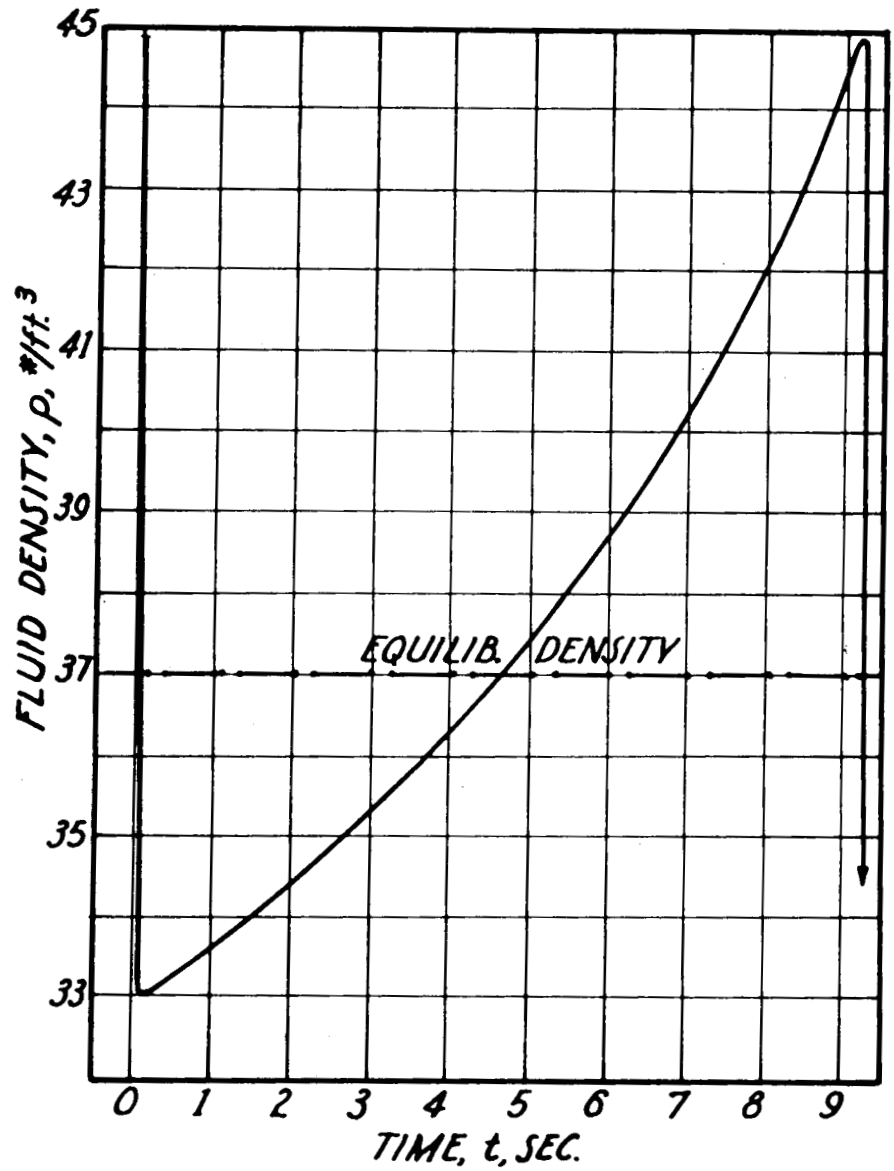

FIG. 11C

Plot of Fluid Density vs. Time; Based on Fig. 11a

[redacted]

a means of damping any such nuclear fluctuations which arise. Such a means is available in the action of delayed neutrons. The damping results from delayed neutrons is analyzed in Case II.

Case II

$\beta = 0.005$, $\tau = 0$, $C = 0.002215$. From the work of Hughes, et al, it has been seen that a maximum value for the fraction of delayed neutrons is about 0.00755. Here β is arbitrarily chosen as 0.005. By choosing different values for k, and considering no bubble-delay time, τ, comparison can be made with Case I as to the effect of delayed neutrons. Unfortunately, for a finite value of β, Equation (22) cannot be integrated analytically; however, a graphical solution can be obtained by the application of the method of isoclines (11) which is used in all the remaining calculations.

Case IIA. $k = k_{co} + Ct$ ($u = -0.0053$ when $\sigma = 1$). For this case Equation (22) yields

$$\frac{1.50211 - 1.20748 \times 10^4 \frac{du}{d\sigma}}{\sigma} - 1 = 2.27827 \times 10^6 \frac{du}{d\sigma} u$$

By choosing appropriate and sufficient number of values for $du/d\sigma$, tracing out the isocline for each, it is possible to trace out the trajectory of Equation (22) since initial conditions are known ($u = -0.0053$ when $\sigma = 1$). The graphical representation of Equation (22) for this instance is given in Figure 12a, curve (1), illustrating no nuclear oscillations, but rather a slow rise in the power which approaches the equilibrium value as time increases. Once having the graphical integration of Equation (22), Equations (23) and (24)

[redacted]

can be used to obtain the existing relationships between the variables. The dependence of σ and ρ on t is indicated in Figures 12b and 12c, curve (1). By comparing the above curves with Figures 6a, 6c, and 6d the damping influence of the delayed neutrons can be observed. Figure 6a is redrawn about the equilibrium point in Figure 12a and is the curve numbered (2). The effect of the delays is thus to eliminate nuclear oscillations, the power now increasing with time and approaching the equilibrium value asymptotically. It follows that the action of the delayed neutrons on time-density changes results in a smoother relationship as shown by comparison of Figure 12c, curve (1) with Figure 6c.

<u>Case IIB</u>. $k = k_{co} + 0.00177 + Ct$ ($u = -0.00353$ when $\sigma = 1$). This is the same as Case IIA except Δk is 0.00177 instead of zero. Hence, the isoclines are the same as in Case IIA, but the initial condition has been changed to $u = 0.00353$ when $\sigma = 1$, which leads to some changes in the nuclear power and density with time relations. Graphical indications of the fluctuations arising here are given in Figures 12a, 12b, and 12c, curves (3). Figure 12b shows that in the case at hand the power will rise faster than in Case IIA, actually rising slightly above the equilibrium value and then approaching equilibrium conditions asymptotically. The influence of the Δk on the time-fluid density relationship causes ρ to decrease somewhat more rapidly with time, but the effect is small as observed by comparing curves (1) and (3) of Figure 12c. Thus a Δk of 0.00177 does not have any particularly adverse effect on the operation of the reactor under the conditions given.

[redacted]

DWG. 12750

Plot of Power Ratio, σ, vs. Excess Reactivity, u,
for Continuously Increasing k and Various Δk,
Considering Delayed Neutron Effects, $\tau = 0$,
C = 0.002215

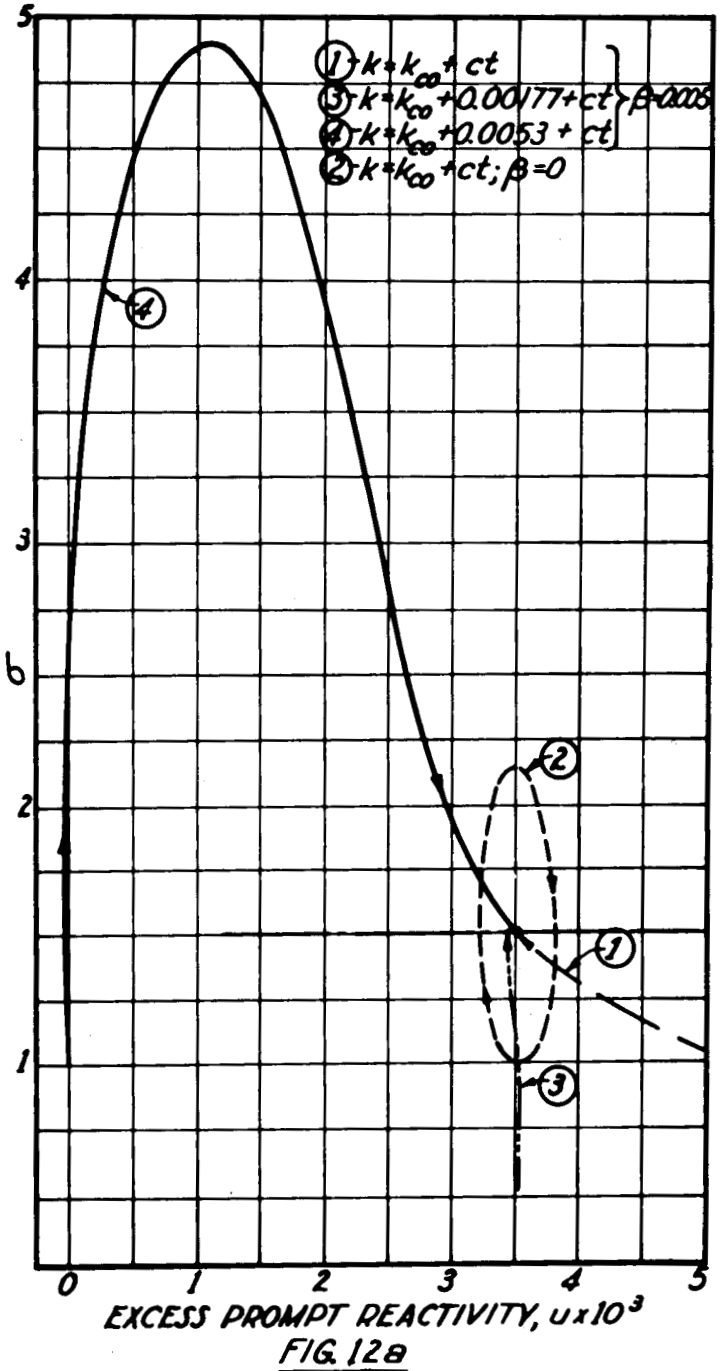

FIG. 12B

[redacted]

DWG. 12751

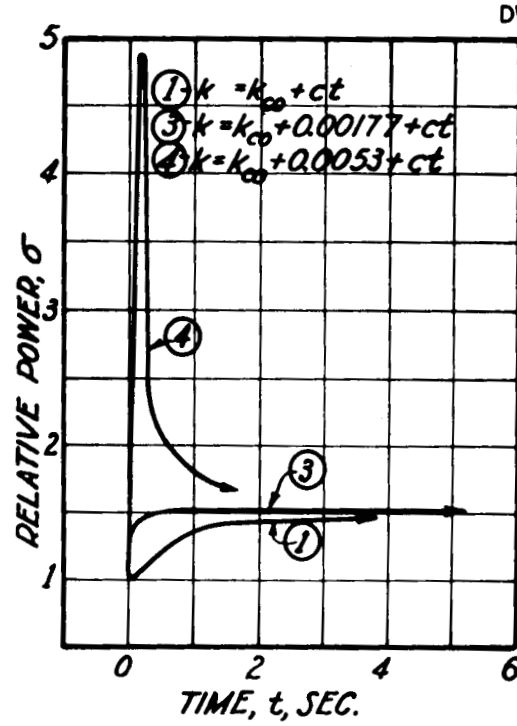

FIG. 12b

Power Ratio vs. Time for Continuous Increase in k, Various Δk, $\beta = 0.005$, $\tau = 0$; Based on Fig. 12a

Plot of Fluid Density vs. Time During which k Increases Linearly; Based on Fig. 12a

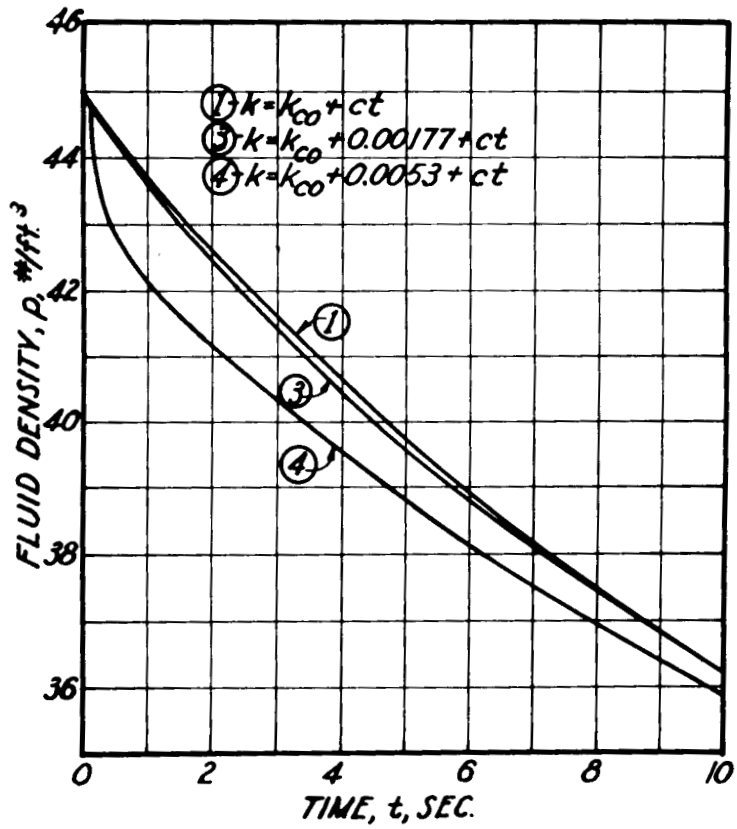

FIG. 12c

[redacted]

Case IIC. $k = k_{co} + Ct$ ($u = 0$ when $\sigma = 1$). Again, the only difference between Case IIA and this one is the presence of a Δk here, so the isoclines of Case IIA can be used, but with a different initial condition. The value of Δk in this situation is 0.0053 compared to 0.00177 in Case IIB. The resulting fluctuations of σ and ρ with time are presented in Figures 12a, 12b, and 12c, given by curves (4). As seen there, no power oscillation occurs, the motion being highly damped. Although the power pulse is higher than in Case IIB, with a correspondingly larger density change, the over-all result does not appear to lead to any control difficulties.

For the situations presented in Case II, the delayed neutrons act as powerful damping factors on any nuclear oscillations which were set up by solution reactivity increases. It remains to study the action of a time delay between nuclear power and corresponding vapor production, since the effect of bubble delay is opposite to that of delayed neutrons. This is studied in Case III.

Case III

$\beta = 0$, $C = 0.002215$, $k = k_{co} + Ct$. This situation corresponds to a linear increase in solution reactivity, with no delayed neutrons present. Two bubble delay times shall be considered, $\tau = 0.004$ and $\tau = 0.01$ seconds. With the values given here, Equation (22) becomes

$$\frac{du}{d\sigma} - N \quad 227.827 \times 10^4 \, u = \frac{1.50464}{\sigma} - 1$$

where

$N = 1.75572 \times 10^{-5}$ when $t = 0.004$ sec.

$N = 4.38931 \times 10^{-5}$ when $t = 0.01$ sec.

[redacted]

The above equations can be graphically integrated by the method of isoclines described above, the results being given in Figure 13a. The resulting curves can be compared with those in Figures 12a, (1), and 6a, which represent situations with and without delayed neutrons respectively, no bubble delay time being considered. Comparing Figures 6a and 13a, it can be seen that the absence of time delay results in nuclear oscillations, whereas a time delay causes the curve to diverge from the closed path. Hence, the action of a bubble delay time causes the nuclear power oscillations to be negatively damped, therefore acting in opposition to the positive damping effect of the delayed neutrons as shown in Figure 12a, (1). For the time delays considered, Figure 13a graphically illustrates that the negative damping factor is not large for the case at hand, but would become more important as τ became larger. Figure 13b indicates the manner in which the nuclear power would behave on a time scale, when $\tau = 0.004$ seconds.

Although it appears that the damping influence of the delayed neutrons outweighs the negative damping effect of bubble time delay, other situations should be examined to see how τ affects reactor operation when delayed neutrons are taken into account. These investigations are given under Case IV.

Case IV

$\beta = 0.00406$; $\tau = 0.01$; $C = 0.002215$. Both bubble delay time and delayed neturons are considered in this case, the fractions of delayed neutrons being taken as 0.00406 for reasons given below. Several values of k shall be studied to weigh the importance of τ.

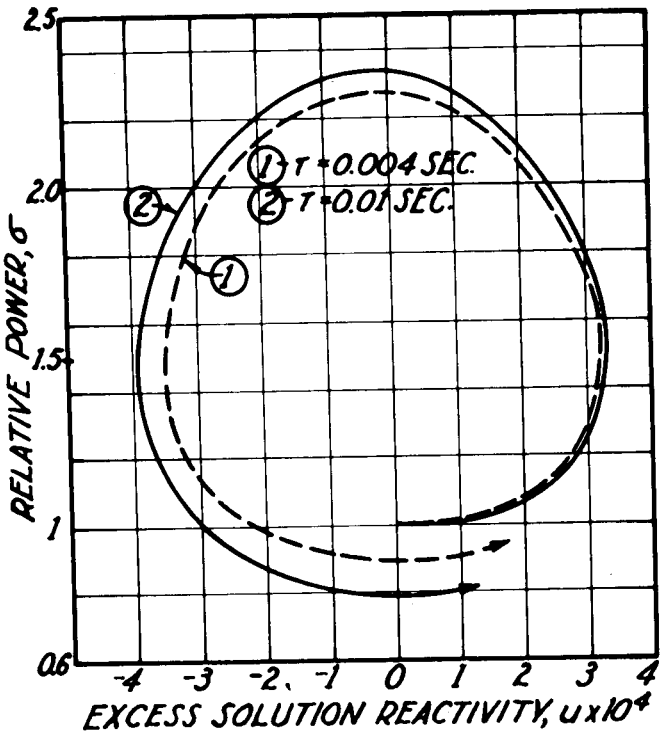

FIG. 13a

Plot of Power Ratio, σ, vs. Excess Reactivity, u, for Continuous k Increase with Time, C = 0.002215, τ = 0.004 and 0.01 Seconds, β = 0

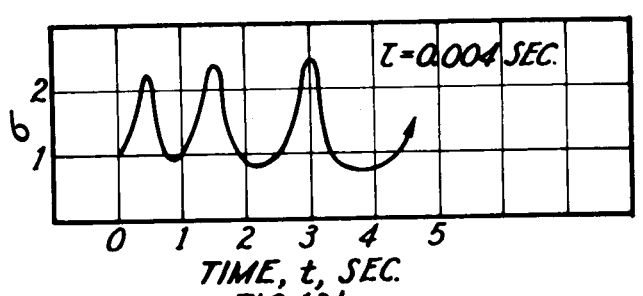

FIG. 13b

Plot of Power Ratio, σ, vs. Time for τ = 0.004, Based on Fig. 13a

[redacted]

Case IVA. $k = k_{co} + 0.0043036$ ($u = 0$ when $\sigma = 1$). For this case, Equation (22) becomes

$$u\left(\frac{du}{d\sigma} - 4.38931 \times 10^{-5}\right) = \frac{M - 0.0043036 \frac{du}{d\sigma}}{\sigma} - 4.38931 \times 10^{-7}$$

where

$$M = 8.48428 \times 10^{-7} \text{ for } C = 0.002215$$

$$M = 6.27829 \times 10^{-7} \text{ for } C = 0$$

Graphically integrating as before, the resulting relations are indicated in Figures 14a and 14b, curves (1). As seen in Figure 14a, the action of the delayed neutrons completely overshadows that given by bubble delay time, since the nuclear oscillation is critically damped. This is illustrated more vividly in Figure 15, which compares the case at hand to a similar one with $\beta = \tau = 0$. The curve in the latter case has been displaced so the equilibrium points of the two situations coincide. Figure 15 shows that the delayed neutrons not only critically damp the nuclear oscillation but also significantly damp the power pulse height.

Case IVB. $k = k_{co} + 0.0043036 + Ct$ ($u = -0.0043036$ when $\sigma = 1$). This case corresponds to a combination of Cases IIC and III, other than slightly different values for Δk and β and the inclusion of the effect of τ. The results are shown in Figures 14a and 14b, curve (2). The effect of adding reactivity continuously to the reactor does not appear to affect the over-all picture, although here the nuclear oscillation is not as highly damped as in Case IVA.

[redacted]

[redacted]

DWG. 12753

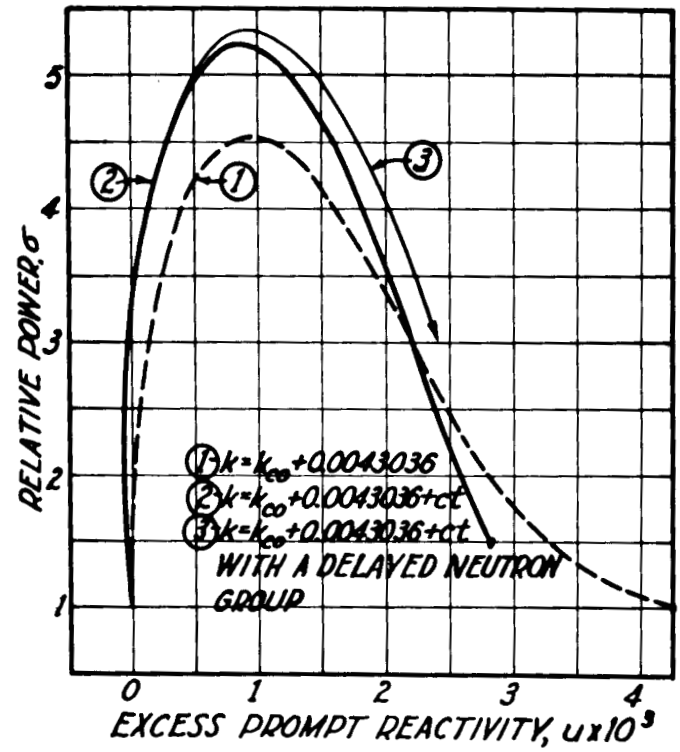

FIG. 14a

Plot of Power Ratio, σ, vs. Prompt Excess Reactivity u, for β = 0.00406, τ = 0.01, C = 0.002215 and 0, and a Delayed Neutron Group with λ = 0.154

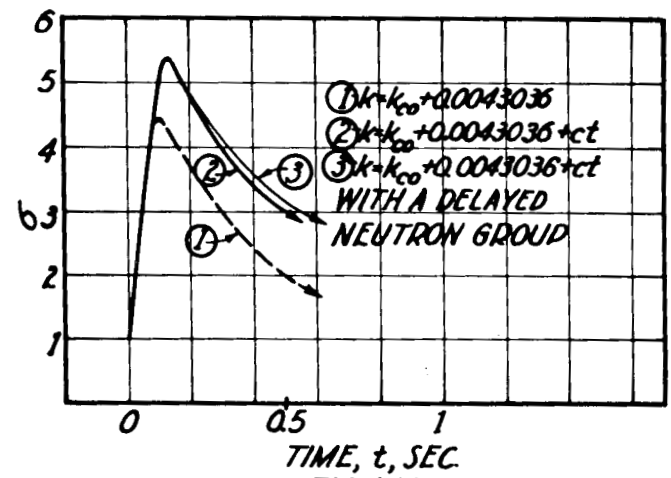

FIG. 14b

Plot of Power Ratio versus Time; Based on Fig. 14a

[redacted]

DWG. 12754

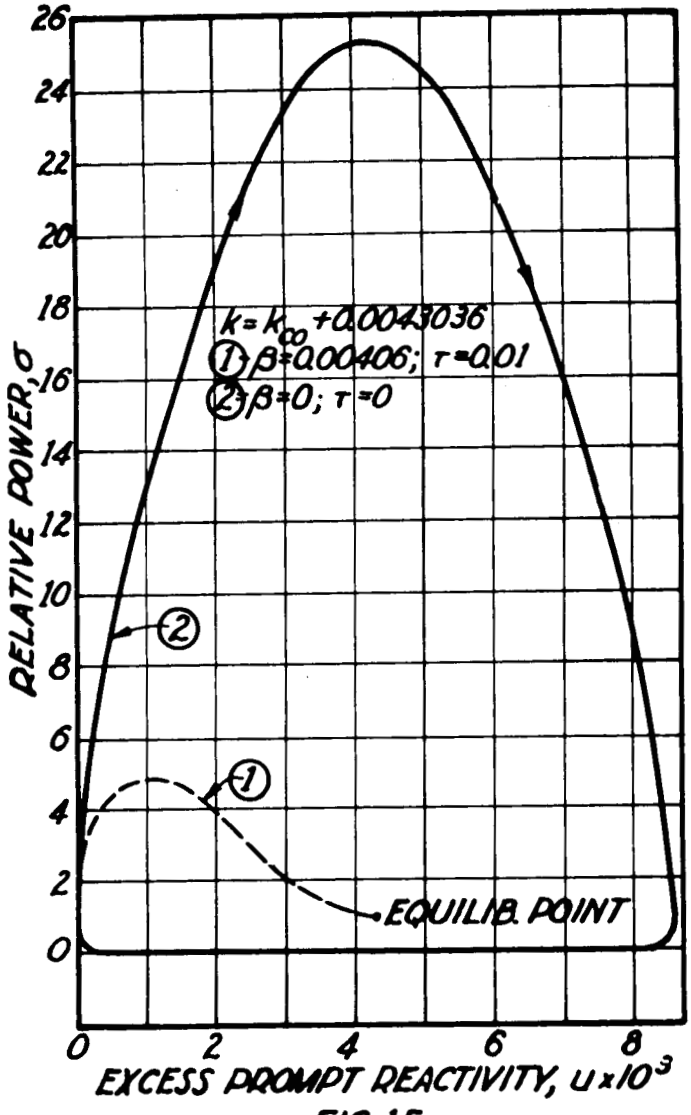

FIG. 15

Plot of Power Ratio vs. Prompt Excess Reactivity for $\Delta k = 0.0043036$, $\beta = \tau = 0$ and $\beta = 0.00406$, $\tau = 0.01$

[redacted]

Case IVC. $k = k_{co} + 0.0043036 + Ct$ ($u = -0.0043036$ when $\sigma = 1$); one group of delayed neutrons, $\lambda = 0.154$. In Equation (16) the delayed neutrons were considered to constitute a steady source of power which effectively kept the power level from dropping below the equilibrium value in the above cases. However, Equation (16) is not generally true for periods of time much longer than 0.1 of a second, and in several of the above cases the time interval of the power pulse exceeded that value. By considering only one group of delayed neutrons, the combination of Equations (7) and (8) yields:

$$\frac{d\sigma}{dt} = \lambda_o \left[(1 - \beta)\, k - k_c\right]\sigma + \lambda_o \beta k_{co}\, e^{-\lambda t} + \lambda_o \lambda \beta k e^{-\lambda t} \int_0^t \sigma e^{\lambda t}\, dt \quad (29)$$

By considering one delayed group with a decay constant of 0.154, the fraction of neutrons which would lie within this value is about 0.00406; hence this value for β.

If Equation (29) is used, Equation (22) is no longer valid, but it can be made to approximate the situation if βk_{co} is considered to vary in the proper manner. Thus, Equation (16) can be used for times which result in no significant differences between Equations (16) and (29). Then by assuming a new value for βk_{co}, the isoclines obtained from Equation (22) are valid until the last two right hand terms of Equation (29) equal the assumed value of $\lambda_o \beta k_{co}$. This involves step processes in which certain isoclines are valid only during certain time intervals. Hence, this is a trial and error graphical method of integrating the modified Equation (22). The results are given in Figures 14a and 14b, curves (3). Comparing these with the results of Case IVB, it can be noted that the effect of the delayed neutron group here helps maintain the power level up, but the over-all influence is small. For Δk's larger than

[redacted]

0.0043036, the power pulse rise would be shorter, and the accuracy of Equation (16) would increase. For a smaller Δk the delayed neutrons would not be as effective in damping oscillations as they appeared previously, but in this case the power rise would be smaller and the corresponding value of the integral expression in Equation (29) would be lower than for the illustration here. It thus appears that delayed neutrons will always cause damping.

Case IVD. $k = k_{co} + 0.0088$ ($u = 0.0045$ when $\sigma = 1$). Because of the value of the Δk, the time interval of the power pulse concerned is so small that Equation (16) is applicable for the major portion of the curve, and direct graphical integration of Equation (22) is possible. The results for this case are given in Figure 16, which illustrates that an increase in the Δk causes a significant change in the power pulse height and in the shape of the resulting curve. Although the fluid density overshoots the equilibrium value, the nuclear oscillation is still highly damped, the minimum fluid density being about 39.4 lb/ft^3. It appears that τ distinctly limits the allowable instantaneous solution reactivity increase, since for too large a Δk the power may rise so rapidly that the lag in corresponding density changes allows the power to increase to very large values.

DISCUSSION

In summarizing the results of this study, it appears that the delayed neutrons act as powerful damping factors on any nuclear power oscillations which might arise, completely overshadowing the influence of a bubble delay time of 0.01 seconds for instantaneous reactivity changes up to 0.009. The main effect of τ is to limit the allowable Δk, but Δk of 0.009 is extreme,

[redacted]
DWG. 12755

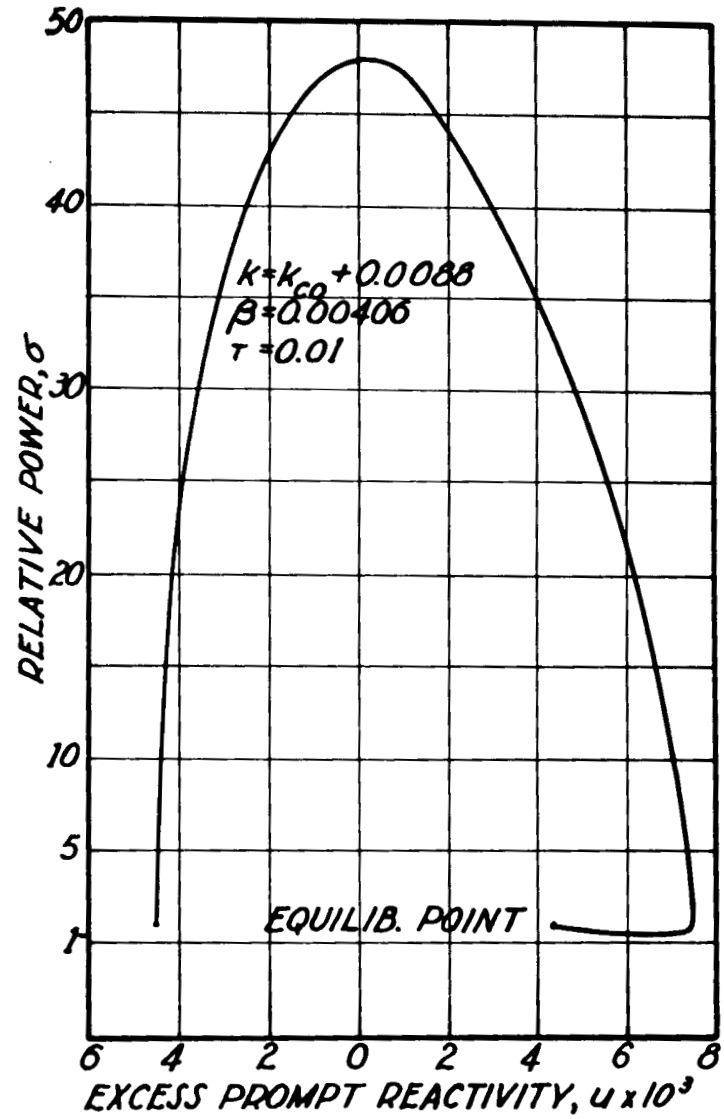

FIG. 16

Power Ratio vs. Prompt Excess Reactivity for
$\Delta k = 0.0088$, $C = 0$, $\beta = 0.00406$,
$\tau = 0.01$

[redacted]

since this corresponds to adding uniformly at once a minimum of 266 pounds of natural uranium to the solution, or 409 pounds of uranyl sulfate. This was obtained by using Equation (25a) to find the value of N_{25} required for a k of 1.069, and then calculating the amount of U^{235} which would have to be added to give this concentration. If k were to increase with time, there are less restrictions as to the allowable solution reactivity increase. For example, with C = 0.002215, Figure 6c indicates that a concentrated solution could be fed into the reactor at 7000 gal/min. for 9 seconds, which corresponds to adding a total of 4380 pounds of natural uranium.

Actual values for τ are, as yet, poorly known and the inherent reactor stability demonstrated would be limited if larger values of τ were used. Martin[12] has recently indicated that small power pulses impressed into an internally heated liquid caused appreciable density changes 0.25 to 1 second following the power pulse, and higher power pulses had a corresponding density change lag of about 0.125 seconds. If these time lags occur in the "boiler" system considered only small Δk can be tolerated. However, values of τ, even though large for small power pulses, may be quite small for high power pulses. This follows since the larger the impressed Δk, the higher the nuclear power pulse, resulting in more rapid superheating of the reactor liquid, which would cause vapor bubbles to form more rapidly than for smaller Δk. $\lambda(\epsilon)$ would thus be a distributed time delay function which depended upon the applied power pulse, or impressed Δk. It would also depend upon the steady-state power, since vapor bubbles would be more numerous at higher operating power, which in turn concerns the effective bubble rise velocity attainable. Decrease in τ with increasing Δk is a desirable relation, inasmuch as relatively large τ allows

[redacted]

a highly damped nuclear oscillation for small Δk, whereas large Δk would require small τ values for reactor stability. The presence of gas bubbles in the reactor fluid may aid considerably in decreasing the value of τ, for although the deuterium and oxygen formed by liquid decomposition would only be about one per cent by volume of the vapor generated, these bubbles may be very numerous and distributed throughout the reactor, forming nuclei for liquid vaporization. Hence, it may well be that although decomposition gases are undesirable in circulating-type reactors, they are quite necessary in boiling types for indirect control purposes. The formation of gases in themselves, while promoting steam bubble formation, would contribute very little to over-all reactor stability since steam vapor would constitute about 99 per cent of the reactor fluid vapor phase.

In this study the percentage of vapor formed by surface boiling was considered negligible. The actual fraction of vapor generated at the surface would be dependent upon the liquid superheat, the surface boiling increasing with the superheat. In ordinary boiling, if there are plenty of points containing adsorbed gases, vapor bubbles form readily, their size being small, and liquid superheating low. It seems reasonable to suppose that a very large number of small gas and vapor bubbles throughout the reactor would be more advantageous in this respect than actual points, and as a result superheating should be low throughout the boiler fluid under this condition. By considering that with each bubble there is an associated volume of liquid, and that heat is transferred by conduction through this liquid to the vapor bubble surface, it can be shown that the amount of superheat decreases considerably as the number of bubbles present increases. In the Los Alamos Water Boiler

[redacted]

there are gas bubbles formed by liquid decomposition, but it may be that most of the gases evolved as such are formed at the liquid surface. It would be desirable to know what percentage of the decomposition gases evolved are generated within the liquid volume, and if there are means by which this percentage can be increased. The operation of the reactor at about 600 psia and the mixing action of the fluid motion should help keep the bubble size small and liquid superheat low, which would aid in keeping the value of τ small, as well as keep the surface boiling effect negligible.

If the liquid superheat is low, practically all the generated heat must appear as vapor bubbles formed within the liquid. To extract a certain amount of heat the bubbles would need to be removed at a certain rate, and for the problem studied, in which 10^6 kw is considered, Equation (3) dictates that the average bubble rise velocity required be about 9 ft/sec. This may be difficult to obtain, since natural rise rates of air bubbles in water are of the order of 1 ft/sec. However, it may be possible to impose upon the reactor fluid itself the necessary circulation velocity which would remove bubbles from the reactor zone at the desired rate. Due to the flux distribution, there would be considerable density gradients across the reactor diameter, which would instigate liquid circulation. Once started, natural circulation could be aided by baffles within the fluid, as well as by some above so condensate would enter near the walls as illustrated in Figure 1. Other than this, if plutonium were being extracted from the reactor solution on a continuous cycle, the exit and inlet pipes for processing purposes could be situated so as to materially aid natural circulation. The bubble velocities attainable by the above or similar methods should be measured

[redacted]

[redacted]

experimentally in order to determine what specific powers are feasible. Operation at high pressure would lower the bubble rise velocity required after a given power output, since the heat content per cubic foot of vapor increases markedly with the pressure.

In a reactor of the type considered, the size of the heat exchanger should be in proportion to the size of the reactor. Thus, the heat exchanger itself should not occupy a volume out of proportion to that of the steam chest, which in the problem considered normally contained about 13,000 cu. ft. of vapor. Assuming an over-all heat transfer coefficient of 1000 Btu/hr-sq.ft-^{o}F and production of 190° C steam, the heat transfer area required would be 34,130 ft^{2}. If 4-inch water tubes were used, the volume occupied by the tubes would be 2850 cu. ft., which does not seem excessive.

The heat exchanger would act as a steam filter, leaving fission and decomposition gases to rise into the space above the exchanger. The lower explosion limit for mixtures of hydrogen, oxygen, and steam is about 22 - 25% electrolytic gas over a range of temperature and pressure of 87° C, 12 psi to 247° C, 700 psi [13]. Hence, below the heat exchanger, where electrolytic gas constitutes about 1 per cent of the mixture, even slow burning is unlikely. However, in passing upward through the exchanger, steam would condense and the percentage of electrylytic gas would rise to combustible proportions. A convenient method of recombining the gases would consist of a number of continuous spark discharge devices placed above the heat exchanger which would ignite the gases as they reached this region. If burning takes place as a series of "pops," it may be necessary to use the alternate system mentioned on page 5.

[redacted]

At 200° C, the heat of the recombination reaction is about 6780 Btu/lb. vapor. Also, if condensate is returned to the heat exchanger at 38.3° C, 3.03×10^6 lbs. of 200 psi steam are generated per hour, during which time 23,200 lbs. of moderator are decomposed. Hence, the heat available for superheating would be 515 Btu/lb., which corresponds to a superheat temperature of 46.6° C (84° F). If the steam generated within the reactor along with the recombined vapor could be used directly for power production and returned as condensate at 38.3° C, the available superheat would be about 30° C (54° F).

In addition to the decomposition gases, fission gases will be formed, e.g., krypton and xenon. The volume of these gases, however, is relatively small so they form no immediate problem, for, even assuming one mole of fission gas formed per mole of U^{235} fissioning, there will only be about 3.4 SCF of fission gases produced per day, which could occupy about 0.165 cu. ft. of a vapor region of about 13,000 ft^3 at 250° C and 580 psi. The fission gases could thus accumulate in the top of the reactor for weeks before venting them to adsorption chambers.

Under steady-state conditions, boiling type reactors offer the optimum as far as maximum temperature at minimum pressure. The pressure developed as a result of solution reactivity changes would depend upon the volume of the vapor chest, while the vapor chest volume would be determined by a balance between the pressures which could be developed by allowable reactivity increases and the difficulties in fabricating vessels having sufficient strength. Thus, having once designated the maximum allowable Δk, it would be possible to calculate the maximum pressure which could be obtained for a

[redacted]

given vapor chest volume; given the size and the pressure the vessel would have to withstand, design problems could be examined. From Figures 3 and 4, the maximum equilibrium pressure which could be developed for Case IVD with $\Delta k = 0.009$, would be about 700 psi. Here the vapor space was normally about 13,000 ft^3. If the volume were increased, the corresponding pressure would decrease, whereas if the volume were decreased, the pressure rise would increase. These factors would need to be balanced against design and fabrication difficulties.

The concentration of U^{235} in the reactor fluid would determine the volume of liquid required for a critical assembly. Increasing the concentration would decrease the volume required, and would also aid reactor safety inasmuch as vapor bubbles would now affect the neutron economy to a higher degree than before. To compare the concentration effect on reactor size and safety, consider three reactors with the same uranium concentration, but different degrees of enrichment. Again, let the steady-state vapor fraction be 0.10; $\rho_o = 44.91$ lb/cu. ft., and $k = k_{co} + 0.02$. The results are presented in Table IV.

TABLE IV

Effect of U^{235} Concentration on Reactor Size and Safety

% U^{235} in Fuel	Critical Radius, (ft.)	Equilibrium Fluid Density for $k = 0.02$ (lb/cu.ft.)	Absolute Pressure at Equilibrium Density (lbs/sq.in.)
0.71 (Natural U)	16	37.0	780
0.8	11.3	39.6	680
1.0	8.02	42.0	625

[redacted]

[redacted]

Table IV illustrates the desirability of enriched fuels, but the advantages obtained would have to be balanced against rising fuel costs as enrichment is increased. Decrease in reactor size would also lessen the fabricating difficulties of building high pressure vessels, but smaller sizes would probably lower natural circulation rates and thus lower the power densities possible. The size of the reactor would be greatly influenced by its intended application; e.g., mobile type reactors would be desired in small packages.

The assumption of constant mass in the steam chest is open to argument, but it seemed the most reasonable preliminary one to make at the time, inasmuch as no experimental data has been observed relating vapor mass and pressure under the transient conditions presented. If the removal rate of heat P_o were lowered, and the vapor mass increased with time, the reactor fluid pressure and temperature would rise until the fluid density decreased enough to make the reactor just critical. Since at the thermodynamic critical point the fluid density is so low that the reactor would be quite subcritical, the maximum pressure that could be expected is about 3200 psi. The above situation can be avoided by an external pressurizer arrangement.

NECESSARY RESEARCH

As already pointed out, there are a number of physical phenomena to be investigated which affect the feasibility of boiling-type reactors. As a result of this study the three factors found to be of primary importance are: the rate at which power can be extracted from the reactor fluid, or specific power output; the effect of transient pressure conditions upon the vapor mass and temperature within the reactor; and the actual form of the bubble time

[redacted]

delay function, associated with the value of τ. The latter is presently under investigation at UCLA.

ALTERNATE SYSTEMS

The system presented in Figure 1 was given so as to be specific, and does not exclude other possible arrangements. It is conceivable that the 250° C generated steam vapor be used directly to drive the turbine if it is possible to remove or catalytically recombine decomposition gases before they enter the turbine. In another design, the steam could be condensed by a heat exchanger in a vessel other than the one containing the reactor unit itself.

Because of corrosion and precipitation problems arising in homogeneous solutions, slurries appear quite attractive as possible reactor fuels since the pH of slurry solutions can be relatively high and the chemical precipitation problem is removed. Operation at higher pressures would thus be possible, resulting in greater thermal efficiency. Slurry solutions may be permissible fuels in boiling reactors since the internal boiling would provide considerable solution agitation to help maintain particles in suspension, and it may be that the presence of particles hotter than the surrounding fluid will aid in the formation of bubbles. Considering use of slurries, the boiling reactor would decrease or eliminate erosion difficulties arising in circulating types.

In addition to those mentioned, there are other design types which could be considered, the most advantageous one to use dependent upon the relative importance of the good and bad features involved.

[redacted]

SUMMARY

As determined here on the basis of developed kinetic equations, boiling-type reactors appear to be inherently stable, the pressure surges decreasing in amplitude as the reactor critical size decreases. Bubble formation delay times are detrimental to the stability, but the delayed neutrons more than offset this effect for a delay time of 0.01 seconds and a Δk of about 0.009 in the problem considered. Boiling reactors offer compactness inasmuch as all high-level radioactivity may be confined to one unit, thereby decreasing maintenance hazards, a salient feature. The power required for pumping may be quite low, aiding over-all economy. Although "boilers" could possibly be made in all sizes, the power density available may limit their applications.

There is considerable experimental work to be performed in the development of boiling reactors, which should be started as soon as possible, inasmuch as the preceding study indicates a degree of stability not appreciated in the past.

[redacted]

[redacted]

NOMENCLATURE

$a = \rho_l / \rho_l - \rho_g$

a, A, b, B, c = constants

B^2 = buckling factor

C = rate at which k linearly increases with time

H = enthalpy of vapor in reactor, Btu, $= V_g \rho_g h_{fg}$

k = material multiplication constant if finite pile had no leakage

k_c = multiplication constant required for criticality, if finite pile had no leakage

k_{co} = k_c evaluated when pile is operating at steady-state power level

$K = \frac{P_o \alpha}{a}$ (see Equation 19)

h_{fg} = enthalpy of vaporization of moderator, Btu/#

L = height of reactor shaped as right circular cylinder

L_o = L evaluated when reactor is operating in the steady-state

m = mass of fluid in reactor, $= \rho V_r$

M^2 = migration area

p = pressure inside reactor, psia

P = nuclear power of reactor, kw

P_o = P evaluated under steady-state conditions

R = radius of reactor, ft.

$\bar{R}$ = average distance a bubble has to travel to reach the steam chest, ft.

t = time, sec.

t_{ave} = average time a bubble spends in reactor proper, sec.

$u = k(1 - \beta) - k_c$ = excess prompt reactivity

$\bar{v}$ = average rise velocity of bubble relative to reactor shell

[redacted]

[redacted]

v_g = fraction of vapor in reactor fluid, $= \frac{V_f}{V_r}$

$v_g(o)$ = v_g evaluated when reactor is in the steady-state

V_f = volume of vapor in reactor proper, cu. ft.

V_l = volume of liquid in reactor proper, cu. ft.

V_r = volume occupied by reactor fluid = $V_f + V_l$

$V_r(o)$ = V_r evaluated when reactor is in the steady-state

V_s = volume of vapor in steam chest, cu. ft.

V_{st} = total volume of vapor in unit, = $V_s + V_f$

$w = 1/(\rho - b)$

α = 3413/3600 Btu/kw-sec.

β = fraction of neutrons from fission which are delayed

β_i = fraction of neutrons having decay characteristic λ_i

ϵ = distributed time delay in bubble formation

λ = decay constant including a number of delayed neutron groups

λ_i = decay constant of delayed neutron emitter

λ_o = reciprocal lifetime of prompt neutrons (Equation 8)

$\lambda(\epsilon)$ = normalized distributed time delay function

ρ = reactor fluid density, lb/cu.ft., = m/V_r

ρ_o = ρ evaluated when the reactor is in the steady-state

ρ_g = density of vapor in reactor, lbs/cu.ft.

ρ_l = density of reactor liquid, lbs/cu. ft.

σ = power ratio = P/P_o

τ = finite time delay in bubble formation

[redacted]

[redacted]

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