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CAN POLARIZATION EFFECTS BE DETECTED IN CAPTURE GAMMA RADIATION?

L. C. Biedenharn, M. E. Rose, G. B. Arfken

I. Introduction

It is well known that the detection of the state of polarization of radiation emitted in nuclear (cascade) transitions can yield useful information concerning the parity of the γ-radiation and is helpful in the determination of decay schemes.[1] It has also been conjectured that the production and subsequent utilization of polarized particles in nuclear transmutations would perhaps be equally useful in providing information concerning the quantum numbers to be assigned to nuclear levels. At present experimental techniques for the production of a beam of polarized particles are very fully developed in the case of neutrons. Such techniques are, of course, limited at present, to slow (S) neutrons. It is therefore pertinent to investigate the possibility of utilizing such polarized neutrons in order to (1) produce other polarized particles and (2) to obtain information on nuclear levels by detecting the emitted radiation with polarization sensitive counters. It is clear that a measurement of total intensity would not be revealing since this intensity would be isotropic.[2]

We consider that the compound state formed by neutron capture decays by photon emission since this is the case of greatest interest. Emission of other types of particles is briefly discussed at the end of this report. The experiment envisaged then involves the use of polarization sensitive detectors

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1. D. R. Hamilton, Phys. Rev. 74, 782 (1948).

2. L. Wolfenstein, Phys. Rev. 75, 1664 (1949).

to measure the intensity and polarization of the emitted photons.[3] The result of this investigation shows that the intensity as measured with any known (practical) polarization-sensitive detector is isotropic and that therefore no additional information is forthcoming by virtue of the neutron polarization. As shown below, this statement is valid for both pure and mixed multipoles.

Specifically, there is no observable anisotropy insofar as the analyzing process can detect only linear polarization. This is the case with all known methods for measuring the photon polarization; e.g., (1) pair production, (2) photoelectric effect or (3) single Compton scattering. The common feature of these observation methods is that they give information only about the state of linear polarization of the photon and cannot distinguish between right- and left-circular polarization, or indeed, distinguish either of these from unpolarized light. It is clear that right vs. left elliptic polarization are similarly indistinguishable. In order to obtain information concerning the radiation emitted and/or the compound state it would be necessary to devise some detection method which discriminates between right vs. left circular polarization; that is, a "nuclear quarter wave plate."

II. Absence of Polarization Effects

Let us consider neutrons having a polarization defined by the polarization vector $\vec{P}_o \equiv \langle \vec{\sigma} \rangle$ — incident upon a target nucleus of spin J_N with the target spins randomly oriented. The relative angular momentum of the system will be taken as zero (S-neutrons). Following the methods originated by

3. M. Deutsch and F. Metzger, Phys. Rev. 74, 1542 (1948); 76, 187 (A) (1949).

Hamilton[4], Goertzel[5], and Falkoff and Uhlenbeck[6] we shall calculate the relative probability $\mathcal{P}(\vec{P}_o, \vec{k}, \vec{e})$ for the emission of a capture gamma ray in the $\vec{k}$ direction with linear polarization $\vec{e}$ following the capture of an S neutron of polarization $\vec{P}_o$. The desired probability is:

$$\mathcal{P}(\vec{P}_o, \vec{k}, \vec{e}) = S \left| \sum_m \langle A(\vec{P}_o, \ell) \,|\, H_1 \,|\, B_m \rangle \cdot \langle B_m \,|\, H_2(\vec{k}, \vec{e}) \,|\, C_{m'} \rangle^* \right|^2 \qquad (1)$$

$A(\vec{P}_o, \ell)$ denotes the probability amplitude for the initial state of the system with ℓ defining the spin state of the target nucleus; similarly, B_m denotes the intermediate state and $C_{m'}$ the final state. S denotes an averaging process over the initial states (ℓ), including phase averaging, and a summation over final states (m'). Also H_1 and H_2 are the Hamiltonians that effect the neutron capture and gamma ray emission respectively.

The sum over the degenerate intermediate states (B_m) in (1) is coherent. It is possible, however, to remove interference terms by a proper choice of a quantization axis. Then (1) takes the form:

$$\mathcal{P}(\vec{P}_o, \vec{k}, \vec{e}) \sim \sum_{\ell, m, m'} \left| \langle A_\ell \,|\, H_1 \,|\, B_m \rangle \right|^2 \left| \langle B_m \,|\, H_2 \,|\, C_{m'} \rangle \right|^2 \qquad (2)$$

This reduction has been demonstrated by other authors[7] for a similar situation

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4. D. R. Hamilton, Phys. Rev. 58, 122 (1940).

5. G. Goertzel, Phys. Rev. 70, 897 (1946).

6. David L. Falkoff and G. E. Uhlenbeck, Phys. Rev. 79, 323 (1950).

7. Stuart P. Lloyd, Phys. Rev. 80, 118 (L), (1950).
J. A. Spiers, Phys. Rev. 80, 491 (L), (1950).
B. A. Lippmann, Phys. Rev. 81, 161 (L), (1951).

wherein a propagation vector for one of the emitted (or absorbed) radiations is specified. Here we are concerned with the case in which the polarization direction (of the neutrons) is specified while the propagation vector is not fixed. That this difference is not essential and that the cross terms in (1) are removed by choosing the axis of quantization along $\vec{P}_0$ is seen directly in the following manner. Assume, for the moment, that the incident neutrons are completely polarized so that $P_0 = 1$. The compound state wave function is then

$$\chi_J^m = C_{J\ \frac{1}{2}\ell}^{\frac{1}{2}\ J_N} \chi_{\frac{1}{2}}^{\frac{1}{2}} \ \chi_{J_N}^{\ell} \tag{3}$$

where the $C_{J\ \frac{1}{2}\ell}^{\frac{1}{2}\ J_N}$ are Clebsch-Gordon or (real) vector addition coefficients[8] as defined by Wigner.[9] An index m, on which the C-coefficients depend, has been suppressed since it is in general given by the sum of the second and third subscripts: $m = \ell + \frac{1}{2}$. If we now consider a given ℓ, it is clear that only one value of m occurs and there are therefore no cross-terms. We can now remove the restriction of complete polarization while recognizing that for incomplete polarization the spin state $\chi_{\frac{1}{2}}^{-\frac{1}{2}}$ of the neutron has random phase relative to the $\chi_{\frac{1}{2}}^{\frac{1}{2}}$ state and the reduction of (1) to (2) still obtains on averaging over this phase.

We can simplify Eq. (3) still further by noting that $\left|\langle A_\ell \,|\, H_1 \,|\, B_m \rangle\right|^2_{Av}$

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8. E. U. Condon and G. H. Shortley, Theory of Atomic Spectra, (Cambridge University Press, 1935), Chap. III.

9. E. P. Wigner, Gruppentheorie, (reprint J. W. Edwards, 1944). We have altered the notation to the extent of replacing s by C. In general $C_{J_3 m_1 m_2}^{J_1 J_2}$ corresponds to the vector addition of J_1 and J_2 to give a resultant J_3, with z-components m_1, m_2 and $m_3 = m_1 + m_2$ respectively.

- to within a common proportionality factor - is just the population of the m^{th} sublevel of the intermediate state. Using Eq. (3), the population of the m^{th} sublevel, call it $B(m, J)$, is:

$$B(m, J) = \left| C^{\frac{1}{2}\ J_N}_{J\ \frac{1}{2}\ (m-\frac{1}{2})} \right|^2 = \frac{1}{2} \pm \frac{m|P_o|}{2J_N + 1} \tag{4}$$

The sign is to be taken + for $J = J_N + \frac{1}{2}$ and - for $J = J_N - \frac{1}{2}$. The departure from uniformity of the populations of the sublevels of the compound state is linear in the magnetic quantum number of these states. This result, which obviously is a direct consequence of the circumstance that spin $\frac{1}{2}$ particles are captured, is of decisive importance for the conclusions to be presented below.

Let us now consider the evaluation of the matrix elements $\langle B_m | H_2 | C_{m'} \rangle$. The Hamiltonian H_2 is $\sum_i \vec{\alpha}_i \cdot \vec{A}$ where $\vec{\alpha}_i$ is the Dirac velocity operator for the i^{th} nucleon and $\vec{A}$ is the vector potential corresponding to a linearly polarized plane wave with the propagation vector $\vec{k}$ and polarization $\vec{e}$ in the observer's coordinates. Since the nuclear transition will correspond to the radiation of a light quantum of definite multipole order (angular momenta, L) and definite parity (electric vs. magnetic radiation depending upon L) it is expedient to expand this plane wave into a sum over all multipoles. Goertzel[5] has done this for vector potential of a circularly polarized plane wave. His result is:

$$\vec{A}(\vec{k}, P) = \pi \sum_{L=1} \sum_{M=-L}^{L} i^L \sqrt{2L + 1}\, \mathcal{D}^{(L)}_{M,P} (\alpha\beta\gamma) \left\{ \vec{A}^m_{LM} + i\, P\, A^e_{LM} \right\} \tag{5}$$

The notation here is: $P = +1$ denotes right circularly polarized while $P = -1$ denotes left circularly polarized waves. The vector potentials $\vec{A}_{LM}$ refer to radiation of 2^L pole multipole order, of substate M, and are given explicitly in reference (5). The superscripts e, m denote electric and magnetic multipoles respectively.

The $\mathcal{D}^{(L)}_{M,P}(\alpha\beta\gamma)$ are the (M^{th}, P^{th}) elements of the rotation matrix of the L^{th} order. The elements (M, $P = \pm 1$) are also given explicitly in reference (5). The α, β, γ are the Euler angles of the coordinate system of the $\vec{A}_{LM}$ relative to the coordinate system of the vectors $\vec{k}$, $\vec{e}$.

A plane polarized wave - whose polarization vector, $\vec{e}$, makes an angle τ with respect to a fixed vector in the plane perpendicular to $\vec{k}$ - may be written:

$$\vec{A}(\vec{k}, \vec{e}) = \cos\tau \sum_P \vec{A}(k, P) + \sin\tau \sum_P (-i)\, P\, \vec{A}(k, P)$$

$$= \sum_P e^{-iP\tau}\, \vec{A}(\vec{k}, P) \tag{6}$$

We can now reduce the matrix element $\langle B_m | H_2 | C_{m'} \rangle$ to a more manageable form.

$$\langle B_m | H_2 | C_{m'} \rangle \sim \sum_L \sum_M \sum_P i^L \sqrt{2L+1}\, e^{-iP\tau}\, \mathcal{D}^L_{M,P}(\alpha\beta\gamma) \cdot$$

$$\cdot \left\langle B_m \left| \sum_i \vec{\alpha}_i \cdot \left\{ \vec{A}^m_{L,M} + i\, P\, \vec{A}^e_{L,M} \right\} \right| C_{m'} \right\rangle \tag{7}$$

Since the terms $\vec{\alpha}_i \cdot \vec{A}_{LM}$ transform under rotation with the rotation matrix $\mathcal{D}^{(L)}$ we may apply the result of Eckart[10] to these matrix elements.

10. C. Eckart, Rev. Mod. Phys. 2, 305 (1930). Also E. P. Wigner, l.c., p. 264.

$$\left\langle B_m \right| \sum_i \vec{\alpha}_i \cdot \left\{ \vec{A}^m_{LM} + i\,P\,\vec{A}^e_{LM} \right\} \left| C_{m'} \right\rangle = \frac{a(JJ'\pi;L)}{\sqrt{2L+1}}\; C^{JL}_{J'mM}\,(i\,P)^{\delta(\pi,L)} \tag{8}$$

The term $a(JJ'\pi;\,L)$ depends upon the total angular momenta (J, J') of the two states involved, upon the multipolarity (2^L) of the emitted quantum and upon the relative parity change ($\pi = -1$ denotes a change in parity; $\pi = +1$, no change.) For $\left\{ \begin{matrix} L \text{ odd}, \pi = - \\ L \text{ even}, \pi = + \end{matrix} \right\}$ we have electric radiation, and oppositely for magnetic radiation. Since parity is a good quantum number, this means that for a given value of L either electric or magnetic 2^L pole radiation may occur, but not both. The factor i P occurs if we have electric radiation. We have introduced the function $i\,P^{\delta(\pi,\,L)}$ where $\delta(\pi,\,L) = \frac{1}{2}(1 + \pi\,(-1)^L)$ which automatically introduces the factor i P for electric multipoles and is unity for magnetic multipoles. The factor $\sqrt{2L+1}$ is introduced for convenience only. The Clebsch-Gordon coefficients, $C^{JL}_{J'mM}$, are zero unless $m + M = m'$; the summation over M in Eq. (7) therefore reduces to a single term.

Introducing these results into Eq. (2) yields the following equation:

$$\rho(\vec{P}_o, \vec{k}, \vec{e}) \sim \sum_{m,M} \left(\tfrac{1}{2} \pm \frac{m|\vec{P}_o|}{2J_N + 1}\right) \cdot \left| \sum_{L,P} i^L\, a(JJ'\pi;L)\, e^{-iP\tau}\, \mathcal{D}^L_{M,P}(\alpha\beta\gamma) \cdot \right.$$

$$\left. \cdot\, (i\,P)^{\delta(\pi,L)} \cdot C^{JL}_{J'mM} \right|^2 \tag{9}$$

We can simplify this if we note (1) that the Euler angle γ corresponds to a different choice of origin for measuring the polarization angle τ and consequently we may put $\gamma = 0$ and (2) that the elements $\mathcal{D}^L_{M,P}(\alpha\beta 0)$ all have the dependence $e^{iM\alpha}$, multiplied by a real function of β, independently of L. As we want only the absolute value we may discard this factor, or what is equivalent, set $\alpha = 0$. The Euler angle β corresponds to the polar angle that $\vec{k}$ makes with respect to $\vec{P}_0$, and to agree with the customary usage we now call this angle ϑ.

The absolute square in Eq. (9) may be written explicitly as:

$$\left| \sum_{L,P} i^{L+\delta} P^{\delta} a(JJ'\pi; L) e^{-iP\tau} \mathcal{D}^{(L)}_{M,P}(0\vartheta 0) C^{JL}_{J'mM} \right|^2$$

$$= \sum_{L \leq L'} \sum_{P,P'} \left\{ i^{L-L'+\delta-\delta'} a(JJ'\pi,L) a^*(JJ'\pi,L') e^{i(P'-P)\tau} + \text{c.c.} \right\} \cdot$$

$$(P)^{\delta(\pi,L)} (P')^{\delta(\pi,L')} \mathcal{D}^{(L)}_{M,P} \mathcal{D}^{(L')}_{M,P'} C^{JL}_{J'mM} C^{JL'}_{J'mM} (1 - \tfrac{1}{2}\delta^{L'}_{L}) \qquad (10)$$

where $\delta^{L'}_{L}$ is the Kronecker symbol and c.c. means complex conjugate.

For practical reasons, one generally considers the case for "pure" multipoles and confines one's attention to a single value of $L \equiv L'$ in the above equation. Since, however, we shall get a null effect for this case, the case of mixed multipoles becomes the important one. We shall forthwith consider mixed multipoles, and obtain the results for a pure multipole by specializing the general case.

The typical terms in Eq. (10) for a mixture of multipoles have the form:

$$\sum_{P,P'} \mathcal{D}^{(L)}_{M,P}\, \mathcal{D}^{(L')}_{M,P'}\, C^{JL}_{J'mM}\, C^{JL'}_{J'mM}\, (1 - \delta^{L'}_{L}/2)\, P^{\delta(\pi,L)}\, (P')^{\delta(\pi,L')} \;\cdot$$

$$\Big\{ \mathcal{R}e\left[a(JJ'\pi;L)\ a^{*}(JJ'\pi,L')\right] \mathcal{R}e\left[i^{L-L'+\delta-\delta'}\ e^{i(P'-P)\tau}\right]$$
$$- \mathcal{I}m\left[a(JJ'\pi,L)\ a^{*}(JJ'\pi,L')\right] \mathcal{I}m\left[i^{L-L'+\delta-\delta'}\ e^{i(P'-P)\tau}\right]\Big\} \tag{11}$$

We note now that the function $L + \delta(\pi,L) = L + \frac{1}{2}(1 + \pi(-1)^{L})$ is always an even integer if $\pi = -1$ and always an odd integer if $\pi = +1$, independent of the value of L. Consequently $L + \delta - (L' + \delta')$ is always an even integer for all values of L and L', and $i^{L-L'+\delta-\delta'}$ is therefore ± 1 in all cases. This sign is of no importance in what follows and will be dropped (it is $+1$ if $L = L'$, however). Also

$$e^{i(P'-P)\tau} = \delta^{P'}_{P} + \delta^{-P'}_{P} \cos 2\tau - i\, P\, \delta^{-P'}_{P} \sin 2\tau \tag{12}$$

Introducing these results into Eq. (10) we get the final form for the absolute square matrix element $\left|\langle B_m \,|\, H_2 \,|\, C_{m'}\rangle\right|^2$.

$$\left|\langle B_m \,|\, H_2 \,|\, C_{m'}\rangle\right|^2 \sim \sum_{L' \leq L'} (1 - \tfrac{1}{2}\delta^{L'}_{L})\, C^{JL}_{J'mM}\, C^{JL'}_{J'mM} \;\cdot$$

$$\cdot \Big\{ \mathcal{R}e\left[a(JJ'\pi,L)\ a^{*}(JJ'\pi,L')\right] \cdot (F^{M}_{L,L'}(\vartheta) + \cos 2\tau \cdot f^{M}_{L,L'}(\vartheta))$$

$$+ \mathcal{I}m\left[a(JJ'\pi,L)\ a^{*}(JJ'\pi,L')\right] \cdot (\sin 2\tau \cdot g^{M}_{L,L'}(\vartheta))\Big\} \tag{13}$$

We have introduced the definitions:

$$F^{M}_{L,L'}(\vartheta) \equiv \sum_{P} P^{L-L'} \mathcal{D}^{(L)}_{M,P}(0\vartheta 0)\,\mathcal{D}^{(L')}_{M,P}(0\vartheta 0) \qquad (14)$$

$$f^{M}_{L,L'}(\vartheta) \equiv \sum_{P} P^{L-L'} \mathcal{D}^{(L)}_{M,P}(0\vartheta 0)\,\mathcal{D}^{(L')}_{M,-P}(0\vartheta 0) \qquad (15)$$

$$g^{M}_{L,L'}(\vartheta) \equiv \sum_{P} P^{L-L'+1} \mathcal{D}^{(L)}_{M,P}(0\vartheta 0)\,\mathcal{D}^{(L')}_{M,-P}(0\vartheta 0) \qquad (16)$$

The relative sign of the three angular functions in the curley bracket in Eq. (13) is an unwieldy function of L, L' and π, which, in view of the result obtained below, has been dropped as unimportant. The functions defined in Eqs. (14), (15) and (16) are well known in the literature, at least for small L, L', (but the notation is not yet common). When $L = L'$ we get from (14) the $F^{M}_{L}(\vartheta)$ given by Falkoff and Uhlenbeck.[6] For $L \neq L'$ the function $F^{M}_{L,L'}(\vartheta)$ has been given by Ling and Falkoff[11], and by Zinnes[12]. Zinnes[12] also gives the function $f^{M}_{L,L'}(\vartheta)$. We are finally in a position to give the desired function $\mathcal{P}(\vec{P}_0, \vec{k}, \vec{e})$. Substituting Eq. (13) into Eq. (9) we get the required result.

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11. D. S. Ling, Jr. and D. L. Falkoff, Phys. Rev. 76, 1639 (1949).
12. Irving Zinnes, Phys. Rev. 80, 386 (1950).

$$\mathcal{P}(\vec{P}_o, \vec{k}, \vec{e}) \sim \sum_{L \leq L'} \sum_M \sum_m (1 - \tfrac{1}{2}\delta_L^{L'})(1 \pm \frac{2m|P_o|}{2J_N + 1})\, C^{JL}_{J'mM}\, C^{JL'}_{J'mM} \cdot$$

$$\cdot \Big\{ \mathcal{R}e \big[a(JJ'\pi, L)\, a^*(JJ'\pi, L')\big] \cdot \big[F^M_{L,L'}(\vartheta) + \cos 2\tau\, f^M_{L,L'}(\vartheta)\big]$$

$$+ \mathcal{I}m \big[a(JJ'\pi, L)\, a^*(JJ'\pi, L')\big] \cdot \big[\sin 2\tau\, g^M_{L,L'}(\vartheta)\big] \Big\} \tag{17}$$

We can simplify this by using a relation originally due to Casimir[13].

$$\sum_m C^{JL}_{J'mM}\, C^{JL'}_{J'mM} = \left(\frac{2J' + 1}{\sqrt{(2L+1)(2L'+1)}} \right) \sum_m C^{JJ'}_{Lm\ M-m}\, C^{JJ'}_{L'm\ M-m}$$

$$= \delta_L^{L'} \cdot \frac{2J' + 1}{2L + 1} \tag{18}$$

In order to simplify further we must obtain some of the properties of the $F^M_{L,L'}(\vartheta)$, $f^M_{L,L'}(\vartheta)$ and $g^M_{L,L'}(\vartheta)$. The desired properties are:

$$F^M_{L,L'}(\vartheta) = (-1)^{L-L'}\, F^{-M}_{L,L'}(\vartheta) \tag{19}$$

$$f^M_{L,L'}(\vartheta) = (-1)^{L-L'}\, f^{-M}_{L,L'}(\vartheta) \tag{20}$$

$$g^M_{L,L'}(\vartheta) = (-1)^{L-L'+1}\, g^{-M}_{L,L'}(\vartheta) \tag{21}$$

$$g^M_{L,L}(\vartheta) = 0 \tag{22}$$

13. H. B. G. Casimir, Archives de Musée Teyler, Series III, VIII, 274 (1936).

These properties all follow immediately from the definitions and the relation[5]:

$$\mathcal{D}^{(L)}_{M,P}(0\vartheta 0) = (-1)^{M+1}\,\mathcal{D}^{(L)}_{-M,-P}(0\vartheta 0) \tag{23}$$

For example, consider the function $F^{M}_{L,L'}(\vartheta)$.

$$F^{M}_{L,L'}(\vartheta) \equiv \sum_{P} P^{L-L'}\,\mathcal{D}^{(L)}_{M,P}\,\mathcal{D}^{(L')}_{M,P} = \sum_{P}(-1)^{2(M+1)}\,P^{L-L'}\,\mathcal{D}^{(L)}_{-M,-P}\,\mathcal{D}^{(L')}_{-M,-P}$$

$$= \sum_{P}(-1)^{2(M+1)+L-L'}\,P^{L-L'}\,\mathcal{D}^{(L)}_{-M,P}\,\mathcal{D}^{(L')}_{-M,P}$$

$$= (-1)^{L-L'}\,F^{-M}_{L,L'}(\vartheta) \tag{24}$$

The other relations are as easily shown.

We need one further relation, the symmetry in M of the sum:

$$H^{M}_{L,L'}(J,J') \equiv \sum_{m} m\, C^{JL}_{J'mM}\, C^{JL'}_{J'mM} \tag{25}$$

We shall prove the result:

$$H^{M}_{L,L'} = (-1)^{L-L'+1}\,H^{-M}_{L,L'} \tag{26}$$

To do this we note that $C^{JL}_{J'mM} = (-1)^{J+J'+L}\,C^{JL}_{J',-m,-M}$. (This relation has been shown by Racah[14].) Thus:

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14. Giulio Racah, Phys. Rev. 62, 438 (1942).

$$H^{M}_{L,L'} \equiv \sum_{m} m\, C^{JL}_{J'mM}\, C^{JL'}_{J'mM} = (-1)^{2(J+J')+L+L'} \sum_{m} m \cdot C^{JL}_{J'-m-M}\, C^{JL'}_{J'-m-M}$$

$$= (-1)^{L-L'+1} \sum_{m} m\, C^{JL}_{J'm-M}\, C^{JL'}_{J'm-M}$$

$$= (-1)^{L-L'+1}\, H^{-M}_{L,L'} \qquad (27)$$

Now consider again the various terms in Eq. (17), using the relations just shown.

$$\mathcal{P}(\vec{P}_o, \vec{k}, \vec{e}) \sim \sum_{L,M} \tfrac{1}{2} \left(\frac{2J'+1}{2L+1} \right) \left| a(JJ'\pi, L) \right|^2 \left(F^{M}_{L,L}(\vartheta) + \cos 2\tau\; f^{M}_{L,L}(\vartheta) \right)$$

$$+ \left(\frac{2|P_o|}{2J_N + 1} \right) \sum_{L<L'} \operatorname{Im} \left[a(JJ'\pi, L)\; a^{*}(JJ'\pi, L') \right] \cdot (\sin 2\tau) \cdot$$

$$\left\{ \sum_{M} H^{M}_{L,L'}\; g^{M}_{L,L'}(\vartheta) \right\} \qquad (28)$$

All the other terms vanish because of the summation over M. For example, take the terms in $F^{M}_{L,L'}(\vartheta)$.

$$F^{M}_{L,L'}(\vartheta)\; H^{M}_{L,L'} = (-)\; F^{-M}_{L,L'}\; H^{-M}_{L,L'}(\vartheta) \qquad (29)$$

Consequently the sum over M of these terms vanishes identically.

From physical considerations it is clear that the first group of terms in Eq. (28) (independent of neutron polarization) must be independent of ϑ and τ. It is easy to show this analytically, using the fact that $\mathcal{D}^{(L)}_{M,M'}(0\vartheta 0) = \mathcal{D}^{L}_{M',M}(0-\vartheta 0)$, which follows from the unitary property of the $\mathcal{D}^{(L)}_{M,M'}(\alpha\beta\gamma)$.

$$\sum_M F^M_{L,L}(\vartheta) \equiv \sum_M \sum_P \mathcal{D}^{(L)}_{M,P}(0\vartheta 0)\,\mathcal{D}^{(L)}_{M,P}(0\vartheta 0)$$

$$= \sum_P \sum_M \mathcal{D}^{(L)}_{P,M}(0-\vartheta 0)\,\mathcal{D}^{(L)}_{M,P}(0\vartheta 0)$$

$$= \sum_P \mathcal{D}^{(L)}_{P,P}(000) = 2 \qquad (30)$$

Similarly we can show that the corresponding sum over the $f^M_{L,L}(\vartheta)$ vanishes identically.

$$\sum_M f^M_{L,L}(\vartheta) = \sum_P \sum_M \mathcal{D}^{(L)}_{M,P}(0\vartheta 0)\,\mathcal{D}^{(L)}_{M,-P}(0\vartheta 0)$$

$$= \sum_P \sum_M \mathcal{D}^{(L)}_{P,M}(0-\vartheta 0)\,\mathcal{D}^{(L)}_{M,-P}(0\vartheta 0)$$

$$= \sum_P \mathcal{D}^{(L)}_{P,-P}(000) = \sum_P \delta^{-P}_{P} = 0 \qquad (31)$$

Finally then we have the result:

$$\mathcal{P}(\vec{P}_o, \vartheta, \tau) \sim \sum_{L} \left(\frac{2J'+1}{2L+1} \right) \left| a(JJ'\pi, L) \right|^2 + \frac{2|P_o|}{2J_N+1} (\sin 2\tau) \cdot$$

$$\sum_{\substack{L<L' \\ M}} \mathrm{Im}\, (a(JJ'\pi L)\ a^*(JJ'\pi L'))\ H^M_{LL'}\ \ g^M_{L,L'}\ (\vartheta) \qquad (32)$$

From (32) it is evident that if the radiation is observed with polarization insensitive detectors the intensity is isotropic and independent of neutron polarization for pure or mixed multipoles. This merely confirms a well-known result as does the fact that in the case of mixture radiation there are no interference contributions to the unpolarized intensity.[11] The only possible effect which might be expected(a priori) to arise from capture of polarized neutrons is an asymmetry which would presumably be observed with polarization-sensitive detectors and this only if the radiation consists of mixed multipoles. However, this possibility is illusory because all the terms $\mathrm{Im}\left[a(JJ'\pi L)\ a^*(JJ'\pi L')\right]$ vanish. That they do indeed all vanish as has recently been shown by Lloyd[15].

This eliminates the last possibility of any detectable γ-ray anisotropy resulting from the neutron polarization, for Eq. (32) now has the simple form, $\mathcal{P}(\vec{P}_o, \vartheta, \tau) \sim$ constant independent of $\vec{P}_o$, ϑ and τ.[16]

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15. Stuart P. Lloyd, Phys. Rev. 81, 161 (L) (1951).

16. This result is consistent with a theorem of Wolfenstein (reference 2) according to which the maximum anisotropy for the case here considered would be $\cos^3 \vartheta$. In the present case the maximum anisotropy is not realized.

It is also clear from the foregoing that the practical restriction to S-neutrons was essential in establishing this result. However, even if fast polarized neutrons were available the capture experiment would not seem to be highly practical for intensity reasons. The S-neutron capture γ-radiation is to all intents and purposes unpolarized and no observable effects arise by virtue of the neutron polarization.[17]

We emphasize again that the foregoing results depend critically on the fact that available polarization detectors can measure only linear polarization.

The question as to alternatives to the n-γ reaction arises in connection with the present problem. The elastic scattering of the neutron by non-zero spin nuclei gives rise, in general, to a partial depolarization to which nothing new can be added here. The emission of α-particles or other spinless particles will give no deviation from isotropy. Proton emission is not probable in view of the limited energy available. Other electromagnetic processes leading to internal pair emission or internal conversion electrons are again not practical - the former because of considerable experimental difficulties to be expected and the latter because the internal conversion coefficients would compete weakly with the high energy and/or low multipole transitions which are to be expected after formation of the compound state.

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17. The possibility of observing polarization effects by selecting the radiation emitted in a special direction is also seen to lead to a negative result. The only special direction at our disposal is the direction $\vec{P}_0$. For gamma-rays emitted at $\vartheta = 0$ only the components $M = \pm 1$ of the radiation appear (see references 4, 11). However, this gives right and left circularly polarized light which, for the detectors available, is equivalent to unpolarized light.