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ORNL SUMMER SHIELDING SESSION

GALE YOUNG, Leader

ON STRAIGHT-AHEAD γ-TRANSMISSION WITH A MINIMUM IN THE CROSS-SECTION

GALE YOUNG

OAK RIDGE NATIONAL LABORATORY

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This report is one of a series on shielding and represents part of the work of a summer group working principally at Oak Ridge National Laboratory. Participants are drawn from Nuclear Development Associates (Navy contract), NEPA, Bureau of Ships and Bureau of Aeronautics (Navy Department), AEC, RAND, General Electric, Argonne and Oak Ridge National Laboratory.

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ON STRAIGHT-AHEAD γ- TRANSMISSION WITH A MINIMUM IN THE CROSS-SECTION

Gale Young

Under certain assumptions about the law of scattering and the variation of the cross-sections with energy, Greuling has obtained solutions of the attenuation problem (1, 2, 3). His results, as they stand, do not include instances where the cross-section can (as with γ rays) go through a minimum as the energy varies. The present note illustrates how different Greuling solutions can be pieced together to construct cases having a minimum.

Introduction

We give here only a brief sketch of the problem, referring to the above references for details. Let σ = scattering cross section, μ = total cross section. It is assumed that scattering distributes the energy of the emerging γ rays (we shall here write in terms of photons, though material particles such as neutrons can be discussed in exactly the same way) uniformly over lower energies, and that all the rays move straight ahead undeviated by scattering. This last assumption leads, of course, to an over-estimate of the penetration.

Let F (x, E) be the number of photons per unit energy range at distance x. Then, with the above assumptions,

$$\frac{\delta F}{\delta x} + \mu F = \int_{E}^{E_0} F(x, t) \frac{\sigma(t)}{t} dt + S(x) \tag{1}$$

The source term S(x) arises from scattering out of the primary or other beams above energy E_0.

A particularly simple case is when $\frac{d\mu}{dE} = -\frac{\sigma(E)}{E}$. Then (1) can be satisfied by a function of x alone, becoming

$$F' + \mu F = F \int_{E}^{E_0} \frac{\sigma(t)}{t}\, dt + S = (\mu - \mu_0) F + S$$

or

$$F' + \mu_0 F = S \quad . \tag{2}$$

Thus all the properties of the medium disappear except μ_0, and it makes no difference how much of μ_0 is contributed by scattering and how much by true absorption. The transmission (for secondaries) of the medium is just as if it had constant absorption equal to μ_0, and no scattering power at all.

In the case of a primary beam at E_0 we have

$$S = \frac{\sigma_0}{E_0} e^{-\mu_0 x} \tag{3}$$

and (2) has the familiar solution

$$F = x S = \frac{\sigma_0 x}{E_0} e^{-\mu_0 x} \quad . \tag{4}$$

The result does, of course, depend upon the strength of the scattering out of the primary beam.

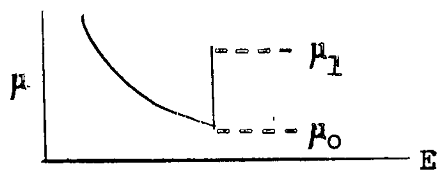
The total number of photons penetrating a distance x is given by

$$Q(x) = e^{-\mu_0 x} + \int_0^{E_0} F\, dE = (1 + \sigma_0 x)\, e^{-\mu_0 x} \quad , \tag{5}$$

so that the "buildup factor" is $1 + \sigma_0 x$. As we have just seen, the quantity σ_0 enters only from the primary beam and not at all from the secondaries.

First Example

Consider now the same situation, except that different cross-section values apply for the primary beam, as indicated in the following sketch.



The source term in (2) is now

$$S = \frac{\sigma_1}{E_o} e^{-\mu_1 x} , \tag{6}$$

and the solution is

$$F = \frac{\sigma_1}{E_o} \frac{e^{-\mu_o x} - e^{-\mu_1 x}}{\mu_1 - \mu_o} \tag{7}$$

with

$$Q = e^{-\mu_1 x} + \frac{\sigma_1}{\mu_1 - \mu_o} \left(e^{-\mu_o x} - e^{-\mu_1 x} \right) . \tag{8}$$

Considering $\mu_1 > \mu_o$, we write (8) as

$$Q(x) = \left[\frac{\sigma_1}{\mu_1 - \mu_o} + e^{-(\mu_1 - \mu_o)x} \left\{ 1 - \frac{\sigma_1}{\mu_1 - \mu_o} \right\} \right] e^{-\mu_o x} \tag{9}$$

where the expression in brackets is the build-up factor referred to the exponential taken at the minimum cross-section; it is unity at $x = 0$ and $\frac{\sigma_1}{\mu_1 - \mu_o}$ for x large. Thus, depending on the constants, it can be either a "build-up" or "build-down" factor. If $\sigma_1 = \mu_1 - \mu_o$, then $Q(x) = e^{-\mu_o x}$ everywhere.

Second Example

We now consider a slightly more complicated case in which the energy range is divided into two regions by $E_c < E_o$. In the top region we shall use a Greuling case with a rising cross-section, and in the bottom region a Greuling case with a falling cross-section.

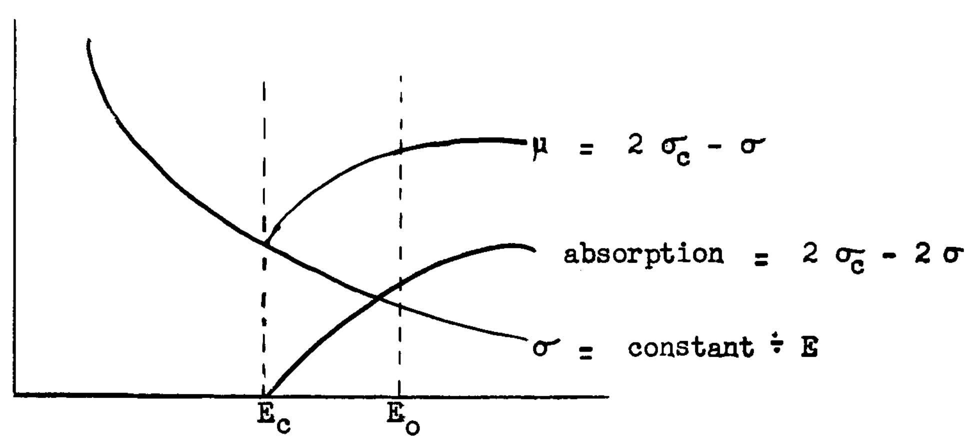
One of the simplest assumptions for the top region is to take

$$\frac{d\mu}{dE} = + \frac{\sigma(E)}{E} \quad .$$ This is the Greuling case $\alpha = -1$ (see above references), and for a source term (3) on the right side of (1) the solution is

$$F(x, E) = \frac{\sigma_o}{E_o} \times e^{-\mu(E)x} \quad , \tag{10}$$

as may be readily verified.

In the bottom region we shall take $\frac{d\mu}{dE} = - \sigma/E$, which is the case described in the introductory section. It may be noted that our assumptions regarding the top and bottom regions include the case with continuous $1/E$ scattering, absorption beginning at E_c and rising hyperbolically with energy, and total cross-section continuous (though with discontinuous derivative) at E_c; as shown in the adjacent sketch



The source fed into the bottom region below E_c is given by

$$\frac{\sigma_o}{E_o} e^{-\mu_o x} + \int_{E_c}^{E_o} F(x, E) \frac{\sigma(E)}{E} dE \tag{11}$$

which with (10) comes out to be

$$\frac{\sigma_o}{E_o} e^{-\mu_c x} \quad . \tag{12}$$

Then, from (4), the solution for $E < E_c$ is

$$F = \frac{\sigma_o}{E_o} x \quad e^{-\mu_c x} \quad ; \tag{13}$$

provided, as assumed here, that μ is continuous across E_c.

The integral fluxes are

$$\left.\begin{array}{llll} (\text{primary}) & Qp & = & e^{-\mu_o x} \\ (E_c < E < E_o) & Q_1 & = & \dfrac{\sigma_o x}{E_o} \displaystyle\int_{E_c}^{E_o} e^{-\mu(E)x} dE \\ (E < E_c) & Q_2 & = & \dfrac{E_c}{E_o} \sigma_o \, x \, e^{-\mu_c x} \quad . \end{array}\right\} \tag{14}$$

The second of these may be written as

$$Q_1 = \frac{\sigma_o x}{E_o} \left(\frac{E}{\sigma}\right)_m \int_{E_c}^{E_o} e^{-\mu x} \frac{\sigma(E)}{E} dE$$

$$= \frac{\sigma_o}{E_o} \left(\frac{E}{\sigma}\right)_m \left(e^{-\mu_c x} - e^{-\mu_o x}\right) \quad , \tag{15}$$

where $\left(\frac{E}{\sigma}\right)_m$ is evaluated at some mean energy E_m between E_c and E_o. If E/σ is an increasing function of E, then $Q_1 < \left(e^{-u_c x} - e^{-u_o x}\right)$. If μ is an increasing function of E, then for x large most of the value of the integral in (15) comes from near the lower limit, and thus asymptotically m tends to c. For x large the dominant term in $Q = Q_p + Q_1 + Q_2$ is Q_2 as given in (14).

Constant Total Cross-section

The above examples have a downward pointed cusp in μ at the minimum, and so presumably give too small a buildup factor. To approach the picture from the other side we consider the case where μ is flat.

A result due to Wigner (4; 3, page 11) can be generalized by considering the Greuling result as α becomes infinite (3, p. 15 and 18); α is the coefficient in the Greuling condition $\alpha \frac{d\mu}{dE} = -\sigma/E$. Let μ be constant, and let σ be an arbitrary function of E. Then, as may be verified in a straight-forward manner by using the recurrence relations among the Bessel functions, the solution of (1) with a source $\frac{\sigma_o}{E_o} e^{-\mu x}$ is

$$F(x, E) = \frac{\sigma_o}{E_o} x e^{-\mu x} \frac{I_1\left(2\sqrt{x\varphi}\right)}{\sqrt{x\varphi}} \tag{16}$$

where

$$\left.\begin{aligned} \varphi(E) &= \int_E^{E_o} \frac{\sigma(t)}{t}\, dt \\ \varphi'(E) &= -\frac{\sigma(E)}{E} \end{aligned}\right\} \tag{17}$$

The integral secondary flux above energy E_2 is

$$\int_{E_2}^{E_o} F(x,\ E)\ dE \ = \ \frac{\sigma_o}{E_o}\ x\ e^{-\mu x} \left(\frac{E}{\sigma}\right)_m \int_{E_2}^{E_o} \frac{I_1(2\sqrt{x\varphi})}{\sqrt{x\varphi}}\ \frac{\sigma}{E}\ dE$$

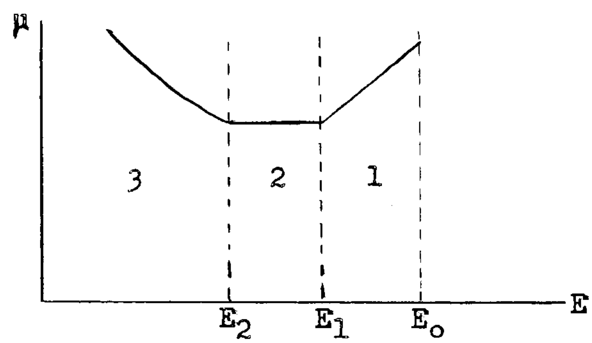
$$= \frac{\sigma_o}{E_o}\left(\frac{E}{\sigma}\right)_m e^{-\mu x}\left[I_o\left(2\sqrt{x\varphi_2}\right) - 1\right]. \qquad (18)$$

For x large most of the integral comes from near the bottom limit, and so asymptotically E_m approaches E_2.

In this case there is, for x large, a $e^{\sqrt{kx}}$ factor in the buildup.

Third Example

We now insert a flat μ central region between the upper and lower regions of the second example above, obtaining



a shape as sketched.

Working down from the initial energy E_o, and letting μ without a subscript denote the central value $\mu = \mu_1 = \mu_2$, we have (primary) $$Q_p = e^{-\mu_o x}$$

(region #1) $$S_1 = \frac{\sigma_o}{E_o} e^{-\mu_o x}$$

$$F_1 = \frac{\sigma_o}{E_o} x\, e^{-\mu(E)x} \qquad \text{from (10)}$$

$$Q_1 = \frac{\sigma_o}{E_o}\left(\frac{E}{\sigma}\right)_{m_1}\left(e^{-\mu x} - e^{-\mu_o x}\right) \qquad \text{from (15)}$$

(region #2) $$S_2 = \frac{\sigma_o}{E_o} e^{-\mu x} \qquad \text{from (12)}$$

$$F_2 = \frac{\sigma_o}{E_o} x\, e^{-\mu x} \frac{I_1(2\sqrt{x\varphi})}{\sqrt{x\varphi}} \qquad \text{from (16)}$$

$$Q_2 = \frac{\sigma_o}{E_o}\left(\frac{E}{\sigma}\right)_{m_2} e^{-\mu x}\left[I_o(2\sqrt{x\varphi_2}) - 1\right] \qquad \text{from (18)}$$

(region #3) Here we find

$$S_3 = S_2 + \int_{E_2}^{E_1} F_2(x,E)\,\frac{\sigma}{E}\,dE$$

$$= S_2 + \frac{\sigma_o}{E_o} e^{-\mu x}\left[I_o(2\sqrt{x\varphi_2}) - 1\right]$$

$$= \frac{\sigma_o}{E_o} e^{-\mu x} I_o(2\sqrt{x\varphi_2}) \qquad (19)$$

Then, with (2), we have $F' + \mu F = S_3$, of which the solution is

$$F_3 = \frac{\sigma_o}{E} x\, e^{-\mu x} \frac{I_1(2\sqrt{x\varphi_2})}{\sqrt{x\varphi_2}} \qquad (20)$$

$$Q_3 = \frac{E_2}{E_o}\sigma_o\, x\, e^{-\mu x} \frac{I_1(2\sqrt{x\varphi_2})}{\sqrt{x\varphi_2}} \qquad (21)$$

For x large, Q_3 is ultimately the dominant term in the total Q, namely

$$\frac{E_2}{E_0}\frac{\sigma_0}{\varphi_2}\frac{1}{2\sqrt{\pi}}\left(\varphi_2\, x\right)^{\frac{1}{4}} e^{-\mu x + 2\sqrt{\varphi_2 x}} \tag{22}$$

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References

1) E. Greuling in Mon P-172, p. 23-28

2) E. Greuling in Mon P-130, p. 7-9

3) G. Young, Mon P-293

4) E. P. Wigner, CH-137

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