Reactor Theory

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ANALYSIS OF AGE MEASUREMENTS IN FINITE SYSTEMS

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Analysis of Age Measurements in Finite Systems

M. E. Rose and A. M. Weinberg

The purpose of the following memorandum is to describe a method whereby from measurements of the age in a finite medium one can deduce the age in an infinite slowing down medium. In particular, although the age thus obtained is properly averaged over the primary energy distribution (fission spectrum), the application of the method of analysis of the data does not require any knowledge of the primary spectrum.

In the following we consider a moderator which has a finite rectangular cross-section but is effectively infinite in length. Designating the latter direction as the z-axis, the age-theory result for the spatial distribution of neutrons due to a plane source is

$$q(z) = \int f(\tau) \frac{e^{-z^2/4\tau}}{\sqrt{4\pi\tau}} \sum_{nm} A_{nm}(x,y) e^{-\beta_{nm}\tau} d\tau \; . \qquad (1)$$

Here $f(\tau)$ is the primary energy distribution normalized on a τ-scale, so that

$$\int_0^\infty f(\tau) d\tau = 1$$

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In terms of the more customary energy distribution N(E) we have

$$f(\tau) = \frac{3 N(E) E \xi (1 - \bar{\mu})}{\lambda^2(E)}$$

and E is expressed in terms of τ by

$$\tau(E) = \frac{1}{3} \int_{E_{In}}^{E} \frac{\lambda^2(E')\, dE'/E'}{\xi(E')(1 - \bar{\mu}(E'))}$$

Denoting the lateral dimensions by a and b we have

$$A_{nm} = \alpha_{nm} \cos \frac{2n+1}{a} \pi x \cos \frac{2m+1}{b} \pi y \, ; \; n, m, = 0, 1, 2 \ldots$$

and α_{nm} are the Fourier coefficients of the source. If the total number of neutrons per cm^2 per sec is $S(x, y)$ then

$$\alpha_{nm} = \frac{4}{ab} \iint dx\, dy\, S(x, y) \cos \frac{2n+1}{a} \pi x \cos \frac{2m+1}{b} \pi y$$

also

$$\beta_{nm} = \pi^2 \left[\frac{(2n+1)^2}{a^2} + \frac{(2m+1)^2}{b^2} \right]$$

From the measurements one obtains q(z) and therefore the moments

$$\overline{z^{2s}} = \frac{\int_{-\infty}^{\infty} z^{2s} q(z)\, dz}{\int q(z)\, dz} = \frac{(2s)!}{s!} \frac{M_s}{M_0} \qquad (2)$$

where

$$M_s = \int \tau^s f(\tau) \varphi(\tau)\, d\tau \qquad (3)$$

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and

$$\varphi(\tau) = \sum_{nm} A_{nm} e^{-\beta_{nm}\tau} \tag{4}$$

One is interested in $\bar{\tau}$, the age in the infinite moderator:

$$\bar{\tau} = \int f(\tau)\,\tau\,d\tau \equiv \int f(\tau)\,\varphi(\tau)\,\tau\,\frac{d\tau}{\varphi(\tau)}$$

In general we can expand $\varphi^{-1}(\tau)$:

$$\varphi^{-1}(\tau) = \sum_0^\infty \frac{\tau^\nu}{\nu!}\left(\frac{d^\nu\varphi^{-1}}{d\tau^\nu}\right)_0$$

Hence

$$\bar{\tau} = \sum_0^\infty \frac{1}{\nu!}\left(\frac{d^\nu\varphi^{-1}}{d\tau^\nu}\right)_0 M_{\nu+1} = M_0 \sum_0^\infty \frac{\nu+1}{(2\nu+2)!}\left(\frac{d^\nu\varphi^{-1}}{d\tau^\nu}\right)_0 \overline{z^{2\nu+2}}$$

In a similar way we find

$$1 = \int f(\tau)\,\varphi(\tau)\frac{d\tau}{\varphi} = M_0 \sum \frac{1}{(2\nu)!}\left(\frac{d^\nu\varphi^{-1}}{d\tau^\nu}\right)_0 \overline{z^{2\nu}}$$

Therefore we obtain for the age in the infinite system

$$\bar{\tau} = \frac{\sum_0^\infty \frac{\nu+1}{(2\nu+2)!}\left(\frac{d^\nu\varphi^{-1}}{d\tau^\nu}\right)_0 \overline{z^{2\nu+2}}}{\sum_0^\infty \frac{1}{(2\nu)!}\left(\frac{d^\nu\varphi^{-1}}{d\tau^\nu}\right)_0 \overline{z^{2\nu}}} \tag{5}$$

wherein all reference to the primary energy distribution has been eliminated.

For simplicity of application it is desirable to arrange matters so that the source contains only one harmonic, say the fundamental, (m = n = 0). This can be done quite easily by

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suitable disposition of the fissionable material. Then

$$\varphi = e^{-\beta_{00}\tau}$$

since a scale factor is of no significance for the moments. The age becomes

$$\bar{\tau} = \frac{\sum_0^\infty \frac{(\nu+1)}{(2\nu+2)!}\,\beta_{00}^{\nu}\,\overline{z^{2\nu+2}}}{\sum_0^\infty \frac{1}{(2\nu)!}\,\beta_{00}^{\nu}\,\overline{z^{2\nu}}} \tag{6}$$

Due to the factorial terms the convergence is quite rapid and in general three or four terms should suffice to evaluate the series in (6) to within about one percent. It may be of interest to note that not only the first moment $\bar{\tau}$ of the fission spectrum on a τ-scale is deducible from measurements of the slowing-down density, but also the higher moments. In fact,

$$\overline{\tau^n} = \int f(\tau)\tau^n d\tau = \frac{\sum_0^\infty \frac{1}{\nu!}\frac{(\nu+n)!}{(2\nu+2n)!}\,\beta_{00}^{\nu}\,\overline{z^{2n+2\nu}}}{\sum_0^\infty \frac{1}{(2\nu)!}\,\beta_{00}^{2}\,\overline{z^{2\nu}}} \tag{7}$$

for $\beta_{nm} = \delta_{n0}\,\delta_{m0}$. Although it would not be practical to attempt to obtain information about the spectrum $f(\tau)$ by determining the higher moments, a knowledge of $\overline{\tau^2}$ e.g., would be useful and could be obtained directly from the age measurements by the method described here.

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