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PRINCIPLES OF NUCLEAR POWER-CHAPTER 22: Control

From: John A. Wheeler

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JOHN A. WHEELER

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PRINCIPLES OF NUCLEAR POWER

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PREFACE

"Principles of Nuclear Power" is intended when complete to be a text book suitable for use in their war work by members of the Plutonium Project who have had a year of post-graduate work in physics or in physical chemistry. It contains three types of material:

1. General background material on the project such as has been presented to TNX personnel in Wilmington in introductory lectures.

2. Fundamental principles of nuclear physics, available here and there for the most part in the published literature, but not assembled so far in a form adapted to project purposes.

3. Principles of the chain reaction and related phenomena. To develop information of this type is the function of the Metallurgical Laboratory, and its success to date is shown by the contents of more than 1700 Chicago reports. The present Manual seeks to present in a clear cut form suitable for purposes of research and design those findings of the laboratory which are directly related to nuclear physics.

Preparation of a Manual summarizing the principles of a nuclear chain reaction was commenced in 1941 while the writer was a consultant to Section S-1 of the National Defense Research Committee at Princeton University. A survey report was issued from Princeton and later abridged and reissued by the Metallurgical Laboratory as CP-293.

On 1942 January 4, Dr. A. H. Compton, Director of the newly constituted Metallurgical Laboratory, asked the writer to continue preparation of the proposed Manual as rapidly as possible consistent with those parts of his work which rated higher priority. During the period of membership in the Metallurgical Laboratory the following sections of the text were issued by the Chicago Information Office as individual reports:

(Preliminary Edition)

Present Chapter classification	Title of report	Chicago numbering
Chapter 1	"Status of Atomic Power"	CP-293
Chapter 4	"Nuclear Matter and Liquid Drop Model"	Memo. 21.
Chapter 5	"Spontaneous Transformation of Nucleus"	Memo. 22.
Chapter 6	"Introduction to Study of Nuclear Transformations"	C-[illegible]
Chapter 24	"Extraction of Products"	C-7
Chapter 25	"Section on fission products"	CC-111
Chapter 13	in part, in report of A. H. Compton for 6 month period ending July, 1942.	

The writer participated in design work at Wilmington to an increasing extent beginning in November, 1942 and was eventually transferred by the Metallurgical Laboratory to the du Pont Company on 1943 March 1 for

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work on the Hanford project. At the time of transfer it was agreed that preparation of the Manual should continue in so far as consistent with more pressing work. Benefits of this policy to the Hanford project were expected and have so far been realized - text material has been used for reference by men engaged in design work, figures drawn for "Principles of Nuclear Power" have appeared also in the Hanford Technical Manual, and the actual preparation of material for writing has provided a systematic means to check over certain aspects of design.

The present chapter is one of those completed at Wilmington. Like other sections of the Manual it is however duplicated and distributed by the Metallurgical Laboratory.

On sections of the Manual so far completed help has been received from A. T. Monk, M. H. Foss, K. Way, and P. F. Gast. Drafting has been done by H. L. Conyers, and at various times G. Nissenbaum, M. Anderson, and R. H. Zipse have given secretarial assistance.

John A. Wheeler
Wilmington.
1944 June 12.

CHAPTER 22. CONTROL

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FUNCTION OF CONTROL 22.1

22.1 FUNCTION OF CONTROL

22.1.1 Need for control

To expect a chain reacting pile to continue at a constant level of operation without attention would be gambling on a great scale. A sudden change in condition of operation such as an alteration of atmospheric pressure or a decrease in the flow of cooling fluid could make the multiplication factor exceed unity. Then the level of the reaction might rise to the point where the structure would be damaged beyond the point of further usefulness. Moreover, construction of a pile for operation without any control would call for an extraordinary niceness of adjustment. One carbon block too many or too few would spell the difference between a divergent or a convergent chain reaction. The poisoning of the chain reaction by the fission products formed or the promotion of the chain reaction by the newly synthesized plutonium would have had to have been automatically compensated with high accuracy in order that the pile could continue to function at a constant rate as these changes went on. Far more practical than any such close-cut design has proved the plan of building a pile oversize and introducing neutron absorbent materials in controllable amounts into the pile structure. The excess size allows leeway against the self-poisoning tendency of the chain reaction and against the generally unfavorable effect on the multiplication factor of temperature and of deposits from the cooling fluid.

22.1.2 Shim, fine control, and safety

Development to date of the use of controls made of neutron absorbent materials has led to the recognition of three functions for these controls, known respectively as shim, fine control and safety. The size of the pile is generally so much in excess of that actually required for operation that a considerable amount of absorbent material must be introduced to bring the pile to the point of steady operation. This material is known as the shim control. As operation goes on, small changes in temperature, pressure, flow of cooling fluid and other variables take place from minute to minute which could be compensated by moving in or out of the pile structure the whole mass of neutron absorbent material. It has proved simpler, however, to move in and out from minute to minute as these changes take place, only a small portion of the whole of the control material. This portion is known as the fine control. In case something unexpected should occur during the operation which resulted in a sudden large increase in the multiplication factor, it would be necessary to insert suddenly into the pile a large additional amount of absorbing material. This material is known as the safety control.

22.1.3 Problems of control

The subject of controls falls under the following major headings:

(1) What signals that the control should be applied? (22.2)
(2) How may absorbent material best be disposed in the pile to achieve this control? (22.3)
(3) How much absorbent is required to compensate a given excess multiplication factor? (22.4 and 22.5)

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FUNCTION OF CONTROL 22.1.3

(4) How much heat will be generated in the absorbent? (22.6)
(5) How quickly does the pile respond to control? (22.7)
(6) How may controls conveniently be operated? (22.8)

22.1.4 Exclude split piles

We limit the present discussion of control in two respects. First, we assume that we always compensate the excess multiplication factor by introducing neutron absorbing material in the pile. Indeed, considerations of mechanical simplicity have so far favored control of the pile by a number of bars sliding in and out of the structure through relatively small channels. In contrast is the early proposal* to regulate the chain reaction by altering the disposition of uranium and moderator. Construct the pile in two halves. Place each on rollers - separate the two or move them together according as it is desired to decrease or increase the reactivity. This is a scheme whose chances for adoption have become less and less as the technical difficulties of designing even a fixed pile structure have become more apparent. Only if we were considering the problem of initiating an explosive chain reaction would we want to go in detail into this type of control.

22.1.5 Neglect effect of controls on power distribution

The other omission from the present discussion is the influence of controls on the distribution of heat production in the pile. To neglect such an effect due to the regulating control is reasonable because this device has a relatively small effect on the multiplication factor. Even less important is the influence of the safety controls on the spacial distribution of power, because these mechanisms act only in case of emergency. In contrast, the shim control will generally be expected to exert a relatively large effect on the distribution of heat production. Part of the multiplication factor to be compensated in this way will change with time and must therefore be balanced by an easily adjustable device, the true shim control. The remainder of the excess multiplication factor will however be used to permit changes in the pattern in which the uranium is loaded and thereby to allow more efficient removal of heat from the pile. The possibilities in this direction are quite important and have been described in Chapter 21. One or another of the methods for adjustment of the loading at periodic intervals will take up the important part of the excess multiplication factor, according to present indications. The true shim control will therefore have relatively so little to do that it, like the other controls, is expected to have no significant influence on the distribution of power release.

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*H. Halban and L. Kowarski, CPB-28, Technological Aspects of Reactions Used as a Source of Power (October, 1941)

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22.2 OCCASIONS FOR CONTROL

The type of control simplest in its design is the <u>shim</u> control. It has to take care of the following situations:

22.2.1 Shim compensates excess uranium

(1) The pile is too large and contains too much uranium. It is decided not to bring the multiplication factor down to the operating value by removing metal from the pile. A shim control must therefore be inserted in the pile to compensate the excess multiplication factor which would otherwise exist. This shim control is left in the pile throughout its operation. However, its position may be altered slightly from day to day or from week to week to compensate certain slow changes in reactivity, as follows:

22.2.2 Adjusted according to temperature

(2) The pile is operated in the beginning at a low level of intensity. As the success of the operation becomes more and more apparent, the level of intensity is increased and the temperature of the uranium in the pile rises. On this account, the multiplication factor decreases. Some of the shim control must therefore be removed from the pile to permit operation at the increased rate. This adjustment is carried out gradually. Evidently the power output can be calibrated in terms of the position of the control. The shim control will correspond to the throttle on an engine.

22.2.3 Compensate poisoning

(3) The poisoning of the chain reaction by the newly formed fission products and the improvement in the reactivity due to the newly formed plutonium are slow changes which work against each other. Which dominates can not be said at present. Reasonable estimates indicate that at a rate of operation as great as 250 megawatts, the resultant of the two effects together will probably not change the excess multiplication factor more than 1% in 1 month. This rate of change has to be compensated by a correspondingly slow adjustment in the position of the shim control.

22.2.4 Compensate absorption by scale

(4) Gradual deposit of chemicals from the cooling fluid in the pile structure will lower the factor of reproduction. This process should occur very slowly. To the extent to which it is appreciable, it will be necessary upon this account also to readjust from time to time the position of the shim control.

22.2.5 Function of fine control

The <u>regulating control</u> or <u>fine control</u> of the pile has to take care of changes in the factor of multiplication which on the one hand are considerably smaller than those compensated by the shim control but which on the other hand take place with much more rapidity. Changes of this type are the following:

22.2.6 Temperature fluctuations

(1) The temperature of the cooling fluid may change from instant to instant. This change will alter the temperature of the pile and thereby its reproduction factor. The alteration in reactivity of the pile on

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this account depends considerably upon the design of the pile but is of the order of magnitude $\sim 4 \times 10^{-5}$ loss in multiplication factor per ^{o}C. rise in temperature.

22.2.7 Fluctuations in fluid pressure

(2) The pressure level at which the cooling fluid is circulated may fluctuate from moment to moment and thereby slightly affect the pile dimensions. Or the amount of cooling fluid within the structure may change, consequently affecting the multiplication factor.

22.2.8 Atmospheric changes

(3) A graphite-uranium pile to the pores of which the atmosphere has access suffers a loss in reproduction factor proportional to the nitrogen content of the structure. On this account, an increase in the barometric pressure by 1 cm. of mercury lowers the factor of multiplication by an amount roughly $\sim 8 \times 10^{-5}$. Barometric changes of considerable magnitude may occur in a space as short as an hour. Compensation of these changes is therefore one of the functions of the fine control.

22.2.9 Reflection of neutrons

(4) The reactivity of a pile is improved by placing reflectors about it. The approach of an individual to a pile will produce a substantial increase in its factor of multiplication unless the pile is very well shielded. Unexpected changes of this kind have to be compensated by the fine control.

22.2.10 Large safety control

The safety control has to take care of sudden changes in the condition of operation as does the fine control but changes of an order of magnitude so much greater that it is not safe or even practical to rely upon the fine control to compensate them. Primary function of the safety control is to stop the chain reaction with the utmost reliability. The safety control must function quickly and must give a large reduction in the factor of reproduction for the following reasons:

22.2.11 Danger from losing water

(1) The cooling fluid may suddenly escape from the pile and thereby lead to a substantial increase in the factor of reproduction. In one design of a water cooled pile, for example, removal of all the water raises the factor of multiplication 2.4%.

22.2.12 Temperature drop

(2) Sudden change from an operating pile to an inactive pile may be associated with a relatively rapid drop in the temperature of the uranium. This decrease in temperature will, in general, bring about a rise in the factor of reproduction. The rise may, under certain circumstances, be as great as 1%. The safety control must be able to overcompensate this rise.

22.2.13 Possible failure of other controls

(3) A catastrophe may suddenly have ejected the fine control and the shim control from the pile structure without damaging the pile itself. On this account, there will be a sudden increase in the pile reactivity.

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OCCASIONS FOR CONTROL 22.2.14

Taken altogether, the possibilities for loss in cooling fluid, sudden drop in temperature, and for loss of all other controls, indicate that the safety control itself should be capable of decreasing the factor of multiplication by a large amount of the order of 4% or possibly more. The safety control should also be capable of making this change in multiplication factor with great rapidity, for the following reasons: 22.2.14 Control must be swift

(1) With an uncompensated multiplication factor as great as 2%, the level of power output of the operating pile will increase at first very rapidly and then by a factor $\sim 10^5$ every second until either the temperature rise compensates the excess multiplication factor or the pile structure is physically destroyed. 22.2.15 Suddenness of overload

(2) Quite apart from some unexpected change in the multiplication factor which may demand a quick stopping of the chain reaction, some mechanical failure may call for an equally rapid stopping of the pile. A stoppage of the flow of cooling fluid through a small portion of the pile, for example, would in the beginning have relatively little effect on the reactivity of the pile as a whole. If this condition were allowed to continue for even a few minutes, however, the structure of the pile might be damaged to such an extent that its further life would be seriously limited. On this account provision has to be made to detect the stoppage of flow or other failure in the tubes of a water cooled pile. 22.2.16 Mechanical failure

(3) Accidental ejection of some of the activated material from the pile into the working space immediately around it may drive away those overseeing the operation of the pile. In this case, the future safety of the pile could not be guaranteed unless it were quickly cut off. 22.2.17 Stop when supervision impossible

(4) Quick stoppage of the chain reaction is a proper response to almost any kind of sabotage of the pile or its accessories. The greatest catastrophe which can be visualized (see Chapter 23) is combustion of the activated uranium and release of the fission products from it to the atmosphere with the possible contamination of many miles of territory. The first step to prevent such combustion is to keep the temperature of the uranium low. This condition requires that the production of further heat from fission be stopped at once. The heat subsequently given out by the radioactivity formed during the previous operation is small enough that circulation of cooling fluid at a reduced rate will suffice to prevent overheating of the uranium. Even if the catastrophe is so great that further circulation of the cooling fluid is entirely impossible, the rise in temperature of the activated uranium will take place at a rate sufficiently reduced so that there may be time to flood the pile and thereby guarantee that the uranium will not catch on fire. Without a rapid safety control, however, it could well be that the temperature of the uranium might rise to values so high that flooding of the pile would not prevent a catastrophe. For all the foregoing reasons, the reliability and rapid insertion of safety controls is one of the most important requirements in the design of the pile. 22.2.18 Prevent major catastrophe

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22.3 SURVEY OF CONTROL THEORY

22.3.1 Complementary methods of describing control

The theory of controls is a special case of the problem of pile reactivity - more complicated but in some way more illuminating. The pile theory , familiar from Chapters 14, 15, and 16, views the central concepts of multiplication factor and buckling of the neutron density as two quite different aspects of the propagation of a chain reaction. However, the complementary nature of the two ideas becomes evident on examining the mode of action of various forms of control.

22.3.2 Large rod considered to affect boundary conditions

As one extreme case, consider a pile - cylindrical for simplicity - along the axis of which we insert a very large cadmium rod. The neutron density extrapolates to zero at the effective boundary of the pile both before and after entrance of the cadmium. When the rod is in place, the concentration of neutrons also extrapolates to zero near the surface of the absorber. The buckling of the neutron density is evidently greater after insertion than before. More neutrons leak to the outside. A considerable number of neutrons also migrate to the new internal boundary and are absorbed there. Both effects lower the overall multiplication factor and cause the power output to fall if it was originally stationary. The action of the control appears as a change in boundary conditions without any alteration in local multiplication factor.

22.3.3 Continuous absorbent alters local k

In the opposite extreme case we control the pile by pumping into its pores a uniform concentration of boron-containing gas. This procedure decreases the local multiplication factor and thereby reduces the overall multiplication factor but does not affect the buckling of the neutron density at all.

22.3.4 Either point of view in intermediate cases

Intermediate between these two extremes is a case where we introduce into the pile a number of rods, wires, or lumps of absorbing material. We have two alternative ways of describing the action of such controls. On the one hand we can say that they introduce internal boundaries into the pile. In this sense they do not affect the local multiplication factor of the medium at all. Their sole action is to increase the buckling of the neutron density and thus to produce both an extra leakage to the outside and a migration of neutrons to the absorber itself. On the other hand we can view scattered lumps of cadmium or other highly absorbent material as giant nuclei whose cross sections are to be measured in units of cm^2 rather than in units 10^{-24} times smaller. We can determine the effect of these "nuclei" in the same way in which we estimate the absorption by the boron gas. The cadmium is considered to alter the local multiplication factor but not the buckling of the neutron density. For effective value of the buckling, we take the same value which prevailed before the controls entered. In other words, we look at the gross overall variation of the neutron density and look apart from its minor irregularities. This procedure is consistent with that followed in analyzing the effect of boron itself, where we look apart from the decrease in neutron density in the immediate vicinity of each absorbent nucleus. We conclude from this discussion that it is often purely a matter of convenience whether we shall describe the effect of controls

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in terms of an alteration in boundary conditions, or a change in local multiplication factor.

22.3.5 The principle of equivalence

The possibility of describing the effect of absorbers in two complementary ways constitutes a fundamental principle in the development of the theory of controls. More specifically, we may formulate the following "Principle of Equivalence": "The absorbent, together with the material dispersed symmetrically around it, is equivalent to a homogeneous multiplying medium of the same volume with a new multiplication factor and a new migration area". This principle is very similar to that used in analyzing the properties of the pile itself. In determining the factor of reproduction for the lattice, we have to apply certain conditions to the neutron density and its derivative at the boundary between the moderator and the fissionable material. The results of this analysis are however summarized in terms of a multiplication factor for the pile medium as a whole, regarded as equivalent to a homogeneous substance.

22.3.6 Plan of application of equivalence principle

The application of the principle of equivalence may be illustrated by an example. A rod of cadmium is passed through a pile. We wish to determine the effect of this rod upon the overall multiplication factor of the pile. We consider together the control rod and a circumscribed cylindrical portion of the pile structure having, for example, a radius of 50 cm. We think of this combination as a lattice unit possessing a definite multiplication factor and migration area. In this way we translate the original problem into two simpler problems. Of these the first is one of pure lattice theory which may be solved by the methods already at our disposal. Procedures for finding the local reactivity are developed with special reference to control rods in Sections 22.4 and 22.5. Then our problem is to determine the reactivity of a pile which contains a core 50 cm in radius with slightly different multiplying properties than the rest of the structure. This question is treated with the method of statistical weights outlined in the remainder of the present section. This treatment allows us to determine with considerable accuracy the effect on the overall reactivity of the pile of a change in the local multiplication factor in one portion of the structure.

22.3.7 Local k independent of location of rod and core

The division of the control problem into two parts owes its convenience to the fact that the two parts are nearly independent of each other. For example, it does not matter in our example whether the control rod and the surrounding core pass through the center of the pile or near its fringe. It does not matter whether the gradient of the neutron density is greater in the direction of the rod or across the direction of the rod. In either case, the basis of our analysis is the same; we determine the local multiplication factor and the migration area once and for all for the combination of rod and core. Just at what point the division of the control problem into two parts will be made is a matter of choice. In the example, the core could have been taken to have a radius of 100 cm instead of the chosen 50 cm. In this case the local multiplication factor of the combination of core and control rod would be different from the value ascribed to the smaller unit. The result of the two methods of analyzing the control problem will however be the same within the limits of accuracy of the principle of equivalence. May, 1944

22.3.8 Local k independent of shape of core

What has been said here of freedom in choosing the size of the region of equivalence applies equally well to our liberty in choice of its shape. Consider, for example, the case in which we are analyzing the effectiveness of parallel control rods spaced in rectangular array. In this case we will naturally take the region associated with one rod to be a square centered on that rod. On the other hand the region will be taken to be a hexagon when the rods are located on a triangular pattern. When we come to the point of determining the loss in local multiplication factor induced by the insertion of a control rod into a region of one or another simple geometrical form, we shall treat the region as equivalent to a cylinder of the same cross sectional area. This is the procedure which we have already followed in the development of lattice theory in Chapters 15 and 16.

22.3.9 Limitations on principle of equivalence

Certain limitations apply to the principle of equivalence just as there are certain limits to the accuracy with which a heterogeneous pile can be described as a homogeneous medium. In the example of control rod and core, it is obviously necessary for the control rod to be surrounded by several lattice units in order to justify treating the core as a uniform multiplying medium. In other words, the radius of the core should certainly exceed a lattice unit. Difficulty also occurs if the region associated with one control rod is comparable in size with the pile itself. Then it is no longer quite accurate to express the equivalence between cylindrical and square regions in terms of area. On this account we shall generally limit our applications to the principle of equivalence to cases where the size of the zone associated with one control rod is intermediate in order of magnitude between the lattice spacing and the width of the pile itself. A further limitation on the principle of equivalence will be apparent on considering as means of control an opaque sheet of cadmium passed through the pile. We might be inclined to consider this sheet, together with a layer of the pile medium on either side of it, as equivalent to a homogeneous multiplying medium with a new multiplication factor and migration area. If this view were correct, the variation of neutron density in one portion of the medium should have an effect on the distribution of neutrons in the other part of the medium. But the disturbance obviously cannot propagate itself through the cadmium. Generalizing from this instance, we can say that the principle of equivalence does not apply when the width of the zone of control is comparable with the width of the opaque neutron absorbent contained within it.

22.3.10 No distinction between fast and slow neutrons in equivalence principle

A further limitation of the principle of equivalence is apparent from the fact that it attempts to describe all the properties of the multiplying medium in terms of the two quantities, local multiplication factor and migration area. This description overlooks the fact that two media may have very different thermal migration areas and very different moderation areas and yet have the same total migration areas and the same local multiplication factor. As a matter of fact however the difference in the behavior of fast neutrons and slow neutrons individually is of little consequence to the equivalence principle. Hardly any two media could be more different in their relative effect on fast and

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slow neutrons than a typical pile and the surrounding graphite reflector. Yet the theory of reflector effectiveness, outlined in Chapter 16, which takes into account only the migration area and multiplication factor in the two media, gives results which differ relatively little from those of a more complete theory which allows for the difference in behavior of fast and slow neutrons.

22.3.11 Design of rod versus location of rod

Apart from the few cases where the principle of equivalence is obviously unsuitable for use, it furnishes a convenient means for dividing the problem of control into two parts so that one can analyze separately the effectiveness of control rod as a control rod and the effectiveness which it derives on account of its location in the pile. The following table lists the magnitude of the depression in the local multiplication factor set up in the typical pile by the insertion of certain simple forms of control.

Table 22.3.12. EFFECT ON LOCAL MULTIPLICATION FACTOR OF TYPICAL TYPES OF CONTROL

Figures refer only to graphite-uranium piles whose designs resemble that assumed in calculations. Reference is therefore made here to Section 22.5 for the precise details of absorber and pile assumed in each case. Radius of zone of equivalence denoted by R, magnitude of effective change of local multiplication factor in this zone by δk_{local}.

Nature of control material	Effect as calculated in Section 22.5		Approximate translation to common size of zone of equivalence	
	R	δk_{local}	R	δk_{local}
Three boron coated tubes which together make black to neutrons a cross sectional area ~3 3/8" x 7/8" or 8.6 cm x 2.2 cm	81.3 cm	0.031	81.3 cm	0.031
Cadmium rod 0.3 cm in radius	30 cm	0.051	81.3 cm	0.007
Cadmium cylinder 30 cm in radius	300 cm	0.0083	81.3 cm	0.1
BF_3 in pores of graphite at 1/100 atmosphere pressure		0.021		0.021

22.3.13 Principle governing effectiveness of location

After this summary of the local effect of various types of absorber, it remains to analyze the influence on overall reactivity due to the location of the control. The effect of location can be treated with high accuracy only by quite complicated mathematical techniques for many dispositions of absorber in the pile which are of practical interest. Such is the case of a single control rod passed through a cylindrical pile

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parallel to its axis but located off center. A centrally located rod or whole group of rods inserted part way into a pile furnish another example of the same kind. It is helpful in such cases to have as a guide the following general principle of analysis: The value of the neutron density at a given point is the quantity which governs the effectiveness of a control located at that point.

22.3.14 Definition of virtual neutron density

It is necessary to be more specific in speaking of neutron density as determiner of effectiveness of control. The properties of the pile may vary from place to place in such a way that point P has a higher density of fast neutrons and yet a lower density of slow neutrons than point Q in the structure. The density of neither special variety of neutron interests us here so much as a quantity which we shall call the "virtual neutron density". This quantity, denoted here by n, is defined completely except for an arbitrary multiplicative constant by the following three conditions:

(1) n vanishes at the effective boundaries of the pile.

(2) n is positive throughout the interior of the pile.

(3) n satisfies the fundamental buckling equation:

$$\partial^2 n/\partial x^2 + \partial^2 n/\partial y^2 + \partial^2 n/\partial z^2 + Bn = 0 \qquad (22.3.14.a)$$

Here the buckling, B, is considered to be a known function of position in the pile. This quantity, when multiplied by the migration area, is equal under steady state conditions to the difference between the local multiplication factor and unity. Knowing the shape of the pile and the local multiplication factor, which is often nearly constant over the pile, we have in (22.3.14.a) a relatively simple means to find how the virtual neutron density varies from point to point. Illustrative examples have appeared in Chapters 14 and 16.

22.3.15 General averaging theorem

Let us now investigate the quantitative relationship between pile reactivity and the value of the virtual neutron density as just defined. We use the quantity, B, to represent the buckling of the neutron density in the pile before the control is introduced. We designate by n the virtual neutron density in this pile. The corresponding quantities after insertion of the control are denoted by B* and n*. Here n* satisfies the equation:

$$\partial^2 n*/\partial x^2 + \partial^2 n*/\partial y^2 + \partial^2 n*/\partial z^2 + B*n* = 0. \qquad (22.3.15.a)$$

We wish to compare conditions before and after the change. We multiply (22.3.15.a) by n, (22.3.14.a) by n*, and subtract. The difference we integrate over the entire volume interior to the pile. A term of the character

$$(n* \partial^2 n/\partial x^2 - n\partial^2 n*/\partial x^2) \qquad (22.3.15.b)$$

gives on integration with respect to x an expression of the form:

$$(n* \partial n/\partial x - n \partial n*/\partial x)\Big|_{\text{lower limit of } x}^{\text{upper limit of } x} \qquad (22.3.15.c)$$

This expression vanishes at both limits because the neutron density extrapolates to zero at the effective surface of the pile. Likewise, the terms containing derivatives of neutron density with respect to y and z give zero contribution after integration. Our series of mathematical operationstherefore finally leaves us with only one term:

$$\int n\, n* \,(B* - B)\, d\,(\text{volume}) = 0 \qquad (22.3.15.d)$$

In words, the average value of the buckling of the neutron density has the same value before and after the introduction of the control, provided that the average is taken with respect to the product of original and final virtual neutron density as weight factor.

From the simple principle of averaging just derived follow some useful results. We shall examine the applications under one or another of two heads according as the controls do or do not considerably change the distribution of neutron density throughout the pile. When the variation of activity through the pile is little affected by the absorber we are able to obtain from the averaging principle a simple and fairly complete account of effectiveness of control as influenced by position and degree of insertion. The situation is ordinarily much more complicated when the controls produce large changes in local reactivity over extended portions of the pile. However, we obtain relatively simple results in this case too, provided that the absorbent or the control rods are to be disposed so as to have maximum effectiveness.

22.3.16 Applications of averaging theorem

In the first group of applications of the averaging principle the alteration in neutron density by the controls is slight. Then the difference between the buckling before and after the change, $B* - B$, is a small quantity of the first order. The term $n\, n*$ may be written as the sum of two contributions of which the first is the square, n^2, of the original neutron density and the second is a correction term, $n\,(n* - n)$, of the first order. The product of two terms of the first order will give a term of the second order which for our purpose is to be neglected. In this approximation we find the equation:

22.3.17 Action of weak control proportional to n^2

$$\int n^2 \,(B* - B)\, d\,(\text{volume}) = 0 \qquad (22.3.17.a)$$

Let control material be spread at a low density in a small volume, V_1, of the whole structure. In this region the buckling drops by an amount δB_1. In the rest of the pile the buckling must therefore increase. The magnitude of this increase δB_{av} is evidently obtained from Eq. (22.3.17.a) and is

$$\delta B_{av} = \frac{n_1^2\, V_1}{\int n^2\, d\,(\text{volume})}\, \delta B_1 \qquad (22.3.17.b)$$

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Put in other terms, this equation states that a slight overall increase in the reactivity of the pile by the amount, δB_{av}, may be compensated through controls by a larger change in reactivity over a limited portion of the pile. The effectiveness of this localized change in reactivity is proportional to (1) the volume of the region of control and (2) the square of the neutron density, n_1^2, at the point of action.

22.3.18 Weight factor and position factor

The proportionality between degree of control and square of the neutron density can be stated in several equivalent forms, whose relative usefulness depends upon the circumstances of the problem under consideration. In this connection the relation between buckling and local multiplication factor can for most purposes be taken to be straight proportionality, as the migration area usually changes relatively little from one region of the pile to another. Thus we translate (22.3.17.b)as follows:

$$\left(\begin{array}{c}\text{Overall loss in k due to a}\\ \text{control which does not great-}\\ \text{ly distort the general dis-}\\ \text{tribution of neutron density}\\ \text{in the pile}\end{array}\right) = \frac{\left(\begin{array}{c}\text{Integral of square of original}\\ \text{expression for virtual neutron}\\ \text{density, multiplied by change}\\ \text{in local multiplication factor}\end{array}\right)}{\left(\begin{array}{c}\text{Integral of square of}\\ \text{virtual neutron density}\\ \text{over whole pile}\end{array}\right)} \qquad (22.3.18.a)$$

$$= \left(\begin{array}{c}\text{Reduction in local k in zone of}\\ \text{equivalence inscribed about con-}\\ \text{trol rod or other absorber}\end{array}\right) \underbrace{\frac{\left(\begin{array}{c}\text{Integral of square of}\\ \text{virtual neutron density}\\ \text{over zone of equivalence}\end{array}\right)}{\left(\begin{array}{c}\text{Integral of square of}\\ \text{virtual neutron density}\\ \text{over whole pile}\end{array}\right)}}_{\begin{array}{c}\text{This ratio is termed the}\\ \text{"weight factor" of}\\ \text{the zone of equivalence}\end{array}} \qquad (22.3.18.b)$$

$$= \underbrace{\left(\begin{array}{c}\text{Reduction in}\\ \text{local k in zone}\\ \text{of equivalence}\end{array}\right) \frac{\left(\begin{array}{c}\text{Volume of zone}\\ \text{of equivalence}\end{array}\right)}{\left(\begin{array}{c}\text{Volume of}\\ \text{whole pile}\end{array}\right)}}_{\begin{array}{c}\text{Reduction in k which would re-}\\ \text{sult if an equivalent amount}\\ \text{of boron were distributed}\\ \text{over the whole pile}\end{array}} \underbrace{\frac{\left(\begin{array}{c}\text{Value of square of virtual}\\ \text{neutron density averaged over}\\ \text{zone of equivalence}\end{array}\right)}{\left(\begin{array}{c}\text{Value of square of virtual}\\ \text{neutron density averaged over}\\ \text{whole pile}\end{array}\right)}}_{\begin{array}{c}\text{This ratio is known as "position}\\ \text{factor" and measures the relative}\\ \text{effectiveness of the control rod}\\ \text{or other absorber due to its}\\ \text{special location in the pile}\end{array}} \qquad (22.3.18.c)$$

[redacted]

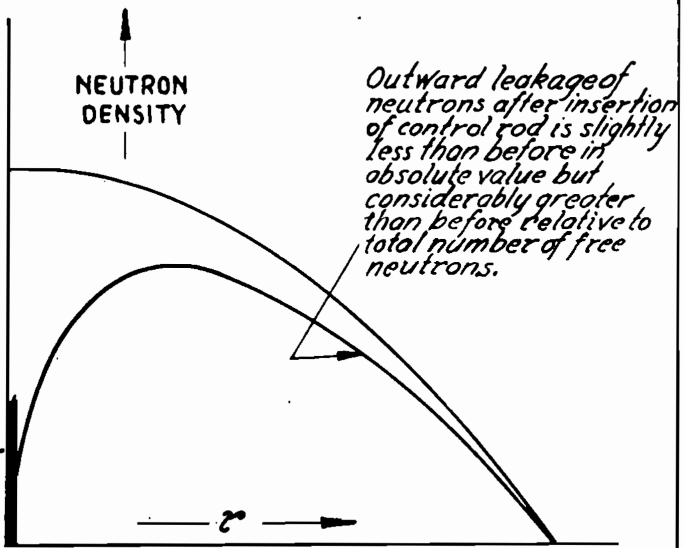
[redacted]

 22.3.18

The position factor as just defined measures the extra effectiveness to be gained by putting the control material in a region of high neutron density. Its value is essentially independent of the size which we choose to attribute to the zone of equivalence circumscribed about the control rod. On the other hand the weight factor depends on the extension of this zone as well as on its location. The reduction in overall multiplication factor is of course independent of the size of the zone of equivalence in either case.

22.3.19 Paradox of high effectiveness

Proportionality of control, not to the first power of the neutron density but to its square, seems at first sight paradoxical. Consider for example a spherical pile. At its center the square of the neutron density is greater by a factor 6.580 than the average value of the square taken over the whole structure. However, the neutron density itself at the center of the pile exceeds its average value only by a factor 3.290. This granted, we undertake two hypothetical experiments. In the first one we spray uniformly throughout the pile enough boron to absorb some given small fraction, say 10^{-5}, of the thermal neutrons. Then the overall multiplication factor will be reduced by 10^{-5}. In the second experiment we use the same amount of boron but spread it over a region about the center of the pile. In this case we have to expect a reduction in overall multiplication factor equal to 6.6×10^{-5}, according to Eq. (22.3.18.c). Yet the boron will absorb only the fraction 3.3×10^{-5} of the neutrons. How does the paradoxical result come about that control material located where it will absorb three times as many neutrons gives six times as much control? A clue to the answer comes from a look at the neutron distribution in a pile containing absorbent at its center (see diagram at right). The neutron density is buckled more strongly than it was in the absence of the absorber.



Consequently, the gradient of the neutron density at the surface of the pile is increased over its normal value. Thus the outward flux of neutrons has been raised above its old level. In other words the control takes neutrons away from the chain reaction by two mechanisms: by direct absorption and by stimulating leakage to the outside. The discrepancy between the factor 6.58 for effectiveness in control and the factor 3.29 for effectiveness in absorbing neutrons has therefore a simple explanation. For every neutron taken up in the control in the special example, two neutrons are lost to the chain reaction, the one absorbed and the other forced out of the surface of the pile by increased leakage. May, 1944

[redacted]

22.3.20 Low effectiveness near surface

What has been said of the extra efficiency of a control at the center of the pile applies just the other way around to a control near the surface of the reactor. Let the absorbent, for example, be put in a region where the neutron density is one tenth the average value. It will absorb only one tenth as many neutrons as it would if it were uniformly dispersed through the structure. At this point in a typical pile the square of the neutron density will have fallen to a fraction of the order of 6×10^{-3} of the average squared value. The drop in multiplication factor will therefore be only $\sim 6 \times 10^{-3}$ times as great, rather than one tenth as great, as that observed in the case of uniform distribution. This lower efficiency is to be understood in the following way. Roughly 94 percent of the neutrons absorbed in the control material would have leaked out of the pile anyway and only the remaining 6 percent represent increased loss of neutrons due to the new absorbent. Like a lightning rod, the object concentrates upon itself the already existing flux load, without very much increasing the total magnitude of that flux. To locate controls near the surface of a pile is evidently inefficient practice.

22.3.21 Position factor

Following this general picture of the effect of position on control, we may make a quantitative study of the magnitude of the position factor in cases of special interest. This quantity is expressed in terms of the virtual neutron density, n, and volume integration over the pile by the formula:

$$\text{position factor} = \frac{n^2 \int d\ (\text{volume})}{\int n^2\ d\ (\text{volume})} \qquad (22.3.21.a)$$

Table 22.3.22 lists expressions for position factor and weight factor in the cases of spherical, cylindrical and rectangular piles. Some examples will illustrate the use of the table.

22.3.23 Effect of few imperfect slugs

EXAMPLE. A cylindrical pile contains 50,000 uranium slugs. The 32 slugs most centrally located have a multiplication factor 1 percent lower than normal. How much is the multiplication factor of the whole structure lowered on this account? From the table we find that the position factor at the center is 7.42. Accordingly, the overall reduction in multiplication factor is $7.42 \times 0.01\ (32/50000) = 4.7 \times 10^{-5}$. If the slugs had been distributed along the whole length of the axis of the pile, the position factor would have been only 3.71 and the loss in k would have been 2.4×10^{-5}.

22.3.24 Effect of small cadmium rod

EXAMPLE. A cylindrical graphite-uranium pile has an effective radius, R, of 510 cm. A cadmium rod 0.3 cm in radius is thrust completely through the pile parallel to its axis. How much is the loss in k when the rod lies (a) on the axis? (b) 255 cm away? This rod produces a drop of 0.051 in local multiplication factor within a zone of equivalence 30 cm in radius, according to the summary in Table 22.3.12 of results derived in Section 22.5. The position factor for a rod thrust completely through the pile depends only on distance, r, from the axis and according to Table 22.3.22 is $3.710\ J_0^{\,2}\ (2.4048\ r/R)$. When the rod lies

Table 22.3.22 POSITION FACTORS AND WEIGHT FACTORS FOR WEAK CONTROLS OR OTHER ABSORBERS LOCATED IN PILES OF SIMPLE GEOMETRICAL FORM. EFFECT OF REFLECTOR MAY BE TAKEN INTO ACCOUNT BY ADDING TO DIMENSIONS OF ACTIVE ZONE THE EFFECTIVE CONTRIBUTION MADE BY THE REFLECTOR.

Shape of pile	Sphere	Cylinder	Rectangular prism
Effective dimensions	radius R	radius R, length h	dimensions a, b, c
Coordinates with origin at geometric center	r	r, z	x, y, z
Virtual neutron density relative to average value	$(\pi^2/3)(R/\pi r)\sin(\pi r/R)$	$2.3163\, J_0(2.4048\, r/R) \cdot (\pi/2)\cos(\pi z/h)$	$(\pi/2)\cos(\pi x/a)(\pi/2)\cos(\pi y/b)(\pi/2)\cos(\pi z/c)$
This ratio at center	$(\pi^2/3) = 3.290$	$2.3163\,\pi/2 = 3.639$	$(\pi/2)^3 = 3.875$
Position factor, square of neutron density relative to average square	$(2\pi^2/3)(R/\pi r)^2 \sin^2(\pi r/R)$	$3.710\, J_0^2(2.4048\, r/R) \cdot 2\cos^2(\pi z/h)$	$2\cos^2(\pi x/a)\, 2\cos^2(\pi y/b)\, 2 \cdot \cos^2(\pi z/c)$
Position factor at center	$(2\pi^2/3) = 6.580$	7.420	8
Limits of a concentrically located volume for which weight factor is readily calculated	r from 0 to R'	r from 0 to R' z from -h'/2 to h'/2	x from -a'/2 to a'/2 y from -b'/2 to b'/2 z from -c'/2 to c'/2
Weight factor for this volume (see Fig.22.3.36 for graphical evaluation)	$(R'/R)-(1/2\pi)\sin(2\pi R'/R)$	$3.710(R'/R)^2[J_0^2(2.4048\,R'/R) + J_1^2(2.4048\,R'/R)]$ $(h'/h + \pi^{-1}\sin\pi h'/h)$	$(a'/a + \pi^{-1}\sin\pi a'/a) \cdot$ $(b'/b + \pi^{-1}\sin\pi b'/b) \cdot$ $(c'/c + \pi^{-1}\sin\pi c'/c)$
Position factor for absorber distributed over the pile with concentration at every point proportional to neutron density at that point	$\dfrac{\int(\sin u/u)^3 u^2 du}{\int(\sin u/u)^2 u^2 du}$ times $\dfrac{\int u^2 du}{\int(\sin u/u) u^2 du}$ $= \dfrac{0.9705(\pi^3/3)}{(\pi/2)\pi} = 2.033$	$1.675\ (4/3) = 2.234$	$\left[\dfrac{\int \sin^3 u\, du \int du}{\int \sin^2 u\, du \int \sin u\, du}\right]^3$ $= (4/3)^3 = 2.371$
Ratio of neutron-effective temperature to central temperature when temperature rise is proportional to neutron density	0.6177	0.6139	$(8/3\pi)^3 = 0.6116$

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along the axis, the loss in overall multiplication factor is therefore $3.710\ (\pi\, 30^2/\pi\, 510^2)\ 0.051 = 0.00066$. At a point half way out from the axis to the surface of the pile the square of the Bessel function has the value $J_o^2 = 0.449$. The loss in overall reactivity is now $0.00066 \times 0.449 = 0.00029$.

22.3.25 Hanford control rods

EXAMPLE. How effective are the various control rods in the Hanford pile? This structure with 2004 tubes loaded with metal is approximately equivalent to a bare pile of dimensions, a = 1060 cm, b = 1060 cm, c = 760 cm. The control rods are 9 in number. They move parallel to the x-axis and are located on a square lattice at the points y = 0, ± 128 cm, z = 0, ± 162 cm. Each rod may be considered to act in the sense of the principle of equivalence on a zone of cross section 128 cm x 162 cm. The individual rods are so constructed that each reduces the local multiplication factor within the zone of action by the amount $\delta k_{local} = 0.031$, as indicated in the first entry of Table 22.3.12. We are interested in knowing the effect on the overall multiplication factor of insertion of the central rod alone; of pushing in only one of the corner rods; and, if possible, the control obtained by driving in all rods together. Each rod may be considered to act in the sense of the principle of equivalence on a zone of cross section 128 cm x 162 cm. This zone is sufficiently large in comparison with the size of the pile that it does not appear entirely reasonable to apply the concept of position factor. Instead we calculate the weight factor of each zone in order to estimate the effectiveness of the rod contained within that zone.

Effect of Central Rod

0.239, integral of $\cos^2 \pi y/b$ from y = -64 cm to y = + 64 cm relative to value of same integral over whole range, 1060 cm, of y (y component of weight factor).

0.402, integral of $\cos^2 \pi z/c$ from z = -81 cm to z = + 81 cm relative to value of same integral over whole range, 760 cm, of z (z component of weight factor).

0.098, product of factors so far gives weight factor of zone of equivalence of central rod.

0.031, reduction of local multiplication factor in this zone according to 22.3.12.

0.0030, product, reduction in overall multiplication factor of pile due to complete insertion of central rod.

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SURVEY OF CONTROL THEORY 22.3.25

Effectiveness of Corner Control Rod

0.206, integral of $\cos^2$ from y = 64 cm to y = 192 cm relative to integral over whole range of y (y component of weight factor).

0.259, integral of $\cos^2$ from z = 81 cm to z = 243 cm relative to integral over whole range of z (z component of weight factor).

0.053, product of preceding two quantities represents weight factor of zone of action of corner control rod.

0.031, total reduction of local multiplication factor within zone of action.

0.0016, product, reduction in overall multiplication factor due to complete insertion of corner control rod.

Effectiveness of All Rods

Proceeding in the manner just described, we find the effectiveness of each of the control rods acting individually:

Number of rods	Location	Effectiveness of one such rod	Total effect if actions were additive
1	central	0.0030	0.0030
4	corner	0.0016	0.0066
2	z = 0, y = ± 128	0.0026	0.0053
2	y = 0, z = ± 162	0.0019	0.0038
9			0.019

When we insert into the pile 9 rods as absorbent as those just considered, we alter so much the distribution of neutron density that we are no longer entitled to use the method of analysis just described. The action cannot be taken as proportional to the square of the original value of the virtual neutron density. A more complete analysis is necessary and is presented later in this section. The total loss in k in introducing all the rods together is there estimated to be 0.017. The small difference between this figure and the total effect, 0.019, of the rods taken individually provides a measure of the so-called "shadowing effect" exerted by one rod or another. This effect is discussed in further detail below. Aside from such fine details of control theory we are evidently in a satisfactory position to calculate the effectiveness of individual control rods thrust completely through a pile.

Partial insertion of a control rod into a pile provides the most practical means yet discovered to obtain fine control of the chain reaction. The precision of control is especially good when the regulating rod enters only a small distance, s, into the effective portion of the

22.3.26
Rod inserted only slightly
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structure. The neutron density vanishes at the effective surface of the pile and reaches at the tip of the rod a value approximately proportional to the distance, s. The square of the neutron density goes as s^2. The integral of this quantity along the inserted length of the rod is therefore proportional to s^3. The control can be made arbitrarily fine by making s sufficiently small.

22.3.27 Partial insertion along diameter

For a more precise estimate of the degree of control in the case of partial insertion, consider the case of a bar of absorbent substance inserted to a depth, s, along the diameter of a cylindrical pile of height, h. The position factor for each infinitesimal element of the rod is obtained from the fifth row of Table 22.3.22. Here the Bessel function is to be evaluated at the point $r = |R - s|$. We suppose that the zone of equivalence of the bar has been chosen to be small in comparison with the size of the pile. Then the weight factor of this zone is

$$\underline{2}\cos^2(\pi z/h)\ \left(\underline{3.710}\int_0^s J_0^2\,ds\right)\underbrace{\left(\begin{matrix}\text{cross section of}\\ \text{zone of equivalence}\end{matrix}\right)/\pi R^2 h}\qquad (22.3.27.a)$$

Volume factor to convert position factor to weight factor

and the loss in overall multiplication factor is

$$\delta k = \frac{7.420}{2.4048\pi}\,\delta k_{\text{local}}\,\frac{\left(\begin{matrix}\text{cross section of}\\ \text{zone of equivalence}\end{matrix}\right)}{Rh}\cos^2(\pi z/h)\int_{x\,=\,2.4048(1-s/R)}^{x\,=\,2.4048} J_0^2(x)\,dx \qquad (22.3.27.b)$$

Here x is an abbreviation for the argument, 2.4048 r/R, of the Bessel function. The lower limit in the integral applies when the rod comes short of reaching the axis of the pile. When it penetrates beyond this point, the integral must be changed in an obvious manner. Values of the integral may be read directly from Fig. 22.3.28.

22.3.29 Example of diametral insertion

EXAMPLE. The cadmium rod of example (22.3.24) reduced the overall multiplication factor by 0.00066 when inserted along the axis of a cylindrical pile 510 cm in radius. We now insert the same rod along a diameter lying in the median plane of the pile. The effective height of the cylinder, which previously made no difference, is now taken to be 760 cm, corresponding to the designed loading of the Hanford piles. How much is the loss in overall multiplication factor due to (a) complete insertion of the rod? (b) insertion by 100 cm? (c) increase of insertion from 500 cm to 520 cm? The rod reduces the local multiplication factor by 0.051 in a zone of equivalence 30 cm in radius. Therefore, the overall loss in k is

$$\delta k = \frac{7.420}{2.4048\pi}\,0.051\,\frac{\pi(30\text{ cm})^2}{510\text{ cm}\ \ 760\text{ cm}}\int J_0^2\,dx$$

$$= 3.65\times 10^{-4}\int J_0^2\,dx \qquad (22.3.29.a)$$

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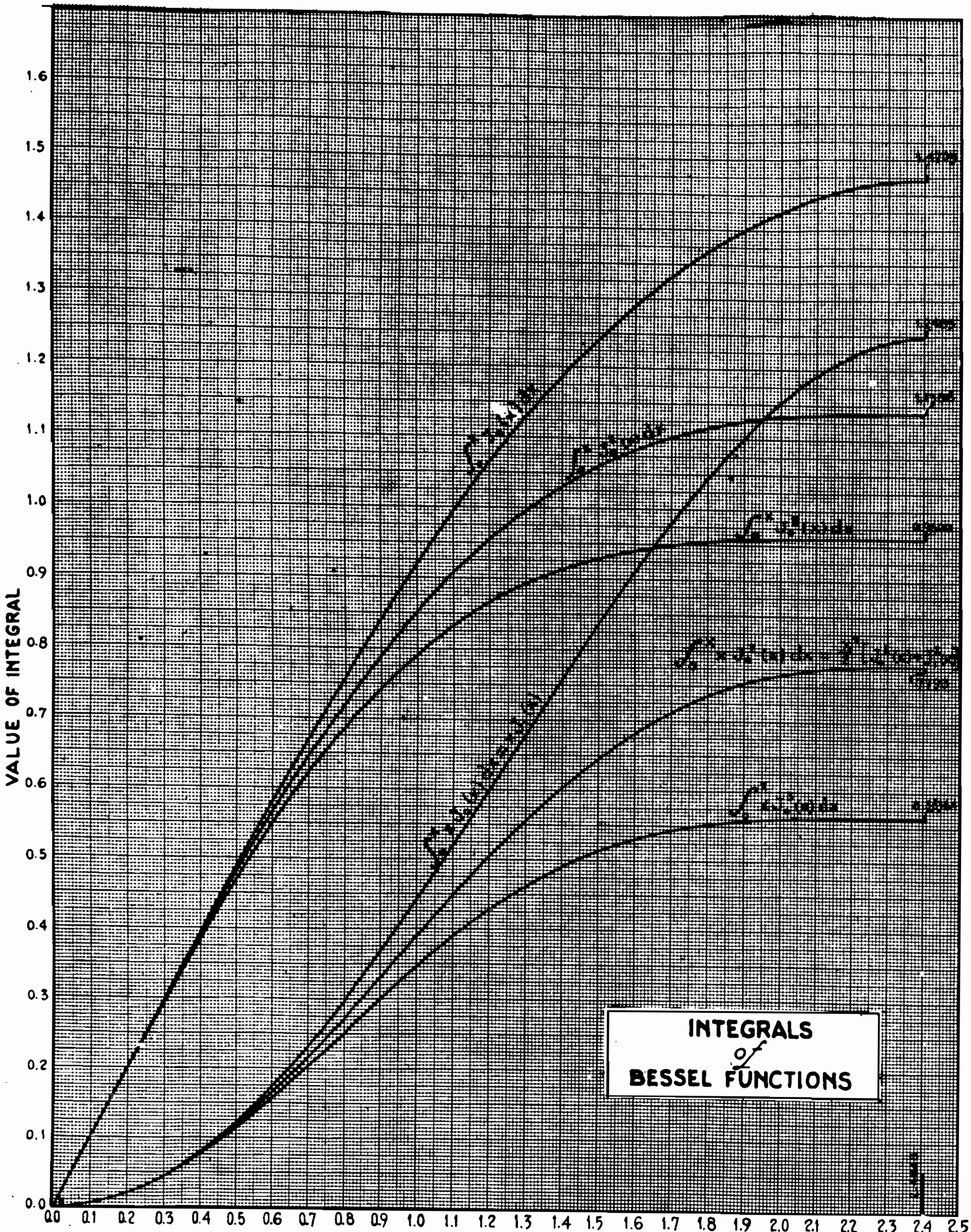
FIGURE - 22.3.28



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A.T.M., H.L.C., 1943-1944.

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This formula is evaluated with the aid of Fig. 22.3.28:

Insertion	Range of x	$\int J_o^2(x)dx$	δk
half way	0 to 2.4048	1.139	4.15×10^{-4}
complete		2.277	8.30×10^{-4}
100 cm	2.4048(1-100/510) to 2.4048	1.139-1.129	3.6×10^{-6}
500cm to 520cm	twice from 0 to 2.4048(10/510)	0.094	3.4×10^{-5}

Two features of the results deserve notice. First, 20 cm of motion of the rod with tip near the center of the pile are roughly ten times as effective as the initial 100 cm of travel. Second, the effect of complete diametral insertion, 8.3×10^{-4}, is greater than that of complete axial insertion, 6.6×10^{-4}, for a pile of the given shape. A closer examination of this point shows that the relative effectiveness of the two positions is the other way around for a cylindrical pile when the ratio of height to diameter exceeds 2.277/2.4048 = 0.945.

Partial introduction of a control rod parallel to the axis of a cylindrical reactor, or parallel to any axis of a rectangular pile, alters the multiplication factor by an easily calculated amount. The virtual neutron density varies with distance of insertion, s, in proportion with the expression $\sin \pi s/h$. The effectiveness varies in proportion to the integral of the square of this expression. Noting that $\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4} \sin 2x$, we have the result: 22.3.30 Partial insertion parallel to axis

$$\frac{\begin{pmatrix}\text{effectiveness of}\\ \text{partial insertion}\end{pmatrix}}{\begin{pmatrix}\text{effectiveness of}\\ \text{complete insertion}\end{pmatrix}} = (s/h) - (1/2\pi)\sin(2\pi s/h) \quad (22.3.30.a)$$

The expression on the right hand side of this equation is plotted in Fig. 22.3.31 and illustrates in graphic form the variation of control power over the range of motion of a rod.

In evaluating the effectiveness of control rods we have thus far continually used the principle of proportionality with the square of the neutron density. An interesting check on this point is provided by unpublished work of Murray*. He has computed by the method of infinite series the effectiveness of an ideal control rod and its dependence upon distance of insertion. His results are plotted in Fig. 22.3.31 and show on the whole a good agreement with the approximate theory of 22.3.30. 22.3.32 Check on approximate calculations

*Letter from F. H. Murray numbered MUC-JM-3, dated 1943 May 18.

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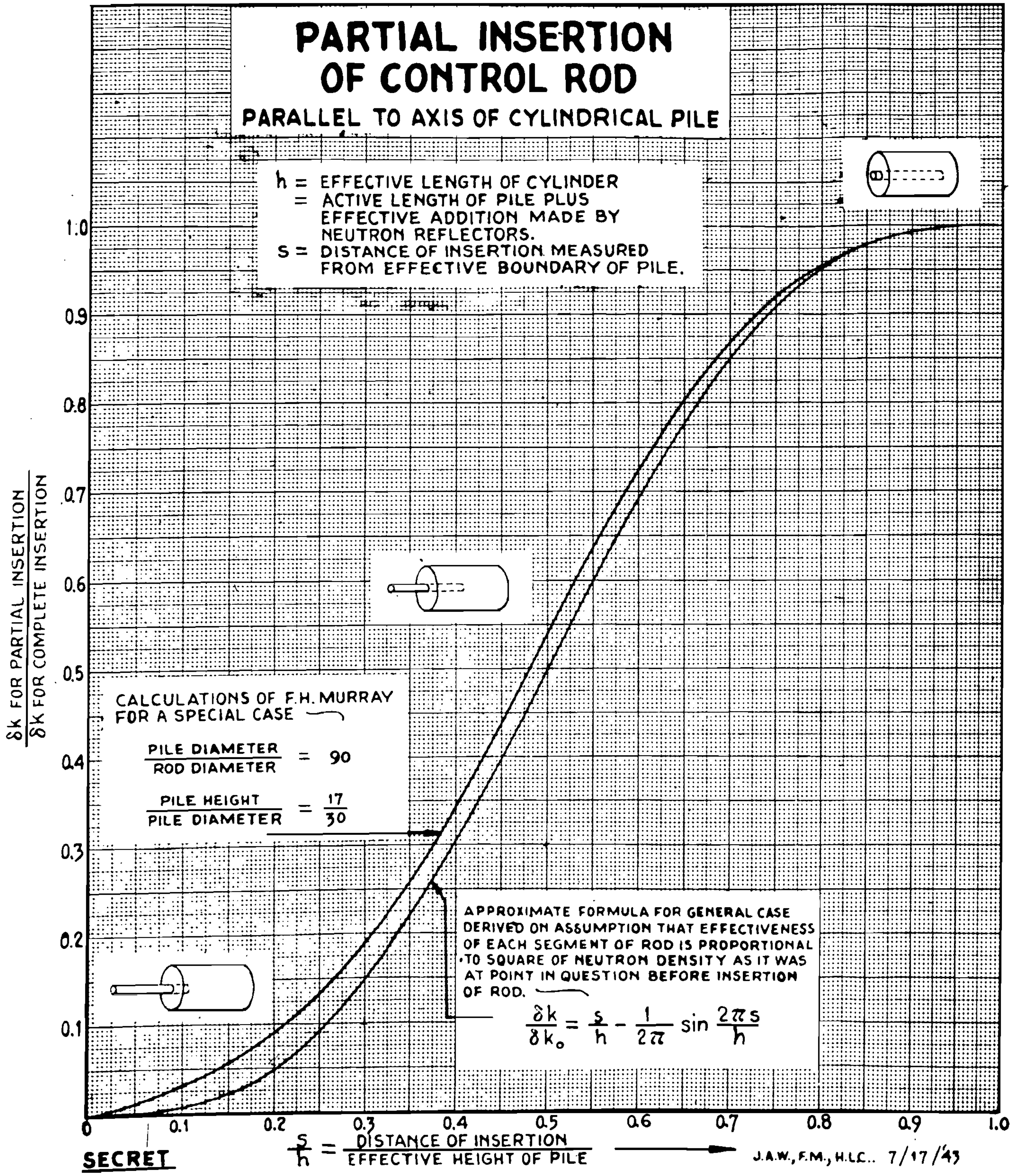
PARTIAL INSERTION
OF CONTROL ROD
PARALLEL TO AXIS OF CYLINDRICAL PILE
h = EFFECTIVE LENGTH OF CYLINDER
= ACTIVE LENGTH OF PILE PLUS EFFECTIVE ADDITION MADE BY NEUTRON REFLECTORS.
s = DISTANCE OF INSERTION MEASURED FROM EFFECTIVE BOUNDARY OF PILE.
CALCULATIONS OF F.H. MURRAY FOR A SPECIAL CASE
$\frac{\text{PILE DIAMETER}}{\text{ROD DIAMETER}} = 90$
$\frac{\text{PILE HEIGHT}}{\text{PILE DIAMETER}} = \frac{17}{30}$
APPROXIMATE FORMULA FOR GENERAL CASE DERIVED ON ASSUMPTION THAT EFFECTIVENESS OF EACH SEGMENT OF ROD IS PROPORTIONAL TO SQUARE OF NEUTRON DENSITY AS IT WAS AT POINT IN QUESTION BEFORE INSERTION OF ROD.
$\frac{\delta k}{\delta k_o} = \frac{s}{h} - \frac{1}{2\pi} \sin \frac{2\pi s}{h}$
$\frac{\delta k \text{ FOR PARTIAL INSERTION}}{\delta k \text{ FOR COMPLETE INSERTION}}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\frac{s}{h} = \frac{\text{DISTANCE OF INSERTION}}{\text{EFFECTIVE HEIGHT OF PILE}}$
J.A.W., F.M., H.L.C.. 7/17/43

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22.3.33 Direction of deviations from approximate theory

Whether the accurate curve for control rod effectiveness will lie above the approximate curve, as in Fig. 22.3.31, or below it, depends entirely on the relationship between the size of the control rod and the radius and height of the pile. In either case the inaccuracy of the approximate theory is due to the distortion of the original neutron distribution by the absorbent. Consider first the case where the length of the cylinder is great in comparison with its diameter. Insertion of the rod half way into the pile lowers the neutron density in the first half of the cylinder relative to its value in the second half. Movement of the rod the rest of the way into the reactor will therefore lower k more than the first part of the stroke. This case is opposite to the one considered by Murray. There the diameter of the cylinder is great in comparison with the length. The partial entrance of the rod pushes the maximum of the neutron distribution out to a ring far away from the axis of the pile. As the rod executes the last half of its motion, it therefore finds itself in a zone of lowered neutron density. No formula is available for the general case. However, the results shown in Fig. 22.3.31 are sufficient to permit an estimate of the order of magnitude of the effect in cases of interest.

22.3.34 Role of experimental calibration

Useful in designing the control system of a pile, the theory just described shows also that the actual functioning of the rods will be complicated by effects such as the shadowing of one absorbent by another or, as in the preceding paragraph, the interaction between different portions of the same rod. In addition we have to expect slight fluctuations in effectiveness due to the lattice structure of the pile itself. The loading of an operating pile will also ordinarily deviate from an ideal geometrical pattern. Precise determination of the relation between reactivity and control rod position in an operating pile is therefore generally done experimentally along the lines described in Chapter 14. The few absolute comparisons so far made between theory and experiment are in reasonable accord (22.5).

22.3.35 Zoning of pile materials

In addition to control rods, certain other neutron absorbing materials exert a small enough effect on the neutron distribution so that they can be studied by the foregoing approximate methods. One instance is the impurity which makes one brand of graphite better than another. There are obvious advantages in putting the better graphite into parts of the pile where it will do the most good. The magnitude of the gain can be calculated in a number of simple cases by the curves of Fig. 22.3.36, due to Morrison, Stephenson and Weinberg*. Al illustrative example is given in the caption.

22.3.37 Position factor for self-poisoning

Initial poisoning or self-promotion of the chain reaction by the products of neutron capture is another instance where we can neglect the distortion in the neutron distribution produced by the change. The magnitude of the local change in reactivity is proportional to the neutron density, and the position factor is proportional to the square of the neutron density. Consequently, the overall effect on k is measured by

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*P. Morrison, J. Stephenson and A. M. Weinberg, CP-761, Piles of Varying Materials, 1943 July 6.

WEIGHT FACTOR

FOR ABSORBENT DISTRIBUTED OVER A CORE CONCENTRIC WITH THE PILE AND GEOMETRICALLY SIMILAR TO IT.

Applicable when properties of core and pile are sufficiently similar that distribution of neutron density is only slightly modified. Curves adapted from Morrison, Stephenson and Weinberg, CP-761.

GENERAL EQUATION FOR OVERALL EFFECTIVE K

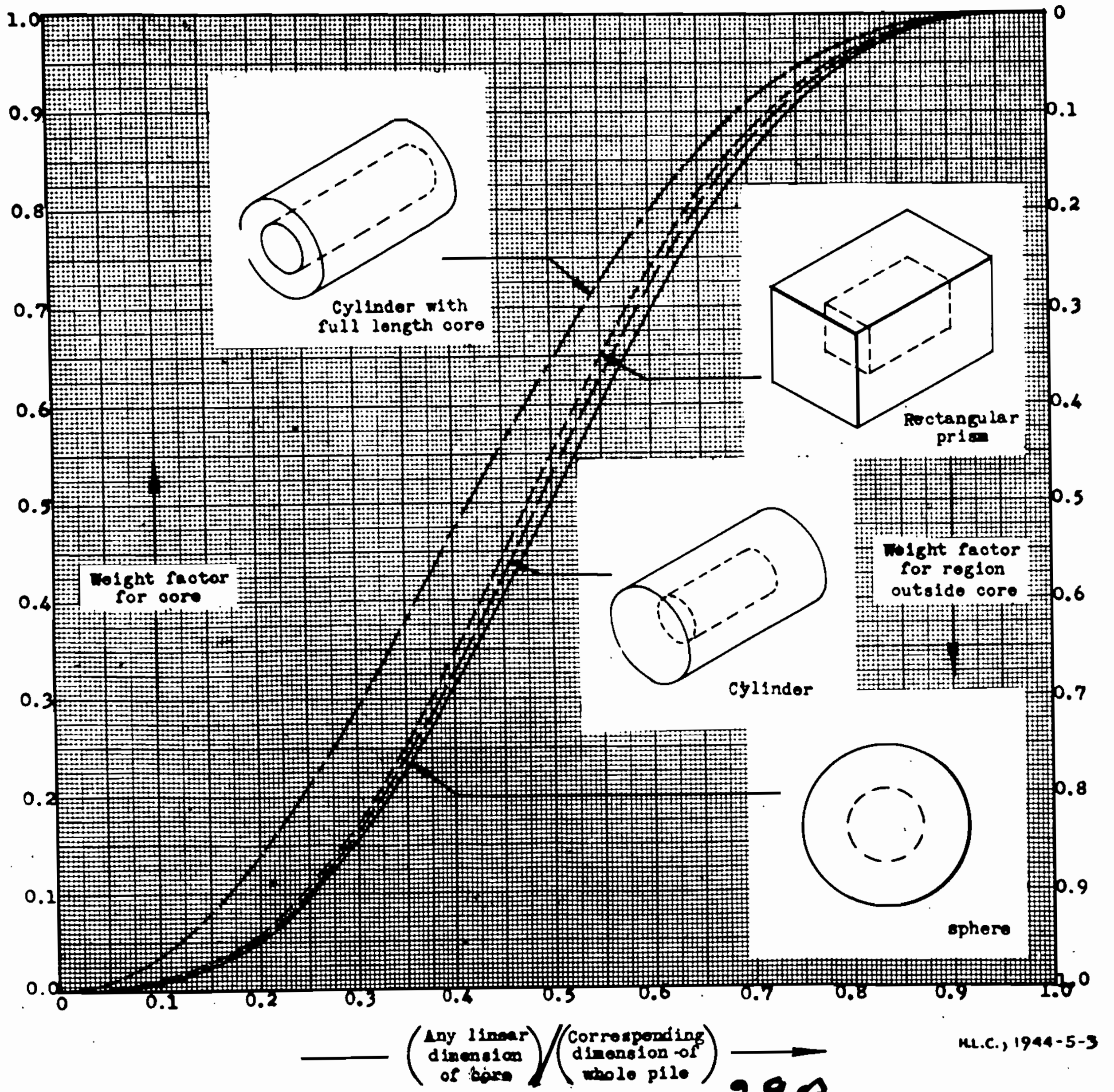
$$\frac{\left(\text{local k for core}\right) - \left(\text{overall effective k for pile}\right)}{\left(\text{migration area for core}\right)}\left(\text{weight factor for core}\right) + \left(\begin{array}{c}\text{corresponding expression for}\\ \text{region outside core}\end{array}\right) = \left(\begin{array}{c}\text{critical buckling calculated from}\\ \text{effective external dimensions of pile}\end{array}\right)$$

SIMPLIFICATION WHEN MIGRATION AREA IS NEARLY SAME IN BOTH REGIONS

$$\left(\begin{array}{c}\text{average local k}\\ \text{for whole pile}\end{array}\right) = \left(\begin{array}{c}\text{local k}\\ \text{for core}\end{array}\right)\left(\begin{array}{c}\text{weight factor}\\ \text{for core}\end{array}\right) + \left(\begin{array}{c}\text{local k out-}\\ \text{side core}\end{array}\right)\left(\begin{array}{c}\text{weight factor for}\\ \text{region outside}\end{array}\right)$$

EXAMPLE

Pile is a rectangular prism 760 cm x 1060 cm x 1060 cm. Core having eighth this volume is prism 380 cm x 530 cm x 530 cm. Local k in core is 1.030, outside is 1.040. Average local k for whole pile is 1.030 x 0.548 + 1.040 x 0.452 = 1.0345.



H.L.C., 1944-5-3

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the volume integral of the third power of the neutron density:

$$\frac{\left(\begin{array}{l}\text{Effect on overall multiplication}\\ \text{factor due to reaction products}\\ \text{as actually distributed in pile}\end{array}\right)}{\left(\begin{array}{l}\text{Effect on overall multiplication}\\ \text{factor due to same amount of re-}\\ \text{action products if they were dis-}\\ \text{tributed uniformly through the pile}\end{array}\right)} = \frac{\int n^3 d\,(\mathrm{vol}) \int d(\mathrm{vol})}{\int nd\,(\mathrm{vol}) \int n^2 d\,(\mathrm{vol})} \qquad (22.3.37.a)$$

The position factor on the right hand side of this equation appears in Table 22.3.22 for piles of simple geometrical form. For example, we find that the fission products produced in a bare rectangular pile are 2.37 times as effective in lowering the multiplication factor as they would be if distributed uniformly through the reactor.

Our last instance of an influence on reactivity distributed through the pile is the temperature effect. Under most circumstances the rise in temperature of the metal in an operating pile can be taken as proportional to the power output or neutron density at the point in question. Then the overall loss in reactivity due to heating of the metal can be expressed in terms of the temperature coefficient of the multiplication factor and the temperature of the pile at the point of maximum power output in the following way: 22.3.38 "Neutron - effective temperature"

$$\left(\begin{array}{c}\text{Loss in multiplication factor}\\ \text{due to temperature elevation}\\ \text{of metal}\end{array}\right) = \left(\begin{array}{c}\text{Change in multiplication}\\ \text{factor per unit temperature}\\ \text{rise in metal}\end{array}\right)$$

$$\left(\begin{array}{c}\text{Temperature elevation of metal}\\ \text{at point of maximum power output}\end{array}\right) \frac{\int n^3 d\,(\mathrm{vol})}{n_{max} \int n^2 d\,(\mathrm{vol})} \qquad (22.3.38.a)$$

The product of the two expressions in the last line may be called the "neutron-effective temperature rise" of the metal. The ratio of neutron-effective temperature to central temperature is given in Table 22.3.22 for piles of simple form. For example, let a rectangular pile run without control to the point where the central metal temperature has risen by 100°C. This operation will lower the reactivity by the same amount as uniform heating of the metal through 61°C.

We have so far considered absorbers which might be located anywhere in the pile, and which did not greatly distort the distribution of neutron density. Then we could treat the effect of the absorbent as proportional to the square of the original value of the neutron density. Now we investigate controls which have a large effect on the reactivity of the pile, and which do not allow that simple principle of calculation. An example is a safety system designed to lower the overall multiplication factor 2 or 3 percent. We will not go into the difficult general case where strong controls are disposed through the pile in arbitrary locations. Instead we shall analyze here the case of greatest importance, where the safety rods are put in positions of maximum effectiveness. In this 22.3.39 Contrast weak and strong controls

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22.3.39

special case the theory of control again takes a relatively simple form.

22.3.40 Optimum disposition of absorbent

Granted that a definite lowering of the multiplication factor is to be achieved, how should controls be disposed in the pile so as to accomplish this result most economically? A discussion of this question is most easily opened by considering the neutron absorbing material to be in the form of a fine powder, such as boron. The effect of a small amount of boron on the reactivity of the pile will be proportional to the square of the neutron density at the point where this absorbent is introduced. Therefore, the first dose of neutron absorbing material will best be put at the center of the pile. If more control is desired, more boron will be located near this point. Soon the amount of boron will be sufficient appreciably to lower the neutron density at the center of the pile. Enough boron must, however, not be introduced at the center of the pile to lower the neutron density there in comparison to its value in the immediately surrounding region. In fact, the boron must always be introduced in such a way as to maintain the neutron density constant over the region which it occupies. If for any reason the neutron density should vary over the poisoned portion of the structure, then a more effective control will result from slight redistribution of boron in which some of it is taken out of the region or lower neutron density and put into the region of higher neutron density. This principle of disposition evidently applies no matter how great is the desired degree of control.

22.3.41 "Region of control"

We arrive at the following picture of the distribution of neutrons and neutron absorbing material in a pile whose multiplying properties were originally everywhere the same. The neutron density vanishes at the surface of the pile and increases as one goes inward, reaching at a certain surface a limiting value. Everywhere inside this surface the neutron density is constant. All the boron lies within this zone to which we therefore give the name "region of most effective control". In this region of control the neutron density is not buckled at all. Consequently, the local multiplication factor there has a constant value. Before introduction of the boron the local k throughout the major portion of the pile will generally also have had a constant value. Thus the change in local multiplication factor will be constant through the region of control, and the boron will be uniformly distributed in this zone. Both the concentration of boron and the size of the optimum region of control will depend upon the amount of reactivity to be compensated.

22.3.42 Concept applies also to rods

When a large number of fine wires or rods take the place of the boron, then for minimum consumption of material they should likewise be limited to the region of most effective control and should be uniformly spaced within it. If instead we pierce the whole structure with rods, we will require a greater amount of neutron absorbent material to get the same degree of control.

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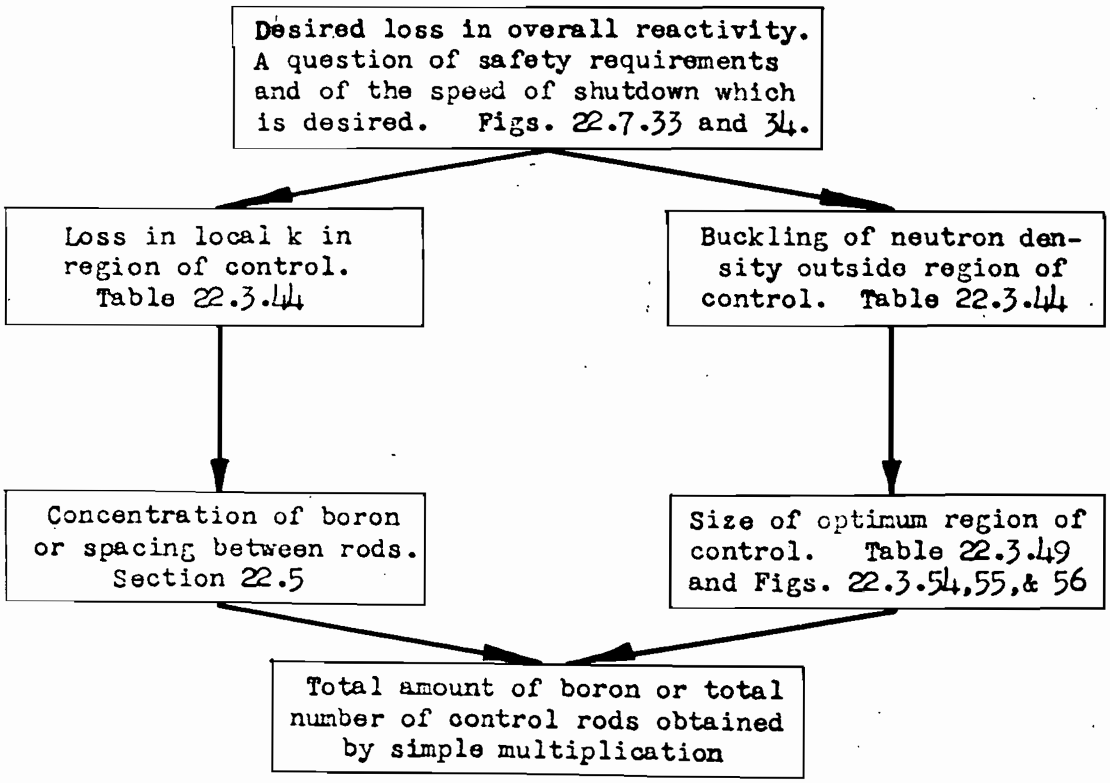
22.3.43 Outline solution of problem of optimum control

The size of the optimum region of control and the proper concentration of absorbent material within it are found along the lines indicated in the following diagram:



The first two steps in the analysis are determination of the loss in local k within the region of control and the buckling of the neutron density outside the region of control, as described in the following table.

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Table 22.3.44 SURVEY OF REQUIREMENTS FOR REGION OF CONTROL

22.3.44 Requirements for region of control

Undesired reactivity to be compensated by absorbers located inside a certain portion of the pile known as the region of control.

This survey analyzes various situations which may occur into terms of (1) local k to be compensated within region of control and (2) buckling which remains outside region of control.

From (1) deduce (3) the concentration of boron required within the region of control (Eq. 22.5.18.d), or the required spacing between control rods in this portion of the pile (Figures 22.5.23).

From (2) deduce (4) the required size of the region of control with the aid of Figures 22.3.54, 22.3.55, and 22.3.56.

From (3) and (4) together determine (5) either the total mass of boron required, or the total number of control rods.

Notation used in this summary	
Factor of multiplication in one generation	k
Buckling of neutron density before absorbent enters, a quantity completely determined by dimensions of pile	B_o
Buckling within region of control after absorbent enters	0
Buckling outside region of control after absorbent enters	B_1
Area of migration of neutrons in one generation	A

CASE I. Pile in steady operation without controls.	
Fraction of neutrons lost by leakage	$A\ B_o$
Value of overall multiplication factor	1
Therefore value of local multiplication factor is	$1 + A\ B_o$

CASE II. Reactivity changes by same amount in all parts of pile due to loss of cooling fluid, decrease of barometric pressure or similar effect. Controls do not enter yet. Denote increase in local k by δk_{local}, increase in migration area by δA.

Value of local multiplication factor after change	$1 + A\ B_o + \delta k_{local}$
Fraction of neutrons lost by leakage	$(A + \delta A)\ B_o$
Value of overall k for pile after change	$1 + \delta k_{local} - B_o\ \delta A$

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[redacted] 33B

[redacted]

Table 22.3.44 - Con'd.

Excess k available for continually increasing power	$k_e = \delta k_{local} - B_o \delta A$
Growth of power output as function of k_e	Fig. 22.7.34
CASE III. Uniform increase in reactivity followed by injection into whole pile of uniform concentration of absorbing material just sufficient to keep reaction at constant level. Procedure inefficient because absorbent near fringes of pile has little effect on reactivity.	
Required concentration of absorbing material expressed as number of boron nuclei per unit volume or number of control rods per square meter or in other suitable units	c
Change in local multiplication factor per unit change in concentration of absorbent, as obtained from Eq.22.5.18.d or Figure 22.5.23. (a negative quantity)	$\frac{dk_{local}}{dc}$
Change in migration area per unit change in concentration of absorbent.	$\frac{dA}{dc}$
Value of local multiplication factor after uniform change in reactivity and subsequent insertion of absorbent	$1 + AB_o + \delta k_{local} + c\,\frac{dk_{local}}{dc}$
Fractions of neutrons lost by leakage under same conditions	$(A + \delta A + c\,\frac{dA}{dc})B_o$
Overall excess multiplication factor, to be adjusted to zero by suitable choice of concentration	$(\delta k_{local} - B_o\delta A) + c(\frac{dk_{local}}{dc} - B_o\,\frac{dA}{dc})$
Expression for required concentration of absorbent. In many piles the buckling is sufficiently small that the terms containing B_o may be neglected	$c = \frac{(\delta k_{local} - B_o\delta A)}{(-\frac{dk_{local}}{dc} + B_o\frac{dA}{dc})}$
CASE IV. Uniform increase in reactivity followed by insertion into region of control of absorbent material just sufficient in amount to maintain reaction at steady level.	
Value of local k in region of control after loss of coolant or similar change and subsequent insertion of absorbent	1
Depression in local k required of absorbent	$\delta k_{abs} = A\,B_o + \delta k_{local}$
Concentration of boron or spacing between control rods in region of control determined by δk_{abs}	**Eq. 22.5.18.d and Figure 22.5.23.**

[redacted] 34B

Table 22.3.44 - Con'd.

Migration area in region of control is altered but precise value is irrelevant because no leakage occurs from region of constant buckling	
Value of local k outside region of control	$1 + A\ B_o + \delta k_{local}$
Leakage factor outside region of control	$A\ B_o + \delta k_{local}$
Buckling of neutron density outside region of control	$B_1 = \dfrac{A\ B_o + \delta k_{local}}{A + \delta A}$
Required size of region of control completely determined by B_1 and dimensions of pile	Figures 22.3.54 22.3.55 22.3.56

CASE V. Uniform increase in reactivity followed by insertion into region of control of absorbent material just sufficient in amount to leave overall excess multiplication factor equal to k_e. In the case of practical importance, k_e is to be negative, of the order of -0.5% or -1.0%, to guarantee quick shutdown of pile. Rate of shutdown given in terms of k_e by Fig. 22.7.34.

Average value of buckling in region of control for case of optimum disposition of absorbent	0
Value of migration area in region of control	Irrelevant
Leakage of neutrons from region of control	0
Value of local k desired in region of control after loss of water and subsequent insertion of absorbent	k_e
Depression in local k in region of control required from absorbing material.	$\delta k_{abs} = A\ B_o + \delta k_{local} - k_e$
Concentration of boron or spacing between control rods determined by δk_{abs}	Figure 22.5.23
Fraction of neutrons available for leakage in portions of pile outside zone of control	$A\ B_o + \delta k_{local} - k_e$
Migration area outside zone of control	$A + \delta A$
Buckling of neutron density outside the zone of control	$B_1 = \dfrac{A\ B_o + \delta k_{local} - k_e}{A + \delta A}$
Required size of region of control completely determined by B_1 and dimensions of pile	Figures 22.3.54 22.3.55 22.3.56

Table 22.3.44 - Con'd.

CASE VI. How much overall excess multiplication factor will result for an arbitrary size and shape of the region of control and an arbitrary - but constant - concentration of absorbers in this region? No general solution of this problem is available, but the problem itself can be translated into a purely mathematical form as follows.

Local multiplication factor and migration area outside zone of control determined by properties of pile. Chapters 15 and 16.	k'_{local}, A'
Local multiplication factor and migration area inside zone of control determined by properties of pile and concentration of boron or spacing between control rods. Eq. 22.5.18.d or Figures 22.5.23.	k''_{local}, A''
Overall excess multiplication factor to be found in terms of above four quantities, dimensions of pile, and dimensions of region of control	k_e
Buckling of neutron density outside region of control	$B = B' = \dfrac{k'_{local} - 1 - k_e}{A'}$
Buckling inside region of control	$B = B'' = \dfrac{k''_{local} - 1 - k_e}{A''}$
Conditions determining variation of neutron density through pile. These conditions can be satisfied all at once only when k_e has one particular value. Determination of this value solves the problem	(a) $\dfrac{\partial^2 n}{\partial x^2} + \dfrac{\partial^2 n}{\partial y^2} + \dfrac{\partial^2 n}{\partial z^2} + Bn = 0$ (b) $n = 0$ at boundaries of pile (c) n positive throughout interior of pile
Growth of power output given as a function of k_e	Fig. 22.7.34

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Having determined as indicated in the preceding table the required loss of local multiplication factor within the zone of control, we are in a position to deduce the necessary concentration of boron or the proper spacing between control rods. For a qualitative treatment of this point it is sufficient to inspect the survey of typical control devices presented in Table 22.3.12. A more detailed analysis may be carried out by the methods of Sections 22.4 and 22.5. 22.3.45 Concentration of absorbent or of rods

The optimum size of the region of control is not given directly in survey Table 22.3.44 but follows from the value given there for the buckling, B_1, of the neutron density outside this region. This quantity, together with the dimensions of the pile, allows us to find the proper size for the region of control. Table 22.3.49 and Figures 22.3.55 and 56 give the relationship between these two quantities in cases where the mathematical analysis may be carried through without undue complications. 22.3.46 Peripheral buckling determines size of region of control

The connection between peripheral buckling, B_1, and required size of the region of control follows in a straight-forward way from these principles: 22.3.47 Principles of determination

(1) The neutron density vanishes at the effective boundaries of the pile.

(2) The neutron density is positive everywhere within the pile.

(3) The neutron density has a constant magnitude within the region of control.

(4) The neutron density and its first derivative are continuous throughout the pile.

(5) The neutron density satisfies outside the region of control the equation:

$$\partial^2 n/\partial x^2 + \partial^2 n/\partial y^2 + \partial^2 n/\partial z^2 + B_1 n = 0 \qquad (22.3.47.a)$$

Only for one very particular size and shape of the region of control is it possible to obtain a representation of the neutron density which will satisfy these conditions. In other words they determine not only the neutron density itself but also the size and shape of the region of control.

The solution of the mathematical problem just formulated is relatively simple when the pile has a form of a sphere or of a cylinder infinite along the axis or of a slab infinite in two directions. Symmetry gives the shape of the region of control in this instance and only the size remains to be found by application of the stated principles. The mathematical details are presented in brief form in Table 22.3.49. 22.3.48 Size of region for simple geometries

22.3.49
Solution for size in simple cases

Table 22.3.49. EXACT CALCULATIONS OF SIZE AND SHAPE OF OPTIMUM REGION OF CONTROL

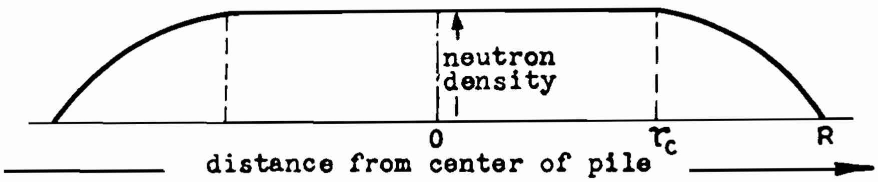
Size depends on:

B_o, buckling of neutron density throughout pile before control is applied. B_o is fixed by dimensions of pile.

B_1, buckling outside region of control after sufficient absorbent is introduced inside to reduce to zero the buckling there. The circumstances of control determine the required value of B_1 as indicated in Table 22.3.44.

Basis of calculations in text, paragraphs 22.3.47 to 22.3.48.

Application of calculations to spherical pile in Fig. 22.3.55, to cylindrical pile in Fig. 22.3.56.



Shape of pile	Slab infinite in two directions	Cylinder infinite in one direction	Sphere
Dimension of pile	half thickness R	radius R	radius R
Dimension of region of control	half thickness r_c	radius r_c	radius r_c
Neutron density before absorbent enters	$\cos(\pi r/2R)$	$J_o(2.4048\ r/R)$	$(R/\pi r)(\sin \pi r/R)$
Buckling B_o throughout pile before absorbent enters	$(1.5708/R)^2$	$(2.4048/R)^2$	$(3.1416/R)^2$
Buckling inside region of control after absorbent enters	0	0	0
Neutron density inside region of control after absorbent enters	1	1	1
Buckling B_1 outside region of control after absorbent enters determined as indicated in Table 22.3.44. For convenience in calculations, we define a new constant b in terms of B_1	$b^2 = B_1$	$b^2 = B_1$	$b^2 = B_1$

Expression for neutron density outside region of control which has buckling b^2 and has zero slope and unit magnitude at $r = r_c$	$\cos b\ (r - r_c)$	$(\frac{\pi b r_c}{2})[J_1(br_c)N_0(br) - N_1(br_c)J_0(br)]$	$(1/br)[\sin b(r-r_c) + br_c \cos b(r-r_c)]$
Condition that neutron density vanish at boundary of pile gives equation from which to find the radius r_c of region of control in terms of the dimensions of the pile and the known quantity $b^2 = B_1$.	$br_c = bR - \pi$	$\frac{J_1(br_c)}{N_1(br_c)} = \frac{J_0(bR)}{N_0(bR)}$	$br_c = \tan(br_c - bR)$ ($\equiv \tan\theta$, where θ is a convenient parameter in terms of which to do calculations)
Graphic presentation of solution	Not given	Fig. 22.3.55	Fig. 22.3.56

22.3.50 Two methods of approximation in general case

In cases where the shape of the region of control is not determined by symmetry considerations, the purely mathematical problem of finding the size and form of this zone is in general quite complicated. It is therefore fortunate that two methods of approximation exist, one of which gives a relatively accurate determination of the required size of the region of control without a precise limit of error, whilst the other gives a quite certain upper limit to the size. Both methods are studied most easily by considering a specific example. A pile having the shape of a rectangular prism is to be controlled by dropping into it absorbent rods which will pass completely through the structure from top to bottom. The vertical variation of the neutron density is represented by a cosine function with the same argument after the introduction of the control rods as before. The buckling of the neutron density in the vertical direction is unaltered. We have therefore only to consider a two dimensional problem: to describe the shape of the region of control by a curve drawn in a horizontal plane. When a large amount of control is desired this curve includes nearly the whole of the pile and has the shape of a rectangle only slightly rounded at the corners. In the opposite case where only a slight degree of control is required the region of optimum disposition is evidently one on which the neutron density has nearly its maximum value, an ellipse. It is difficult to find a set of curves which form the proper transition between the small ellipse and the large pseudo-rectangle. We therefore approximate the region of control by a rectangle. Between it and the boundaries of the pile is a band of constant width. A region of control of this shape is not quite the optimum. Consequently, the area of this rectangle must evidently be somewhat larger than the surface of the proper zone of control. Within the inner rectangle we reduce the buckling of the neutron density to zero by insertion

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of a sufficient concentration of absorbent material. By the line of reasoning outlined in Table 22.3.44 we know the buckling, B_1, outside this region. Let us take the width of this band to be $\pi/2B_1^{\frac{1}{2}}$. We shall see that this choice guarantees that the region of control will be big enough.

22.3.51 Certain upper limit to size of region of control

To prove that we have a certain upper limit to the size of the region of control, we shall show that the pile requires internal sources of neutrons to keep it operating. We shall locate these sources along the four diagonal lines which run from each corner of the rectangle of control to the corresponding corner of the pile. Everywhere within the pile except on these lines the neutron density will satisfy the conditions of 22.3.47, provided we write:

(1) $n = 1$ within inner rectangle.

(2) $n = \sin\left[B_1^{\frac{1}{2}}\left(\begin{matrix}\text{perpendicular distance from}\\ \text{nearest boundary of pile}\end{matrix}\right)\right]$ within peripheral band.

Even at the diagonals themselves the expressions for the neutron density in different regions join together continuously. Only the first derivative is discontinuous. Here is the important point. The sense of the discontinuities in the normal derivative of the neutron density is such as to indicate a net outward flux of neutrons from these diagonals. We conclude that sources are required to make the pile function. In the absence of such sources we therefore have a certain upper limit for the size of the region of control required to shut down the pile.

22.3.52 Possibility of improved upper limit

Wigner* has generalized the foregoing method of guaranteeing a safe size for the region of control. The pile is divided up into regions in each one of which the neutron density is represented by an expression that satisfies the conditions of 22.3.47. The expressions must be such that the boundary conditions at each line of join of two regions can only be satisfied by placing there a source of neutrons. Then it is certain that the pile will not function with a region of control of the selected size and shape. If it is impossible to choose the regions and to set up the solutions in such a way as to evade sinks at one or another boundary line, then the region of control is not big enough to insure safety and must be enlarged. By following out in detail this procedure of Wigner's it is possible in principle to improve to an arbitrary degree of precision on the rectangular approximation to the region of control. However, for present purposes the rectangular approximation will suffice.

22.3.53 Good approximation to size of region

When a reasonably accurate idea of the size of the region of control is preferred to a certain upper limit to its magnitude, then another method of analysis offers itself. To develop this method we go back to the fundamental equation for the virtual neutron density:

$$\partial^2 n/\partial x^2 + \partial^2 n/\partial y^2 + \partial^2 n/\partial z^2 + Bn = 0 \qquad (22.3.53.a)$$

*E.P. Wigner, private communication of unpublished work.

Here the buckling, B, is zero within the region of control and equal to B_1 outside. We multiply the equation through by n and integrate by parts, obtaining the relation:

$$B_1 = \frac{\int_{\text{whole pile}} \left[(\partial n/\partial x)^2 + (\partial n/\partial y)^2 + (\partial n/\partial z)^2\right] d\,(\text{vol})}{\int_{\substack{\text{peripheral}\\ \text{region}}} n^2 d\,(\text{vol})} \qquad (22.3.53.b)$$

It will be noted that the numerator on the right hand side does not explicitly contain any reference to the size of the region of control while the denominator contains this magnitude in a quite evident way. Imagine therefore that we know the proper expression for the neutron density or a good approximation to it. Then Eq. (22.3.53.b) provides a means to evaluate the required size of a region of control of any selected shape. Naturally, the better chosen the approximate expression for the neutron density and the more reasonable the assumed shape, the more accurate will be the size of the region of control calculated from in this way. To employ the present method of approximation to the greatest advantage, it is desirable that the assumed expression for the neutron density should have discontinuities neither in its functional value nor in its first derivative. Such a function is readily developed in our example of a rectangular pile with a rectangular region of control surrounded by a border of constant width, w. The approximate function, n, is chosen as follows:

(1) $n = 1$ within inner rectangle.

(2) $n = \sin\left[(\pi/2w)\left(\begin{matrix}\text{perpendicular distance from}\\ \text{nearest boundary of pile}\end{matrix}\right)\right]$ within peripheral band, except in corners.

(3) $n = \sin\left[(\pi/2w)\left(\begin{matrix}\text{perpendicular distance from}\\ \text{nearest boundary of pile}\end{matrix}\right)\right]$

$\sin\left[(\pi/2w)\left(\begin{matrix}\text{perpendicular distance from}\\ \text{nearest boundary of pile}\end{matrix}\right)\right]$ in corner sections of peripheral band.

It is evident that this choice for the function, n, satisfies our conditions of continuity. The integral (22.3.53.b) is now easily evaluated to give a relation between peripheral buckling and size of the region of control. The size calculated from this relation is generally smaller than that deduced by the more conservative method described in the previous paragraph. This point is illustrated by reference to Fig. 22.3.54 where there are given in graphical form results of the calculations just outlined. Accurate calculations for the cases of sphere and cylinder along the lines of Table 22.3.49 are likewise presented in Figs. 22.3.55 and 22.3.56. We conclude that we possess adequate means to determine the size of the region of control in problems of practical interest.

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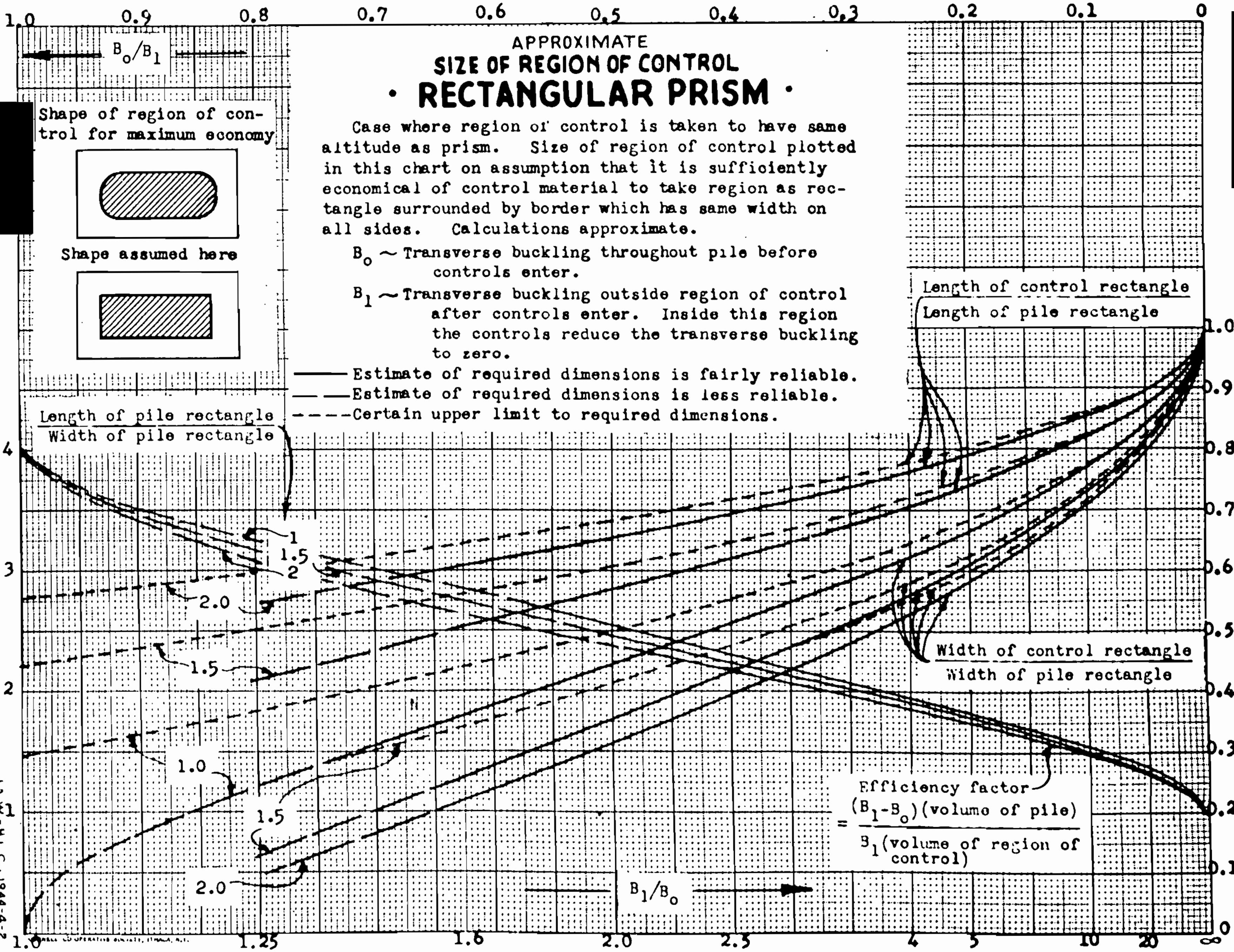

J.A.W.-H.L.C., 1944-4-2

FIGURE-22.3.54

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43B

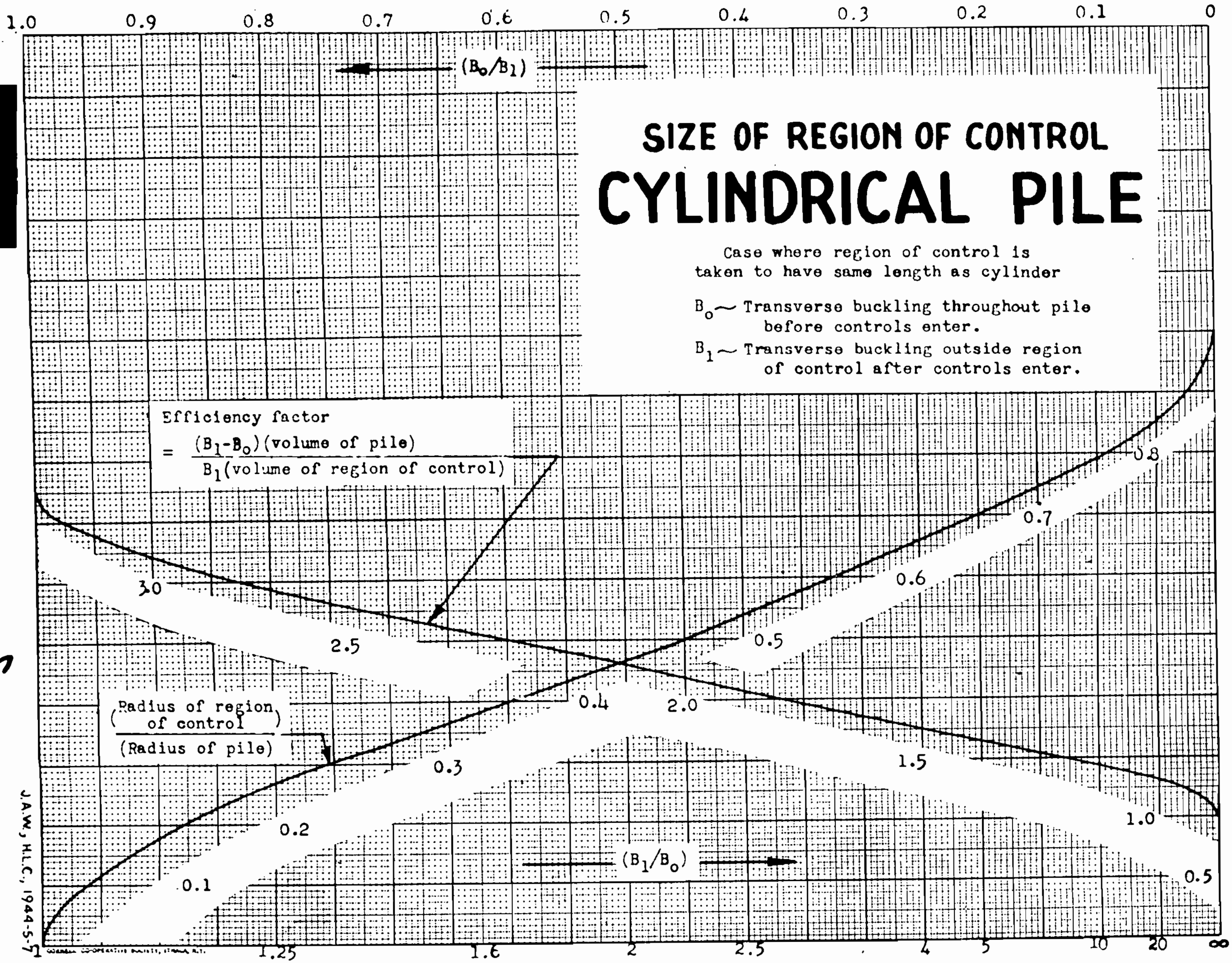

FIGURE-22.3.55

44 B

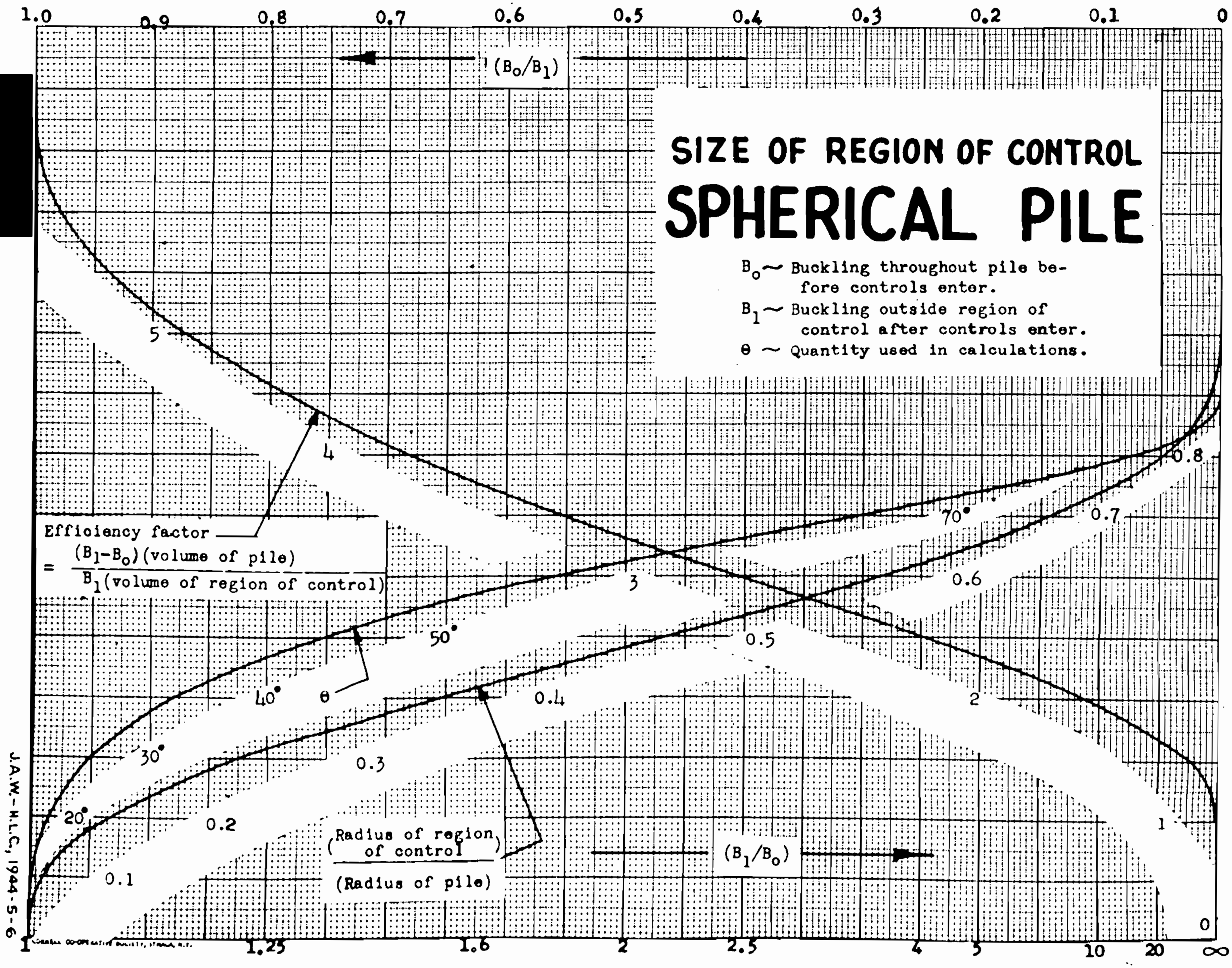

FIGURE-22.3.56

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22.3.57 Safety control of Hanford pile as example

It may provide a helpful summary of the considerations in this section to outline their application to the problem of quick shutdown of the Hanford water cooled pile. Experimental results available at the time of design indicated that loss of water by piping failure or otherwise would cause an increase in the local multiplication factor about 2.5 percent. It was considered necessary to be able to over-compensate such an increase in k with enough control material to give an overall multiplication factor less than unity by about 1.0 percent. Such a deficit, according to the curve of Fig. 22.7.34, will cause the fission power output to fall to 5 percent of its original value in 80 seconds. It was not certain at the time of design whether the pile would contain 1500 tubes or 2004 tubes. The larger number of tubes would call for a larger region of control but no greater or less concentration of absorbent within the region of control. The control system was therefore designed on the basis of 2004 tubes loading. Corrected for the contribution of the graphite reflector, the effective dimensions of the pile could be considered in a certain approximation to be those of a rectangular prism 30 + 700 + 30 = 760 cm long and 41 + 978 + 41 = 1060 cm square. The reactivity was assumed to be the same throughout the structure, with no allowance for central poisoning which might be introduced to give more nearly uniform power distribution. The control rods completely penetrate the pile. Therefore, the vertical component of the buckling remains unchanged and we have to consider only the horizontal components of the buckling.

22.3.58 First step in analysis of safety system

We model our analysis of the Hanford safety control system along the pattern of Table 22.3.44, CASE IV, as follows:

Quantity	Magnitude
Buckling, $(\pi/760)^2 + 2(\pi/1060)^2$, of neutron density before absorbent enters	$17.1\times10^{-6} + 17.6\times10^{-6}$ $= 34.7 \times 10^{-6}\,cm^{-2}$
Migration area	587 cm^2
Local k before loss of water (product of last two items plus unity)	1.0204
Assumed gain in local k on loss of water	0.0250
Local k assumed after loss of water	1.0454
Migration area after this loss	607 cm^2
Vertical component of buckling	$8.8 \times 10^{-6}\,cm^{-2}$
Fraction of neutrons lost by leakage out of top and bottom of pile, product of last two quantities	0.0053
Excess of local k over this leakage factor	1.0401
Overall multiplication factor for whole pile to give quick shutdown	0.9900
Local k to be compensated within region of control; given by difference	0.0501
Transverse leakage factor required in peripheral zone	0.0501
Migration area there	607 cm^2
Transverse buckling of neutron density required outside zone of control. (quotient of last two items)	$82.6 \times 10^{-6}\,cm^{-2}$
Transverse buckling before entrance of controls or loss of water	$25.9 \times 10^{-6}\,cm^{-2}$

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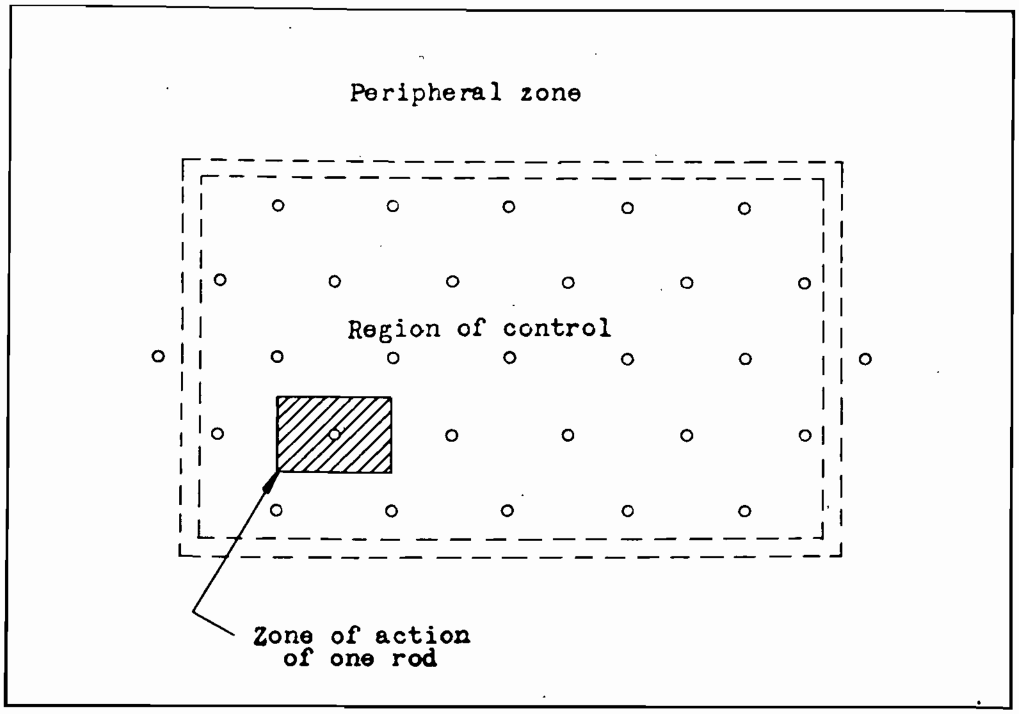
22.3.59 Calculated size of region of control

The size of the region of control is set by the transverse buckling in the peripheral portion of the pile. According to the table, this quantity has the value, $B_1 = 82.6 \times 10^{-6}$, as compared to the original value of the transverse buckling, $B_o = 25.9 \times 10^{-6}$. For the ratio, B_o/B_1, we have the number 0.313. This is one of the quantities we require in order to use Fig. 22.3.54. The other, the ratio 1060/760 = 1.395 of pile length and width, falls between the values given in the chart in question. We therefore interpolate, and find for dimensions of the region of control the values listed below:

Fully loaded Hanford pile	Dimensions of pile, cm	Dimension ratios from Fig. 22.3.54		Dimensions of region of control, cm	
		Probable	Upper limit	Probable	Upper limit
Width	760	0.512	0.545	389	414
Length	1060	0.650	0.672	689	714
Area	$8.05\times10^5 cm^2$			$2.68\times10^5 cm^2$	$2.96\times10^5 cm^2$

22.3.60 Agreement as to size with final design

The layout of the safety system as finally designed is illustrated in the following diagram, drawn to 1/100 of full size. The outline of the pattern deviates slightly from a rectangle in the direction to be



expected from a more accurate calculation of the shape of the region of control. The 29 rods each have a zone of action $50\frac{1}{4}$" x 32" or 127.5 cm x 81.3 cm, comprising an area of $1.037 \times 10^4 cm^2$. The area of the region of control is therefore $29 \times 1.037 \times 10^4 = 3.01 \times 10^5 cm^2$, very slightly in excess of the maximum requirement listed in the above table, and 12 percent above the probable needed area.

22.3.61 More than adequate control power

In addition to an adequate size, the final safety system has more than enough local control power. The required reduction in local reproduction factor in the region of control is 0.0501, according to the analysis of 22.3.58. This is to be accomplished by rods each of which acts on a zone of $1.037 \times 10^4 cm^2$, equivalent in area to a circle with radius R = 57.5 cm. The rods as finally designed are steel tubes, $2\frac{1}{4}$ inches in outside diameter, with 3/16 inch wall thickness, and contain $1\frac{1}{2}$ percent of boron by weight. A rod of this design, acting in a zone of the given size, is calculated in 22.5 to lower the local multiplication factor by 0.0605, 1 percent more than the required amount. We conclude that the Hanford system of emergency control, under the assumed conditions of operation, has a considerable margin of safety.

22.3.62 Shadowing effect

We have completed the discussion of rods located in an optimum region of control and may now discuss briefly the much more complicated case of strong absorbers disposed in some other pattern. Consider as a simple example the interaction between two control rods. Insertion of the first one lowers the relative value of the neutron density nearby, raises it farther away. The second control rod will therefore exert less effect than before if it is close to the first rod, more effect if at some distance. The shadowing effect of one rod on a neighbor is illustrated by experiments of Zinn and Anderson* on the first chain reacting pile built at Chicago. The controls were two strips of cadmium 8 feet long and 2 inches wide. The effect of either alone was enough to depress the reactivity below the critical level. Consequently, a special procedure was employed to measure the effects of the rods individually and in combination. With both removed, the overall multiplication factor of the pile was adjusted exactly to unity by means of supplementary controls. Then the desired strip was inserted and a reading was obtained from a galvanometer connected to an ionization chamber in the pile. The reading furnished a measure of the neutron density which in turn was inversely proportional to 1 - k (Chapter 14). The loss in reactivity could therefore be measured up to an unknown constant of proportionality. This constant drops out when the shadowing effect is evaluated percentage-wise, as in the following table. Here the influence on the shadowing effect due to separation of the controls is quite apparent.

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*W. H. Zinn and H. L. Anderson, CP-510, Physics Research for Month Ending 1943 March 6.

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SURVEY OF CONTROL THEORY

22.3.62

First Chicago chain-reacting pile. In critical condition with both experimental rods removed	Second rod 45 inches from first rod		Second rod 90 inches from first rod	
	Galvanometer reading	Reciprocal proportional to δk	Galvanometer reading	Reciprocal proportional to δk
First rod alone	6.27 cm	0.159	6.27 cm	0.159
Second rod alone	4.73	0.211	6.28	0.159
Sum of individual effects		0.370		0.318
Observed effect of both together	2.96 cm	0.338	3.30 cm	0.303
Decrease due to shadowing		8.6%		4.7%

An approximate estimate of the magnitude of the shadowing effect is of some interest in the case of the Hanford regulating and shim control system. The sum of the effects on k of the 9 rods, considered individually, amounts to 0.019, according to the analysis of 22.3.25. The actual effect of all the rods inserted together may in principle be determined along the lines sketched in Table 22.3.44, CASE VI. We have to choose by trial and error such a value of the deficit, $-k_e$, in overall reproduction factor, that we can just satisfy conditions 22.3.47 on the neutron density. Specifically, we have to solve the fundamental buckling equation 22.3.14.a in a two-dimensional region 760 cm x 1060 cm. In the inner portion of this region, a space 487 cm x 383 cm, the buckling is less than outside by an amount which is the product of the migration area and the loss in local k due to the controls. To make this problem mathematically manageable, we replace the inner region by a cylinder of the same cross section, and the rectangular pile by a cylindrical reactor of the same transverse buckling. The radii of the two cylinders are respectively 244 cm and 474 cm. In each the neutron density is represented by a Bessel function, the argument in which depends on the local buckling. The two functions must have the same value and slope at $r = 244$ cm. This condition fixes the value of the buckling in each region. Comparing the value in the outer region with the value before insertion of the 9 controls, we have a measure of their total effect. We multiply the change in buckling by the migration area and find for loss in k the figure 0.017 ± 0.001. Here the uncertainty arises from the imperfect equivalence between cylinders and rectangles. The difference between the total drop in k and the sum, 0.019, or the individual decrements is 0.002 ± 0.001, a measure of the shadowing effect in the Hanford control system. In magnitude this effect is comparable to that of 1 of the 9 rods. 22.3.63 Magnitude for Hanford control system

When a number of rods enter a cylindrical pile in a pattern like that of Fig. 22.3.65, it is again possible to calculate their combined effect by an application of the properties of Bessel functions. However, we shall do no more with this case than to refer to contour diagram which shows how the distribution of neutron density is affected by the control rods. 22.3.64 Ring of control rods

May, 1944

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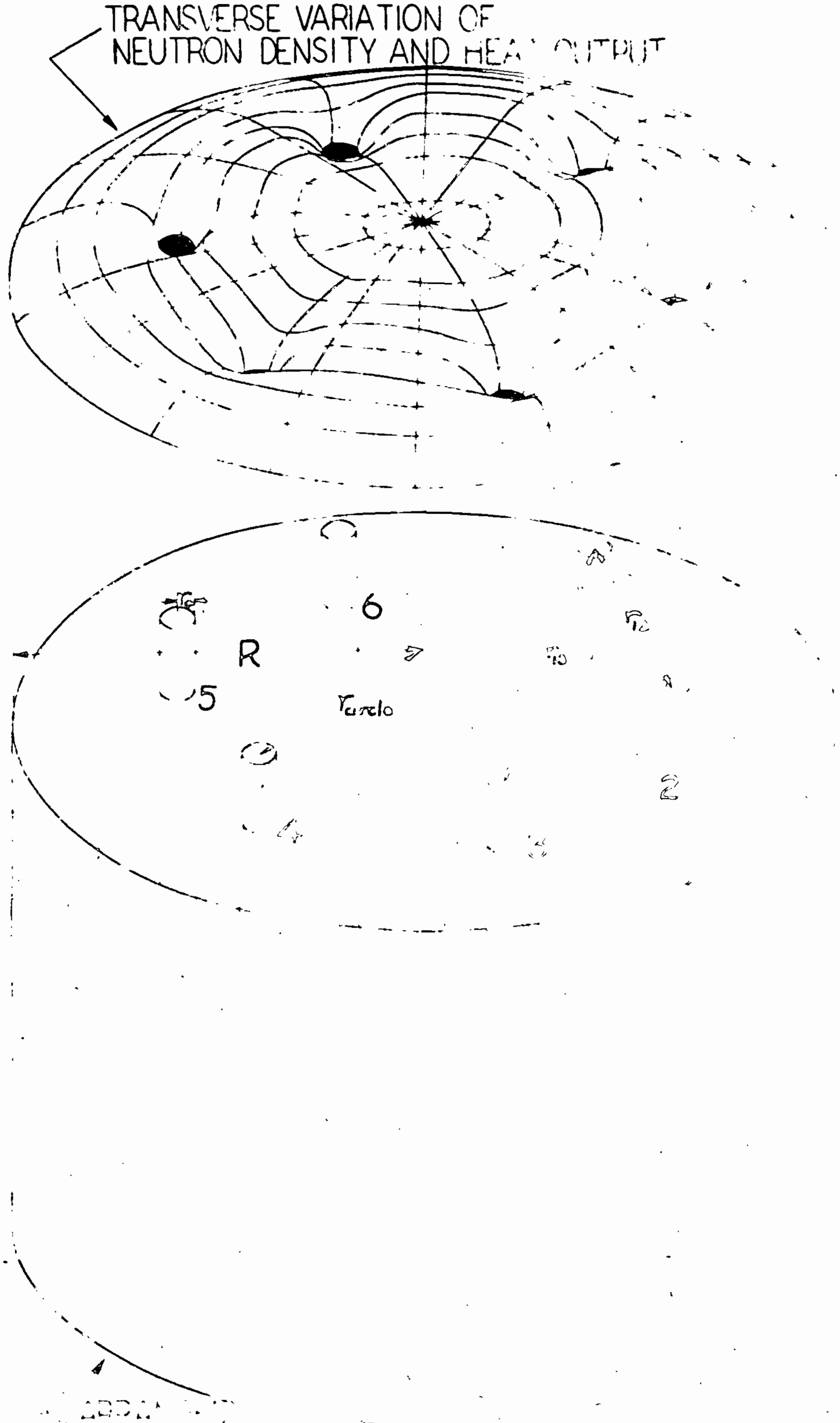
FIGURE 22

RING OF CONTROL RODS



SURVEY OF CONTROL THEORY

22.3.66 Sheet absorbers

We conclude the survey of control theory presented in this section by considering briefly sheet absorbers, a type of control which has so far not found application. Study of the mechanism of action of this type of control will permit us to give a simplified account of the effect of control rods themselves, and thus prepare the way for the more nearly complete theory of rods given in Sections 22.4 and 22.5.

22.3.67 Sheets change boundary conditions

The effect on the reactivity of a pile due to a sheet of material opaque to neutrons is most simply considered as an alteration in boundary conditions, with no change at all in local reproduction factor. The presence of the sheet buckles the neutron density more strongly than before. Thus more neutrons escape by migration to the outside or to the new absorber. The product of the increase in buckling and the migration area gives the loss in overall multiplication factor:

$$\delta k = A\,\delta B \qquad (22.3.67.a)$$

22.3.68 Example of sheet controls

The actual evaluation of the increase in buckling due to control sheets of arbitrary form is a complicated mathematical problem. For certain simple types of geometry, however, the solution may be found by the method of separation of variables. Fig. 22.3.69 illustrates the case of a cylindrical pile. In this figure all the examples represent instances where the neutron density can be expressed in the form of a product of functions, each of which depends on only one of the three cylindrical coordinates:

- r, distance from the axis.
- z, distance from the median plane.
- θ, angle of rotation about the axis.

Thus we have

$$n = \cos g\,(z - z_0)\cos n\theta\, J_n(br) \qquad (22.3.68.a)$$

where

- n is an integer, 0, 1, 2. . . .
- b is a constant so chosen as to make the neutron density vanish at the proper value of r.
- g and z_0 are constants so chosen as to make the neutron density vanish at the proper values of z.

The effect of the sheet controls is analyzed as follows:

Longitudinal buckling of (22.3.68.a),	g^2.
Transverse buckling,	b^2.
Buckling before sheets entered, expressed in terms of height, h, of cylinder and its radius, R,	$(\pi/h)^2 + (2.4048/R)^2$

Change in overall multiplication factor due to sheets,

$$\delta k = A\,(g^2 + b^2 - \pi^2/h^2 - 2.4048^2/R^2) \qquad (22.3.68.b)$$

[redacted]

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CONTROLS OF SHEET CHARACTER

FIGURE 22·3·69

THE EFFECTIVENESS OF OPAQUE SHEETS OF NEUTRON ABSORBING MATERIAL IN STOPPING THE CHAIN REACTION DEPENDS UPON THE FINENESS WITH WHICH THEY SUBDIVIDE THE VOLUME OF THE PILE. THE EFFECTIVENESS IS EXPRESSED IN TERMS OF THE MULTIPLICATION FACTOR, k, WHICH THE PILE MEDIUM ITSELF MUST POSSESS IN ORDER JUST TO PERMIT A CHAIN REACTION TO PROCEED WITH THE GIVEN CONTROL. THE EFFECT OF A SHEET CAN ALSO BE OBTAINED BY MEANS OF A LARGE NUMBER OF CLOSELY SPACED RODS OF ABSORBENT MATERIAL.

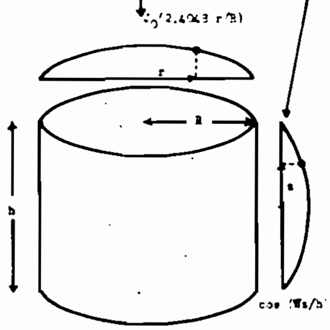
Radial variation of neutron density

Axial variation of neutron density

Constant times neutron density at the point with coordinates r,z,θ

Buckling of the neutron density

Evaluation for a pile 700 cm high 500 cm in radius migration area = 700 cm^2



PILE WITH NO CONTROL

$J_0(2.4048\ r/R) \cos(\pi z/h)$

$$\frac{k-1}{\text{migration area}} \equiv B = \left(\frac{2.4048}{R}\right)^2 + \left(\frac{\pi}{h}\right)^2$$

$B = (23.13 + 20.14) \times 10^{-24}$

$= 43.27 \times 10^{-24}$

$k = 1.0303$

ONE RADIAL AND ONE AXIAL NODE

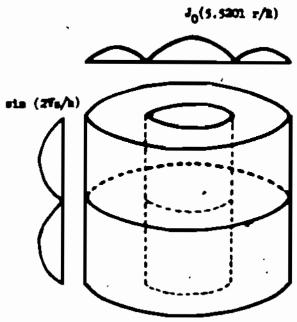
$J_0(5.5201\ r/R) \sin(2\pi z/h)$

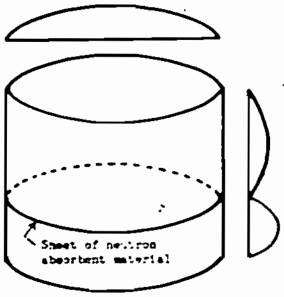
$$\frac{k-1}{\text{migration area}} = B = \left(\frac{5.5201}{R}\right)^2 + \left(\frac{2\pi}{h}\right)^2$$

$B = 202.45\ \text{cm}^{-2} \times 10^{-6}$

$k = 1.1417$

Increase in k required for operation with controls in is 0.1114





NON-CENTRAL SHEET PARALLEL FACES

Buckling of neutron density in upper region not as great as that in lower region. Hence with a medium of sufficiently great multiplying power a chain reaction may develop in upper region, though the action is cut off in lower region. Example illustrates importance of uniform subdivision of volume of pile.

ONE RADIAL AND ONE ANGULAR NODE

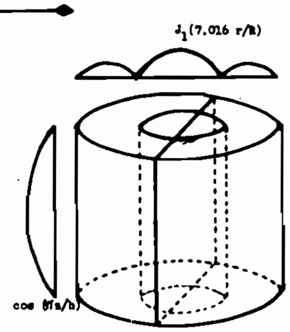
$J_0(7.016\ r/R) \cos(\pi z/h) \cos\theta$

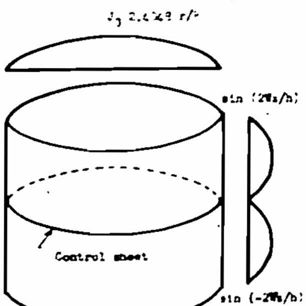
$$\frac{k-1}{\text{migration area}} = B = \left(\frac{7.016}{R}\right)^2 + \left(\frac{\pi}{h}\right)^2$$

$B = 217.34\ \text{cm}^{-2} \times 10^{-6}$

$k = 1.1519$

Change in k, 0.1216





CENTRAL SHEET PARALLEL FACES

$J_0(2.4048\ r/R) \sin(2\pi z/h)$

$$\frac{k-1}{\text{migration area}} = B = \left(\frac{2.4048}{R}\right)^2 + \left(\frac{2\pi}{h}\right)^2$$

$B = 103.7 \times 10^{-6}\ \text{cm}^{-2}$

$k = 1.0726$

Change in k required to counterbalance control is 0.0423

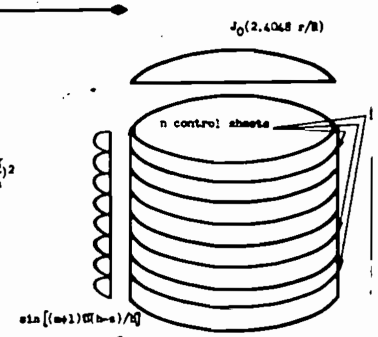
MANY SHEETS PARALLEL FACES

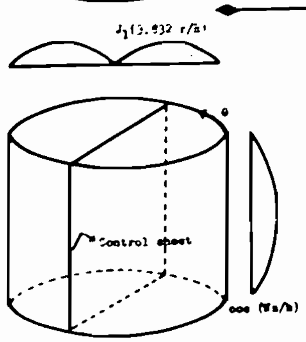
$J_0(2.4048\ r/R) \sin\left[(n+1)\pi(h-z)/h\right]$

$$\frac{k-1}{\text{migration area}} = B = \left(\frac{2.4048}{R}\right)^2 + (n+1)^2\left(\frac{\pi}{h}\right)^2$$

$B = 92.53 \times 10^{-6} + 20.14 \times 10^{-6}(n+1)^2\ \text{cm}^{-2}$

Change in k, $0.0141\ (n^2 + 2n)$





CENTRAL SHEET PARALLEL AXIS

$J_1(2.832\ r/R) \cos(\pi z/h) \cos\theta$

$$\frac{k-1}{\text{migration area}} = B = \left(\frac{3.832}{R}\right)^2 + \left(\frac{\pi}{h}\right)^2$$

$B = 76.88 \times 10^{-6}\ \text{cm}^{-2}$

$k = 1.0552$

Change in k required, 0.0249. Relative effectiveness of single sheet parallel or perpendicular to axis depends on ratio h/R.

MANY SHEETS THROUGH AXIS

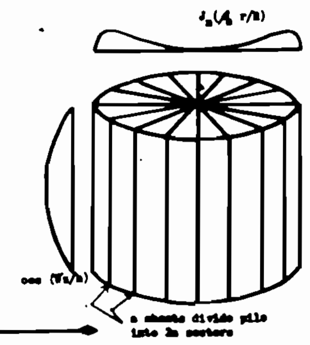
$J_n(\beta_n\ r/R) \cos(\pi z/h) \cos n\theta$

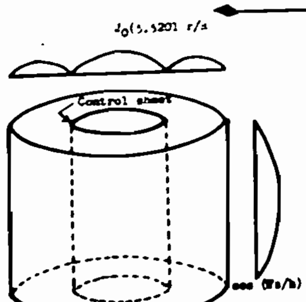
$$\frac{k-1}{\text{migration area}} = B = \left(\frac{\beta_n}{R}\right)^2 + \left(\frac{\pi}{h}\right)^2$$

$\beta_n \sim n + 1.86\ n^{1/3}$

$B \sim (n/R)^2 + (\pi/h)^2$

Change in k, $\sim 0.0028\ n^2$





CYLINDRICAL SHEET PARALLEL AXIS

$J_0(5.5201\ r/R) \cos(\pi z/h)$

$$\frac{k-1}{\text{migration area}} = B = \left(\frac{5.5201}{R}\right)^2 + \left(\frac{\pi}{h}\right)^2$$

$B = 142.03 \times 10^{-6}\ \text{cm}^{-2}$

$k = 1.0994$

Change in k is 0.0691. The same effect is achieved by a large number of control rods closely spaced around the circle in question.

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MANY CONCENTRIC CYLINDRICAL SHEETS

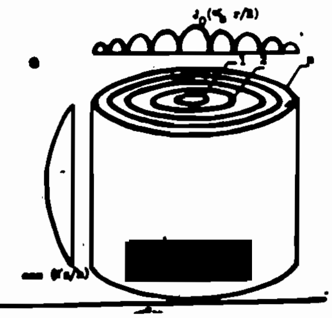
$J_0(\alpha_n\ r/R) \cos(\pi z/h)$

$$\frac{k-1}{\text{migration area}} = B = \left(\frac{\alpha_n}{R}\right)^2 + \left(\frac{\pi}{h}\right)^2$$

$\alpha_n \sim (n - 1/4)$

$B \sim (n - 1/4)^2\left(\frac{\pi}{R}\right)^2 + \left(\frac{\pi}{h}\right)^2$

Change in k, $\sim 0.0276\ n^2$



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Evaluation of this formula in Fig. 22.3.69 for the case of a pile 700 cm high and 500 cm in radius gives an impression of the relative effectiveness of various forms of absorbent sheets. Maximum efficiency of control evidently results when the pile is subdivided into portions of comparable size. The control is no safer than its weakest link - the reduction in overall k in the largest subdivision of the pile.

As final and most important example of a sheet control, we consider a large cylinder of neutron absorbing material passed through a cylindrical pile along its axis. We simplify in four respects as compared with the fuller theory of control rods: 22.3.70 Idealized cylindrical control rod

(1) The rod is at the center of the pile.
(2) The pile is cylindrical.
(3) We assume the rod is opaque to both fast and thermal neutrons.
(4) We assume that the neutron density falls to zero at the surface of the rod.

The longitudinal buckling of the neutron density has the same value, $(\pi/\text{height of pile})^2$, before and after the rod enters. Consequently, we need only consider the effect of the rod on the transverse variation of neutron density. This variation was originally described by the Bessel function, $J_0(2.4048\ r/R)$, where R is the radius of the pile (see Fig. 22.3.71). After the absorbent cylinder enters, the radial variation of the neutron density is described outside the cylinder by a linear combination of the regular and irregular Bessel functions of order zero, which correspond to the sine and cosine functions of trigonometrical analysis:

$$n = f\,J_0(br) + g\,N_0(br). \qquad (22.3.70.a)$$

Here f and g are numerical coefficients and the square of the quantity, b, represents the radial buckling of the neutron density. This quantity can be considered to be known if we prescribe in advance how much the rod is to lower the overall multiplication factor:

$$\begin{aligned} b^2 &= (\text{original buckling}) + (\text{change in buckling}) \\ &= (2.4048/R)^2 + (\delta k/A) \qquad (22.3.70.b) \end{aligned}$$

We find the size of cylinder required to produce the prescribed change in k by the following reasoning: 22.3.72 Solution for radius of rod

(1) The neutron density vanishes at the surface of the pile. Hence $-g/f = J_0(bR)/N_0(bR)$.
(2) The neutron density vanishes at the surface, $r = r_0$, of the cylindrical control. Consequently, $-g/f = J_0(br_0)/N_0(br_0)$
(3) Quantities equal to the same quantity are equal to each other:

$$J_0(br_0)/N_0(br_0) = J_0(bR)/N_0(bR) \qquad (22.3.72.a)$$

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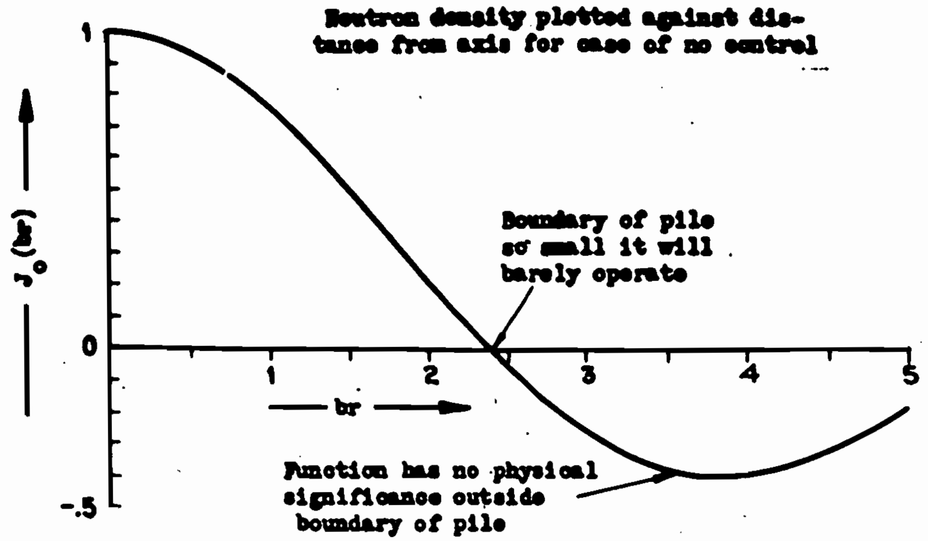
52.

FIGURE 22.3.71

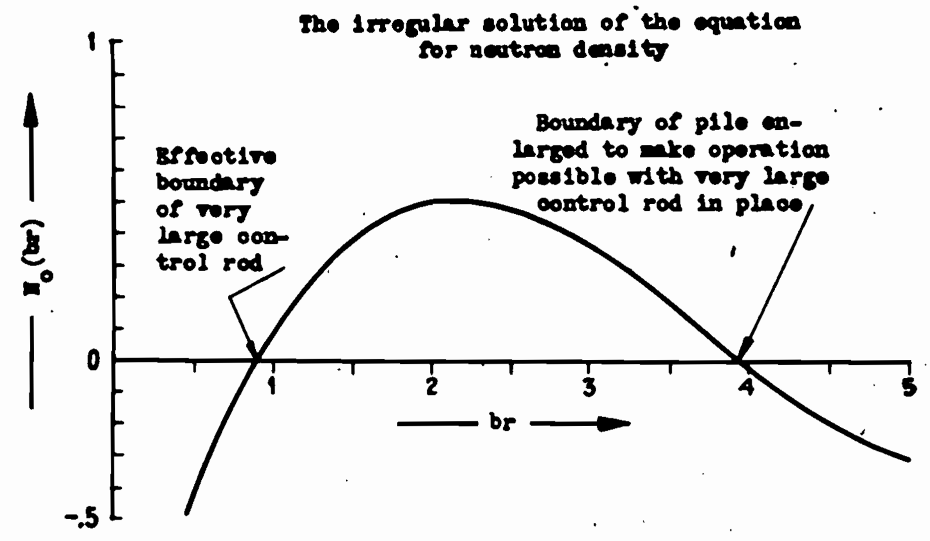
EFFECT OF CONTROL ROD ON NEUTRON DENSITY



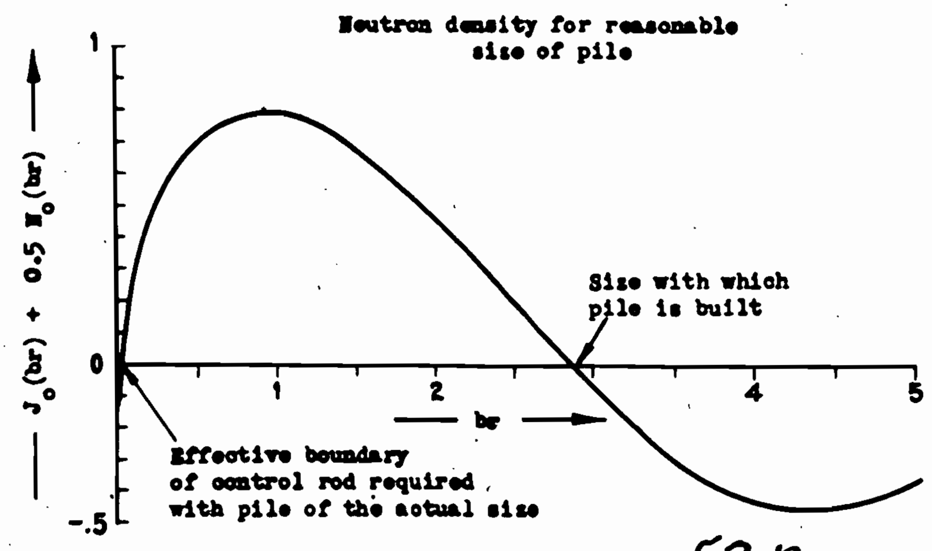
Radial variation of neutron density in a cylindrical pile without control rod is proportional to the regular Bessel function of 0 order of br. The square of the quantity b represents the radial buckling of the neutron density. It is connected with the multiplication factor by the equation for total buckling:

$$\frac{k-1}{\text{migration area}} = \text{buckling}$$

$$= b^2 + \left(\frac{\pi}{\text{height of pile}}\right)^2$$



With the same amount of radial buckling, the neutron density can be equally well represented by the irregular Bessel function of order 0. Its relation to the regular function is much the relation of the sine function to the cosine function. It represents the variation of the neutron density in case a very large control rod is inserted along the axis of the pile.



For a pile of arbitrary size, the neutron density must vanish at the outer boundary. Neither J_0 nor N_0 alone will, in general, satisfy this condition. However, the actual neutron density satisfies the same second order differential equation as J_0 and N_0 and must therefore be a linear combination of them. Such a linear combination, $J_0 + 0.5\ N_0$, is illustrated in this example.

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J.A.W., M.A., 1943-5-1

22.3.72

The last equation gives the unknown radius, r_o, in terms of the two known quantities, the radius, R, of the pile and the final value of the transverse buckling, b^2. The relation is readily applied, either by a process of trial and error with the aid of a table of Bessel functions, or directly by use of the outer nomographic spiral in Fig. 22.5.23.

22.3.73 Difficulty of strong control with a single rod

EXAMPLE. The overall multiplication factor is to be reduced by 1.5 percent in an operating cylindrical pile 500 cm in radius where the neutrons have a migration area of 700 cm^2. How big a cylindrical control should be inserted along the axis? The original transverse buckling was $(2.4048/500)^2 = 23.1 \times 10^{-6} cm^{-2}$. The required gain in buckling is $0.015/700 = 21.4 \times 10^{-6}$, making the total transverse buckling $b^2 = 44.5 \times 10^{-6} cm^{-2}$. The quantity, b, has the value $6.68 \times 10^{-3} cm^{-1}$ and the variable, $bR = 6.68 \times 10^{-3} \times 500 = 3.34$. We solve (22.3.72.a) for br_o with the aid of the outer spiral in Fig. 22.5.23, finding $br_o = 0.34$. Dividing this quantity by $b = 6.68 \times 10^{-3} cm^{-1}$, we obtain for the required radius of the absorbent cylinder the figure $r_c = 52$ cm. In this instance the radius is sufficiently large in comparison with the square root of the migration area to make the present approximate theory reasonably reliable. It is apparent from the result how difficult it is to obtain a large amount of control with a single control rod. A number of smaller rods are much more convenient. Their effect on the local multiplication factor cannot however be estimated reliably with the present simplified form of control rod theory.

22.3.74 Recapitulation

The survey of control theory presented in this section began by distinguishing two factors in the action of an absorbent. The first of these, the reduction in local multiplication factor, was briefly discussed and will be examined in more detail in Section 22.5. The second factor, the influence of the location of the control on the overall multiplication factor, was analyzed in detail both for individual rods in arbitrary locations and for groups of rods disposed in patterns of optimum effectiveness. Finally, a discussion of sheet controls has led to a simplified account of the action of a large control rod and has prepared the way for the fuller theory of control rods presented in the next two sections.

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22.4 EFFECTIVE RADIUS OF CONTROL BAR

22.4.1 Actual vs. ideal control bars

The ideal control bar is a perfect absorber of neutrons of all energies and reduces to zero the density of neutrons of every velocity at its surface. An actual control bar differs from an ideal control bar in three respects: it does not have equal action on neutrons of all energies, it is not a perfect absorber of neutrons, and the neutron density outside the control bar does not extrapolate to zero at the surface of the bar. Let us consider each of these complications in turn.

22.4.2 Appreciable absorption only for thermal neutrons

The action of the control substance on fast neutrons will in general be negligible in comparison with its power of absorbing thermal neutrons. A fast neutron reacts with a nucleus with an effective cross-section at most of the order of magnitude of the geometrical cross-section of the nucleus, itself, i.e., roughly $\sim 10^{-24}$ cm^2. On this account, one substance has little to recommend it as compared to any other substance for use as absorbent of fast neutrons. The cross-section of the control bar material for slow neutrons, however, will normally be very large in order to allow the bar to take up the least possible space in the pile. The cross-section for thermal neutron capture in boron and mercury being of the order of magnitude of 10^{-21} cm^2 and that in cadmium and gadolinium being even larger, the control bar will ordinarily absorb to an important extent only those neutrons which have reached thermal energy. We shall therefore have to define the effective radius of the control bar only for neutrons of thermal energy.

22.4.3 Definition of effective radius

The control bars of interest will have cross-sectional dimensions small in comparison with the size of the pile and even small in comparison to the spacing between bars. This circumstance simplifies the definition of the effective radius of the bar for interaction with thermal neutrons. With the center line of the bar as axis, describe about the bar a cylinder whose radius, r, is considerably larger than the width of the bar. At the same time we require r to be small in comparison with the spacing between bars. Thus the volume of pile proper to one control bar bears to the volume within the cylinder a ratio of the order of (spacing between bars/r)^2, or (radius of pile/r)^2, a ratio which is very large relative to unity. Consequently, we may neglect in comparison to the total neutron output of the pile the absorption and production of neutrons in that portion of the structure which is comprised between the surface of the bar and the cylinder. The absorption of neutrons by the bar will therefore be measured by the net flux of neutrons through the surface of the cylinder. The surface of the cylinder is proportional to $2\pi r$. The net flow in across unit area of the cylinder is proportional to ∂(neutron density)/∂r. The flux depends on the properties of the bar but is independent of the radius of the cylinder. Consequently we have the relation,

$$r \; \partial(\text{neutron density})/\partial r = \text{constant} \qquad (22.4.3.a)$$

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Integrating, we find the general behavior of the neutron density:

$$\text{neutron density} = \text{constant} \ln (r/r_{eff}), \qquad (22.4.3.b)$$

for values of r which are at the same time large in comparison with the size of the bar and small in comparison with the spacing between bars. The first constant depends upon the power output of the pile and is of no concern here. The constant of integration, r_{eff}, however, is completely determined by the size, shape, and material of the control bar. It defines the effective radius of the bar. How to determine the effective radius of a control bar is the subject of the following paragraphs. The subsequent section, 22.5, then develops the relationship between the effective radius and the actual degree of control exerted by the bar.

22.4.4 Ideal control rod

An ideal control rod may be considered to be a cylindrical adsorber at the surface of which the neutron density vanishes. The neutron population in the neighborhood of such a bar of radius, r_o, will vary as $\ln (r/r_o)$. Comparing this expression with (22.4.3.b), we conclude that the effective radius of a control bar of any cross-section is equal to the radius of that ideal control rod which will have the same effect on the neutron density.

22.4.5 Completely absorbing rod embedded in moderator

No actual control rod reduces to zero the density of neutrons at its surface. At the face of a perfect absorber the neutron population, n, satisfies not the condition n = o, but rather the relationship

$$n = (\lambda/3^{\frac{1}{2}})\, \partial n/\partial(\text{normal}) \qquad (22.4.5.a)$$

where λ is the mean free path of a neutron in the enveloping moderator (cf. Chap. 11). The neutron density evidently extrapolates to zero only some distance within the absorber. To find the effective radius of the rod, we insert the expression, $n = \text{constant} \ln (r/r_{eff})$, into (22.4.5.a) and evaluate the normal derivative of the neutron density at the actual radius, r_o, of the rod:

$$\ln (r_o/r_{eff}) = (\lambda/3^{\frac{1}{2}}\, r_o) \qquad (22.4.5.b)$$

We solve this equation for the effective radius of the rod and find

$$r_{eff} = r_o \exp(-\lambda/3^{\frac{1}{2}}\, r_o) \qquad (22.4.5.c)$$

When the radius of the rod is large in comparison with the mean free path of neutrons in the surrounding moderator, we have the approximate result,

$$r_{eff} = r_o - \lambda/3^{\frac{1}{2}} \qquad (22.4.5.d)$$

This limiting formula conforms to the observation that the neutron density extrapolates to zero $\lambda/3^{\frac{1}{2}}$ cm inside the surface of a flat piece of cadmium. Equation (22.4.5.c) is presented in graphical form in Fig. 22.4.6.

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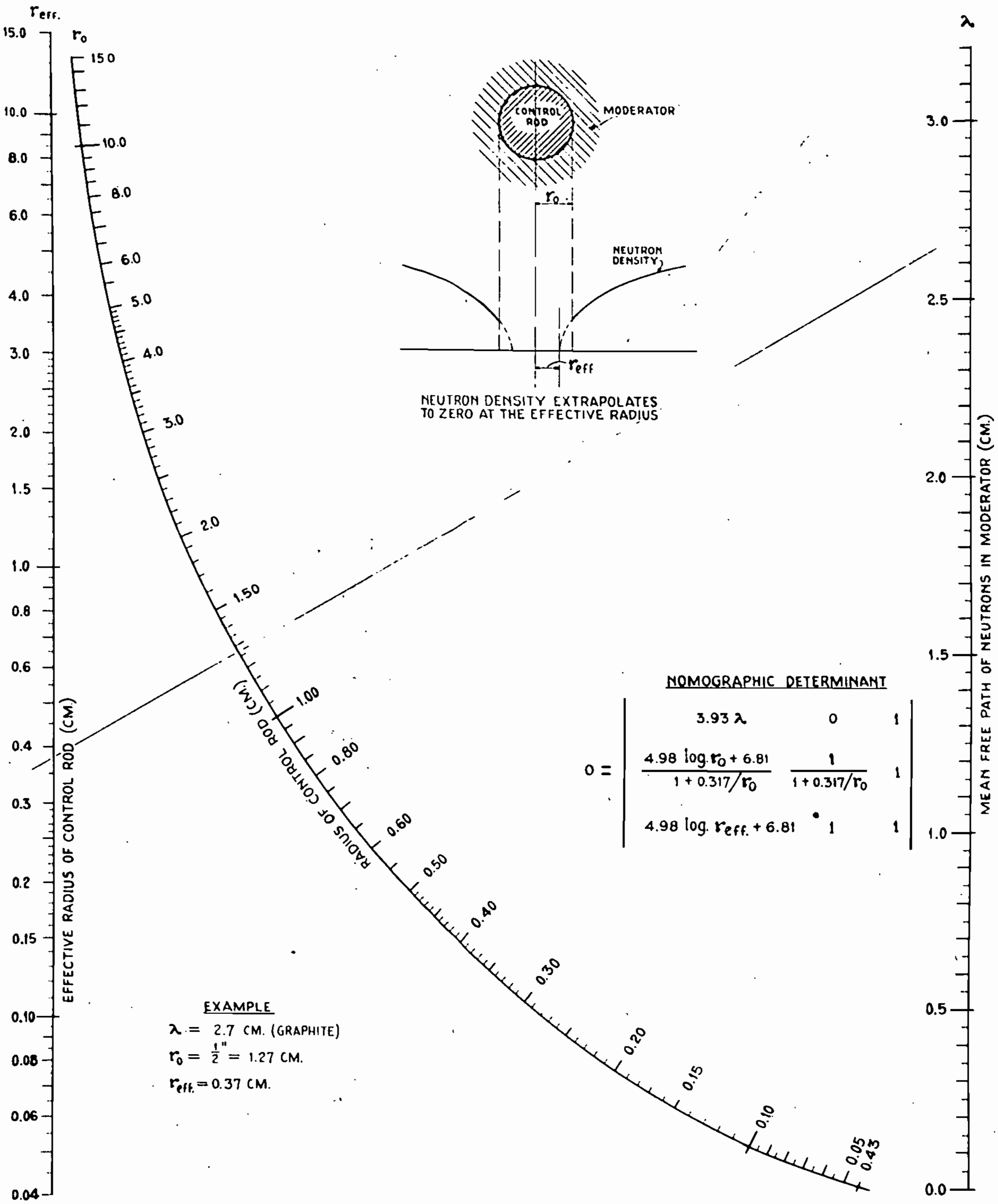
CIRCULAR CONTROL ROD

EFFECTIVE RADIUS OF A COMPLETELY ABSORBENT CONTROL ROD IMBEDDED IN A MODERATOR

$$r_{eff.} = r_0 e^{-\frac{\lambda}{3\frac{1}{2} r_0}}$$

[redacted]



NOMOGRAPHIC DETERMINANT

$$0 = \begin{vmatrix} 3.93\lambda & 0 & 1 \\ \dfrac{4.98 \log r_0 + 6.81}{1 + 0.317/r_0} & \dfrac{1}{1 + 0.317/r_0} & 1 \\ 4.98 \log r_{eff.} + 6.81 & 1 & 1 \end{vmatrix}$$

EXAMPLE

$\lambda = 2.7$ CM. (GRAPHITE)
$r_0 = \frac{1}{2}'' = 1.27$ CM.
$r_{eff.} = 0.37$ CM.

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FIGURE-22.4.6

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[redacted]

EFFECTIVE RADIUS OF CONTROL BAR 22.4.7

EXAMPLE: A cadmium rod 1.27 cm in radius is passed into a graphite-uranium pile through a snug hole. The mean free path of thermal neutrons in the moderator is 2.7 cm. How much is the effective radius of the rod? From Fig. 22.4.6 we find $r_{eff} = 0.37$ cm. 22.4.7 Example of imbedded rod

The surrounding moderator has an influence on the effective radius of a control rod, as is apparent in the preceding example. The extent of this influence becomes clearer when the moderating material is cut away from the immediate neighborhood of the rod. Thus, let a cadmium rod of radius, r_o, be inserted into a hole of considerably greater radius, r_1. In the moderator the neutron density will be represented by the expression $n = n_o \ln (r/r_{eff})$. In the cavity, as in a hollow space filled with black-body radiation, the neutron density will be constant except as it is depressed in the immediate neighborhood of the rod. This depression will show up when we observe the number of neutrons moving away from the rod, but will be very little apparent in the flux to the rod. To calculate this flux we shall therefore adopt for the neutron density the same value, $n_o \ln(r_1/r_{eff})$, which prevails at the inner surface of the moderator, and shall multiply this value with the factor, (velocity of neutrons/4), which gives the neutron flow across a unit of area. On this basis we find that a unit of length of the control rod absorbs neutrons at the rate 22.4.8 Rod in cavity

$$2\pi r_o . (v/4) . n_o \ln(r_1/r_{eff}) \qquad (22.4.8.a)$$

per second. These neutrons are supplied by diffusion in through the moderator. The diffusion coefficient is $v\lambda/3$ (Chap. 11). The net flow inward across 1 cm. of length of the cylinder is given by the product

$$\text{(surface).(diffusion coefficient).(density gradient)}$$

$$= 2\pi r \ . \ (v\lambda/3) \ . \ (n_o/r) = 2\pi(v\lambda/3)n_o \qquad (22.4.8.b)$$

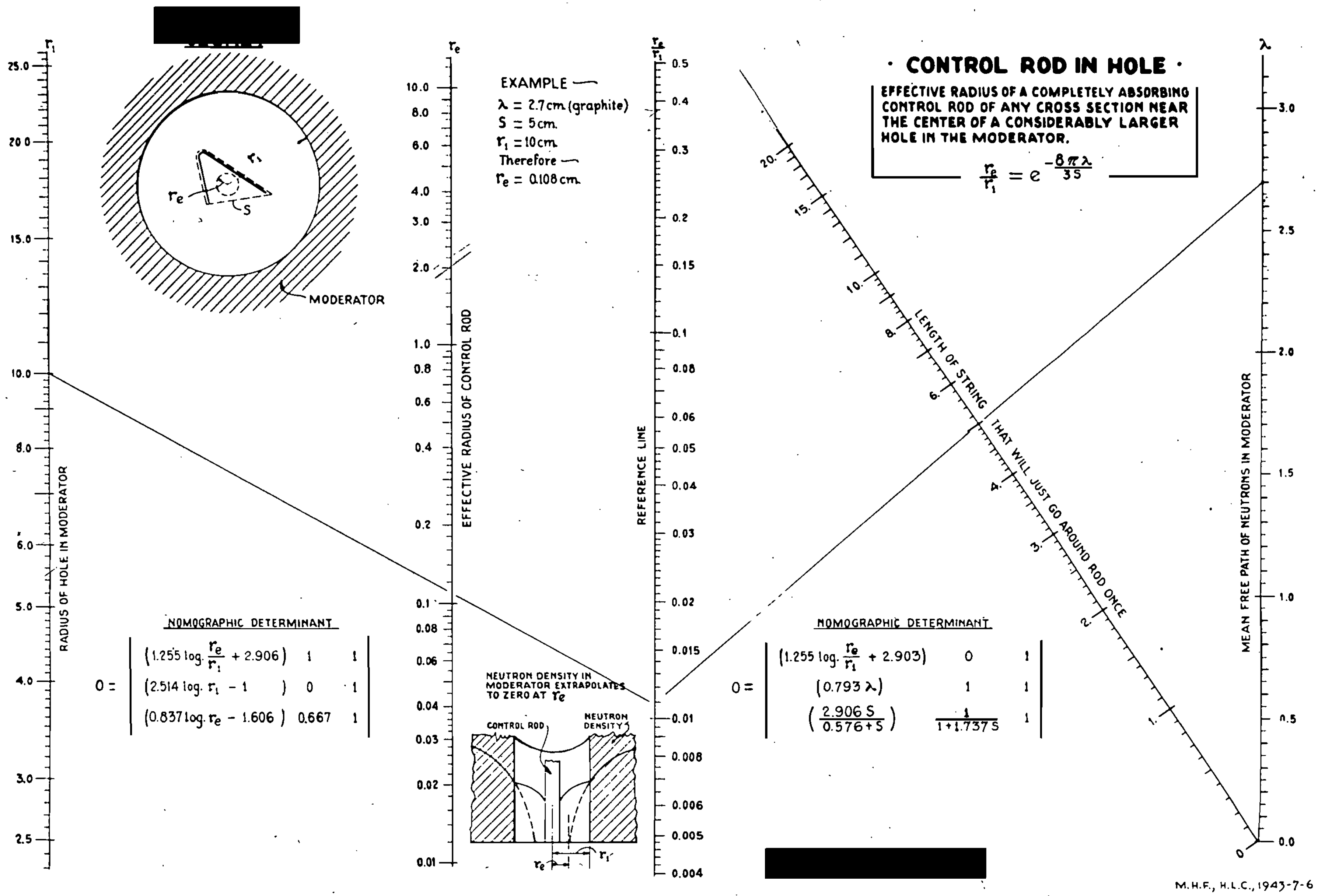
Equating the absorption (22.4.8.a) to the net inward flow (22.4.8.b), we obtain a relation giving the effective radius of the control rod:

$$r_{eff} = r_1 \exp(-4\lambda/3 \ r_o) \qquad (22.4.8.c)$$

It is apparent from the mathematical form of the relationship that the effectiveness of a control is increased by a surrounding gap. Neutrons reach the rod with greater ease when we remove the moderator which blocks their way.

A bar of arbitrary cross-section passing through a considerably larger hole in the moderator acts on the same principles as the rod just discussed. The fact that the rod was circular affected only the calculation of the absorbing surface per unit of length of the rod, $2\pi r_o$. We have now only to replace this figure by the corresponding figure, s, for 22.4.9 Bar of any shape in large hole Aug, 1943

FIGURE-22.4.10



the perimeter of a bar of the cross-section in question. In this way we obtain the result

$$r_{eff} = r_1 \exp(-8\pi\lambda/3s) \qquad (22.4.9.a)$$

This relation appears in graphical form in Fig. 22.4.10. The figure illustrates that the perimeter is to be determined by putting a measuring tape once about the bar. A neutron which once crosses the line of the tape will be absorbed, whether or not cadmium touches the tape at that point.

22.4.11 Example of bar in hole

EXAMPLE: Through a graphite-uranium pile passes a hole 17.7 cm square for the insertion of a control bar. The bar is located nearly centrally in the hole. It is made by creasing a long strip of cadmium 2.94 cm wide along its center line and bending it into a right angle. What is the effective radius of the control bar? The perimeter is $s = 1.47 + 1.47 + \sqrt{2} \times 1.47 = 5$ cm. The square cavity may be taken with sufficient accuracy to be equivalent to a circular hole of the same cross-section: $\pi r_1^2 = (17.7 \text{ cm})^2$, from which $r_1 = 10$ cm. The mean free path of neutrons in the graphite is 2.7 cm. Employing the nomograph of Fig. 22.4.10, we find from these numbers that the effective radius of the bar is 0.11 cm.

22.4.12 Small bar imbedded in moderator

When a control bar is small in comparison to a mean free path, the population of neutrons in its neighborhood is little affected by its presence. The neutron density varies as

$$n = n_0 \ln(r/r_{eff}) = n_0\left[\ln(r/\lambda) + \ln(\lambda/r_{eff})\right] \qquad (22.4.12.a)$$

We conclude that the constant term in the density, $\ln(\lambda/r_{eff})$, is large in comparison with the variable term, $\ln(r/\lambda)$. This conclusion has two consequences. First the ratio, $\lambda/r_{eff} = \exp \ln(\lambda/r_{eff})$, must be exceedingly great and therefore the effective radius of the bar must be very small in comparison with the mean free path. Second, it will be sufficiently accurate for a first approximation to neglect the variable part of the neutron density. Then the flux into a unit length of the control bar will be given by the product

(neutron density).(velocity/4).(perimeter of bar defined as in

Figure 22.4.10) $= n_0 \ln(\lambda/r_{eff})\ (vs/4)$ (22.4.12.b)

Equating the rate of loss of neutrons to the net diffusion inward through the moderator, as given by equation 22.4.8.b, we obtain an approximate relation for the effective radius of a bar whose width is small in comparison with a mean free path:

$$r_{eff} \sim \lambda \exp(-8\pi\lambda/3s) \qquad (22.4.12.c)$$

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EFFECTIVE RADIUS OF CONTROL BAR

22.4.12

A more detailed calculation can be made along the same line of reasoning used in the derivation of Fick's law (Chap. 11). The result involves Euler's constant, $\gamma = 1.781$:

$$r_{eff} = (e^{\frac{1}{2}}\lambda/2\gamma)\ \exp\ (-8\pi\lambda/3s)$$
$$= 0.462\ \lambda\ \exp\ (-8\pi\lambda/3s). \qquad (22.4.12.d)$$

This expression for the effective radius of a fine control bar imbedded in a moderator can be evaluated with the aid of Fig. 22.4.10. It is only necessary to replace the value of the radius of the cavity by the value, 0.462(neutron's mean free path), which appears in Eq. (22.4.12.d.). The discrepancy by a factor more than 2 between the approximate formula (22.4.12.c) and the more nearly accurate Eq. (22.4.12.d) is not a cause for concern: the degree of control exerted by a bar is more nearly proportional to the logarithm of its effective radius than to the radius itself. In other words, the important part of expressions (22.4.12.c) and (22.4.12.d) is the quantity in the exponent, which agrees in the two cases. This agreement emphasizes that the effectiveness of a fine control bar depends only upon its perimeter, as defined in Fig. 22.4.10 and is otherwise independent of its shape.

22.4.13
Strip control

Considerable practical interest attaches to a control having the form of a long thin strip. (1) When such a strip is immersed in the moderator, and has a width small in comparison with a mean free path, its effective radius will depend only on its perimeter and will be given by Eq. (22.4.12.d). (2) When the strip passes through the pile in a cavity large in comparison with its own width, then, whether wide or narrow, it has an effective radius given by Eq. 22.4.9.a and Fig. 22.4.10. In both these cases an ant crawling around the strip will find himself bombarded at every point with the same neutron flux, and underfoot the rate of heat production will not vary from place to place on his circuit. The situation is different when the strip is imbedded in the moderator and when its width is comparable with or larger than a mean free path. Under these conditions neutrons arrive at the edges of the strip considerably more frequently than they hit its center. The problem is closely related to one of electrostatics. A charge given to a long thin electrical conductor of width w will distribute itself with a surface density proportional to $1/(1-(2x/w)^2)^{\frac{1}{2}}$, where x is the distance from the center line. The concentration of charge at the two edges of the strip is quite marked. The correspondence between the neutron problem and the electrostatic problem is further apparent from the following table:

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EFFECTIVE RADIUS OF CONTROL BAR

Basis of comparison	Neutron density, n	Electrical potential, V
Differential equation valid at distances small in comparison with spacing of control rods	$\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} = 0$	$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$
Condition at surface of strip	$n = \frac{\lambda}{3^{\frac{1}{2}}} \frac{\partial n}{\partial(\text{normal})}$	$V = 0$
Variation some distance from strip of width w	$n \sim \ln(r/r_{eff})$	$V \sim \ln(4r/w)$
Coordinate normal to plane of strip	y	y
Coordinate in plane of strip measured in normal direction from center line	x	x
Definition of a new complex variable, θ, in terms of which the desired function has a relatively simple form. Width of strip = w.	$x + iy = \frac{1}{2} w \sin\theta$	$x + iy = \frac{1}{2} w \sin\theta$
The function itself is given in the region outside the rod by the real part of the expression listed	$n + im = c - i\theta + \sum_{k=1}^{\infty} c_k \exp(2ik\theta)$	$V + iU = i\theta$
The boundary condition expressed in terms of the new variable, θ. Equation applies only at surface of strip	$n\lvert\cos\theta\rvert = \frac{-2\lambda}{3^{\frac{1}{2}} w} \frac{dm}{d\theta}$	$V = 0$

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EFFECTIVE RADIUS OF CONTROL BAR

22.4.13

When the mean free path of the neutrons is very small in comparison to the dimensions of the control, then the difference between the boundary conditions of the two problems can be neglected. Under these conditions the effective radius of the strip will be the same for the two problems. Referring to line 3 of the preceding table, we find the limiting formula for strips large in comparison with a mean free path,

$$r_{eff} \sim \text{width}/4 \qquad (22.4.13.a)$$

The error in this limiting formula will be of the order of a mean free path and will be appreciable when the width of the strip itself is of the same order of magnitude. Then a more complete treatment is required. Such an analysis has been given by K. Way, P. F. Gast, and John A. Wheeler, whose results are presented here for the first time. The method of treatment is outlined briefly in the following table and the conclusions are presented in graphical form in Fig. 22.4.15.

Table 22.4.14. OUTLINE OF SOLUTION OF PROBLEM OF STRIP CONTROL

22.4.14 Solution of strip problem

Physical content	Mathematical formulation
Expression of boundary condition for neutron density in terms of Fourier coefficients c and c_k.	$\frac{2\lambda}{3^{\frac{1}{2}}w}\left(1 - \sum_{k=1}^{\infty} c_k\, 2k \cos 2k\theta\right) = \lvert\cos\theta\rvert \left(c + \sum_{k=1}^{\infty} c_k \cos 2k\theta\right)$
Definition of a new unknown function	$f(\theta) = 1 - \sum_{k=1}^{\infty} c_k\, 2k \cos 2k\theta$
Fourier expansion of known function to be used as the kernel of [illegible] integral operator	$\ln 2\lvert\sin(\theta-\theta')\rvert = -\sum_{k=-\infty}^{+\infty} (1/2\lvert k\rvert)\, e^{ik(\theta-\theta')}$ (omitting $k = 0$)
Int[illegible] operator, L, defined by [illegible] on [illegible] periodic [illegible] $g(\theta)$	$L\left[g(\theta)\right] = \int_{-\pi}^{\pi} \ln 2\lvert\sin(\theta-\theta')\rvert\, g(\theta')\, d\theta'/\pi$

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EFFECTIVE RADIUS OF CONTROL BAR

Integral equation for f (θ) derived from boundary condition, line 1	$f_1(\theta) = \frac{3^{1/2}w}{2\lambda}\,\lvert\cos\theta\rvert\,(c + L\,[f(\theta)])$
Solve for f (θ) by recursion process	$f(\theta)/c = \frac{3^{1/2}w}{2\lambda}\lvert\cos\theta\rvert + \left(\frac{3^{1/2}w}{2\lambda}\right)^2 \lvert\cos\theta\rvert L \lvert\cos\theta\rvert + \left(\frac{3^{1/2}w}{2\lambda}\right)^3 \lvert\cos\theta\rvert L \lvert\cos\theta\rvert L \lvert\cos\theta\rvert + \cdots$
Define average value for an arbitrary periodic function, g (θ)	$\overline{g(\theta)} \equiv (1/\pi)\int_{-\pi/2}^{\pi/2} g(\theta)\,d\theta$
Average value of f (θ) follows from definition of f (θ), line 2	$\overline{f(\theta)} = 1.$
Applying condition in preceding line to solution already obtained for f (θ), line 6	$1/c = \frac{3^{1/2}w}{2\lambda}\,\overline{\lvert\cos\theta\rvert} + \left(\frac{3^{1/2}w}{2\lambda}\right)^2 \overline{\lvert\cos\theta\rvert L (\lvert\cos\theta\rvert)} + \cdots$
Asymptotic form of neutron density at considerable distance from control strip	$\ln(r/r_{eff}) \sim$ real part of $(c - i\theta)$ $\sim$ real part of (c-i arc sin 2 (x + iy)/w) $\sim \ln e^c + \ln(4r/w) = \ln(4e^c r/w)$
Value of effective radius of strip	$(w/4)e^{-c}$
Series for value of c used in construction of Fig. 22.4.15	$1/c = (3^{1/2}w/$[illegible]$) - (3^{1/2}w/$[illegible]$)^2($[illegible]$ - 2\ln 2) - (3^{1/2}w/$[illegible]$)^3$ [illegible] $(\ln 2)^2$ [illegible] $\ln 2 - ($[illegible]$) - ($[illegible]$^2/18$[illegible]$) - \cdots$

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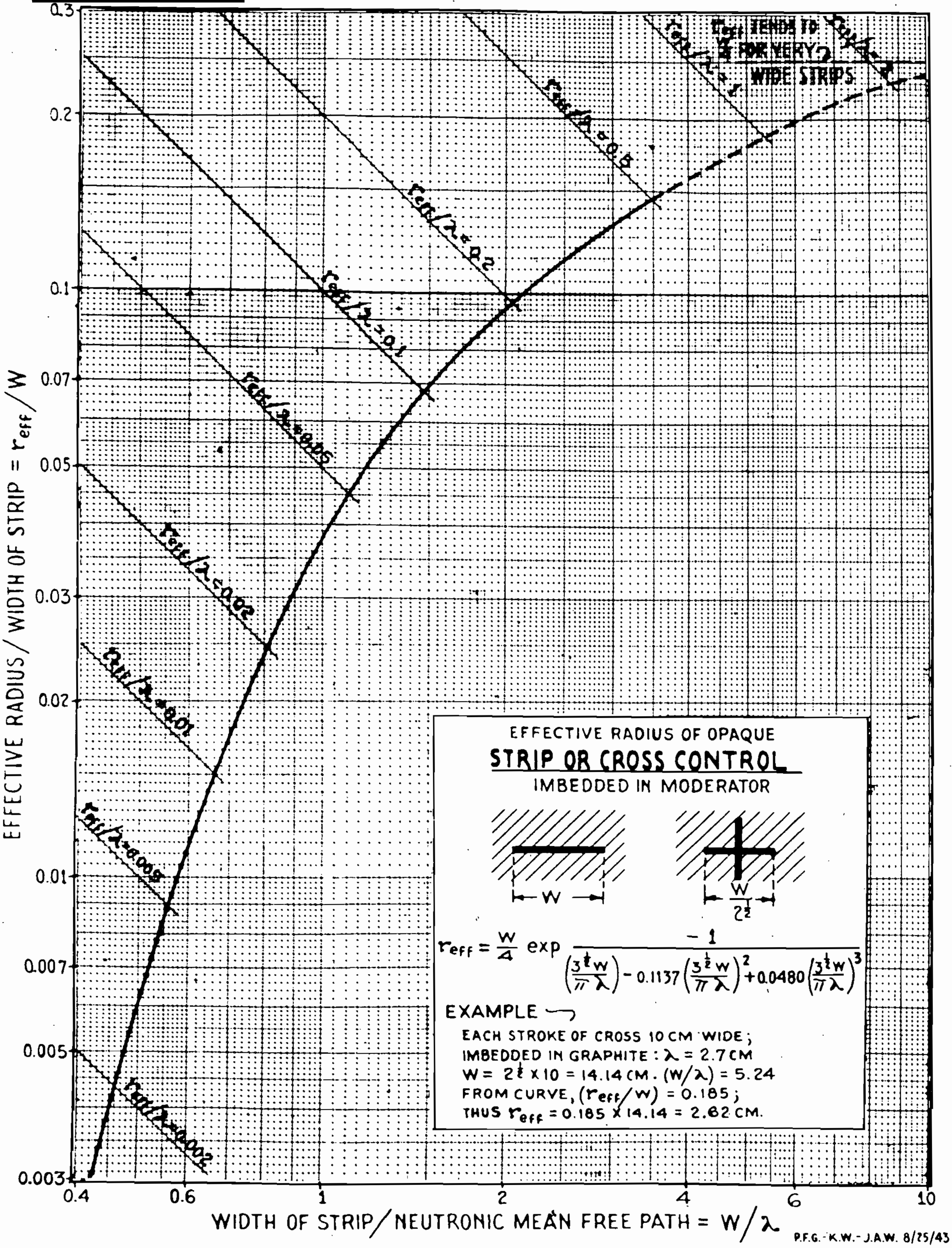
FIGURE-22.4.15



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Draw a plus sign, +, each stroke of which has a length t. From a die of this shape will be extruded a bar which has considerable value as a control. It requires for its motion only a small space in the pile. We shall take the effective radius of such a bar the value of the effective radius for a strip of width $w = 2^{\frac{1}{2}} t$. On this basis we are able to read off the effective radius from Fig. 22.4.15. This approximate procedure is justified, not by an accurate evaluation of the effective radius, which would be laborious, but by its correctness in the following limiting cases: (a) When the dimensions are small in comparison with a mean free path, and the bars are imbedded in the moderator, the effective radius is determined solely by the perimeter, as defined in Fig. 22.4.10. The perimeters will be equal for the two shapes when $2^{\frac{1}{2}} t = w$. (b) The same is true for bars of greater dimensions when they are passed in turn through a cavity whose opening is several times the size of the bars. (c) When the two bars are wide in comparison with a mean free path and are imbedded in the moderator the results of electrostatic theory can be applied, as in (22.4.13.a). The electrical potential of a charged strip varies at considerable distances as $\ln(4r/w)$. The potential of the extruded bar has the asymptotic form, $\ln(4r/2^{\frac{1}{2}} t)$. The two will therefore have the same effect when $2^{\frac{1}{2}} t = w$.

22.4.16 Bar with cross section of a plus sign

A bar of rectangular cross section has an effective radius whose value is readily estimated in the following special cases: (a) The bar is imbedded in the moderator and is small in comparison with a mean free path. Its effectiveness depends only on its perimeter. We use Eq. (22.4.12.d) and Fig. 22.4.10. (b) The bar is passed through a hole whose radius is considerably larger than the dimensions of the bar. The perimeter is again determining. Eq. (22.4.9.a) and Fig. 22.4.10 apply. (c) The bar is imbedded in the moderator and is large in comparison with a mean free path. One can apply electrostatic theory, as in 22.4.13. Professor E. P. Adams, of Princeton University, has kindly investigated this problem and communicated his results in the form of a letter. He finds that the effective radius of the bar is connected with its width and thickness by the parametric relations

22.4.17 Rectangular bar

$$\frac{\text{width}}{4\ \text{effective radius}} = E(\sin\theta) - \cos^2\theta\, K(\sin\theta)$$

$$\frac{\text{thickness}}{4\ \text{effective radius}} = E(\cos\theta) - \sin^2\theta\, K(\cos\theta), \qquad (22.4.17.a)$$

where E and K are the complete elliptic integrals. These relations are presented in convenient graphical form in Fig. 22.4.18. There an attempt is made to correct for the finite mean free path of the neutrons by diminishing the dimensions of the bar on each side and each face by the distance $\lambda/3^{\frac{1}{2}}$. This procedure is only reasonable when the dimensions are large in comparison with the mean free path. For the case when they are of the same order of magnitude as the mean free path, accurate values of the effective radius are not available.

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EFFECTIVE RADIUS OF CONTROL BAR

22.4.17

22.4.18 Weakly absorbing bar

In contrast to all the control bars so far considered in this section is the case of a partially absorbent rod. Let us first consider rods whose absorbing power is very small, so that a neutron has a high probability to go through the rod without capture. Denote by r_{av} the value of the distance from the axis to the point where the average neutron absorbed in the rod may be considered to have started its last flight. The number of neutrons absorbed in unit length of the control is then given by the product,

$$\left\{\begin{array}{l}\text{number, N, of}\\ \text{absorbent nuclei}\\ \text{per cm of length}\\ \text{of control}\end{array}\right\}\left\{\begin{array}{l}\text{absorption}\\ \text{cross section,}\\ \text{a, per nucleus}\end{array}\right\} v\, n(\text{at } r_{av}) \qquad (22.4.18.a)$$

The number absorbed will be replaced by an equal number diffusing in through the moderator,

$$2\pi r.\ (v\lambda/3).\ dn/dr. \qquad (22.4.18.b)$$

Expressing the neutron density in the form $n(r) = n_o \ln (r/r_{eff})$ we put the foregoing equality in the terms:

$$Na \ln (r_{av}/r_{eff}) = 2\pi\lambda/3. \qquad (22.4.18.c)$$

From this equation follows the effective radius of a weakly absorbing bar

$$r_{eff} = r_{av} \exp (-2\pi\lambda/3Na) \qquad (22.4.18.d)$$

The applications of this equation are listed in the following table:

Weakly absorbing control bar with absorption cross section Na per unit length	Expression for effective radius of bar	Reference for value of r_{av} inserted in 22.4.18.d
Bar of any cross sectional shape is imbedded in moderator and its girth is small in comparison with a mean free path	$0.462\,\lambda \exp(-2\pi\lambda/3Na)$	(22.4.12.d)
Bar of any cross sectional shape passed through the center of a hole of radius r_1	$r_1 \exp(-2\pi\lambda/3Na)$	(22.4.8.c)

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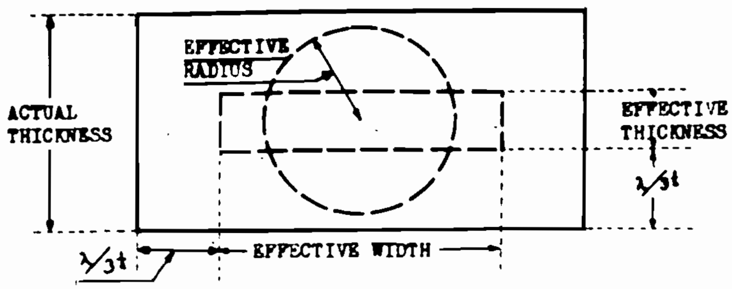
FIGURE-22.4.18

EFFECTIVE RADIUS OF AN

OPAQUE RECTANGULAR CONTROL ROD

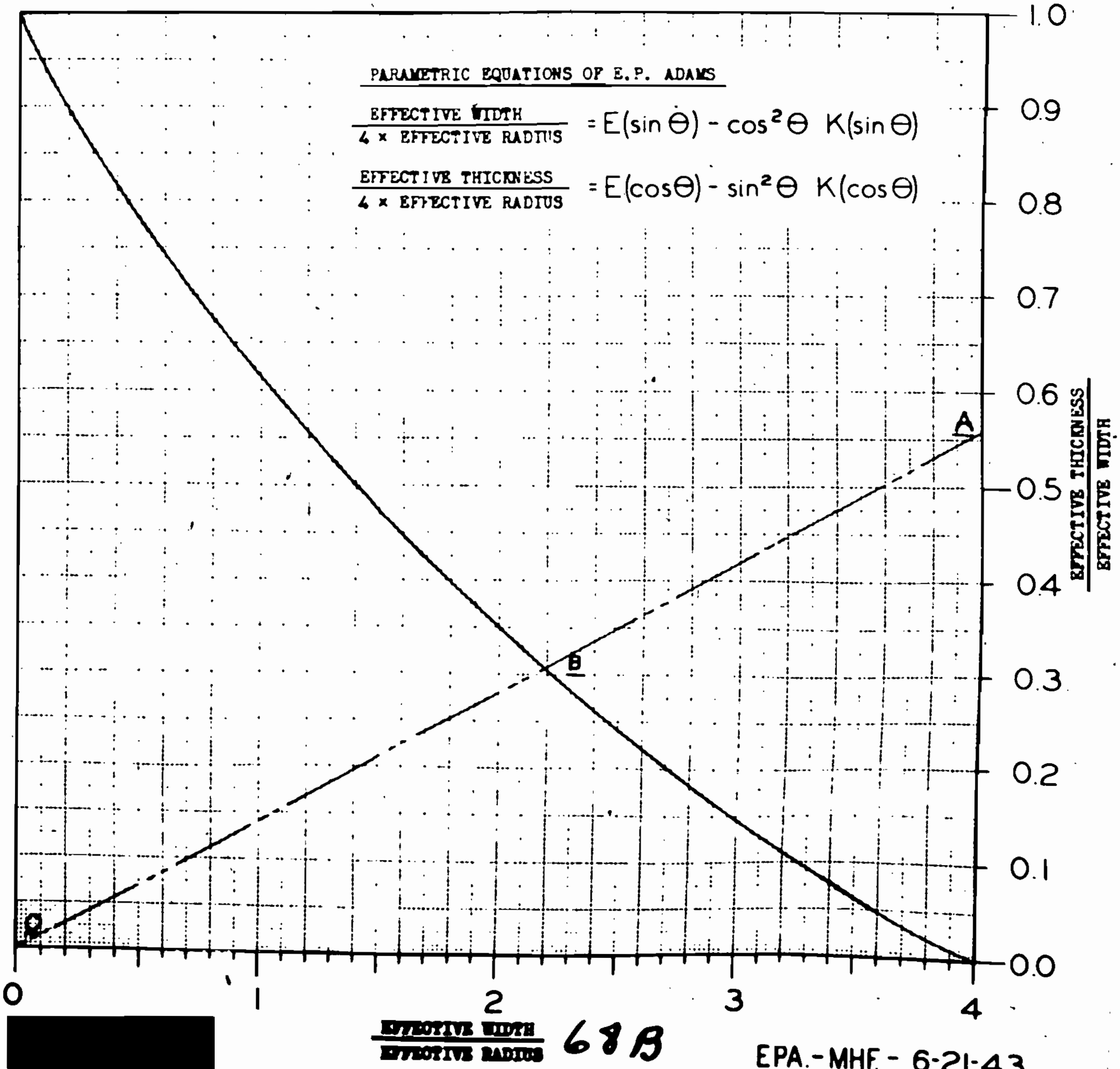
WHICH IS LARGE IN COMPARISON WITH THE MEAN FREE PATH, λ, OF THE NEUTRONS IN THE SURROUNDING MODERATOR



EXAMPLE

A boron steel bar 8 cm. by 12 cm. in graphite

λ = 2.56 cm., $\lambda/3^{\frac{1}{2}}$ = 1.48 cm.; thus the effective thickness is 5.04 cm. and the effective width is 9.04 cm. The ratio is 0.558 (point A on the chart). By means of line A-O find point B and read effective width/ effective radius = 2.19 . The effective radius is 4.13 cm.



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68 B

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Circular bar of radius r_0, or nearly circular bar of same average radius, imbedded in moderator	$r_0 \exp(-2\pi\lambda/3Na)$	(22.4.5.c)

22.4.19 Example of weakly absorbing rod

Example: An aluminum rod 0.3 cm in radius, of density 2.7 gm/cm^3, passes through a hole 5 cm square in a graphite uranium pile. What is the effective radius of the rod? Aluminum of the given density contains $(2.7\ \text{gm/cm}^3)(6.02 \times 10^{23}/27\ \text{gm}) = 6 \times 10^{22}$ nuclei per cm^3, each with a thermal absorption cross section about $0.22 \times 10^{-24}\text{cm}^2$. A neutron which traverses a diameter will be captured with a probability $6 \times 10^{22} \times 0.22 \times 10^{-24} \times 0.6 = 0.79 \times 10^{-2}$. The rod may therefore be considered to be weakly absorbing. We apply the second line of the preceding table. We find the equivalent radius of the hole from the equation $\pi r_1^2 = (5\ \text{cm})^2$, $r_1 = 2.82$ cm. The mean free path of neutrons in graphite is 2.7 cm. The absorption cross section per unit length of the rod is $6 \times 10^{22}\text{cm}^{-3}\pi(0.3\ \text{cm})^2 0.22 \times 10^{-24}\text{cm}^2 = 3.73 \times 10^{-3}$cm. The effective radius of the control rod is therefore $r_{eff} = 2.82\ \text{cm} \exp(-2\pi \times 2.7/3 \times 3.73 \times 10^{-3}) = 2.82\ \text{cm} \exp(-1517) \sim 3 \times 10^{-658}$cm. This number appears ridiculously small until one recalls that primary physical significance attaches to the neutron density, constant times $\ln(r/r_{eff})$, rather than to the effective radius itself. Thus, we find that the neutron population varies as $1517 + \ln(r/2.82\ \text{cm})$. The value increases by 1 part in 1517 when r goes from 2.82 cm to 2.718×2.82 cm $= 7.66$ cm. This variation is perfectly reasonable in view of the small absorbing power of the aluminum. Moreover, as we shall see in 22.5, the rod will produce an observable deficit in the reproduction factor of a pile.

22.4.20 Rod of intermediate absorbing power

When the absorption of neutrons in a control rod is neither very great nor very small, a more detailed treatment is required in order to obtain the effective radius. The neutron density varies within the rod as the Bessel function, $J_0(i\mathcal{H} r)$ (cf. Chap. 14). Here $\mathcal{H}$ is the macroscopic coefficient of absorption of neutrons in the material of the rod (cf. Chap. 11). The true absorption coefficient is given by the product of the concentration, c, and the absorption cross section, a. From the absorption coefficient we can find the number of neutrons absorbed per cm of length of the rod:

$$\frac{\text{number of neutrons absorbed per cm of length per second}}{\text{density of neutrons at surface of rod}} = \frac{cav\int^{r_0} J_0(i\mathcal{H} r)\, 2\pi r\,dr}{J_0(i\mathcal{H} r_0)}$$

$$= -2\pi r_0 i\, J_1(i\mathcal{H} r_0)\, cav/\mathcal{H} J_0(i\mathcal{H} r_0) \qquad (22.4.20.a)$$

The absorption will be balanced by the flux of neutrons diffusing inward through the moderator:

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$$\frac{\text{number of neutrons diffusing in per cm of length per second}}{\text{density of neutrons at inner face of moderator}} = \frac{2\pi r \cdot (v\lambda/3)\cdot(1/r)}{\ln (r_1/r_{eff})} \qquad (22.4.20.b)$$

For the problems of interest it will generally be a sufficient approximation to equate the density of neutrons at the surface of the rod to the density at the inner face of the moderator. This will certainly be the case when the two are in contact. We therefore equate expressions (a) and (b) and solve for the effective radius of the control rod, introducing the quantity N to represent the number of absorbing nuclei per unit length of the bar:

$$r_{eff} = r_1 \exp -\left\{\frac{2\pi\lambda}{3\,Na}\;\frac{i\varkappa r_o J_o(i\varkappa r_o)}{2J_1(i\varkappa r_o)}\right\} \qquad (22.4.20.c)$$

The second fraction in the exponent is plotted as a function of $\varkappa r_o$ in Chap. 15. It represents the ratio between the neutron density at the surface of the rod and the average density through the interior of the rod. Except for this factor, our result agrees with that derived in (22.4.18.d) for a weakly absorbing bar.

Example: For a structural purpose, an aluminum rod 5 cm in radius is passed through a snug hole in a graphite uranium pile. Regarded as a control rod, the aluminum has what effective radius? For absorption cross section of the aluminum we use $0.22 \times 10^{-24}\text{cm}^2$; and for total cross section, $1.6 \times 10^{-24}\text{cm}^2$. The nuclear concentration is $6 \times 10^{22}/\text{cm}^3$. For value of the macroscopic absorption coefficient (Chap. 11) we find $\varkappa = 6 \times 10^{22} \times 10^{-24} (3 \times 1.6 \times 0.22)^{\frac{1}{2}} (1 - 2 \times 0.22/5 \times 1.6) = 5.8 \times 10^{-2}\text{cm}^{-1}$. Thus the product $\varkappa r_o$ is 0.29. From the proper graph in Chap. 15 we find that the ratio of density at the face of the rod to average density through its interior is 1.011. The absorption cross section per unit of length is $Na = \pi (5\text{cm})^2\, 6 \times 10^{22}\text{cm}^{-3}\, 0.22 \times 10^{-24}\,\text{cm}^2 = 1.03$ cm. The effective radius of the rod is $r_{eff} = 5$ cm exp $(-2\pi \times 2.7 \text{ cm} \times 1.011/3 \times 1.03 \text{ cm}) = 0.020$ cm. 22.4.21 Example of rod of intermediate absorption

The foregoing discussion overlooks a number of conceivable types of control bar. However, almost any case to be anticipated can be treated with sufficient approximation by comparison with a known case. For example, there passes through a snug hole in a graphite-uranium pile a long strip of cadmium bent into the form of an equilateral triangle. It will be reasonable to take the effective radius of this bar to be equal to the effective radius of a circular rod having the same cross sectional area. Upper and lower limits on the correct figure can be obtained by comparison with rods whose bounding circles will either just circumscribe or just be circumscribed by the triangle. If the moderator is not in contact with the bar, one can calculate its effective radius on the two extreme assumptions that (a) the cavity is large in comparison with the size of the bar or (b) the moderator fills the cavity. It is then possible, with a little judgment, to estimate the position of the actual effective radius 22.4.22 Types of bar not considered in relation to the two extreme figures. Finally it is to be recalled that the deficit in reproduction factor caused by a control rod is more closely related to the logarithm of the effective radius than to the effective radius itself, so that an error of small percentage in the effective radius will not significantly affect the final conclusions.

22.4.23 Comparative survey of shapes of control bars

Table 22.4.23. Comparison of various control bars having the same effective radius.

Design of control bar	Approximate dimensions required to give	
	r_{eff} = 0.05 cm	r_{eff} = 5 cm
Cadmium rod imbedded in graphite	1.12 cm radius (22.4.12.d) 0.62 cm radius (22.4.5.c)*	6.3 cm radius (22.4.5.c)
Cadmium rod in hole 10 cm square through graphite	0.76 cm radius (22.4.9.a)	———
Cadmium strip in hole 10 cm square through graphite	2.4 cm wide (22.4.9.a)	———
Cadmium strip imbedded in graphite	3.5 cm wide (22.4.12.d) 2.2 cm wide (22.4.15)	23 cm wide (22.4.15)
Cadmium extruded in form of plus sign imbedded in graphite	maximum dimension 1.6 cm (22.4.16)	maximum dimension 16 cm (22.4.16)
Same in hole 10 cm square	1.7 cm (22.4.16)	———
Aluminum rod imbedded in graphite	5.4 cm radius (22.4.20.c)	———
Hole in graphite, filled with water	4.8 cm radius (22.4.20.c)	14 cm radius (22.4.20.c)
Hollow iron pipe 5 cm in radius, imbedded in graphite	wall thickness 0.23 cm (22.4.18.d)	———

*The upper figure is more nearly correct, as is evident on comparison with the case directly below. The figure 0.62 cm is derived on the assumption that ($\lambda/3^2$) (dn/dr) = n at the boundary. This result of diffusion theory is not valid for very small control rods. The derivation of (22.4.12.d) is free of this assumption.

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22.5 EFFECTIVENESS OF CONTROL RODS

22.5.1 Useable theory of effectiveness

The most satisfactory method of evaluating the effectiveness of control rods is that due to Wigner, Weinberg and Williamson* and summarized in mathematical form by Eq. (22.5.12.a) below. In this section we shall outline the derivation of the formula, show that it is physically reasonable, describe methods for using it, and check it against some of the available experimental material. This done we shall have completed the last step in presenting the theory of control rods in a useable form along the lines discussed in Section 22.3 and especially in the program laid down in Table 22.3.44.

22.5.2 Assumptions underlying theory

It is fortunate that the treatment of Wigner, Weinberg, and Williamson gives fairly accurate results because all other practical known methods of approximation tend to over-estimate the action of a control rod. The only approximations remaining in the present theory are believed to influence but little the accuracy of the results: the pile, apart from the control rods themselves, is treated as a homogeneous medium, and the moderation of neutrons in this medium is described by simplified model. Apart from these deficiencies the theory takes into account the following factors: (1) A difference exists between the actual radius of the rod and the radius which is effective in the action of the rod on thermal neutrons. The relationship between the effective radius and the actual constitution of the rod has been analyzed in detail in the preceding section. (2) The lowering of the density of thermal neutrons in the neighborhood of the rod is treated by the standard methods of diffusion theory. (3) It is taken into account that few fast neutrons are produced in regions where the thermal neutron density is low. The rod is assumed to have no direct action on the fast neutrons. Finding a simple way to take this fact into account was the principal advance made by the theory of Weinberg, Wigner, and Williamson on previous work. Murray** had evaluated with high accuracy the effectiveness of control rods in certain special cases of interest, but the method of calculation was too complicated for quick and general application. Plass*** had applied to the problem of safety control of the Hanford pile the standard lattice theory described in Chapter 15, adding however a correction for the non-uniform distribution of nascent thermal neutrons. Here again the treatment was accurate but complicated as compared to that which is now available.

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*E. P. Wigner, A. M. Weinberg, and R. R. Williamson, CP-1461 and Erratum, Efficiency of Control Rods which Absorb only Thermal Neutrons, 1944 February 24.

**F. H. Murray, CP-742, The Critical Value of the Reproduction Constant k for a Cylindrical Pile with a Control Rod which Absorbs only Thermal Neutrons, 1943 June 22.

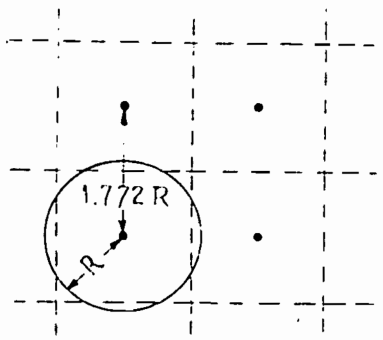
***Gilbert Plass, CP-964, Physics Research Report for Month ending 1943 September 25.

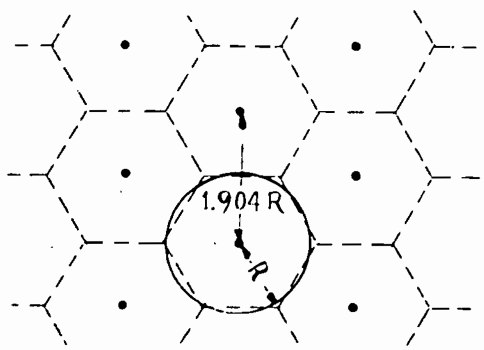
22.5.3 Absorption of fast neutrons assumed negligible

When a control rod acts appreciably on fast neutrons, its effectiveness can be calculated in a straight-forward way by simple modification of the theory to be presented below. In this case the control rod has to be described by two effective radii, one for fast neutrons and the other for thermal neutrons. However, unless the rod contains a large amount of water or other powerful moderator, its effect on fast neutrons is ordinarily negligible. It is therefore more a matter of practical utility than of mathematical difficulty when we limit the following discussion to rods which act only on thermal neutrons.

22.5.4 Problem is one of local effectiveness

In accordance with the principle of equivalence of Section 22.3, we shall be interested in the effective reduction in local multiplication factor induced by the rod in a region of the pile symmetrically circumscribed around it. For present purposes, it is sufficient to replace this zone of equivalence by a cylinder of the same area. The relationship between the radius, R, of this cylinder and the spacing between control rods is illustrated in the diagram at the left for two simple control rod patterns. It is possible to go beyond the problem of local effectiveness and to obtain by methods of analysis similar to those below the overall change in the reactivity of a pile of cylindrical form, along whose axis a single control rod is inserted. However, this case possesses such special symmetry that it is of little practical interest. Moreover, the results thus directly obtainable for such a specialized geometry follow also from the principle of equivalence as soon as one knows the equivalent reduction in local multiplication factor induced by a control rod of the same design in the circumjacent zone of control. It is therefore reasonable to limit the discussion to the fundamental problem of local effectiveness.





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In the analysis of the control problem we consider neutrons to belong to one or other of two categories: thermal neutrons and fast neutrons. Each category is described by the mean area of migration of a neutron during the interval of time while it remains a member of that group. One of these areas, the moderation area, A_{mod}, will be identified with the quantity of the same name familiar from the more detailed account of the moderation process given in Chapter 12. The other, the thermal migration area, A_{th}, is the quantity which enters into the standard theory of the

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pile of Section 15.5. Our approximation is to assume that we can treat the properties of a fast neutron as being invariable over the whole period before it is transformed into a thermal neutron. This approximation allows us to use for fast neutrons the same simple type of diffusion theory which we are accustomed to apply to thermal neutrons.

22.5.6 Moderation and absorption densities

We let the function of position, q_{th}, represent the number of thermal neutrons absorbed per cm^3 per second. This quantity will be independent of time in our present application because we are considering a pile in which the neutron output has attained a steady level. A second function of position, q_f, will represent the number of neutrons which leave the fast group by absorption or by becoming thermal in one cm^3 in the course of one second. In conformity with pile theory, we denote by p the ratio between the number of neutrons which become thermal and the number which cease to be fast; p represents the probability of escaping resonance absorption. Just as we are interested in the number, p, of slow neutrons per fast neutron so we shall also be concerned with the product, $f\eta\epsilon$, which represents the net number of new fast neutrons created in one generation per thermal neutron absorbed in the preceding generation. This product is directly expressible in terms of the local multiplication factor, k_{local}, of the pile medium and the probability, p, of escaping resonance absorption:

$$f\eta\epsilon = k_{local}/p \qquad (22.5.6.a)$$

22.5.7 Conditions for stationary state

In a stationary state we have for each group of neutrons an equation of the form:

$$\begin{pmatrix}\text{number of neutrons of}\\ \text{one type destroyed per}\\ \text{cm}^3\text{ per second}\end{pmatrix} = \begin{pmatrix}\text{number of neutrons of}\\ \text{that type diffusing into}\\ \text{that cm}^3\text{ per second}\end{pmatrix} + \begin{pmatrix}\text{number of neutrons}\\ \text{of that type created}\\ \text{per cm}^3\text{ per second}\end{pmatrix} \qquad (22.5.7.a)$$

We note that the rate of diffusion of neutrons is directly proportional to the product of the buckling of the neutron density and the migration area. Thus we obtain a straight-forward mathematical expression of the above equation:

$$q_f = A_{mod}(\partial^2 q_f/\partial x^2 + \partial^2 q_f/\partial y^2 + \partial^2 q_f/\partial z^2) + f\eta\epsilon q_t \qquad (22.5.7.b)$$

$$q_{th} = A_{th}(\partial^2 q_t/\partial x^2 + \partial^2 q_t/\partial y^2 + \partial^2 q_t/\partial z^2) + p\, q_f \qquad (22.5.7.c)$$

We can obtain a solution of this pair of equations by setting the thermal neutron capture density, q_{th}, equal to an arbitrary function, Z, of constant buckling.

$$\partial^2 Z/\partial x^2 + \partial^2 Z/\partial y^2 + \partial^2 Z/\partial z^2 + (\text{buckling})\cdot Z = 0, \qquad (22.5.7.d)$$

provided that we take the numerical value of the buckling to be one or other of the two solutions of the quadratic equation:

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$$\begin{vmatrix} (1 + A_{th} \cdot \text{buckling}) & -(k_{local}/p) \\ -p & (1 + A_{mod} \cdot \text{buckling}) \end{vmatrix} = 0 \qquad (22.5.7.e)$$

A solution of this equation may be called a "proper value" of the neutron buckling.

Table 22.5.8. SOLUTIONS OF THE QUADRATIC EQ.(22.5.7.e) FOR THE PROPER BUCKLING OF THE NEUTRON DENSITY IN THE NEIGHBORHOOD OF A CONTROL ROD

22.5.8 Solutions for proper buckling

The local multiplication factor is close to unity in all our applications. Thus, a special degree of accuracy attaches to the approximate expressions given for the buckling, which we shall therefore use hereafter.

Description	Mathematical Expression	
Accurate expression for solutions of quadratic equation for buckling	$-\frac{1}{2}(A_t^{-1} + A_m^{-1}) \pm \left[\frac{1}{4}(A_t^{-1} + A_m^{-1})^2 + A_t^{-1}A_m^{-1}(k-1)\right]^{\frac{1}{2}}$	
Approximate expressions for the two solutions	$(k_{local} - 1)/A$	$-(A_t^{-1} + A_m^{-1})$
Abbreviation for this value of buckling	b^2	None used
Designation employed for functions of position having the stated buckling	$Z(x,y,z)$	$\tilde{Z}(x,y,z)$
Physical terminology for function of this form in expression for neutron density	principal term	transient term

The general expression for the neutron density is a linear combination of two terms which respectively possess the two proper values of the buckling:

22.5.9 Expressions for moderation and absorption densities

$$q_f = gZ - h\tilde{Z} \qquad (22.5.9.a)$$

$$q_{th} = pgZ + (phA_{mod}/A_{th})\,\tilde{Z} \qquad (22.5.9.b)$$

Here g and h are numerical coefficients to be chosen to make the solutions satisfy the boundary conditions. The functions, Z and $\tilde{Z}$, may be considered to depend only upon the distance, r, from the center of the control rod in the problems of interest. They will therefore be represented by linear combinations of regular and irregular Bessel functions, the proper combinations also to be determined by the boundary conditions:

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Z is a linear combination of $J_o\ (br)$ and $N_o\ (br)$;

$\widetilde{Z}$ is a linear combination of $J_o\ (i\sqrt{A_t^{-1} + A_m^{-1}}\ r)$

and $i\ H_o^{(1)}\ (i\sqrt{A_t^{-1} + A_m^{-1}}\ r)$.

22.5.10 Zero slope at surface of zone of equivalence

Our problem is to relate the effectiveness of the control rod to the size of the zone of the pile on which it has to act. We shall take this zone to be equivalent to a cylinder of radius, R, at the surface of which the radial gradients of densities of fast and slow neutrons are both zero. This boundary condition for q_f and q_{th} implies the same boundary condition for the two constituent functions, Z and $\widetilde{Z}$:

$$dZ/dr = 0 \text{ at } r = R$$
$$d\widetilde{Z}/dr = 0 \text{ at } r = R \qquad (22.5.10.a)$$

We recall that the derivative of a Bessel function of order of zero is a Bessel function of the same type and of the first order. We thus deduce from (22.5.10.a) the linear combinations of Bessel functions required to give zero gradient at the boundary of the zone of control:

$$Z = \frac{N_1\ (bR)}{J_1\ (bR)}\ J_o\ (br) - N_o\ (br) \qquad (22.5.10.b)$$

$$\widetilde{Z} = i\ H_o^{(1)}(i\sqrt{A_m^{-1} + A_t^{-1}}\ r) - \frac{H_1^{(1)}(i\sqrt{A_m^{-1} + A_t^{-1}}R)}{-i\ J_1(i\sqrt{A_m^{-1} + A_t^{-1}}R)}\ J_o(i\sqrt{A_m^{-1} + A_t^{-1}}\ r)$$
$$(22.5.10.c)$$

22.5.11 Fast neutron density finite inside rod

A further condition on the neutron density follows from our assumption that the control rod has no appreciable action on fast neutrons. The density, q_f, must therefore behave in a perfectly regular way near the point $r = 0$, although the functions, Z and $\widetilde{Z}$, individually both become infinite there:*

$$Z \text{ (for small r)} \doteqdot \text{constant} + (2/\pi)\ \ln\ (1.122/br) \qquad (22.5.11.a)$$

$$\widetilde{Z} \text{ (for small r)} \doteqdot (2/\pi)\ \ln\ (1.122/\sqrt{A_m^{-1} + A_t^{-1}}\ r) + \text{constant.}$$
$$(22.5.11.b)$$

The two functions evidently have singularities of the same magnitude. They must therefore be subtracted to give an acceptable expression for the moderation density, q_f. We have therefore to set equal to each other the coefficients, g and h, in the linear combination of Eq. (22.5.9.a)

---

*Actually the solutions in question are not valid inside the control rods. The procedure outlined in (22.5.11) represents an incorrect but nevertheless reasonably accurate means to express the boundary conditions for fast neutrons at the surface of the rod.

Our last condition requires that the thermal neutron density, q_{th}, of Eq. (22.5.9.b) should extrapolate to zero at the effective radius, $r = r_{eff}$, of the control rod. Into the formula for q_{th}, we insert the expression (22.5.10.b and 22.5.10.c) for Z and $\tilde{Z}$, and we divide through by the common factor, $pg = ph$. In this way we arrive at the fundamental equation of Wigner, Weinberg and Williamson for the effectiveness of a control rod when acting on a zone of equivalent radius R. 22.5.12 Fundamental control equation

$$\left[N_o(br_{eff}) - \frac{N_1(bR)}{J_1(bR)} J_o(br_{eff})\right] = \frac{A_{mod}}{A_{th}} \left[i\ H_o^{(1)}(i\sqrt{A_m^{-1} + A_t^{-1}}\ r_{eff}) - \frac{H_1^{(1)}(i\sqrt{A_m^{-1} + A_t^{-1}}\ R}{-i\ J_1(i\sqrt{A_m^{-1} + A_t^{-1}}\ R}\ J_o\ (i\sqrt{A_m^{-1} + A_t^{-1}}\ r_{eff})\right] \qquad (22.5.12.a)$$

In this equation we may take as known quantities the quantity, R, the effective radius, r_{eff}, the partial migration areas, A_{mod} and A_{th}, and their sum A. Then we are in a position to solve for the unknown quantity, b, and thus to find the effective change in the local multiplication factor, δk_{local}, produced by the action of the control rod:

$$\delta k_{local} = b^2 A. \qquad (22.5.12.b)$$

That the effective change in local multiplication factor due to a control rod is properly represented by the last equation follows from a comparison of conditions before and after insertion. The pile under consideration had in the beginning a local multiplication factor, k_{local}. So far as the truly local behavior of the pile is concerned, the same value holds after the rod goes in. But at the boundaries of the zone of control, the neutron density now has zero radial gradient; and the longitudinal gradient of the neutron density has also been taken to be zero in our treatment. Consequently, the zone of control, from the point of view of the surrounding portions of the pile, is equivalent to a homogeneous medium in which the neutron density is constant. The apparent buckling is therefore zero, and the effective local multiplication factor is 1. We conclude that the rod has lowered the effective local multiplication factor by the difference between k_{local} and 1. This difference is equal to the product b^2A by virtue of our original definition of b^2 (Table 22.5.8). Consequently, Eq.(22.5.12.a) for b provides a straight-forward means to evaluate the effectiveness of a control rod under the conditions assumed in our treatment. 22.5.13 Interpretation of control equation

We will obtain from Eq.(22.5.12.a and 22.5.12.b) a proper account of the effect of a control rod whether or not the overall variation of neutron density satisfies the conditions of our derivation. There we considered for sake of convenience a stationary state, zero longitudinal buckling, and zero radial gradient at the boundary of the zone of control. But according to the principle of equivalence of Section 22.3, the value 22.5.14 General applicability of control equation of the effective change in local multiplication factor will be substantially the same whether or not there is an appreciable longitudinal buckling of the neutron density, whether or not there is a radial gradient at the boundary of the zone of control after the rod enters and whether the neutron activity is constant, rising or falling. The quantity, δk_{local}, is determined by the properties of the control rod and by the size of the zone of control but is independent of conditions in the rest of the pile.

22.5.15 Two reasonable tests

It is in order to investigate the reasonableness of the complicated control (Eq. 22.5.12.a) before proceeding to apply it. We shall, therefore, test whether it gives correct results in two simple limiting cases: (1) a control rod large in comparison with the square root of the migration area; (2) many very small control rods.

22.5.16 Limiting case of large rods

When the control rod and the size of the zone of control are both large, we have a slowly varying function of position for the density of thermal neutrons, and therefore also for the density of sources of fast neutrons. The migration of fast neutrons during moderation will not alter the essential character of the distribution function. This, being zero for thermal neutrons at the effective radius of the control rod, will be nearly zero for fast neutrons at the same point. In this sense the effective radius of a large control rod can be considered as practically the same for fast and slow neutrons. We can therefore class all neutrons together and describe them by a single function, n (r), of the form:

$$n(r) = c' J_o(br) + c'' N_o(br). \qquad (22.5.16.a)$$

In order that such an expression should vanish at $r = r_{eff}$, it is necessary that

$$-c'/c'' = N_o(br_{eff})/J_o(br_{eff}); \qquad (22.5.16.b)$$

and in order for the neutron density to have zero gradient at $r = R$, it is necessary that

$$-c'/c'' = N_1(bR)/J_1(bR). \qquad (22.5.16.c)$$

Consequently, both requirements will be satisfied together only when the buckling, $b^2 = \delta k_{local}/A$, meets the condition

$$\frac{N_o(br_{eff})}{J_o(br_{eff})} - \frac{N_1(bR)}{J_1(bR)} = 0. \qquad (22.5.16.d)$$

This simplified equation for the effectiveness of a large control rod is a limiting case of the general control rod Eq. (22.5.12.a). To see this, it is only necessary to note that the Hankel functions on the right hand side of that equation become exponentially small for large values of their arguments, $\sqrt{A_m^{-1} + A_t^{-1}}\, r_{eff}$ and $\sqrt{A_m^{-1} + A_t^{-1}}\, R$. On neglecting the

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right hand side of the equation, we obtain the relation (22.5.16.d) in which there is no reference to the difference in properties of fast and slow neutrons. We conclude that we have only to use the one-group theory of neutrons and simply to treat a large rod as equivalent to an internal boundary of the pile in order to obtain an adequate account of the effectiveness of the rod. We have already applied this principle of analysis in 22.3.72 to the case of a large cylindrical rod on the axis of a cylindrical pile.

22.5.17 Limiting case of very small rods

The opposite limiting case of a pile controlled by a large number of narrow cadmium strips presents a straight-forward question of utilization of thermal neutrons. Consider a 1 cm length of the zone of control of one of these strips. The uranium or moderator nuclei in this volume in the course of 1 second absorb neutrons to the number

$$\pi R^2 \begin{pmatrix}\text{number}\\ \text{of nuclei}\\ \text{per unit}\\ \text{volume}\end{pmatrix} \begin{pmatrix}\text{absorption}\\ \text{cross}\\ \text{section per}\\ \text{nucleus}\end{pmatrix} \begin{pmatrix}\text{density}\\ \text{of}\\ \text{thermal}\\ \text{neutrons}\end{pmatrix} \begin{pmatrix}\text{velocity}\\ \text{of}\\ \text{thermal}\\ \text{neutrons}\end{pmatrix}$$

where πR^2 is the cross sectional area of the zone of control of the strip. A 1 cm length of the strip itself absorbs neutrons at the rate

$$(\text{perimeter})\ \tfrac{1}{4} \begin{pmatrix}\text{density}\\ \text{of}\\ \text{thermal}\\ \text{neutrons}\end{pmatrix} \begin{pmatrix}\text{velocity}\\ \text{of}\\ \text{thermal}\\ \text{neutrons}\end{pmatrix},$$

the factor $\frac{1}{4}$ taking account of the distribution in direction of neutron velocities (Chapter 11). Comparing the absorption due to the strip with the number of neutrons available for absorption, we find the loss in local multiplication factor:

$$\delta k_{local} = \frac{(\text{perimeter of strip})}{4\pi R^2 \begin{pmatrix}\text{thermal absorption cross section of}\\ \text{unit volume of normal pile material}\end{pmatrix}} \qquad (22.5.17.a)$$

We can translate this result into terms of the thermal migration area, A_{th}, and the mean free path, λ, of a thermal neutron with respect to scattering by means of the relationship of Section 15.5:

$$A_{th} = \frac{\lambda}{3\begin{pmatrix}\text{thermal absorption cross section of}\\ \text{unit volume of normal pile material}\end{pmatrix}} \qquad (22.5.17.b)$$

The effectiveness of the strip is therefore:

$$\delta k_{local} = \frac{(\text{perimeter of control})}{4\lambda}\ \frac{A_{th}}{\pi R^2} \qquad (22.5.17.c)$$

We now test whether the general formula for control rod effectiveness due to Wigner, Weinberg and Williamson reduces to the above simple result in the case of narrow closely spaced control strips. We take advantage of two simplifying factors: 22.5.18 Check on general control equati

(1) The effective radius of a small control is given, according to Eq. (22.4.12.d) by the expression:

$$r_{eff} = 0.462\lambda \quad \exp(-8\pi\lambda/3 \text{ perimeter}) \qquad (22.5.18.a)$$

(2) In all the Bessel functions in the fundamental control equation the arguments are small, so that we can use the approximations:

$$J_o(br_{eff}) \doteq 1$$

$$N_1(bR)/J_1(bR) \doteq -4/\pi b^2R^2$$

$$N_o(br_{eff}) \doteq -(2/\pi) \ \ln(1.122/br_{eff})$$

$$i\,H_o^{(1)}(i\sqrt{A_m^{-1} + A_t^{-1}}\, r_{eff}) \doteq (2/\pi)\ \ln(1.122/\sqrt{A_t^{-1} + A_m^{-1}}\, r_{eff})$$

$$J_o(i\sqrt{A_m^{-1} + A_t^{-1}}\, r_{eff}) \doteq 1$$

$$H_1^{(1)}(i\sqrt{A_m^{-1} + A_t^{-1}}R)/i\ \ J_1(i\sqrt{A_m^{-1} + A_t^{-1}}R) \doteq 4/\pi(A_m^{-1} + A_t^{-1})\ R^2$$

(22.5.18.b)

We enter these expressions into the fundamental control Eq.(22.5.12.a), multiply through by the factor $\pi A_{th}/4A$, rearrange, and find that the change in multiplication factor is given by the following equation:

$$(A_{th}/\delta k_{local}\ R^2)^{\#} + (A_{th}A_{mod}^2/A^2R^2) = (4\pi\lambda/3 \text{ perimeter})^{\#}$$
$$+ (A_{mod}/2A)\ \ln(2.43/\sqrt{A_m^{-1} + A_t^{-1}}\lambda) + (A_{th}/2A)\ \ln(2.43/b\lambda)$$

(22.5.18.c)

When the cadmium strips are quite narrow, then the terms marked with #'s become very great in comparison with the other terms in this equation. Neglecting the latter terms, we find that the general equation reduces to the special equation (22.5.17.c) for the effectiveness of narrow cadmium strips. We see that the action of a control in this limiting case reduces to a simple matter of utilization of thermal neutrons, having nothing to do with the migration which takes place during moderation. However, the accuracy of our evaluation of the thermal utilization in this limiting case is obviously limited by our assumption that the pile can be treated as homogeneous. In a lattice structure a fine cadmium May, 1944 strip will actually have a different control power according as it is located on the surface of the uranium or in the moderator midway between lumps of fissionable material. Therefore, in estimating the loss in k in an inhomogeneous pile due to absorbers fine enough to distort the neutron density only slightly, we shall replace the limiting form (22.5.17.c) of the general control equation by the following more nearly accurate formula:

$$\delta k = \frac{\begin{pmatrix}\text{neutron}\\ \text{density}\\ \text{of}\\ \text{absorber}\end{pmatrix}\begin{pmatrix}\tfrac{1}{4}\text{ of the surface of the absorber, if it is}\\ \text{opaque; or, if it consists of scattered nuclei,}\\ \text{the sum of their absorption cross sections}\end{pmatrix}}{\begin{pmatrix}\text{average}\\ \text{neutron}\\ \text{density at}\\ \text{uranium}\end{pmatrix}\begin{pmatrix}\text{sum of cross}\\ \text{sections of all}\\ \text{uranium nuclei}\\ \text{in zone of}\\ \text{equivalence}\end{pmatrix} + \begin{pmatrix}\text{average}\\ \text{neutron}\\ \text{density in}\\ \text{moderator}\end{pmatrix}\begin{pmatrix}\text{sum of cross}\\ \text{sections of all}\\ \text{moderator nuclei}\\ \text{in zone of}\\ \text{equivalence}\end{pmatrix}} \tag{22.5.18.d}$$

22.5.19 Comparison of large and small rods

We have confirmed mathematically the reasonableness of the general control equation in the two limiting cases of very large and very small rods, and are now in a position to summarize the basis of the check in a simple physical picture. Fig. 22.5.21 shows the distribution of fast and slow neutron transformation densities, q_f and q_{th}, in the two extreme cases, together with a similar curve due to Weinberg, Wigner and Williamson for an intermediate case not much different from that realized in practice. In the case of the large control rod we see how little difference it makes whether the rod does or does not act on fast neutrons. In the case of the very small rod, however, the situation is quite different. The fast neutron density is practically constant over the whole of the zone of action when the control absorbs only thermal neutrons, and the rod is roughly only half as effective in this case as it would be if it absorbed both fast and slow neutrons. The following table presents in more detail the chief points of interest in the two limiting cases.

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Table 22.5.20 EFFECTIVENESS OF VERY SMALL AND VERY LARGE CONTROL RODS

The neutron density in the neighborhood of these rods is indicated in Fig.22.5.21. The following values have been adopted for the physical quantities entering into these purely illustrative examples. Migration area, $A = 662\ cm^2$, $A_{th} = 331\ cm^2$, $A_{mod} = 331\ cm^2$, $\sqrt{A_t^{-1} + A_m^{-1}} = 0.078\ cm^{-1}$. The mean free path of a thermal neutron with respect to scattering in the pile medium is taken to be $\lambda = 2.7$ cm. The absorption cross section per unit volume of pile medium is given by the expression, $\lambda/3A_{th} = 2.7\ cm/3 \times 331\ cm^2 = 0.00272\ cm^2/cm^3$. Inspection of the values of δk for the small control rod shows that this rod absorbs almost as many neutrons as it would if the density of available neutrons were not affected by its presence. It is almost exactly half as effective as it would be if it absorbed both fast and slow neutrons. For the case of the large rod the situation is quite the reverse. The effectiveness varies little whether the rod absorbs only thermal neutrons or fast neutrons as well. In either case, however, the action of the large control rod is much diminished by its effect in lowering the density of neutrons near it.

Characteristics of limiting cases	Very small rods	Very large rods
Actual radius of rod	0.30 cm	30 cm
Effective radius of control rod	$1.25\ e^{-12}$ cm	28.4 cm
Radius of zone of control	30 cm	300 cm
Thermal neutron absorption cross section of all the nuclei of the pile medium contained in a unit length of the zone of control	7.69 cm^2/cm	769 cm^2/cm
$\frac{1}{4}$ perimeter of control rod, a measure of the equivalent cross section of a unit length of the rod with respect to thermal neutrons	0.471 cm^2/cm	47.1 cm^2/cm
Ratio of last two quantities gives loss in k to be expected from control rod on assumption it absorbs no fast neutrons and that thermal neutron density is constant over zone of control	$\delta k = 0.0613$	$\delta k = 0.0613$
Loss in k calculated by accurate theory (Eq. 22.5.12.a)	$\delta k = 0.0510$	$\delta k = 0.0083$
Loss in k which would be produced if rods were black to fast as well as to slow neutrons (Eq.22.5.16.d)	$\delta k = 0.1020$	$\delta k = 0.0087$

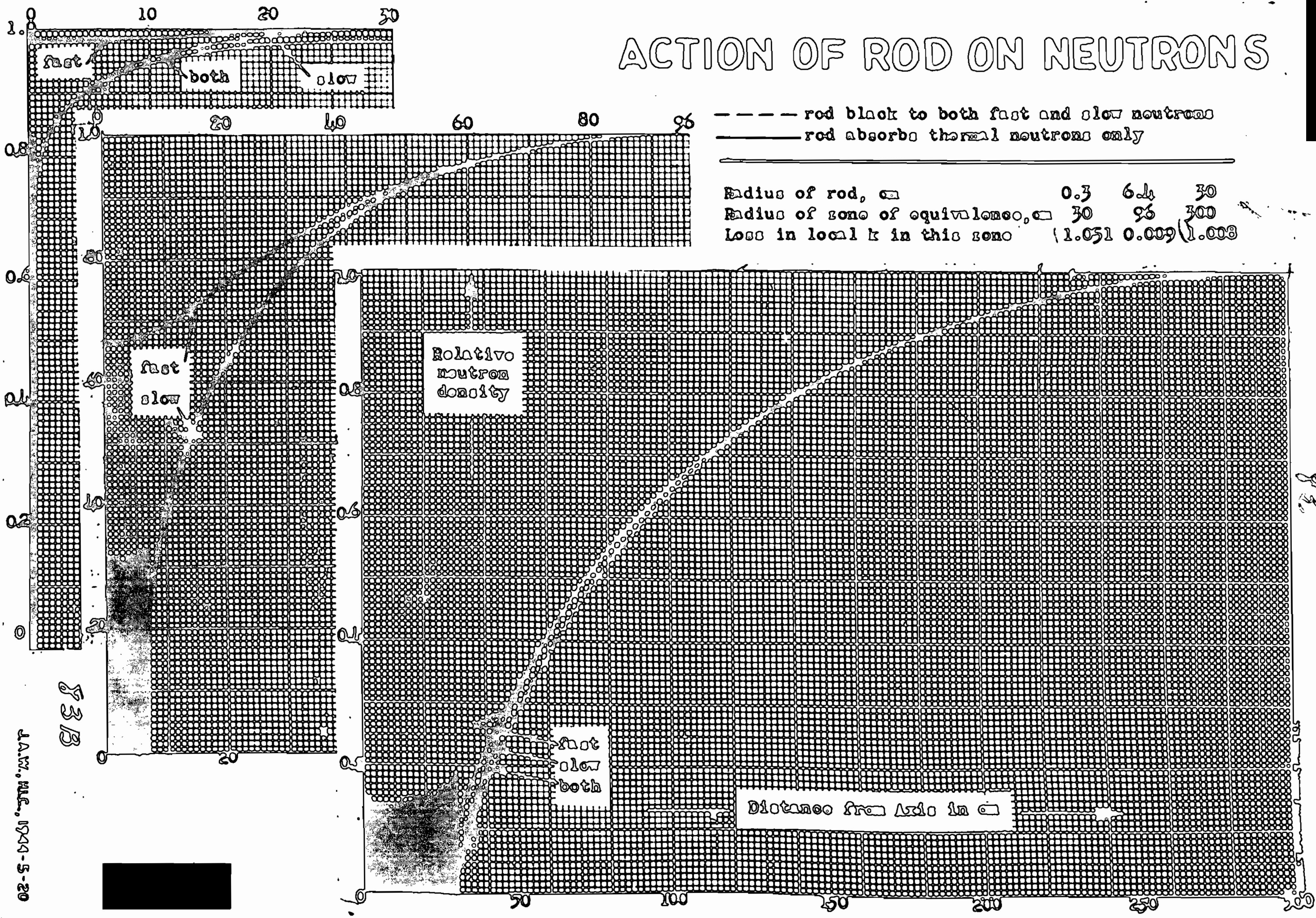

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FIGURE-22.5.21

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FIGURE - 22.5.23

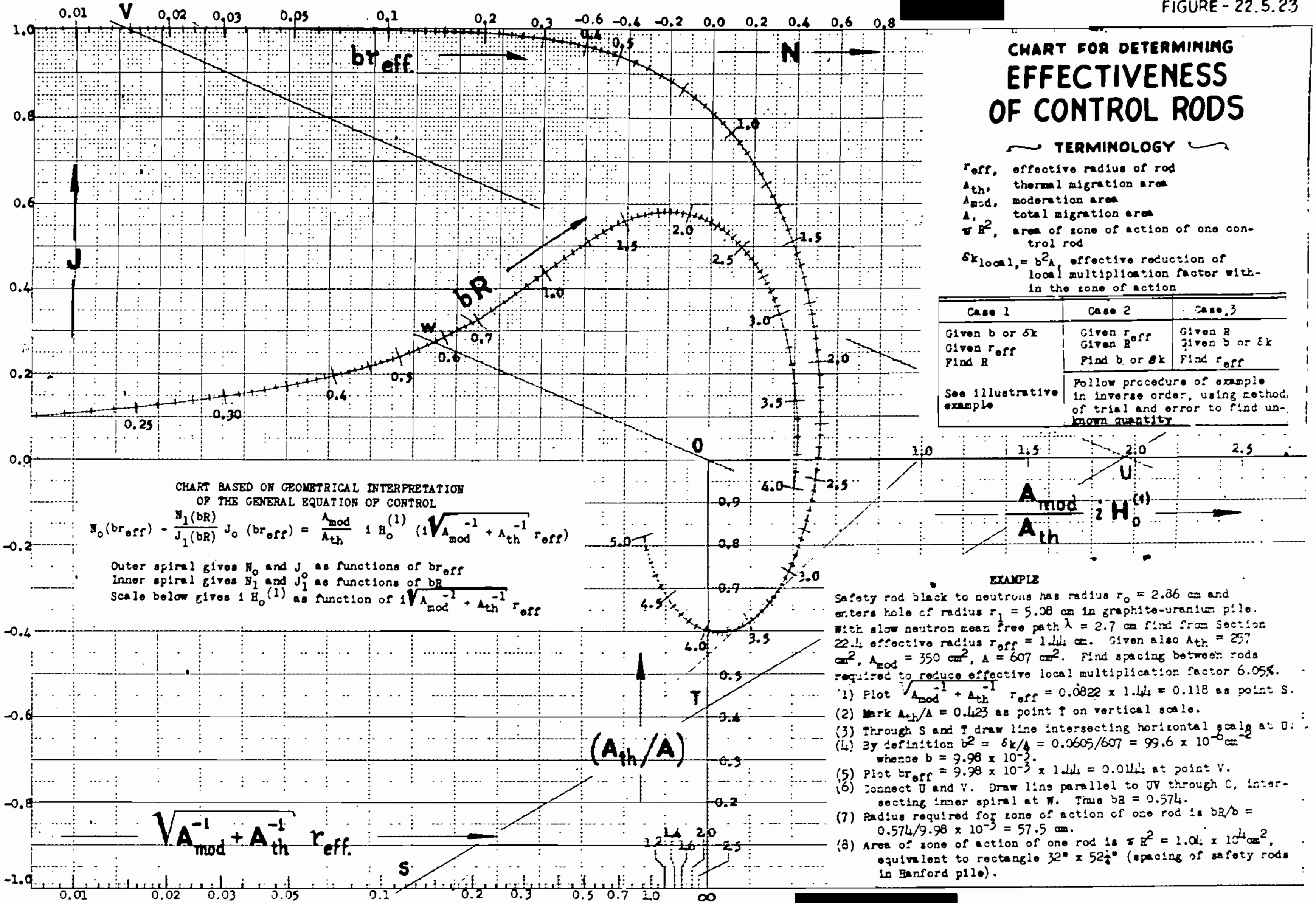

CHART FOR DETERMINING EFFECTIVENESS OF CONTROL RODS

TERMINOLOGY

r_{eff}, effective radius of rod
A_{th}, thermal migration area
A_{mod}, moderation area
A, total migration area
πR^2, area of zone of action of one control rod
$\delta k_{local}, = b^2 A$, effective reduction of local multiplication factor within the zone of action

Case 1	Case 2	Case 3
Given b or δk Given r_{eff} Find R	Given r_{eff} Given R Find b or δk	Given R Given b or δk Find r_{eff}
See illustrative example	Follow procedure of example in inverse order, using method of trial and error to find unknown quantity	

CHART BASED ON GEOMETRICAL INTERPRETATION OF THE GENERAL EQUATION OF CONTROL

$$N_0(br_{eff}) - \frac{N_1(bR)}{J_1(bR)} J_0(br_{eff}) = \frac{A_{mod}}{A_{th}}\ i\,H_0^{(1)}\left(i\sqrt{A_{mod}^{-1}+A_{th}^{-1}}\ r_{eff}\right)$$

Outer spiral gives N_0 and J_0 as functions of br_{eff}
Inner spiral gives N_1 and J_1^0 as functions of bR
Scale below gives $i\,H_0^{(1)}$ as function of $i\sqrt{A_{mod}^{-1}+A_{th}^{-1}}\ r_{eff}$

EXAMPLE

Safety rod black to neutrons has radius $r_0 = 2.86$ cm and enters hole of radius $r_1 = 5.08$ cm in graphite-uranium pile. With slow neutron mean free path $\lambda = 2.7$ cm find from Section 22.4 effective radius $r_{eff} = 1.44$ cm. Given also $A_{th} = 257$ cm^2, $A_{mod} = 350$ cm^2, $A = 607$ cm^2. Find spacing between rods required to reduce effective local multiplication factor 6.05%.

(1) Plot $\sqrt{A_{mod}^{-1}+A_{th}^{-1}}\ r_{eff} = 0.0822 \times 1.44 = 0.118$ as point S.
(2) Mark $A_{th}/A = 0.423$ as point T on vertical scale.
(3) Through S and T draw line intersecting horizontal scale at U.
(4) By definition $b^2 = \delta k/A = 0.0605/607 = 99.6 \times 10^{-6}$ cm^{-2} whence $b = 9.98 \times 10^{-3}$.
(5) Plot $br_{eff} = 9.98 \times 10^{-3} \times 1.44 = 0.0144$ at point V.
(6) Connect U and V. Draw line parallel to UV through O, intersecting inner spiral at W. Thus $bR = 0.574$.
(7) Radius required for zone of action of one rod is $bR/b = 0.574/9.98 \times 10^{-3} = 57.5$ cm.
(8) Area of zone of action of one rod is $\pi R^2 = 1.04 \times 10^4$ cm^2, equivalent to rectangle 32" x 52½" (spacing of safety rods in Hanford pile).

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22.5.22

Evaluation of the loss in k due to a control rod is relatively simple when the rod is either very small or very large, so that we can employ limiting equations (22.5.18.d) or (22.5.16.d). However, it is usually necessary to apply the general control equation (22.5.12.a), which contains eight different Bessel functions. Fortunately, the second term on the right hand side is quite negligible in most practical cases, so that three Bessel functions drop out. The remaining equation may then be solved by a process of trial and error for whatever quantity happens to be the unknown. Alternatively it is possible to express the formula in nomographic form, as in Fig. 22.5.23, and solve graphically for the unknown. The example in the figure illustrates the method of solution. 22.5.22 Method of application of control theory

Having put the theory of controls in a useable form, we shall apply it to discuss cases of interest. In all of the following examples the reactor is constructed of graphite and uranium. The mean free path for diffusion of thermal neutrons in the graphite is taken to be 2.7 cm. Values of moderation area and thermal migration area vary according to design of pile and presence or absence of coolant. Even for identical conditions the adopted values of the migration areas sometimes differ from one example to another, reflecting the uncertain history of a quantity which has been unusually difficult to calculate reliably. Comparison with experiment has been made when possible. Data on the control systems of the Clinton operating pile and Hanford test pile have not been included because of the complication of deducing from the observed control power with the irregular loading pattern the absolute effectiveness of the rods* 22.5.24 Examples of control rod effectiveness

<u>Safety rods for Hanford pile</u>. Outside diameter $2\frac{1}{4}$ inches, outside radius 2.86 cm. Walls 3/16 inch thick, contain $1\frac{1}{2}$ percent of boron by weight, making rod effectively opaque to thermal neutrons. Rod enters hole in graphite with diameter of 4 inches, radius 5.08 cm. Effective radius from Eq. (22.4.8.c) is $r_{eff} = r_1 \exp(-4\lambda/3r_o) = 5.08 \exp(-4 \times 2.7/3 \times 2.86) = 1.44$ cm. Zone of action of rod 32 by $50\frac{1}{4}$ inches, equivalent in area to a circle of radius $R = 57.5$ cm. Thus $bR/br_{eff} = 57.5/1.44 = \underline{39.9}$. Adopt $A_{mod} = 350\ cm^2$, $A_{th} = 257\ cm^2$, $A = 607\ cm^2$, $A_{th}/A = \underline{0.423}$, $(A_m^{-1} + A_t^{-1})^{\frac{1}{2}} = 0.0822\ cm^{-1}$; this times r_{eff} gives $\underline{0.118}$. Use underlined figures in nomograph of Fig. 22.5.23, finding $\underline{bR = 0.574}$. This divided by R gives $b = 9.98 \times 10^{-3}$. Loss in local multiplication factor is $\delta k = b^2 A = 99.6 \times 10^{-6} \times 607 = \underline{0.0605}$. See 22.3.57-61 for discussion of overall loss in k due to 29 such rods. 22.5.25 Hanford safety rods

<u>Simulated safety rod</u>. We carry out the following calculations with the idea of comparing them with experimental results reported by Morrison in CP-1389 and discussed by Wigner, Weinberg and Williamson in CP-1461, 1944 February 24. Hanford lattice as in preceding example. Tube covered with cadmium to give rod a radius $r_o = 2.08$ cm, imbedded in graphite. Effective radius by Eq. (22.4.5.c) is $r_o \exp(-\lambda/3^{\frac{1}{2}} r_o) = 0.99$ cm. Work by Placzek and Seidel reported in MT-5 would suggest a lower figure, $r_o \exp(-0.710\lambda/r_o) = 0.83$ cm, which we adopt for sake of conservativeness. The comparison with experiment is most conveniently carried out by asking how small a zone of action is required to make the reduction in local k equal to 0.191. We have $b^2 = \delta k/A = 0.191/607 = 315 \times 10^{-6}\ cm^{-2}$, whence $b = 17.8 \times 10^{-3}\ cm^{-1}$ and $br_{eff} = 17.8 \times 10^{-3} \times 0.83$ 22.5.26 Simulated safety rod - theory

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*However, a letter from P. F. Gast to J. A. Wheeler, 1944 May 23, reports 270 ih calculated control power of upper regulating rod of Clinton pile as compared to 272 ih measured by H. Jones (CP-1300).

$= \underline{0.0147}$. This quantity, together with $A_{th}/A = \underline{0.423}$ and $(A_m^{-1} + A_t^{-1})^{\frac{1}{2}}$ $r_{eff} = 0.0822 \times 0.83 = \underline{0.068}$ is used in Fig. 22.5.23 to deduce $bR = 0.55$. Thence we conclude that the zone of action should have a radius $\underline{R = 31 \text{ cm}}$. However, according to the experiments, the rod gives the desired degree of control when it acts on a zone of radius R = 33 cm. Thus the effectiveness of the rod is observed to exceed slightly the calculated value, possibly partly due to absorption of some resonance neutrons by the cadmium.

22.5.27 Simulated safety rod experiment

The figure R = 33 cm for radius of zone of action is deduced from Morrison's exponential experiment by reasoning quite independent of control rod theory, as follows. Square pile of extrapolated side 272.2 cm; has same transverse buckling, $266.5 \times 10^{-6} \text{cm}^{-2}$, as cylindrical pile of radius 147.3 cm. Equivalence of square to cylinder demonstrated by Wigner, Weinberg and Williamson in CP-1461. Neutron density falls off as ionization chamber moves vertically upward in the exponential pile, reaching fraction 1/2.71828 of original value in 82.7 cm when rod is absent, in 71.6 cm when rod is present. Hence, longitudinal buckling changed by $(1/71.6)^2 - (1/82.7)^2 = (195 - 147) \times 10^{-6} = 48 \times 10^{-6} \text{cm}^{-2}$. Total buckling not altered. Hence transverse buckling after rod enters is $b^2 = (266.5 + 48) \times 10^{-6} = 315 \times 10^{-6} \text{cm}^{-2}$, corresponding to a value $b = 17.8 \times 10^{-3} \text{cm}^{-1}$. Radial neutron distribution has stated buckling and vanishes at r = 147.3 cm, or $br = 17.8 \times 10^{-3} \times 147.3 = 2.615$. It is therefore proportional to $J_0(br) - N_0(br)\, J_0(2.615)/N_0(2.615)$. Maximum of this function occurs for br = 0.583, or r = R = 32.9 cm. This quantity represents radius of zone of control, within which by the principle of equivalence the neutron density can be considered to have been reduced to constancy: effective transverse buckling in this zone is zero; actual transverse buckling in this zone is 315×10^{-6}; hence effective reduction in local k is $315 \times 10^{-6} \times 607 = 0.191$.

22.5.28 Poison slugs

Poison slugs to flatten neutron distribution over central portion of Hanford pile. Case where 1500 columns of metal are loaded, giving pile an effective radius of 510 cm, and resultant overall excess multiplication factor is 1 percent. Same migration area as in first example above. Transverse buckling to be compensated over region of flattening is $b^2 = (2.4048/510 \text{ cm})^2 + (0.01/\text{migration area of } 607 \text{ cm}^2) = (22.28 + 16.46) \times 10^{-6} = 38.7 \times 10^{-6} \text{cm}^{-2}$. Thus $b = 6.22 \times 10^{-3} \text{cm}^{-1}$. Slugs 10 percent cadmium, 90 percent lead, 1.7 cm radius, black to thermal neutrons. Effective radius calculated as in preceding example, neglecting effect of surrounding water film: $r_{eff} = 1.7 \text{ cm} \exp(-0.710 \times 2.7 \text{ cm}/1.7 \text{ cm}) = 0.55$ cm. Thus $(A_m^{-1} + A_t^{-1})^{\frac{1}{2}} r_{eff} = 0.0822 \times 0.55 = \underline{0.0452}$ and $br_{eff} = \underline{3.42 \times 10^{-3}}$. These values, together with $A_{th}/A = \underline{0.423}$ used in Fig. 22.5.23. Deduce bR = 0.485. Divide by b and obtain $\underline{R = 78 \text{ cm}}$, radius of zone of action of each column of poison slugs, corresponding to an area, πR^2, equal to $1.91 \times 10^4 \text{cm}^2$. This area spans 42 lattice units. The whole zone of flattening may be shown to contain 302 tubes. Hence 302/42 or 7 columns of poison slugs are considered necessary to accomplish the flattening.

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22.5.29 Hanford control rods

Control rods of Hanford pile. Following calculations made with older figures for migration area: $A_{mod} = 365$ cm^2, $A_{th} = 295$ cm^2, $A = 660$ cm^2, $A_{th}/A = 0.447$, $(A_m^{-1} + A_t^{-1})^{\frac{1}{2}} = 0.0783$ cm^{-1}. Rod shown in Fig. 22.8.10. Two boron coated tubes 7/8 inch in diameter, centers separated by $2\frac{1}{2}$ inches. Between them is a third and larger boron coated tube which almost completely blocks the intervening space against passage of neutrons. Consequently, treat rod as having perimeter $2 \times 2.5 + \pi \times 7/8 = 7.75$ inches or 19.7 cm. The rod traverses a hole in the graphite having a cross section 2×4 inches, equivalent in area to a cylinder of radius 4.05 cm. From Eq. (22.4.9.a) obtain $r_{eff} = \underline{4.05 \text{ cm}} \exp(-8\pi \times 2.7 \text{ cm}/3 \times 19.7) = 1.25$ cm. Zone of control of one rod 64 inches by $50\frac{1}{4}$ inches, equivalent in area to cylinder of radius $R = \underline{81.3 \text{ cm}}$. Thus $bR/br_{eff} = 81.3/1.25 = \underline{65.0}$. Also $(A_m^{-1} + A_t^{-1})^{\frac{1}{2}} r_{eff} = 0.0783 \times 1.25 = \underline{0.0979}$. Use nomograph of Fig. 22.5.23. By trial and error find solution $bR = \underline{0.557}$, whence $b = 0.557/81.3 = 6.85 \times 10^{-3}$, $b^2 = 47.0 \times 10^{-6}$. Loss in local reactivity is $\delta k = b^2 A = 47 \times 10^{-6} \times 660 = \underline{0.031}$. Loss in overall reactivity evaluated in 22.3.25 as function of control rod position.

22.5.30 Comparison of −, +, 0 rods

Effect of shape of rods. John Marshall has measured and reported in CP-718 the relative effectiveness of rods with the cross section of the signs −, + and 0. All had the same maximum extension, 9.05 cm, and went through a hole in the Argonne graphite-uranium pile 4 inches square equivalent in area to a cylinder of radius, $r_1 = 5.74$ cm. We calculate the effective radius from the formula: $r_{eff} \doteq r_1 \exp(-8-\lambda/3 \text{ perimeter})$. The measurements of degree of control in the three cases were essentially relative, so we adopt for size of the zone of equivalence the arbitrary figure, $R = 100$ cm, for common basis of the three calculations. Also adopt values, $A_{mod} = 320$ cm^2, $A_{th} = 320$ cm^2, $A = 640$ cm^2, $A_{th}/A = 0.5$, $(A_m^{-1} + A_t^{-1})^{\frac{1}{2}} = 0.0791$ cm^{-1}. The calculations are summarized below and compared with Marshall's observations. The agreement between the two is reasonably good.

Shape of bar	Perimeter in cm	Effective radius in cm	Ratio $\frac{bR}{br_{eff}}$	b^2 from Figure 22.5.23	Effectiveness of control relative to + bar: Calculated	Observed	Observed loss in reactivity in inhours due to 5 foot insertion
−	18.1	1.65	60.7	35.6×10^{-6} cm^{-2}	0.87	0.90	33.2
+	25.6	2.38	42.1	40.9	1.00	1.00	36.9
0	28.4	2.58	38.7	42.3	1.03	1.06	39.1

A less satisfactory agreement results when we give up the picture of a small rod in a large hole and try the opposite idealization where we regard the bar as immersed in the moderator. The results of this calculation are listed below.

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Shape of bar	Method of calculation of r_{eff}	Effective radius in cm	Ratio $\frac{bR}{br_{eff}}$	b^2 from Figure 22.5.23	Effectiveness of control relative to + bar	
					Calculated	Observed
-	Fig.22.4.15	1.28	78.4	32.9×10^{-6} cm^{-2}	0.82	0.90
+	Fig.22.4.15	2.24	44.6	40.0	1.00	1.00
0	Eq.(22.4.5.o)	3.20	31.2	46.7	1.17	1.06

Liquid controls. Weinberg has calculated and reported in CS-1033 the loss in k to be expected from pipes 4 inches in diameter and spaced on a regular lattice, finding 5 percent drop in reproduction factor when the pipes contain pure water, and 6 percent when the water carries 8 grams of NH_3 per liter. Experiments related to this point have been performed by Anderson and his associates*. They found that an aluminum cylinder 12 feet long, 3 inches in diameter, with a 1/16 inch wall, when filled with a 4.57 percent aqueous solution of NH_3, cut the reactivity of the Argonne pile half as much as a cadmium strip of the same length, 1 7/8 inches wide. The detailed results follow. No theoretical analysis 22.5.31 Liquid controls

Inhours converted into excess reactivity here by factor 2.32×10^{-5}	Critical position of control rod in meters	reactivity in inhours	10^5 times change in overall k
No absorber	3.782	115.88	
Aluminum tube	3.741	113.63	0
Al tube and cadmium strip	2.705	56.08	134
Al tube and water	3.214	83.83	69.1
Tube and 3.29% NH_3 solution	3.170	81.33	74.9
Tube and 4.5% NH_3 solution	3.154	80.43	77.0

will be attempted here.

Gaseous control. Example of use of Eq.(22.5.18.d) for an absorbent whose location in the lattice unit influences its control power. Graphite pile so designed that this moderator absorbs 13 percent of the thermal neutrons. Control obtained by filling pores with BF_3 at 1/100 atmosphere. One cm^3 of graphite contains 1.6 grams of carbon and 0.27 cm^3 of voids. We take 460 cm^2 as the cross section of all the boron nuclei in one mole or 22,000 cm^3 of BF_3. The cross section of 1 gram of graphite for 22.5.32 Control by BF_3

*H. L. Anderson, L. Seren, W. Sturm and W. E. Moyer, CP-1088, Physics Research Report for month ending 1943 November 23.

neutrons of the same velocity is taken to be $2.15 \times 10^{-4}\text{cm}^2$. We obtain for loss in local multiplication factor the result:

$$\delta k = \frac{0.01\ (0.27\ \text{cm}^3/22000\ \text{cm}^3)}{1.6\ \text{gm}\ (2.15 \times 10^{-4}\text{cm}^2/\text{gm})}\ 0.13 = 0.021 \qquad (22.5.32.a)$$

With this survey of typical control devices, we complete the account of the effectiveness of individual absorbers. This theory, together with the treatment of disposition of controls given in Section 22.3 suffices to analyze the effect of controls on the reactivity of a pile. We have now to study the effect of the pile on the controls. 22.5.33 Analysis of effectiveness completed

22.6 HEATING OF CONTROLS

22.6.1 Heating of order of 35 kw in Hanford control rod

Control rods in a pile of high output absorb nuclear radiations, become heated, and have to be cooled. The cooling requirement is as important in the design of the rod as it is in the construction of the pile itself. Estimation of the magnitude of the heat produced in the rod is the subject of this section. We shall first relate the neutron flux into the rod with the production of neutrons in the neighboring portions of the pile, then evaluate the heat production per neutron absorbed, and finally estimate the total heat production in a rod of simple design. This rod, inserted in a pile of Hanford design, develops at most about 35 kw of heat.

22.6.2 Heating related to change in local k

The absorption of neutrons by a control rod is more directly related to its effect on the local multiplication factor than to its influence on the overall reactivity. This conclusion follows from the discussion of 22.3.19. There an absorber located at the center of a pile was seen to take up only half the proportion of neutrons which might have been expected from its actual effect on k; but when the same control was located near the fringes of the pile, it absorbed a fraction of all the neutrons ten times greater in magnitude than the depression it caused in the overall reproduction constant. Yet the depression in local k is the same in both cases. This quantity furnishes the simplest starting point for an analysis of the heating problem. It gives the number of neutrons absorbed in the rod relative to the number of neutrons generated in the surrounding zone of control.

22.6.3 Rate of capture of neutrons by rod

Consider for example the central control rod of the Hanford pile, already discussed in 22.3.25 and 22.5.29. The rate of generation of neutrons in the zone of control of the rod is proportional to the heat output. The concentration of power production in tubes near the center of the pile is limited to 1 kw/cm by reason of corrosion by hot water. The tubes are spaced on a square lattice with 21.28 cm between centers. Thus heat is liberated per unit volume of the pile at a rate not exceeding $(1 \text{ kw/cm})/(21.28 \text{ cm})^2 = 2.21 \text{ watts/cm}^3$. Each fission which contributes to this heat output releases about 200 Mev of energy and 2.2 neutrons, signifying the generation of 1 neutron per 90 Mev of heat. Consequently, the number of neutrons generated per cm^3 of pile material per second is at most about $(2.21/90)$ watts/Mev cm^3. In absolute units this rate is $1.24 \times 10^{11}/\text{cm}^3$ sec, but left in its present mixed units the quantity will be more useable. The zone of control is 128 cm x 162 cm in cross section, and the rod in question absorbs the fraction 0.031 of the neutrons produced in this region. The rate of absorption of neutrons by the rod is therefore $128 \text{ cm} \times 162 \text{ cm} \times 0.031 \times (2.21 \text{ watts/cm}^3)/90 \text{ Mev} = 1420 \text{ watts}/90 \text{ Mev}$ per cm of length of the control. Let E be the number of Mev of energy released in the rod per neutron absorbed by the rod. Then the neutronic heating of the central portion of the control is $(E \text{ Mev}/90 \text{ Mev})\ 1.42 \text{ kw/cm}$. Other parts of the rod receive less heat by a factor which on the average is not far from $2/\pi$. The length of the rod in the effective portion of the pile is 1060 cm.

Consequently, we take the total heat generated in the control by neutron capture to be at most about (E Mev/90 Mev) $(2/\pi)$ 1060 x 1.42 = (E Mev/90 Mev) 956 kw. We now require an estimate of the energy release, E.

22.6.4 Three sources of heat

Neutrons absorbed in the control will cause the liberation of energy in the form of kinetic energy of nuclear fragments, beta rays, gamma rays and neutrinos. The first two radiations are of short range. Their heating effect will therefore ordinarily be localized in the control substance. Of the gamma ray energy only a fraction will ordinarily be dissipated in the rod. And the neutrinos will produce no measurable heating effect in the pile. The relative contribution of the three radiations of significant heating power depends upon the design of the rod. As illustration, we consider an idealized version of the Hanford control rod of Fig. 22.8.10. We consider the boron to be distributed throughout an aluminum rod in the form of a solid solution, to the extent of 3 percent by weight. We take the rod to have a diameter of 3 inches, radius of 3.81 cm, and to contain a hole for flow of water 1 inch in diameter, 1.27 cm in radius. From the following table we deduce an energy release of E = 2.8 Mev in the rod per neutron absorbed by the rod. We conclude that neutron absorption produces heat in the rod at the rate (2.8 Mev/90 Mev) 956 kw = 30 kw.

Table 22.6.5. HEAT RELEASED PER NEUTRON ABSORBED IN SIMPLE FORM OF CONTROL ROD

Rod 3 inches in diameter, with hole 1 inch in diameter inside it, containing water. Composition by weight: 3% B - 97% Al. This example illustrates how the heat due to the presence of the control can be localized within the rod itself. It is only necessary to choose as absorbent an element such as boron or lithium which undergoes fission but has no gamma ray emitting fission products. In contrast, an element such as cadmium releases the heat of condensation of the neutrons in the form of gamma rays. These radiations escape in considerable measure from a control rod. Use of a cadmium rod might therefore result in local overheating of neighboring portions of the pile.

Constituent	Boron	Aluminum
Density	0.081 gm/cm^3	2.6 gm/cm^3
Cross section per gm for absorption of thermal neutrons	39 cm^2/gm	0.0051 cm^2/gm
Partial linear absorption coefficient	3.2/cm	0.013/cm
Total linear absorption coefficient	3.21/cm	
Fraction of neutrons absorbed in given element	0.996	0.004
Order of magnitude estimate of energy of individual gamma rays given off on neutron capture	none	~3 Mev

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Table 22.6.5 - Con'd.

Constituent	Boron	Aluminum
Linear absorption coefficient for such gamma rays in Al-B alloy		0.08/cm
Product of radius of rod by total absorption coefficient		
(1) of gamma rays		0.30
(2) of neutrons		12.2
Fraction of gamma rays absorbed in rod deduced from last two quantities with aid of figure in Section 15.2 (neglecting effect of hole through rod)		~ 0.5
Total gamma ray energy given off as result of capture of neutron		9.7 Mev
Amount of this energy released in rod (product of last two rows)		~ 5 Mev
Amount of beta ray energy released in rod per neutron captured by given element	none	~ 1 Mev
Energy of fission	2.8 Mev	none
Energy released within and absorbed by rod		
(1) per neutron absorbed in the given element	2.8 Mev	~ 6 Mev
(2) per neutron absorbed in rod (product of (1) with figure above for fractional absorption due to given element)	2.8 Mev	0.02 Mev
Total energy released within and absorbed by rod per neutron absorbed by rod	E = 2.8 Mev	

22.6.6 Moderator as standard of reference for two heating effects

To the heat developed in a control rod by neutron capture we have to add that due to moderation of neutrons and absorption of gamma rays generated in the surrounding portion of the pile. These contributions are proportional in magnitude to the similar heating effects which occur in the moderator itself. For example, in a typical graphite-uranium pile, gamma ray absorption in the carbon accounts for roughly 4 percent of the power, and neutron moderation for about 2 percent, according to the discussion of this question in Chapter 19. At the center of the Hanford pile the gamma ray heating of the graphite will amount at most to about $0.04 \times 2.21 = 0.088$ watts/cm^3 or (0.088 watts/cm^3)/(1.6 gm/cm^3)= 0.055 watts/gm, and the heating by neutron moderation will be half as great.

We will obtain a conservative account of the effect of gamma rays on the control rod if we neglect the shielding of the inner portions of the control by its exterior. Also we shall adopt as a reasonable approximation for photons in the energy range of importance the assumption that mass absorption coefficient is independent of atomic number. Thus we estimate for gamma ray heating per unit mass of the central length of the rod the same figure which applies to the moderator, 0.055 watts/gm. The mass per unit length is $\pi\,(3.81^2\,\text{cm}^2 - 1.27^2\,\text{cm}^2)\ 2.7\ \text{gm/cm}^3 + \pi\,(1.27^2\,\text{cm}^2)\ 1\ \text{gm/cm}^3 = 109 + 5 = 114\ \text{gm/cm}$. Thus the heat per unit length is of the order of (0.055 watts/gm) x (114 gm/cm) = 6.3 watts/cm. 22.6.7 Gamma ray heating

By a similar procedure of comparison with the graphite we estimate the heating incident to moderation of neutrons in the rod: 22.6.8 Heating incident to moderation

Moderator	Aluminum	Oxygen	Hydrogen
Average fraction of neutron energy lost in elastic encounter (a measure of moderating power per nucleus) = $2M_1M_2/(M_1 + M_2)^2$	$2 \times 27/(28)^2$	$2 \times 16/(17)^2$	$2 \times 1/2^2$
This quantity divided by atomic weight gives a measure of the moderating power per gm	$2/(28)^2$	$2/(17)^2$	$2/2^2$
Corresponding measure of moderating power per gm of C	$2/(13)^2$	$2/(13)^2$	$2/13^2$
Ratio of last two quantities gives moderating power relative to graphite on a mass basis	0.215	0.585	42.2
Maximum heat production in graphite due to neutron moderation	~0.028 watts per gm	~0.028	~0.028
Maximum heat production in rod materials due to moderation, product of last two rows	~0.006 watts per gm	~0.016	~1.18
Amount of given element per unit length of rod	~109 gm/cm	~4.5	~0.56
Contribution of individual elements to heating by moderation, per unit length of rod, product of last two rows	~0.65 watts per cm	~0.07	~0.66
Total heating by moderation in central section of rod	~1.4 watts/cm		

It is apparent in this example that the heating by neutron moderation is small on two counts: first, only a relatively small fraction of the power output of the pile appears as kinetic energy of neutrons; and June, 1944 second, the control rod, gram for gram, is less effective than the graphite in taking up this energy.

The total heating of the simplified control rod by neutron capture, gamma ray absorption and neutron moderation together is evaluated in the following summary: 22.6.9 Total heat release by summation

Source of heat	Paragraph in which discussed	Over central portion of rod	Whole rod (preceding column time $(2/\pi)$ 1060 cm)
Neutron capture	(22.6.4)	44 watts/cm	30 kw
Gamma ray absorption	(22.6.7)	$\sim$6.3 watts/cm	$\sim$4 kw
Neutron moderation	(22.6.8)	$\sim$1.4 watts/cm	$\sim$1 kw
Total		52 watts/cm	35 kw

It is clear from this table that neutron capture is the major source of heat production in the control. We were therefore justified in giving a rather crude account of the contribution from gamma ray absorption and neutron moderation. A more accurate treatment would have required us to allow for the depression in local neutron density and gamma ray production brought about by the rod in its immediate neighborhood. However, this lowering effect has already automatically been taken into account in our evaluation of the major part of the heat production. The reduction in local multiplication factor by the rod and the production of heat in the rod are quantities which are affected in the same way by alterations in the distribution of neutrons near the rod. It follows that our estimate of the total heat development in the rod is reasonably reliable. Our figure of 35 kw for maximum output from a simplified version of the central control rod of the Hanford pile may be compared with the designed flow of 10 gallons of water per minute or 750 cc/second through the cooling system of each rod. The temperature rise of the water should therefore be relatively small, (35000 watts) x (0.24 cal/sec watt)/(750 gm/sec) = 11°C.

Partial insertion of a control rod results always in a smaller total heating but sometimes in a greater local heating. The tip of the absorbent acts like a lightning rod and receives a high neutron flux. This phenomenon is illustrated in the curves of Figs. 22.6.11 and 22.6.12. The local heating at the tip is greatest when roughly half the length of the rod is in the pile. The bearing on the design of the rod is twofold. First, the local rate of heat transfer to the coolant must be sufficient to prevent undue temperature rise. Second, the tip of a cadmium rod may rebroadcast in the form of gamma rays to the surrounding pile a too high fraction of the heat of condensation of the neutron stream. Slug jackets already near the critical temperature for corrosion by hot water may become overheated and fail. This possibility is an argument against use of absorbers which emit gamma rays on neutron capture. This point and the requirement of high heat transfer rate near the tip were both taken into account in the design of the Hanford control rods. 22.6.10 Concentration of heating at tip

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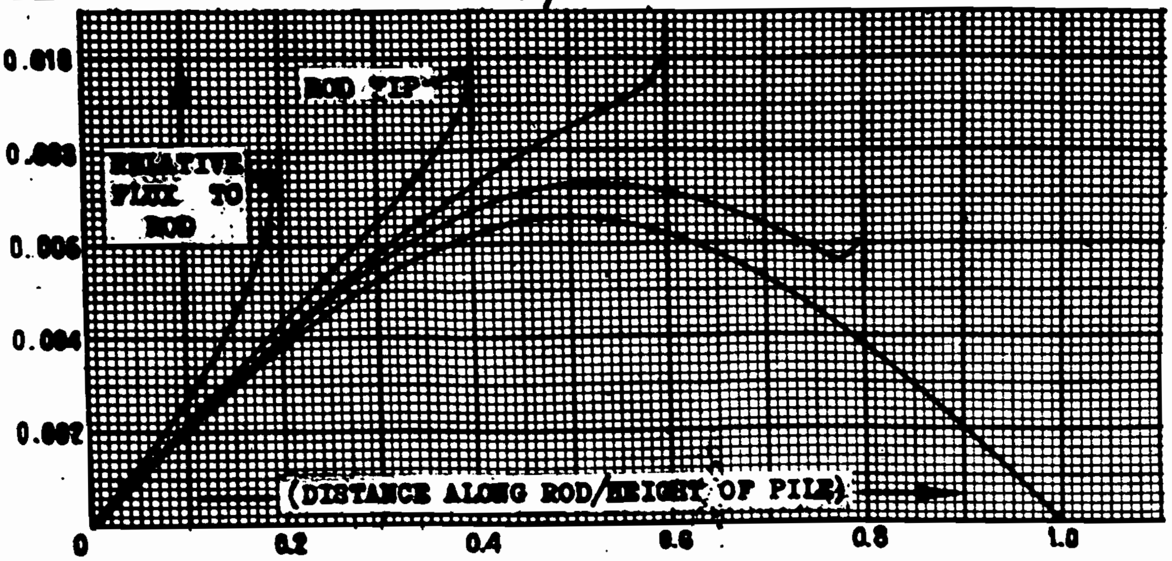
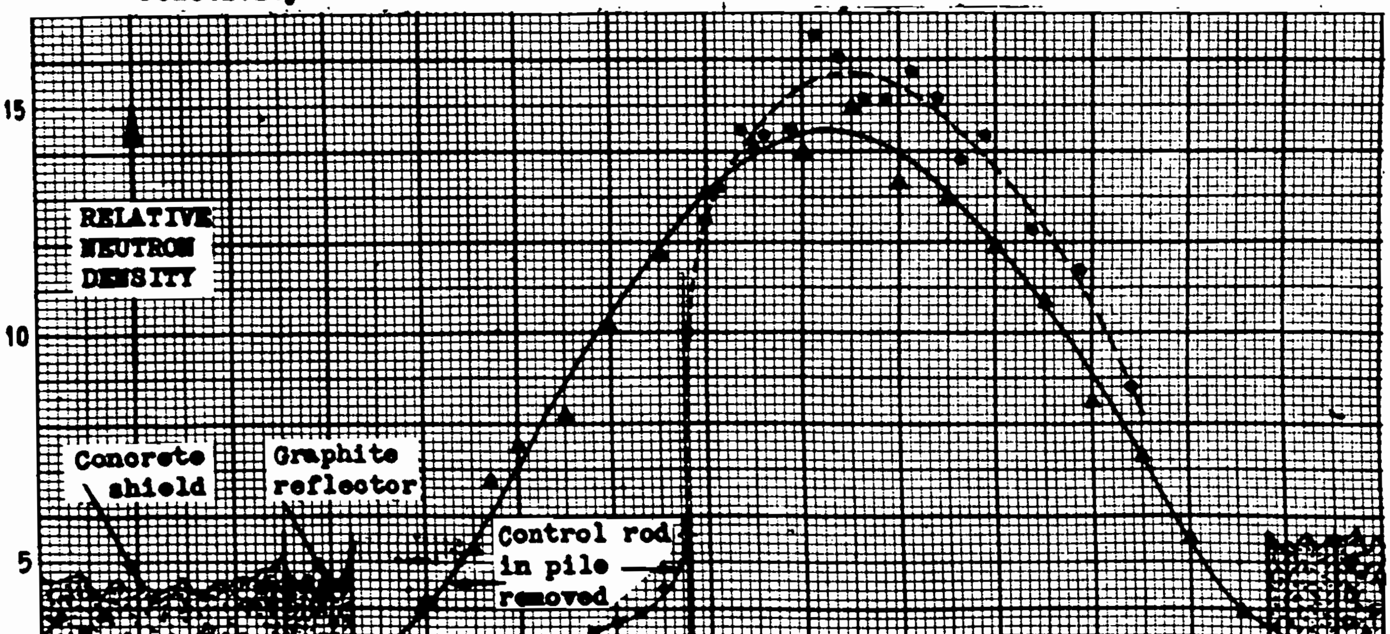


• HEATING OF RODS •

ABOVE —Neutron flux to control rod as calculated by F. H. Murray, MUC-FM-2, for rod inserted along axis of cylindrical pile to fraction 0.2, 0.4, 0.6, 0.8, 1.0 of height of pile. Relative dimensions - pile height: pile radius: rod radius = 17: 15: 1/12. Rod treated as opaque to both slow and fast neutrons. Quantity plotted, $hA2\pi r(\partial n/\partial r)\iiint n\,dx\,dy\,dz$, when multiplied by neutron output of pile, gives fraction of all neutrons which would be absorbed in full length, h, of rod if everywhere it received as much flux as it does at the point in question.

BELOW —Neutron flux along axis of control rod as measured in Argonne Forest pile by Leo Seren, W. Sturm, W. E. Moyer, CP-1088, 1943 November 23. Rod of 0.020 inch cadmium, 3 inches wide, mounted on board of same width. Slight asymmetry in distribution observed when rod is removed is due to presence, 5 feet away, of another rod used to compensate the excess reactivity which would otherwise result.



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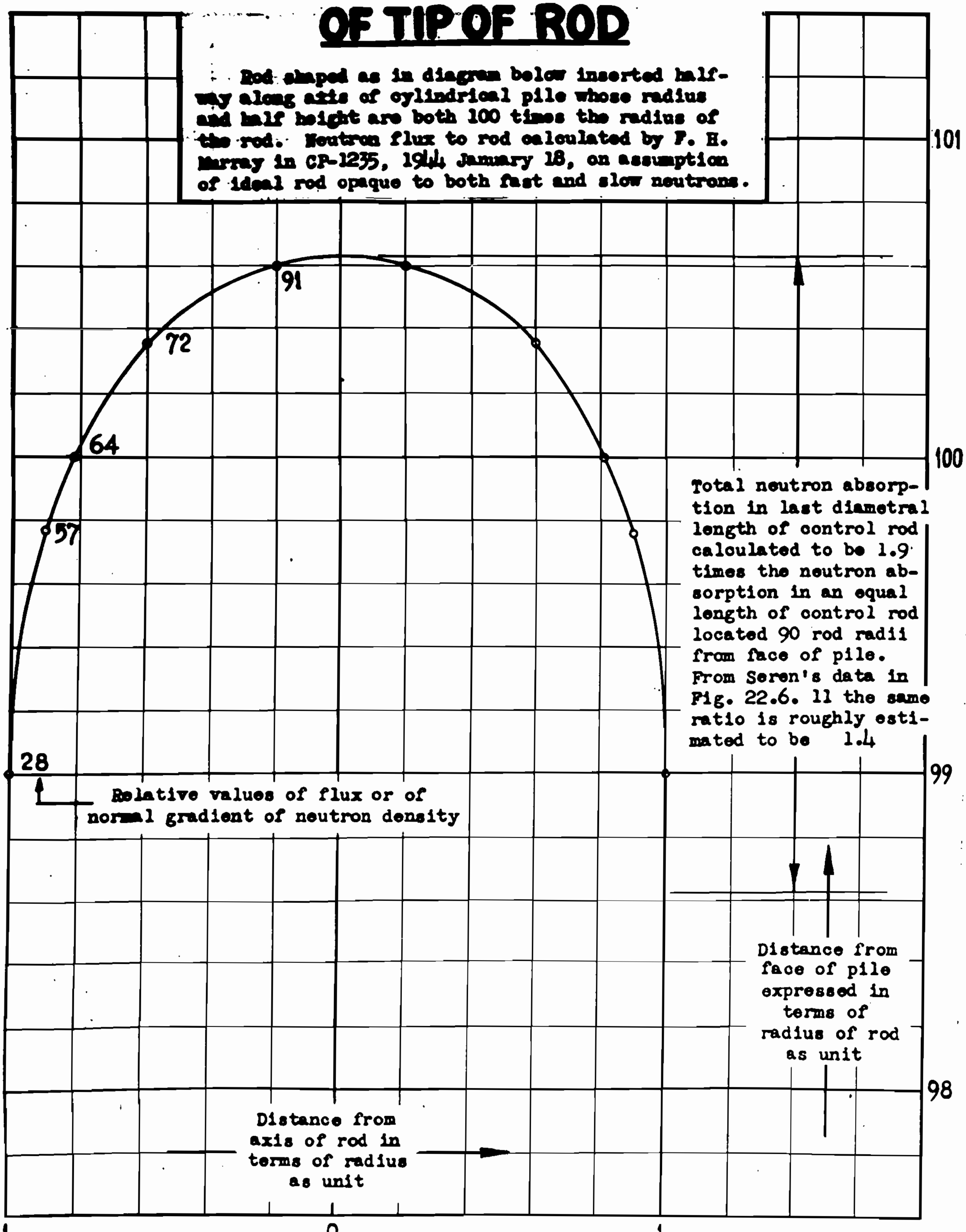
HEATING OF TIP OF ROD



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22.7 SPEED OF CONTROL

22.7.1 Speed of safety control

The designed speed of control is a balance between what is desirable and what is practicable. In a pile which contains 100 tons of uranium and operates at 500 megawatts, the average temperature of the metal will rise about 50°C. a second and the temperature at the center of the pile will increase 100°C. a second. A very few seconds of operation at such a level without cooling will result in permanent damage to the pile. From the point of view of heat transfer, therefore, it must be possible to shut off the chain reaction in a time at most of the order of a few seconds. From the point of view of nuclear physics, however, there exists a still more pressing requirement for speed in control in a water cooled pile. Sudden loss of all the water in the pile will result in an increase of the multiplication factor by an amount of the order of 2%. The rate of reaction will then rise by many factors of ten in a single second. Then, not only will the usefulness of the pile be destroyed, but also the possible vaporization or combustion of the activated uranium will create a radioactive hazard of the greatest magnitude. The safety control must, therefore, act in a time short in comparison with the period necessary for loss of water from the pile.

22.7.2 Speed of fine control

Quite another order of speed is desirable for the fine control of the pile. This control compensates momentary changes in the multiplication factor of the order of 10^{-4} and can at most produce a change in reproduction factor in the neighborhood of 10^{-3}. The speed of regulating control may, therefore, be of the order of 10^{-4} k units per second as contrasted with the speed of the safety rods in the neighborhood of 5×10^{-2} k units per second.

22.7.3 Excess multiplication factor defined

It is in order now to consider in further detail the factors relating the rate of growth or decay of the chain reaction with the excess or deficit of the reproduction factor. The excess multiplication factor, k_e, is defined as the difference between the average local reproduction factor available in a pile structure and that value of the reproduction factor which would be just sufficient to keep the chain reaction running at a steady level. For example, let the pile be of such a size that a local multiplication factor of 1.03 will just balance the leakage of neutrons from the structure and maintain the power output constant. Let the actual local reproduction factor be 1.05. Then, the excess reproduction factor, k_e, equals 0.02, and is available for increasing the rate of reaction.

22.7.4 Delayed emission of neutrons important

How the reproduction of neutrons takes place and how the growth of the neutron content of the pile is connected with the multiplication factor has been discussed in a general way in Chapter 14. There it was pointed out that the rate of growth is affected in an important way by the phenomenon of delayed neutron emission. Of the 2.2 secondary neutrons liberated per fission, a fraction, f, of the order of 0.006

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only becomes free a few seconds after the actual act of fission. Before we make a detailed study of the effect of these delayed neutrons, let us examine qualitatively their influence upon the rate of growth or decay of the chain reaction,

Large excess k

22.7.5 Case of rapidly rising activity

Consider first the case where there is considerable excess reproduction factor and the concentration of neutrons in the pile rises every second by a factor large in comparison with unity. Under these conditions, the delayed neutrons which become free at one instant have originated from fissions which took place several seconds earlier. Then the rate of reaction was negligible in comparison with the value which it has now reached. Consequently, the delayed neutrons may be considered to make no appreciable contribution to the chain reaction. We have therefore to subtract from the excess multiplication factor, k_e, the contribution, f, of the delayed neutrons, in order to find the effective excess multiplication factor, $k_e - f$. If we arbitrarily call the neutron concentration in the pile unity at the beginning of one generation, then the effective concentration at the end of that generation will be $1 + (k_e - f)$. At the end of two generations, the concentration, in the same terminology, will be $\left[1 + (k_e - f)\right]^2$, and at the end of n generations, $\left[1 + (k_e - f)\right]^n$. In all the cases in which we shall be interested, the quantity $(k_e - f)$ will be small in comparison with unity. Consequently, we can write the expression $1 + (k_e - f)$ in the form $\exp (k_e - f)$. The neutron concentration at the end of n generations will have arisen above its original value by the factor

$$\left[\exp (k_e - f)\right]^n = \exp n (k_e - f) \qquad (22.7.5.a)$$

The number of generations will be connected with the time, t, of operation, and the lifetime, τ, of one generation by the relation

$$n = t/\tau \qquad (22.7.5.b)$$

We conclude that the rate of reaction rises exponentially with time in proportion with the expression

$$\exp \left[(k_e - f)\ t/\tau\right] \qquad (22.7.5.c)$$

22.7.6 Example $k_e = 0.016$

EXAMPLE: In the water cooled pile described in Chapter 21, the life time of one generation of neutrons is 1.2×10^{-3} seconds. A group of control rods is suddenly removed so that the excess multiplication factor, k_e, increases from 0.000 to 0.016. What is the approximate rate of rise of the neutron activity?

Taking for the fraction of delayed neutrons the value f = 0.006, we have

$$k_e - f = 0.010$$

Expression (22.7.5.c) takes the form

$$\exp\left(\frac{0.010\ t}{1.2 \times 10^{-3}}\right) = \exp 8.2\ t$$

The reaction therefore rises by a factor 22.7.7 Activity multiplied by 3600 in one second

$$\exp\ (8.2) = 3600$$

in one second. Under these conditions, the rate of rise of the reaction is so great that we are entitled to neglect the effect on the reaction at one instant of the delayed neutrons emitted at an earlier instant, as we have in fact assumed. This approximation will be still better if the excess multiplication factor is greater.

Large deficit in k

In the opposite extreme case, where a pile in steady operation suddenly experiences a large loss in multiplication factor, the activity at first falls off exponentially toward zero. It does not continue to drop indefinitely at this rate, however. The radioactive nuclei formed during the steady operation of the pile will continue to give off neutrons, and these neutrons will be multiplied. Consequently, the rate of drop of activity will be limited by the action of the delayed neutrons. In order to give an approximate discussion of this effect, let us denote by P the rate of production of neutrons during the steady operation of the pile. Then in the same terminology the rate of production of delayed neutrons will be fP. Since radioactive equilibrium has been attained, the same rate of emission will apply for the first few seconds after the control rod is introduced. The delayed neutrons, after emission, find themselves in a pile whose reproduction factor, $(1 + k_e)$, is less than unity (k_e is negative). This expression does not, however, represent the effective multiplication factor because it assumes that all fission processes taking place after the insertion of a control rod result in instantaneous neutrons. Actually, of course, of the neutrons which result from these later fission processes, the fraction, f, is delayed by some seconds and will not contribute to the reproduction factor. Consequently, the effective multiplication factor is $1 + (k_e - f)$, as in 22.7.5. 22.7.8 Large deficit in k

We can now summarize the situation a second or so after the insertion of the control rod in the following terms; Radioactive nuclei eject fP neutrons per second into a medium whose effective reproduction factor, $1 + (k_e - f)$, is less than unity. Each of these neutrons, therefore, produces a convergent family tree, the total number of neutrons in which is 22.7.9 Multiplication of delayed neutrons

$$1 + (1 + k_e - f) + (1 + k_e - f)^2 + \ldots\ldots = \frac{1}{1 - (1 + k_e - f)}$$

$$= \frac{1}{(-k_e) + f} \qquad (22.7.9.a)$$

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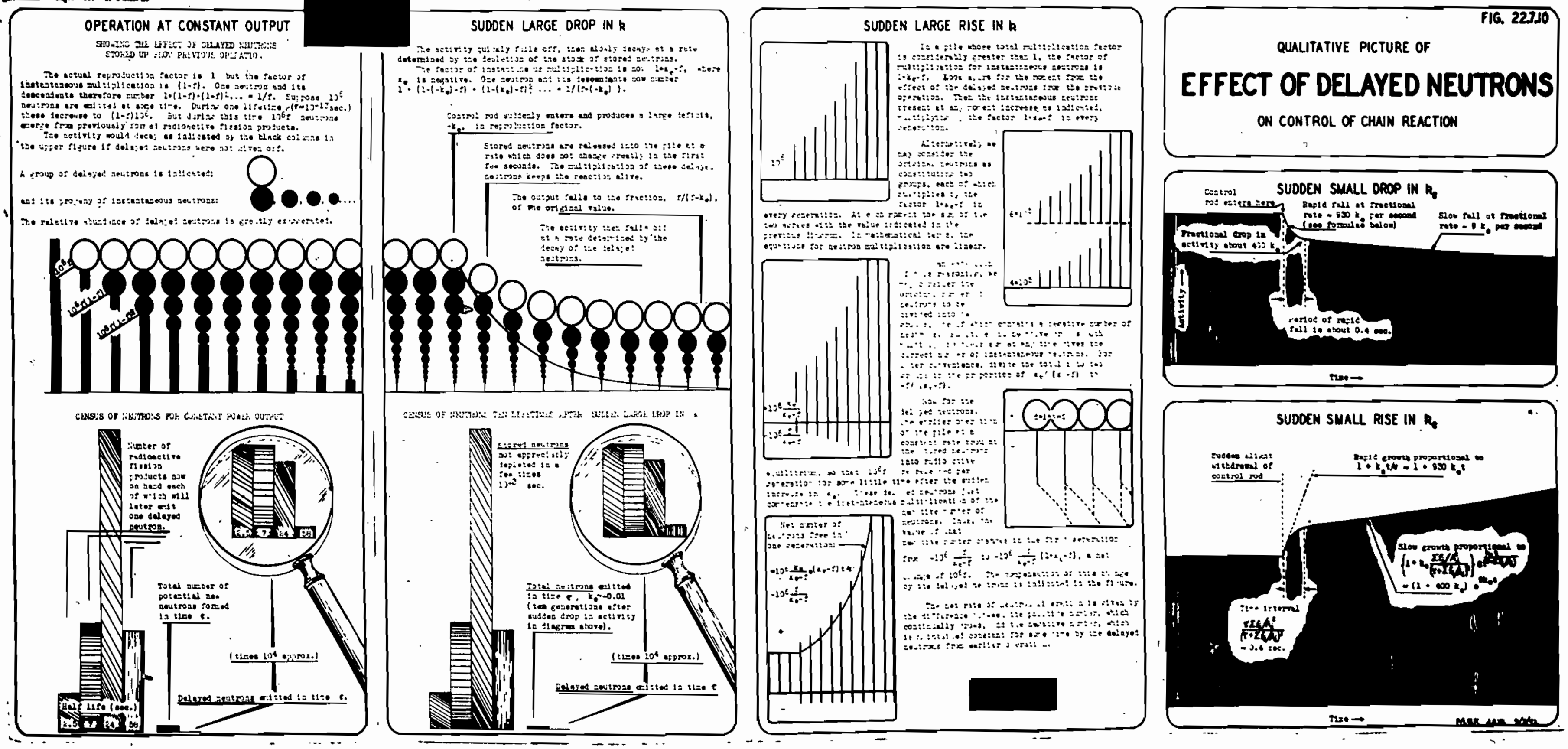

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By virtue of this multiplicative effect, the source of f neutrons per second will give rise to a production in the medium $f/[(-k_e) + f]$ P neutrons per second. Consequently, we can say that in a very short interval after the introduction of the control rod, the neutron activity falls to the fraction $f/[(-k_e) + f]$ of its original value. Subsequently the activity falls off very much more slowly at a rate determined by the lifetime of the radioactive nuclei responsible for delayed neutron emission. (See Figs. 22.7.10, 22.7.33, and 22.7.34)

22.7.11 Example of sudden drop

EXAMPLE: Into a pile are suddenly inserted control rods which lower the reproduction factor from its operating value to a figure 2% lower. By what factor does the rate of fission drop? Taking for the fraction of delayed neutrons the value $f = 0.006$, we conclude that the rate of fission quickly drops after the insertion of a control rod to the fraction $0.006/(0.020 + 0.006)$, or 23% of its original value, and then slowly falls off.

Small excess k

22.7.12 Case of slowly rising activity

Contrast with the case of a sudden large change in multiplication factor, the situation which arises on a sudden small change of reactivity in a steadily operating pile. Under this condition, the rate of reaction will rise only slowly with time. Consequently, the neutron production at one instant of time receives an important contribution from the radioactive fission products formed several seconds earlier when the reaction differed very little in intensity from its present value. In this way the past history of the structure plays an important role in governing the future development of a chain reaction, in contrast to the case where the multiplication factor rises by an amount large compared to the contribution of the delayed neutrons.

22.7.13 Analysis of neutron production

If a small and constant excess in the reproduction factor is maintained over a long period of time, then the level of the reaction will asymptotically approximate an exponential rise, represented by an expression of the form $e^{\lambda t}$. The value of the constant, λ, is simply related to the value, k_e, of the excess multiplication factor, the value τ, of the mean life of a neutron in the pile, and the number and decay periods of the radioactive nuclei which emit delayed neutrons.* Let the decay constant for the radioactive nuclei in question be represented by A, so that the half life for emission of the delayed neutrons is 0.693/A. Let the number of neutrons present in the pile at a given instant of time arbitrarily be called unity. The number of free neutrons in the pile at the end of the subsequent small time interval, dt, will be given by the sum of the following contributions:

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*E. P. Wigner, CP-351, On the Variations in the Power Output of a Running Pile.

$1 + (k_e - f)\ (dt/\tau)$, secondary neutrons produced instantaneously by multiplication of neutrons present at the beginning of the time interval. The fraction, f, of the potential new neutrons formed in the given time interval are "stored" and therefore will contribute to the multiplication process only some seconds later on. They are therefore not included in the present figure. However, the neutrons "stored" during an earlier period of operation will now be making their contribution and must be taken into account.

$f\ (1 - \lambda/A)\ (dt/\tau)$, the number of "stored" neutrons emitted during the time interval, dt, and thereafter kept alive by reproduction. If the pile were in operation at a steady rate, this number would have the value, $f\ dt/\tau$. However, the rate of formation of radioactive nuclei has been increasing each second by the fractional amount, λ. The average interval between the formation of a radioactive nucleus and the liberation of a delayed neutron will be $1/A$. Consequently, the rate of reaction at the time of formation was lower approximately by the factor $(1 - \lambda/A)$.

$1 + \left[k_e - (\lambda f/A)\right]\ (dt/\tau)$, the total number of free neutrons in the pile after the lapse of the time dt.

It follows from the foregoing analysis that the fractional rate of rise of the reaction in one generation is

$$k_e - \lambda f/A. \qquad (22.7.14.a)$$

However, the fractional rise in rate of operation during the lifetime of one generation follows also from the originally stated exponential expression for the activity, and is $\lambda\tau$. The equivalence of these two alternative expressions for the rise in the reaction gives a condition from which to find the value of the growth constant, λ : 22.7.14 Evaluation of growth constant

$$k_e - \lambda f/A = \lambda\tau \qquad (22.7.14.b)$$

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We conclude that the rate of growth of activity in the pile increases exponentially in time with the factor of proportionality

$$\lambda = \frac{k_e}{\tau + (f/A)} \quad ; \qquad (22.7.14.c)$$

thus,

$$\text{Activity} \sim \exp \frac{k_e\, t}{\tau + (f/A)} \qquad (22.7.14.d)$$

In other words, we can say that the phenomenon of delayed neutron emission effectively increases the lifetime of one generation from τ to $\tau + (f/A)$, provided that the excess or deficit in the reproduction factor is small in comparison with the fraction, f, of delayed neutrons.

22.7.15 Generalization to several decay periods

Available experimental evidence* indicates that there are probably four species of radioactive fission products of uranium which release neutrons. We have, therefore, to express the total fraction of delayed particles as the sum, $f = f_1 + f_2 + f_3 + f_4$, the four contributions, each of which is described by its own characteristic decay constant, A_1, A_2, A_3, or A_4. The generalization of the above discussion is simple and leads to the result

$$\text{Activity} \sim \exp \frac{k_e\, t}{\tau + \sum (f_i/A_i)} \qquad (22.7.15.a)$$

22.7.16 Observation of slow rise

Experiments with the chain reacting pile** have verified the existence of the predicted slow exponential rise or fall of activity with time following shortly after a sudden small increase or decrease in the multiplication factor. Later work has shown that the exponential rise or decay factor is directly proportional to the excess reproduction factor when k_e is small compared with f. Anderson reports*** the result

$$3.04 \times 10^{-5} \ (\text{fractional rise in activity per hour}) = k_e \qquad (22.7.16.a)$$

For example, when the excess multiplication factor is $k_e = 1 \times 10^{-5}$, then the fractional rise in activity per hour is $(1 \times 10^{-5})/(3.04 \times 10^{-5}) = 0.329$. Thus, in 1/10 of an hour the activity rises by 3.3%. At the

*A. H. Snell, V. A. Nedzel, and H. W. Ibser, C-81, A Study of the Delayed Neutrons Associated with Uranium Fission.

**E. Fermi, CP-413, Experimental Production of a Divergent Chain Reaction.

***H. L. Anderson in CS-655, Meeting of Laboratory Council.

end of one hour, the activity increases to the factor exp 0.329 = 1.39 times its original value.

22.7.17 Evaluation of lifetime effective for small k_e

The observed correlation between the excess multiplication factor and rate of growth of activity gives some information about the magnitude of the delayed neutron effect. Comparing the theoretical formula

$$k_e = (\tau + \sum f_i/A_i)\ (\text{fractional rise in activity per hour})/3600 \qquad (22.7.17.a)$$

with the observed result

$$k_e = 3.04 \times 10^{-5}\ (\text{fractional rise in activity per hour}) \qquad (22.7.17.b)$$

we conclude that for small excess or deficit in the reproduction factor, the effective lifetime of one generation has the value

$$\tau + \sum f_i/A_i = 3.04 \times 10^{-5} \text{ hours} = 0.1095 \text{ seconds} \qquad (22.7.17.c)$$

In contrast, the actual mean life of a neutron in one form of water cooled pile, for example, is only 1.22×10^{-3} seconds. In other words, we can say that the delayed neutrons effectively slow down the process of growth or decay of the chain reaction by a factor of the order of 100. This fact greatly simplifies the problem of control of a chain reacting pile.

Initial growth of activity

22.7.18 Activity immediately after change in k

The discussion so far has concerned the rate of growth or decay of activity at some time after a sudden change has been made in the reproduction factor, a change which may be either large, as in 22.7.5-11, or small, as in 22.7.11-17. Immediately after the insertion of a control rod, however, the asymptotic formulae for the rate of change of activity will not apply. In contrast, the rate of fission will increase or decrease by the fractional amount, $k_e(t/\tau)$, in the first few generations immediately after the location of the control rod is altered. In fact, during this short time interval, the level of activity will not greatly change. Consequently, the delayed neutrons given off as a result of the earlier steady operation of the pile contribute to the development of the chain reaction in these first few generations just as effectively as if they had been given off instantaneously. Thus we are entitled over this short time interval to disregard the delayed character of some of the neutrons and use the simple theory of multiplication, according to which the neutron activity will rise with time in proportion with the expression $e^{k_e t/\tau}$.

22.7.19 Advance warning of rise

From what has just been said, we can make a more detailed picture of the reaction just after the control rod is withdrawn slightly from a pile operating at a constant level. For example, let the increase be

$k_e = 1 \times 10^{-5}$, and let the mean life of a neutron in the pile be $\tau = 1.22 \times 10^{-3}$ seconds. Then, in the first few generations, the activity will rise at the fractional rate, 8.2×10^{-3}/sec. or 29.5/hr. The chain reaction will quickly adjust itself to the new conditions, however. The rate of rise will quickly drop off and approach the asymptotic value 0.329 per hour. This sudden increase in neutron activity in the pile furnishes, it will be seen, a kind of advance warning of the slower but inexorable rise later to take place if the value of the multiplication factor is not brought down again. This warning phenomenon, pointed out by Wigner,* is a second blessing bestowed by the existence of the delayed neutron effect.

Characteristics of Delayed Neutron Emitters

22.7.20 Number of delayed neutrons

The observed rate of slow rise or fall of activity when the excess multiplication factor is very little furnishes a means to estimate the absolute number of delayed neutrons relative to all neutrons from fission. Snell, Nedzel, and Ibser** have measured the decay of secondary neutron emission. They have decomposed the observed curve into the form of four exponentials. The periods of these four decay curves are listed in the following table, together with the relative values of the abundance f_i of each group. However, the difficulty of measuring the total number of neutrons made it possible to give only an approximate figure for the absolute value of the sum,

$$f = \sum f_i \sim 0.01. \qquad (22.7.20.a)$$

However, from the experiments on the effect of a control rod on a chain reacting pile (22.7.16), we obtain a considerably more precise value for a quantity closely connected with the number of delayed neutrons;

$$\sum f_i/A_i + \tau = 0.1094 \text{ seconds} \qquad (22.7.20.b)$$

The value of the mean life, τ, of a neutron in the pile in question is estimated to be approximately 0.0014 second. We conclude that the sum is

$$\sum f_i/A_i = 0.108 \quad . \qquad (22.7.20.c)$$

Possession of this datum makes it possible to readjust the absolute values of the ratios f_i given by Snell, Nedzel, and Ibser, as indicated in Table 22.7.21. In view of various experimental uncertainties, two sets of values are given such that the calculated values of $\sum f_i/A_i$ bracket the experimental value.

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*E. P. Wigner, CP-351, On Variations in the Power Output in a Running Pile.

**Snell, Nedzel, and Ibser, C-81, A Study of the Delayed Neutrons Associated with Uranium Fission.

Table 22.7.21. Number and period of hold-up of delayed neutrons. The quantities, f_i, give the number of delayed neutrons relative to the total of all neutrons resulting from fission. Each quantity, A_i, represents the rate of decay of the radioactive fission product responsible for the given group of delayed neutrons. The absolute values of the A_i and the relative values of the f_i come from the work of Snell, Nedzel and Ibser. Two sets of absolute values of the f_i are given, such that the calculated values of the sum $\sum_i f_i/A_i$ bracket the value, 0.108 sec., deduced from the curve of response of the chain reacting pile to a small change in multiplication factor.

i	parent fission product	A_i in sec^{-1}	half life, $0.693/A_i$	values adjusted to $f = 0.005$			values adjusted to $f = 0.008$		
				f_i $\times 10^{+3}$	f_i/A_i $\times 10^{+3}$	$f_i A_i$ $\times 10^{+6}$	f_i $\times 10^{+3}$	f_i/A_i $\times 10^{+3}$	$f_i A_i$ $\times 10^{+6}$
1	unknown	0.28	2.5	1.69	6.05	473	2.71	9.67	760
2	unknown	0.099	7	1.69	17.10	167	2.71	27.4	268
3	unknown	0.029	24	1.42	48.8	41	2.27	78.3	66
4	unknown	0.012	58	0.19	15.6	2.3	0.30	25.0	3.6
$f = \sum f_i$. When the value of k_e is large in comparison with f, the quantity $k_e - f$ determines the asymptotic rate of growth of activity				0.005			0.008		
$\sum f_i/A_i$. When the absolute value of k_e is small in comparison with f, the "effective lifetime" of one generation is $\tau + \sum f_i/A_i$				87.6×10^{-3} sec.			140.4×10^{-3} sec.		
$\sum f_i/A_i^2$. Continued steady operation followed by a sudden small increase of k_e leads after the lapse of a long time to a rate of fission greater than the original figure by the factor $\left\{1 + k_e \frac{\sum f_i/A_i^2}{(\tau + \sum f_i/A_i)^2}\right\}$. $\exp \frac{k_e t}{(\tau + \sum f_i/A_i)}$				3.43 $sec.^2$			5.09 $sec.^2$		
$\sum f_i A_i$. If a sudden burst of fission suddenly produces 10^6 potential secondary neutrons, then the number $10^6 f$ will be delayed, and of these the number $10^6 \sum f_i A_i dt$ will be released in an immediately following small time interval, dt				0.684×10^{-3} sec^{-1}			1.10×10^{-3} sec^{-1}		

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SPEED OF CONTROL 22.7.22

Sudden Burst of Activity

In all cases so far considered, the chain reaction was in operation at a constant level up to the moment when the multiplication factor was suddenly changed. In contrast, let us ask how the activity will vary following a sudden burst of fission.* Such a burst may come about through short irradiation of a pile by a cyclotron. Alternatively, a water cooled pile may suddenly lose its water. The multiplication factor will then increase by a considerable amount, the activity will rise at a rapid rate until checked by the sudden entry of the safety controls. If the rise in activity is sufficiently great in comparison with the original rate of operation, the effect of this sequence of events will be equivalent to a short burst of irradiation. The characteristic feature of a burst is the abnormally low percentage of stored neutrons relative to instantaneous neutrons. This circumstance has, as a consequence, that the effective multiplication factor following the burst is less than the quantity, k_e, by the amount, f. If k_e at that time is negative, or is positive but very small, the activity will rapidly decay after the burst. This rapid decay will not continue indefinitely, however, because some of the radioactive nuclei formed during the burst will provide a source of neutrons for some seconds afterward. 22.7.22 Activity after sudden burst

The growth and decay of the neutron-emitting nuclei can be followed in more detail. Let N potential new neutrons be formed during the burst. Of these the number Nf will not be emitted at the time of the activation but only later. Of the Nf_i neutrons "stored" in nuclei with a radioactive decay constant, A_i, the fraction, $A_i dt$, will be emitted in a small time interval, dt, just subsequent to the burst. Consequently, the total rate of release of delayed neutrons just after the burst will be $N(\Sigma f_i A_i)$ per second. Each of these neutrons will be multiplied into a convergent chain of neutrons provided that the effective excess multiplication factor, $k_e - f$, is negative. The total number of neutrons in one such chain will be $1/[(-k_e) + f]$. Taking into account the delayed neutrons themselves and the neutrons from the fissions they produce, we can say that shortly after the end of the burst the total rate of emission of neutrons will be 22.7.23 Rapid fall followed by slow fall or slow rise

$$\frac{N(\Sigma f_i A_i)}{(-k_e) + f} \qquad (22.7.23.a)$$

The activity will continue slowly to fall off at a rate determined by the period of the delayed neutrons. Eventually, the radioactive nuclei formed during this subsequent period of activity will themselves begin to contribute an appreciable number of delayed neutrons. If the excess multiplication factor is slightly above unity, these delayed neutrons will be sufficient in number gradually to build up the power output at an ever increasing rate, tending asymptotically to a time dependence proportional to the expression

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*H. W. Ibser, John H. Manley, and John A. Wheeler, C-65, Burst Method of Determining Approach to a Self-Sustaining Reaction.

[redacted]

[redacted]

$$\exp \frac{k_e\, t}{\tau + \sum f_i/A_i} \qquad (22.7.23.b)$$

If, on the other hand, the multiplication factor is slightly less than one, the activity will continue to decay after the burst and will be represented asymptotically by the same formula with k_e negative.*

22.7.24 Example of sudden burst

EXAMPLE: Sudden loss of water from a 250 megawatt water cooled pile results in an increase 0.02 in the excess multiplication factor. Half a second later, the safety rods have taken effect and reduced the excess multiplication factor to $k_e = -0.01$. To what level is the rate of fission carried by this sequence of events?

During the period of rapid rise of activity, the effective excess multiplication factor is approximately

$$k_e - f = 0.020 - 0.006 = 0.014 \qquad (22.7.24.a)$$

The fractional rate of rise of activity per second is approximately

$$\frac{(k_e - f)}{\tau} = \frac{0.014}{1.22 \times 10^{-3}} = 11.5 \text{ sec}^{-1} \qquad (22.7.24.b)$$

The time required for the activity to increase by the factor, e, is 0.087 sec. After the time, t = 0.5 seconds, the rate of fission has risen by a factor whose value can be obtained from Fig. 22.7.10. The factor is

$$\frac{k_e}{(k_e - f)}\, e^{(k_e - f)t/\tau} - \frac{f}{(k_e - f)} \qquad (22.7.24.c)$$

$$= 1.43 \times 310 - 0.43 = 443$$

Immediately after the insertion of the safety controls, the effective excess multiplication factor becomes

$$-0.01 - 0.006 = -0.016$$

The activity falls off at the fractional rate

$$0.016/1.22 \times 10^{-3} \text{ sec} = 13.1 \text{ sec}^{-1}$$

The time required for the activity to decrease by the factor, e, is 0.076 second. The mean time during which the activity may be considered to have the peak value, 443 fold of the original figure, is

*Curves illustrating these phenomena are given in the preceding reference, C-65.

[redacted]

$$0.087 + 0.076 = 0.163 \text{ sec.}$$

The number of potential neutrons formed during the burst of activity is consequently equal to the number formed during operation at the original level for an interval of

$$443 \times 0.163 = 72.2 \text{ seconds}$$

Those among these neutrons which are delayed will be liberated in the period shortly after the burst at the rate (last line of Table 22.7.21)

$$\sum f_i A_i = 0.82 \times 10^{-3}/\text{sec.} \qquad (22.7.24.d)$$

These neutrons are released in a medium whose effective excess multiplication factor is $k_e - f = 0\text{-}0.016$. Taking into account the multiplication of the delayed neutrons by the medium, we conclude that the rate of fission shortly after the conclusion of the burst differs from the rate of fission during the original steady operation by the factor

$$\frac{72.2 \text{ sec.} \times 0.82 \times 10^{-3} \text{ sec}^{-1}}{0.016} = 3.7 \qquad (22.7.24.e)$$

in accordance with Eq. 22.7.23.a. In a first approximation we can, therefore, say that the power output rises from 250 megawatts to 110,000 megawatts at the end of the first half second and then suddenly drops off as the control rods go in to a level of 920 megawatts and finally slowly decays. For a more precise determination of the power output at each instant, one must, of course, take into account the fact that a portion of the heat is liberated not simultaneously with the act of fission but only after the lapse of some time due to the radioactive decay of the various fission fragments and the breakdown of 92-239 and 93-239. This effect produces a correction in the foregoing figures only of the order of magnitude of 10%.

Theory of Delayed Neutron Effects

We have seen in a qualitative way the effect of the delayed neutrons in retarding the response of the chain reacting pile to a sudden alteration in multiplication factor, and the way in which they prolong the reaction long after it would otherwise have been expected to drop to a negligible level. In order to consider these effects in more detail, we may introduce equations describing quantitatively the change with time of the rate of fission and of the number of "stored" neutrons:

22.7.25 Equations for storage and release of neutrons

s_i = number of neutrons "stored" in radioactive nuclei with the decay constant, A_i. Table 22.7.21 indicates the four radioactive periods involved.

τ = mean life of a neutron in the pile after the moment of emission.

F = number of potential new neutrons formed during the mean life time, τ, of one generation.

f_i = fraction of these neutrons "stored" in radioactive nuclei with the decay constant, A_i.

k_e = excess multiplication factor.

In the small time interval, dt, the fraction, A_i dt, of the delayed neutrons, s_i, will become free. On this account, the number of "stored" neutrons will decrease by the amount, $s_iA_i dt$. New stored neutrons will, however, be formed during the same time interval. The number of generations is dt/τ. In each generation, F potential new neutrons are formed, of which the fraction, f_i, are delayed. The increment in stored neutrons is, therefore, $f_i F (dt/\tau)$. Balancing this new production against the loss by decay, we conclude that the number of "stored" neutrons increases by the amount

$$ds_i = \left\{-A_is_i + f_i F/\tau\right\} dt \qquad (22.7.25.a)$$

In a similar manner, we can calculate the change, dF, in the number, F, of potential new neutrons formed per generation. So far as instantaneous multiplication of neutrons is concerned, the effective reproduction factor is $k_e - f$. In the (dt/τ) generations which occur in the time interval, dt, the rate of production of potential new neutrons increases on this account by the fractional amount $(k_e - f)(dt/\tau)$. In addition, the delayed neutrons released during this same time interval produce fission and thereby form additional potential new neutrons to the number $\sum s_iA_i dt$. Altogether, we have for the increase in number of potential new neutrons formed per generation the quantity

$$dF = \left\{(k_e - f)(F/\tau) + \sum A_is_i\right\} dt \qquad (22.7.25.b)$$

The equation (22.7.25.b) and the four equations (22.7.25.a) govern the change with time of the five quantities F, s_i.

22.7.26 The characteristic solutions

We can investigate the equations for neutron storage by the method of characteristic solutions. We can assume as trial solution

$$s_i = s_i^I e^{\lambda^I t} \qquad (22.7.26.a)$$

and

$$F = f^I e^{\lambda^I t} \qquad (22.7.26.b)$$

where the constants s_i^I and λ^I are to be determined by substituting these expressions into the differential equations (22.7.25.a and b). From equations (22.7.25.a), we find

[redacted]

$$s_i^I = \frac{f_i F^I/\tau}{A_i + \lambda^I} \qquad (22.7.26.c)$$

Inserting these values for the quantities s_i^I and the expression (22.7.26.b) for F in the other differential equation (22.7.25.b), we find the following relationship connecting the fractional rate of growth, λ^I, with the decay periods of the delayed neutrons and the excess multiplication factor:

$$\lambda^I\tau + \sum_i \frac{\lambda^I f_i}{A_i + \lambda^I} = k_e \qquad (22.7.26.d)$$

From this so-called "characteristic equation" follow the results we have already found in the extreme cases of very large and very small change in reproduction factor. For example, when the rate of growth of activity is very small compared to the decay periods of the delayed neutrons, we find

22.7.27 Relations for growth constant

$$\lambda(\tau + \sum f_i/A_i) = k_e \qquad (22.7.27.a)$$

in agreement with the considerations of 22.7.15. On the other hand, when the rate of growth, λ, is large in comparison with the decay constants, A_i, we obtain the opposite limiting formula

$$\lambda\tau + f = k_e \qquad (22.7.27.b)$$

Certain accurate and simple relations between the growth constants follow from the theory of algebraic equations. When equation (22.7.26.d) is expressed in polynomial form, the coefficients of the various powers of λ are symmetric functions of the five roots, λ^I....... λ^V. Thus we find

$$\lambda^I + \lambda^{II} + \ldots\ldots + \lambda^V = (k_e - f)/\tau - \sum_i A_i \qquad (22.7.27.c)$$

$$\frac{1}{\lambda^I} + \frac{1}{\lambda^{II}} + \ldots\ldots + \frac{1}{\lambda^V} = \frac{\tau + \sum f_i/A_i}{k_e} - \sum_i \frac{1}{A_i} \qquad (22.7.27.d)$$

$$\lambda^I \lambda^{II} \lambda^{III} \lambda^{IV} \lambda^V = A_1 A_2 A_3 A_4 \; k_e/\tau \qquad (22.7.27.e)$$

These relations are of service below.

In the general case of an arbitrary value of excess multiplication factor, k_e, the characteristic equation (22.7.26.d) can only be solved by numerical or graphical methods. Figure 22.7.29 presents a graph of the

22.7.28 Five characteristic solutions

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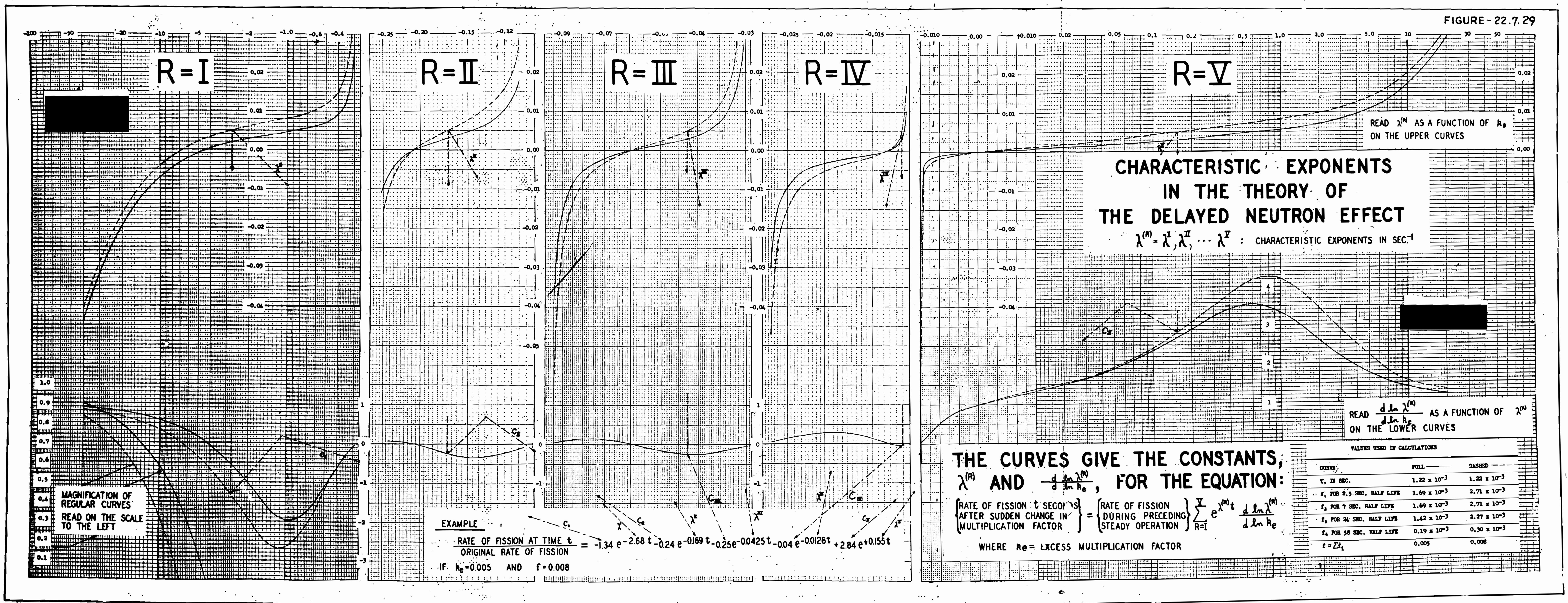

VALUES USED IN CALCULATIONS

CURVE	FULL ———	DASHED - - -
τ, IN SEC.	1.22×10^{-3}	1.22×10^{-3}
f_1 FOR 2.5 SEC. HALF LIFE	1.69×10^{-3}	2.71×10^{-3}
f_2 FOR 7 SEC. HALF LIFE	1.69×10^{-3}	2.71×10^{-3}
f_3 FOR 24 SEC. HALF LIFE	1.42×10^{-3}	2.27×10^{-3}
f_4 FOR 58 SEC. HALF LIFE	0.19×10^{-3}	0.30×10^{-3}
$f = \Sigma f_i$	0.005	0.008

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left-hand side of this equation as a function of λ. It will be noted that for any given value of k_e, we have not one solution but five solutions for λ : λ^{I}, λ^{II}, λ^{III}, λ^{IV}, λ^{V}. In other words, the differential equations (22.7.25.a and b) are satisfied by any one of five independent solutions. Of these solutions the fifth, for example, has the form

$$F = F^{V} e^{\lambda^{V} t} \qquad (22.7.28.a)$$

$$s_i = s_i^{V} e^{\lambda^{V} t} = f_i \frac{F^{V}/\tau}{A_i + \lambda^{V}} e^{\lambda^{V} t} \qquad (22.7.28.b)$$

where F^{V} is an arbitrary numerical constant. It is quite reasonable that we should have five independent solutions, each with one arbitrary constant, for we have five differential equations which govern the change with time of the number of free neutrons and the number of neutrons "stored" with each of the four decay periods. In general, the problem which we are considering can be stated in these terms: Given the values of F and the four quantities s_i at the time $t = 0$, to find the values of these five quantities at any later moment. We evidently require all five adjustable constants in order to satisfy the given five initial conditions.

22.7.30 Orthogonality of characteristic solutions

In order to solve the problem just outlined, we shall make use of the relationship of "orthogonality" which exists between the characteristic solutions corresponding to two different growth constants, $\lambda^{(R)}$ and $\lambda^{(S)}$:

$$\tau F^{(R)} F^{(S)} + \sum_i \frac{\tau^2 A_i}{f_i} s_i^{(R)} s_i^{(S)} \begin{cases} (1) \text{ vanishes when } R \neq S \\ (2) \text{ equals } (F^{(R)})^2 d\, k_e/d\lambda^{(R)} \text{ when } R = S \end{cases} \qquad (22.7.30.a)$$

This orthogonality relation follows on (1) substitution of the two characteristic solutions of the form (22.7.26.a and b) in the basic differential equations (22.7.25.a and b) and (2) doing the algebra outlined in the following table:

Differential equations (22.7.25.a and b) as applied to characteristic solutions	Multiply equation by
$\lambda^{(S)} F^{(S)} = (k_e - f) F^{(S)}/\tau + \sum A_i s_i^{(S)}$	$\tau F^{(R)}$
$\lambda^{(S)} s_i^{(S)} = f_i F^{(S)}/\tau - A_i s_i^{(S)}$	$(\tau^2 A_i/f_i) s_i^{(R)}$
$\lambda^{(R)} F^{(R)} = (k_e - f) F^{(R)}/\tau + \sum A_i s_i^{(R)}$	$-\tau F^{(S)}$
$\lambda^{(R)} s_i^{(R)} = f_i F^{(R)}/\tau - A_i s_i^{(R)}$	$-(\tau^2 A_i/f_i) s_i^{(S)}$

[redacted]

SPEED OF CONTROL

22.7.31

Multiplication of the equations as indicated and summations lead to the result

$$(\lambda^{(S)} - \lambda^{(R)})\left\{\tau F^{(R)}\, F^{(S)} + \sum \frac{\tau^2 A_i}{f_i}\, s_i^{(R)}\, s_i^{(S)}\right\} = 0 \qquad (22.7.30.b)$$

When the two characteristic growth constants are different, the expression in brackets must evidently vanish. Thus the first part of the orthogonality relation is proved. On the other hand, when $\lambda^{(R)} = \lambda^{(S)}$, then the left-hand side of the orthogonality relation reduces by virtue of (22.7.26.c) to the product of $(F^{(R)})^2$ by the sum

$$\tau + \sum_i \frac{\tau^2 A_i}{f_i}\left\{\frac{f_i/\tau}{A_i + \lambda^{(R)}}\right\}^2 \qquad (22.7.30.c)$$

This expression may be obtained by differentiating the left-hand side of the characteristic equation (22.7.26.d) with respect to $\lambda^{(R)}$. Consequently it represents the derivative $d\, k_e/d\, \lambda^{(R)}$. The second part of the orthogonality relation is thereby proved.

We wish to find the state of the pile at any time when the value of the reproduction factor is fixed and when we are given at the initial instant, $t = 0$, (1) the number of stored neutrons, s_i, of each decay period and (2) the total number, F, of potential new neutrons formed per generation. The solution of this general problem* will be obtained by a superposition of the five characteristic solutions:

22.7.31 General expression for state of pile

$$F(t) = \sum_{R=I}^{V} F^{(R)}\, e^{\lambda^{(R)} t} \qquad (22.7.31.a)$$

$$s_i(t) = \sum_{R=I}^{V} s^{(R)}\, e^{\lambda^{(R)} t} = \sum_{R=I}^{V} \frac{f_i\, F^{(R)}/\tau}{A_i + \lambda^{(R)}}\, e^{\lambda^{(R)} t} \qquad (22.7.31.b)$$

We have only to find the five unknown constants, $F^{(R)}$, in order to complete the solution of the problem. For this purpose we set $t = 0$ in the five equations and apply the orthogonality relationship as follows. We multiply the first equation by $\tau F^{(S)}$ and the other four equations by $(\tau^2 A_j/f_j) s_j^{(S)} = F^{(S)} \tau A_j/(A_j + \lambda^{(S)})$. We add the results of the multiplication, use the orthogonality condition, and find

$$F^{(S)}\left\{\tau F(0) + \sum s_j(0) \frac{\tau A_j}{A_j + \lambda^{(S)}}\right\} = (F^{(S)})^2\, d\, k_e/d\, \lambda^{(S)} \qquad (22.7.31.c)$$

*H. W. Ibser, John H. Manley, and John A. Wheeler, C-65, Burst Method of Determining Approach to a Self-Sustaining Reaction. The more general case where the power level is low and the contribution of the spontaneous neutrons has to be taken into account is treated by E. P. Wigner, CP-351, On Variations of the Power Output in a Running Pile.

This equation tells the value of the typical unknown constant, $F^{(S)}$. Thus we are able to find from the state of the pile at the moment $t = 0$ the conditions obtaining at any later time. Explicitly, we have the result

$$F(t) = \sum_{R=I}^{V} \left\{ F(0) + \sum_j s_j(0) \frac{A_j}{A_j + \lambda^{(R)}} \right\} \cdot \tau \, (d\lambda^{(R)}/d\, k_e) \, e^{\lambda^{(R)} t} \qquad (22.7.31.d)$$

$$s_i(t) = \sum_{R=I}^{V} \left\{ F(0) + \sum_j s_j(0) \frac{A_j}{A_j + \lambda^{(R)}} \right\} \cdot \frac{f_i}{A_i + \lambda^{(R)}} (d\lambda^{(R)}/d\, k_e) \, e^{\lambda^{(R)} t} \qquad (22.7.31.e)$$

22.7.32 Case where activity has been constant

In the case of greatest interest, the pile has been in steady operation at a constant level up to the moment $t = 0$, when a sudden change is made in the reproduction factor. Up until the moment of change, the delayed neutrons are in equilibrium with the instantaneous neutrons. Quantitatively, this condition means that the number, s_i, of neutrons stored with the decay constant, A_i, is

$$s_i(0) = F(0) \, f_i/A_i \, \tau, \qquad (22.7.32.a)$$

as follows from (22.7.26.c) or as is apparent on a little consideration. The expression

$$\left\{ F(0) + \sum s_j(0) \frac{A_j}{A_j + \lambda^{(R)}} \right\} \qquad (22.7.32.b)$$

which appears in (22.7.31.d) will therefore reduce to the form

$$(F(0)/\tau) \left\{ \tau + \sum \frac{f_j}{A_j + \lambda^{(R)}} \right\} \qquad (22.7.32.c)$$

The bracketed quantity has the value $k_e/\lambda^{(R)}$, according to the characteristic equation (22.7.26.d). We make use of these reductions to simplify the general equation (22.7.31.d) for the rate of production of neutrons. We find a simple result in the present case, where the pile has been in operation at a constant level and where a sudden change has been made in k_e:

[redacted]

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FISSION AFTER SUDDEN WITHDRAWAL OF CONTROL

FIGURE 22.7.33

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——— 0.5 PERCENT DELAYED NEUTRONS

- - - - - - 0.8 PERCENT DELAYED NEUTRONS

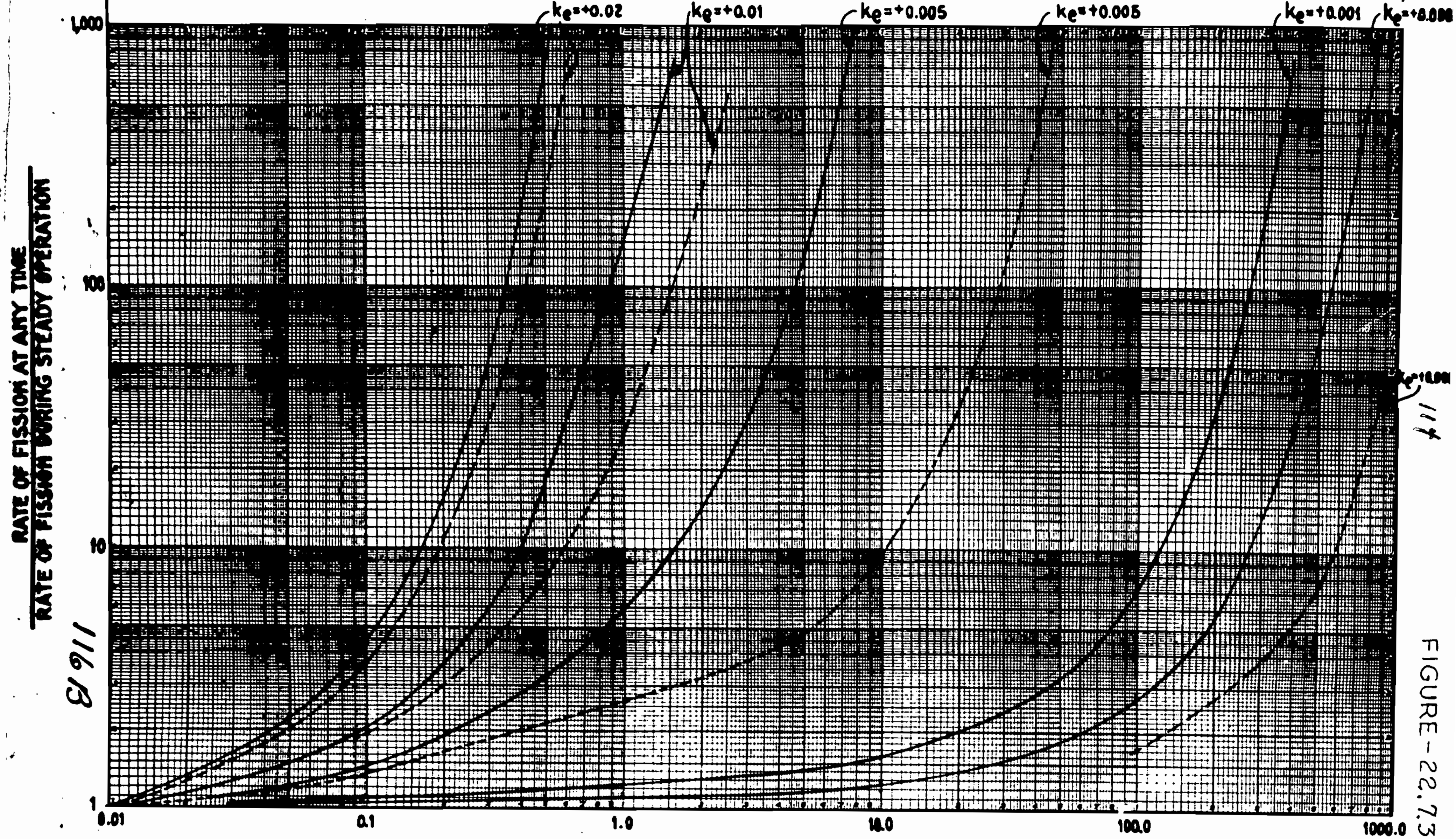

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FIGURE-22.7.33

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FIGURE 22.7.34

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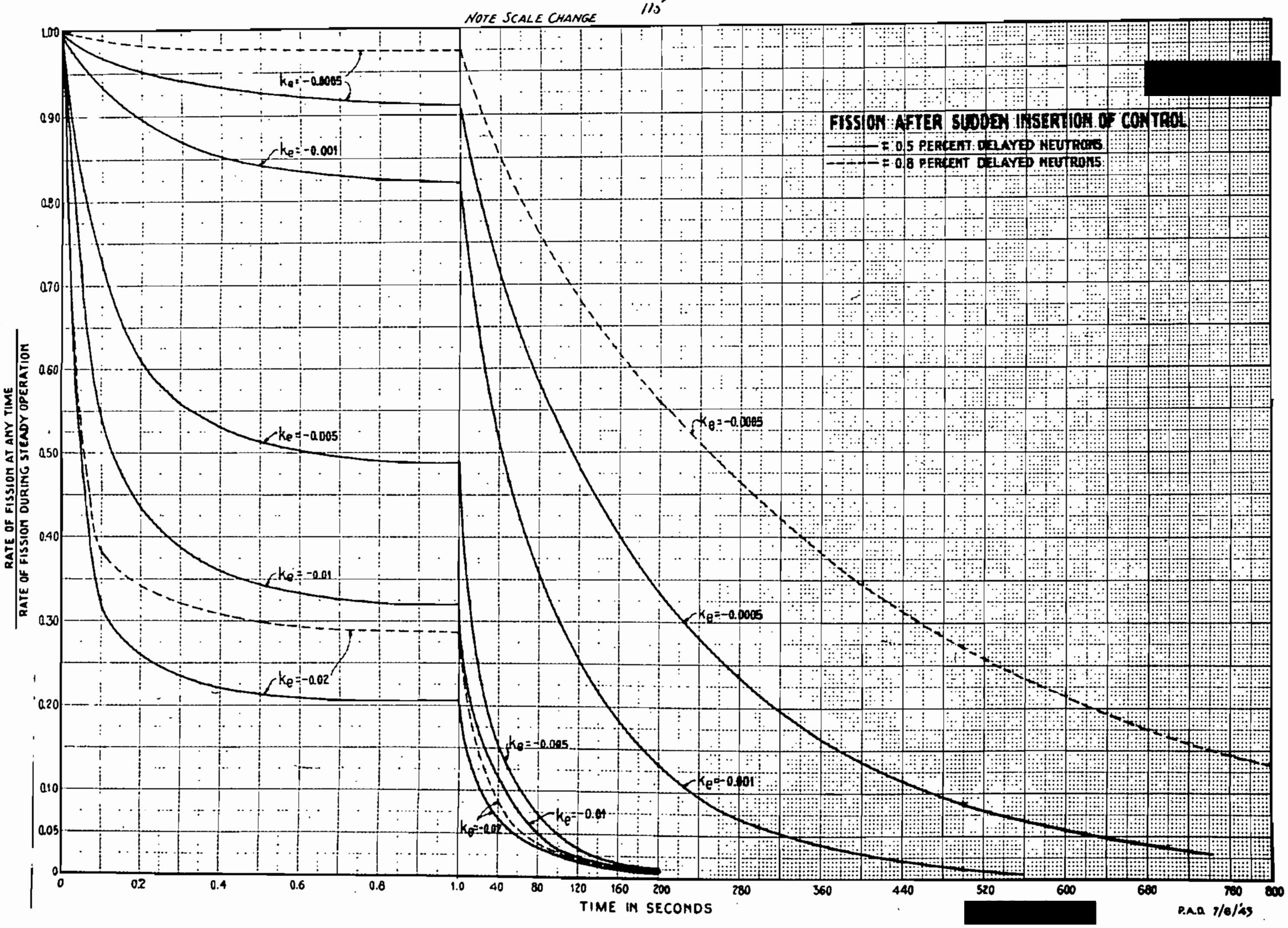

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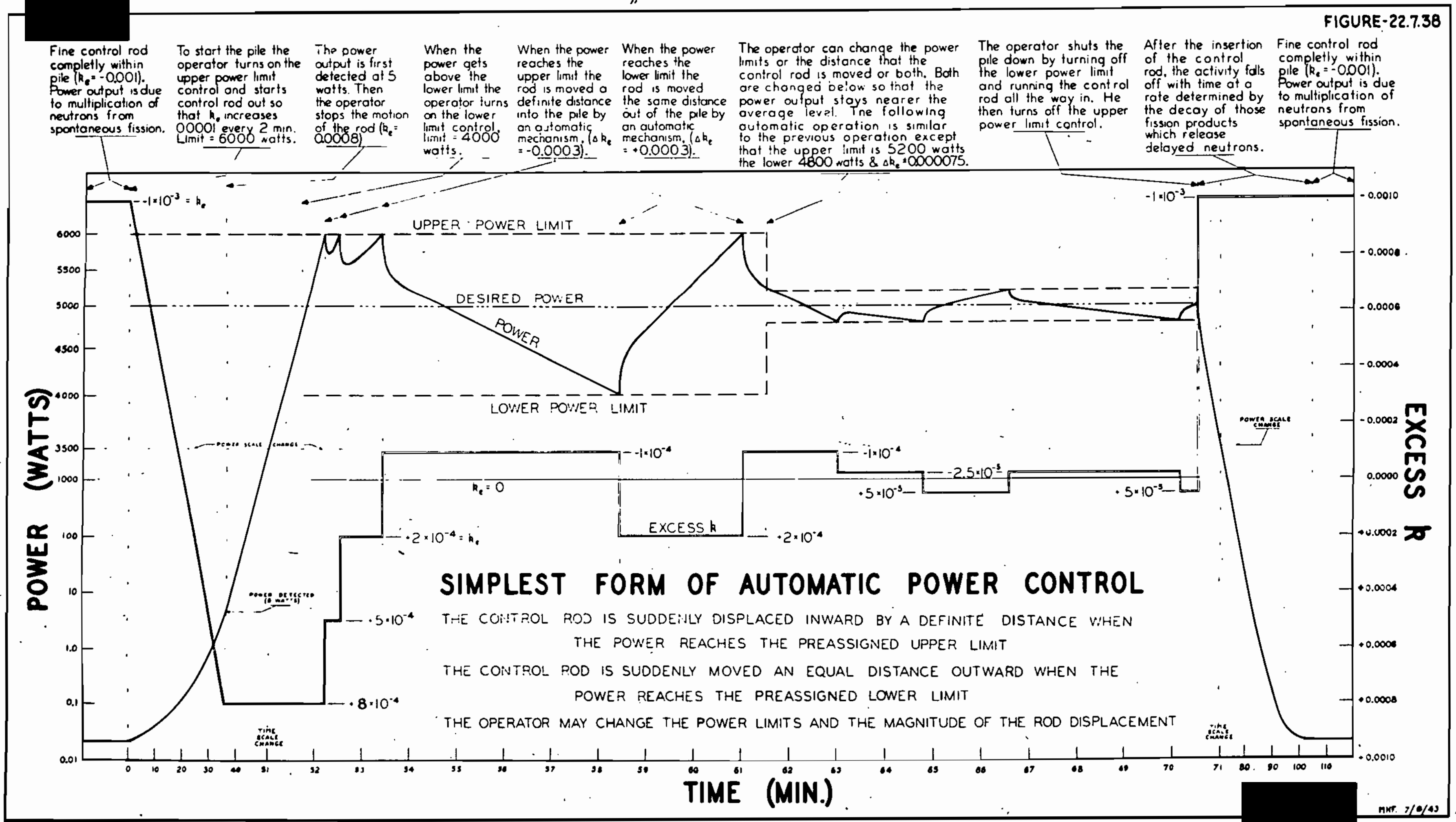

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Automatic Control

How to keep a pile running at a nearly constant output is one of the most important problems of control theory. The precise position of the control rod which is required for steady operation will not be known in advance; moreover, this position will change from time to time as alterations in temperature, barometric pressure, and content of cooling fluid affect the multiplication factor. There is, therefore, a need for an automatic mechanism to keep the output constant within certain preassigned limits. It will be the fine control whose motion will have to be governed in such a way as to accomplish this end. 22.7.36 Function of automatic control

The simplest form of automatic operation is provided by the so-called limit control. An ionization chamber or proportional counter in the pile is connected to an amplifying circuit. When the current flowing shows 1% below the preassigned value, a relay is activated and the control rod is pulled out a small distance. The multiplication factor, therefore, rises slightly. The pile continues to function with the new value of the multiplication factor. If the activity again falls off (see Figure 22.7.38), the same relay will soon be called to function again and the control rod will be pulled out still further. By a series of such operations the fine control will eventually be brought to a position corresponding to an effective reproduction factor slightly greater than unity. The activity will begin to rise. When it reaches a value 1% greater than the preassigned figure, a second relay is activated. Thereby, the fine control is pushed into the pile a small distance. The activity begins to fall. When it gets to 99% of the preassigned level, the lower limit relay again functions. Thus the output of the pile oscillates to and fro as the control rod is moved back and forth between two positions relatively close to each other. The time for one cycle of this process of rise and fall depends upon the magnitude of the displacement given to the control rod by the operation of the relay. Figure 22.7.38 illustrates the situation in the case where the activation of the relay brings about a change in k of 7.5×10^{-5}. It is seen that the power output will fluctuate with a period, depending upon the closeness of upper and lower limits. The period is in the neighborhood of several minutes in the example illustrated. The limiting type of control evidently provides an effective means to keep the output of the pile within preassigned limits. 22.7.37 Simplest form: limit control

The development of automatic control mechanisms for various industrial applications has shown in recent years an increasing tendency to make use of the principle of anticipation, in contrast to the simpler scheme of the limiting control. The regulating device is governed not only by the magnitude of the quantity which is to be controlled but also by the rate of change of this quantity. Consider a case where the quantity is rising rapidly but has not yet reached the preassigned upper limit. In this case the limiting control does nothing to stop the rise. The anticipatory control, however, is governed by the rate of rise. It already starts to retard the reaction before the upper limit has been 22.7.39 Anticipatory control

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$$\frac{\begin{Bmatrix}\text{rate of fission}\\ \text{after change}\end{Bmatrix}}{\begin{Bmatrix}\text{rate of fission}\\ \text{before change}\end{Bmatrix}} = \frac{F(t)}{F(0)} = \sum_{R=I}^{V} \frac{k_e \, d\,\lambda^{(R)}}{\lambda^{(R)} d\, k_e}\, e^{\lambda^{(R)}t} \qquad (22.7.32.d)$$

$$= \sum_{R=I}^{V} (d \ln \lambda^{(R)}/d \ln k_e)\, e^{\lambda^{(R)}t}$$

All the quantities used in this equation can be read off the curves in Figure 22.7.29. An illustrative example set forth in the figure shows the use of this equation to determine the rate of fission. Typical curves for the activity as a function of time after the sudden insertion or removal of the control rod are presented in Figures 22.7.33 and 22.7.34. Similar curves showing the variation of activity with time after a sudden burst of irradiation have been calculated and are available in C-65.

22.7.35 Total number of after-fissions

The total number of fissions occurring after the control rods enter the pile is a quantity of some importance. It is determined by integrating the rate of fission as given by (22.7.32.d). The result of the integration is evidently

$$\sum_{R=I}^{V} \frac{k_e \, d\,\lambda^{(R)}}{\lambda^{(R)} d\, k_e} \; \frac{1}{-\lambda^{(R)}} = \frac{k_e \, d}{d\, k_e} \sum_{R=I}^{V} \frac{1}{\lambda^{(R)}} \qquad (22.7.35.a)$$

The last sum is evaluated in equation (22.7.27.d). We differentiate it as indicated and find the result,

$$\frac{\begin{Bmatrix}\text{total number of}\\ \text{neutrons liberated}\\ \text{after } k_e \text{ drops}\end{Bmatrix}}{\begin{Bmatrix}\text{number of potential}\\ \text{neutrons formed per}\\ \text{second during operation}\end{Bmatrix}} = \frac{\begin{Bmatrix}\text{total number of}\\ \text{after-fissions}\end{Bmatrix}}{\begin{Bmatrix}\text{original number}\\ \text{of fissions per}\\ \text{second}\end{Bmatrix}} =$$

$$= \frac{\int_0^{\infty} F(t)\, dt}{F(0)} = \frac{\tau + \sum f_i/A_i}{-k_e} = \frac{0.11 \text{ sec}}{-k_e} \qquad (22.7.35.b)$$

For example, let the insertion of the control make a deficit, $-k_e =$ 0.01, in the reproduction factor. Then the total number of subsequent fissions is the same as the number which would have occurred in 11 more seconds of normal operation.

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22.7.40

reached. Electronic circuits have been developed by the group working on controls at the Chicago Metallurgical Laboratory under the leadership of V. C. Wilson and W. P. Overbeck, circuits which give the control rod a motion depending not only upon the level of neutron activity but also upon the rate at which this activity is changing with time. The theory governing the choice of constants in the electronic circuits has been developed by Christy.* This type of control proves to function satisfactorily.**

22.7.40 Pile well suited to control

Two features of the operating pile stand out from the foregoing discussion of the time dependence of activity. First, the delayed neutrons give the period of response to control a quite reasonable magnitude. Although the process of fission occurs in a time of the order of magnitude of 10^{-21} seconds, every other step in the chain reaction tends to lengthen the effective time interval of multiplication: the process of moderation extends it to $\sim 10^{-4}$ seconds, the process of thermal diffusion brings the time up to $\sim 10^{-3}$ seconds, and the delayed neutrons stretch it to ~ 0.1 second. Secondly, the control mechanism operates most conveniently not on the temperature of the cooling fluid itself, but on the neutron activity, i.e., essentially the rate of rise of temperature. In other words, it is possible to correct a sudden rise in neutron activity long before the temperature of the emergent water has had an opportunity to respond. This feature of a chain reacting pile increases greatly the stability of operation.

22.7.41 New developments

Subsequent to the writing of the main part of this section, five developments have occurred which are connected with the subject of speed of control and which may be summarized briefly here: (1) further study of delayed neutron emission, (2) more accurate experimental measurement of the relation between period and excess reactivity, (3) more complete treatment of decay of activity after shutdown when finite time is required for entrance of control rods, (4) treatment of self-stabilization of homogeneous heavy water pile by reason of negative temperature coefficient, (5) treatment of same problem for graphite piles.

22.7.42 Further study of decay of delayed neutrons

Decay of delayed neutron activity following a one second cyclotron irradiation has been studied with improved precision by Snell, Sampson, and Levinger.*** The counting rate drops sharply in the initial second after stoppage of bombardment, thus indicating that one emitter has a period less than a second. Inability to follow precisely this initial decay fortunately does not much affect the accuracy of estimates of the total number of delayed neutrons or the precision of calculations of their effect on the reactivity of a pile. Snell and his associates extrapolate the activity back to zero time in a reasonable way and find that it falls

*R. F. Christy, CP-349, Slow Control of a Chain Reaction.

**W. P. Overbeck, CT-669, Control System for the Argonne Pile.

***Arthur H. Snell, Milo B. Sampson, and J. S. Levinger, CP-1014, Further Work on the Possible Use of the Delayed Neutrons for Detection of Coating Failures in the W Pile.

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off in relative magnitude in proportion with the following expression:

$$0.57 \exp(-1.38t) + 0.40 \exp(-0.154t) + 0.024 \exp(-0.030t) + 0.002 \exp(-0.014t) \qquad (22.7.42.a)$$

The corresponding mean lives are 0.7 second, 6.5 seconds, 33 seconds, and 72 seconds, and the half lives, also in seconds, are 0.5, 4.5, 23 and 50.

22.7.43 Chemical identification of emitters

In other experiments* Snell and his co-workers have identified the 50 second delayed neutron emitter as a radioactive bromine nucleus and the 23 second source as an iodine activity. The activities were generated by brief irradiation of uranium at the Chicago cyclotron. The lifetime of the two groups in question was sufficient to carry out one chemical operation. The reaction chosen could be made sufficiently characteristic to make the identification fairly certain. Beta ray activities with periods similar to these have been observed by other workers. However, it is necessary to be cautious about concluding the identity of a known beta ray emitter with a delayed neutron source. The latter will lie at or near the head of a fission chain. On the other hand a beta ray activity of short period is only isolated chemically and studied satisfactorily when it is descended from an earlier fission product of reasonable lifetime.

22.7.44 Relation between period and excess reactivity

Combining the new results on decay of sources of delayed neutrons with observations on the rate of growth of power output of the Argonne pile, Fermi** has been able to describe the connection between period and excess reactivity with improved precision. He applies relation (22.7.26.d) between growth constant and excess k (1) taking the periods and relative strengths of the delayed neutron groups from the work of Snell's group, (2) adjusting the relative strength of delayed and instantaneous neutrons to fit the Argonne data, and (3) eliminating reference to the absolute value of the multiplication factor because that quantity cannot at present be measured with a precision at all approaching the accuracy of determinations of period and of relative changes in reactivity. On this last account the experimental results have been put into a form which involves only relative values of the excess multiplication factor, k_e:

$$\frac{k_e \text{ (for period of } T \text{ seconds)}}{\left(\begin{array}{l}\text{limiting value observed in case}\\ \text{of very long periods for product}\\ \text{of period in hours and excess } k\end{array}\right)} = \left(\begin{array}{l}\text{excess reactivity expressed}\\ \text{in "inhours" (definition)}\end{array}\right)$$

$$= \frac{54}{T} + \frac{33}{T + 0.7} + \frac{1139}{T + 6.5} + \frac{1793}{T + 34} + \frac{585}{T + 83} \qquad (22.7.44.a)$$

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*A. Snell, CP-961, Report for Month ending 1943, September 25.

**Personal communication by P. Morrison of results transmitted by E. Fermi on 1944, March 29.

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[redacted]

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The first term in this formula is a measure of the effect of the instantaneous neutrons. The relative magnitude of this term will change by an amount of the order of magnitude of 30 percent between graphite piles of different composition. Such a change will evidently have only little effect on the overall value of the right hand side of the equation. On this account Fermi's relationship between period and relative reactivity should apply to all graphite piles with an accuracy of the order of magnitude of 1 percent for periods of 20 seconds or more. For shorter periods on the other hand the difference between one pile and another will increasingly manifest itself because the first term in 22.7.44.a eventually dominates.

22.7.45 Effect of finite time of insertion of rods

How the shutdown of a pile is influenced by the speed of insertion of the control rod is a question of some practical interest which has been treated by Schwinger.* When the rods enter instantly the activity falls off with time as illustrated in Figs. 22.7.33 and 34. Actually the safety rods of the Hanford pile require a period of the order of 2 to $2\frac{1}{2}$ seconds to fall into position. The events during this time can be analyzed by neglecting as in the case of instantaneous insertion (22.7.8) the production or decay in the number of neutron emitters which takes place in so short an interval. We are then dealing with a constant source of neutrons undergoing multiplication in a medium whose reproduction factor changes with time. All other aspects of the delayed neutrons are overlooked. Schwinger has treated this straight-forward problem of transient equilibrium. He has given the solution in graphical form for the particular case where the control rod crosses the active zone of the pile in a time of 0.93 seconds. His curves show that the power output has fallen at the end of half a second to a level about half way between its original figure and the final value for transient equilibrium with rods all the way in; i.e., half way to the fraction $0.006/(-k_e + 0.006)$ of the original power. From this result it is apparent that the speed of shutdown of the Hanford pile during the entrance of the safety rods is limited more by the rate at which the controls enter than it is by neutronic characteristics of the chain reaction.

22.7.46 Stabilizing effect of temperature coefficient

The considerations on stability of operation and speed of shutdown so far developed assume that the multiplication factor is determined entirely by external conditions such as rod movement and water supply. The internal temperature of the pile is however an important additional factor in stabilizing its operation. The coefficient of the reactivity with respect to temperature change has already been discussed in Chapter 16 for piles of various types. The application of these results has so far been explored in any detail only for the homogeneous heavy water pile and for graphite piles.

22.7.47 Self-stabilization of heavy water pile

Schwinger has analyzed the behavior to be expected from a 30 ton pile constituted of a homogeneous mixture of heavy water and uranium. The pile material is conceived to be a slurry and is circulated by means of special pumps to heat exchangers and back to the reactor. Schwinger

*Julian Schwinger, CP-884, 1943 August 16, The Effect of the Insertion of a Safety Control Rod on the Power Output of the Pile.

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considers the case where the pile is operating at 800 megawatts and half the pump equipment suddenly fails. In this case the temperature of the slurry rises and thereby decreases the reactivity of the structure.* He** takes into account the effect of the change in reactivity on the power output, assuming that 2/3 percent of the neutrons given off in each generationarise from radioactive fission products and that 1/3 percent of the neutrons originate from the dissociation of deuterium by gamma rays. He concludes that subsequent to the pump failure the temperature of the slurry will rise at most by 14°C and that this rise will occur in a period of the order of 47 seconds. Subsequently the power output of the pile will stabilize at a new and lower level. In other words the negative temperature coefficient of the chain reaction is an important factor in the stability of the proposed slurry pile.

22.7.48 Self-stabilization of graphite piles

The important position of graphite-uranium piles in the plutonium project gives special interest to the discussion of their stability during operation. In this connection we may note the similarity between the Clinton and Hanford structures as regards temperature coefficient. For both piles the dominating effect is a loss in the reactivity of 2×10^{-5} per °C rise in the effective average temperature of the metal. In neither of the two rather different piles does the temperature of the graphite exert any important influence on the reactivity. On this account we can say that the temperature of the graphite will follow the power output of the pile but will not affect it. This circumstance reduces the analysis of the thermal stabilization of these piles to a two-fold problem: (1) the effect upon the temperature of the metal of unbalance between heat production and cooling and (2) the effect of the temperature of the metal by way of the temperature coefficient upon the rate of change of the power output. A simple mathematical analysis of the interaction of these two effects shows that the temperature and power output of the Clinton pile when displaced from a state of equilibrium oscillate about that state with a period of the order of 17 minutes while a similar disturbance in the Hanford pile will be followed by a gradual drift back to equilibrium.

22.7.49 Equations of self-stabilization

We use the following notation to describe the conditions in the pile:

T, the value of the effective average temperature of the metal in the pile, taken with respect to the inlet temperature of the coolant as point of reference. The uranium acts as a thermostat and ultimately stabilizes its power output at a level which will just maintain this temperature elevation at the value $T = T_o$. The numerical magnitude of T_o depends upon the position at which the control rods are locked.

P, power output. In a steady state at the temperature $T = T_o$ the pile gives off the power $P = P_o$.

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*The effect of temperature on the reactivity of a homogeneous uranium-heavy water pile has been treated by Gale Young in CP-807, 1943 August 6.

**These results of Schwinger are briefly summarized in the Chicago report for Nuclear Physics Research for the month ending 1943 October 23, CP-1016.

[redacted]

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H, heat capacity of the uranium. We neglect the transfer of heat from uranium to graphite and via graphite to the coolant as having only a small effect on the self-stabilization of the pile.

t, elapsed time.

δT, difference between instantaneous temperature elevation and equilibrium temperature elevation, T_o.

δP, difference between instantaneous power output and equilibrium power output.

The flow of heat from the metal to the coolant is proportional to the temperature difference between the two. When the temperature elevation is T_o, the rate of flow is P_o. Consequently, when the temperature difference increases by the amount δT the rate of heat transfer will increase by the amount $(P_o/T_o)\,\delta T$. On the other hand it will tend to warm up due to the increase, δP, in the power output of the metal. The difference between the two opposing tendencies is measured by the quantity, $\delta P - (P_o/T_o)\,\delta T$. This expression represents the excess of heat production in the metal over escape of heat from the metal and therefore determines, together with the heat capacity of the metal, the rate of warming of the uranium:

$$d\,\delta T/dt = (\delta P - (P_o/T_o)\,\delta T)/H \qquad (22.7.49.a)$$

Having thus evaluated the effect on temperature of changes in power output, we have to find the effect on power output due to temperature. When the metal warms up by the amount, δT, the reactivity drops by the amount $-2 \times 10^{-5}\,\delta T$. We divide this change in k by the effective lifetime, 0.1 second, of one generation of neutrons (22.7.17.c) in order to obtain the fractional decrease in power output which occurs in the course of one second. Thus we arrive at the second fundamental equation of the problem of self-stabilization:

$$d\,\delta P/dt = -2 \times 10^{-4}\,P_o\,\delta T \qquad (22.7.49.b)$$

22.7.50 Characteristic solutions

In order to investigate the rate of approach to equilibrium after a disturbance in the operating conditions of the pile, we try whether we can satisfy equations 22.7.49.a and 22.7.49.b by a solution of the form:

$$\begin{aligned} \delta P &= P_1 \exp(\lambda t) \\ \delta T &= T_1 \exp(\lambda t) \end{aligned} \qquad (22.7.50.a)$$

In order to make such a solution satisfy the equations in question, we may have to give λ a positive value. In this case we will conclude that the pile is unstable when the control rods are locked into position as we have assumed. Then some control mechanism will have to be provided in order to prevent a catastrophe. On the other hand, if λ must be negative in order to permit a solution of the form in question, then the pile will come smoothly to equilibrium after a disturbance. Finally,

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if λ is complex we will conclude that the temperature and power output of the pile will oscillate periodically after a disturbance. We substitute the trial solution (22.7.50.a) into the differential equations and discover that it will give a proper account of a disturbance only if the characteristic exponent λ is a solution of the quadratic equation:

$$\begin{vmatrix} \lambda & 2 \times 10^{-4} P_o \\ -1/H & \lambda + (P_o/T_oH) \end{vmatrix} = 0 \qquad (22.7.50.b)$$

The two characteristic roots are therefore:

$$\lambda = -(P_o/2\,T_oH) \pm \left[(P_o/2\,T_oH)^2 - (2 \times 10^{-4}\,P_o/H)\right]^{\frac{1}{2}} \qquad (22.7.50.c)$$

22.7.51 Oscillations when power exceeds critical valu

From the nature of the characteristic exponents, we conclude that the pile is never unstable and can at most be oscillatory. Oscillations will take place after a disturbance if the quantity within the square brackets in 22.7.50.c is negative. The first term in those brackets involves the ratio between the equilibrium power output and the equilibrium temperature. This ratio is independent of the power output itself and depends only on the type of cooling system in use. The second term, however, is directly proportional to the power output. Consequently, for a pile with any fixed rate of circulation of coolant the response to a thermal disturbance will never be oscillatory if the power output is sufficiently low and will always be oscillatory if the power output is sufficiently high. The critical power output at which the transition occurs between the one type of response and the other is given by the equation:

$$P_{critical} = (1250^{o}C\ sec)\ H\ (P_o/T_oH)^2 \qquad (22.7.51.a)$$

Adopting the numerical values shown in Table 22.7.52, we find that the critical output of the Clinton pile for oscillation is (1/40) megawatt and for the Hanford pile is 400 megawatts. The first figure is far below normal operating conditions at Clinton and the second is considerably above the output initially expected at Hanford. Consequently, oscillations of temperature and power are to be anticipated in the one pile following a disturbance and a simple relapse to normal conditions to be expected in the other case. The quantitative and qualitative features of the response are presented in further detail in Table 22.7.52 and in Fig. 22.7.53. Qualitative experimental information on the behavior of the Clinton pile confirms in a general way the conclusions obtained by this simplified mathematical analysis.* This confirmation of our general method of approach gives some assurance that the Hanford piles will show the very smooth and rapid self-stabilization properties predicted for them above and illustrated in Fig. 22.7.53. May, 1944

*A more detailed analysis has since been carried out for the Clinton pile by L. W. Nordheim, M-CP-1611, 1944 May 15. He finds satisfactory agreement with observation. A discussion with him of the corresponding Hanford problem indicates that allowance for the finer details of the delayed neutron decay and for finite heat conductivity of the uranium should not markedly change the conclusions reported in Fig. 22.7.53.

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Table 22.7.52. THERMAL SELF-STABILIZATION OF COOLED PILES

Full flow of coolant maintained and position of control rods kept constant while temperature and power output of pile approach or oscillate about a steady state value. Multiplication factor assumed to drop 2×10^{-5} per °C rise in effective average temperature of metal and to be practically unaffected by graphite temperature. Uranium acts as thermostat and stabilizes its power output at a level which will just maintain the temperature required to compensate the available excess k. Graphite temperature assumed to follow power output but not to affect it. Heat capacity of metal 0.030 cal/gm°C or (1/8) megawatt sec/metric ton °C. Figures below conventionalized; accurate values depend upon loading pattern. Conclusions illustrated in Fig. 22.7.53.

Quantity	Symbol	Clinton	Hanford	Hanford
Normal power output	P_o	1 Mw	1 Mw	250 Mw
Metric tons of uranium		40	160	160
Heat capacity of metal	H	5Mw sec/°C	20Mw sec/°C	20Mw sec/°C
Ratio of normal power to heat capacity gives initial rate of cooling on shutdown	P_o/H	0.2 °C/sec	0.05 °C/sec	12.5 °C/sec
Order of magnitude of mean temperature elevation of metal	T_o	100°C	0.4°C	100°C
Fractional decrease per second in case of shutdown or rise per second in case cooling fails while pile is running	P_o/HT_o	2×10^{-3}/sec	0.125/sec	0.125/sec
Reciprocal of last row defines cooling period in seconds	HT_o/P_o	500 sec	8 sec	8 sec
Cooling medium		air	water	water
Critical power output	$1250\,P_o^2/T_oH$	1/40 Mw	400 Mw	400 Mw
Temperature coefficient of k		-2×10^{-5}/°C	-2×10^{-5}/°C	-2×10^{-5}/°C
Temperature coefficient of growth constant, allowing 0.1 sec effective mean life per neutron generation		-2×10^{-4}/sec°C	-2×10^{-4}/sec°C	-2×10^{-4}/sec°C
Rate of fall of growth constant with time in absence of cooling (preceding row times P_o/H)		$4 \times 10^{-5}/\text{sec}^2$	$10^{-5}/\text{sec}^2$	$2.5 \times 10^{-3}/\text{sec}^2$

Continued on next page

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Table 22.7.52 - Con'd.

Quantity		Clinton	Hanford	Hanford
Characteristic exponents in solution (22.7.50.c) of stabilization problem $-(P_o/2HT_o) \pm [(P_o/2HT_o)^2 - \text{preceding row}]^{\frac{1}{2}}$				
Case of two real exponents	Bigger exponent represents essentially the rate at which uranium temperature adjusts itself to a value which will carry to coolant the heat as it is produced	no real exponents	$-0.125\ \sec^{-1}$ (reciprocal is 8 seconds)	$-0.10\ \sec^{-1}$ (reciprocal is 10 seconds)
	Smaller exponent represents essentially the rate at which power output and temperature subsequently hand-in-hand adjust themselves to value at which excess k is just compensated.		$-8 \times 10^{-5}\ \sec^{-1}$ (reciprocal is 3.5 hours)	$-2.5 \times 10^{-2}\ \sec^{-1}$ (reciprocal is 40 seconds)
Case of two complex exponents	Real part represents rate of decay of oscillations	-1×10^{-3} sec (reciprocal is 16.67 minutes)	no complex exponents	no complex exponents
	Complex part gives circular frequency of oscillations	6.25×10^{-3} radians/second		
	Latter quantity divided into 2π gives period of one oscillation	16.74 minutes		
	Factor by which amplitude of oscillation is decreased in one period	$\exp(16.74/16.67) = 2.73$		
General solution of stabilization problem contains two arbitrary constants, C and D, (or C and t_o) δP in Mw, δT in °C, t in seconds		$\delta T = C\exp(-1\times10^{-3}t) \cos 6.25 \times 10^{-3} (t - t_o - 226)$ $\delta P = C\exp(-1\times10^{-3}t) \cos 6.25\times10^{-3} (t - t_o)$	$\delta T = C\exp(-0.125t) + D\exp(-8\times10^{-5}t)$ $\delta P = 16\times10^{-4} C \exp(-0.125t) + D\exp(-8\times10^{-5}t)$	$\delta T = C\exp(-0.025t) + 2D\exp(-0.100t)$ $\delta P = 2C\exp(-0.025t) + D\exp(-0.100t)$

[redacted]

THERMAL SELF-STABILIZATION OF COOLED PILES

Intervals of 4 seconds

Intervals of 100 seconds

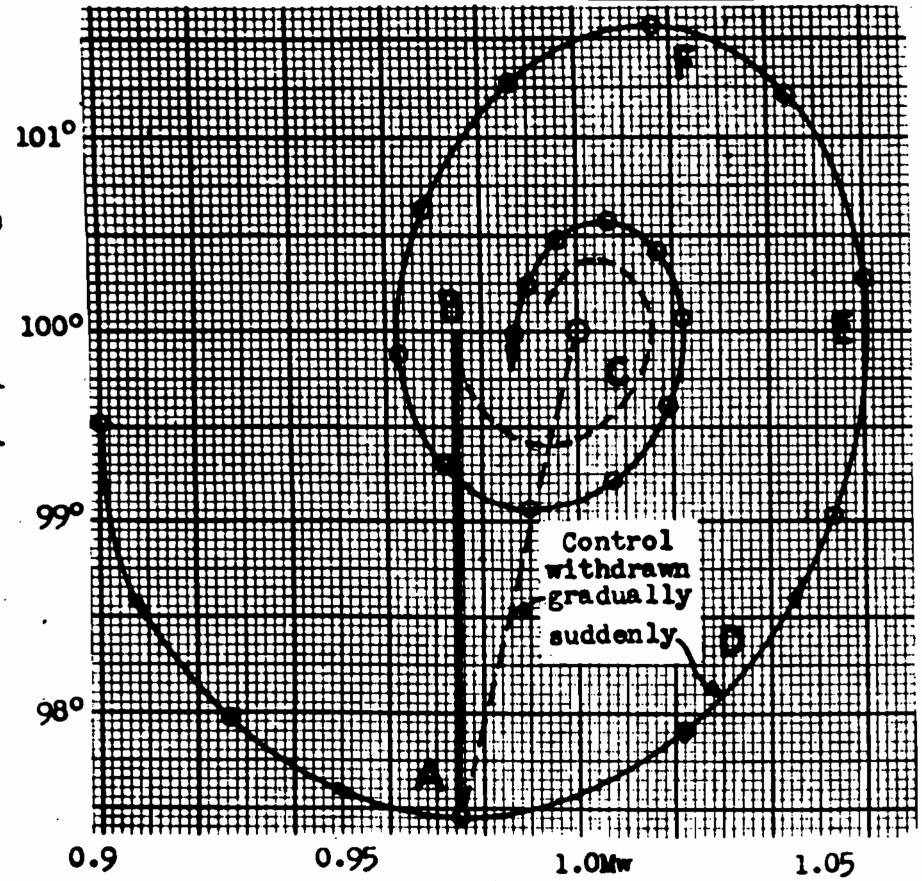
Intervals of 1 hour

ADO Pile in equilibrium at lower temperature has rod suddenly displaced to position corresponding to equilibrium at higher temperature.

OEFO Pile in equilibrium until rod is quickly moved out and as quickly restored, thus suddenly raising heat production.

ABCO Pile in equilibrium at lower power output until temperature of coolant suddenly drops.

Clinton air cooled pile with controls fixed at point such that average metal temperature will ultimately come to equilibrium at 100°C.



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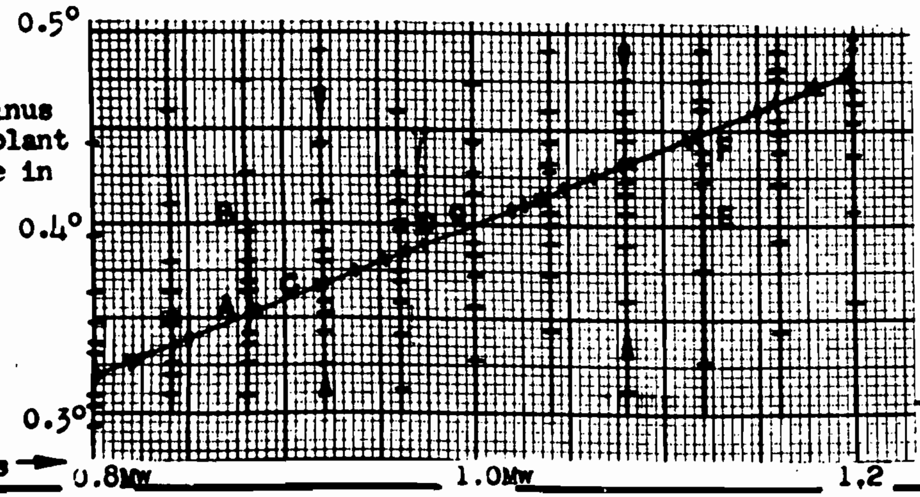
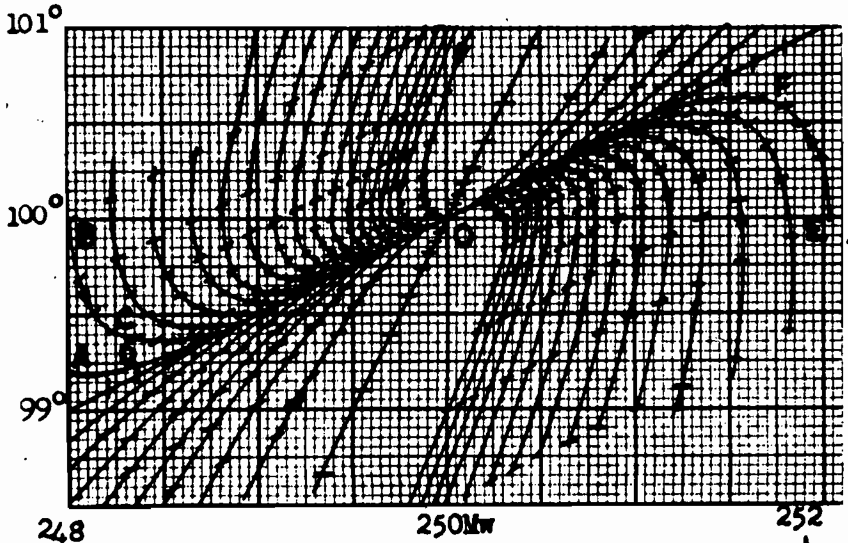
Metal temperature minus initial coolant temperature in °C.



Heat in Megawatts



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Hanford water cooled pile with controls fixed at point corresponding to equilibrium elevation of metal temperature of {100°C. 0.4°C.

FIGURE 22.7.53

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22.8 MECHANISM OF CONTROL

22.8.1 Development of control mechanism

The evolution of the control mechanism in the hands of the Chicago Metallurgical Laboratory and the du Pont Company has paralleled the transition of the pile from an experimental tool to a production unit. Control rods of adequate mechanical strength and neutron absorbing power have been developed. Mechanisms to drive the rods have been designed, built and found to operate satisfactorily. The freedom of operation of these mechanisms has been limited by automatic circuits designed to respond immediately to changes in the conditions within the pile, and to guarantee that the operation of the pile will meet all agreed-upon standards of safety and good practice.

22.8.2 Outline: principles of design; construction; operation

In this section we shall first summarize the considerations of reliability and health hazard which have influenced the choice of means of control, not only in the Argonne Forest pile and the Clinton semi-works pile but also in the full scale production unit at Hanford. Then we shall describe the Hanford control system as illustration of these principles. Next comes a brief account how the philosophy of control circuits has come to depend largely on the pile itself for self-stabilization, in contrast to the earlier reasoning in favor of completely automatic regulation of rod positions. Finally a description of the Hanford control room, the start-up of the production pile, its normal operation, and its shutdown will complete our discussion of the mechanism of control.

22.8.3 Induced radioactivity as factor in choice of absorbent

Induced radioactivity is the first of the factors which influence the choice between gaseous, liquid and solid types of control. A substance which reduces the multiplication factor of a 250 megawatt pile by 1 percent undergoes neutron induced transformations at a rate equal to the product of the following factors:

0.01	transmutation per neutron
2.2	neutrons per fission
$1/200 \times 1.6 \times 10^{-6}$	fissions per erg
250×10^{13}	ergs per second
1.7×10^{17}	nuclear changes per second, or 0.74 grams atoms per month, a rate of transformation equal to 4.6×10^{6} curies

Transformation at this rate will induce an intense radioactivity in all but a few elements. To search for a control material which will not have a health hazard on this score we inspect the possible reactions of neutrons under four heads:

(1) Neutron capture by a stable isotope to form an active isotope with mass number one unit higher. High radioactivity will result unless the radiations from the new nucleus have low energy or its life time is very long in comparison with the proposed time of use of the control material. Among instances of this type none is known where the initial nucleus has a high

enough capture cross section to make a satisfactory absorbent.

(2) Neutron capture by a stable isotope to form another stable isotope would be the perfect reaction if it alone could be guaranteed. However, isotope separation at this time is neither cheap enough nor complete enough to allow isolation of isotopes for control purposes. And amongst the stable isotopes will always be one which, if it captures neutrons at all, gives rise to an active nucleus. We therefore exclude reactions of type (2) for the same reasons which applied to group (1).

(3) Neutron taken up by fission reaction; fission products radioactive. The radioactivity is too great in the case of the splitting of U-235 and Pu-239, quite apart from the absurdity of using a neutron multiplier as a control. The other slow neutron fission reactions which result in active products, N-14 (n, p) C-14 and O-17 (n, α) C-14, have too low cross sections to supply effective absorption mechanisms.

(4) Neutron taken up by fission reaction; fission products stable. Only two known elements, lithium and boron, lead to this type of transformation, but both have big enough cross sections, $90 \times 10^{-24} cm^2$ and $700 \times 10^{-24} cm^2$, to be very effective absorbers. Absence of induced radioactivity gives them an edge over all other elements for use in controls.

22.8.4 Difficulties of gaseous controls

Boron and lithium being the only strong absorbers free of induced radioactivity, and neither of these substances being capable of use in gaseous form except in combination with some other element, it follows that a gaseous control will become radioactive. Granted that leaks in the pile are inevitable, the active gas will create a problem in whatever way it is handled. If above atmospheric pressure, it will escape to some extent all the time and in case of mechanical failure will rush out. The evident danger to personnel has so far excluded this type of system. The other possibility, operation below atmospheric pressure, will draw in all the time a certain amount of contaminating air. Arrangement has to be made to separate the control gas from the air. Both these gases are active and they may carry with them radioactive contaminants from the pile. Difficulty under these conditions of servicing the separation equipment has proved a convincing argument against this alternative scheme for using an absorbent gas. And any form of gaseous control presents not only a radioactive hazard but also a general menace to safety, because it may fail to halt the chain reaction if for any reason the membrane of the pile is ruptured. Both considerations have limited attention to liquid and solid absorbers.

22.8.5 Liquid rejected except for final safety device

Liquid controls have the same disadvantages as gaseous controls. Any pump used to adjust the level of the liquid and the loss in multiplication factor may become difficult to service owing to deposited radioactivity. The more important objection is the chance of failure of the piping. Whether it corrodes through or fails by shock, there

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results a two-edged danger. The absorbent fluid may drain away to the exterior of the pile or to a point in the reflector. Then the power output of the pile will rise out of control. Or the break may occur in the center of the structure and allow the poison to be soaked up in the graphite. In this case it may not be possible to clean out the pile. Its usefulness is destroyed. Only if all other emergency control devices have failed and the pile is on the way to possible self-destruction has it been considered reasonable in designing the Hanford pile to make provision for injecting neutron absorbent fluid. On this account the liquid control is known as "a final safety device". Such an emergency control obviously may have to function in an overheated pile. A liquid alone might be boiled away. The fluid in the Hanford final safety control therefore contains dissolved boron. This absorber will be deposited out on the walls of the tube under boiling conditions. In this respect the liquid control is reliable only to the extent that it is equivalent to a solid control.

22.8.6 Choice of solid rods

Piles built to date are controlled by solid absorbers. Rigid objects are reliable in action and they confine radioactive hazard to a definite region of space. The principal possibilities for solid controls are sheets, rods, wires and shot. Choice between these forms has been based on a compromise between requirements of control theory, protection against radioactivity, and mechanical simplicity. A single control rod 5 feet in diameter has no more effect on the reactivity of the pile than 9 control rods 4 inches in diameter. A hole 5 feet in diameter leading out from a pile would obviously create very difficult problems of shielding and personnel protection quite apart from the mechanical difficulties of operation of a rod of this size. The rods of smaller size are evidently much the more practical. Whether the control systems of the future will go still further in this direction and operate by means of wires drawn in and out of the pile is an open question. The Hanford design did not go to this extreme partly because experience at the Argonne and at Clinton had so far proven only the reliability of large control rods and partly also because it was desired not to have too many individual units passing through the pile. The 4-3/8 inch space between graphite stringers set a natural upper limit to the size of control rods. It was fortunate that with this upper limit on diameter, the required number of rods was sufficiently great to guarantee satisfactory performance in case of accidental failure of a single absorber, yet small enough not unduly to complicate the design of the machinery for operation of the controls.

22.8.7 Hanford system as example

One solution of the control problem along the lines just discussed is illustrated by the Hanford system. It consists of two regulating rods, 7 shim rods, an additional shim through replacement of uranium slugs by poison cylinders, 29 safety rods, and a final safety device which injects into the pile a boron-containing solution. The principal features of these controls are summarized in Table 22.8.8, and Fig. 1.4.8 shows the layout of the rods relative to the pile. A more complete description of the method of insertion is best combined with an account of the internal structure of the rods.

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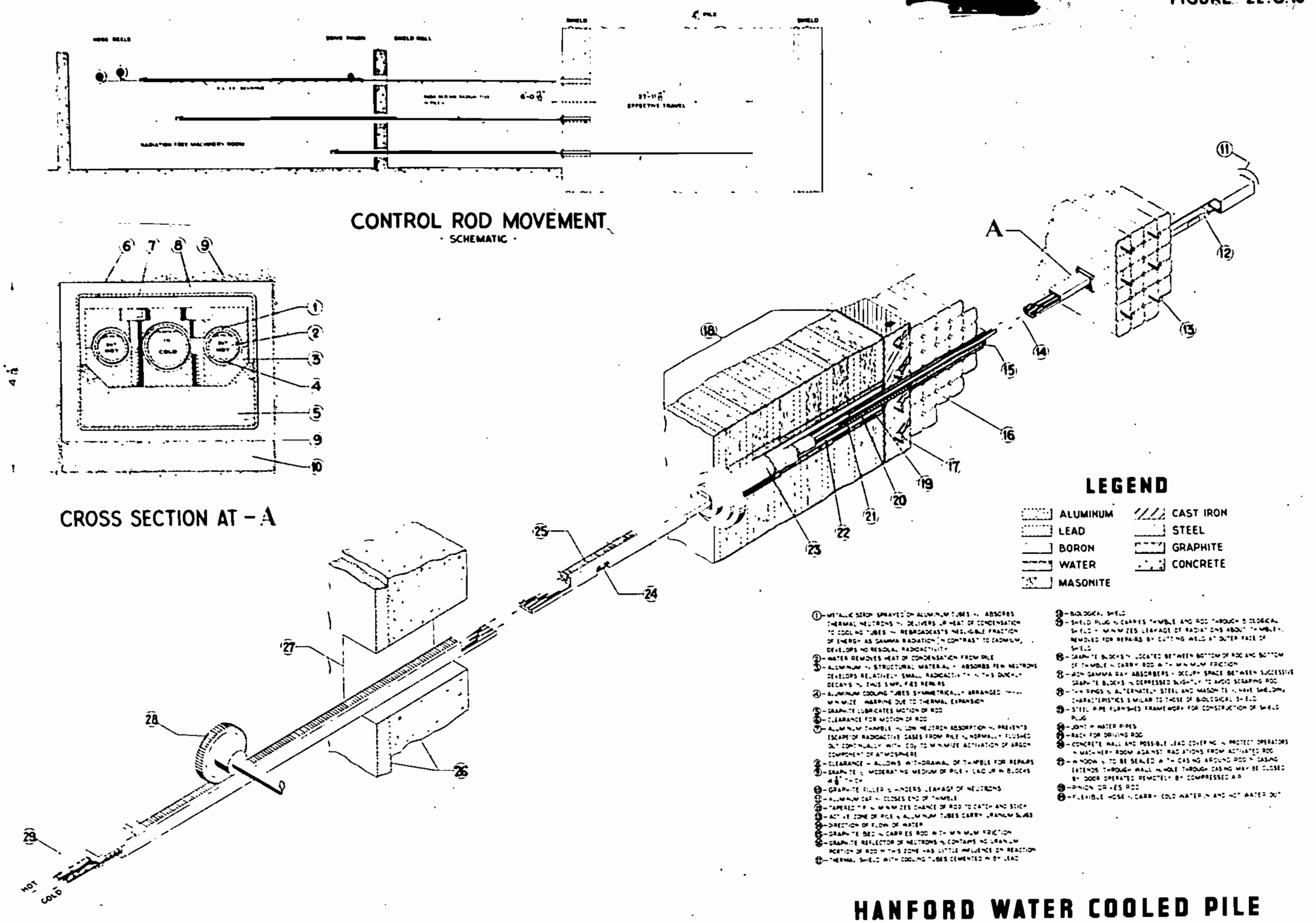
FIGURE - 22.8.10



CONTROL ROD MOVEMENT
· SCHEMATIC ·

CROSS SECTION AT – A

LEGEND

ALUMINUM	CAST IRON
LEAD	STEEL
BORON	GRAPHITE
WATER	CONCRETE
MASONITE	

1 — METALLIC BORON SPRAYED ON ALUMINUM TUBES — ABSORBS THERMAL NEUTRONS — DELIVERS UP HEAT OF CONDENSATION TO COOLING TUBES — REBROADCASTS NEGLIGIBLE FRACTION OF ENERGY AS GAMMA RADIATION (IN CONTRAST TO CADMIUM), DEVELOPS NO RESIDUAL RADIOACTIVITY

2 — WATER REMOVES HEAT OF CONDENSATION FROM PILE

3 — ALUMINUM — STRUCTURAL MATERIAL — ABSORBS FEW NEUTRONS DEVELOPS RELATIVELY SMALL RADIOACTIVITY — THIS QUICKLY DECAYS — THIS SIMPLIFIES REPAIRS

4 — ALUMINUM COOLING TUBES SYMMETRICALLY ARRANGED [illegible] MINIMIZE WARPING DUE TO THERMAL EXPANSION

5 — GRAPHITE LUBRICATES MOTION OF ROD

6 — CLEARANCE FOR MOTION OF ROD

7 — ALUMINUM THIMBLE — LOW NEUTRON ABSORPTION — PREVENTS ESCAPE OF RADIOACTIVE GASES FROM PILE — NORMALLY FLUSHED OUT CONTINUALLY WITH CO_2 TO MINIMIZE ACTIVATION OF ARGON COMPONENT OF ATMOSPHERE

8 — CLEARANCE — ALLOWS WITHDRAWAL OF THIMBLE FOR REPAIRS

9 — GRAPHITE — MODERATING MEDIUM OF PILE — LAID UP IN BLOCKS 4 3/8" THICK

10 — GRAPHITE FILLER — HINDERS LEAKAGE OF NEUTRONS

11 — ALUMINUM CAP — CLOSES END OF THIMBLE

12 — TAPERED TIP — MINIMIZES CHANCE OF ROD TO CATCH AND STICK

13 — ACTIVE ZONE OF PILE — ALUMINUM TUBES CARRY URANIUM SLUGS

14 — DIRECTION OF FLOW OF WATER

15 — GRAPHITE BED — CARRIES ROD WITH MINIMUM FRICTION

16 — GRAPHITE REFLECTOR OF NEUTRONS — CONTAINS NO URANIUM PORTION OF ROD IN THIS ZONE HAS LITTLE INFLUENCE ON REACTION

17 — THERMAL SHIELD WITH COOLING TUBES CEMENTED IN BY LEAD

18 — BIOLOGICAL SHIELD

19 — SHIELD PLUG — CARRIES THIMBLE AND ROD THROUGH BIOLOGICAL SHIELD — MINIMIZES LEAKAGE OF RADIATIONS ABOUT THIMBLE — REMOVED FOR REPAIRS BY CUTTING WELD AT OUTER FACE OF SHIELD

20 — GRAPHITE BLOCKS — LOCATED BETWEEN BOTTOM OF ROD AND BOTTOM OF THIMBLE — CARRY ROD WITH MINIMUM FRICTION

21 — IRON GAMMA RAY ABSORBERS — OCCUPY SPACE BETWEEN SUCCESSIVE GRAPHITE BLOCKS — DEPRESSED SLIGHTLY TO AVOID SCRAPING ROD

22 — THIN RINGS — ALTERNATELY STEEL AND MASONITE — HAVE SHIELDING CHARACTERISTICS SIMILAR TO THOSE OF BIOLOGICAL SHIELD

23 — STEEL PIPE FURNISHES FRAMEWORK FOR CONSTRUCTION OF SHIELD PLUG

24 — JOINT IN WATER PIPES

25 — RACK FOR DRIVING ROD

26 — CONCRETE WALL AND POSSIBLE LEAD COVERING — PROTECT OPERATORS IN MACHINERY ROOM AGAINST RADIATIONS FROM ACTIVATED ROD

27 — WINDOW — TO BE SEALED WITH CASING AROUND ROD — CASING EXTENDS THROUGH WALL — HOLE THROUGH CASING MAY BE CLOSED BY DOOR OPERATED REMOTELY BY COMPRESSED AIR

28 — PINION DRIVES ROD

29 — FLEXIBLE HOSE — CARRY COLD WATER IN AND HOT WATER OUT

HANFORD WATER COOLED PILE

CONTROL ROD

FOR SHIM AND FINE CONTROL

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MECHANISM OF CONTROL

22.8.9 Boron in shim and regulating rods

The shim and regulating rods are exactly alike and are built as shown in Fig. 22.8.10. The neutron-absorbing element is a film of boron flame-sprayed in the form of an aluminum-boron mixture onto the outside of three long parallel aluminum tubes bound together as a single unit. Transmutation of the boron by neutrons is calculated to weaken the absorbing power of the films by only a few percent in the course of a year's operation. The advantages of boron from the point of view of heat generation have already been reviewed in Section 22.6. Boron does not rebroadcast the heat of condensation of the neutrons in the form of gamma rays. In this respect this element is to be preferred to cadmium. It is much easier to remove power liberated in the control rod than it is to take care of surplus heat produced in the center of the pile where the normal cooling means are already loaded to the limit. Furthermore, boron remains a solid at temperatures where cadmium has melted and perhaps permanently damaged the surrounding reactor. Finally, the thermal output of one boron coated rod of Hanford design is at most of the order of magnitude of 35 kw.

22.8.11 Design for case of heat removal

Heat generation in the control is kept small and removal of this heat is simplified by the chosen design. Aluminum is used for the framework of the rod because of its low neutron absorbing power, which prevents significant generation of nuclear energy, and its low density, which reduces the production of heat via absorption of gamma rays from the pile. From both sources together the heat is only a fraction of that generated in the boron itself. Transfer of this heat to the coolant is promoted by the intimate contact between the boron and the underlying aluminum tube. The rod is cooled by water. Ten gallons flow per minute down the central aluminum tube and return through the two outer tubes. This arrangement minimizes the danger of warping the rod through thermal expansion. Each aluminum tube is about 74 feet long; 34 feet of its course is in the control rod proper and the remainder through a 40 foot extension on the rod. This extension carries a rack which meshes with a driving pinion. The aluminum tubes are connected at the end of the combined rod to flexible hoses which roll up or unroll from reels according as the rod is moving out of or into the pile.

22.8.12 Thimble protects against radioactive gas

Other features of the control rod design are intended to reduce exposure of personnel to sources of radioactivity. The rod moves in an aluminum thimble. It prevents the escape from the pile of helium or any admixed radioactive gases. This construction makes it unnecessary to have gas-tight sliding fits. However, air present in the thimble will be displaced by the entrance of a control rod and create a health hazard. For this reason provisions are made by which carbon dioxide can be introduced into the thimble in a continuous stream. This gas develops an activity about one hundred times less than that of air.

22.8.13 Other protection for personnel

Escape of radiations from the pile is minimized by providing for each rod a special passageway through the shield, similar to the arrangement by which the 2004 water tubes enter the front face of the pile. However, nearly 1/8 inch clearance is allowed to insure freedom of motion. So much radiation escapes through this crack during the operation of the

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Table 22.8.8 CHARACTERISTICS OF CONTROL SYSTEM OF HANFORD WATER-COOLED PILE

Purposé of control device	Fine control	Day-to-day shim	Long period shim	Normal safety	Extra safety
Neutron absorbent material	boron	boron	cadmium	boron	boron
Carried as	coating on 3 Al tubes	coating on 3 Al tubes	10% Cd-90% Pb alloy	1.5% B in steel	aqueous solution
Structural element	hollow Al bar	hollow Al bar	formed into slugs	hollow rod	thin walled Al tube
δk for one rod or tube	0.0016	up to 0.0030	up to 0.0025	up to 0.0031	up to ~0.0034
Number of rods or rubes	1 normal, 1 standby	7 rods	up to 30 of 2004 tubes	29 rods	29 rod wells
δk for all rods or tubes	0.003	0.014	~0.035	~0.040	~0.045
Outside dimensions of movable element or solution	9.2cm x 4.45cm	9.2cm x 4.45 cm	1.8cm radius	2.86cm radius	4.29cm radius
Outside dimensions of Al guides	10.2cm x 7.8cm	10.2cm x 7.8 cm	2.2cm radius	4.45cm radius	4.45cm radius
Spacing of rods or tubes	128cm vertical, 162cm horizontal	128cm vertical, 162cm horizontal	down to 106 cm by 106 cm	128cm perpendicular to flow, 81cm parallel to flow	
Effective length in pile	1060 cm	1060 cm	760 cm	1060 cm	1060 cm
Direction of introduction relative to uranium columns	horizontal, perpendicular	horizontal, perpendicular	horizontal, parallel	vertical, perpendicular	vertical, perpendicular
Method of operation	differential coupled on one end to fast electric drive, on other to slow electric drive	hydraulic motor driven normally by pump, in emergency by accumulator	same procedure used for charging uranium slugs	gravity, when deactivation of magnetic latch releases cable winch	solution driven in by compressed air when valve is opened
Speed of travel	low speed adjustable to 0.01, 0.02, 0.05 or 0.10 inch/sec; high speed adjustable to 0.5, 1.0, 2.0 or 3.0 inch/sec.	3 inches/second normal in or out motion; 30 inches/second for emergency insertion	half hour to empty tube of uranium slugs and reload poison slugs	1.7 seconds for nearly complete entry, 0.7 sec. more for last 5 ft. of travel	3 3/4 seconds to begin to enter, about 10 sec. to become effective

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reasonable amount may be seen from the following tabulation of the type of changes in k which these rods must be able to compensate:

Loss in multiplication factor due to heating of pile operating at 500 megawatts	~0.004
Order of magnitude of change in multiplication factor due to poisoning or self-promotion in 100 days' operation at 250 megawatts	~0.010
Deficit in multiplication factor required for rapid shutdown of chain reaction	~0.005

22.8.17 Additional shim via poison slugs

Over and above the shim control due to the 9 control rods an ample margin of adjustment for long period changes in k is provided by the possibility of replacing uranium slugs by poison slugs. The possibilities thus to improve the power distribution in the pile and to gain increased yield are discussed more fully in Chapter 18. The poison slugs for use in the Hanford pile are made of an alloy of 10% Cd, 90% Pb by weight, placed in aluminum cans similar to those which protect the uranium itself from the corrosive action of water. The high cadmium content makes the rods effectively opaque to thermal neutrons. A higher cadmium content would be of little advantage. On the other hand, a lower proportion of this absorbent would weaken the control power per slug, require more slugs, and therefore cut down the number of uranium slugs available for plutonium production.

22.8.18 Safety control

In contrast to the Hanford controls so far discussed, the 29 safety rods enter the pile vertically. They are calculated to reduce the overall multiplication factor about 4 percent, accounted for as follows:

Possible increase in multiplication factor due to complete loss of water	0.02
Deficit in k required for quick shutdown	0.01
Extra margin of safety in design	0.01

The margin of safety guarantees shutdown of the pile even in case of failure of several of the rods.

22.8.19 Structure of rods

The safety controls are hollow steel rods containing 1.5 percent of boron by weight. The wall thickness is 3/16 inch and the outside diameter is $2\frac{1}{4}$ inches. Each rod is $40\frac{1}{4}$ feet long and weighs 250 lbs. The hollow structure reduces the shock to the overhead framing on sudden stoppage of the rods.

22.8.20 Controlled fall under gravity

Means have been provided to drop the safety rods into the pile quickly and surely. Each safety rod is supported by a steel cable, (Fig. 14.8). This cable passes over a pulley and around a drum. The magnetically operated latch normally prevents rotation of this winch under the pull of the rod. When the current fails, either through electrical breakdown or because a safety relay has been tripped, then the magnetic solenoid releases the latch. The drum starts to turn and the safety rod falls into the pile through an aluminum guide tube inserted in the graphite. The safety rods can be removed from the pile by

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winches. Free fall carries the tip well past the center of the pile in the first 1.7 seconds. The last 7 feet of fall require 0.7 seconds. During this period, the rotation of the winch is decelerated by a device which closes the exit part of a hydraulic pump connected to the same shaft. A buffer located on the pile itself (see Fig. 14.8) supplies another means to stop the rod in case the cable or winch fail.

Positions of the safety rods intermediate between complete removal 22.8.21
and complete insertion have been provided for use in certain tests. An electric motor is connected to each winch. The motors are normally used in withdrawing the safety rods from the pile and may also be used to lower them. Any rod can be stopped at any point in its motion in or out by an automatic brake in the motor. But whether the rods are all the way out or only part way out, they are all released in case of emergency and drop to the safe position.

Partial removal possible

The arrangements for adjusting the position of the safety rods 22.8.22
during certain tests in no sense makes these rods equivalent to shim rods. No special cooling provision is made. During the normal operation of the pile every rod must for this reason be in its up position. Then the tip is retracted into the thermal shield on top of the reactor to a point only slightly above the graphite reflector.

No cooling of safety rods

Three measures protect personnel from health hazards associated 22.8.23
with the safety rods: First, like the control rods, they enter gas tight thimbles. A continuous circulation of carbon dioxide is maintained in these wells to prevent air from entering and becoming activated. Second, personnel is restricted from coming into direct line with the beam of radiation which escapes through the annulus around the drop safety rods where they pierce the shield. Finally, the bottoms and tops of the hollow safety rods are plugged with steel to prevent escape of pile radiations through the controls themselves when they occupy either the in or the out position. Only when a rod is partially inserted into the reactor does radiation have a free course through its interior. Then the ionization level about the pile greatly exceeds the tolerable value. However, the period of excess dosage will normally be limited to the 24 seconds while the rods are dropping.

Protection against radiation

In case of failure of the safety rods, the thimbles provided for 22.8.24
them are filled with an aqueous solution containing about 1.5 percent by weight of $Na_2B_4O_7$. The solution is driven out of storage tanks by compressed air and makes its way into the pile through spiral passages cut in the shield plug - a hollow fitting which guides the safety rods through the biological shield, and which itself snugly fills a stepped hole in this shield. The solution has an effect on k comparable to that of the safety rods themselves. With this brief account of the final safety device we complete our survey of the construction of the Hanford control system.

Final safety device

pile that individuals are excluded from the control side of the pile at these times. When the reaction is cut off, the radiation through this crack is not sufficient to give a tolerance dose. An additional reason for excluding workers from the rod area is the activity of the bars themselves as they come out of an active portion of the pile. The induced radioactivity is sufficient under certain conditions to produce one foot away from the rod an ionization level in the neighborhood of 10,000 roentgen units per hour. The aluminum activity decays very rapidly. After about a week the rod is calculated to be safe for approach to a foot distance. Access to the rod driving machinery is guaranteed in spite of the activity of the rods themselves and notwithstanding the radiations which escape from the pile. The protecting element is a wall, constructed as shown in Fig. 22.8.10. Through it the rods pass in small holes which may be closed by remotely operated lead shields. This wall, together with the special ports through the shield and the thimble in the pile, reduce the radioactive hazard to the point where it does not interfere with the operation of the rods.

22.8.14 Electric regulating rod drive

The shim and regulating rods, alike in construction, heat production, and induced radioactivity, differ only in drive and speed of operation. Only one regulating rod is used at a time, the other being maintained as a spare. The acting fine control is connected by rack and pinion to a differential gear. An electric motor geared to one side of the differential drives the rod in or out at one inch per second. The other side of the differential meshes with a second motor which moves the rod in or out at 1/100 inch per second. As experience is gained, it will be easy to change the gear ratio and to alter the speeds for either slower or faster drive. The operation of the motors is remotely controlled by switches on the control panel. The two speed drive makes it possible to adjust the position of the regulating rod quickly and with considerable accuracy.

22.8.15 Hydraulic shim rod drive

In contrast to the regulating rod, shim rod racks and pinions are driven in or out by hydraulic motors at a speed of 3 inches per second. The hydraulic motor gets its power from oil at a pressure about 1000 lbs per square inch. A selector switch controls the rate of flow of the oil and the direction of rotation of the motor. Interlocking mechanisms guarantee that only one shim rod can be withdrawn from the pile at a time. This provision minimizes the chance that the power output of the pile will increase at a dangerously high rate. In case of emergency, however, all 7 shim rods are pushed into the pile simultaneously at a speed of 30 inches per second. For this purpose a weighted hydraulic accumulator is provided. It stores under high pressure sufficient oil to drive all 7 hydraulic motors. Considered as a safety device, the shim rods function even in case of failure of electric power. This certainty of operation is the reason for adopting hydraulic drive.

22.8.16 Control power of shim and regulating rods

The regulating and shim rods together total 9 in number. They are disposed in a rectangular lattice to make 3 rows of 3 bars each. The upper corner rods are chosen for fine control because of their relatively low effect on k. Introduction of all 9 rods is calculated to lower the reproduction factor about 1.7 percent (22.3.63). That this is a

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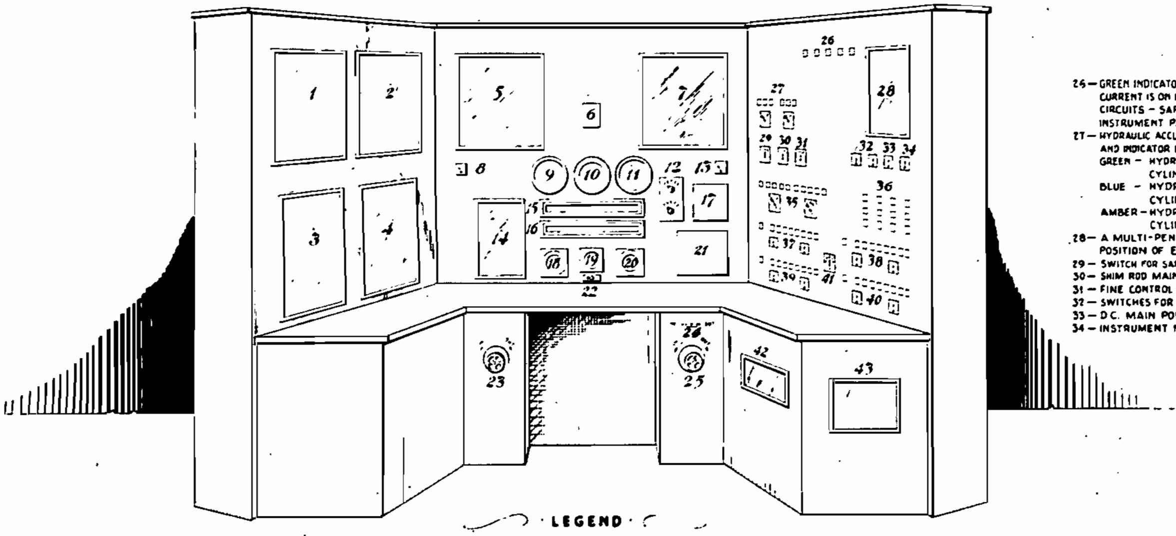
FIGURE, 22.8.27



LEGEND

1 — CONTINUOUS RECORD OF THE INLET AND EXIT TEMPERATURES OF THE COOLING FLUID.
2 — CONTINUOUS RECORD OF SELECTED TEMPERATURES IN THE METAL AND GRAPHITE.
3 — CONTINUOUS RECORD ON MOVING GRAPH PAPER OF THE FLOW OF COOLING FLUID.
4 — CONTINUOUS RECORD OF THE PRESSURE DIFFERENTIAL IN THE COOLING FLUID ACROSS THE PILE.
5 — CONTINUOUS RECORD OF THE POSITION OF WHICHEVER FINE CONTROL IN USE.
6 — LEVEL CONTROL; USED TO ADJUST THE POWER LEVEL DURING AUTOMATIC OPERATION.
7 — CONTINUOUS RECORD ON MOVING GRAPH PAPER OF THE POSITION OF EACH SHIM ROD.
8 — THIS SWITCH IS TO BE USED IN AN EMERGENCY TO PUT SHIM AND FINE CONTROL RODS ALL THE WAY INTO THE PILE.
9 — DIAL SHOWS POSITION OF HYDRAULIC FINE CONTROL ROD.
10 — CLOCK GIVES TIME OF DAY.
11 — DIAL SHOWS POSITION OF ELECTRICAL FINE CONTROL ROD.
12 — COARSE ADJUSTMENT OF ZERO FOR GALVANOMETER # 16
13 — THIS SWITCH IS USED IN AN EMERGENCY TO PUT ALL SHIM RODS ALL THE WAY INTO THE PILE
14 — CONTINUOUS RECORD ON MOVING CHART OF THE LEVEL OF OPERATION
15 — GALVANOMETER, INDICATES THE LEVEL AT WHICH THE PILE IS OPERATING
16 — GALVANOMETER, INDICATES SMALL CHANGES IN THE LEVEL OF THE OPERATION WHEN ZERO IS PROPERLY ADJUSTED
17 — SLIDE WIRE COARSE CONTROL OF ZERO FOR GALVANOMETER # 16
18 — SHUNT FOR GALVANOMETER # 15
19 — INTERVAL TIMER FOR USE IN PILE CALIBRATION.
20 — SHUNT FOR GALVANOMETER # 16
21 — SLIDE WIRE FINE CONTROL OF ZERO FOR GALVANOMETER # 16.
22 — SWITCH FOR # 19
23 — MANUAL ADJUSTMENT FOR HYDRAULIC DRIVE FINE CONTROL ROD.
24 — MANUAL FINE ADJUSTMENT FOR ELECTRIC DRIVE FINE CONTROL ROD.
25 — MANUAL COARSE ADJUSTMENT FOR ELECTRIC DRIVE FINE CONTROL ROD.

LEGEND

26 — GREEN INDICATOR LIGHTS; WHEN LIGHTED CURRENT IS ON IN EACH OF THE FOLLOWING CIRCUITS – SAFETY, SHIM RODS, D.C., AND INSTRUMENT PANEL.
27 — HYDRAULIC ACCUMULATOR CONTROL SWITCHES AND INDICATOR LIGHTS —
  GREEN – HYDRAULIC ACCUMULATOR CYLINDER IS FULL.
  BLUE – HYDRAULIC ACCUMULATOR CYLINDER IS LESS THAN ¾ FULL.
  AMBER – HYDRAULIC ACCUMULATOR CYLINDER IS LESS THAN ½ FULL
28 — A MULTI-PEN RECORDER INDICATES THE POSITION OF EACH SAFETY ROD.
29 — SWITCH FOR SAFETY ROD LATCHING SOLENOID.
30 — SHIM ROD MAIN POWER SWITCH.
31 — FINE CONTROL MAIN POWER SWITCH
32 — SWITCHES FOR SHIM ROD OPERATING PUMPS
33 — D.C. MAIN POWER SWITCH.
34 — INSTRUMENT MAIN POWER CONTROL SWITCH
35 — SWITCHES AND INDICATOR LIGHTS FOR FINE CONTROL RODS
  LIGHTS:
  GREEN – ALL THE WAY IN
  AMBER – INTERMEDIATE
  WHITE – NORMAL OPERATING RANGE
  BLUE – INTERMEDIATE
  RED – ALL THE WAY OUT
36 — SAFETY ROD POSITION INDICATORS.
  LIGHTS:
  RED – ALL THE WAY OUT
  BLUE – INTERMEDIATE
  GREEN – ALL THE WAY IN
37 — CONTROLS AND INDICATOR LIGHTS FOR A SHIM ROD
  RIGHT HAND SWITCH IS A COARSE CONTROL
  LEFT HAND SWITCH IS A FINE CONTROL
  LIGHTS:
  BLUE – SHIM ROD CAN BE MOVED
  GREEN – SHIM ROD IS ALL THE WAY IN
  WHITE – SIX WHITE LIGHTS INDICATE HOW FAR ROD IS WITHDRAWN
  RED – SHIM ROD IS ALL THE WAY OUT
38 — CONTROLS AND INDICATOR LIGHTS FOR ANOTHER SHIM ROD, - SIMILAR TO # 37.
39 — CONTROLS AND INDICATOR LIGHTS FOR ANOTHER SHIM ROD, - SIMILAR TO # 37.
40 — CONTROLS AND INDICATOR LIGHTS FOR ANOTHER SHIM ROD, - SIMILAR TO # 37.
41 — SHIM ROD SELECTOR SWITCH, PERMITS MOTION OF ONE SHIM ROD AT A TIME.
42 — MAINTENANCE CONTROL PANEL HAS —
  SWITCHES TO PERMIT OPERATION OF SHIM RODS WHEN FINE CONTROL IS NOT ON AUTOMATIC OPERATION.
  SWITCHES TO PERMIT MOTION OF SHIM RODS PAST LOCKED STOPS.
  SWITCHES TO SYNCHRONIZE THE INDICATORS WITH ROD POSITIONS.
43 — SUPERVISOR'S LOCKED PANEL HAS —
  SWITCHES, WHEN UNLOCKED, PERMITS THE REMOVAL OF SHIM RODS PAST THE CORRESPONDING STOPS. THUS THE SHIM RODS CANNOT BE MOVED OUTWARD WITHOUT CONSENT OF THE SUPERVISOR.

PILE CONTROL PANEL

AS DEVELOPED FOR CLINTON SEMI-WORKS PLANT

- JULY, 1943 -

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H.L.C., 1943-7-5

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MECHANISM OF CONTROL 22.8.25

The philosophy of operation of the controls, especially the regulating rod, has changed in the period between the start-up of the first experimental pile and the design of the Hanford production units. In the beginning no one could exclude the possibility that the multiplication factor would increase with rising temperature. The reaction might be intrinsically unstable, and a major catastrophe was not out of the question. In this state of uncertainty it was obviously necessary to place a heavy burden of responsibility on the control system. It should be able automatically to keep the power level within preassigned tolerances and to prevent the temperature from approaching the critical point for instability. Consequently, much effort went into the design of ingenious circuits and mechanisms which would so far as possible take out of the hands of the operator the routine job of pile regulation. Concurrently experiments were in progress to determine the temperature coefficient of the reactivity (Chap. 16). When measurements made it clear that the multiplication factor of both the Clinton semi-works unit and the Hanford water cooled piles would decrease on heating, a change took place in the principle of design. The limitations so far imposed by electronic relays on the freedom of the operator were much lightened, and the electrical circuits were simplified. The operator now has no greater possibility than before to cause a catastrophe; only today safety is assured more by the self-stabilizing property of the neutron reaction than by the design of the regulating rod circuits. 22.8.25 Change in philosophy of control circuits

The nature of the operator's job is apparent from a brief account of the operation of the Hanford control system. The management of the regulating, shim and safety rods is concentrated in the control room seen in Fig. 14.8. For this purpose the room is provided not only with switches to regulate the motion of the rods but also with indicators to record their positions and with instruments to keep track of operating conditions within the pile and in the water system. Most of the switches for remote control of the rods are located on a control panel. The panel illustrated in Fig. 22.8.27 was developed for the one megawatt air cooled pile at Clinton and is somewhat simpler than the corresponding Hanford panel. The Hanford control room contains in addition a panel board which gives readings on the pressure of the water to each tube of the pile and another panel which indicates the temperature of the water emerging from each tube. Other instruments record the temperature of the graphite at various points in the pile, the temperature rise of the whole water supply on passage through the pile, and the power output of the reactor. 22.8.26 Control room

On the control panel itself are concentrated the key indicators. A galvanometer scale in the center of the board registers a spot of light which moves markedly to right or left when the activity in the pile rises or falls slightly with respect to a preassigned level. Pointers indicate the position of the regulating rod and shim rods. These pointers are driven by so-called "Selsyn" motors which rotate in synchronism with generators of the same construction which are coupled directly to the corresponding rods. Position can be read to 0.1 inch in 400 inches with the aid of the two pointers on each dial. In addition 22.8.28 Control panel

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MECHANISM OF CONTROL 22.8.31

We now have the pile operating at the desired power level. Here it remains so long as the multiplication factor is constant and the flow and inlet temperature of cooling are unvaried. A small change in any one of these three quantities will produce a temporary disturbance in the temperature of the metal and the power output of the pile. First the temperature of the uranium will adjust itself to a new value in a time of the order of 12 seconds. Then the power output and temperature will hand in hand shift to the new equilibrium figures in a period of the order of magnitude of 40 seconds, as illustrated in Fig. 22.7.53. So long as initial equilibrium condition and final equilibrium condition are satisfactory from an operating point of view, the smooth thermal self-stabilization of the pile itself guarantees that the conditions during the intermediate period of transition will also be satisfactory. The problem of fluctuations in output therefore reduces itself to the question of possible fluctuations in the temperature or supply of cooling water. Taking 100°C as the order of magnitude of the elevation of the effective average temperature of the metal above the level of the inlet water, we conclude that a change of 1°C in the temperature of the water coming from the river will result in an alteration of the order of 1 percent in the power output of the pile. A comparable change in heat generation will be induced by a 1 percent alteration in water flow. Neither type of variation should occur quickly and neither should be the source of any concern. It is not even necessary to change the position of the regulating rod. If one chooses to do so, however, he can hold the power output constant to better than 1 percent. The operating characteristics of the pile are evidently thoroughly satisfactory. 22.8.31 Steady operation

After a long period of normal operation the level of product concentration will reach the point where some of the uranium is due for discharge. At such a time, and also on occasions when repairs or other changes are necessary, the pile is shut down by running into it the regulating rod and the shim rods. The operator can also cut off power production by manually tripping one of the safety devices provided for emergency stoppage of the reaction. Except in cases of danger, such emergency shutdown will normally be avoided. Thermal shock to pile and equipment should be kept at a minimum to avoid damage to the reactor. On this account the operator at the control rod should extinguish the reaction by introducing the control rods and by moving them sufficiently slowly so that the power output drops approximately 1 megawatt per minute. Thus equipment will have a period of the order of 4 hours to adjust itself to changes in temperature. 22.8.32 Normal shutdown

Emergency stoppage of the chain reaction is accomplished in the Hanford pile by any one of three separate and quite distinct safety devices: 22.8.33 Emergency shutdown

(1) Rapid insertion of the shim rods driven by the hydraulic fluid stored in the accumulators.

(2) Dropping the safety rods.

(3) Discharging borax solution into the thimble wells of the drop safety rods.

June, 1944

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MECHANISM OF CONTROL

22.8.33

These emergency devices are activated by abnormality in certain pile operating conditions. Information on these and other conditions is also transmitted through an annunciator system to the operator himself. He is then in a position to take further measures if he considers this action desirable. However, he is not at liberty to prevent the operation of the safety devices when conditions arise which have been previously agreed upon as dangerous. The following list of these conditions is adapted from Chapter VI of the Hanford Engineer Works Technical Manual:

(1) The shim rods are driven into the pile under any one of the following circumstances:

High or low water pressure on any one of the 1500 (or 2004) tubes in the pile.
High exit water temperature.
High power output signalled from the ion chamber which normally controls the regulating rod.
Low level of hydraulic fluid in any one of the three accumulators provided for emergency drive of shim rods.
Low flow of cooling water through control rods.
Operator presses release button.

(2) Automatic circuits drive in the shim rods and also drop the safety rods in any one of the following events:

Low pressure on chilled or unchilled water headers.
High power output as signalled from any one of three ion chambers.
Power failure.
Operator presses release button.

(3) Borax solution is forced into the safety rod wells in any one of the following circumstances:

Failure of the safety rods to operate.
Extremely low pressure on inlet water headers.
Manual signal by operator.

None of the three safety devices just mentioned is operated in case certain abnormal conditions develop which are not immediately dangerous. However, the operator is notified by the sounding of an alarm and he can then exercise his judgment whether or not to set off one of the emergency controls. Among the conditions about which he thus receives intelligence are the following:

Low pressure on cooling water to thermal shield.
Approach to high or low level of hydraulic fluid in accumulators which connect with shim rods.
Low level in reserve oil supply for accumulators.
Low level of water stored in high tank for emergency cooling of pile.

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to this means of indicating rod position there are rows of colored lights which accomplish the same end in a less accurate manner. Lights flash red when the safety rods are in the out position and shine green when they have been dropped into the pile. Switches are at the hand of the operator by means of which he can move in or out the regulating rod or any chosen shim rod. Most critical element on the control panel is the release button. It is only to be touched in an emergency to cause the safety rods to fall.

22.8.29 Operation seen from control room

The control room is the nerve center of the pile building. Standing there we can watch the whole sequence of normal operation, including start-up, maintenance of power at constant level, and shutdown.

22.8.30 Normal start-up of pile

Preliminary to start-up the full water supply is sent through the pile and all instruments and controls are checked. Measurements have been made to determine the local multiplication factor; sufficient surplus metal has been loaded in the pile to compensate the loss in reactivity which will occur when the pile is heated. Additional excess multiplication factor may be provided on the first run to compensate possible poisoning by fission products. After the magnitude of this effect shall have been determined the loading will be made in such a way as to take this effect into account more exactly. Now the pile is ready to go. The following description of the next steps in the start-up are taken from Chapter VI of the Hanford Technical Manual, pp. 619-621. "The drop safety rods are withdrawn. This should produce essentially no change in the power output of the pile since k is still below 1.000 (actually about 0.990). After withdrawing the regulating rod the shim rods are withdrawn successively until the k of the pile, as determined from previous experiments, is about 1.002. The pile power then starts to rise from the shutdown value of about 10^{-4} watts and should reach 100 kw in about 10 minutes, doubling intensity every 20 seconds. During this period, the rate of rise is checked from time to time to see if it is too large or too small. If the power output is rising too fast, the regulating rod is inserted into the pile by an amount to check the power output rise back to the desired level. If the rise is too slow, the shim rod is pulled out further. After a power output of about 100 kw is reached, it may be desirable to reduce the rate of rise to two-thirds or a half that used in bringing the power output up to this level. This is done by making a suitable adjustment with either the control rod or with a shim rod. As soon as the power level has reached several thousand kw, the various temperatures of the pile and of the exit water have risen by an observable amount. Due to the negative temperature coefficient of k, the raising of the pile temperature reduces the rate of rise of the power output so that the shim rod, or rods, must be pulled out additional amounts to maintain the desired rate rise. The magnitude of this effect is such that k is reduced by about 0.003 between zero power level and a power level of 250,000 kw. This requires the removal of about one and a half shim rods. As the desired power level is approached, the rate of rise in power must be further decreased by shoving in the regulating rod. The power output is then controlled at 250,000 kw by moving the regulating rod in or out, depending on whether the power level is above or below the control point".

[redacted]

[redacted]

MECHANISM OF CONTROL 22.8.34

With this description of normal and emergency shutdown we have completed our survey of the control system of the Hanford pile. This survey illustrates the mechanism and operation of control devices. It complements the analysis earlier in this chapter of the principles of control. Both theory and practice reveal the neutronic chain as a reaction susceptible to close and reliable regulation.

22.8.34 Neutronic control reliable in theory and practice

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