CL-574 (2)
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PILE THEORY LECTURE NOTES

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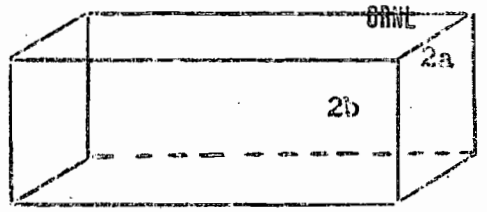
C. 2 Application to diffusion length measurements

At great distance from source, no production of thermal neutrons. Hence



$$\Delta n - \varkappa^2 n = 0$$

$$\therefore \quad n_{mn} = A_{mn} \cos m \frac{\pi x}{2a} \cos n \frac{\pi y}{2b} e^{-z/c_{mn}}$$

$$-\frac{m^2\pi^2}{4a^2} - \frac{n^2\pi^2}{4b^2} + \frac{1}{c^2_{mn}} - \varkappa^2 = 0 \qquad \text{where } m, n \text{ are odd integers}$$

$$\varkappa^2 = \frac{1}{c^2_{mn}} - \pi^2\left(\frac{m^2}{4a^2} + \frac{n^2}{4b^2}\right)$$

For 1.1 harmonic, if a = b,

$$(1) \qquad \boxed{\varkappa^2 = \frac{1}{c^2} - \frac{\pi^2}{2a^2}}$$

Hence from measurement of c (and a) one can get $\varkappa^2$. Note that

$$(2) \qquad \varkappa^2 = \frac{\Delta n}{n}$$

is the constant value which the normalized Laplacian has; the quantity $\frac{\Delta n}{n}$ is a measure of the curvature of n at each point. It is positive for absorption, negative for production.

To consider higher harmonics, suppose distribution in z = o plane is n(x, y). Then we can expand n(x, y) in a Fourier series:

$$n(x, y) = \sum_n \sum_m A_{mn} \cos \frac{m\pi x}{2a} \cos \frac{n\pi y}{2b}$$

$$\int_{-b}^{b}\int_{-a}^{a} \cos n(x, y) \cos \frac{m\pi x}{2a} \cos \frac{n\pi y}{2b}\, dxdy = A_{mn} \int_{-b}^{b}\int_{-a}^{a} \cos^2 \frac{m\pi x}{2a} \cos^2 \frac{n\pi y}{2b}\, dxdy$$

$$\therefore A_{mn} = \frac{\iint n(xy) \quad c\ c\ dxdy}{\iint \cos^2 \cos^2 dxdy} = \frac{1}{4ab} \iint n(x,y) \cos \frac{m\pi x}{2a} \cos \frac{m\pi y}{2b}\, dxdy$$

This is somewhat academic as it stands because a pure primary thermal source is unknown. For point sources, $n(x, y, o)$ is a δ-function, $\delta(x)$ $\qquad \delta(x) = o,\ x \neq o$

$$\int_{-a}^{a} \delta(x)\ f(x)\, dx = f(o)$$

$$\delta(x) = \lim_{r \to o} e^{-\left(\frac{x}{r}\right)^2}$$

$$\int \delta(x) - (\xi)\ f(x)\, dx = f(\xi)$$

C.3 <u>Actual determination of $\varkappa$</u>

In practice cannot make measurements only where n's are thermal. In this case,

$$\Delta nv - \varkappa^2 nv + \frac{3}{\lambda} q = o$$

where q = number becoming thermal/cm^3/sec ≡ "nascent thermal density". If cd. sheet is placed at x = o, have effectively placed a sink at x = o. New thermal distribution, n'v obeys

$$\Delta n'v - \varkappa^2 n'v + \frac{3}{\lambda} q = o$$

Introduce $\nu \equiv n - n'$, then

(1) $$\boxed{\Delta \nu - \varkappa^2 \nu = o}$$

which is identical with C. 2 (1) and so can use same equations as in C.2. Hence by measurement with and without sheet can eliminate effect of q. This is method actually used for measurement of $\varkappa$.

Exercise: Find the distribution of thermal neutrons far from the source in an infinitely long block of diffusion length $\varkappa_o$ if a "sandwich" of thickness ℓ of a material with different diffusion length $\varkappa_1$ is inserted.

Exercise: Find the distribution of thermal neutrons in a finite block of length ℓ for a point source at $x = y = z = o$.

<u>Corrections in formula for $\varkappa$</u>

1) Absorption

$$\varkappa = N\sqrt{3\sigma\sigma_a}\left(1 - \frac{2}{5}\frac{\sigma_a}{\sigma}\right)$$

2) Forward scattering (isotropic scattering in <u>c</u>. of <u>g</u>. system) and absorption

$$\varkappa = N\sqrt{3\sigma\sigma_a}\left(1 - \frac{2}{5}\frac{\sigma_a}{\sigma}\right)\left(\sqrt{1 - \overline{\cos\theta}}\right)$$

$\overline{\cos\theta} = \frac{2}{3M}$ M = mass of scattering atoms

If scatterer is a "molecule", $\sigma_t \overline{\cos\theta} = \frac{2}{3}\left(\frac{\sigma_1}{M_1} + \frac{\sigma_2}{M_2}\right)$

$$\overline{\cos\theta} = \frac{2}{3}\,\frac{\sigma_1 M_2 + \sigma_2 M_1}{(\sigma_1 + \sigma_2)\,M_1 M_2}$$

This is valid <u>outside</u> region of chemical binding!

To derive cos θ

φ = angle in c.g. system $\qquad \theta = \theta(\varphi)$

θ = angle in lab. system

$$\overline{\cos\theta} = \frac{1}{2}\int_{-1}^{1}\cos\theta\, d(\cos\varphi) = \frac{2}{3A}, \text{ since}$$

$$\cos\theta = \frac{1 + A\cos\varphi}{(1 + A^2 + A\cos\varphi)^{\frac{1}{2}}}$$ (This may be left as an exercise.)

In previous derivation of D, assumed that history of particle before it made collision in volume element dv was immaterial. Actually those which came from right direction on previous collision are somewhat more likely to continue in right direction on next flight. Moreover, if there is n gradient, then weighting in right direction due to those from above dv more than compensates weighting in wrong direction due to those from below dv. We could thus calculate velocity <u>vector</u> distribution out of dv and from that the transport equation.

The result of this calculation is that

$$D = \frac{1}{3} \frac{\lambda v}{\sqrt{1 - \overline{\cos\theta}}}$$

and λ is changed correspondingly. The quantity

$$\frac{\lambda}{\sqrt{1 - \overline{\cos\theta}}}$$

is often called the "transport" mean free path.

3) Boundary correction

"Augmentation length $= \lambda/\sqrt{3}$. Can get λ from this. This correction arises in considering transition from diffusion to "transport" flow, where λ is fairly appreciable. At a boundary with a perfect absorber, condition is that Fick's law flow and transport flow must match.

Problem: Show that in 2 dimensions the diffusion coefficient is given by $D = \frac{\lambda v}{2}$

Note on 1-dimensional diffusion. (See Bethe, Rev. Mod. Physics, Vol. 9)

Suppose r atoms move to right, ℓ to left. Then

$$\begin{cases} (1) & \dfrac{dr}{dx} = -\left(\dfrac{\sigma_s}{2} + \sigma_a\right) r + \dfrac{\sigma_s}{2} \ell \\ (2) & -\dfrac{d\ell}{dx} = -\left(\dfrac{\sigma_s}{2} + \sigma_a\right) \ell + \dfrac{\sigma_s}{2} r \end{cases}$$

scattering results in reversal of direction 1/2 of time.

$$(3) \quad \therefore \quad r'' = -\left(\sigma_a + \frac{\sigma_s}{2}\right) r' + \frac{\sigma_s}{2}\left(\left(\frac{\sigma_s}{2} + \sigma_a\right)\ell - \frac{\sigma_s}{2} r\right)$$

$$= -\left(\sigma_a + \frac{\sigma_s}{2}\right) r' - \frac{\sigma_s^2}{4} r + \frac{\sigma_s}{2}\left(\frac{\sigma_s}{2} + \sigma_a\right)\ell$$

Then from (1) x (3): $r'' = \left(\frac{\sigma_s}{2} + \sigma_a\right)^2 r - \frac{\sigma_s^2}{4} r = \left(\sigma_s\sigma_a + \sigma_a^2\right) r$

$$= (\sigma_s + \sigma_a)\sigma_a r$$

Diffusion coef. = λ in 1-dimension $\left(= \dfrac{1}{\sigma_a + \sigma_s}\right)$

$$\therefore \frac{1}{\sigma_a + \sigma_s} r'' = \sigma_a r$$

In one dimensional "r" "ℓ" analysis, the boundary correction turns out to be just λ - (instead of $\lambda/\sqrt{3}$), since at a boundary (on right) there are no ℓ particles; hence, from equations (1) and (2)

$$\frac{dn}{dx} = -(\sigma_s + \sigma_a)\, n \qquad \text{at boundary}$$

$$= -r/\lambda$$

Hence extrapolation distance is just λ in 1-dimensional model.

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