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PILE THEORY LECTURE NOTES

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MS. for N.T. Bray 2/1/54
SUPERVISOR CENTRAL FILES
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Small Controls

In our previous discussion of controls we have assumed that the control is so large that the thermal density extrapolates to zero at a distance $\lambda/\sqrt{3}$ which is rather small compared to the rod dimensions. Mathematically this means that a boundary condition for the pile equation is nv = 0 at r_o, the effective radius of the rod; such a boundary condition implies that the neutron density is very greatly perturbed by the insertion of the control rod. If the rod radius is actually less than $\lambda/\sqrt{3}$, this method is clearly inapplicable; two possible procedures are then available for computing the effectiveness of the rod, viz.

a) We could still demand that the neutron density extrapolate to zero at a distance $\lambda/\sqrt{3}$ inside the rod surface. This condition could be expressed in terms of the slope of the density at the geometric boundary:

$$\frac{\lambda}{\sqrt{3}} \frac{d(nv)}{dr} = -(nv) \quad \text{at the rod boundary.}$$

The perturbed neutron density which satisfies this condition can be calculated as before, and the critical radius is again determined by the first root of a fairly complicated transcendental equation. However, the validity of this method depends on what justification there is for assuming a $\lambda/\sqrt{3}$ extrapolation distance even when the rod is not a perfect absorber. Certainly for very small rods which practically do not affect the density, the $\lambda/\sqrt{3}$ rule is not at all obvious.

b) An alternative (and simpler) scheme is to consider the control as an absorbing foreign substance which decreases the multiplication factor (and therefore the Δ) of each cell through which it passes, but does not perturb the neutron distribution appreciably. The effect of such a rod on the Δ of the pile can be calculated by weighting the Δ_i of each cell with the statistical weight of the cell. The results obtained this way are useful in interpreting poisoning experiments in the going pile, where absorption cross-sections are measured by determining the change in Δ due to the presence of an impurity of known mass. This method is applicable if the control is not a black body.

Consider an infinitely long, thin control rod which passes th[redacted] cell of an infinitely long cylindrical pile, critical

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radius R, Laplacian (without the rod) Δ_o*. The Δ of the i-th cell is reduced to a value

$$\Delta' = \Delta_{oi} + \Delta_{ci}$$

whose value will be estimated below. Hence the Δ of the whole pile is

$$\Delta = \sum_j w_j \Delta_j \tag{1}$$

where w_j is the statistical weight of the j-th cell and the summation is over the whole pile. Since the Laplacian of each cell, except the one through which the control passes, is assumed to be unchanged by the presence of the control, we can write

$$\Delta = \sum_j w_j \Delta_{oj} + w_i \Delta_{ci} = \Delta_o + w_i \Delta_{ci} \tag{2}$$

By definition of the statistical weight,

$$w_i = \frac{(nv)_i^2 V_i}{\sum_j (nv)_j^2 V_j} \quad \left(= \frac{(nv)_i^2}{\sum_j (nv)_j^2} \right. \quad \text{if } V_i \text{ is the same for all cells.)} \tag{3}$$

where $(nv)_i$ is the average density in the i-th cell. For the cylindrical pile under consideration,

$$nv = J_o(\sqrt{|\Delta_o|}\, r) \;;$$

if V_i is the effective volume of the i-th cell, and r_i is the distance of the i-th cell from the center of the pile, then

$$w_i = \frac{J_o^2(\sqrt{|\Delta_o|}\, r_i)\, V_i}{2\pi \int_0^R r J_o^2(\sqrt{|\Delta_o|}\, r)\, dr} = \frac{J_o^2(\sqrt{|\Delta_o|}\, r_i)\, V_i}{J_1^2(\sqrt{|\Delta_o|}\, R)\, V} \tag{4}$$

where V is the volume of the pile.

To calculate Δ_{ci} we must take into account the small-scale distribution of neutrons in the cell, and the position of the rod in the cell. If the rod is at a position in the cell where the density of thermal neutron is F times the average density of neutrons in the metal, the ratio $\frac{\text{neutrons absorbed by rod}}{\text{neutrons absorbed by metal}}$ is

$$\varphi = \frac{N_c\, \sigma_{ac}\, V_c\, F}{N_o\, \sigma_{a_o}\, V_o} \tag{5}$$

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* Δ_o is negative.

where $N_c \sigma_{ac}$ and $N_o \sigma_{ao}$ are the macroscopic thermal absorption cross-sections of the rod and metal respectively, and V_c and V_o are the rod and metal volumes in the cell. The per cent of slow neutrons absorbed by the metal without the rod is f_o, the thermal utilization; that is,

$$\frac{1}{f_o} = \frac{\text{slow neutrons absorbed in metal + moderator}}{\text{slow neutrons absorbed in metal}}$$

while with the control,

$$\frac{1}{f} = \frac{\text{slow neutrons absorbed in metal + moderator + control}}{\text{slow neutrons absorbed in metal}}$$

$$= \frac{1}{f_o} + \varphi$$

or,

$$(6) \qquad \varphi = \frac{+\delta f}{f^2} .$$

Since $k = \eta p \epsilon f$, $\quad \delta k = \delta(k-1) = \eta p \epsilon \delta f$ and so

$$(7) \qquad \delta(k-1) = -M^2 \delta \Delta_i = M^2 \Delta_{c_i} = kf \varphi \cancel{M^2} = kf \frac{N_c \sigma_{ac} V_c}{N_o \sigma_{ao} V_o} F$$

or

$$(8) \qquad \Delta_{c_i} = \frac{N_c \sigma_{ac} V_c}{N_o \sigma_{ao} V_o} \frac{Ffk}{M^2}$$

Substituting (4) and (8) into (2), we obtain

$$(9) \qquad \Delta = \Delta_o + \frac{N_c \sigma_{ac} V_c V_i}{N_o \sigma_{ao} V_o V} \frac{kFf}{M^2} \frac{J_o^2(\sqrt{\Delta_o} r_i)}{J_1^2(\sqrt{\Delta_o} R)}$$

The ratio V_i/V_o represents the volume ratio of cell to metal, the ratio V_c/V the volume ratio of control to pile. If we call the first ratio ρ_u, the second ρ_c, we have for the change in Δ caused by the rod

$$(10) \qquad \frac{N_c \sigma_{ac} \rho_c \rho_u}{N_o \sigma_{ao}} \frac{Ffk}{M^2} \frac{J_o^2(\sqrt{\Delta_o} r_i)}{J_1^2(\sqrt{\Delta_o} R)}$$

In this form it is seen that the effect of the rod does not depend on our rather arbitrary assumption that the rod changed the k of only the i-th cell.

The quantity F will be calculated later when we discuss the distribution in the individual cells; for a homogeneous mixture it is unity, and for the usual sized cells it varies between 1.3 and 2 depending on how close to the edge of the cell the rod is placed. Since $f \sim .85$, it is sufficient for most purposes to put $Ff \approx 1$.

The Danger Coefficient

This discussion of the change in the k of a cell caused by a control rod can be carried over immediately to calculate the effect of any impurity. Hence the loss in k due to an impurity is given by

$$(11) \qquad -\frac{\delta k}{k} = Ff\,\frac{N_c\,\sigma_{ac}V_c}{N_o\,\sigma_{a_o}V_o} = Ff\,\frac{\sigma_{ac}}{A_c}\,\frac{A_o}{\sigma_{a_o}}\,\frac{M_c}{M_o}$$

where $\frac{M_c}{M_o}$ is the mass ratio of impurity to metal. The quantity

$$\frac{\sigma_{ac}}{A_c}\,\frac{A_o}{\sigma_{a_o}}$$

which is the ratio of absorption cross sections per unit mass of impurity and metal is called the "danger coefficient" by Fermi. It should be noted (11) that in computing the loss in k due to a given impurity, both F, the $\frac{\text{average neutron density in the impurity}}{\text{average neutron density in the metal}}$ and f the thermal utilization enter. Usually these factors are neglected, mostly because the uncertainties in the absorption cross-sections are greater than the errors introduced by neglecting f.

PROBLEM: Show that the ratio, ρ, of excess leakage out of the sides of a long cylindrical pile of radius R (caused by the introduction of a control rod, radius r_o) to the leakage into the rod is

$$\rho = -\frac{\pi\xi}{2}\,C_o r_{o,1R}\left[1-\left(\frac{2.405}{\xi}\right)^2\right]-\frac{2.405^2}{\xi^2}$$

where

$$C_o r_{o,1R} = Y_o\,(\xi r_o/R)\,J_1(\xi) - Y_1(\xi)\,J_o(\xi r_o/R)$$

and ξ is the first root of

$$\frac{Y_o\left(\xi\frac{r_o}{R}\right)}{J_o\left(\xi\frac{r_o}{R}\right)} = \frac{Y_o(\xi)}{J_o(\xi)}$$

Show that for $\frac{r_o}{R} = .0037$, $\rho = .657$.

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