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# TEMPERATURE EFFECT IN HOMOGENEOUS PILE

Gale Young
December 9, 1943

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## Abstract

Some idealized considerations of the temperature field in the homogeneous pile are given. It is crudely estimated that the effective mean temperature rise of the pile to be used in calculating the k loss is something like 3/4 the rise in temperature of the slurry in passing through the pile.

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In CP-807, part II, some estimates were made on the increase in size of the homogeneous pile with uniform heating. Actually the situation is more complicated, since the temperature rise will be different in different parts of the tank.

## Temperature Coefficient of B

In the pile equation $\nabla^2 n + Bn = 0$ the quantity $B = \frac{k-1}{\tau + L^2}$ is a function of the local slurry temperature. Upon referring to the above report it is seen that an estimate for $\frac{1}{f(T)} = \frac{B(20)}{B(T)}$ is given by the product (second column of Table I)·(expression 20)·(expression 21). The quantity f(T) is plotted in Fig. I of the present report. It is reasonably linear in T, the fractional decrease being about $2 \times 10^{-3}$ per degree.

## Temperature Profile

The currents and temperatures in the tank present a difficult problem, and we here consider only some idealized situations. Consider a pile in the shape of a general cylinder, with the slurry flowing through parallel to the axis and having a velocity V which varies over the plane transverse to the axis. The neutron density has approximately a cosine factor along the axis and a factor N of some sort in the transverse plane. If the streams do not mix sidewise in going through the tank the temperature rise T at any point in a transverse plane is proportional to

$N/V$. The loss in B at any point is proportional to T, and thus the effective average B loss is proportional to the integral over the pile cross-section $J = \iint TN^2 = \iint N^3/V$.

If we hold N and the integral $I = \iint V$ fixed, then the total power output, the total flow, and the mean slurry temperature upon mixing after it emerges from the pile will all be constant; and we can consider the effect on J of varying the velocity profile V. The Euler equation for the calculus of variations problem of minimizing J with I held constant is $-\frac{N^3}{V^2} + \text{constant} = 0$, which says that for the smallest B loss V should vary as $N^{3/2}$. Thus T should vary as $N^{-\frac{1}{2}}$, so that the pile is kept cooler along the central axis and allowed to run warmer near the lateral walls.

To see how sharp this minimum is we consider one transverse direction of a pile with rectangular cross-section. N thus has a factor cos x for this direction, and we compare the velocity profiles $V = A_m \cos^m x$ for several values of m. This involves the integrals

$$I = A_m \int_0^{\pi/2} \cos^m x \cdot dx$$

and

$$J = \frac{1}{A_m} \int_0^{\pi/2} \cos^{3-m} x \cdot dx.$$

Upon adjusting $A_m$ to keep I equal always to $\pi/2$ we obtain the results

| m | $A_m$ | J | $J/J_{min}$ | $(J/J_{min})^2$ |
|---|---|---|---|---|
| 0 | 1 | 2/3 | 1.37 | 1.88 |
| 1 | $\pi/2$ | .5 | 1.03 | 1.06 |
| 1.5 | 1/.557 | .487 | 1 | 1 |
| 2 | 2 | .5 | 1.03 | 1.06 |

The entries in the last column refer to the rectangular cross-section with two independent dimensions and thus will serve roughly for any actual pile shape. It is to be noted that under the assumed condition of constant total flow the theoretically best profile (m = 1.5) is only slightly better than the constant temperature profile (m = 1). It might be expected that some approximation to a constant temperature over any transverse cross-section of the pile would be realized in practice because (a) turbulent sidewise mixing tends to equalize temperature differences, (b) the turbulence causes elements of the tank liquid to follow winding paths so that the integrated heat produced along the different paths is more nearly equal than for the separated parallel paths considered above, and (c) if the temperature becomes higher in the center of the pile than near the edges this will tend to set up natural convection circulation somewhat as considered in CP-779, with the effect of increasing the upward velocity in the center of the tank and decreasing it nearer the edges, and thus tending to reduce the temperature inequalities.

If the pile really were a cylinder with the temperature stratified in transverse planes, and if the temperature rise of the liquid through the pile were sufficiently small, then the effective average pile temperature rise for computing the loss in B and the required mass of

P-9 would be .5 of the overall rise in liquid temperature through the pile. However, because of (a) imperfect temperature equalization in the transverse planes which may leave the temperature higher in the center, and (b) longitudinal flattening at the high temperature end, this fraction will be increased. Perhaps .7 or .8 will serve as a guess, though of course no real estimate can yet be made.

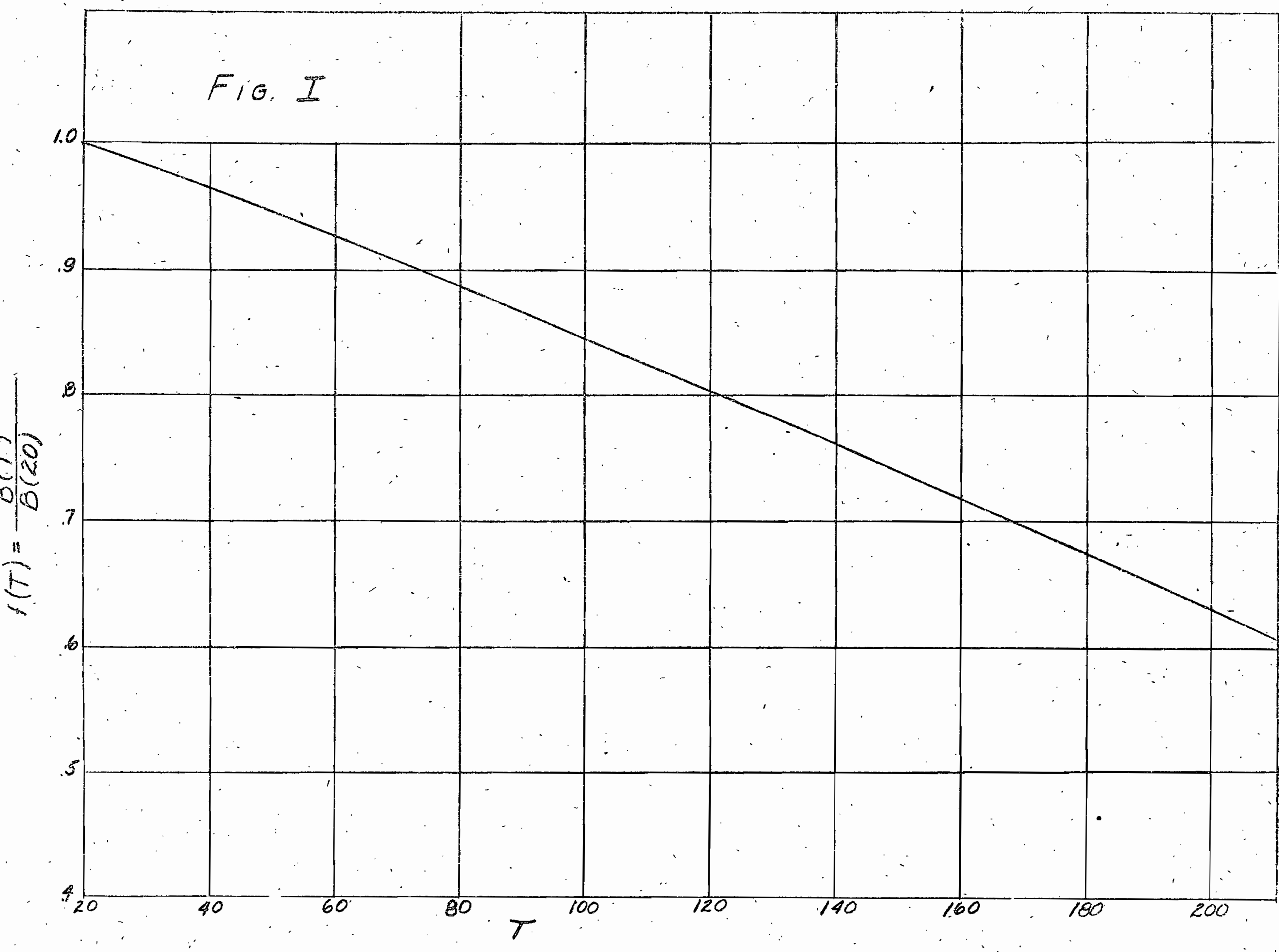


FIG. I